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The Evolution of Contracts and Property Rights

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Abstract

I apply stochastic stability (Kandori, Mailath and Rob, Econometrica 1993, and Young Econometrica 1993) to a contracting game. There are three stages to this game. In the first, a surplus sharing transfer is made. This is followed by a relationship specific investment, and finally bargaining over the gross surplus from investment. In the stochastically stable outcome, the investing party gets (almost) all of the surplus in the final stage, and makes the efficient investment. However, the transfer is set so that over half of the net surplus goes to the party who does not make an investment. That is, while hold up does not occur, it's possibility increases the surplus received by the noninvesting party.

1 Introduction

Conventions, accepted behavior when multiple agents are involved, are present in many aspects of life, including contracting. Some conventions, in contracting and more generally, are explicit, while other are implicit.[1] Almost by definition, conventions regarding incomplete contracts must be both. They must be explicit in the portion of the relationship which is guided by the written contract, and implicit in the portion of the relationship which is not. Explicit and implicit contractual conventions have been studied, but only in isolation from each other. This paper investigates the simultaneous evolution of explicit and implicit conventions in a fairly standard incomplete contracting problem. The evolutionary process is modeled via stochastic stability.[2]

Young (1998) studies a contracting game in which a pair of agents each name a contract, and enter into it if they name the same contract. Young's main result is that the stochastically stable convention splits the surplus equally.[3] Because these contracts are enforcible, Young's result is a statement about explicit conventions. Ellingsen and Robles (2001) (see also Troger (2000) and Dawid and MacLeod (2000)) study a game in which one agent makes a relationship specific investment, after which he and another agent bargain over the surplus. Ellingsen and Robles (2001) find that evolution leads to

[1] Young (1998) contains examples. For one, employment contracts are generally explicit concerning wages, but not concerning bonuses. Consequently, conventions concerning wages are explicit, while those concerning bonuses are implicit.

[2] Kandori, Mailath and Rob (1993), Young (1993) and Noldeke and Samuelson (1993).

[3] Approximately by the Kalai-Smordinsky (1975) solution, which in the linear context considered here, is an even split.

a convention with an implicit assignment of efficient property rights and, consequently, efficient investment. This paper studies the evolution of conventions in a contracting game which includes relationship specific investment, *implicit* post investment property rights, and *explicit* sharing of surplus.

The contracting game is between a buyer and a seller. It consists of a monetary transfer, a (seller specific) investment by the buyer, and a price at which the good is sold. The only portion of the game which is contractible is the transfer. The game begins with the two agents suggesting a transfer. If they suggest different transfers, then there is no contract, and the game ends. Otherwise, the transfer is made, and the agents enter into a relationship. The buyer then makes an observable, but noncontractible, investment. Finally, the two agents bargain over the price of the good.

The evolutionary model consists of two populations: one of buyers and one of sellers. Each period, agents from the two populations meet and play the contracting game. Over time, agents come to choose optimal responses to the actions taken in the other population. There are many outcomes of the contracting game such that if all agents in both populations play according to that outcome, then no agent in either population has an incentive to change his actions. Such outcomes are conventions. Agents also change their actions by 'mutating.' A mutating agent chooses his actions at random. Mutations are taken to be quite rare, so that the two populations spend most of the time in a convention. In fact, the presence of mutations implies that the populations spend most of the time in a very specific set of conventions. These are the stochastically stable conventions.

A simple composite of the results in Young (1998) and Ellingsen and Robles (2001) would suggest: efficient implicit post investment property rights, efficient investment, and a transfer that shares surplus evenly. The logic of such a composite result requires first letting property rights and investment evolve, and only then applying evolution to the determination of the transfer. A more proper approach must study the simultaneous determination of transfer, investment and post investment bargaining. Interestingly, property rights and investment evolve much more quickly than the division of surplus. Hence, stochastic stability in the contracting game does imply efficient property rights and investment.

However, while stochastic stability precludes hold up, the possibility of hold up drives the selection of the stochastically stable transfer. Stochastic stability is, at least partially, a statement about the number of mutants who must play contrary to a convention in order to change the optimal choices of other agents. With this in mind, consider a convention with efficient investment and property rights. Suppose some sellers mutate to suggest the same transfer, but demand a higher price. This attempted hold up makes the contract less attractive to the buyers, and if enough sellers demand a higher price, then the convention dissolves. Of course, the larger the transfer, the more a buyer has to loose from being held up. Hence, hold up decreases the stability of contracts which give too much of the surplus to the sellers. On the other hand, starting from the same candidate convention, the buyers might mutate to suggest a different transfer. If sellers anticipate the ability to hold up buyers following this new transfer, then this new transfer will look very attractive to them. Consequently, for a contract to be stable against these mutations, it must give a large share of the surplus to sellers. It turns out that this second issue is much more important. In a stochastically stable convention the (non-investing) sellers receive a larger share of the net surplus than the (investing) buyers. Further, the larger the perceived benefits to the sellers of hold up, the larger the share of the surplus which they receive.

The rest of the paper is organized as follows. The contracting game is defined in Section 2. Section 3 presents the evolutionary dynamic and some preliminary results. Section 4 states and discusses the main results and some Corollaries. Proofs are relegated to the Appendix.

2 The Contracting Game

There are two players, the buyer (B) and the seller (S,) who play a three stage game. In stage one, each player suggests a transfer T_i, ($i = B, S$) to divide surplus given the anticipated continuation. If the agents suggest different transfers, then the game ends. If $T_B = T_S$, then the buyer gives this transfer to the seller, and the game proceeds. In stage two, the buyer makes an investment specific to the seller's product. There are two levels of investment: I^0 and I^* with $I^* > I^0$. An investment I creates a surplus of $V(I)$, with $V^* = V(I^*)$ and $V^0 = V(I^0)$. I^* is the efficient investment; $V^* - I^* > V^0 - I^0$. Both the seller's cost of production, and the inefficient investment level, I^0, are normalized to zero. In the third stage, players bargain, via the Nash demand game, over the price P. Let P_i be the price demanded by player i. The buyer's payoff is

$$\pi_B = \begin{cases} 0 & \text{if } T_B \neq T_S \\ V(I) - I - P_B - T & \text{if } T_B = T_S = T \text{ and } P_B \geq P_S \\ -I - T & \text{otherwise} \end{cases} \tag{1}$$

The seller's payoff is

$$\pi_S = \begin{cases} 0 & \text{if } T_B \neq T_S \\ P_S + T & \text{if } T_B = T_S = T \text{ and } P_B \geq P_S \\ T & \text{otherwise} \end{cases} \tag{2}$$

A pure strategy for a buyer is a triplet (T_B, I, P_B). A pure strategy for a seller is a pair $(T_S, P(I))$, where $P(I)$ maps investments by the buyer into the set of possible prices. For the sake of the evolutionary analysis to follow, it is assumed that for any investment level I, the only possible demands are $\mathcal{P}(I) = \{\Delta, \frac{1}{2}V(I), V(I) - \Delta\}$. While restrictive, $\mathcal{P}(I)$ allows a standard form of hold up ($P = \frac{1}{2}V(I)$,) and an efficient assignment of property rights ($P = \Delta$.)

Assumption 1 *A)* $2\Delta < \min\{V^0, I^*, (V^* - I^* - V^0)\}$

B) $\frac{1}{2}V^* - I^* < V^0 - \Delta$

Part (A) of the assumption, is an assumption that Δ is small, relative to both efficient investment and the gains from efficient investment. Part (B) is weaker than $\frac{1}{2}V^* - I^* < \frac{1}{2}V^0$, which is required for there to a hold up problem, if, for example, P is determined through bargaining a la Rubinstein (1982).

Let $\bar{T}_S = V^* - I^* - \Delta$ and $\bar{T}_B = -(V^* - \Delta)$. If a player i suggests $\bar{T}_i$, then the lowest payoff he can receive is zero. The grid of allowable transfers is $\mathcal{T}(\phi) = \{\bar{T}_B, \bar{T}_B + \phi, ..., \bar{T}_S - \phi, \bar{T}_S\}$. It is assumed here, as in Young (1998), that agents choose only strictly individually rational strategy. That is, if a buyer (resp. seller) plays (T, I, P) (resp. $(T, P(\cdot))$ then $V(I) - I - P_B - T_B > 0$ (resp. $\max_I\{T + P(I)\} > 0$.) Clearly, then a buyer will choose only $T < \bar{T}_S$, and a seller will only choose $T > \bar{T}_B$.

Our objective is to predict an outcome in the contracting game. Let us consider then, the outcomes which are possible. If a buyer and seller suggest different transfers T_i, then the outcome is $([T_B, T_S])$. If they suggest $T_B = T_S = T$, demand different prices P_i, and the buyer invests I, then the outcome is denoted $(T, I, [P_B, P_S])$. If the previous case is modified so that $P_B = P_S = P$, then the outcome is denoted (T, I, P). An outcome (T, I, P) is a *convention*, if there are off path beliefs which make it self enforcing.[4] This requires: $P + T \geq 0$, $V(I) - I - P - T > 0$ and $V(I) - I - P \geq \Delta$.

[4]A convention is supportable as the outcome from a subgame perfect equilibrium.

