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The Valuation of Public Goods: Why Do We Work?

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Abstract

Conventional analysis of public goods provision treats income as exogenous. We explore the implications of allowing leisure demand and human capital accumulation to be endogenously determined. We prove that the standard practice of treating income as exogenous results in an unambiguous downward measurement bias of society's value for public goods. Our results are true even under the ideal condition of perfect demand revelation. For individuals who have high demand for public goods relative to what society collectively provides, the level of public goods is too low as well as their level of earned income.

I. Introduction

In his famous article, Samuelson (1954) presented necessary conditions for the socially optimal provision of private and public goods. For private goods, marginal rates of transformation in production must equal marginal rates of substitution in consumption for each individual producer and consumer. For public goods, the analogous condition is that (vertically) aggregated marginal willingness to pay must be equilibrated to the marginal cost of provision. Ending on a somewhat pessimistic note, Samuelson noted that no decentralized pricing mechanism will yield the optimal levels of public goods, because "it is in the selfish interest of each person to give *false* signals, to pretend to have less interest in a given collective consumption activity than he really has, etc." Recognition of this incentive to understate one's value for a public good spawned a literature dedicated to constructing mechanisms that induce truthful public good demand revelation (*e.g.* Clarke (1971) and Groves and Ledyard (1977)). On another front, the analysis of public goods provision through benefit-cost analysis continues to center around Samuelson's vertical summation condition within the welfare framework of Kaldor (1939) and Hicks (1939).

Absent from Samuelson's analysis and the many extensions into the welfare analysis of public goods provision is the explicit consideration of the demand for leisure or it's mirror image, the supply of labor. While a closely related issue, the relationship between the optimal allocation of public goods and distribution of wealth, has been analyzed[1], explicit consideration of leisure demand/labor supply and its implications for the analysis of public goods provision is missing from the literature. We contend that by failing to explicitly recognize the demand for leisure, a different problem arises in the analysis of public goods provision. In what follows, we prove that the

[1]Musgrave (1969) raises the problem that one cannot separate issues of optimal public goods provision and optimal distribution of wealth. Bergstrom and Cornes (1983) provide a general class of preferences that avoid Musgrave's problem.

conventional analysis found in the literature, which treats labor supply as exogenous, results in a downward bias in the true social values for public goods, *even under truthful revelation*. The inability to *individually* buy public goods means that individuals make different leisure/labor choices and human capital investments than would be made if they were able to individually choose the level of public goods.[2] There is a heretofore unrecognized parallel input market failure that corresponds to the output market failure; inability to *individually* buy the public good will result in a reduced desire to generate income, with resulting reductions in labor supply and human capital investment.

The intuition for our contention is simple, and is most readily grasped in comparing individuals of two distinct types. Consider first Wanda Lott, who has very strong preferences for ordinary goods and does not care much about public goods (*e.g.* environmental quality). In her pursuit of market goods consumption, Wanda will want to generate a high income to finance her desired high consumption levels. She is likely to invest in human capital and also to work long hours at her job to acquire the goods she desires in such abundance. She will do this because she knows that, if she gets the income to do it, she can have the goods she wants.

In stark contrast to Wanda Lott is Sten (for strong environmentalist) who has strong preferences for environmental quality and only modest desires for ordinary goods. If environmental/public goods (say, species preservation, air quality and CO_2 abatement, for concreteness) *could* be bought like an ordinary good, Sten would generate the income to buy it. He would be observed buying it, until the marginal values per dollar spent were equated across each argument of his utility function (exhibiting a large total valuation for the environment and a high marginal valuation for any quantity of the environment substantially smaller than at his optimal

[2]Samuelson assumes this interconnectedness away: “Provided economic quantities can be divided into two groups, (1) *outputs* or goods which everyone always wants to maximize and (2) inputs or factors which everyone always wants to minimize, we are free to change the algebraic signs of the latter category and from then on to work only with ‘goods,’ knowing that the case of factor inputs is covered as well.”

bundle). However, Sten knows he cannot, in fact, affect environmental quality by his individual demands; he is too small to make a difference, say, in saving a species or in CO_2 abatement. So, Sten, and others like him, consume their small demands for ordinary goods and *reduce their income* by increasing their leisure, a variable they can affect. Indeed, Sten is also likely to invest less in education than Wanda Lott and those like her. Sten will do this because he knows that, even if he gets the income to do it what he really wants, he cannot have the goods he wants.

