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Labor Hoarding, Superior Information, and Business Cycle Dynamics

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Abstract:
In this paper, we test whether labor-hoarding environments with basic and augmented laws of motion provide an adequate explanation for observed business cycle dynamics. The basic law of motion assumes that the information set used by economic agents to forecast future forcing variables includes only the history of forcing variables. Augmented laws of motion assume that the information set is superior and include both forcing and hidden exogenous variables. We show that the labor-hoarding environment with the basic law of motion fails to replicate observed business cycle facts, while the environment with augmented laws of motion successfully matches these facts.

Keywords: Hidden variable, law of motion, trend-cycle decomposition.

JEL Classification Codes: E32, E37.

1. Introduction

The objective of real business cycle (RBC) studies is to evaluate the ability of dynamic, stochastic, general equilibrium artificial economies to account for observed business cycle facts. This evaluation proceeds as follows. First, the RBC researcher specifies a set of market features (e.g. preferences, technology, and market clearing conditions) that describes the environment in which agents form their decisions. These decisions rely on forward-looking rules. The researcher also specifies a law of motion for the forcing variables that depict the stochastic nature of the artificial economy. This law of motion is required to forecast the future forcing variables involved in the decision rules. Second, the researcher characterizes the equilibrium allocation of some key variables. As such equilibrium usually does not possess an analytical solution, the allocation is approximated using numerical methods. This approximation requires that values be assigned to all parameters. Generally, the researcher calibrates the parameters underlying the market features using long-run averages and previously published estimates. The researcher also calibrates the parameters of the law of motion for forcing variables from estimates on historical data. Finally, the researcher assesses the ability of its artificial economy to account for business cycle facts by confronting certain statistics computed from the artificial economy to those found in historical data.

This evaluation is thus a joint test of the calibrated market features and of the estimated law of motion. In this context, the business cycle statistics computed from the artificial economy may not match the data, not because market features are inadequately described, but simply because the law of motion is misspecified. In RBC studies, the standard law of motion involves only forcing variables. This presumes that the relevant information set used by economic agents to forecast future forcing variables includes exclusively the history of forcing variables. It seems most likely, however, that the law of motion used by agents in the actual economy incorporates other exogenous variables. If this is the case, the relevant information set is superior to the researcher's basic information set, because it includes the history of both forcing and other exogenous variables.

Unfortunately, ignoring the extra relevant information (i.e. the other exogenous variables) is akin to omitting forcing variables and may lead to serious mismeasurements of the statistics computed using the artificial economy. A number of studies have recognized this issue. Shiller (1972) provides an early description of the superior information problem. Hansen and Sargent (1982, 1980) develop econometric techniques to estimate forward-looking decision rules with unobservable, or hidden, exogenous variables. Campbell and Deaton (1989), Campbell and Shiller (1987), Flavin (1993), and Normandin (1999) account for superior information in partial equilibrium environments. Finally, Boileau and Normandin (2001) and King and Watson (1996) extend the analysis to general equilibrium RBC environments.

Boileau and Normandin (2001) demonstrate that adequate laws of motion for forcing variables are obtained by augmenting the basic law of motion with endogenous variables. In these augmented laws, the feedbacks from lagged endogenous variables to current forcing variables reflect the existence of additional exogenous variables. This follows from the fact that, under the assumed market features, all the relevant information is summarized by agents' optimal decisions. This is most attractive, since it allows us to avoid selecting the appropriate additional exogenous variables. That is, it only requires knowledge of forcing and endogenous variables, and not of those other exogenous variables. Using augmented laws of motion, these authors also document that the labor-hoarding environment of Burnside, Eichenbaum, and Rebelo (1993) offers the best account of aggregate employment volatility and dynamics.

In this paper, we extend their analysis in two crucial directions. First, we enlarge the analysis by studying the properties of several key macroeconomic variables. More precisely, we assess whether an artificial economy with labor hoarding can explain observed business cycle fluctuations for output, consumption, and investment. Also, we confront the business cycle properties predicted by the labor-hoarding environment with a basic law of motion to those predicted with augmented laws of motion. This evaluation is important to ensure that the labor-hoarding environment with augmented laws of motion succeeds in matching the behavior of the key macroeconomic aggregates. This contrasts with Boileau and Normandin (2001) whom focus exclusively on aggregate employment.

Second, we examine the cyclical fluctuations of output, consumption, and investment under two distinct measures of the cycle. These series display both cyclical and secular movements. Consequently, it is important to carefully distinguish between trend and cycle components. Our first measure of the cycle corresponds to growth rates, which is widely used in the RBC literature (e.g. Burnside and Eichenbaum 1996; Christiano and Eichenbaum 1992). Our second measure is the cyclical components extracted using the cycle definition of Beveridge and Nelson (1981). This measure has been recently emphasized in the RBC literature (e.g. Rotemberg and Woodford 1996). This contrasts with Boileau and Normandin (2001) whom use a single measure of the cycle, because aggregate employment exhibits only cyclical fluctuations.

Our empirical results reveal that the labor-hoarding environment of Burnside, Eichenbaum, and Rebelo (1993) with a basic law of motion numerically and statistically understates observed volatility for all key macroeconomic aggregates. This holds for both measures of the cycle, but it is especially pronounced for the cyclical components of the various series. Also, the predicted cross-correlations numerically and statistically undervalue observed cross-correlations between output and consumption and between output and investment. Moreover, these predicted correlations often display the wrong sign for the growth rates of investment and for the cyclical components of consumption. Finally, the predicted responses to both technology and government expenditure growth shocks are significantly different from observed ones for all aggregates. This is robust to the choice of the cycle measure, and is particularly evident for the cyclical components of the key macroeconomic aggregates. Hence, the labor-hoarding environment with a basic law of motion fails to reproduce observed business cycle facts.

In contrast, our results show that the labor-hoarding environment with augmented laws of motion tracks remarkably well observed volatility, cross-correlations, and dynamic responses. These findings hold for all the key macroeconomic aggregates and both measures of the cycle. Hence, the labor-hoarding environment with augmented laws of motion successfully matches observed business cycle facts.

The paper is organized as follows. Section 2 presents the labor-hoarding environment, its calibration, and its solution with basic and augmented laws of motion for forcing variables. Section 3 documents empirical results for the growth rates of output, consumption, and investment. Section 4 discusses results for the cyclical components of our macroeconomic aggregates. Section 5 concludes.

2. The Economic Environment

Our analysis is based on both unrestricted and restricted vector autoregressions (VARs) for selected variables. As in most of the RBC literature, these VARs are obtained using calibrated decision rules and estimated laws of motion for forcing variables. In what follows, the decision rules are derived using the market features of the Burnside, Eichenbaum, and Rebelo (1993) labor-hoarding environment. The laws of motion are estimated under two distinct assumptions. First, as is standard, we assume that the law of motion involves exclusively the forcing variables. Second, we assume that the law of motion contains not only forcing variables, but also other exogenous variables.

2.1 The Market Features

The main features are that labor is indivisible, employment is preset, and effort is adjusted to ensure that the labor market clears. These features are summarized in the following social planning problem:

$$\max E_t \left[\sum_{j=0}^{\infty} \beta^j \left[\ln(C_{t+j}) + \eta N_{t+j} \ln(H - \zeta - W_{t+j} f) + \eta (1 - N_{t+j}) \ln(H) \right] \right], \quad (1.1)$$

$$C_t + I_t + G_t = Y_t, \quad (1.2)$$

$$K_{t+1} = I_t + (1 - \delta) K_t, \quad (1.3)$$

$$Y_t = K_t^{\alpha} (Z_t N_t W_t f)^{1-\alpha}, \quad (1.4)$$

where E_t represents the conditional expectation operator, Y_t is output, C_t is consumption, I_t is investment, K_t is the capital stock, N_t is the fraction of the population that is employed, and W_t is effort. The variables G_t and Z_t correspond to stochastic government expenditures and labor-augmenting technology. Also, the subjective discount factor β, the preference parameter η, the time endowment H, the fixed time cost ζ, the fixed shift length f, the depreciation rate δ, and the capital share α are the underlying parameters of the market features. Equation (1.1) describes the planner's objective, equation (1.2) is the aggregate resource constraint, equation (1.3) defines investment, and equation (1.4) describes the aggregate production function.

