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Comparison of Urban Equilibria:
The Conflict Between Efficiency and Equity
in the Choice of the Central City's Majority

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IN THE CHOICE OF THE CENTRAL CITY'S MAJORITY

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ABSTRACT

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JEL Classification: H73, R12, R14.

Key words: urban, equilibria, welfare

Suggested running title: Urban Equilibria.

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ABSTRACT

A circular metropolitan area consists of a central city and a suburbs. Households sort over the two jurisdictions based on public service levels and their costs of commuting to the metropolitan center. Using numerical simulations, we show (1) there typically exist two equilibria: one in which the poor form the voting majority in the central city and the other in which the rich form the majority in the central city; (2) there is an efficiency *v.* equity trade-off as to which equilibrium is preferred; (3) if the central city contains only poor households, equity favors expanding the central city to include rich households; and (4) as a metropolitan population grows, it is likely to select the equilibrium in which poor households form the city's majority.

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1. INTRODUCTION

In Tiebout's (1956) seminal model of fiscal competition, households sort themselves between communities based on their benefits from the public service, and the resulting allocation of households to communities is first-best efficient.[1] In Tiebout's model, the public service is financed by a residency tax and community boundaries are flexible. Hamilton (1975, 1976) shows that, if the residency tax is replaced by a property tax, zoning can allow households to sort across local jurisdictions to achieve first-best efficiency. Alternatively, Elickson (1971), Yinger (1982), Epple, Filimon and Romer (1984, 1993) and Epple and Romer (1991) describe communities as having fixed boundaries and a property tax, but no zoning powers. In such models, the public service and the property tax are capitalized into the land price in a jurisdiction, distorting the location and housing decisions of individual households.[2] In general, the equilibrium outcomes in such economies are not first-best efficient.

In contrast to Tiebout's model, Mills (1967) and Muth (1969) present a spatial model of the monocentric city: income sorting occurs from the interaction of commuting costs and land demand. If land demand is sufficiently income elastic, the saving achieved by the purchase of land further from the city's center is greater for the rich households and compensates them for the associated increase in commuting cost.[3] In this case poor households win the bid for land near the city's center and vote for low levels of public services. Mills (1967), Muth (1969), and Mills and Lubuele (1997) suggest that this sorting mechanism is a key factor in the concentration of the poor into U.S. central cities and in the resulting low level of public-service quality in those cities. Conversely, if commuting costs increase with income and if land demand is unresponsive to income changes, rich households outbid poor households for locations closer to the city's center. Wheaton (1977) and Glaeser, Kahn, and Rappaport (2000) find evidence that the income elasticity of land demand is quite small and as a result conclude that this type of sorting cannot play a major role in explaining the centralization of the poor.

In general, households choose communities based on both public services and commuting costs. We consider a stylized model in which the central city has an exogenous boundary and is surrounded by the suburbs. All households must commute to the central business district which is located at the center of the city. The model has two income-classes. Rich households have higher commuting costs per mile than poor households and land demand is relatively income inelastic; *ceteris paribus* rich households outbid poor households for land near the city center. The public service in each community is determined by voting. We find multiple equilibria in which each income class controls one jurisdiction. In one equilibrium, poor households are the majority in the central city, voting low public services in the city; in the second equilibrium, rich households are the majority in the central city, voting high public services there.

Which equilibrium is preferred? If the city boundary is free to adjust until each jurisdiction contains only one income class, efficiency is clearly higher in the equilibrium in which rich households reside in the city: there is perfect matching of households with their desired public service and commuting costs are minimized. From an equity standpoint, however, poor households obtain the most utility if they form the majority in the city and if the city is large enough to have vacant land at its jurisdictional boundary - this arrangement decreases competition for urban land and thereby lowers the rent that the poor must pay for living in the city.

If the city boundary is fixed, the form of the efficient equilibrium depends on the city size, and the efficiency-equity trade-off continues to be present. For example, if the size of the cental city is small so that the city contains one income class and the suburbs contains both income classes, efficiency is higher in the equilibrium in which the city contains only poor households and the suburbs contains poor and rich households, with rich households forming the suburban majority. This suggests that getting a good match of rich households with high public service levels is more important than trying to reduce the resources spent on commuting. In contrast, equity considerations favor the equilibrium in which the city contains only rich households, because this arrangement enables the poor households to get their preferred public service at a lower rent.

The jurisdictional boundary between the central city and the suburbs is neither chosen by the market nor is it fixed; it is a variable which is potentially chosen by the policy-maker. For example, in 1993 Memphis (USA) proposed merging with its near suburbs and a similar exercise was completed by Toronto (Canada) in 1998. One motivation for such a change is strategic: by merging with its suburbs a central city is able to enlarge its commercial tax base and reduce tax competition. We consider another, perhaps more fundamental, feature of boundary changes: the change in the sorting of households by income between the city and suburbs induces a change in rents. Our simulations suggest that, if the status-quo is the equilibrium in which poor households congregate in the central city, expanding the city so that it contains some rich households benefits poor households. In our model, taxes are residency taxes so that this benefit to poor households does not arise because rich households pay more taxes; poor households benefit because their rents fall.

Which of the two possible equilibria - poor households or rich households forming the central city's majority - is more likely? We consider the comparative statics of an increase in the metropolitan population in the presence of a fixed city boundary. When the population is small,

all households live in the central city; there is a majority of poor households and the public service is low. As the population increases, city rents increase and the edge of urban development moves towards the city boundary. While there is still some undeveloped land in the city, some rich households "jump to the suburbs" to form a new community with a high public service. This suggests that the equilibrium in which poor households congregate in the city is likely to be the observed equilibrium. As the population grows, this equilibrium shifts from being inefficient but equitable, to being efficient but inequitable. As the population further expands, high rents plus commuting costs induce rich households to start to move back to the city, and the city changes again to have a majority of rich households.

This paper extends the model of de Bartolome and Ross (2000). In de Bartolome and Ross, all households demand the same lot size, and the emphasis is on how capitalization allows an equilibrium to exist in which both communities contain both income levels. In this paper we compare the welfare properties of equilibria which are possible in a spatial model in which fiscal competition and commuting costs are present. Because comparisons are made between different equilibria, it is difficult to do calculus-based comparisons. We therefore use a computable general equilibrium model. We also use a very simple utility function so that the intuition is highlighted.

The paper is structured as follows: Section 2 presents the theoretical model and the possible equilibria. Section 3, 4 and 5 present the simulations. Section 6 concludes.

2. THE MODEL

A household h lives in a community and obtains utility U from consuming a privately-provided good c^h and from a public service g provided by the community: $U(c^h, g)$. The privately-provided good is the numeraire good. The household's demand for lot size a^h is assumed to be exogenous[4] and the non-land components of housing are included as part of the private good: therefore housing *per se* does not enter the utility function.

Our interest is in the welfare comparisons of different equilibria. This comparison is made particularly simple if we restrict attention to a utility function of consumer surplus form:

$$U = c^h + \beta^h V(g).$$

Because we want the public service to appear normal or to be more valued by households of higher income, we make β^h to be a function of endowed income M^h :

$$\beta^h \equiv \beta(M^h) \;;\quad \beta'(M^h) > 0 \;.$$

In this description, households differ in the tastes for the public service, and their tastes vary systematically with endowed income.

All households commute to the central business district which is located at the city's center and, for analytical convenience, is assumed to have no area. Household h has a fixed time endowment which he can use either for working or for commuting. If he lives at the city center, he spends no time commuting and his income is M^h . If he lives at distance d from the metropolitan center, the price of a unit of land is $r(d)$ and his income is reduced by the opportunity cost of the commute. The time spent commuting is proportional to d and the opportunity cost of a unit of his time is proportional to M^h , so that his commuting cost is tM^hd .

The community provides the public service g.[5] The public service shows constant returns to community size, and the cost of providing a unit of the public service to each resident is one unit of numeraire per resident; the public service is financed by a residency tax g. Therefore the consumption of the private good by the household if he locates distance d from the city center is

$$c^h = M^h - a^h r(d) - tM^h d - g.$$

The utility of the household if he locates at distance d from the metropolitan center in a community providing public service g is

$$M^h - a^h r(d) - tM^h d - g + \beta(M) V(g).$$

The marginal change in utility associated with a change in d is obtained by differentiating as:

$$\frac{d}{dd}\left[M^h - a^h r(d) - tM^h d - g + \beta(M^h) V(g) \right] = -a^h \frac{dr}{dd} - tM^h.$$

$-a^h\, dr/dd - tM^h$ is the benefit gained by the household - the saving on land expenditure less the increase in the commuting cost - if he moves marginally further out. The household chooses his location so that the marginal benefit is zero, or

$$-\frac{dr}{dd} = \frac{tM^h}{a^h}.$$

Provided some household is residing at distance d from the metropolitan center, $dr/dd < 0$ and the commuting disadvantage of locating further from the metropolitan center is capitalized in the fall in the land rent.