Sten's true willingness to pay, *if he could purchase public goods in the same way he can private goods,* would be much larger than his apparent willingness to pay at the low income he generates in a world in which he cannot so purchase public goods. After all, Sten is poor because he optimally chose not to work much or accumulate human capital...it is not, though we shall discuss it that way in the analytical section, that there are varying regimes in which Sten can or cannot vary his labor supply or human capital investment. It is that the *same* market failure that causes non-optimal provision of public goods in the market setting also creates a market failure in input markets; the failure to generate income when that income cannot buy what we would like. At this point, many economists might quip that "if you're going to talk the talk, you had better be willing to walk the walk," dismissing Sten's truthful revelation as incredible. In fact in an extreme case, such as Sten, his stated willingness to pay for the host of public goods he cares about may indeed add up to implausibly large shares of his current income. Though Sten's values may be high relative to current income, they might not be so large relative to the income he *would have generated* were he able to purchase the goods he desires in the amounts he desires as does Wanda Lott in her pursuit of private goods consumption.

Having argued that public good market failures result generally in input market failures that imply under-valuation of public goods via usual (Samuelsonian) measurement methods, we now move to a formal representation of our claims. In Section II we consider the public goods valuation problem introduced above, allowing for short-run variations in labor supply that *do* occur when the levels of public goods are changed. Section III extends the analysis to the longer-term adjustments in human capital investment that *do* take place when the levels if public goods are changed. Section IV provides further discussion, observation, and possible extensions.

II. Welfare Analysis with Leisure Demand

Before beginning the formal analysis, note that standard welfare analysis for valuing a change in environmental quality typically treats income as being fixed. This implicitly assumes that leisure demand, hence its additive inverse, labor supplied, is constrained to the status quo level. It is quite clear generally that if we constrain a person to the same level of leisure/labor supply, then the gain in utility for an increase in environmental quality will be less than if we allow for adjustment in leisure versus labor. In turn, the compensating variation/willingness to pay for the increase under constrained leisure choices will be less than willingness to pay under a regime of free choice of leisure. Similarly for the same reduction, equivalent variation/willingness to accept compensation will also be less than if we had allowed for leisure adjustment. One may be led to conclude that what we point to will only be a problem for those individuals whose leisure/labor choices are affected by changes in environmental quality. However as we show below, the problem generally affects anyone whose leisure choice is influenced by changes in wealth.

Our model is very similar to general models that treat environmental quality as a quantity rationed public good. A representative consumer's preferences are defined over a vector of market goods, X, leisure, L, and environmental quality, q.[3] Though we use a representative agent in developing our analytical points, we are not assuming that everyone has the same preferences.

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[3]We focus on environmental quality as opposed to general public goods because environmental quality often displays the mixed benefits, "use values" and "non-use values" and there are often clashes between private uses of an environmental good (e.g. whale meat, scrimshaw) and public uses (e.g. species preservation).

Rather our observations can be developed using the case of a single consumer. While our model is similar in spirit to the typical analysis, the addition of leisure differs from the standard analysis. Some, but not necessarily all, market goods and leisure are assumed necessary goods.[4] Our consumer maximizes utility by choosing levels of market goods and leisure in a single period. The consumer is endowed with money wealth, W, and an amount of time, H, to use working and pursuing leisure. Labor income and wealth are completely used toward the purchase of market goods. The formal problem is stated as follows.

$$\max_{\{X,\ L\}} \ U(X,q,L) \quad s.t. \quad pX + Lw \leq Hw + W,\ 0 \leq X,\ 0 < L \leq H,\ q = q^0, \tag{1}$$