The forward-looking decision rules that characterize the competitive equilibrium allocation can be found by solving the planning problem (1). This planning problem, however, does not possess an analytical solution for general values of the underlying parameters. As is standard practice, we obtain an approximate solution by applying the method developed in King, Plosser, and Rebelo (1987, 1988). We implement this method as follows. First, we divide all growing variables by the level of technology to ensure the existence of a deterministic steady state, and express all variables in terms of percentage deviations from their steady state. Then, we obtain a system of linear difference equations by log-linearizing the first-order conditions of the social planning problem around the steady state. Finally, this system is solved as in Blanchard and Kahn (1980) to yield decision rules for selected nonpredetermined and predetermined (state) variables:

$$\widehat{\mathbf{m}}_t \equiv \mathbf{m}_t - \theta_{11}\mathbf{p}_t = \theta_{12}\mathbf{s}_t + \theta_{13}E_t\left[\sum_{j=1}^{\infty}\lambda^{-j}\mathbf{s}_{t+j}\right], \tag{2.1}$$

$$\widehat{\mathbf{p}}_{t+1} \equiv \mathbf{p}_{t+1} - \theta_{21}\mathbf{p}_t = \theta_{22}\mathbf{s}_t + \theta_{23}E_t\left[\mathbf{s}_{t+1}\right] + \theta_{24}E_t\left[\sum_{j=1}^{\infty}\lambda^{-j}\mathbf{s}_{t+j}\right], \tag{2.2}$$

where $\mathbf{m}_t = (\,y_t \quad c_t \quad i_t\,)'$ is the vector of nonpredetermined variables, $\mathbf{p}_t = (\,k_t \quad n_t\,)'$ is the vector of predetermined variables, and $\mathbf{s}_t = (\,z_t \quad g_t\,)'$ is the vector of forcing variables. The variables are $y_t = \ln(\widetilde{Y}_t/\widetilde{Y})$ for $\widetilde{Y}_t = Y_t/Z_t$, $c_t = \ln(\widetilde{C}_t/\widetilde{C})$ for $\widetilde{C}_t = C_t/Z_t$, $i_t = \ln(\widetilde{I}_t/\widetilde{I})$ for $\widetilde{I}_t = I_t/Z_t$, $k_{t+1} = \ln(\widetilde{K}_{t+1}/\widetilde{K})$ for $\widetilde{K}_{t+1} = K_{t+1}/Z_t$, $n_{t+1} = \ln(N_{t+1}/N)$, $z_t = \ln(\widetilde{Z}_t/\widetilde{Z})$ for $\widetilde{Z}_t = Z_t/Z_{t-1}$, and $g_t = \ln(\widetilde{G}_t/\widetilde{G})$ for $\widetilde{G}_t = G_t/Z_t$—where $\widetilde{Y}$, $\widetilde{C}$, $\widetilde{I}$, $\widetilde{K}$, N, $\widetilde{Z}$, and $\widetilde{G}$ are the steady state values. The parameters λ and θs are complex functions of the underlying parameters as well as $\widetilde{Z}$ and $\widetilde{G}/\widetilde{Y}$. We obtain values for them using the calibration of Boileau and Normandin (2001) and Burnside and Eichenbaum (1996): $\beta = 1.03^{-0.25}$, $\eta = 3.89$, $H = 1369$, $\zeta = 60$, $f = 324.8$, $\delta = 0.021$, and $\alpha = 0.344$. We also use a sample of U.S. seasonally adjusted quarterly data over the 1960:II to 1993:IV period to set $\widetilde{Z} = 1.0031$ and $\widetilde{G}/\widetilde{Y} = 0.125$ (see Data Appendix).

2.2 The Basic Law of Motion

To obtain the restricted VARs, decision rules (2) must be expressed exclusively in terms of observed variables. This requires a law of motion to construct the expectations of future forcing variables that appear in decision rules (2). As a benchmark, we posit the law of motion:[1]

$$\begin{pmatrix} z_t \\ g_t \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \rho \end{pmatrix} \begin{pmatrix} z_{t-1} \\ g_{t-1} \end{pmatrix} + \begin{pmatrix} u_{zt} \\ u_{gt} \end{pmatrix}$$

or

$$\mathbf{s}_t = \Pi_s \mathbf{s}_{t-1} + \mathbf{u}_t, \tag{3}$$

[1] A similar law is used in Burnside and Eichenbaum (1996), Burnside, Eichenbaum, and Rebelo (1993), and Christiano and Eichenbaum (1992).

where $\Omega_s = E\left[\mathbf{u}_t \mathbf{u}_t{}'\right]$. This basic law of motion implies that the relevant information set used to forecast future forcing variables contains exclusively the history of forcing variables.

An estimated version of this law of motion is used to construct the expectations of future forcing variables. OLS estimation yields $\rho = 0.969$, $E\left[u_{zt}u_{zt}\right] = 0.000084$, $E\left[u_{gt}u_{gt}\right] = 0.000689$, and $E\left[u_{zt}u_{gt}\right] = -0.000048$. The constructed expectations are then substituted in decision rules (2) to yield:

$$\widehat{\mathbf{m}}_t^b = \varphi_{ms}\mathbf{s}_t, \tag{4.1}$$

$$\widehat{\mathbf{p}}_{t+1}^b = \varphi_{ps}\mathbf{s}_t. \tag{4.2}$$

The parameters φs are functions of the calibrated parameters in (2) and the estimated parameters in (3): $\varphi_{ms} = \theta_{12} + \theta_{13}\left[I_s - \lambda^{-1}\Pi_s\right]^{-1}\lambda^{-1}\Pi_s$ and $\varphi_{ps} = \theta_{22} + \theta_{23}\Pi_s + \theta_{24}\left[I_s - \lambda^{-1}\Pi_s\right]^{-1}\lambda^{-1}\Pi_s$, where I_s is an identity matrix. The superscript b indicates that these are predicted variables using the basic law of motion.

The restricted VARs are built from the basic law of motion (3) and reduced forms (4). For example, we obtain a restricted VAR for output as follows. First, we write the reduced form for output contained in (4.1) as:

$$\widehat{y}_t^b = \varphi_{yz}z_t + \varphi_{yg}g_t, \tag{5}$$

where φ_{yz} and φ_{yg} are the elements on the first line of φ_{ms}. Then, we use the law of motion (3) to produce:

$$\begin{pmatrix} z_t \\ g_t \\ \widehat{y}_t^b \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \rho & 0 \\ 0 & \phi & 0 \end{pmatrix} \begin{pmatrix} z_{t-1} \\ g_{t-1} \\ \widehat{y}_{t-1}^b \end{pmatrix} + \begin{pmatrix} u_{zt} \\ u_{gt} \\ u_{yt}^b \end{pmatrix}$$

or

$$\mathbf{x}_{yt}^b = \Phi_y^b \mathbf{x}_{yt-1}^b + \mathbf{u}_{yt}^b, \tag{6}$$

where $\phi = \varphi_{yg}\rho$ and $u_{yt}^b = \varphi_{yz}u_{zt} + \varphi_{yg}u_{gt}$. Using this method, we obtain restricted VARs for all nonpredetermined and predetermined variables.

2.3 Augmented Laws of Motion

The basic law of motion (3) imposes the restrictive assumption that the relevant information set that agents use to forecast future forcing variables includes only the history of those forcing variables. It seems plausible, however, that agents possess extra relevant information to construct these forecasts. In what follows, we assume that this extra information is embodied in a single hidden variable h_t.[2]

For exposition purposes, consider the following law of motion:

$$\begin{pmatrix} z_t \\ g_t \\ h_t \end{pmatrix} = \begin{pmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{pmatrix} \begin{pmatrix} z_{t-1} \\ g_{t-1} \\ h_{t-1} \end{pmatrix} + \begin{pmatrix} v_{zt} \\ v_{gt} \\ v_{ht} \end{pmatrix}$$

or

$$\mathbf{w}_t = \Pi_w \mathbf{w}_{t-1} + \mathbf{v}_t, \tag{7}$$

where $\Omega_w = E\left[\mathbf{v}_t \mathbf{v}_t{}'\right]$. This law of motion implies that the relevant information set includes the history of both forcing and hidden variables. Agents make their decisions from (2) and construct their expectations of future forcing variables from (7) to obtain:

$$\widehat{\mathbf{m}}_t = \vartheta_{mw} \mathbf{w}_t, \tag{8.1}$$

$$\widehat{\mathbf{p}}_{t+1} = \vartheta_{pw} \mathbf{w}_t. \tag{8.2}$$

The parameters ϑ_{mw} and ϑ_{pw} are functions of the parameters in (2) and (7): $\vartheta_{mw} = \theta_{12} e_s + \theta_{13} e_s \left[I_w - \lambda^{-1}\Pi_w\right]^{-1} \lambda^{-1}\Pi_w$, $\vartheta_{pw} = \theta_{22} e_s + \theta_{23} e_s \Pi_w + \theta_{24} e_s \left[I_w - \lambda^{-1}\Pi_w\right]^{-1} \lambda^{-1}\Pi_w$, where I_w is an identity matrix and e_s is defined such that $\mathbf{s}_t = e_s \mathbf{w}_t$.

Unfortunately, it is difficult to built the required restricted VARs from the law of motion (7) and reduced forms (8), because they include the hidden variable h_t. The omission of this variable from (3) and (4) suggests that h_t is either unknown or unobservable by the researcher. To circumvent this problem, we apply the method developed in Boileau

[2] This variable can be interpreted as a factor common to several hidden variables.

and Normandin (2001). This method allows us to obtain laws of motion and associated reduced forms that contain only observables. Under the null hypothesis that decision rules (2) are valid, reduced forms (8) indicate that agents reveal their expectations of future forcing variables through their decisions about $\widehat{\mathbf{m}}_t$ and $\widehat{\mathbf{p}}_{t+1}$. Then, an adequate law of motion for forcing variables is obtained by replacing the hidden variable h_t by any variable included in either $\widehat{\mathbf{m}}_t$ or $\widehat{\mathbf{p}}_{t+1}$. This result in a law of motion and reduced forms that are augmented by agents' superior information.