Observe that it is possible for a seller to receive a payoff of zero in a convention. This possibility can lead to complications, which we avoid with the following assumption.[5]

Assumption 2 *If* $P \in \mathcal{P}(I^*)$ *and* $T \in \mathcal{T}(\phi)$, *then* $P + T \neq 0$.

3 Evolution

The contracting game has multiple pure strategy subgame perfect equilibria, many of which suggest a self enforcing convention. The objective of an evolutionary analysis is to establish the convention most likely to arise endogenously. This convention is found using a stochastic evolutionary dynamic as introduced by Young (1993) and Kandori, Mailath and Rob (1993). Noldeke and Samuelson (1993) provide the extension to extensive form games which is used here.

There is a population of agents, which is split into two subpopulations, one of buyers, and one of sellers. Each subpopulation is of size N. Each period every agent is matched with each agent in the opposite subpopulation to play the contracting game. In any given period, an agent is characterized by a pure strategy, and her beliefs. Agents are required neither to hold beliefs which are correct, nor to always choose best responses to their beliefs. However, agents are restricted to play only strictly individually rational strategies, even after mutation.

[5] See Footnote 16 in the Appendix.

There are two types of histories after which a buyer must have beliefs: $\emptyset$ (the null history) and (T, I) for some agreed upon transfer T and investment level I. A seller must have beliefs following these histories, and also following T for any agreed upon transfer T. Let $\nu(\cdot|\emptyset)$ specify a buyer's belief over the transfers suggested by sellers, and $\nu(\cdot|(T, I))$ specify a buyer's belief over the price demanded by sellers following (T, I). Let $\sigma(\cdot|\emptyset)$ and $\sigma(\cdot|(T, I))$ be beliefs for sellers which are defined analogously. Let $\sigma(\cdot|T)$ be a seller's belief over the investment levels to follow an agreed upon transfer T. Both σ and ν are probability distributions over actions taken by agents in the opposite subpopulation, which are dependent upon the history to that point. We say that an agent *expects* an action following some history if she assigns probability one to that action. For example, a buyer expects P following (T, I) if $\nu(P|(T, I)) = 1$.

A state θ specifies how many agents in each subpopulation have each possible combination of belief and strategy. The set of possible states, denoted Θ, is finite.[6] With each state there is an associated probability distribution over terminal nodes.

Beliefs and strategies evolve in two manners: by *adaptation* to the current environment, and by random *mutation*. Adaptation occurs in the following manner. Every period each agent has an i.i.d chance of updating his beliefs and strategy. An updating agent observes the distribution on terminal nodes in that period, updates his beliefs

[6]Since there are a finite number of agents, each with a finite set of strategies, there are but a finite number of possible strategy profiles within each population. The set of beliefs are restricted to those which reflect these profiles.

based upon this information (beliefs following unreached decision nodes are unchanged) and chooses a best response to his new beliefs. Updating works on behavioral strategies; if an agent is already playing a best (behavioral) response following some node, then she continues to do so. If not, she chooses one of the available best responses, each with positive probability. Agents' beliefs and strategies are also subject to mutation. Every period, each agent has an i.i.d. probability ϵ of mutating. When an agent mutates, his beliefs and strategy are chosen from an exogenously specified distribution which gives full support to all of that agent's possible belief/strategy combinations. The updating draw and mutation together form a Markov chain over the state space Θ in which every transition has positive probability. Hence there is an ergodic distribution $\mu(\epsilon)$.

Stochastic stability is used as the solution concept. The set of stochastically stable states, denoted Θ^*, are those assigned positive probability by the limit distribution $\mu^* = \lim_{\epsilon \to 0} \mu(\epsilon)$. Characterization of the stochastically stable states is facilitated through the use of two weaker concepts: *locally stable sets*, and *absorbing sets.* A set Q is absorbing (w.r.t. updating) if: it is impossible for the population to depart Q without mutations, and for any two states in Q, it is possible for the population to move between these states without mutation. If an absorbing set is a singleton, then it's unique element is an *equilibrium.*

Proposition 1 *All absorbing sets of the contracting game are singletons.*

Because of Proposition 1, we may speak of equilibria, rather than absorbing sets. Let

Υ be the set of equilibria. While useful, the restriction to equilibria is not sufficient; an equilibrium may have multiple outcomes, and so does not necessarily yield a convention.

Local stability is sufficient to restrict attention to conventions. For $\theta \in \Theta$, let $\xi^0(\theta) \subset \Upsilon$ be the set of equilibria, which can be reached from θ through updating alone.[7] For $l > 0$ let $\xi^l(\theta)$ be the elements in Υ which can be reached from some element of $\xi^{l-1}(\theta)$ with updating and no more than a single mutation. If $\theta' \in \xi^l(\theta)$, then a sequence of l transitions between equilibria, each of which required only one mutation, can move the population from θ to θ'. $\xi(\theta) = \cup_{l \geq 0} \xi^l(\theta)$ is the set of equilibria which can be reached from θ with a sequence of single mutation transitions.

Definition 1 *A set of states Y is locally stable if $\forall \theta \in Y$, $\xi(\theta) = Y$.*

A locally stable set is impossible to escape with a single mutation, and does not contain any proper subset with this property. Locally stable sets contain only equilibria.

We now turn to the relationship between conventions and locally stable sets. Let $\rho(\theta)$ denote the unique outcome within θ.[8] For an outcome ρ, the ρ-component is the set $\{\theta \in \Upsilon | \rho(\theta) = \rho\}$.

Recall that a convention is an outcome (T, I, P) such that $P + T > 0$, $V(I) - I - P - T > 0$ and $V(I) - I - P > \Delta$. An outcome is an *efficient convention* if in addition there is efficient investment, $I = I^*$, and efficient assignment of property rights, $P = \Delta$.

[7]If $\theta_1 \in \xi^0(\theta_2)$, then θ_2 is in the *basin of attraction* of θ_1.

[8]If there are multiple outcomes within θ, then $\rho(\theta)$ is undefined.

Clearly, in an efficient convention, $-\Delta < T$.

Proposition 2 *Every locally stable set contains the ρ-component for an efficient convention ρ.*

Proposition 2 follows from essentially the same arguments as Proposition 3 from Ellingsen and Robles (2001). In particular, if we pretend that the transfer is fixed, then the contracting game is the same game as that in Ellingsen and Robles except for a constant offset in payoffs. As there, single mutation transitions suffice to arrive at efficient property rights and investment.

Proposition 2 does not quite say that if ρ is an efficient convention, then the ρ-component is locally stable.[9] However, it is sufficient for our purposes. Local stability (and hence stochastic stability) must treat with entire ρ-components.[10] This allows us to focus the remaining analysis on conventions. Hence, to say that a convention ρ is e.g. stochastically stable is to say that the ρ-component is in the stochastically stable set. Further, we may determine the stochastically stable set by considering only transitions between different efficient ρ-components.

[9] And in fact, this is not quite true. Efficient ρ-components which do not give almost all of the surplus to either buyer or seller are locally stable.

[10] As shown in Samuelson (1994), one state in a locally stable set is stochastically stable if and only if every other element of the locally stable set is stochastically stable.

4 Main Result

From Proposition 2 it is but a small step to know that stochastic stability yields a convention with efficient property rights and investment.[11] The one remaining question then is, what share of the surplus does each player receive? Before answering this question, it is useful to define some expressions.

$$H \equiv V^* + V^0 - 2\Delta. \tag{3}$$

H represent the desirability of hold up for sellers: $V^* - \Delta$ is the highest price a seller can charge in the final stage. The individual rationality of buyers implies that (in the limit as $\phi \to 0$) the highest transfer after which sellers might hope to charge $V^* - \Delta$ is $V^0 - \Delta$.[12] Hence H, the sum of these two terms, is the highest payoff that sellers can hope to receive from holding up a buyer.

It is easiest to characterize the stochastically stable convention in two separate cases. We say that *efficient investment is small* if

$$\frac{I^* - \Delta}{V^* - I^*} \leq \frac{(V^* - I^*)}{H + (V^* - I^*)}. \tag{4}$$

[11]There might in fact be two stochastically stable conventions. However, if so, then they will differ only in that one has a transfer which is ϕ higher.

[12]Of course, buyers can always expect $P = V^* - \Delta$ following any transfer which is not currently agreed upon. However, the relevant issue is, does this make a difference when calculating how hard it is to replace one efficient convention with another. What makes H important is that such beliefs are only relevant for transfers less than $V^0 - \Delta$. Proposition 3 in Appendix C.

Otherwise efficient investment is large. Some feeling for Inequality 4 might be found rewriting H as $(V^0+I^*)+(V^*-I^*)-2\Delta$. Written so, we can see that either decreasing V^0, or increasing V^*-I^* makes the inequality easier to satisfy. With this in mind, we can read Inequality 4 as, I^* is small relative to the net benefit of investment $(V^*-I^*-V^0)$ and the net value of the relationship $(V^*-I^*.)$

Results are stated in terms of the share of surplus received by the seller. In the case of small efficient investment, F_1 approximates the seller's share.

$$F_1 \equiv \frac{H}{H+(V^*-I^*)} \tag{5}$$

Note that $\frac{1}{2} < F_1 < \frac{2}{3}$.