There are two income levels. Poor households have income M_1 and rich households have income M_2: $M_1 < M_2$. Associated with each income level, M_1 and M_2, is a lot size a_1 and a_2 respectively: as noted earlier, a_1 and a_2 are exogenous and $a_1 < a_2$. We assume

$$\frac{tM_2}{a_2} > \frac{tM_1}{a_1} . \tag{1}$$

If both households live in a community, when rich households are at their preferred location,

$$-\frac{dr}{dd} - \frac{tM_2}{a_2} = 0$$

and , using Inequality (1), $- a_1\, dr/dd - tM_1 > 0$, or poor households benefit by moving further out.[6] Conversely, if poor households are at their preferred location,

$$-\frac{dr}{dd} - \frac{tM_1}{a_1} = 0$$

and, using Inequality (1), $- a_2\, dr/dd - tM_2 < 0$, or rich households benefit by moving further in. Hence, there is income sorting within the jurisdiction, with the rich households living on the "inside" (closer to the metropolitan center) and poor households living on the "outside" (further from the metropolitan center).[7]

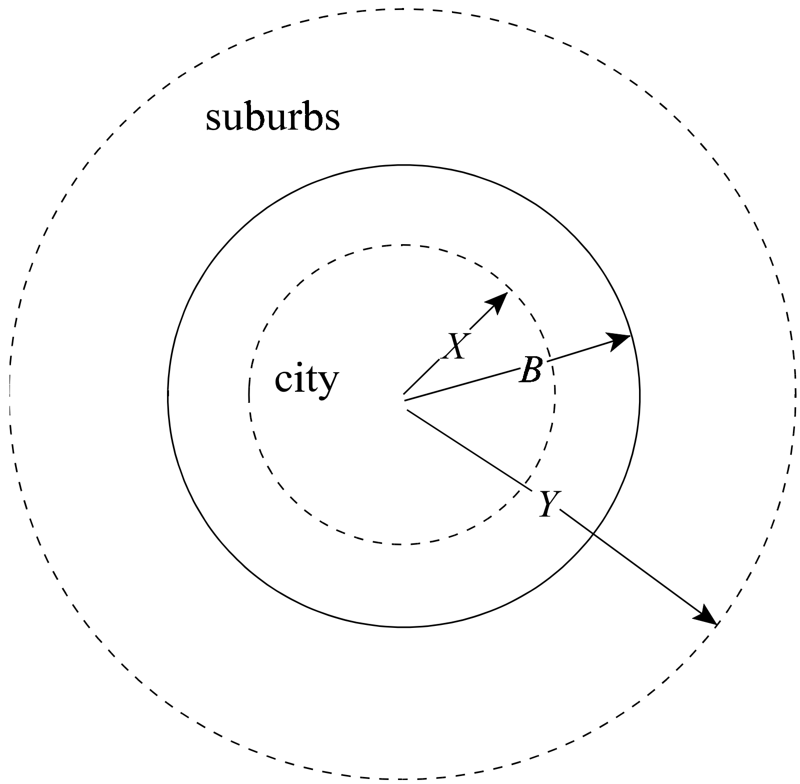


Figure 1: the metropolitan area

The metropolitan area is comprised of a circular city surrounded by the suburbs. The jurisdictional boundary between the city and the suburbs has radius B . There is the possibility of undeveloped land at the fringe of the city if either poor or rich households are unwilling to pay more than the reservation rent to live in the city. The limit of development in the city has radius X . If

$X < B$:	there is undeveloped land at the fringe of the city;
$X = B$:	there is no undeveloped land in the city.

Remembering that rich households live on the inside of the city and poor households live on the outside, denote the boundary between rich and poor households in the city as occurring at distance x from the center. If

$x = 0$:	only poor households live in the city;
$0 < x < X$:	rich and poor households live in the city;
$x = X$:	only rich households live in the city.

In the suburbs, the limit of development is distance Y from the city center and we assume that the outer jurisdictional boundary is sufficiently distant from the metropolitan center that all households live in the city or in the suburbs. The boundary between the rich and poor households in the suburbs occurs at distance y from the center. If

$y = B$:	only poor households live in the suburbs;
$B < y < Y$:	rich and poor households live in the suburbs;
$y = Y$:	only rich households live in the suburbs.

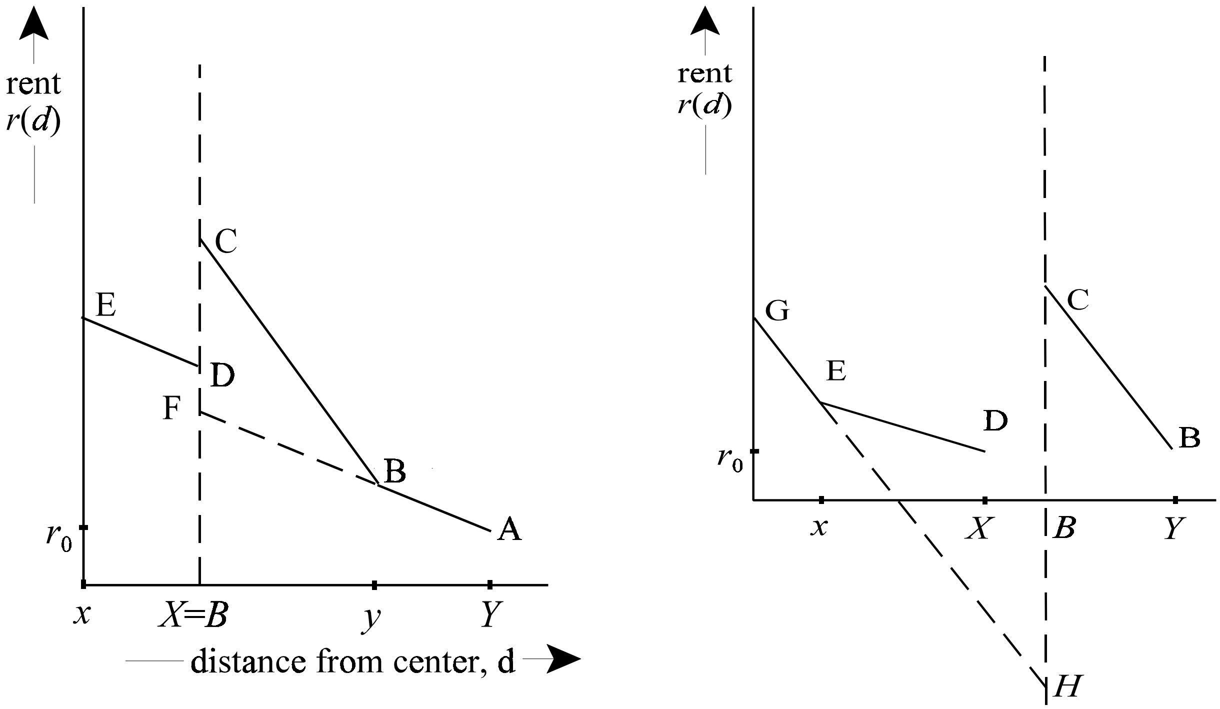
There are N households of which a fraction 2 are poor. Equating land demand and land supply requires:

$$\pi X^2 + \pi (Y^2 - B^2) = \theta N a_1 + (1 - \theta) N a_2 \; ; \tag{2}$$

$$\pi x^2 + \pi (y^2 - B^2) = (1 - \theta) N a_2 . \tag{3}$$

The model is now summarized descriptively; the formal algebraic formulation is presented in the Appendix.

1. Rent continuity: rent is continuous in a jurisdiction. If it were discontinuous, a household living on the side of relatively high rent could increase his utility by moving across the discontinuity to the side of low rent: his rent would change discontinuously but his commuting cost would change only marginally.[8]
2. No migration: no household can achieve higher utility by moving to another location. This implies that, if an income class resides in both communities, the rents are such that a household in that income class is indifferent between the communities. If an income class does not reside in a community, rents are such that a household in that income class cannot increase his utility by moving into the community.
3. Reservation land price: the reservation price of land is r_0. If a community contains no undeveloped land, the rent at the limit of development is at least r_0. If a community contains undeveloped land, the rent at the limit of development is r_0.
4. Determination of the public service level. The public service level in each community is determined by majority voting; households vote myopically, taking the rent schedules as given.
5. Model closure. We assume that rent is paid to absentee landlords.[9]
6. The population in each community is considered to be a continuous variable.



(a) (b)

Figure 2: illustrative rent schedules

In Figure 2 we illustrate the model with two equilibrium rent profiles based on the central city having a majority of poor households and the suburbs having a majority of rich households. In Figure 2(a), the city contains only poor households; it is sufficiently small that some poor households reside in the outer suburban areas. The rent at the limit of suburban development is r_0: this rent anchors the rent schedules. As the location moves inwards from Y, the commuting advantage to poor households is capitalized in the rent and the rent rises at rate tM_1/a_1 . At distance y from the metropolitan center, residents become rich and the rent gradient rises to tM_2/a_2 . *ABF* is interpreted as the bid-rent curve of a poor household in the suburbs, and *BC* is interpreted as the bid-rent curve of a rich household in the suburbs.

As the location moves across the jurisdictional boundary, the public service changes from the level desired by the rich households to the level desired by poor households: poor suburban households are willing to pay the premium *FD* to move across the jurisdictional boundary. This premium ensures that the rent exceeds the reservation rent r_0 and there is no undeveloped land. The rent schedule in the city is *DE* and along *DE* the rent rises at rate tM_1/a_1. Rich households do not live in the city because the rent exceeds their willingness to pay for the lower public service.

In Figure 2(b), the city boundary has expanded to include all poor households and some rich households. There are no poor households in the suburbs. The rent at the suburban fringe is r_0 and rises at rate tM_2/a_2 along *BC* as the location moves towards the city center. As the location crosses the jurisdictional boundary, the rent would have to fall by *CH* (and become negative in this example) if a rich household were to be willing to live on the city side of the boundary. *HEG* is the bid-rent curve of a rich household in the city.