We illustrate this procedure using output. First, we write the reduced form for output contained in (8.1) as:

$$\widehat{y}_t = \vartheta_{yz} z_t + \vartheta_{yg} g_t + \vartheta_{yh} h_t, \tag{9}$$

where ϑ_{yz}, ϑ_{yg}, and ϑ_{yh} are the elements on the first line of ϑ_{mw}. Second, we rewrite this reduced form as:

$$\begin{pmatrix} z_t \\ g_t \\ \widehat{y}_t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \vartheta_{yz} & \vartheta_{yg} & \vartheta_{yh} \end{pmatrix} \begin{pmatrix} z_t \\ g_t \\ h_t \end{pmatrix}$$

or

$$\mathbf{x}_{yt} = \Upsilon_y \mathbf{w}_t. \tag{10}$$

Third, we substitute (7) in (10) to obtain a VAR for output:

$$\begin{pmatrix} z_t \\ g_t \\ \widehat{y}_t \end{pmatrix} = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix} \begin{pmatrix} z_{t-1} \\ g_{t-1} \\ \widehat{y}_{t-1} \end{pmatrix} + \begin{pmatrix} u_{zt} \\ u_{gt} \\ u_{yt} \end{pmatrix}$$

or

$$\mathbf{x}_{yt} = \Gamma_y \mathbf{x}_{yt-1} + \mathbf{u}_{yt}, \tag{11}$$

where $\Gamma_y = \Upsilon_y \Pi_w \Upsilon_y^{-1}$ and $\mathbf{u}_{yt} = \Upsilon_y \mathbf{v}_t$. The first two equations of (11) form the law of motion for forcing variables augmented by output. In this augmented law of motion, the feedbacks from lagged $\widehat{y}_t$ to current forcing variables reflect the effects of the lagged h_t on contemporaneous forcing variables highlighted in the true law of motion (7). The last equation of (11) states that the innovation of output is a function of the innovations of forcing and hidden variables: $u_{yt} = \vartheta_{yz}u_{zt} + \vartheta_{yg}u_{gt} + \vartheta_{yh}u_{ht}$. This formulation is in accord with the notion that $\widehat{y}_t$ fully summarizes the relevant information.

Fourth, given that the augmented law of motion contains all the relevant information, we estimate an unrestricted VAR for output. OLS estimation yields $\gamma_{11} = 0.333$, $\gamma_{12} = 0.015$, $\gamma_{13} = 0.012$, $\gamma_{21} = -0.226$, $\gamma_{22} = 0.971$, $\gamma_{23} = -0.064$, $\gamma_{31} = 0.078$, $\gamma_{32} = 0.001$, and $\gamma_{33} = 1.013$. It also yields $E[u_{zt}u_{zt}] = 0.000070$, $E[u_{gt}u_{gt}] = 0.000686$, $E[u_{yt}u_{yt}] = 0.000023$, $E[u_{zt}u_{gt}] = -0.000042$, $E[u_{zt}u_{yt}] = -0.000035$, and $E[u_{gt}u_{yt}] = 0.000030$.[3] The constructed expectations are then substituted in the decision rule for output contained in (2.1) to yield:

$$\widehat{y}_t^a = \varphi_{yz}z_t + \varphi_{yg}g_t + \varphi_{yy}\widehat{y}_t. \tag{12}$$

The parameters φs are functions of the calibrated parameters in decision rules (2.1) and the estimated parameters in the unrestricted VAR (11). Also, the superscript a indicates that these variables are predicted using an augmented law of motion.

Finally, the restricted VAR for output is obtained using the augmented law of motion (11) and the reduced form (12). That is, we rewrite (12) as:

$$\begin{pmatrix} z_t \\ g_t \\ \widehat{y}_t^a \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \varphi_{yz} & \varphi_{yg} & \varphi_{yy} \end{pmatrix} \begin{pmatrix} z_t \\ g_t \\ \widehat{y}_t \end{pmatrix}$$

or

$$\mathbf{x}_{yt}^a = \Theta_y \mathbf{x}_{yt}. \tag{13}$$

Then, the restricted VAR for output is:

$$\begin{pmatrix} z_t \\ g_t \\ \widehat{y}_t^a \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix} \begin{pmatrix} z_{t-1} \\ g_{t-1} \\ \widehat{y}_{t-1}^a \end{pmatrix} + \begin{pmatrix} u_{zt} \\ u_{gt} \\ u_{yt}^a \end{pmatrix}$$

[3] Both the Aikaike and Schwartz criteria are consistent with first-order VARs for output, consumption, investment, employment, and capital. Estimates of these unrestricted VARs can be obtained from the authors.

or

$$\mathbf{x}^a_{yt} = \Phi^a_y \mathbf{x}^a_{yt-1} + \mathbf{u}^a_{yt}, \tag{14}$$

where $\Phi^a_y = \Theta_y \Gamma_y \Theta_y^{-1}$ and $\mathbf{u}^a_{yt} = \Theta_y \mathbf{u}_{yt}$. These five steps are used to obtain restricted VARs for all nonpredetermined and predetermined variables.

3. Results: Growth Rates

In this section, we evaluate whether the labor-hoarding environment with either basic or augmented laws of motion explains a number of business cycle facts. As is frequent in the RBC literature, we define the cycle component of a series from its growth rate. The business cycle facts refer to volatility, cross-correlations, and dynamic responses of the growth rates of output, consumption, and investment.

3.1. Volatility and Cross-Correlation

Throughout, we analyze several measures of volatility and correlation. The first measure of volatility is the standard deviation for the growth rates of output $\sigma_{\Delta \ln(Y)}$, consumption $\sigma_{\Delta \ln(C)}$, and investment $\sigma_{\Delta \ln(I)}$. The second measure corresponds to the ratios of standard deviations $\sigma_{\Delta \ln(C)}/\sigma_{\Delta \ln(Y)}$ and $\sigma_{\Delta \ln(I)}/\sigma_{\Delta \ln(Y)}$. The measures of correlation are the cross-correlations at different lags and leads of the growth rate of output with the growth rates of consumption and investment: $\text{corr}\,[\Delta \ln(C_t), \Delta \ln(Y_{t+k})]$ and $\text{corr}\,[\Delta \ln(I_t), \Delta \ln(Y_{t+k})]$, where $k = -4$, -2, -1, 0, 1, 2, and 4.

The observed measures of volatility and correlation are computed using usual sample estimators of standard deviation and correlation on our U.S. quarterly data. The predicted measures are constructed using the identities $\Delta \ln(Y_t) = \Delta y_t + z_t + \ln(\widetilde{Z})$, $\Delta \ln(C_t) = \Delta c_t + z_t + \ln(\widetilde{Z})$, and $\Delta \ln(I_t) = \Delta i_t + z_t + \ln(\widetilde{Z})$, as well as the appropriate restricted VARs for nonpredetermined and predetermined variables (see Technical Appendix). We also calculate the p-value of a $\chi^2(1)$ distributed statistic of the test that the ratio of predicted to observed measures is unity. For this test, we treat the observed measure as a constant and the predicted one as a random variable, where the variance of the predicted measure accounts for the uncertainty of the estimated parameters in either basic or augmented laws of motion.

Table 1 confronts observed and predicted measures of volatility. For output, the volatility predicted with augmented laws of motion is both statistically and numerically closer to the observed volatility. The sample estimate for $\sigma_{\Delta \ln(Y)}$ is 0.856 percent. The volatility (p-value) predicted with the basic law of motion is 0.787 percent (0.000). Accordingly, the ratio of predicted to observed volatility is 91.9 percent and statistically different from unity. The volatility (p-value) predicted with augmented laws of motion, however, is 0.825 percent (0.813). The ratio of predicted to observed volatility is 96.4 percent and insignificantly different from unity.

For consumption, the volatility predicted with either the basic or augmented laws of motion is numerically and statistically close to the observed volatility. The sample estimate for $\sigma_{\Delta \ln(C)}$ is 0.512 percent. The volatility (p-value) predicted by the environment with the basic law of motion is 0.488 percent (0.229) and the volatility (p-value) predicted with augmented laws of motion is 0.538 percent (0.879). This pattern extends to the measure of relative volatility $\sigma_{\Delta \ln(C)}/\sigma_{\Delta \ln(Y)}$, where the volatility predicted with both basic and augmented laws of motion statistically reproduces the observed volatility.

Finally, for investment, the volatility predicted with the basic law of motion significantly understates the observed volatility. In contrast, the volatility predicted with augmented laws of motion insignificantly overstates the observed volatility. More precisely, the sample estimate for $\sigma_{\Delta \ln(I)}$ is 2.294 percent. The volatility (p-value) predicted with the basic law of motion is 1.763 percent (0.000), while the volatility (p-value) predicted with augmented laws of motion is 3.010 percent (0.463). This pattern extends to the measure of relative volatility $\sigma_{\Delta \ln(I)}/\sigma_{\Delta \ln(Y)}$.