The corresponding Lagrange function for this problem is given in (2) and the necessary conditions are given in (3).

$$\mathcal{L} = U(X,q,L) + \lambda(Hw + W - pX - Lw) + \lambda_H(H - L) + \lambda(q^0 - q) \tag{2}$$

$$\begin{gathered}
(a)\ \frac{\partial U}{\partial X} \leq \lambda p,\ 0 \leq \lambda,\ 0 \leq X\ c.s.\ \lambda X = 0 \\
(b)\ \frac{\partial U}{\partial L} = \lambda w + \lambda_L,\ 0 \leq \lambda_L,\ 0 < L \leq H\ c.s.\ \lambda_L(H - L) = 0 \\
(c)\ \frac{\partial U}{\partial q} = \lambda_q,\ \lambda_q > 0 \\
(d)\ pX + Lw = Hw + W
\end{gathered} \tag{3}$$

The ordinary demands for market goods will depend on prices, the wage rate, environmental

[4]The marginal utility of necessary goods tends to infinity as the level tends to zero.

quality, and wealth, $X^m = X^m(p,w,q,W)$, $L^m = L^m(p,w,q,W)$.[5] While these ordinary demands dictate the observable behavior, they do not provide the proper insights into the monetary measures used in welfare analysis. In order to develop proper welfare measures, we consider the dual problem of minimizing expenditures on market goods and leisure subject to the level of q, the level of utility, and the constraint on the range of leisure.

$$\min_{\{X,\ L\}} \quad pX + Lw \quad s.t. \quad U(X,q,L) = U^0,\ q = q^0,\ 0 \le L \le H \tag{4}$$

The first order conditions will mirror those from the utility maximization problem with the exception that the multipliers now are expressed in monetary, as opposed to utility, units.

$$\begin{gathered}
(a)\ \frac{\partial U}{\partial X}\gamma \le p,\ 0 \le \gamma,\ 0 \le X\ c.s.\ \gamma X = 0 \\
(b)\ \frac{\partial U}{\partial L}\gamma + \gamma_L = w,\ 0 \le \gamma_L,\ 0 < L \le H\ c.s.\ \gamma_L(H - L) = 0 \\
(c)\ \frac{\partial U}{\partial q}\gamma = \gamma_q,\ \gamma_q > 0 \\
(d)\ U(X,q,L) = U^0
\end{gathered} \tag{5}$$

The solutions to the dual problem depend on prices, the wage rate, the level of environmental quality, and the level of utility, $X^h = X^h(p,w,q,U)$, $L^h = L^h(p,w,q,U)$.[6] Using the demands, we can easily represent the amount of wealth adjustment that would leave our consumer indifferent between obtaining an increase in environmental quality from an initial level q^0 to a new, higher level of environmental quality q^1. As in standard welfare analysis that treats income as exogenous, the

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[5]The *m* superscript refers to ordinary, Marshallian solutions.

[6]The *h* superscript refers to the solution to the dual problem, the compensated or Hicksian demands.

wealth adjustment that makes the consumer indifferent is referred to as compensating variation and is equal to the difference in the minimized expenditures.

$$\begin{aligned} CV &= p\cdot[X^h(p,w,q^0,U^0) - X^h(p,w,q^1,U^0)] \\ &+ w\cdot[L^h(p,w,q^0,U^0) - L^h(p,w,q^1,U^0)] \end{aligned} \tag{6}$$

In order to contrast (6) with a standard welfare analysis that treats leisure demand as fixed, we must constrain the choice of our consumer while providing the same increase in environmental quality. The minimization problem will be identical to the problem in (5) with the exception that leisure is constrained to the status quo level of leisure which we will refer to as L^0. Now the only choice variables in the expenditure minimization problem are the levels of the market goods, X. Given the constraint we alter our notation to reflect this constraint, $X_S^h = X_S^h(p,L^0,q,U^0)$, letting the S superscript refer to the "standard" notion of compensated demand where leisure is fixed. With our new notation, we can express the standard notion of compensating variation found in the welfare economics literature.

$$\begin{aligned} CV_S &= p\cdot[X_S^h(p,L^0,q^0,U^0) - X_S^h(p,L^0,q^1,U^0)] \\ &+ w\cdot[L^0 - L^0] \end{aligned} \tag{7}$$

As our intuition suggests, the relationship between the standard compensating variation (7) and compensating variation with flexible leisure demand (6) is easy to establish.