There is undeveloped land in the city and the poor households at the limit of development pay rent r_0 ; rent rises at rate tM_1/a_1 along *DE*. At distance *x* from the city center, rent equals the willingness to pay of a rich suburban household to live at *x*, and rich households live at the locations closer to the city center. The number of rich households, and the level of the rent schedule *HEG* , is determined endogenously. If the willingness of a rich household to pay for the lower public service were to fall, rich households would migrate to the suburbs, shifting the suburban rent schedule to the right and shifting the city rent schedule to the left. The distance *CH* would increase until it equaled the willingness of the rich household to pay for the city public service. More undeveloped land would open up.

There are many possible equilibrium configurations corresponding to which income class forms the majority in the city and whether the city includes one or both income classes, which income class forms the majority in the suburbs and whether the suburbs includes one or both income classes, and whether there is undeveloped land in the city.[10] In our simulations, we found that the equilibrium configurations change as the jurisdictional boundary changes or as the metropolitan population changes; at each given value of the jurisdictional boundary, we found no more than two equilibria. These are described in the next sections.

3. SIMULATIONS

3.1 Analytical Framework

The utility function of household *h* is specified as

$$c^{h} + sign(\rho)\ A\ (M^{h})^{\delta}\ g^{\rho}$$

where A , * and D are parameters, * > *0,* $|\rho| < 1$ and $\rho \neq 0$.[11] The household's demand for *g* is

$$g = (|\rho| A)^{\frac{1}{1-\rho}}\ (M^{h})^{\frac{\delta}{1-\rho}}.$$

We represent the income distribution by assuming that there are equal numbers of poor and rich households, and choose the income of households in each group to be (close to) the median income of the bottom- and top- half of the U.S. income distribution in 1990. Table 1 shows the assumed values of the model parameters and , for comparison, the observed values for U.S. metropolitan areas.

Parameter	Variable Name	Model Value	Societal Value
Poor households as fraction of population	2	0.5	0.5
Income of poor household ($ per year) [a]	M_1	15,000	16,523
Income of rich household ($ per year) [a]	M_2	45,000	46,725
Metropolitan population (households) [b]	N	270,000	266,389
Average lot size (acres) [c]		0.3333	0.3402
Lot size of poor household (acres) [d]	a_1	0.2833	
Lot size of rich household (acres) [d]	a_2	0.3833	
Commute time per mile as fraction of work day [e]	t	0.0125	
Public service demand parameter [d]	*	1.4	
Public service demand parameter [d]	D	-1.0	
Public service demand parameter [d]	A	3.0	
Reservation rent [d]	r_0	0	

a. Societal figures are total 1990 money income at 25th and 75th percentile in metropolitan income distribution (*Money Income of Households, Families and Persons in the United States 1990*, Table 2) multiplied by *(1-J)* where *J* is average federal income tax rate for federal tax return with adjusted gross income equal to money income of respective household (*Individual Income Tax Returns* 1990). Model value represents earned income of a household with zero commuting costs.

b. Societal metropolitan population calculated as 1991 number of owner-occupied units in all central cities plus suburbs (*American Housing Survey for the United States 1991*, Tables 8-3 and 9-3), divided by number of metropolitan areas (*Statistical Abstract of the United States 1998*, Table 40)

c. Metropolitan land area calculated as 1991 occupied housing units in all central cities multiplied by median lot size in central city plus same 1991 figure for suburban units (*American Housing Survey for the United States* 1991, Tables 8-3 and 9-3), divided by the number of metropolitan areas *(Statistical Abstract of the United States 1998, Table 40)*. Societal average lot size is metropolitan land area divided by societal metropolitan population, see note b.

d. See text

e. Fraction of 8-hour workday spent commuting to metropolitan center if household lives one mile from city center. Figure is based on an average travel speed of 20 miles per hour and a round trip commute.

Table 1: Parameter and Population Values

The population and average lot size are chosen to be close to the values observed in the population. The lot size for poor and rich households is adjusted down and up from the average lot size, respectively, in order to be consistent with a 0.3 income elasticity of demand for land - an elasticity value which is consistent with the recent estimates by Glaeser, Kahn, and Rappaport (1999).[12] The population and housing demand parameters imply that the area of developed land in the metropolitan area is 90,000 acres: this translates to a radius (*Y*) of 6.69 miles if the city has no vacant land.

The values for *D* and * are chosen so that the price and income elasticities of demand for the public service are -0.5 and 0.7, respectively, which is consistent with current estimates in the literature (see Ross and Yinger (1999)). These parameter values (and *A=3)* imply that poor households vote a public service level of 1452 ($ per year) and rich households vote a public service level of 3132 ($ per year). The "average" calculated value for the share of income spent on public service is therefore (1452+3132)/(1500 + 4500) = .076 [13]; in contrast, the societal value is 0.13.[14] When we raised the value of the parameter *A* so that the simulated value lay closer to the observed value, there was little effect on the results except that we ceased to find an equilibrium configuration in which both poor and rich households live in the city and in which all city land is developed.[15] In order to illustrate an equilibrium with this form, we decided to use the lower value of *A*.

Our focus is on comparing equilibria and what is important is the relative value of rents. Therefore, for convenience, we set the reservation rent to zero.

The simulations of this section investigate how the equilibrium outcomes change as the city's jurisdictional boundary, *B*, is increased from 0.5 miles to 6.5 miles in increments of 1 mile. For each value of *B*, we found two equilibria: an equilibrium in which poor households are the majority in the city and rich households are the majority in the suburbs *and* an equilibrium in which the majorities are reversed. The former configurations are grouped together as Case 1, the latter configurations are grouped together as Case 2.[16]

3.2 Case 1: Poor are majority in city, rich are majority in suburbs

Table 2 presents the simulation results for the equilibrium in which poor households are the city's majority and rich households are the suburban majority. This might be considered the common U.S. outcome, with high levels of public services in the suburbs. The first row shows the city's jurisdictional boundary (assigned exogenously). The second row shows the equilibrium case (using labels which are to be discussed) which arises for the given boundary size. The next row shows the average rent;[17] the next two rows show the utility achieved by poor and rich households. The next four rows show some characteristics of the metropolitan area; the number of poor and rich households living in the city is shown in Table 4. The final two rows show the utility change a poor and rich household would experience if he were to move from the community in which he resides to the other community.

Metropolitan population, $N = 270{,}000$

Attributes of metropolitan area	Symbol	Equilibrium values						
City jurisdictional boundary (miles from center)	B	0.5	1.5	2.5	3.5	4.5	5.5	6.5
Equilibrium Case Number (label from text)		1.1	1.1	1.1	1.1	1.2	1.3	1.3
Average rent per household ($ per year)		756	783	852	969	558	398	413
Utility of poor households ($ per year)		9,941	9,941	9,941	9,941	11,006	11,158	11,088
Utility of rich households ($ per year)		35,465	35,405	35,293	35,136	34,973	34,761	34,451
Boundary between income groups in city (miles)	x	0	0	0	0	1.11	2.46	3.15
Boundary of city development (miles)	X	0.5	1.5	2.5	3.5	4.5	5.01	5.38
Boundary between income groups in suburbs (miles)	y	5.10	5.29	5.66	6.16	6.69	7.07	7.62
Boundary of suburban development (miles)	Y	6.69	6.69	6.69	6.69	6.69	7.07	7.62
Poor household's utility change on moving from city to suburbs ($/yr)		0	0	0	0	-1,065	-1,288	-1,321
Rich household's utility change on moving from suburbs to city ($/yr)		-1,591	-1,531	-1,419	-1,262	0	0	0

Table 2: Simulation results for Case 1

As the city's jurisdictional boundary moves outwards, the structure of the equilibrium takes on three forms. The rent profile for each equilibrium structure is sketched in Figure 3. In a region occupied by poor (rich) households, the rent rises at rate tM_1/a_1 (tM_2/a_2) as the location moves closer to the metropolitan center. Therefore, in interpreting this and later figures, we can use the slope of the rent schedule to infer the income of a household living at a location: poor (rich) households live in the region where the rent schedule is relatively flat (steep).

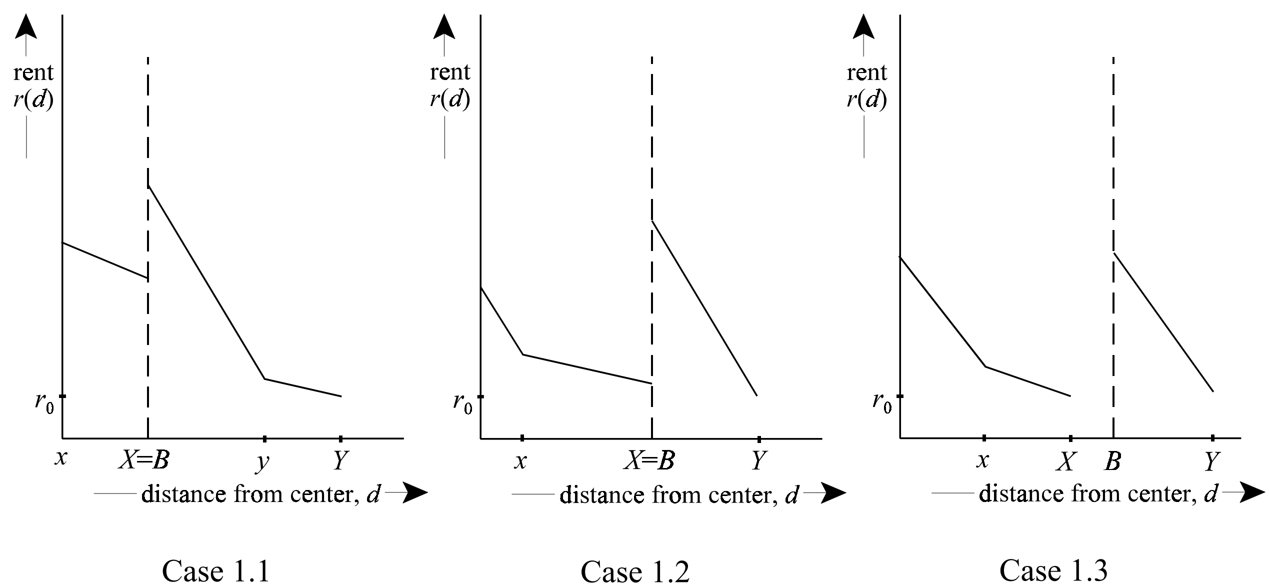


Figure 3: rent schedules for Case 1.