In sum, the labor-hoarding environment with the basic law of motion generally fails to replicate observed business cycles volatility. In contrast, the environment with augmented laws of motion statistically replicates observed business cycles volatility.

Table 2 compares observed and predicted measures of cross-correlation. For consumption, the environments with basic and augmented laws of motion generate a contemporaneous correlation that is close to the observed one. Specifically, the observed correlation is 0.764, while the correlations (p-values) predicted with basic and augmented laws of motion are 0.813 (0.437) and 0.747 (0.870). Thus, the ratios of predicted to observed correlations are 106.4 percent and 97.8 percent, and are insignificantly different from unity. At other leads and lags, the environment with the basic law of motion generates correlation that significantly underpredicts observed correlation. For example, at $k = -2$ and $k = 2$, the observed correlations are 0.296 and 0.275. The correlations (p-values) predicted with the basic law of motion are 0.061 (0.000) and 0.003 (0.000). Accordingly, the ratios of predicted to observed correlations are only 20.6 percent and 1.1 percent, and are significantly different from unity. The correlations (p-values) predicted with augmented laws of motion are 0.146 (0.609) and 0.196 (0.768), such that the ratios of predicted to observed correlations are 49.3 percent and 71.3 percent, and are both insignificantly different from unity.

For investment, the labor-hoarding environment with the basic law of motion predicts cross-correlations that are always different from observed cross-correlations. The environment with augmented laws of motion predicts cross-correlations that are statistically closer to observed ones. More precisely, the observed contemporaneous correlation is 0.827. The correlations (p-values) generated with basic and augmented laws of motion are 0.928 (0.000) and 0.804 (0.915). At $k = -2$ and $k = 2$, the observed correlations are 0.173 and 0.197, while the correlations (p-values) predicted with the basic law of motion are -0.042 (0.000) and 0.000 (0.000), whereas those predicted with augmented laws of motion are 0.159 (0.862) and 0.130 (0.353).

Overall, the cross-correlations generated by the labor-hoarding environment with the basic law of motion also deviate from observed cross-correlations. The environment with augmented laws of motion, however, generally replicates them.

3.2. Dynamic Responses

We now study the dynamic responses of the growth rates of output, consumption, and investment to both positive technology and government expenditure growth shocks. The observed responses are computed using the identities $\Delta \ln(Y_t) = \Delta y_t + z_t + \ln(\widetilde{Z})$, $\Delta \ln(C_t) = \Delta c_t + z_t + \ln(\widetilde{Z})$, $\Delta \ln(I_t) = \Delta i_t + z_t + \ln(\widetilde{Z})$, and $\Delta \ln(G_t) = \Delta g_t + z_t + \ln(\widetilde{Z})$, as well as the appropriate unrestricted VARs for nonpredetermined and predetermined variables. For example, the unrestricted VAR for output is similar to (11):

$$\mathbf{x}_{yt} = \Gamma_y \mathbf{x}_{yt-1} + \mathbf{u}_{yt}. \tag{15}$$

The predicted responses are computed from the relevant identities and restricted VARs (see Technical Appendix). We also calculate the p-value from a $\chi^2(1)$ distributed statistic of the test that the difference between predicted and observed responses is null. For this test, we treat the observed response as a constant and the predicted one as a random variable, where the variance of the predicted response accounts for the uncertainty of the estimated parameters in either the basic or augmented laws of motion.

Figures 1 and 2 depict observed and predicted dynamic responses. For output, the observed responses to both technology and government expenditure growth shocks exhibit an increase at impact followed by a decay. The dynamic responses predicted by the environment with either basic or augmented laws of motion numerically replicate this pattern well. The responses generated with the basic law of motion, however, are almost always significantly different from observed ones. With the exception of the first quarter, the responses generated with augmented laws of motion are insignificantly different from observed responses.

For consumption, the observed and predicted responses to both technology and government expenditure growth shocks display an increase at impact, followed by a second peak, and then a decay. As for output, the responses generated with the basic law of motion are most of the time significantly different from observed responses, while responses generated with augmented laws of motion are almost always insignificantly different from observed ones.

Finally, for investment, the observed responses to a technology growth shock show a large increase at impact, followed by a trough and a return to the steady state. The observed responses to a government expenditure growth shock show a small increase at impact, followed by a peak, a trough, and a gradual return to the steady state. The responses to both shocks predicted with the basic law of motion display a large increase at impact, followed by a trough and a return to the steady state. With one exception, these responses are significantly different from observed ones. The responses to a technology growth shock predicted with augmented laws of motion exhibit a large increase at impact followed by a peak, a trough, and a return to the steady state. Except for the first quarter, these responses are insignificantly different from observed responses. The responses to a government expenditure growth shock display a reduction at impact, followed by a peak, a trough, and a return. These responses are often insignificantly different from observed responses.

Thus, the predicted responses computed with both the basic and augmented laws of motion predict observed dynamics fairly well. However, responses predicted with the basic law of motion are significantly different from observed responses, while those generated with augmented laws of motion are insignificantly different from observed responses.

4. Results: Cyclical Components

In this section, we evaluate whether the labor-hoarding environment with basic and augmented laws of motion account for business cycle facts, when the cycle component of a series corresponds to the cycle definition of Beveridge and Nelson (1981). We show that the labor-hoarding environment with the basic law of motion mostly fails to reproduce observed business cycle facts, while the environment with augmented laws of motion almost always replicates them.

4.1 Volatility and Cross-Correlation

The first measure of volatility is a standard deviation for the cyclical components of the logarithms of output $\sigma_{\ln(Y)^c}$, consumption $\sigma_{\ln(C)^c}$, and investment $\sigma_{\ln(I)^c}$. The second measure corresponds to the ratios of standard deviations $\sigma_{\ln(C)^c}/\sigma_{\ln(Y)^c}$ and $\sigma_{\ln(I)^c}/\sigma_{\ln(Y)^c}$. The measures of correlation are the cross-correlations at different lags and leads of the cyclical component for the logarithm of output with the cyclical components of the logarithms of consumption and investment: $\text{corr}\left[\ln(C_t)^c, \ln(Y_{t+k})^c\right]$ and $\text{corr}\left[\ln(I_t)^c, \ln(Y_{t+k})^c\right]$, where $k = -4, -2, -1, 0, 1, 2,$ and 4.

The measures of volatility and correlation are computed using the cycle definition of Beveridge and Nelson (1981):

$$\ln(Y_t)^c = -\lim_{h\to\infty} E_t\left[\ln(Y_{t+h}) - \ln(Y_t) - h\gamma_y\right], \tag{16.1}$$

$$\ln(C_t)^c = -\lim_{h\to\infty} E_t\left[\ln(C_{t+h}) - \ln(C_t) - h\gamma_c\right], \tag{16.2}$$

$$\ln(I_t)^c = -\lim_{h\to\infty} E_t\left[\ln(I_{t+h}) - \ln(I_t) - h\gamma_i\right], \tag{16.3}$$

where γ_y, γ_c, and γ_i are the unconditional mean growth of output, consumption, and investment. The observed measures are obtained by evaluating (16) from the appropriate unrestricted VARs for nonpredetermined and predetermined variables. The predicted measures are computed from the relevant definitions and restricted VARs (see Technical Appendix). As before, we also calculate the p-value of a $\chi^2(1)$ distributed statistic of the test that the ratio of predicted to observed measures is unity.

Table 3 compares observed and predicted measures of volatility. For output, the volatility predicted with the basic law of motion severely understates the observed volatility. In contrast, the volatility predicted with augmented laws of motion is both statistically and numerically close to the observed volatility. This observed volatility $\sigma_{\ln(Y)^c}$ is 0.133. The volatility (p-value) predicted with the basic law of motion is 0.013 (0.000). Accordingly, the ratio of predicted to observed volatility is only 9.8 percent and significantly different from unity. The volatility (p-value) predicted with augmented laws of motion, however, is 0.124 (0.973). The ratio of predicted to observed volatility is 93.2 percent and insignificantly different from unity.

For consumption, the volatility predicted with augmented laws of motion is both statistically and numerically closer to the observed volatility. The observed volatility $\sigma_{\ln(C)^c}$ is 0.290. The volatility (p-value) predicted by the environment with the basic law of motion is 0.016 (0.000) and that predicted with augmented laws of motion is 0.175 (0.808). However, for $\sigma_{\ln(C)^c}/\sigma_{\ln(Y)^c}$, the environments with both basic and augmented laws of motion predict a relative volatility close to the observed one. This occurs because the environment with the basic law of motion undervalues the volatility for both output and consumption.

Finally, for investment, augmented laws of motion also outperform the basic law of motion. The observed volatility $\sigma_{\ln(I)^c}$ is 0.426. The volatility (p-value) predicted with basic and augmented laws of motion are 0.037 (0.000) and 0.320 (0.980). The environments with both basic and augmented laws of motion generate a relative volatility $\sigma_{\ln(I)^c}/\sigma_{\ln(Y)^c}$ close to the observed one. As for consumption, this arises because the environment with the basic law of motion underpredicts the volatility for both output and investment.