Theorem 1 *Let $(\tilde{T}, I^*, \Delta)$ denote a stochastically stable convention. Let $\Lambda_S \equiv \frac{\tilde{T}+\Delta}{(V^*-I^*)}$ denote the (non investing) seller's share of net surplus in the stochastically stable outcome. For any level of approximation $\lambda > 0$, if ϕ is sufficiently small, and N is sufficiently large, then:*

If investment is small (Inequality 4 holds,) then $|\Lambda_S - F_1| < \lambda$.

For the sake of understanding Theorem 1 let us presume that while agents may choose the transfer to suggest, any agreed upon transfer must be followed by (I^*, Δ). In this case, individual rationality implies that $\pi_S = T + \Delta \geq 0$ and $\pi_B = V^* - I^* -$

$(T+\Delta) \geq 0$ or that $-\Delta \leq T \leq V^* - I^* - \Delta$.[13] To displace a convention (T, I^*, Δ) with a convention (T', I^*, Δ), requires that a sufficient number of buyers or sellers mutate to play (T', I^*, Δ). If it is sellers who mutate, then the proportion that much mutate is r^S such that $r^S(V^* - I^* - (T' + \Delta)) = (1 - r^S)(V^* - I^* - (T + \Delta))$. This expression indicates that buyers would be willing to play T'. Clearly, r^S is increasing in T'. Given the constraint of individual rationality, r^S is minimized when $T' = -\Delta$. When $T' = -\Delta$, r^S satisfies $r^S(V^* - I^*) = (1 - r^S)(V^* - I^* - (T + \Delta))$. This value for r^S can be thought of as a the stability of (T, I^*, Δ) to buyer optimism. We see the prize which draws optimistic buyers is $V^* - I^*$, the net value of the relationship. Because buyers make the investment, they can not try to hold up sellers. Hence, even when investment and price are not fixed at (I^*, Δ) it remains the case that optimistic buyers hope for no more than $V^* - I^*$.

Sellers, on the other hand, do not make the investment and can hope to hold up buyers. As stated above, when buyers mutate to a new transfer, the most that sellers can hope to receive from holding up buyers is $H = V^* + V^0 - 2\Delta$. They are always incorrect in this hope, but this is nonetheless the prize they chase. Hence, the value of r^B which can thought of as the stability against seller optimism of (T, I^*, Δ) satisfies $r^B H = (1 - r^B)(T + \Delta)$. Now if H and $V^* - I^*$ were equal, then r^S and r^B would be

[13]Strict individual rationality requires strict inequalities. However, Theorem 1 is most easily understood with an appeal to the limit as $\phi \to 0$. Hence, throughout this discussion, it is this case which is considered.

symmetric, and the most stable transfer would be $T = \frac{1}{2}(V^* - I^*) - \Delta$ which would split the net surplus equally. However, since H is always greater than $(V^* - I^*)$, the seller gets over half of the net surplus. Further, the seller's share is larger, the greater his hoped for payoff from holding up the seller.

We now turn to the case when investment is *large*, $\frac{I^*-\Delta}{V^*-I^*} > \frac{V^*-I^*}{H+V^*-I^*}$. In this case, the best that can be found is a pair of bounds on the seller's share, which the following two fractions provide.

$$\underline{F}_2 \equiv \frac{H}{H + (V^* - \Delta)} \tag{6}$$

$$\overline{F}_2 \equiv \frac{H}{H + (\frac{1}{2}(V^* + I^*) - \Delta)} \tag{7}$$

Observe that $\frac{1}{2} < \underline{F}_2 < \overline{F}_2 < \frac{2}{3}$.

Theorem 2 *Let $(\tilde{T}, I^*, \Delta)$ denote a stochastically stable convention. Let $\Lambda_S \equiv \frac{\tilde{T}+\Delta}{(V^*-I^*)}$ denote the (non investing) seller's share of net surplus in the stochastically stable outcome. For any level of approximation $\lambda > 0$, if ϕ is sufficiently small, and N is sufficiently large, then:*

If investment is large, *then $\underline{F}_2 - \lambda < \Lambda_S < \overline{F}_2 + \lambda$.*

Whether investment is large or small, that which moves sellers is their desire for H, the prize for a successful hold up. Hence, H enters $\underline{F}_2$ and $\overline{F}_2$ in the same way it entered F_1. However, when investment is large, there is a force operating on the the buyers which is stronger than their desire to grab the entire surplus, $V^* - I^*$. Rather, what moves buyers in this case, is their desire to avoid being held up. If a seller attempts to hold up an unsuspecting buyer, then there is disagreement at the the price setting stage. This leaves the buyer with a loss of $-(T+I^*)$. When efficient investment is large, avoiding this loss can be much more important than chasing after $V^* - I^*$. Because this loss depends upon the transfer, we can not write down it's exact strength as an incentive. However, since individual rationality implies $T \leq V^* - I^* - \Delta$, we can be sure that the strength of this incentive is less than $V^* - \Delta$, the term which appears in $\underline{F}_2$. On the other hand, the loss from being held up decrease as T decreases. For $T \leq \frac{1}{2}(V^* - I^*) - \Delta$ the incentive to avoid being held up becomes too weak to matter. Adding I^* to this transfer yields the term in $\overline{F}_2$.

In order to better understand how different parameters determine the distribution of surplus, I present two limiting results.

Corollary 1 *As* $I^* \to 0$, $\Lambda_S \to \frac{V^*+V^0}{V^*+2V^0}$.
As $V^* - I^* - V^0 \to 0$ *and* $I^* \to 0$, $\Lambda_S \to \frac{2}{3}$.
As $V^0 \to 0$ *and* $I^* \to 0$, $\Lambda_S \to \frac{1}{2}$.

As I^* becomes vanishingly small, V^0 comes to represent the seller's ability to hold up the buyer. Adding $V^* - I^* - V^0 \to 0$, we might think that the whole issue of investment become irrelevant, so that Young (1998) would suggest an even split. However, we see just the opposite. While investment makes no difference in the surplus from the relationship, it is in this case that the seller has strongest incentive to hold up the buyer.

On the other hand, adding $V^0 \to 0$, all of the surplus is generated from investment. However, the buyer really has no hold up ability, and so receives only half of the surplus.

We turn next to the opposite extreme, when the magnitude of I^* dwarfs the other parameters.

Corollary 2 *Holding $V^* - I^*$ constant, as V^* and $I^* \to \infty$, $\Lambda_S \to \frac{1}{2}$.*

In this case, as I^* dominates, an absolute cap is put on the power gained from the sellers ability to hold up the buyer. While the seller will still attempt hold up, hold up is so costly to the buyers that his attempts to avoid it leave the agents with an even split.

A Proofs of Propositions 1 and 2

Proof of Proposition 1*All communication classes are singletons.*

From Ellingsen and Robles (2001, Lemma 2) if the transfer were fixed at $T = 0$, then all communication classes would be singletons. The only difference between fixed transfers $T = 0$ and $T' \neq 0$ is a constant offset in payoffs, which leaves incentives unchanged. Hence, a nonsingleton communication classes must involve multiple transfers. Consider a nonsingleton communication class in which different transfers, including $\hat{T}$ are suggested. Since agents are switching between transfers, if there is a buyer who invests I^* following $\hat{T}$, then there must be a state within the communication class in which only one buyer suggests $\hat{T}$ followed by I^* and at least one seller suggests $\hat{T}$. In this state, let all sellers update, this will lead them all to choose the same price following $\hat{T}, I^*$. In the next state in which $\hat{T}, I^*$ is played, let all buyers update, this will lead them to all choose the same price following $\hat{T}, I^*$. These steps are irreversible and can also be applied if I^0 follows $\hat{T}$. Hence, all agents in both populations suggest the same price following any T, I which occur in the communication class. Hence for any transfer T which is played, and for both I, the payoff which is expected following (T, I) is fixed for every buyer. Hence, for every transfer which is played, either only one investment is played following that transfer, or both investments lead to the same payoff for buyers. Hence, for buyers, the payoff expected for agreement on a transfer is constant, and strictly positive by individual rationality. Since the communication class is nonsingleton, there must be some state in which a particular transfer is most attractive to the sellers. In this state let all sellers update. Follow this by allowing all buyers to update. Since all sellers are suggesting the same transfer, the buyers will all switch to suggesting that transfer as well. Now if the sellers receive a nonnegative payoff in this state, then no agent has an incentive to change his strategy. If the sellers receive a negative payoff in this state, then let them all update, they will never suggest this transfer again. Either of these possibilities contradicts the presumption of a nonsingleton communication class. ♣

The proof of Proposition 2 is more involved, and will be accomplished through a series of lematta.

Lemma 1 *Let θ be an equilibrium. In θ:*

1) All agents receive a nonnegative payoff.

2) all agents in the same subpopulation receive the same payoff.

3) If T is sometimes agreed upon, I always follows T, and at least one seller demands P following (T, I) then $T + P \geq 0$.

Further, if payoffs for both populations are strictly greater than zero, then:

4) the same set of transfers are suggested by the two subpopulations.

5) The same set of prices are demanded by both subpopulations following any (T, I) which occurs in θ.