For small and medium sized central cities (B less than or equal to 3.5 miles), equilibrium has the form of Case 1.1: only poor households reside in the central city; all rich households reside in the "inner suburbs"; some poor households reside in the "outer suburbs"; there is no vacant land in the city. As the city radius increases from 0.5 to 3.5 (miles), poor households are attracted by the lower public service in the city and move from the suburbs into the city: poor households are willing to pay a premium for this move and average rent increases accordingly.

The rent paid at the suburban fringe, r_0, anchors the utility achieved by poor households, and their utility does not change as the boundary expands. The efficiency gain (from better matching of poor households with their desired public service) is captured as rent by landlords. Because rich households are being pushed away from the center and have high commuting costs, their utility falls.

As the jurisdictional boundary moves outwards from 3.5 to 4.5 (miles), the equilibrium structure changes to the form of Case 1.2. The city has grown sufficiently large that it can contain all poor households and some rich households; the remaining rich households live in the suburbs and there is no undeveloped city land. The level of the rent schedule in the city is now determined by the rent required to make a rich household indifferent between the communities. Rich households in the city need to be compensated for the lower public service, and average rents fall accordingly. The utility of poor households is no longer determined by what happens at the suburban edge; the lower city rent raises their utility. The utility of rich households is determined by the utility they obtain in the suburbs: their rent plus commuting costs has increased and their utility falls.

When B moves from 4.5 to 5.5 (miles), vacant land appears near the city boundary, and the equilibrium moves to the form of Case 1.3. The city contains all poor and some rich households, the suburbs contains only rich households and there is undeveloped city land. As the city's jurisdictional boundary expands and rich households in-migrate into the city, the boundary between the rich and poor households in the city, x, moves out so that rich households moving to x have to incur higher commuting costs. This lowers the rent at x. The rent paid by poor households falls faster than their commuting costs increase. The overall effect is a decrease in average rent and an increase in utility of poor households. The utility of rich households continues to be determined by what happens at the suburban edge: their commuting costs are increasing and their utility falls.

Finally, when *B* increases from 5.5 to 6.5 (miles), more rich households migrate into the city to avoid the longer commute; this pushes up the rent in the city at the locations occupied by rich households. The rent of poor households is anchored by the reservation price of land.. Poor households are being pushed further out and, with the rent at the edge of development fixed, their utility falls. Rich households in the suburbs continue to incur higher commuting costs and the utility of rich households, determined by what happens in the suburbs, continues to decrease.

3.3 Case 2: Rich are majority in city; poor are majority in suburbs

Table 3 presents the simulation results for the equilibrium in which rich households are the city's majority and poor households are the suburban majority. When the jurisdictional boundary *B* is set to values between 0.5 and 4.5 (miles), Case 2.1 arises: the city contains only rich households but it is so small that it cannot contain all the rich households so that some members of this group reside in the suburbs with the poor households; there is no undeveloped city land.

Metropolitan Population, *N = 270,000*

Attributes of metropolitan area	Symbol	Equilibrium values						
City jurisdictional boundary (miles from center)	*B*	0.5	1.5	2.5	3.5	4.5	5.5	6.5
Equilibrium Case Number (label from text)		2.1	2.1	2.1	2.1	2.1	2.2	2.2
Average rent per household ($ per year)		762	838	989	1,216	1,518	544	536
Utility of poor households ($ per year)		10,843	10,843	10,843	10,843	10,843	10,781	10,629
Utility of rich households ($ per year)		33,527	33,527	33,527	33,527	33,527	35,883	35,883
Boundary between income groups in city (miles)	*x*	0.5	1.5	2.5	3.5	4.5	5.07	5.07
Boundary of city development (miles)	*X*	0.5	1.5	2.5	3.5	4.5	5.07	5.07
Boundary between income groups in suburbs (miles)	*y*	5.07	5.07	5.07	5.07	5.07	5.5	6.5
Boundary of suburban development (miles)	*Y*	6.69	6.69	6.69	6.69	6.69	7.02	7.83
Poor household's utility change on moving from suburbs to city ($/yr)		-3,383	-3,155	-2,927	-2,698	-2,470	-537	-385
Rich household's utility change on moving from city to suburbs ($/yr)		0	0	0	0	0	-2,571	-3,085

Table 3: Simulation results for Case 2

At B = *5.5* and *6.5* (miles), the equilibrium structure changes to Case 2.2: all rich households live in the city, all poor households live in the suburbs, and vacant land appears in the city. The two communities are income-segregated. The rent profiles for the two equilibrium structures are sketched in Figure 4.

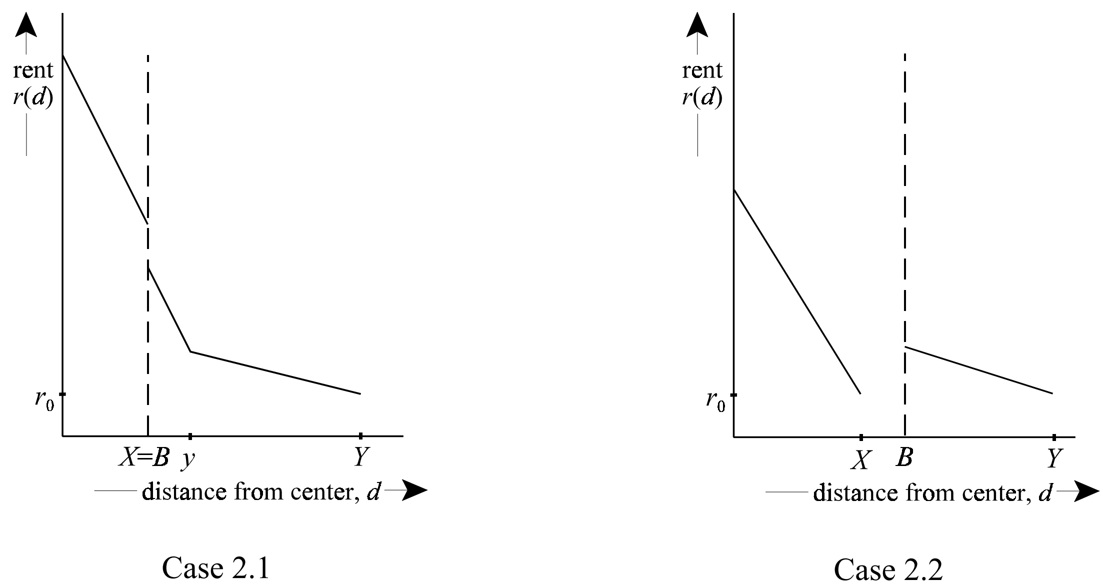


Figure 4: rent schedules for Case 2.

As the city's boundary moves out from 0.5 to 4.5 (miles), rich households in the suburbs move into the city: because rich households control the public service level in the city, rich households are willing to pay a premium to move to the city and rents on the city side of the jurisdictional boundary exceed rents on the suburban side. Average rents therefore increase. However, utilities do not change: the boundary between the rich and poor households in the suburbs does not move, so the utilities of both rich and poor households stay anchored at their suburban levels. The efficiency gain of better matching of the rich households with their desired public service is entirely captured as rent by landlords.

When B is set to 5.5 and 6.5 miles, the equilibrium structure is Case 2.2. There are no longer rich households in the suburbs bidding to get into the city, and rents in the city fall: the rent decrease increases the utility achieved by rich households. Of course, as B increases in this range, poor households commute further and their utility decreases.

4. WELFARE COMPARISONS

4.1 Efficiency comparisons

Because we have specified an utility function with the property that the marginal utility of income is unity, the "as if" shifting of income from absentee landlords to resident households does not change the total surplus. Therefore our measure of efficiency is the sum of the average utility of resident households plus the average land rent.

The first-best efficient outcome is to separate the rich and poor households into different communities, so that there is perfect sorting of households by their taste for the public service (Tiebout sorting). Because commuting costs increase with income, rich households should be placed in the city and poor households should be placed in the suburbs. The city boundary should be set at 5.1 (miles) and there should be marginal undeveloped city land. The efficiency measure (average utility plus average land rent) for this equilibria is 23,915 ($ per household per year).