In sum, the labor-hoarding environment with the basic law of motion grossly understates observed business cycle volatility. This confirms the facts documented in Rotemberg and Woodford (1996) for a standard RBC economy. However, the environment with augmented laws of motion numerically and statistically replicates observed volatility.

Table 4 confronts observed and predicted measures of cross-correlation. For consumption, the environment with the basic law of motion generates cross-correlations that display the wrong sign and are significantly different from observed cross-correlations. More specifically, the observed contemporaneous correlation is 0.679. The correlation (p-value) predicted with the basic law of motion is -0.101 (0.000). Thus, the ratio of predicted to observed correlations is -14.7 percent and significantly different from unity. Conversely, the correlation (p-value) predicted with augmented laws of motion is 0.970 (0.248), such that the ratio of predicted to observed correlations is 142.8 percent and insignificantly different from unity. This pattern extends to all lags and leads. For example, the observed correlations at $k = -2$ and $k = 2$ are 0.671 and 0.654. The correlations (p-values) predicted with the basic law of motion are -0.129 (0.000) and -0.151 (0.000). Accordingly, the ratios of predicted to observed correlations are -19.2 percent and -23.1 percent, and are significantly different from unity. The correlations (p-values) predicted with augmented laws of motion are 0.948 (0.254) and 0.929 (0.286), such that the ratios are 141.3 percent and 142.0 percent and insignificantly different from unity.

For investment, the labor-hoarding environment with either basic or augmented laws of motion predicts cross-correlations that are insignificantly different from observed ones. The environment with the basic law of motion, however, greatly undervalues observed correlations. The observed contemporaneous correlation is 0.173. The correlation (p-value) generated with the basic law of motion is 0.023 (0.624) and that generated with augmented laws of motion is 0.226 (0.991). Also, the observed correlations at $k = -2$ and $k = 2$ are 0.146 and 0.185. The correlations (p-values) predicted with the basic law of motion are 0.037 (0.690) and 0.067 (0.652), whereas the correlations predicted with augmented laws of motion are 0.205 (0.989) and 0.237 (0.992).

Overall, the cross-correlations generated by the labor-hoarding environment with the basic law of motion frequently fail to reproduce observed cross-correlations. The environment with augmented laws of motion, however, always matches them.

4.2 Dynamic Responses

We finally document the dynamic responses of the cyclical components of the logarithms of output, consumption, and investment to both positive technology and government expenditure growth shocks. The observed responses are computed using the cycle definition (16), as well as the appropriate unrestricted VARs for nonpredetermined and predetermined variables. The predicted responses are computed from the relevant definition and restricted VARs (see Technical Appendix). As before, we also calculate the p-value from a $\chi^2(1)$ distributed statistic of the test that the difference between predicted and observed responses is null.

Figures 3 and 4 display observed and predicted dynamic responses. For output, the observed responses to a technology growth shock show an increase at impact and an hump-shaped return. The observed responses to a government expenditure growth shock exhibit a reduction at impact and a gradual return to the steady state. The responses predicted with the basic law of motion display the wrong sign at impact and substantially understate observed responses. The predicted responses to a technology growth shock show a small decrease at impact and a rapid return to the steady state, while the predicted responses to a government expenditure growth shock show a slight increase at impact followed by a hump-shaped return. Clearly, for both shocks, the differences between predicted and observed responses are always significantly different from zero. The dynamic responses predicted with augmented laws of motion, however, track observed responses extremely well, such that the differences between predicted and observed responses are always insignificantly different from zero.

For consumption, the observed responses to a technology growth shock display an increase at impact followed by a further increase after three quarters. The observed responses to a government expenditure growth shock exhibit a reduction at impact followed by a return to the steady state. The responses predicted with the basic law of motion significantly undervalue these responses. As for output, the responses predicted with augmented laws of motion are always insignificantly different from observed responses.

Finally, for investment, the observed responses to both technology and government expenditure growth shocks show a large increase at impact, followed by a reduction and a return to the steady state. The responses to both shocks predicted with the basic law of motion display a small increase at impact, followed by a hump-shaped return to the steady state. Once again, these responses are significantly smaller than the observed ones. In contrast, the responses predicted with augmented laws of motion are always insignificantly different from observed responses.

Hence, the predicted responses computed with the basic law of motion greatly undervalue observed responses. This accords with our empirical results that predicted measures of volatility grossly understate observed measures. The responses predicted with augmented laws of motion, however, replicate observed dynamic responses remarkably well.

5. Conclusion

In this paper, we test whether an artificial economy with labor hoarding provides an adequate explanation for observed business cycle dynamics. Importantly, our evaluation is performed using two different descriptions for the law of motion of the economy's forcing variables. The first assumes that the information set used to forecast future forcing variables contains exclusively the history of these forcing variables. This leads to a basic law of motion that only includes forcing variables. The second assumes that the relevant information set is superior and includes not only forcing variables but also hidden exogenous variables. This leads to augmented laws of motion that include both forcing and endogenous variables, where the endogenous variables replace hidden variables.

We show that omitting the hidden variables leads to serious mismeasurements of the business cycle. More precisely, we find that the labor-hoarding environment with a basic law of motion predicts volatility, cross-correlations, and dynamic responses of key macroeconomic aggregates that substantially deviate from observed ones. This holds whether we define the cycle as growth rates or as Beveridge-Nelson cyclical components, but it is especially severe for the latter. Hence, the labor-hoarding environment with a basic law of motion fails to reproduce observed business cycle facts. In contrast, our results reveal that the labor-hoarding environment with augmented laws of motion tracks observed statistics of all the key macroeconomic aggregates remarkably well. These findings are robust to the choice of the cycle measure. Hence, the environment with augmented laws of motion successfully matches observed business cycle facts.

Technical Appendix

The VARs

With the exception of the observed measures of volatility, all our computations are based on VARs. The unrestricted VARs for output, consumption, investment, employment, and capital are:

$$\mathbf{x}_{jt} = \Gamma_j \mathbf{x}_{jt-1} + \mathbf{u}_{jt}, \tag{A.1}$$

where $\Omega_{j\ell} = E[\mathbf{u}_{jt}\mathbf{u}_{\ell t}{}']$ for $j,\ \ell = y,\ c,\ i,\ n,\ k$. Also, $\mathbf{x}_{yt} = (\, z_t \quad g_t \quad \widehat{y}_t \,)'$, $\mathbf{x}_{ct} = (\, z_t \quad g_t \quad \widehat{c}_t \,)'$, $\mathbf{x}_{it} = (\, z_t \quad g_t \quad \widehat{i}_t \,)'$, $\mathbf{x}_{nt} = (\, z_t \quad g_t \quad \widehat{n}_{t+1} \,)'$, and $\mathbf{x}_{kt} = (\, z_t \quad g_t \quad \widehat{k}_{t+1} \,)'$. Some of our computations are based on the stacked version:

$$\begin{pmatrix} \mathbf{x}_{yt} \\ \mathbf{x}_{ct} \\ \mathbf{x}_{it} \\ \mathbf{x}_{nt} \\ \mathbf{x}_{kt} \end{pmatrix} = \begin{pmatrix} \Gamma_y & 0 & 0 & 0 & 0 \\ 0 & \Gamma_c & 0 & 0 & 0 \\ 0 & 0 & \Gamma_i & 0 & 0 \\ 0 & 0 & 0 & \Gamma_n & 0 \\ 0 & 0 & 0 & 0 & \Gamma_k \end{pmatrix} \begin{pmatrix} \mathbf{x}_{yt-1} \\ \mathbf{x}_{ct-1} \\ \mathbf{x}_{it-1} \\ \mathbf{x}_{nt-1} \\ \mathbf{x}_{kt-1} \end{pmatrix} + \begin{pmatrix} \mathbf{u}_{yt} \\ \mathbf{u}_{ct} \\ \mathbf{u}_{it} \\ \mathbf{u}_{nt} \\ \mathbf{u}_{kt} \end{pmatrix}$$

or

$$\mathbf{X}_t = \mathbf{\Gamma}\mathbf{X}_{t-1} + \mathbf{U}_t. \tag{A.2}$$

The relevant covariance matrices are $\mathbf{\Omega} = E\left[\mathbf{U}_t\mathbf{U}_t{}'\right]$ and $\mathbf{\Sigma} = E\left[\mathbf{X}_t\mathbf{X}_t{}'\right]$, where $\mathbf{\Sigma} = \mathbf{\Gamma}\mathbf{\Sigma}\mathbf{\Gamma}' + \mathbf{\Omega}$.