Proposition 1: For an increase in environmental quality from q^0 to a new higher level q^1, it will generally be true that $CV_S \le CV$. Furthermore, in most cases $CV_S < CV$.

Proof: *Part 1.* The first statement in proposition 1 is very easy to establish. First note that by definition $p\cdot X^h(p,w,q^0,U^0) + w\cdot L^h(p,w,q^0,U^0)$ equals $p\cdot X_S^h(p,L^0,q^0,U^0) + w\cdot L^0$. Given the respective minimization problems that provide the demands in (6) and (7), we know that it must be the case that $p\cdot X^h(p,w,q^1,U^0) + w\cdot L^h(p,w,q^1,U^0)$ is weakly less than $p\cdot X_S^h(p,L^0,q^1,U^0) + w\cdot L^0$. Combining $p\cdot X^h(p,w,q^0,U^0) + w\cdot L^h(p,w,q^0,U^0) = p\cdot X_S^h(p,L^0,q^0,U^0) + w\cdot L^0$ with the fact that

$- p \cdot X_S^h(p,L^0,q^1,U^0) + w \cdot L^0 \le - p \cdot X^h(p,w,q^1,U^0) + w \cdot L^h(p,w,q^1,U^0)$ leads to the fact that $CV_S \le CV$.

Part 2. Part 1 of the proof establishes the weak inequality. To prove the second statement we need only establish the conditions under which $CV_S = CV$. $CV_S = CV$ when $L^h(p,w,q^0,U^0) = L^h(p,w,q^1,U^0)$, that is when compensated leisure demand does not respond to the change in q. Compensated leisure demand from minimization problem (4) will differ from ordinary leisure demand in problem (1) once q is changed from the initial level. The two diverge since as q is increased, wealth must be decreased in the compensated case. There will be an income effect if leisure responds to changes in wealth, regardless of whether leisure is a normal good, *i.e.* increases with wealth, or an inferior good in the same sense. In the case of wealth sensitive leisure demand of either form (normal or inferior), $L^h(p,w,q^0,U^0) = L^h(p,w,q^1,U^0)$ only when the substitution effect exactly cancels out the effect of the reduction in income for the compensated demands. In the case where leisure demand is independent of wealth, then $L^h(p,w,q^0,U^0) = L^h(p,w,q^1,U^0)$ only when there is no substitution effect. Both of these cases are the razor's edge, particularly the case where leisure demand is sensitive to wealth effects.

Proposition 1 is the basis for our claim that there is an inherent systematic bias towards under-provision of public goods by standard welfare analysis that treats leisure/labor decisions as fixed when considering the analysis of public goods provision. Our result is quite general in that no unusual restrictions need to be placed on preferences to obtain our results. Similarly, the model could trivially be extended to the case where we are considering a vector of public goods as opposed to focusing exclusively on a single environmental good.

Our analysis readily extends to the case of willingness to accept compensation. If we begin with the higher level of environmental quality, q^1 and consider a reduction to the lower level q^0, willingness to accept from both the leisure restricted and unrestricted minimization problems are forms of compensating variation with the reference utility level at the level of utility provided from solving maximization problem 1 where environmental quality is q^1. We can extend Proposition 1 to Corollary 1.

$$\begin{aligned} WTA &= p \cdot [X^h(p,w,q^0,U^1) - X^h(p,w,q^1,U^1)] \\ &\quad + w \cdot [L^h(p,w,q^0,U^1) - L^h(p,w,q^1,U^1)] \end{aligned} \tag{8}$$

$$
\begin{aligned}
WTA_S &= p\cdot[X_S^h(p,L^0,q^0,U^1) - X_S^h(p,L^0,q^1,U^1)] \\
&\quad + w\cdot[L^0 - L^0]
\end{aligned}
\tag{9}
$$

Corollary 1: For an decrease in environmental quality from q^1 to a new lower level q^0, it will generally be true that $WTA_S \leq WTA$. Furthermore, in most cases $WTA_S < WTA$.