Proof:(1) Otherwise an agent would suggest $\bar{T}_i$ which guarantees him a zero payoff. (2) If not, then an agent receiving a lower payoff would imitate one who was receiving a higher one. (3) Observe that sellers get some average of zero (when different transfers are suggested), T when the transfer is agreed upon, but prices demanded are incompatible, and $T + P$ otherwise. Hence if this last quantity is less than zero, then the payoff conditional on T is less than zero, which we know is not possible. (4) Suggesting a transfer which the other population does not suggest guarantees a zero payoff, which is avoided by assumption. (5) From Ellingsen and Robles (2001, Lemma 1) this is true if the transfer is fixed at $T = 0$. In an equilibrium, it is always the same buyers (resp. sellers) playing T, I (resp. T) so that in an equilibrium of the contracting game it is as if agents suggesting T are playing the investment/demand game within a smaller population, with payoffs offset by a constant of T, from which the result follows. ♣

For $\theta \in \Theta$, let $H(\theta)$ denote the set of outcomes which occur in θ. Let $\rho^D = ([\bar{T}_S, \bar{T}_B])$ be the disagreement outcome. Let $\bar{\Upsilon} = \{\theta \in \Upsilon | H(\theta) = \{(T, I, P)\} \text{ or } \{\rho^D\}\}$. Call the elements of $\bar{\Upsilon}$ conventional states.

Lemma 2 *Let θ_1 be an equilibrium, with T, I, P' and T, I, P'' ($P' \neq P''$) both elements of $H(\theta)$. Then $\exists \theta_2 \in \xi(\theta_1) \cap \bar{\Upsilon}$ such that $\rho(\theta_2) = (T, I, P) \in H(\theta_1)$.*

Proof: Denote by P' the lowest price demanded by either population following (T, I) and by P'' the highest. Let one buyer who was suggesting the transfer T and demanding P' following T, I mutate, and change his play only in that he now demands P'' following (T, I). This makes (T, I, P'') the only best response for sellers. Let all sellers update to play T, I, P''. Then let all buyers update; this leaves us at an equilibrium with the unique outcome of (T, I, P''). ♣

Lemma 3 *Let θ_1 be an equilibrium in which the transfer T_1 is sometimes agreed upon. If a unique investment level and price follow the transfer T_1 (i.e. $(T_1, I', P'), (T_1, I'', P'') \in H(\theta_1)$ implies that $I' = I''$ and $P' = P''$,) then $\exists \theta_2 \in \xi(\theta_1) \cap \bar{\Upsilon}$.*

Proof: Of course if only the transfer T is suggested, then θ_1 is a conventional state, and the proof is completed. Assume that this is not so. Denote by (T_1, I_1, P_1) the outcome which occurs when T_1 is agreed upon, and by $(T_2, I_2.P_2)$ some other outcome which occurs in θ_1. We know that $V(I_1) - I_1 - T_1 - P_1 > 0$, or buyers would never play so. Let one seller who was suggesting T_2 mutate to suggest T_1 and follow I_1 with P_1. This increases the probability of being matched with a seller suggesting T_1, which makes playing (T_1, I_1, P_1) the unique best response for buyers. Let them all update, and switch accordingly. If $T_1 + P_1 > 0$, then as soon as sellers update, they will all switch to playing T_1 with P_1 following I_1, since only T_1 yields a payoff greater than zero. If, on the other hand, $T_1 + P_1 = 0$, then the population is already at an equilibrium, since all transfers net sellers a payoff of zero. From this equilibrium, let a single seller mutate to play $(T_1, P(I_1) = P_1)$. This increases every buyer's payoff, and leaves every seller's payoff unchanged. Hence this is another equilibrium. By repeating this process, the population arrives at a conventional state with outcome (T_1, I_1, P_1). ♣

Lemma 4 *If θ_1 is an equilibrium with a unique price demanded following any transfer and investment (i.e. $T, I, [P_B, P_S]) \in H(\theta_1)$ implies that $P_B = P_S$,) then $\exists \theta_2 \in \xi(\theta_1) \cap \bar{\Upsilon}$.*

Proof: If there exists a transfer which is sometime agreed upon, and then followed by a unique investment level, then an application of Lemma 3 completes the proof. Assume this is not so. There are two cases to consider. Let us first presume that $\exists (T, I_1, P_1), (T, I_2, P_2) \in H(\theta_1)$ such that $P_2 + T < P_1 + T$. This implies that $0 < P_1 + T$. Let a single buyer mutate from playing (T, I_2, P_2) to playing (T, I_1, P_1). This make playing T, and following (I_1) with P_1 a requirement of any best response by a seller. Let all sellers update. Now allow all buyers to update, and we are left at a convention with outcome (T, I_1, P_1). If on the other hand, $P_2 + T = P_1 + T$, then again let a single buyer mutate from playing (T, I_2, P_2) to playing (T, I_1, P_1). This changes no agent's payoffs, and so again we are at an equilibrium. Proceeding in this manner, a sequence of single mutation transitions lead to an equilibrium in which T is sometimes agreed upon, and only (I_1, P_1) ever follows T. One may now apply Lemma 3 to complete the proof. ♣

Lemma 5 *For all $\theta \in \Upsilon$, $\xi(\theta) \cap \bar{\Upsilon}$ is nonempty.*

Proof: Simply a collection of Lemmas 2, 3, and 4. ♣.

Lemma 6 *Let θ_1 and θ_2 be two conventional states, with $\rho(\theta_1) = \rho(\theta_2)$. $\xi(\theta_1) = \xi(\theta_2)$*

Proof: Let $\hat{\rho}$ denote the common outcome, then in both θ_1 and θ_2, agents must have off path beliefs which support play of $\hat{\rho}$. Hence, one by one, agents might have their beliefs in θ_1 replaced through mutation by those in θ_2. The reverse is true, so that $\theta_1 \in \xi(\theta_2)$ and vice versa. ♣

Lemma 7 *Let θ_1 be a convention in which the agreed upon transfer is T.*

1) If $T < -\Delta$ then then $\exists \theta_2 \in \xi(\theta_1)$ such that $\rho(\theta_2) = \rho^D$.

2) If $T \geq -\Delta$ then $\exists \theta_2 \in \xi(\theta_1)$ such that $\rho(\theta_2) = (T, I^*, P^*)$

Proof: Let $\rho(\theta_1) = (T, I_1, P_1)$. Let $I_2 \neq I_1$ be the other investment level. Let us first observe that if $T \geq V^0 - \Delta$, then $(I_1, P_1) = (I^*, \Delta)$ since otherwise $V(I_1) - I_1 -$

$P_1 - T_1 \leq 0$. Of course if $(I_1, P_1) = (I^*, \Delta)$, then the proof is done so assume that this is not so. For the remainder of the proof, observe that if $(I_1, P_1) \neq (I^*, \Delta)$, then $V(I_2) - I_2 - \Delta > V(I_2) - I_1 - P_1$. Hence $\exists \theta'$, not necessarily an equilibrium, which can be reached from θ_1 through some single mutation transitions, such that $\rho(\theta') = (T, I_2, \Delta)$. To accomplish this, first let all sellers drift to expect Δ to follow (T, I_2). Then allow a single buyer to mutate to play (T, I_2, Δ). Allow all buyers to update, and the population has arrived at the state θ'. If $T < -\Delta$, then $T + \Delta < 0$, and θ' is not an equilibrium. Let all sellers update to $\bar{T}_B$, and we are at an equilibrium in which every action yields a payoff of zero. From here, let each of the buyers mutate one by one to choose $\bar{T}_S$, and we have arrived at a disagreement convention through a series of one mutation transitions. If $T \geq -\Delta$ and $I_2 = I^*$, then we are done. If $T \geq -\Delta$ and $I_2 = I^0$, then we can apply the above argument to get from θ' to the desired θ_2 with $\rho(\theta_2) = (T, I^*, \Delta)$. ♣

Lemma 8 *If* $\rho(\theta_1) = \rho^D$ *and* $-\Delta < T < V^* - I^* - \Delta$, *then* $\exists \theta_2 \in \xi(\theta_1)$ *such that* $\rho(\theta_2) = (T, I^*, \Delta)$.

Proof: For a transfer T satisfying the requirements of the lemma, let all sellers drift to expect Δ to follow (T, I^*). Allow single buyer mutate to play (T, I^*, Δ). Let sellers update, they will all switch to playing $(T, P(I^*) = \Delta)$. Allow the remaining buyers to update, and we have arrived at the desired θ_2. ♣

Proof of Proposition 2*Every locally stable set contains the* ρ*-component for an efficient convention* ρ

Every locally stable set contains at least one equilibrium. From Propositions 7 and 8 we know that if θ is an equilibrium, then $\exists \theta' \in \xi(\theta)$ with $\rho(\theta')$ an efficient convention. Hence, every locally set contains at least one equilibrium θ' with $\rho(\theta')$ an efficient convention, and by Lemma 6, the locally stable set must contains the entire $\rho(\theta')$-component. ♣

B A Theorem on Stochastic Stability

For $\theta \in \Upsilon$, a θ-tree is a collection of directed edges $(\theta' \to \theta'')$ $(\theta', \theta'' \in \Upsilon)$ such that $\forall \theta' \in \Upsilon \setminus \{\theta\}$, there is a unique directed path of edges from θ' to θ. For $\theta', \theta'' \in \Upsilon$, and $(\theta' \to \theta'')$ an edge, let $C((\theta_1 \to \theta_2))$ be the smallest number of mutations required for a transition from θ' to θ''. $C(E)$ is the cost of the edge E. The cost of a θ-tree is the sum of the costs of the edges. The stochastic potential of $\theta \in \Upsilon$, is the minimum cost over all θ-trees. The following Theorem is due to Young (1993, Theorem 4).