The restricted VARs are constructed from the reduced forms and the law of motion $\mathbf{w}_t = \Pi_w \mathbf{w}_{t-1} + \mathbf{v}_t$, where $\Omega_w = E[\mathbf{v}_t\mathbf{v}_t']$. When superior information is ignored, $\mathbf{w}_t = \mathbf{s}_t = (\, z_t \quad g_t \,)'$, $\Pi_w = \Pi_s$, and $\Omega_w = \Omega_s$. When superior information is considered, $\mathbf{w}_t = (\, z_t \quad g_t \quad h_t \,)'$. In both cases, the restricted VARs are:

$$\mathbf{x}^r_{jt} = \Phi^r_j \mathbf{x}^r_{jt-1} + \mathbf{u}^r_{jt}, \tag{A.3}$$

where $\Omega^r_{j\ell} = E\left[\mathbf{u}^r_{jt}\mathbf{u}^r_{\ell t}{}'\right]$ for $r = b,\ a$. Note that $\mathbf{u}^r_{jt} = \Theta_j \mathbf{u}_{jt} = \Theta_j \Upsilon_j \mathbf{v}_t$ for appropriately defined Θ_j and Υ_j. Finally, the stacked version is:

$$\begin{pmatrix} \mathbf{x}^r_{yt} \\ \mathbf{x}^r_{ct} \\ \mathbf{x}^r_{it} \\ \mathbf{x}^r_{nt} \\ \mathbf{x}^r_{kt} \end{pmatrix} = \begin{pmatrix} \Phi^r_y & 0 & 0 & 0 & 0 \\ 0 & \Phi^r_c & 0 & 0 & 0 \\ 0 & 0 & \Phi^r_i & 0 & 0 \\ 0 & 0 & 0 & \Phi^r_n & 0 \\ 0 & 0 & 0 & 0 & \Phi^r_k \end{pmatrix} \begin{pmatrix} \mathbf{x}^r_{yt-1} \\ \mathbf{x}^r_{ct-1} \\ \mathbf{x}^r_{it-1} \\ \mathbf{x}^r_{nt-1} \\ \mathbf{x}^r_{kt-1} \end{pmatrix} + \begin{pmatrix} \mathbf{u}^r_{yt} \\ \mathbf{u}^r_{ct} \\ \mathbf{u}^r_{it} \\ \mathbf{u}^r_{nt} \\ \mathbf{u}^r_{kt} \end{pmatrix}$$

or

$$\mathbf{X}_t^r = \mathbf{\Phi}^r \mathbf{X}_{t-1}^r + \mathbf{U}_t^r. \qquad (A.4)$$

The covariance matrices are $\mathbf{\Omega}^r = E\left[\mathbf{U}_t^r \mathbf{U}_t^{r\prime}\right]$ and $\mathbf{\Sigma}^r = E\left[\mathbf{X}_t^r \mathbf{X}_t^{r\prime}\right]$, where $\mathbf{\Sigma}^r = \mathbf{\Phi}^r \mathbf{\Sigma}^r \mathbf{\Phi}^{r\prime}$ $+\mathbf{\Omega}^r$.

Volatility and Correlation of Growth Rates

The observed measures of volatility and correlation for the growth rates of output, consumption, and investment are computed from sample estimates of variances and covariances in our U.S. quarterly data. Variances are computed as:

$$\sigma_x^2 = \frac{1}{T}\sum_{t=1}^{T}\left(x_t - \overline{x}\right)^2, \qquad (A.5)$$

where $\overline{x} = (1/T)\sum_{t=1}^{T} x_t$ for $x_t = \Delta\ln(Y_t)$, $\Delta\ln(C_t)$, $\Delta\ln(I_t)$. Covariances are computed as:

$$\text{cov}[x_t, \Delta\ln(Y_{t+k})] = \frac{1}{T-|k|}\sum_{t=1}^{T-|k|}\left(x_t - \overline{x}\right)\left(\Delta\ln(Y_{t+k}) - \overline{\Delta\ln(Y)}\right), \qquad (A.6)$$

for $x_t = \Delta\ln(C_t)$, $\Delta\ln(I_t)$.

The predicted measures of volatility and correlation are computed from the stacked VAR (A.4). First, we use the relations $\Delta\ln(Y_t) = \Delta y_t + z_t + \ln(\widetilde{Z})$, $\Delta\ln(C_t) = \Delta c_t + z_t + \ln(\widetilde{Z})$, $\Delta\ln(I_t) = \Delta i_t + z_t + \ln(\widetilde{Z})$ to define the vector of predicted demeaned growth rates: $\mathbf{W}_t^{rg} = \Delta\mathbf{m}_t^r + e' z_t$, where $\mathbf{W}_t^g = (\,\Delta\ln(Y_t) \quad \Delta\ln(C_t) \quad \Delta\ln(I_t)\,)'$, $\mathbf{m}_t = (\,y_t \quad c_t \quad i_t\,)'$ and $e = (\,1 \quad 1 \quad 1\,)$. Second, we employ the identities $\mathbf{m}_t = \widehat{\mathbf{m}}_t + \theta_{11}\mathbf{p}_t$ and $\mathbf{p}_{t+1} = \widehat{\mathbf{p}}_{t+1} + \theta_{21}\mathbf{p}_t$ to write $\mathbf{W}_t^{rg}$ in terms of $\mathbf{X}_t^r$:

$$\mathbf{W}_t^{rg} = \Psi_0^r \mathbf{X}_t^r + \Psi_1^r \mathbf{X}_{t-1}^r + \Psi_2^r \sum_{j=0}^{\infty} \theta_{21}^j e_p \mathbf{X}_{t-2-j}^r, \qquad (A.7)$$

where $\Psi_0^r = e_m + e' e_z$, $\Psi_1^r = -e_m + \theta_{11} e_p$, and $\Psi_2^r = -\theta_{11} + \theta_{11}\theta_{21}$. Also, e_m, e_p, and e_z are defined by $\widehat{\mathbf{m}}_t = e_m \mathbf{X}_t$, $\widehat{\mathbf{p}}_{t+1} = e_p \mathbf{X}_t$, and $z_t = e_z \mathbf{X}_t$, where $\mathbf{p}_t = (\,n_t \quad k_t\,)'$. Finally, we use (A.7) to compute the necessary moments of $\mathbf{W}_t^{rg}$. In particular, we compute:

$$\begin{aligned} E\left[\mathbf{W}_t^{rg}\mathbf{W}_t^{rg\prime}\right] =& (\Psi_0^r \otimes \Psi_0^r)\text{vec}\left[\mathbf{\Sigma}^r\right] + (\Psi_1^r \otimes \Psi_1^r)\text{vec}\left[\mathbf{\Sigma}^r\right] + \\ & (\Psi_2^r \otimes \Psi_2^r)\left[I_{p^2} - \theta_{21}\otimes\theta_{21}\right]^{-1}(e_p \otimes e_p)\text{vec}\left[\mathbf{\Sigma}^r\right] + \\ & (\Psi_1^r \otimes \Psi_0^r \mathbf{\Phi}^r)\,\text{vec}\left[\mathbf{\Sigma}^r\right] + (\Psi_2^r \otimes \Psi_0^r)\left[I_{pq} - \theta_{21}\otimes\mathbf{\Phi}^r\right]^{-1}(e_p \otimes \mathbf{\Phi}^{r\,2})\text{vec}\left[\mathbf{\Sigma}^r\right] + \\ & (\Psi_2^r \otimes \Psi_1^r)\left[I_{pq} - \theta_{21}\otimes\mathbf{\Phi}^r\right]^{-1}(e_p \otimes \mathbf{\Phi}^r)\text{vec}\left[\mathbf{\Sigma}^r\right] + \Omega_{01}^r \text{vec}\left[\mathbf{\Sigma}^r\right] + \\ & (\Psi_0^r \mathbf{\Phi}^r \otimes \Psi_1^r)\text{vec}\left[\mathbf{\Sigma}^r\right] + (\Psi_0^r \otimes \Psi_2^r)\left[I_{pq} - \mathbf{\Phi}^r \otimes \theta_{21}\right]^{-1}(\mathbf{\Phi}^{r\,2} \otimes e_p)\text{vec}\left[\mathbf{\Sigma}^r\right] + \\ & (\Psi_1^r \otimes \Psi_2^r)\left[I_{pq} - \mathbf{\Phi}^r \otimes \theta_{21}\right]^{-1}(\mathbf{\Phi}^r \otimes e_p)\text{vec}\left[\mathbf{\Sigma}^r\right] + \Omega_{02}^r \text{vec}\left[\mathbf{\Sigma}^r\right], \end{aligned} \qquad (A.8)$$

where $\text{vec}[\Omega^r_{01}] = \left((\theta_{21}e_p \otimes \mathbf{\Phi}^r)' \otimes (\Psi^r_2 \otimes \Psi^r_2)\right) \left[I_{p^3q} - (\theta_{21} \otimes I_q)' \otimes (I_p \otimes \theta_{21})\right]^{-1}$ $\text{vec}\left[(I_p \otimes e_p)\left[I_{pq} - \theta_{21} \otimes \mathbf{\Phi}^r\right]^{-1}\right]$ and $\text{vec}[\Omega^r_{02}] = \left((\mathbf{\Phi}^r \otimes \theta_{21}e_p)' \otimes (\Psi^r_2 \otimes \Psi^r_2)\right)$ $\left[I_{p^3q} - (I_q \otimes \theta_{21})' \otimes (\theta_{21} \otimes I_p)\right]^{-1} \text{vec}\left[(e_p \otimes I_p)\left[I_{pq} - \mathbf{\Phi}^r \otimes \theta_{21}\right]^{-1}\right]$. Finally, I_p, I_q, I_{p^2}, I_{pq}, and I_{p^3q} are identity matrices, where p and q refer to the dimensions of $\mathbf{p}_t$ and $\mathbf{X}_t$.