If the jurisdictional boundary is not 5.1 miles, should the city have a majority of rich households or a majority of poor households? If the city radius is small, the income class which lives in the city is also present in the suburbs where it obtains a non-optimal public service level. In contrast, all households of the income class which forms the majority in the suburbs live in the suburbs and obtain their preferred public service level. Relative efficiency involves a trade-off as to which income-class should be perfectly matched with its preferred public service, and considerations of commuting cost. We denote the public service voted by poor (rich) households as g_1 (g_2). Our specific utility function implies that matching is more important for rich households: the benefit to a rich household of being matched with g_2 instead of with g_1 exceeds the benefit to a poor household of being matched with g_1 instead of with g_2 ; or

$$[\beta(M_2)V(g_2) - g_2] - [\beta(M_2)V(g_1) - g_1] > [\beta(M_1)V(g_1) - g_1] - [\beta(M_1)V(g_2) - g_2] .$$

In Case 1, when the city radius is small (4.4 miles or less), some poor households live in the city and enjoy their desired public service and some poor households live in the suburbs, obtaining a higher-than-desired public service level; in contrast, all rich households live in the suburbs and enjoy their desired public service level. There is therefore a perfect match of rich households with their desired public service (and a bad match of some poor households with their desired public service). Conversely, in Case 2, some rich households live in the city, but some rich households and all poor households live in the suburbs; matching with desired public services is therefore bad for rich households and perfect for poor households. Summarizing, for small cities, Case 1 has a better match of rich households to their desired public service; it also has higher commuting costs.

Metropolitan population, $N = 270{,}000$

Attributes of metropolitan area	Equilibrium values						
City jurisdictional boundary, *B* (miles from center)	0.5	1.5	2.5	3.5	4.5	5.5	6.5
CASE 1 (poor are majority in city):							
Efficiency measure (Average utility plus average rent) ($ per household per year)	23,459	23,456	23,469	23,507	23,548	23,357	23,182
City population of poor households	1,774	15,969	44,357	86,940	135,000	135,000	135,000
City population of rich households	0	0	0	0	6,443	31,726	52,207
CASE 2: (rich are majority in city)							
Efficiency Measure (Average utility plus average rent) ($ per household per year)	22,947	23,022	23,174	23,401	23,703	23,876	23,791
City population of poor households	0	0	0	0	0	0	0
City population of rich households	1,311	11,803	32,785	64,258	106,222	135,000	135,000

Table 4: Efficiency comparison of Case 1 and Case 2 equilibria

Table 4 presents the efficiency measures for the equilibria achieved under Cases 1 and 2, as well as the number of poor and rich households living in the city at each boundary size. If the city radius is small (3.5 miles or less), efficiency is higher when the poor are the majority in the city. If the city boundary is 0.5 miles, the efficiency difference is 512 ($ per households per year). We conclude that, for small cities, good matching of rich households with their desired public service is more important than considerations of commuting cost.

As the jurisdictional boundary expands, the number of households living in the city increases, fewer suburban households are being mismatched with their desired public service, and the importance of matching decreases. However, for Case 1, commuting costs increase as all (or almost all) rich households are being forced to live further out. In consequence, the welfare difference between the two cases falls. When the jurisdictional boundary is 4.5 miles, in the Case 2 equilibrium 80% of rich households are able to live in the city and obtain their desired public service; commuting cost considerations dominate and efficiency is higher under Case 2.

For jurisdictional boundaries of 5.5 and 6.5 miles, the city is able to contain all the households of an income class, and matching is not an issue. Minimization of commuting costs dominates, and higher efficiency is obtained when rich households form the majority in the city (Case 2). This is true even if there is undeveloped land.

To reinforce the importance of good sorting of rich households into the community providing their desired public service, we compare Case 1 outcomes with city radii of 3.5 and 6.5 miles respectively. The efficiency measure drops by 325 ($ per household per year). In the former case, 48,060 poor households live in the suburbs and get more public service than desired; in the latter case, approximately the same number of rich households (52,207) live in the city getting less public service than desired. Further analysis shows that approximately 2/3 of the drop in efficiency is associated with the perfect matching with public services changing from being with rich households to being with poor households, and approximately 1/3 of the drop is associated with the higher commuting costs.[18] [19]

4.2 Equity comparisons

Our measure of equity is the Rawlsian welfare function, *max min* $[U_1, U_2]$ where U_i is the utility achieved by a household with income M_i. Comparison of Tables 2 and 3 shows that the best outcome for poor households is for poor households to be the majority in the city (Case 2) and for the city boundary to be 5.5 miles. The utility of a poor household in this equilibria is 11,158 ($ per year) while the utility of a poor household in the first-best efficient outcome is only 10,843 ($ per year).

If the city radius is small (3.5 miles or less), Tables 2 and 3 show that poor households achieve utility 9,941 ($ per year) in the Case 1 equilibrium and 10,843 ($ per year) in the Case 2 equilibrium. Equity therefore places rich households in the city and makes poor households the majority in the suburbs. An important difference between the efficiency and equity analyses concerns the treatment of rent. Efficiency is concerned with total surplus: any gain which accrues to landlords is included in the analysis and rent therefore is considered "as if" returned to households. Equity is concerned with the surplus accruing to poor households: rent paid by poor households is considered lost and not considered "as if" returned. If the city radius is 3.5 miles or less, there are poor households in the suburbs in both equilibria. Poor households at the suburban fringe pay the same rent and have the same commuting costs in both equilibria. Under Case 2, poor households in the suburbs obtain their desired public service. Under Case 1, it is the poor households living in the city who receive their desired public service, but they are prevented from getting the benefit by the competition of poor suburban households; the benefit is capitalized into land rents and paid to absentee landlords. Poor households have higher utility, therefore, in the Case 2 equilibrium.

If the city radius is large (4.5 miles or larger), Case 1 has all poor households living in the city and it is rich households who compete for city land. The low public service in the city makes living in the city relatively unattractive to rich households, creating a "barrier to entry" for rich households. This drives down the rent to benefit poor households. The benefit of good matching of poor households with their desired public service is no longer captured by landlords as rent. Case 1 is preferred for equity.

Our equity finding has a policy implication. Many U.S. metropolitan areas stylistically resemble Case 1.1 in which only poor households reside in the city and in which rich and poor households reside in the suburbs. In this case, expanding the city boundary to include the near suburbs (i.e. to shift the equilibrium from Case 1.1 to Case 1.2 or Case 1.3) is good policy for city residents as it increases their utility. It should be stressed that this utility gain accrues to poor families not because of tax shifting - all city households pay the same tax - but because of rent changes.

4.3 Conflict between efficiency and equity

The discussion in the two previous subsections has shown that efficiency and equity favor different city majorities. Table 5 summarizes the discussion.

Metropolitan population, $N = 270{,}000$

Attributes of metropolitan area	Equilibrium values						
City jurisdictional boundary, *B* (miles from center)	0.5	1.5	2.5	3.5	4.5	5.5	6.5
<u>*CITY MAJORITY PREFERRED FOR*</u>:							
Efficiency	poor	poor	poor	poor	rich	rich	rich
Equity	rich	rich	rich	rich	poor	poor	poor

Table 5: Efficiency *v.* equity conflict

Our simulations show that a city boundary of near five miles is preferred for both equity and efficiency reasons. However, there is conflict as to which income class should form the city's majority. Efficiency favors the equilibrium in which rich households are the city's majority: there is perfect matching of households with their public service levels and commuting costs are minimized. In contrast, equity favors the equilibrium in which poor households are the city's majority: poor households have low commuting costs and obtain their desired public service level, and the presence of a few rich households in the city lowers city rents.

The conflict between efficiency and equity - shown in Table 5 to be present at all the city sizes considered - is driven by the rivalness of city space. For efficiency, what primarily matters is the matching of rich households with their preferred public service; this implies that rich households should only be placed in the city if it is large enough to contain them all (or almost all). Equity is concerned with poor households. For equity, poor households should only be placed in the city if it is large enough to contain them all: otherwise competition from poor households in the suburbs bids up land rents and the benefit of good matching of poor city households with their preferred public service is expropriated by landlords.

5. SUBURBANIZATION IN A GROWING CITY

Current metropolitan areas have grown out of much smaller cities. Does the likely path of development select an equilibrium which is efficient or which is equitable? Our presumption is that increases in the population are accommodated by marginal changes in the boundary between the income groups and not by large population shifts between the communities. Put differently, once an equilibrium configuration of majorities is established, we presume that it is maintained as the population grows (provided the configuration continues to be an equilibrium).

Historically, when metropolitan populations were small compared to the size of the city, all households lived in the city and poor households formed the majority. As the metropolitan population grows, rents increase in the city, the limit of urban development moves outwards, and the poor households who reside at the edge of development are pushed further from the city center. Eventually such growth will lead to the development of suburban communities. One possibility is that poor households are the first households to move into the suburbs. Poor households have no incentive to set up a suburban community while there is still undeveloped city land - if the household at the edge of the city were to move, he would get the same public service, pay the same rent and incur higher commuting costs. Unless rich households "jump over" undeveloped city land to form a new community in the suburbs, the suburbs are not developed until the limit of development moves across the jurisdictional boundary; at this instant poor households who reside at the edge of urban development spill into the suburbs. With poor households leaving the city, rich households become the city majority.[20]

The alternative possibility is that the rich households are the first households to move into the suburbs. By setting up a new community in the suburbs, they obtain their desired public service and face lower rents, but incur higher commuting costs. If they are sufficiently sensitive to the public service level, rich households choose to suburbanize while there is still undeveloped city land[21] corresponding to Case 1.3 in our simulations. In this scenario, rich households establish themselves in the suburbs, leaving poor households to control the city. Given our presumption that the equilibrium configuration of majorities does not change spontaneously (provided it continues to be an equilibrium), rich households control the city only if Case 1.3 does not arise while the metropolitan population is too small to completely fill the city.