Theorem 3 *An equilibrium θ is stochastically stable if and only if no other equilibrium has lower stochastic potential.*

We do not directly use Theorem 3, but rather a consequence of it. $L \subset \Upsilon$ is a *mutation connected set* if $\forall \theta \in L$, $L \subseteq \xi(\theta)$. Let $\mathcal{L}$ be a collection of disjoint mutation connected sets, such that for every locally stable set L, there is $L' \in \mathcal{L}$ such that $L' \subseteq L$. For $L', L'' \in \mathcal{L}$, an $\mathcal{L}$-edge from L' to L'', is a collection of directed edges between equilibria,

$\beta = \{(\theta_1 \to \theta_2), (\theta_2 \to \theta_3)...(\theta_{k-1} \to \theta_k)\}$ such that $\theta_1 \in L'$ and $L'' \subset \xi(\theta_k)$. Under these circumstances, we define an $(L, \mathcal{L})$-tree, as a collection of $\mathcal{L}$-edges, such that $\forall L' \in \mathcal{L} \setminus \{L\}$ there is a unique directed path of $\mathcal{L}$-edges from L' to L. We do not define a cost for $\mathcal{L}$-edges, but directly define the cost of $\mathcal{L}$-trees. Let η be an $\mathcal{L}$-tree, and let $E(\eta) = \{E = (\theta' \to \theta'') | \exists \beta \in \eta \text{ with } E \in \beta\}$. The $\mathcal{L}$-cost of an $\mathcal{L}$-tree η is $\sum_{E \in E(\eta)} (C(E) - 1)$. The $\mathcal{L}$-potential of $L \in \mathcal{L}$ is the minimum cost over $(L, \mathcal{L})$-trees.

Theorem 4 *Let $\mathcal{L}$ be a collection of disjoint mutation connected subsets of Υ, such that for every locally stable set L, $\exists L' \in \mathcal{L}$ with $L' \subseteq L$. $\theta \in \Upsilon$ is stochastically stable, if and only if $\theta \in \xi(\theta^*)$ for $\theta^* \in L^*$ and L^* is an element of $\mathcal{L}$ with lowest $\mathcal{L}$-potential.*

Proof: Let L^* have lowest $\mathcal{L}$-potential, $\theta^* \in L^*$, and let η be an $(L^*, \mathcal{L})$-tree which achieves this lowest potential. Let $\mathcal{L}_0 = \{L^*\}$, and for $i > 0$, let $\mathcal{L}_i$ be the elements $L \in \mathcal{L}$ such that η contains an $\mathcal{L}$-edge from L to some $L' \in \mathcal{L}_{i-1}$. For $\mathcal{G}$ a collection of edges, let $B(\mathcal{G}) = \{\theta | \theta = \theta^* \text{ or } (\theta \to \theta') \in \mathcal{G}\}$. Define $\xi^{-1}(A) \equiv \{\theta \in \Upsilon | \exists \theta' \in A \text{ with } \theta' \in \xi^1(\theta)\}$ as the inverse image of $\xi^1(\cdot)$. We now construct a θ^*-tree $\bar{\eta}$ which I claim has lowest stochastic potential. Let $\eta_0 = E(\eta)$. Let $\bar{\mathcal{L}}_i = \{\theta | \theta \in L \in \mathcal{L}_i\}$. Construct η_i^0 so that $\forall \theta \in [\xi^{-1}(\bar{\mathcal{L}}_i \cap B(\eta_i))] \setminus B(\eta_i)$, $\exists! (\theta \to \theta') \in \eta_i^0$. Further, choose this $\theta' \in \bar{\mathcal{L}}_i \cap B(\eta_i)$ and such that $C(\theta \to \theta') = 1$. Include no other edges in η_i^0. We define two sets of edges, $\bar{\eta}_i^j$ and η_i^j (for $j \neq 0$,) simultaneously. Define $\bar{\eta}_i^j = \eta_i \cup (\cup_{k=1}^j \eta_i^k)$. Construct η_i^j so that $\forall \theta \in [\xi^{-1}(B(\eta_i^{j-1}))] \setminus B(\bar{\eta}_i^{j-1})$, $\exists! \theta'$ such that $(\theta \to \theta') \in \eta_i^{j-1}$. Further, choose this $\theta' \in B(\eta_i^{j-1})$ and such that $C(\theta \to \theta') = 1$. Include no other edges in η_i^j. For

$i > 0$, let $\eta_i = \cup_j \bar{\eta}^j_{i-1}$. Finally, let $\bar{\eta} = \cup_i \eta_i$. We observe that η_0 is a collection of edges which provide escape from every locally stable set but L^*. η^0_0 is empty. Hence, $B(\eta^0_0) = \{\theta^*\}$. η^1_0 consists of all edges $(\theta \to \theta^*)$ which are single mutation transitions and depart from equilibria from which an edge does not already depart in η_0. η_1 then results from repeatedly adding edges for single mutation transitions which eventual lead to θ^*. Hence η_1 has all of the edges in η_0, plus an edge for a single mutation transition departing from every θ such that $\theta^* \in \xi(\theta)$. Proceeding from here, we see that η_i consist of all the edges in η_{i-1} plus single mutation transition from every θ such that $\theta_1(\beta) \in \xi(\theta)$, where $(\theta_1(\beta) \to \theta_2(\beta))$ is the first edge in β, the $\mathcal{L}$ edge departing some set $L \in \mathcal{L}_{i-1}$. Further, for every $\theta \in B(\eta_i) \cap \bar{\mathcal{L}}_{i-1}$, there is a sequence of edges which lead from θ to θ^*. Since Θ is finite, this process then eventually yields a θ^*-tree. We observe, that every equilibrium but one must have an edge departing from it for any θ-tree. Each of these edges must have a cost of at least one. Let $\bar{M}$ denote the cardinality of Υ. By construction, $\bar{\eta}$ minimizes $\sum_{E \in \mathcal{G}}(C(E)-1) = (\sum_{E \in \mathcal{G}} C(E)) - (\bar{M}-1)$ for any collection of edges $\mathcal{G}$ which provides an escape from all but one of the locally stable sets. Since any tree must do this, and $\bar{M}$ is a constant, θ^* has lowest stochastic Υ potential, and is stochastically stable. From Samuelson (1994) we know that θ^* is in a locally stable set, and that the stochastically stable set must include that entire locally stable set, $\xi(\theta^*)$.

♣

Theorem 4 and Proposition 2 indicate that it is possible to find the stochastically stable set through trees with edges which include the efficient conventions.

C Resistances between Conventions

To apply Theorem 4 we must determine the number of mutations required to replace an efficient convention with another. What is required is to move the population to a state from which a sequence of single mutation transitions suffices to reach another efficient convention. From Lemmas 7 and 8 we know that it suffices to move the population to a convention with a different transfer. Because positive payoffs are always decreased with disagreement, there is always a convention which looks more attractive than an equilibrium with multiple outcomes. Hence, it suffices to consider only transitions between conventions.

Unlike the transitions discussed in Appendix A, the number of mutations required to upset an efficient convention will in general depend upon the population size N. However, there is an $r \in (0,1)$, such that if any proportion greater than r of, e.g., the buyers mutate appropriately, then the sellers will all want to change their strategy and choose the newly suggested transfer. The number of mutations needed to make this transition, is then the smallest integer strictly larger than rN.[14] Clearly, for large

[14] For a single mutation transition, this proportion is zero.

enough N, we can work with the proportions r which I term resistances.[15]

There are broadly speaking, two types of transitions between conventions which bear consideration: *direct* and *indirect*. Consider a transition between two efficient conventions (T, I^*, Δ) and (T', I^*, Δ). Let θ_1 be the first equilibrium on the path of this transition such that $\rho(\theta_1) \neq (T, I^*, \Delta)$. If $\exists(I, P)$ such that $\rho(\theta_1) = (T', I, P)$, then it is a direct transition. If $\exists(I, P)$ such that $\rho(\theta_1) = (T, I, P)$ then it is an indirect transition. We focus first on direct transitions.

There are two possible means of affecting a direct transition. A new transfer might be made attractive if agents mutate to suggest it, or the current transfer might be made unattractive by changing behavior following it. Clearly this second is accomplished by changing the price. The worst price that sellers (resp. buyers) can demand is $V(I) - \Delta$ (resp. Δ.) Since the price is already Δ in an efficient convention, there is no point to having buyers mutate in this manner. The buyer's payoff is linear in the number of sellers choosing different strategies. Hence, there is no point in considering transitions which involve both types of mutations: either mutations which decrease the payoff for the old transfer are more effective, or mutations which increase the payoff for the new transfer are more effective. These observations are collected in Lemma 9.