Finally, for $k = 1$, 2, and 4, we compute:

$$
\begin{aligned}
E\left[\mathbf{W}^{rg}_t \mathbf{W}^{rg\,\prime}_{t-k}\right] =& (\Psi^r_0 \otimes B_k)\text{vec}[\mathbf{\Sigma}^r] + (\Psi^r_1 \otimes \Psi^r_2 \theta^{k-1}_{21} e_p)\text{vec}[\mathbf{\Sigma}^r] + \\
& (\Psi^r_2 \otimes \Psi^r_2)\left[I_{p^2} - \theta_{21} \otimes \theta_{21}\right]^{-1}(e_p \otimes \theta^k_{21} e_p)\text{vec}[\mathbf{\Sigma}^r] + \\
& (\Psi^r_1 \otimes B_k \mathbf{\Phi}^r)\text{vec}[\mathbf{\Sigma}^r] + \\
& (\Psi^r_2 \otimes B_k)\left[I_{pq} - \theta_{21} \otimes \mathbf{\Phi}^r\right]^{-1}(e_p \otimes \mathbf{\Phi}^{r\,2})\text{vec}[\mathbf{\Sigma}^r] + \\
& \Omega^r_{k1}\text{vec}[\mathbf{\Sigma}^r] + (\Psi^r_0 \otimes \Psi^r_2)\left[I_{pq} - \mathbf{\Phi}^r \otimes \theta_{21}\right]^{-1}(\mathbf{\Phi}^r \otimes \theta^{k-1}_{21} e_p)\text{vec}[\mathbf{\Sigma}^r] + \\
& (\Psi^r_1 \otimes \Psi^r_2)\left[I_{pq} - \mathbf{\Phi}^r \otimes \theta_{21}\right]^{-1}(\mathbf{\Phi}^r \otimes \theta^k_{21} e_p)\text{vec}[\mathbf{\Sigma}^r] + \Omega^r_{k2}\text{vec}[\mathbf{\Sigma}^r], \quad (A.9)
\end{aligned}
$$

where $\text{vec}[\Omega^r_{k1}] = \left((e_p \otimes \mathbf{\Phi}^r)' \otimes (\Psi^r_2 \otimes \Psi^r_2)\right) \left[I_{p^3q} - (\theta_{21} \otimes I_q)' \otimes (I_p \otimes \theta_{21})\right]^{-1}$ $\text{vec}\left[(I_p \otimes \theta^{k-1}_{21} e_p)\left[I_{pq} - \theta_{21} \otimes \mathbf{\Phi}^r\right]^{-1}\right]$ and $\text{vec}[\Omega^r_{k2}] = \left((\mathbf{\Phi}^r \otimes \theta^{k+1}_{21} e_p)' \otimes (\Psi^r_2 \otimes \Psi^r_2)\right)$ $\left[I_{p^3q} - (I_q \otimes \theta_{21})' \otimes (\theta_{21} \otimes I_p)\right]^{-1} \text{vec}\left[(e_p \otimes I_p)\left[I_{pq} - \mathbf{\Phi}^r \otimes \theta_{21}\right]^{-1}\right]$. Finally, $B_1 = \Psi^r_0 \mathbf{\Phi}^r + \Psi^r_1$, $B_2 = \Psi^r_0 \mathbf{\Phi}^{r\,2} + \Psi^r_1 \mathbf{\Phi}^r + \Psi^r_2 e_p$, and $B_4 = \Psi^r_0 \mathbf{\Phi}^{r\,4} + \Psi^r_1 \mathbf{\Phi}^{r\,3} + \Psi^r_2 e_p \mathbf{\Phi}^{r\,2} + \Psi^r_2 \theta_{21} e_p \mathbf{\Phi}^r + \Psi^r_2 \theta^2_{21} e_p$.

Dynamic Responses of Growth Rates

The observed dynamic responses are computed using the unrestricted VARs, as well as the relations $\Delta\ln(Y_t) = \Delta y_t + z_t + \ln(\widetilde{Z})$, $\Delta\ln(C_t) = \Delta c_t + z_t + \ln(\widetilde{Z})$, $\Delta\ln(I_t) = \Delta i_t + z_t + \ln(\widetilde{Z})$, and the identities $\mathbf{m}_t = \widehat{\mathbf{m}}_t + \theta_{11}\mathbf{p}_t$, and $\mathbf{p}_{t+1} = \widehat{\mathbf{p}}_{t+1} + \theta_{21}\mathbf{p}_t$. As an example, consider the dynamic responses of output growth. First, we recursively compute the responses of y_t, z_t, n_{t+1}, and k_{t+1} using the appropriate unrestricted VARs:

$$R_{y,j} = e_3 \Gamma^j_y \Lambda_y \bar{e}' + \theta_{yn} R_{n,j-1} + \theta_{yk} R_{k,j-1}, \quad (A.10)$$

$$R_{z,j} = e_1 \Gamma^j_y \Lambda_y \bar{e}', \quad (A.11)$$

$$R_{n,j} = e_3 \Gamma^j_n \Lambda_n \bar{e}' + \theta_{nn} R_{n,j-1} + \theta_{nk} R_{k,j-1}, \quad (A.12)$$

$$R_{k,j} = e_3 \Gamma^j_k \Lambda_k \bar{e}' + \theta_{kn} R_{n,j-1} + \theta_{kk} R_{k,j-1}. \quad (A.13)$$

Then, the observed dynamic responses are computed as:

$$R_{\Delta\ln(Y),j} = R_{y,j} - R_{y,j-1} + R_{z,j}, \quad (A.14)$$

where $e_1 = (1 \quad 0 \quad 0)$, $e_2 = (0 \quad 1 \quad 0)$, and $e_3 = (0 \quad 0 \quad 1)$. Also, θ_{yn} and θ_{yk} are the elements on the first line of θ_{11}, θ_{nn} and θ_{nk} are the elements on the first line of θ_{21}, and θ_{kn} and θ_{kk} are the elements on the second line of θ_{21}. Note that $\Omega_{yy} = \Lambda_y \Lambda_y{}'$, $\Omega_{nn} = \Lambda_n \Lambda_n{}'$, and $\Omega_{kk} = \Lambda_k \Lambda_k{}'$, where Λ_y, Λ_n, and Λ_k are lower triangular matrices with positive elements on their diagonals. This allows us to extract orthogonal innovations $\mathbf{e}_{yt} = \Lambda_y^{-1} \mathbf{u}_{yt}$, where $\mathbf{e}_{yt} = (e_{zt} \quad e_{gt} \quad e_{yt})'$. The technology growth shock is measured by a positive one standard deviation in e_{zt}, $\bar{e} = e_1$. The government expenditure growth shock is measured as the sum of a positive one standard deviation in e_{zt} and e_{gt}, $\bar{e} = e_1 + e_2$, since $\Delta \ln(G_t) = \Delta g_t + z_t + \ln(\widetilde{Z})$.

Similarly, the predicted dynamic responses are constructed using the restricted VARs and relevant relations and identities. For output growth, we first compute the responses of y_t, z_t, n_{t+1}, and k_{t+1} using the appropriate restricted VARs:

$$R^r_{y,j} = e_3 \Phi_y^{r\,j} \Lambda_y^r \bar{e}' + \theta_{yn} R^r_{n,j-1} + \theta_{yk} R^r_{k,j-1}, \tag{A.15}$$

$$R^r_{z,j} = e_1 \Phi_y^{r\,j} \Lambda_y^r \bar{e}', \tag{A.16}$$

$$R^r_{n,j} = e_3 \Phi_n^{r\,j} \Lambda_n^r \bar{e}' + \theta_{nn} R^r_{n,j-1} + \theta_{nk} R^r_{k,j-1}, \tag{A.17}$$

$$R^r_{k,j} = e_3 \Phi_k^{r\,j} \Lambda_k^r \bar{e}' + \theta_{kn} R^r_{n,j-1} + \theta_{kk} R^r_{k,j-1}. \tag{A.18}$$

Then, we compute the predicted responses as:

$$R^r_{\Delta \ln(Y),j} = R^r_{y,j} - R^r_{y,j-1} + R^r_{z,j}. \tag{A.19}$$

Note that $\Omega^r_{yy} = \Lambda^r_y \Lambda^{r}_y{}'$, $\Omega^r_{nn} = \Lambda^r_n \Lambda^{r}_n{}'$, and $\Omega^r_{kk} = \Lambda^r_k \Lambda^{r}_k{}'$, where Λ^r_y, Λ^r_n, and Λ^r_k are lower triangular matrices with positive elements on their diagonals.