Corollary 1 is a direct extension of Proposition 1 and so no proof is required. We do not use the phrase equivalent variation to describe willingness to accept in order to avoid confusion. It is true that for the minimization problem that is unrestricted in leisure, WTA will equal the equivalent variation for the change from q^0 to q^1. However if we consider the restricted problem, WTA_S will not equal equivalent variation since the utility level obtained in the restricted maximization problem will not be as great as that obtained in the unrestricted problem. In the next section we present a simple life cycle model that allows for human capital accumulation and show that our results from this section carry over similarly.

Before moving onto our model with human capital investment, consider the implications of the model and results from this section. First, the key results revolve around the way in which earned income responds to changes in the rationed level of the public good environmental quality. Let us contrast this result with the situation where environmental quality is not individually rationed. If someone like Sten could *individually* choose the level of environmental quality, he would choose a higher level than the level set by the collective choice of society given his personal marginal cost. In turn in order to pay for this higher level of environmental quality, Sten would work more. So in a very real sense, the level of public good provided by society is too low for Sten and in turn we can say that Sten's earned income is also too low. The standard conceptual framework completely misses this important point.

III. Welfare Analysis with Leisure Demand and Human Capital Investment

Our human capital model is a simple two-period model in which our consumer spends some time in school, *S*, in the first period in order to earn more in the second period. In the first period, our consumer spends his time endowment pursuing leisure, working, and going to school. The wage in the first period is a low skilled wage, $\underline{w}$. If no schooling is undertaken, the second period wage is the low skilled wage. Investment in education exhibits diminishing returns to scale. Formally we have that for $S = 0$, $w(S) = \underline{w}$ and for $S > 0$, $w(S) > \underline{w}$ subject to continuity of $w(S)$, $w'(S) > 0$, and $w''(S) < 0$. The maximization problem is given in (10), the Lagranian function in (11), and the accompanying necessary conditions are in (12).

$$\begin{gathered}\max_{\{X_1,\ L_1,\ S,\ X_2,\ L_2\}} \quad U(X_1,q_1,L_1) + \beta U(X_2,q_2,L_2) \\ s.t.\ \ p_1X_1 + (L + S)\underline{w} + \beta p_2X_2 + \beta w(S)L_2 \le H\underline{w} + \beta H w(S) + W, \\ 0 < L_1 + S \le H, \quad 0 < L_2 \le H, \quad q_i = q_i^0,\ i = 1,\ 2\end{gathered} \tag{10}$$

$$\begin{aligned}\mathcal{L} = {} & U(X_1,q_1,L_1) + \beta U(X_2,q_2,L_2) \\ & + \lambda\left[H\underline{w} + \beta H w(S) + W - p_1X_1 - (L + S)\underline{w} - \beta p_2X_2 - \beta w(S)L_2\right] \\ & + \lambda_1(H - L_1 - S) + \lambda_2\beta(H - L_2) + \lambda_{q_1}(q_1^0 - q_1) + \lambda_{q_2}\beta(q_2^0 - q_2)\end{aligned} \tag{11}$$

$$
\begin{aligned}
&(a)\ \frac{\partial U}{\partial X_i} = \lambda p_i \\
&(b)\ \frac{\partial U}{\partial L_1} = \lambda \underline{w} + \lambda_1,\ 0 < L_1,\ L_1 + S \le H\ c.s.\ \lambda_1(H - L_1 - S) = 0 \\
&(c)\ \frac{\partial U}{\partial L_2} = \lambda w(S) + \lambda_2,\ 0 < L_2,\ L_2 \le H\ c.s.\ \lambda_2(H - L_2) = 0 \\
&(d)\ \frac{\partial U}{\partial q_i} = \lambda_{q_i} \\
&(e)\ p_1 X_1 + (L + S)\underline{w} + \beta p_2 X_2 + \beta w(S) L_2 \le H\underline{w} + \beta H w(S) + W \\
&(f)\ \underline{w} - w'(S)\beta(H - L_2) \le 0
\end{aligned}
\tag{12}
$$