Lemma 9 *Consider a minimal mutation direct transition from an efficient convention with transfer T_1 which results in a convention with transfer $T_2 \neq T_1$.*

[15] This applies to Theorem 4 since $\frac{rN-1}{N} \to r$ as $N \to \infty$.

If mutations are to buyers, then all the mutants suggest the transfer T_2*.*

If the mutation is to sellers, then either all of the sellers suggest the transfer T_2*, or all of the sellers demand a price of* $V^* - \Delta$ *following the current transfer and investment.*

Lemma 9 identifies three different types of direct transition. Let us focus first on a transition from (T, I^*, Δ), to a convention with transfer $T' \neq T$, which is affected by mutating buyers who suggest the transfer T'. Clearly the sellers can't know what payoff will result from a match in which T' is agreed upon. But they do have beliefs. What concerns us is the nature of these beliefs in a minimum resistance transition. Presume that sellers hold such beliefs, and define $u_S(T')$ such that $T' + u_S(T')$ is the payoff that sellers expect from a match in which T' is agreed upon. That is, the resistance for this transition is r such that $(1 - r)(T + \Delta) = r(T' + u_S(T'))$. To make r small, $u_S(T')$ should be as large as possible. Of course, one can alway suppose that sellers expect $(I^*, V^* - \Delta)$ following T'. However, it does not follow that if such beliefs are used to define $u_S(T')$, and hence r, that a proportion r of mutants would be sufficient to cause the desired transition. A problem might arise because individual rationality constrains the mutating buyers choice of strategy. This might imply that when buyers switch to a strategy which suggests T', their beliefs might be contradicted in such a manner that they end up switching back to their previous strategy. So $u_S(T')$ is as large as possible while avoiding this problem. We consider also a transition caused by sellers mutating to suggest T'. Let $u_B(T')$ be defined so that $u_B(T') - T'$ is the payoff buyers expect in such a transition of minimum resistance. That is, the minimum resistance of such a transition should be r such that $(1-r)(V^* - I^* - \Delta - T) = r(u_B(T') - T')$.[16]

Proposition 3 *Consider a minimum resistance transition from an efficient convention (T, I^*, Δ) to a convention with transfer $T' \neq T$. Define $u_S(T')$, and $u_B(T')$ as above.*

If sellers mutate to suggest T' then $u_B(T') = V^* - I^* - \min\{P \in \mathcal{P}(I^*)\}$ such that $u_B(T') - T' < V^* - I^*$.

If buyers mutate to suggest $T' < V^0 - \Delta$, then $u_S(T') = V^* - \Delta$.

If buyers mutate to suggest $T' \geq V^0 - \Delta$, then $u_S(T') = V^* - I^* - \Delta$.

Proof: Transitions which change the transfer from T by suggesting a new transfer T' follows three steps. (1) A proportion r of one subpopulation (the leaders) mutates to a new transfer T'. (2) The agents in the other subpopulation expect a higher payoff following T', and so all switch to T'. (3) The remaining agents in the leader population who did not mutate in step one switch to playing T'. With this in mind, we first show that the suggested values of $u_i(T')$ are feasible, and then show that no larger value is feasible. Consider first $u_S(T)$. Let $P' = \min\{P \in \mathcal{P}(I^*) | V^* - I^* - P' - T' < V^* - I^*\}$. Note that $u_S(T') = V^* - I^* - P'$ is the value suggested by the Proposition, and that $T' + P' > 0$. Let all buyers drift to expect P' following (T', I^*). Let sellers mutate to play

[16] It is in the statement of Proposition 3 that Assumption 2 plays a role. If $P' + T = 0$, then we might wonder if demanding P' following (T, I^*) is individually rational for sellers. This depends upon what they play following (T, I^0). The best (from their perspective) this could be would be $P(I^0) = V^0 - \Delta$. Because $V^0 - \Delta - \frac{1}{2}V^*$ may be either positive or negative, we can't know the individual rationality of $-T = \frac{1}{2}V^* = P'$ for the sellers. Assumption 2 allows the statement of Proposition 3 without reference to this issue.

$(T', P(I^*) = P')$. If a sufficient proportion of sellers have mutated, the buyers update to play (T', I^*, P'). At this point updating by the sellers completes the transition. This demonstrates that the suggested value for $u_S(T)$ is feasible. By assumption $u_S(T) - T' = V^* - I^*$ is not feasible, so we ask is it possible that $u_S(T) - T' > V^* - I^*$. The only way for buyers to receive above $V^* - I^*$ is for sellers to receive a negative payoff. In this case, the non-mutating sellers would not imitate the mutants and the mutants would eventually imitate the nonmutants. Hence if the buyers needed a draw of $u_S(T) - T' > V^* - I^*$ to switch strategies, then they will eventually switch back, and $u_S(T) - T' > V^* - I^*$ is not feasible. Now consider $u_B(T')$. If $T' < V^0 - \Delta$, then the mutating buyers play (T', I^0, Δ), while the sellers expect $(I^*, V^* - \Delta)$ following T' and correctly expect Δ following (T', I^0). All the sellers update as soon as they see T'. Then all the buyers update before the sellers can switch back. This leaves the population at an equilibrium with outcome (T', I^0, Δ). Hence $u_B(T') = V^* - \Delta$ is feasible. It is obviously the highest possible value. If $T' \geq V^0 - \Delta$, then the mutating buyers play (T', I^*, Δ), while the sellers expect $(I^0, V^0 - \Delta)$ following T' and correctly expect Δ following (T', I^*). All the sellers update as soon as they see T'. Then all the buyers update before the sellers can switch back. This leaves the population at an equilibrium with outcome (T', I^*, Δ). The only way that $u_B(T')$ could be larger is if the sellers expected I^* to follow T'. If $T' \geq V^0 - \Delta$, then individual rationality assures that I^0 will not be played following T'. Hence the only way that $u_B(T')$ could be greater is if the sellers correctly anticipated I^*

following T', but then they would have to be incorrect concerning the price. Hence, there would be disagreement concerning the price following T, I^*. At this point, if the sellers update, not getting the payoff they expected, they will return to the starting convention. Further, nothing could make the mutated buyers demand a high enough price to change this fact, so that eventually the sellers will return to the previous convention. Hence if $T' \geq V^0 - \Delta$, then $u_B(T') > V^0 - \Delta$ is not possible. ♣

I now wish to define three important transfers: $T^L = \min\{T \in \mathcal{T}(\phi) | T > -\Delta\}$, $T^0 = \max\{T \in \mathcal{T}(\phi) | T < V^0 - \Delta\}$ and $T^H = \bar{T}_S - \phi$. The lowest and highest transfers within efficient conventions are T^L and T^H respectively. The highest transfer to which a buyer who intended to invest I^0 would agree is T^0. Let $\rho^M \equiv (T^M, I^*, P^M)$ where $T^M, P^M \in \arg\min\{T^M + P^M | T^M + P^M > 0, T^M \in \mathcal{T}(\phi), P^M \in \mathcal{P}(I^*)\}$. ρ^M is the convention which maximizes buyer's payoff. Let $\delta^M = T^M + P^M$, $\delta^0 = V^0 - \Delta - T^0$, and $\delta^L = -\Delta - T^L$. Clearly $0 < \delta^j < \phi$ for $j = L, M, 0$.

Let $R^B(T, T')$ denote the resistance for a transition from (T, I^*, Δ) to a convention with transfer T' when *buyers* mutate to suggest T'. $R^B(T, T') = r$ such that $r(T' + u_B(T')) = (1 - r)(T + \Delta)$ which solves to $R^B(T, T') = \frac{T+\Delta}{T+T'+u_B(T')+\Delta}$. To minimize this expression, we must maximize $T' + u_B(T')$. There are two possibilities: $T' = T^0 \equiv V^0 - \Delta^0 - \delta$ and $u_B(T^0) = V^* - \Delta$, or $T' = T^H \equiv V^* - I^* - \Delta - \phi$ and $u_B(T^H) = V^0 - \Delta$. Obviously, the first possibility results in a lower resistance. Let $R^B(T) \equiv \min_{T'} R^B(T, T')$.

Proposition 4 $R^B(T,T') = \frac{T+\Delta}{T+T'+u_B(T')+\Delta}$.

If $T \neq T^0$, *then* $R^B(T) = R^B(T,T^0) = \frac{T+\Delta}{T+V^*+V^0-\Delta-\delta^0}$.

$R^B(T^0) = R^B(T^0, T^0 - \phi) = \frac{V^0-\delta^0}{V^*+2(V^0-\Delta-\delta^0)-\phi}$.

Similarly, let $R^S(T,T')$ denote the resistance for a transition from (T, I^*, Δ) to a convention with transfer T' when *sellers* mutate to suggest T'. $R^S(T,T') = r$ such that $r(-T' + u_B(T')) = (1-r)(V^* - I^* - \Delta - T)$, which solves to $R^S(T,T') = \frac{V^*-I^*-\Delta-T}{V^*-I^*-\Delta-T-T'+u_B(T')}$. By assumption, this expression is minimized when $T' = T^M$ and $u_B(T^M) - T^M = V^* - I^* - \delta^M$. Let $R^S(T) \equiv \min_{T'} R^S(T,T')$.