To consider these possibilities, we approximate the path of development by the comparative statics of increasing the metropolitan population *N* in the presence of a fixed city boundary *B*=5.0 miles; other parameter values are maintained at the values shown in Table 1.[22] With a boundary at 5.0 miles, the city can contain all households until the population exceeds 150,811. At a very small population (not shown in the table), all households live in the city and the suburbs are uninhabited: the potential cost of commuting from the suburbs deters households from setting up a new community in the suburbs. Table 6 shows the form of the different equilibria as *N* is increased in increments of 100,000 households. At a metropolitan population of 100,000 households, there is an equilibrium of the form of Case 1.3: although the metropolitan population is still too small to fill the city, rich households "jump over" undeveloped city land to form a new community in the suburbs. Our earlier discussion, therefore, suggests that rich households become established in the suburbs, or that the path of development selects the Case 1 equilibrium - poor households form the city's majority.

City jurisdictional boundary: $B = 5.0$ (miles)

	Equilibrium outcomes								
Metropolitan population, N (households)	100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
CASE 1(poor are majority in city)									
Equilibrium Case Number (label from Section 3)	1.3	1.3	1.2	1.1	1.1	1.1	1.1	1.1	
Average rent per household ($ per year)	217	332	498	1,209	1,238	1,284	1,337	1,394	
Utility of poor households ($ per year)	11,523	11,311	11,052	9,669	9,488	9,325	9,176	9,036	
Utility of rich households ($ per year)	35,544	35,152	34,769	34,217	33,794	33,407	33,050	32,707	
Efficiency measure (Average utility plus average rent) ($ per year)	23,750	23,564	23,408	23,151	22,879	22,650	22,448	22,266	
CASE 2 (rich are majority in city)									
Equilibrium Case Number (label from Section 3)	2.2	2.2	2.1	2.1	2.1	2.1	2.1	2.1	2.1
Average rent per household ($ per year)	320	466	1,644	1,554	1,535	1,548	1,577	1,615	1,658
Utility of poor households ($ per year)	11,036	10,925	10,775	10,570	10,390	10,227	10,077	9,938	9,807
Utility of rich households ($ per year)	37,000	36,280	33,351	32,818	32,349	31,925	31,536	31,173	30,832
Efficiency measure (Average utility plus average rent) ($ per year)	24,337	24,068	23,707	23,249	22,904	22,624	22,383	22,170	21,978
CITY MAJORITY PREFERRED FOR:									
Efficiency	rich	rich	rich	rich	rich	poor	poor	poor	
Equity	poor	poor	poor	rich	rich	rich	rich	rich	

Table 6: Equilibria as the metropolitan population increases

Table 6 shows that, at $N = 100,000$ households, two equilibria exist: Case 1.3 (poor households are the city majority), and Case 2.2 (rich households are the city majority). We consider Case 2.2 to be unlikely. Its realization presupposes that the path of development allows rich households to become established as the city's majority, or that poor households migrate to the suburbs. However, as discussed above, the incentives do not favor such a migration - while poor households form the city majority, migrating to the suburbs makes a poor household worse-off.

As the metropolitan population further expands from 100,000 to 800,000 households, the possible equilibria with the poor forming the city's majority moves from Case 1.3 to Case 1.2 to Case 1.1. Average income in the city decreases. Descriptively, a growing metropolitan population is similar to a contracting city boundary.

The last two rows of Table 6 confirm the conflict between efficiency and equity. At metropolitan populations between 100,000 and 300,000, the Case 1 equilibrium is inefficient but equity-preferred. It is inefficient because, in the alternative Case 2 equilibrium, the population is small enough that all (or almost all) rich households can live in the city, so that a city majority of rich households can give good matching of rich households with their preferred public service level, and low commuting costs. The Case 1 equilibrium is equity-preferred because the presence of some rich households in the city keeps down the city rent to benefit the poor households in the city.

At metropolitan populations between 400,000 and 500,000 households, the Case 1 equilibrium is inefficient and inequitable. It is inefficient because the alternative Case 2 equilibrium still gives good matching for over half of the rich households with their preferred public service level, and low commuting costs. It is inequitable because the poor households in the suburbs bid up city rents, so that the gains to poor city households of being matched with their preferred public service are expropriated by landlords.

At metropolitan populations between 600,000 and 800,000 households, a smaller proportion of rich households can be housed in the city: the importance of good matching of rich households with their preferred public service favors them all being placed in the suburbs, and the Case 1 equilibrium becomes efficient. However, it continues to be inequitable due to the suburban poor bidding up the city rent.

At a population of 900,000 households and above, under Case 1 the boundary between rich and poor households in the suburbs would be far from the metropolitan center; rich suburban households would have such high rents and commuting costs that they gain by moving "back" to the city even at the low public service levels chosen by the current city residents. In equilibrium, poor households are outbid in the city and move to the suburbs - and the only equilibrium is Case 2.1. This incentive for rich households to move back to the city is broadly consistent with the gentrification that has been observed in many large U.S. cities over the last couple of decades.

To summarize this section, our simulation suggests that, as the population grows, rich households migrate to form a new community beyond the city's jurisdictional boundary. This suggests that an inner city with a majority of poor households is the likely case for many cities. Whether this arrangement is efficient or equitable continues to depend on the size of the central city relative to the population. At large populations, Case 1 cannot be supported and rich households move back into the city.

6. CONCLUSION

This paper has examined a monocentric urban model in which the metropolitan area is divided into two jurisdictions -a central city and a surrounding suburb - and there are two income classes. There are multiple equilibria: those in which poor households are the city's majority and those in which rich households are the city's majority. We find that there is often a conflict as to which equilibrium is preferred. For small central cities or large metropolitan population sizes, efficiency favors the poor being the city's majority and equity favors the rich being the city's majority. For large central cities or small metropolitan population sizes, the ordering is reversed. This conflict arises from the basic spatial allocation problem and the competition for land. Because of the importance of matching rich households with their preferred public service, efficiency favors an equilibrium in which rich households are the majority in a community that is large enough to contain all rich households although this may involve higher commuting costs. Alternatively, equity favors an outcome in which the poor are a majority in a community that can contain all poor households. Otherwise, poor households that reside in the rich community will bid up the price for land in the poor community, and landlords will capture all benefits from the matching of poor households to their desired level of public services. Although the model is necessarily stylized, we believe it highlights important trade-offs in urban policy and in the growth of metropolitan areas.

APPENDIX: FORMAL PRESENTATION OF THE MODEL

We denote the city as jurisdiction c and the suburbs as jurisdiction s. If households of income M_i $(i = 1,2)$ with land demand a_i locate in jurisdiction j $(j = c, s)$, they achieve the same utility at all points d at which they locate, or

$$M_i - a_i r(d) - tM_i d - g + \beta(M_i)V(g) = constant\,.$$

Hence the land expenditure plus commuting cost paid by a household of income M_i locating at d in the community j is

$$c_{ij} = a_i r(d) + tM_i d$$

where c_{ij} is a constant.[23] Instead of solving for the rent function $r(d)$, it is convenient instead to solve for the constants c_{2c}, c_{1c}, c_{2s} and c_{1s}.

Rent is continuous in a jurisdiction. If both income levels live in the suburbs, rent continuity at y implies

$$B < y < Y: \qquad \frac{c_{2s} - tM_2 y}{a_2} = \frac{c_{1s} - tM_1 y}{a_1}\quad.$$

If $y = B$, only poor households live in the suburbs. If a rich household were to move to the suburbs, he would achieve his highest utility by locating just across the jurisdictional boundary, and would pay rent $(c_{1s} - tM_1B) / a_1$ In this case, we interpret $(c_{2s}-tM_2B) / a_2$ as the rent he would pay if he were to move and therefore set :

$$B = y: \qquad \frac{c_{2s} - tM_2 y}{a_2} = \frac{c_{1s} - tM_1 y}{a_1}.$$

If $y = Y$, only rich households live in the suburbs. If a poor household were to move to the suburbs, he would achieve his highest utility by locating at the fringe of development, or pay rent $(c_{2s} - tM_2 Y)/a_2$. In this case we interpret $(c_{1s} - tM_1 Y)/a_1$ as the rent a poor household would pay if he were to move to the suburbs, and we set:

$$y = Y: \qquad \frac{c_{2s} - tM_2 Y}{a_2} = \frac{c_{1s} - tM_1 Y}{a_1}.$$

Combining these cases, we can write:

$$B \# y \# Y: \qquad \frac{c_{2s} - tM_2 y}{a_2} = \frac{c_{1s} - tM_1 y}{a_1}. \qquad \text{(A.1)}$$

Similarly, rent continuity at x in the city implies

$$0 \# x \# X: \qquad \frac{c_{2c} - tM_2 x}{a_2} = \frac{c_{1c} - tM_1 x}{a_1}, \qquad \text{(A.2)}$$

where, if $x = 0$, c_{2c} / a_2 is interpreted as the rent a rich household would pay if he were to move to the city and, if $x = X$, $(c_{1s} - tM_1X) / a_1$ is interpreted as the rent a poor household would pay if he were to move to the city.