The first order conditions for this problem are qualitatively similar to those in (3) with the exception of (f). (f) summarizes the condition for optimal human capital investment. Given the conditions of *w(S)*, the amount of human capital investment is inversely related to the demand for leisure in the second period. The dual problem with human capital investment must be slightly modified in form from the minimization problem presented in (4) because the value of the endowment of time in the second period, *w(S) H* is endogenously determined. The dual problem is stated as follows.

$$
\begin{aligned}
&\min_{\{X_1,\ L_1,\ S,\ X_2,\ L_2\}} \quad p_1 X_1 + (L + S)\underline{w} + \beta p_2 X_2 + \beta w(S) L_2 - \beta H w(S) \\
&\qquad s.t.\quad U(X_1,q_1,L_1) + \beta U(X_2,q_2,L_2) = V^0, \\
&\qquad 0 < L_1 + S \le H,\quad 0 < L_2 \le H,\quad q_i = q_i^0,\ i = 1,\ 2
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
\mathfrak{L} = {} & p_1 X_1 + (L + S)\underline{w} + \beta p_2 X_2 + \beta w(S) L_2 - \beta H w(S) \\
& + \gamma[V^0 - U(X_1,q_1,L_1) - \beta U(X_2,q_2,L_2)] \\
& + \gamma_1(H - L_1 - S) + \gamma_2 \beta(H - L_2) \\
& + \gamma_{q_1}(q^0 - q) + \gamma_{q_2}\beta(q_2^0 - q_2)
\end{aligned}
\tag{14}
$$

The first order conditions that result from (13) and (14) will be virtually the same as those conditions presented in (12) excepting from the difference in multipliers which are now expressed in dollar units as opposed to present value utility units.

$$
\begin{aligned}
&(a)\ p_i = \gamma \frac{\partial U}{\partial X_i} \\
&(b)\ \underline{w} + \gamma_1 = \gamma \frac{\partial U}{\partial L_1},\ 0 < L_1,\ L_1 + S \le H\ c.s.\ \gamma_1 (H - L_1 - S) = 0 \\
&(c)\ w(S) + \gamma_2 = \gamma \frac{\partial U}{\partial L_2} = ,\ 0 < L_2,\ L_2 \le H\ c.s.\ \gamma_2 (H - L_2) = 0 \\
&(d)\ \gamma \frac{\partial U}{\partial q_i} = \gamma_{q_i} \\
&(e)\ U(X_1, q_1, L_1) + \beta U(X_2, q_2, L_2) = V^0 \\
&(f)\ \underline{w} - w^{\prime}(S) \beta (H - L_2) \le 0
\end{aligned}
\tag{15}
$$

Note in particular that condition (12f) and (15f) are identical with the exception that the respective demands for leisure are the compensated demand for period two leisure in (15f) and the ordinary demand for leisure in (12f). Compensated demand for schooling is inversely related to the compensated demand for leisure in the second period. The form of compensating variation for the change in environmental quality from q_2^0 to a higher level q_2^0 can be expressed using the differences in the argmin at the two levels of environmental quality. In order to distinguish from the compensating variation presented in section 1, we use a *w* superscript to denote the problem in which the second period wage is endogenous.