Proposition 5 $R^S(T,T') = \frac{V^*-I^*-\Delta-T}{V^*-I^*-\Delta-T-T'+u_B(T')}$

If $T \neq T^M$, *then* $R^S(T) = R^S(T,T^M) = \frac{V^*-I^*-\Delta-T}{2(V^*-I^*)-T-\Delta-\delta^M}$.

There remains one type of direct transition. This last occurs when, starting from an efficient convention (T, I^*, Δ), buyers drift to expect $P = V^0 - \Delta$ following (T, I^0), after which sellers mutate to play $(T, P(I) = V(I) - \Delta)$. That is, they continue to make the same transfer, but demand all but Δ of the post investment surplus. If $T \geq \Delta$, then the buyers' individual rationality rules out playing $(T, I, V(I) - \Delta)$ for both values of I. Hence, given their beliefs, they will never switch to playing (T, I^0, P) for any $P \in \mathcal{P}(I^0)$. This assures that if $T \geq \Delta$, and sellers mutate as described, then the only way for buyers to escape a negative payoff is to change their transfer to one which is not currently suggested. This will net them a payoff of zero. Let $R^d(T)$ represent the resistance for upsetting a convention in this manner. $R^d(T)$ is r such that

$(1-r)(V^*-\Delta)-I^*-T=0$. This expression follows because the payoff for suggesting a different transfer is zero, and the buyers only get $V^*-\Delta$ when they are matched with non-mutants, but must pay the investment and transfer in all matchings. Of course, the buyers can guarantee themselves a payoff of $\Delta - T$ by playing (T, I^0, Δ). Hence if $T < \Delta$, then this form of transition is not possible. This case is denoted with a resistance of ∞.

Proposition 6 *If* $T \geq \Delta$, *then* $R^d(T) = \frac{V^*-I^*-T-\Delta}{V^*-\Delta}$

If $T < \Delta$, *then* $R^d(T) = \infty$.

We now turn to indirect transitions. Starting from an efficient convention $\rho = (T, I^*, \Delta)$ an indirect transition would first arrive at a convention $\rho_i = (T, I_i, P_i)$ with the same transfer. Then from the convention ρ_i, the populations move to some convention $\rho' = (T', I', P,)$. The only purpose in the transition from ρ to ρ_i is to make the final transition to ρ' easier. This requires that the subpopulation which is *not* mutating in the transition from ρ_i to ρ' has a lower payoff in ρ_i than in ρ. This implies that sellers mutate in this final transition. Therefore there is no increase in resistance from presuming that $\rho' = \rho^M$. This final transition is caused by having sellers mutate to either a new convention, or to an attempt to grab all of the surplus. In the first case, we have already established that $\rho' = \rho^M$ offers the biggest draw to buyers. In the second case, any convention with a different transfer is equally good, they all offer a zero payoff to the buyers.

Proposition 7 *A least resistance indirect transfer can be constructed to end at ρ^M.*

There are nonetheless many different types of indirect transfers. However, it happens that only transitions beginning at conventions with a transfer $T > \frac{1}{2}(V^* - I^*) - \Delta$ are relevant. The following Proposition restricts attention to only one one type of indirect transition.

Proposition 8 *If $T \geq \frac{1}{2}(V^* - I^*) - \Delta$, and $P \neq \Delta$, then (T, I, P) is not strictly individually rational for buyers. The resistance of any transition to such a convention is infinite.*

Proof: By assumption $2\Delta < \min\{I^*, (V^* - I^* - V^0)\}$. Hence if $T \geq \frac{1}{2}(V^* - I^*) - \Delta$, and $P \geq \frac{1}{2}V(I)$, then $V(I) - I - P - T \leq \frac{1}{2}V(I) + \Delta - I - \frac{1}{2}(V^* - I^*) < 0$ since $\frac{1}{2}V^0 + \Delta < \frac{1}{2}(V^* - I^*)$ (if $I = I^0$) and $\Delta < \frac{1}{2}I^*$ (if $I = I^*$.) ♣

Let us then consider an indirect transfer which starts at (T, I^*, Δ), passes through (T, I^0, Δ) and then moves on to ρ^M. We presume that buyers correctly expect Δ to follow (T, I^0). In this circumstance, the resistance r, to change (T, I^*, Δ) to (T, I^0, Δ) satisfies $(1 - r)(V^* - \Delta) - I^* - T = V^0 - \Delta - T$. Of course, if $V^0 - \Delta - T \leq 0$, then buyers will not play I^0 following T, making such a transition impossible. The resistance r for the second part of this indirect transition, from (T, I^0, Δ) to ρ^M satisfies either $(1-r)(V^0 - \Delta - T) = r(V^* - I^* - \delta^M)$ or $(1-r)(V^0 - \Delta) - T = 0$. Algebra reveals that the first expression yields a smaller r. Let $R^0(T)$ denote the resistance for the transition from (T, I^*, Δ) to (T, I^0, Δ) and let $R^{0L}(T)$ be the resistance of the transition from (T, I^0, Δ) to ρ^M.

Proposition 9 *If* $V^0 - \Delta > T$ *then* $R^0(T) = \frac{V^* - I^* - V^0}{V^* - \Delta}$.

If $V^0 - \Delta \leq T$ *then* $R^0(T) = \infty$.

$R^{0L}(T) = \frac{V^0 - \Delta - T}{V^* - I^* + V^0 - \Delta - \delta^M - T}$

We let $R^i(T)$ denote the summed resistances for the two legs of an indirect transition.

Proposition 10 *If* $T \geq \frac{1}{2}(V^* - I^*) - \Delta$, *then* $R^i(T) = R^0(T) + R^{0L}(T)$.

D Construction of the $\mathcal{L}$-Tree

We now attempt to find which of the resistances is lowest for different conventions, and which efficient convention is most difficult to upset. The first question we ask is, when is $R^d(T) < R^S(T)$? Since both expressions have the same numerator, we can see that $R^d(T) < R^S(T)$ if and only if $T - \Delta + \delta^M > V^* - \Delta - 2I^*$.

Proposition 11 $R^d(T) \leq R^S(T)$ *if and only if* $T + I^* + \delta^M \geq V^* - I^*$.

This expression has some nice intuition. The bigger the investment or the transfer, the the fewer sellers must attempt hold up in order to upset an efficient convention. The expression approximately says that if the cost of hold up $(T + I^*)$ is greater than the value of the relationship $(V^* - I^*,)$ then it is easy for sellers to cause disagreement.

Conversely, if the value of the relationship is greater than the cost of hold up, then it is easier for sellers to suggest a new agreement.

Proposition 12 *If* $\min\{V^0, \frac{1}{2}(V^* - I^*)\} \leq T + \Delta$ *then* $R^i(T) \geq \min\{R^S(T), R^d(T)\}$.

Proof: If $V^0 - \Delta \leq T$, then $R^i(T) = \infty$. So assume $V^0 > T + \Delta \geq \frac{1}{2}(V^* - I^*)$. Fix T, and set $X = T + \Delta$. Note that if $I^* + T + \delta^M > V^0$, then $R^i(T) > \frac{(V^*-I^*)-X}{V^*-\Delta}$. Consequently $R^i(T) - R^d(T) > \frac{(V^*-I^*)-X}{V^*-\Delta} - \frac{(V^*-I^*)-X}{V^*-\Delta} = 0$. On the other hand, if $I^* + T + \delta^M \leq V^0$, then $R^i(T) \geq \frac{(V^*-I^*)-X}{V^*-I^*+V^0-X-\delta^M}$. Consequently, $R^i(T) - R^S(T) \geq \frac{(V^*-I^*)-X}{V^*-I^*+V^0-X-\delta^M} - \frac{(V^*-I^*)-X}{2(V^*-I^*)-X-\delta^M} > 0$ since $V^* - I^* > V^0$. ♣

Evidently $R^B(T)$ is increasing in T while both $R^d(T)$ and $R^S(T)$ are decreasing in T. Let $R^B_0(T) = \lim_{\phi\to 0} R^B(T)$, and $R^S_0(T) = \lim_{\phi\to 0} R^S(T)$. Define τ such that $R^B_0(\tau) = \min\{R^S_0(\tau), R^d(\tau)\}$ and $\tilde{\tau} = \arg\max_{T\in\mathcal{T}(\phi)}\{\min\{R^B(T), R^S(T), R^d(T)\}\}$.[17] Clearly $\lim_{\phi\to 0}|\tau - \tilde{\tau}| = 0$. As one might suspect, the transfer $\tilde{\tau}$ is very important. Given Proposition 12, if ϕ is sufficiently small, and $\tau > \frac{1}{2}(V^* - I^*) - \Delta$, then the convention $(\tilde{\tau}, I^*, \Delta)$ is the most difficult convention to upset. Furthermore, it is almost possible to construct a $(\tilde{\tau}, I^*, \Delta)$ tree using only the easiest transition out of every efficient convention. It is now to demonstrate that $\tau > \frac{1}{2}(V^* - I^*) - \Delta$.

Let $X_\tau \equiv \tau + \Delta$. Let X_S be defined such that $R^B_0(X_S - \Delta) = R^S_0(X_S - \Delta)$, and let X_d be defined such that $R^B_0(X_d - \Delta) = R^d(X_d - \Delta)$. Clearly, $X_\tau = \min\{X_S, X_d\}$.