Denoted the public service in the city as g_c and the public service in the suburbs as g_s Equilibrium requires that the rent schedule adjusts so that no household can obtain more utility by moving between jurisdictions. If $x = 0$, there are no rich households in the city or equilibrium requires that a rich suburban household obtains at least as much utility in the suburbs as he could obtain if he were to move to the city, or

$$x = 0: \qquad M_2 - c_{2s} - g_s + \beta(M_2)V(g_s) \geq M_2 - c_{2c} - g_c + \beta(M_2)V(g_c). \qquad \text{(A.3a)}$$

Similarly, if there are rich households in the city and in the suburbs

$$0<x \text{ and } B<y\text{:}\quad M_2 - c_{2s} - g_s + \beta(M_2)V(g_s) = M_2 - c_{2c} - g_c + \beta(M_2)V(g_c); \qquad \text{(A.3b)}$$

and if there are no rich households in the suburbs

$$B = y\text{:}\quad M_2 - c_{2s} - g_s + \beta(M_2)V(g_s) \le M_2 - c_{2c} - g_c + \beta(M_2)V(g_c). \qquad \text{(A.3c)}$$

Similar equations apply for poor households:

$$x = X\text{:}\quad M_1 - c_{1s} - g_s + \beta(M_1)V(g_s) \ge M_1 - c_{1c} - g_c + \beta(M_1)V(g_c); \qquad \text{(A.4a)}$$

$$x<X \text{ and } y<Y\text{:}\quad M_1 - c_{1s} - g_s + \beta(M_1)V(g_s) = M_1 - c_{1c} - g_c + \beta(M_1)V(g_c); \qquad \text{(A.4b)}$$

$$y=Y\text{:}\quad M_1 - c_{1s} - g_s + \beta(M_1)V(g_s) \le M_1 - c_{1c} - g_c + \beta(M_1)V(g_c). \qquad \text{(A.4c)}$$

The rent paid at the limit of development in the suburbs is the reservation rent r_0 . If the suburbs contains poor households,

$$y < Y\text{:}\quad \frac{c_{1s} - tM_1 Y}{a_1} = r_0 ;$$

if the suburbs contains only rich households

$$y = Y\text{:}\quad \frac{c_{2s} - tM_2 y}{a_2} = r_0 .$$

Using Equation (A.1), this is collapsed to the single relationship:

$$y \# Y\text{:}\quad \frac{c_{1s} - tM_1 y}{a_1} = r_0 . \qquad \text{(A.5)}$$

Similarly, if there is undeveloped land in the city, the rent at the limit of development must be the reservation rent r_0. If there is no undeveloped land, the rent at the city's boundary must equal or exceed r_0. If the city contains only rich families

$$x = X < B: \qquad \frac{c_{2c} - tM_2X}{a_2} = r_0 \;;$$

$$x = X = B: \qquad \frac{c_{2c} - tM_2B}{a_2} \geq r_0 \;.$$

If the city contains poor households

$$x < X < B: \qquad \frac{c_{1c} - tM_1X}{a_1} = r_0;$$

$$x < X = B: \qquad \frac{c_{1c} - tM_1B}{a_1} \geq r_0.$$

Using Equation (A.2), these equations are collapsed to

$$X < B: \qquad \frac{c_{1c} - tM_1X}{a_1} = r_0; \tag{A.6a}$$

$$X = B: \qquad \frac{c_{1c} - tM_1B}{a_1} \geq r_0. \tag{A.6b}$$

The public service in each community is determined by voting. We assume that households vote myopically by taking the rent schedules as given. If the suburbs has a majority of poor households, voting sets the public service in the suburbs g_s to maximize their utility or

$$\frac{\pi(Y^2 - y^2)}{a_1} \geq \frac{\pi(y^2 - B^2)}{a_2}: \qquad \beta(M_1)V'(g_s) = 1. \tag{A.7a}$$

Conversely, if the suburbs has a majority of rich households, voting sets the public service level in the suburbs as

$$\frac{\pi\,(Y^2 - y^2)}{a_1} < \frac{\pi\,(y^2 - B^2)}{a_2}: \qquad \beta(M_2)\,V'(g_s) = 1; \tag{A.7b}$$

Similarly in the city the public service level g_c is set by the majority as

$$\frac{\pi\,(X^2 - x^2)}{a_1} \geq \frac{\pi\,x^2}{a_2}: \qquad \beta(M_1)\,V'(g_c) = 1. \tag{A.8a}$$

$$\frac{\pi\,(X^2 - x^2)}{a_1} < \frac{\pi\,x^2}{a_2}: \qquad \beta(M_2)\,V'(g_c) = 1. \tag{A.8b}$$

Summarizing, the variables to be solved for are: c_{2c}, c_{1c}, c_{2s}, c_{1s}, g_c, g_s, x, X, y, and Y. The equation system corresponds to the Equations (2) - (3) and (A.1) - (A.8). The equilibrium outcome is a function of the city boundary B and the other parameters.

The solution to the equation system is only a solution if the ordering of distances satisfies

0 #x #X #B #y #Y .

In addition we require that consumption is non-negative

$x < X$:	$M_1 - c_{1c} - g_c \geq 0$;
$0 < x$:	$M_2 - c_{2c} - g_c \geq 0$;
$y < Y$:	$M_1 - c_{1s} - g_s \geq 0$;
$B < y$:	$M_2 - c_{2s} - g_s \geq 0$;
	$g_c \geq 0$;
	$g_s \geq 0$.

APPENDIX : POSSIBLE EQUILIBRIUM CONFIGURATIONS

With a majority of poor households, it is impossible for there to be a majority of rich households in both the city and in the suburbs. The three possible cases are represented in the figure below:

		SUBURBS MAJORITY	
		poor	*rich*
URBAN AREA MAJORITY	*poor*	*Case 3*	*Case 1*
	rich	*Case 2*	

Figure : classes of equilibria

These cases are further broken down into possible sub-cases in the matrix of matrices shown below.

NO UNDEVELOPED LAND IN CITY | SOME UNDEVELOPED LAND IN CITY

CASE 1:

poor majority in city, rich majority in suburbs

No undeveloped land in city:

city \ suburbs	poor only	both	rich only
poor only	**A**	**1.1**	**1.4**
both	**B**	**1.5**	**1.2**
rich only	**B**	**B**	**A**

Some undeveloped land in city:

city \ suburbs	poor only	both	rich only
poor only	**A**	**D**	**1.6**
both	**B**	**D**	**1.3**
rich only	**B**	**B**	**A**

CASE 2:

rich majority in city, poor majority in suburbs

No undeveloped land in city:

city \ suburbs	poor only	both	rich only
poor only	**A**	**B**	**B**
both	**2.3**	**C**	**B**
rich only	**2.4**	**2.1**	**A**

Some undeveloped land in city:

city \ suburbs	poor only	both	rich only
poor only	**A**	**B**	**B**
both	**2.5**	**C**	**B**
rich only	**2.2**	**D**	**A**

CASE 3:

poor majority in city and suburbs

No undeveloped land in city:

city \ suburbs	poor only	both	rich only
poor only	**A**	**E**	**B**
both	**3**	**E**	**B**
rich only	**B**	**B**	**A**

Some undeveloped land in city:

city \ suburbs	poor only	both	rich only
poor only	**A**	**D**	**B**
both	**D**	**D**	**B**
rich only	**B**	**B**	**A**

Figure 4: labeling of possible equilibria

The outer matrix is to be interpreted as follows. The first column corresponds to the cases in which there is no undeveloped city land; the second column corresponds to the cases in which there is some undeveloped city land. The first row corresponds to Case 1 (poor households form the majority in the city and rich households form the majority in the suburbs); the second row corresponds to Case 2 (rich households form the majority in the city and poor households form the majority in the suburbs); and the third row corresponds to Case 3 (poor households form the majority in the city and in the suburbs).

Within each cell of the outer matrix is a matrix. The first column of this submatrix corresponds to the case when the suburbs contains only poor households; the second column corresponds to the case when the suburbs contains both poor and rich households; and the third column corresponds to the case when the suburbs contains only rich households. Similarly, the first row corresponds to the case when the city area contains only poor households; the second row corresponds to the case when the city contains both poor and rich households; and the third row corresponds to the case when the city contains only rich households;

The boxes are shaded if they do not represent possible equilibria. They are coded with a letter which gives the cause as to why the configuration is inconsistent with an equilibrium:

A: The composition is inconsistent with the metropolitan population. I.e., it is not possible for city and suburbs to both contain only poor households or only rich households.

B: The composition is inconsistent with assumed voting majorities. E.g., Case 1 (poor households form majority in both areas) is inconsistent with the city having only rich households.

C: If rich households vote the public service in the city, they are willing to pay more than poor households for both the commuting advantage of the city and for the public service of the city. Therefore rich households outbid poor households for locations in the city, so that either the city contains only rich households or the suburbs contain only poor households. (Proposition A below).

D: If a group forms the majority in the city, any member of that group that resides in the suburbs could raise their utility by moving to undeveloped land in the city. This move would provide the suburban member with lower community costs, a no-worse fit in terms of public service provision, and no increase in housing price since the undeveloped land must be priced at the reservation value of land. Hence, if there is undeveloped land in the city, poor (rich) households cannot form the majority in the city and live in the suburbs. (Proposition B below).

E: If poor households are the majority in both jurisdictions, they vote the same public service level in each jurisdiction. Rich households have a higher willingness to pay for the commuting advantages of the city and, with the same public service in each jurisdiction, outbid poor households for city homes. Hence, if poor households are present in the city, there can be no rich households in the suburbs. (Proposition C below).

The unshaded boxes correspond to the possible equilibria. The number inside of these boxes labels the case.

The propositions are shown overleaf. Denote the city's public service as g_c and the suburban public service as g_s.