$$
\begin{aligned}
CV^{w} = {} & p_1[X_1(p,q^0,\underline{w},V) - X_1(p,q^1,\underline{w},V)] \\
& + \beta p_2[X_2(p,q^0,\underline{w},V) - X_2(p,q^1,\underline{w},V)] \\
& + [L(p,q^0,\underline{w},V) - L(p,q^1,\underline{w},V)]\underline{w} \\
& + [S(p,q^0,\underline{w},V) - S(p,q^1,\underline{w},V)]\underline{w} \\
& + \beta w(S(p,q^0,\underline{w},V))[L_2(p,q^0,\underline{w},V) - L_2(p,q^1,\underline{w},V)] \\
& - \beta[w(S(p,q^1,\underline{w},V)) - w(S(p,q^0,\underline{w},V))]L_2(p,q^1,\underline{w},V) \\
& + \beta H[w(S(p,q^1,\underline{w},V)) - w(S(p,q^0,\underline{w},V))]
\end{aligned}
\qquad \textbf{(16)}
$$

The relevant contrast in this section is between the compensating variation derived in (16) and a restricted version in which we constrain schooling after the change in environmental quality to the initial level of schooling, $S(p,q^0,\underline{w},V) = S^0$. We can define a the parallel compensating variation that we distinguish by the *S* super script.

$$
\begin{aligned}
CV_S^{w} = {} & p_1[X_1(p,q^0,\underline{w},S^0,V) - X_1(p,q^1,\underline{w},S^0,V)] \\
& + \beta p_2[X_2(p,q^0,\underline{w},S^0,V) - X_2(p,q^1,\underline{w},S^0,V)] \\
& + [L(p,q^0,\underline{w},S^0,V) - L(p,q^1,\underline{w},S^0,V)]\underline{w} \\
& + [S^0 - S^0]\underline{w} \\
& + \beta w(S(p,q^0,\underline{w},S^0,V))[L_2(p,q^0,\underline{w},S^0,V) - L_2(p,q^1,\underline{w},S^0,V)] \\
& - \beta[w(S(p,q^1,\underline{w},S^0,V)) - w(S(p,q^0,\underline{w},S^0,V))]L_2(p,q^1,\underline{w},S^0,V) \\
& + \beta H[w(S(p,q^1,\underline{w},S^0,V)) - w(S(p,q^0,\underline{w},S^0,V))]
\end{aligned}
\qquad \textbf{(17)}
$$

Using the same reasoning as in the first model, we know that $CV_S^{w} \leq CV^{w}$ and the parallel versions of willingness to accept will produce $WTA_S^{w} \leq WTA^{w}$. From these inequalities we can also infer a bias toward under provision.

Combining the inferences from this and the preceding section, we see that looking merely at *apparent* aggregated willingness-to-pay, as in the traditional Samuelsonian approach that underlies modern benefit-cost analysis, fails to properly value public goods, environmental quality being emphasized here. There are, however, a number of additional implications to which we now turn.

IV. Discussion of Implications and Extensions

For most people, the choices regarding human capital accumulation are not heavily influenced by the levels of collectively provided environmental goods. However there are individuals, like Sten, who greatly care about the environment and consider the levels of environmental quality chosen over time by the social collective when making *specific* human capital decisions. Sten can pursue different paths of human capital accumulation, the choice depending on the expected social equilibrium outcome. One possible outcome is that the divergence from the socially optimal level of the public good is not too far from what Sten really desires. In this case Sten might make minor adjustments in his human capital decisions and his leisure/goods consumption, although he would prefer to have more environmental quality.

Another outcome is that society, from Sten's perspective, chooses far too low of a level of environmental quality. Since Sten cannot unilaterally choose the level of environmental quality and he does not care much about consumption goods, Sten may choose not to accumulate much at all in the way of human capital. That is, he may effectively "drop-out" in the jargon of the '60s.