[17] For ease of exposition I presume that there is only one argmax. Observe that if there are two, then they adjacent to each other, and still quite close to τ.

Recall that $H \equiv V^* + V^0 - 2\Delta$.

Proposition 13 $X_S = (V^* - I^*)\frac{H}{H+(V^*-I^*)}$.

It is not so easy to find an expression for X_d, because the solution is quadratic. However, we know that there is an $\alpha \in (0,1)$ such that $X_d = \alpha(V^* - I^*)$. This allows us to write an expression, which while not in closed form, is nonetheless informative.

Proposition 14 $X_d = (V^* - I^*)\frac{H}{H+(\alpha V^*+(1-\alpha)I^*-\Delta)}$.
Here $\alpha \equiv \frac{X_d}{(V^*-I^*)} \in (\frac{1}{2}, 1)$.

Proof: Algebra yields $X_d = (V^* - I^*)\frac{H}{H+I^*+X_d-\Delta} = (V^* - I^*)\frac{H}{H+I^*+\alpha(V^*-I^*)-\Delta} = (V^* - I^*)\frac{H}{H+(\alpha V^*+(1-\alpha)I^*-\Delta)}$. We observe that X_d is decreasing in α. Setting $\alpha = 1$, yields $X_d = (V^* - I^*)\frac{H}{H+V^*-\Delta} > \frac{1}{2}(V^* - I^*)$, while setting $\alpha = 0$, yields $X_d = (V^* - I^*)\frac{H}{H+I^*-\Delta} < V^* - I^*$. Hence $\alpha \equiv \frac{X_d}{(V^*-I^*)} \in (\frac{1}{2}, 1)$. ♣

Note that $X_\tau \equiv \min\{X_d, X_S\} > \underline{X} \equiv (V^* - I^*)\frac{H}{H+(V^*-\Delta)}$ Let $\underline{R}(T) = \min\{R^S(T), R^B(T), R^d(T), R^i(T)\}$. From the above, we can see that (for ϕ sufficiently small) $\underline{R}(\tilde{\tau}) \approx R^B(\tau) > R_0^B(\underline{X} - \Delta) = \frac{V^*-I^*}{V^*-I^*+V^*+V^0-3\Delta}$.

Proposition 15 *For ϕ sufficiently small, if $\tau \geq V^0 - \Delta$ then $R^B(T^0, T^L) < \underline{R}(\tilde{\tau})$*

Proof: $X_S \geq \tau + \Delta \geq V^0$, or $V^0 < (V^* - I^*)\frac{V^*+V^0-2\Delta}{V^*+V^0-2\Delta+V^*-I^*}$. Hence $R^B(T^0, T^L) = \frac{V^0}{V^*+V^0-2\Delta} < \frac{V^*-I^*}{V^*+V^0-2\Delta+V^*-I^*} < \frac{V^*-I^*}{V^*+V^0-2\Delta+V^*-I^*-\Delta} = R^B(\underline{X} - \Delta) < \underline{R}(\tilde{\tau})$. ♣

Proposition 16 *As $\phi \to 0$, $R^B(T^L, T^H) \to 0$.*

Proof: $R^B(T^L, T^H) = \frac{T^L+\Delta}{T^L+V^*-I^*+V^0-\Delta-\phi} = \frac{\delta^L}{V^*-I^*+V^0-2\Delta-\phi+\delta^L}$ with $\delta^L < \phi$. ♣

Proposition 17 *If efficient investment is small, then* $\frac{\tau+\Delta}{V^*-I^*} = F_1$

If efficient investment is large, then $\underline{F_2} < \frac{\tau+\Delta}{V^*-I^*} < \overline{F_1}$

Proof: We know that $\tau + \Delta = X_\tau = \min\{X_S, X_d\}$, hence the Proposition follows from the demonstration that $X_S \leq X_d$ if and only if I^* is small and the observations that: $X_S = (V^* - I^*)F_1$, and $(V^* - I^*)\underline{F_2} < X_d < (V^* - I^*)\overline{F_2}$. The second observation follows because if $\alpha \in (\frac{1}{2}, 1)$ then $V^* > (1-\alpha)I^* + \alpha V^* > \frac{1}{2}(V^* + I^*)$. Both $R_0^S(T)$ and $R^d(T)$ are decreasing in T, and $R_0^S(T) \geq R^d(T)$ if and only if $T \geq V^* - 2I^*$. Therefore, either $X_S \leq X_d \leq V^* - 2I^* + \Delta$ or $X_S > X_d > V^* - 2I^* + \Delta$. Hence, $X_\tau = X_S$ if and only if $X_S = \frac{H(V^*-I^*)}{H+(V^*-I^*)} \leq (V^* - I^*) - (I^* - \Delta)$ or $1 - \frac{I^*-\Delta}{V^*-I^*} \geq \frac{H}{H+V^*-I^*}$ or $\frac{V^*-I^*}{H+V^*-I^*} \geq \frac{I^*-\Delta}{V^*-I^*}$. Which is to say, that $X_\tau = X_S$ if and only if efficient investment is small. ♣

Proof of Theorems 1 and 2

We proceed by using Theorem 4 and constructing an $\mathcal{L}$-tree which we then argue must have lowest $\mathcal{L}$ potential. Let $\mathcal{L}$ consist of the ρ-components for either ρ an efficient convention or $\rho = \rho^M$. From Proposition 2 this set $\mathcal{L}$ satisfies the conditions of Theorem 4. For $x \in \{B, S, i, d\}$, let $\mathcal{T}_x$ consist of the transfers $\{T \in \mathcal{T}(\phi) | -\Delta < T < V^* - I^* - \Delta\}$ such that $\underline{R}(T) = R^x(T)$. We observe the following facts about these regions: there is a T^1, $T^H > T^1 \geq \tau$, such that $\mathcal{T}_d = \{T \in \mathcal{T}(\phi) | T^1 \leq T \leq T^H\}$ and $\mathcal{T}_S = \{T \in \mathcal{T}(\phi) | \tau \leq T \leq T^1\}$. Also there are T^2, T^3 with $T^L < T^2 < T^3 < \min\{\tau, T^0\}$ such that $\mathcal{T}_i \subset \{T \in$

$\mathcal{T}(\phi)|T^2 \leq T \leq T^3\}$, while $\{T \in \mathcal{T}(\phi)|\Delta \leq T \leq \tau\} \setminus \mathcal{T}_i \subseteq \mathcal{T}_B$. First presume that $\tau > T^0$. Let us construct an $\mathcal{L}$ graph as follows: if $T \in \mathcal{T}_i \cup \mathcal{T}_S$, then include $(T, I^*, \Delta) \rightarrow \rho^M$. If $T \in \mathcal{T}_d$, then include $(T, I^*, \Delta) \rightarrow (\tilde{\tau}, I^*, \Delta)$. Include $(T^0, I^*, \Delta) \rightarrow (T^0 - \phi, I^*, \Delta)$ and for $T \in \mathcal{T}_B \setminus \{T^0\}$, include $(T, I^*, \Delta) \rightarrow (T^0, I^*, \Delta)$. If there is convention with multiple $\mathcal{L}$-edges departing from it, then arbitrarily delete all but one. This graph has a lowest resistance transition leaving from every efficient ρ-component. From here, delete the transition leaving $(\tilde{\tau}, I^*, \Delta)$, and replace the transitions leaving (T^0, I^*, Δ), and (T^L, I^*, Δ) with $(T^0, I^*, \Delta) \rightarrow (T^L, I^*, \Delta)$ and $(T^L, I^*, \Delta) \rightarrow (T^H, I^*, \Delta)$. Finally, if $\rho^M \neq (T^L, I^*, \Delta)$, then add $\rho^m \rightarrow (\tilde{\tau}, I^*, \Delta)$. This results in an $(\tilde{\tau}, \mathcal{L})$-tree and By Propositions 15 and 16, this tree has lower $\mathcal{L}$-cost than any $\mathcal{L}$-tree which is not rooted at $(\tilde{\tau}, I^*, \Delta)$. Hence $(\tilde{\tau}, I^*, \Delta)$ has lowest $\mathcal{L}$ potential, and since $R^B((\tilde{\tau}, I^*, \Delta))$ is bounded away from zero, it follows that the $(\tilde{\tau}, I^*, \Delta)$-component is locally stable for sufficiently large N. Hence the $(\tilde{\tau}, I^*, \Delta)$-component is the stochastically stable set. If instead $\tau \leq T^0$, then the transition $(T^0, I^*, \Delta) \rightarrow \rho^M$ is the least resistance transition out of (T^0, I^*, Δ). Here essentially the same tree construction as above will yield a $(\tilde{\tau}, \mathcal{L})$-tree with lower $\mathcal{L}$-cost than any $\mathcal{L}$-tree which is not rooted at $(\tilde{\tau}, I^*, \Delta)$. The difference is that if $\rho^M \neq (T^L, I^*, \Delta)$, then this tree has $(T^0, I^*, \Delta) \rightarrow \rho^M$ instead of $(T^0, I^*, \Delta) \rightarrow (T^L, I^*, \Delta)$. Hence, the same conclusions follow. Theorems 1 and 2 now follow from Proposition 17. ♣

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