PROPOSITION A: *If rich households form the majority in the city and live in the suburbs, there can be no poor households in the city.*

PROOF: Rich households choose the city's public service. Equilibrium requires that a rich household obtains the same utility at x in the city and at y in the suburbs:

$$\max_g \; M_2 - a_2 r(x) - tM_2 x - g + \beta(M_2)V(g) \;=\; M_2 - a_2 r(y) - tM_2 y - g_s + \beta(M_2)V(g_s)\,,$$

or $$a_2 r(x) + tM_2 x \;\geq\; a_2 r(y) + tM_2 y\,. \tag{B.1}$$

If rich households form the majority in the city, poor households must form the majority in the suburbs. The utility of a poor household at y is

$$\max_g \; M_1 - a_1 r(y) - tM_1 y - g + \beta(M_1)V(g)\,.$$

Using Inequality (B.1),

$$\begin{aligned}
&\max_g \; M_1 - a_1 r(y) - tM_1 y - g + \beta(M_1)V(g) \\
&\quad \geq \max_g M_1 - a_1 r(x) - tM_1 x - \left[\frac{a_1}{a_2} tM_2(y - x) + tM_1(y - x)\right] - g + \beta(M_1)V(g) \\
&\quad > M_1 - a_1 r(x) - tM_1 x + g_c + \beta(M_1)V(g_c)\,,
\end{aligned}$$

A poor household obtains more utility in the suburbs than he could obtain in the city, or no poor households live in the city.

PROPOSITION B: *If a group forms the majority in the city and also lives in the suburbs, there can be no undeveloped urban land.*

PROOF: The proof is by contradiction.

(a) Assume that rich households form the majority in the city and live in the suburbs, and that there is undeveloped city land. If rich households are the majority in the city, poor households must form the majority in the suburbs. A poor household achieves the same utility at y or at Y, or

$$M_1 - a_1 r(y) - tM_1 y - g_s + \beta(M_1)V(g_s) = M_1 - a_1 r_0 - tM_1 Y - g_s + \beta(M_1)V(g_s),$$

or $$a_1 r(y) + tM_1 y = a_1 r_0 + tM_1 Y. \tag{B.2}$$

Rich households are the majority in the city and choose the city's public service. A rich household residing at x in the city achieves utility

$$\max_g M_2 - a_2 r_0 - tM_2 x - g + \beta(M_2)V(g).$$

Using Equation (B.2) and Inequality (1):

$$\max_g M_2 - a_2 r_0 - tM_2 x - g + \beta(M_2)V(g)$$

$$= \max_g M_2 - a_2 r(y) - tM_2 y + \left[\frac{a_2}{a_1} tM_1(Y - y) + tM_2(y - x)\right] - g + \beta(M_2)V(g)$$

$$> M_2 - a_2 r(y) - tM_2 y - g_s + \beta(M_2)V(g_s),$$

or the rich household achieves more utility at x than at y, which is inconsistent with the assumed equilibrium.

(b) Assume that poor households form the majority in the city and live in the suburbs, and that there is undeveloped city land. A poor household at X achieves utility as

$$\max_{g} M_1 - a_1 r_0 - tM_1 X - g + \beta(M_1)V(g) .$$

But $X < Y$, or

$$\max_{g} M_1 - a_1 r_0 - tM_1 X - g + \beta(M_1)V(g) > M_1 - a_1 r_0 - tM_1 Y - g_s + \beta(M_1)V(g_s) .$$

The poor household achieves higher utility at X than at Y, which is inconsistent with the assumed equilibrium.

PROPOSITION C: *If poor households are the majority in both areas, the suburbs contain no rich households.*

PROOF: Poor households vote the public service levels in each area and obtain the same utility in each area, or for poor households living at x and y

$$\max_{g} M_1 - a_1 r(x) - tM_1 x - g + \beta(M_1)V(g) = \max_{g} M_1 - a_1 r(y) - tM_1 y - g + \beta(M_1)V(g) .$$

or $$a_1 r(x) + tM_1 x = a_1 r(y) + tM_1 y , \tag{B.3}$$

and $$g_c = g_s = g .$$

A rich household living at x in the city achieves utility

$$M_2 - a_2 r(x) - tM_2 x - g + \beta(M_2)V(g) .$$

If no rich households live in the city, $x=0$ and the above expression is the utility a rich household achieves if it moves to the city. A rich household living at y in the suburbs achieves utility

$$M_2 - a_2 r(y) - tM_2 y - g + \beta(M_2)V(g) .$$

If no rich households live in the suburbs, $y = B$ and the above expression is the utility a rich household achieves if it moves to the suburbs. Using Equation (B.3) and Inequality (1)

$$\begin{aligned} & M_2 - a_2 r(x) - tM_2 x - g + \beta(M_2)V(g) \\ & \qquad = M_2 - a_2 r(y) - tM_2 y + a_2\left(\frac{tM_2}{a_2} - \frac{tM_1}{a_1}\right)(y - x) - g + \beta(M_2)V(g) \\ & \qquad > M_2 - a_2 r(y) - tM_2 y - g + \beta(M_2)V(g). \end{aligned}$$

A rich household has higher utility in the city than in the suburbs, or there can be no rich households in the suburbs.

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ENDNOTES

1. Ross and Yinger (1999) survey this literature.

2. Inefficiencies include the impact of the property tax on housing consumption and the influence of community heterogeneity on voting outcomes.

3. Wheaton (1976) and Sasaki (1990) provide a comparative static analysis of this equilibrium.

4. The assumption of fixed housing size greatly simplifies the problem and allows us to avoid well-known existence problems associated with stratified local public-finance equilibria (see Epple, Filimon and Romer (1984, 1993)).

5. For ease of presentation, the jurisdiction is assumed to provide a public service and not a public good. The public service shows constant returns to community size. It is straightforward to change the publically-provided good from a public service to a public good.

6. If *dr/dd* is interpreted as the slope of the bid-rent curve of the household, Inequality (1) implies that rich households have steeper bid-rent curves then poor households, or rich households outbid poor households for the locations closer to the metropolitan center.

7. This assumption is consistent with the findings of Glaeser, Kahn, and Rappaport (1999).

8. A formal proof is provided in de Bartolome and Ross (2000).

9. Because of the specific form of the utility function, it is straightforward to change this assumption to allow each household to receive an equal share of the total rent paid as a lump-sum transfer. To do this, simply sum the average rent and the quoted utility levels in Tables 2 and 3. We prefer to have rents paid to absentee landlords as the alternative suggests that households in one community benefit from property value increases in the other community.

10. A list of all such possible equilibria is available from the authors on request. de Bartolome and Ross (2000) show that an equilibrium with strict income sorting always exists.

11. In many studies, D is required to be positive, which implies a price elasticity that exceeds unity. We generalize the utility function to allow for negative values of D and price elasticities that are less than one.

12. Sensitivity analysis was performed using a lower and higher value (0.1 and 0.5) for the implied income elasticity of land demand, and there was little effect on the results.

13. This figure is a crude average and ignores possible variations in the number of households living in low- and high- service communities.

14. Societal figure calculated as local government expenditure financed from local government own-revenue (*Census of Governments Volume 4 Number 5* 1992, Table 3) divided by personal

income times one minus the average federal income tax rate (*Individual Income Tax Returns* 1990, Table 1.1).

15. Using the labels described later in the text, the equilibrium at $B = 4.5$ miles in Table 2 changed from Case 1.2 to Case 1.3.

16. At each value of B, we found only one equilibrium within each Case. Because the two income classes have been chosen to be the same size, we found no equilibrium in which one income-class forms the majority in both communities. We also did not find an equilibrium in which both communities contain both income classes.

17. This is the average rent paid per household. If the value quoted in the table is T ($ per household per year): because the average lot size is 0.3333 (acres per household), the average rent per acre per year is $3T$. Because the reservation rent has been set to zero, this is interpreted as the average rent premium (above the reservation rent) paid for one acre of land in the metropolitan area.

18. Consumer surplus from public services falls by 216 ($ per household per year). Commuting costs increase by 109 ($ per household per year).

19. We have compared above the efficiency of the two equilibria achieved at a given city boundary. For completeness, we now consider the effect of an increase in the city's jurisdictional boundary with consideration initially focused on Case 1 (city majority of poor households). As the boundary moves from 0.5 to 4.5 miles, efficiency does not change very much: the improvement of the matching of poor households with public services is almost exactly balanced by the increase in the commuting costs because the rich are being pushed further out. Initially (as the boundary moves from 0.5 to 1.5 miles), the increase in commuting costs dominate and efficiency falls. However, as the city becomes larger, the geometry of the problem means that, for each mile the boundary moves out, the area of the city expands by a larger amount and the average distance of rich households from the city center increases by a smaller amount. In consequence, there is a larger gain in the matching of poor households with their desired public services and a smaller increase in the commuting cost of rich households. Between 1.5 and 4.5 miles, the improvement in matching dominates and efficiency increases slightly.

Continuing with Case 1, as the boundary increases from 4.5 to 6.5 miles, some rich households are migrating into the city, lowering their commuting costs; however, the rich households remaining in the suburbs are being pushed further out, raising their commuting costs. The two effects approximately offset and commuting costs are approximately constant. However, the rich households moving into the city obtain less public service than they desire: matching of rich households with their desired public service worsens and efficiency falls.

For Case 2, increasing the boundary from 0.5 to 5.5 miles gives better matching of rich households with their public service level and no (or little) increase in commuting costs: welfare increases accordingly. Beyond 5.5 miles, commuting costs increase but there is no change in matching: efficiency falls.

20. With equal numbers of poor and rich households, the rich become the majority in the city immediately the poor spill over into the suburbs.

21. Although the causation is quite different, this result resembles the leapfrog development pattern that may appear in models of urban growth where some land is left vacant in the interior because its option value for future development exceeds its value in current use. For some examples in the literature, see Arnott and Lewis (1979), Capozza and Helsley (1989) and Wheaton (1982).

22. Technically, to ensure that there is a metropolitan majority of poor households, we assume that there is one more poor household than rich household.

23. If households of income M_i do not live in the community j, we interpret below c_{ij} to be the rent plus commuting cost which a household of income M_i would pay if he were to move into the community j.