However, still another choice under these conditions is for Sten to accumulate human capital that is complementary with influencing society's environmental quality choices. For instance Sten could work for an environmental advocacy group in a variety of positions. The fact that pay within these organizations is low suggests that benefits in addition to monetary compensation accrue to workers like Sten. Sten's income may be low, but his preferences for the environment are very strong. To work at low pay for such an advocacy group, for Sten, is completely rational.[7]

While it is true that the direction of the bias in the individual valuation for public goods is unambiguous (true values are always greater than apparent willingness-to-pay, unless increments to the public good have zero or negative marginal value[8]), this does not mean that socially-optimal increases in public goods levels would necessarily win a popular vote. Incorporating willingness-to-pay after allowing for alterations in the labor/leisure and human capital investment decisions, as we do here, does not eliminate the possibility that a majority might have increases in costs that exceed their incremental benefits.

An interesting implication of moving from an under-provision of public goods to the larger socially-optimal quantity that would exist with proper valuation, is the impact on private goods markets. The under-provision of a public good implies that there will be an under-provision of private good complements and an over-provision of private good substitutes for the that public good. For example, failure to build enough lighthouses will result in reduced demand for boats, hence reduced equilibrium quantities of boats. A substitute for cleaning up urban air quality might be moving to a large lot in a distant suburb, hence failure to clean up urban air will (non-optimally) exacerbate the exodus to the suburbs and to ex-urban areas.

Of potentially great importance is the implication for selecting the appropriate social rate of

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[7]While our examples and analyses are specific to environmental issues, the results and observations carry over to a much larger class of examples. Advocates for other social causes that involve publicly rationed goods can also be described by our analysis.

[8]Negative marginal values might be possibilities for, say, national defense (depending on one's "theory of peace," fears of global destruction, and so on) but in the environmental context this would be unusual. Ranchers wishing to eliminate wolves, in the face of wolf reintroduction plans is not really a counter-example, but rather a case where the costs of the policy might be inappropriately distributed. That is, the rancher could have a positive wolf preservation demand that might be greatly exceeded by the costs he must pay to receive that benefit; full compensation for lost cattle might reveal his positive preservation value.

discount to be used in evaluating long-term projects (Flores and Graves (2001)). Those placing high values on public goods currently do not earn the large incomes they would have earned if they could have directly purchased the goods they care about. They also will not have saved as much as they would have...because they would *like* to be able to save environmental resources, not merely financial goods which they don't care much about (and about which their children, raised as they will be, won't care much about). If they *could* save for a better environment in the future for their children they would, but, as argued throughout, their inability to do that *individually* results in both smaller incomes and reduced savings. But, with proper valuation, there would be much more total saving, which means that the appropriate social discount rate is much smaller than currently presumed.

How important, quantitatively, are the qualitative points made here likely to be? We believe that there is substantial undervaluation of public goods, particularly public goods without "special interest support" (e.g. national defense). Consider the case of environmental quality. Assume that roughly four percent of GDP is being spent to obtain current environmental quality levels ($400 billion out of about a $10 trillion economy). Suppose, further (though we very much doubt this to be true) that, with currently employed valuation methods, that this expenditure results in the correct apparent environmental quality (matching aggregate marginal willingness-to-pay to marginal provision costs). This presumes, very conservatively, that environmental quality is a very small component of both utility and expenditure.

It is not implausible to argue that there might be a ten to twenty percent larger income, if people could buy environmental quality like they can buy ordinary goods. Suppose income would be only ten percent larger. One might plausibly argue that income elasticities of demand for environmental quality are greater than unity but less than two, say, 1.5. With these conservative assumptions, the $400 billion in apparent benefits would understate the true benefits by $60 billion dollars. We feel that one could plausibly argue that such numbers might well be an order of magnitude larger than suggested by this conservative illustration. An extension of the present effort could fruitfully examine the quantitative significance of the observations made here in greater depth.

We have here provided a rationale for believing that the traditional approaches to valuing public goods are flawed by failing to allow for labor supply and human capital adjustments that would be made were people able to buy public goods as they do ordinary goods. There is likely to be a quantitatively important mis-allocation of resources, for both public and private goods as a result. A subsidiary implication is that the social rate of discount currently in use is too large, particularly when applied to public good/environmental projects.

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