Volatility and Correlation of Cyclical Components

The observed measures of volatility and correlation for the cyclical components of the logarithms of output, consumption, and investment are computed from the stacked VAR $(A.2)$ using the cycle definition (16). First, we use the relations $\ln(Y_{t+h}) - \ln(Y_t) = y_{t+h} - y_t + \sum_{j=1}^{h} z_{t+j} + h \ln(\widetilde{Z})$, $\ln(C_{t+h}) - \ln(C_t) = c_{t+h} - c_t + \sum_{j=1}^{h} z_{t+j} + h \ln(\widetilde{Z})$, and $\ln(I_{t+h}) - \ln(I_t) = i_{t+h} - i_t + \sum_{j=1}^{h} z_{t+j} + h \ln(\widetilde{Z})$ to define the vector of cyclical components $\mathbf{W}^c_t = -\lim_{h\to\infty} E_t \left[\mathbf{m}_{t+h} - \mathbf{m}_t + e' \sum_{j=1}^{h} z_{t+j}\right] = \mathbf{m}_t - e' \sum_{j=1}^{\infty} E_t \left[z_{t+j}\right]$, where $\mathbf{W}^c_t = (\ln(Y_t)^c \quad \ln(C_t)^c \quad \ln(I_t)^c)'$. Second, we employ the identities $\mathbf{m}_t = \widehat{\mathbf{m}}_t + \theta_{11} \mathbf{p}_t$ and $\mathbf{p}_{t+1} = \widehat{\mathbf{p}}_{t+1} + \theta_{21} \mathbf{p}_t$ to write $\mathbf{W}^c_t$ in terms of $\mathbf{X}_t$:

$$\mathbf{W}^c_t = \Psi_0 \mathbf{X}_t + \Psi_1 \mathbf{X}_{t-1} + \Psi_2 \sum_{j=0}^{\infty} \theta_{21}^j e_p \mathbf{X}_{t-2-j}, \tag{A.20}$$

where $\Psi_0 = e_m + e' e_z \left[I_q - \mathbf{\Gamma}\right]^{-1} \mathbf{\Gamma} - \theta_{11} e_p$, $\Psi_1 = -\theta_{11}\theta_{21} e_p$, and $\Psi_2 = -\theta_{11}\theta_{21}^2$. Finally, we use $(A.20)$ to compute the necessary moments of $\mathbf{W}_t^c$. These computations use equations similar to $(A.8)$ and $(A.9)$.

Similarly, the predicted moments are computed from the stacked VAR $(A.4)$. We use the required relations and identities to write the vector of predicted cyclical components in terms of $\mathbf{X}_t^r$:

$$\mathbf{W}_t^{rc} = \Psi_0^r \mathbf{X}_t^r + \Psi_1^r \mathbf{X}_{t-1}^r + \Psi_2^r \sum_{j=0}^{\infty} \theta_{21}^j e_p' \mathbf{X}_{t-2-j}^r, \tag{A.21}$$

where $\Psi_0^r = e_m + e' e_z \left[I_q - \mathbf{\Phi}^r\right]^{-1} \mathbf{\Phi}^r - \theta_{11} e_p$, $\Psi_1^r = -\theta_{11}\theta_{21} e_p$, and $\Psi_2^r = -\theta_{11}\theta_{21}^2$. Then, we use $(A.21)$ to compute the necessary moments of $\mathbf{W}_t^{rc}$, where the computations use equations similar to $(A.8)$ and $(A.9)$.

Dynamic Responses of Cyclical Components

The observed dynamic responses are computed using the unrestricted VARs, as well as the relations $\ln(Y_t)^c = y_t - \sum_{j=1}^{\infty} E_t\left[z_{t+j}\right]$, $\ln(C_t)^c = c_t - \sum_{j=1}^{\infty} E_t\left[z_{t+j}\right]$, $\ln(I_t)^c = i_t - \sum_{j=1}^{\infty} E_t\left[z_{t+j}\right]$, and the identities $\mathbf{m}_t = \widehat{\mathbf{m}}_t + \theta_{11}\mathbf{p}_t$ and $\mathbf{p}_{t+1} = \widehat{\mathbf{p}}_{t+1} + \theta_{21}\mathbf{p}_t$. As an example, consider the dynamic responses of the cyclical components of the logarithm of output. First, we construct the infinite sum of expected future technology growth using the unrestricted VAR for output: $\sum_{j=1}^{\infty} E_t\left[z_{t+j}\right] = B_y \mathbf{x}_{yt}$, where $B_y = e_1 \left[I_3 - \Gamma_y\right]^{-1} \Gamma_y$ and I_3 is an identity matrix. Second, we use the responses $(A.10)$, $(A.12)$, and $(A.13)$. We also construct the responses of $\mathbf{x}_{yt}$ using the unrestricted VAR for output:

$$R_{x_y,j} = \Gamma_y^j \Lambda_y \bar{e}'. \tag{A.22}$$

Finally, the observed dynamic responses are computed as:

$$R_{\ln(Y)^c,j} = R_{y,j} - B_y R_{x_y,j}. \tag{A.23}$$

Similarly, the predicted dynamic responses are constructed using the restricted VARs and relevant relations and identities. For output, we first construct the infinite sum using the restricted VAR for output: $\sum_{j=1}^{\infty} E_t\left[z_{t+j}\right] = B_y^r \mathbf{x}_{yt}$, where $B_y^r = e_1 \left[I_3 - \Phi_y^r\right]^{-1} \Phi_y^r$. We then use the responses $(A.15)$, $(A.17)$, and $(A.18)$, as well as the responses of $\mathbf{x}_{yt}$ constructed using the appropriate restricted VAR:

$$R_{x_y,j}^r = \Phi_y^j \Lambda_y^r \bar{e}'. \tag{A.24}$$

Finally, the predicted dynamic responses are computed as:

$$R_{\ln(Y)^c,j}^r = R_{y,j}^r - B_y^r R_{x_y,j}^r. \tag{A.25}$$

Data Appendix

This appendix describes the U.S. seasonally adjusted quarterly data covering the 1960:II to 1993:IV period. The Citibase data mnemonics are presented on the right-hand side of the following definitions:

Pop $= P16$ is the civilian noninstitutional population aged 16 or older, which is expressed in thousands of persons for the last month of each quarter;

P $= (GCN + GCS)/(GCNQ + GCSQ)$ is the implicit 1987 deflator for consumption in nondurables and services;

C $= [(GCN + GCS) \times 1000000]/(Pop \times P)$ is per capita consumption of nondurables and services;

G $= [(GGNN + GGOSA + GGSN + GGSA) \times 1000000]/(Pop \times P)$ is per capita government expenditures on nondurables and services;

I $= [(GIF + GCD + GGE - GGNN - GGOSA - GGSN - GGSA) \times 1000000]/(Pop \times P)$ is per capita gross private domestic fixed investment plus consumer durables plus government durables and structures;

N $= (LHOURS \times 1000 \times 52)/(4 \times Pop)$ is per capita total hours worked, which is constructed from the quarterly average of the manhours employed per week reported in the household survey;

Y $= C + G + I$ is per capita output;

K $= I_{-1} + (1 - \delta)K_{-1}$ is per capita capital stock, where $K_0 = k_0 Z_{-1}$ for k_0 set to its steady state value;

W is labor effort constructed from the first-order condition $\frac{\eta N_t f}{H - \zeta - W_t f} = (1 - \alpha)\frac{Y_t}{C_t W_t}$;

Z is technology constructed from the production function $Y_t = K_t^{\alpha}\left(Z_t N_t W_t f\right)^{1-\alpha}$.

References

Blanchard, O.J. and C.M. Kahn, 1980, The solution of linear difference models under rational expectations, *Econometrica* **48**, 1305–1311.

Beveridge, S. and C.R. Nelson, 1981, A new approach to the decomposition of economic time series into permanent and transitory components with particular attention to measurement of the 'business cycle', *Journal of Monetary Economics* **7**, 151–174.

Boileau, M. and M. Normandin, 2001, Aggregate employment, real business cycles, and superior information, Accept/Revise at *Journal of Monetary Economics.*

Burnside, C. and M. Eichenbaum, 1996, Factor hoarding and the propagation of business cycle shocks, *American Economic Review* **86**, 1154-1174.

Burnside, C., M. Eichenbaum, and S. Rebelo, 1993, Labor hoarding and the business cycle, *Journal of Political Economy* **101**, 245–273.

Campbell, J.Y. and A. Deaton, 1989, Why is consumption so smooth?, *Review of Economic Studies* **56**, 357–374.

Campbell, J.Y. and R.J. Shiller, 1987, Cointegration and tests of present value models, *Journal of Political Economy* **95**, 1062–1088.

Christiano, L.J., and M. Eichenbaum, 1992, Current real-business-cycle theories and aggregate labor-market fluctuations, *American Economic Review* **82**, 430–450.

Flavin, M., 1993, The excess smoothness of consumption: Identification and interpretation, *Review of Economic Studies* **60**, 651–666.

Hansen, L.P. and T.J. Sargent, 1982, Instrumental variables procedures for estimating linear rational expectations models, *Journal of Monetary Economics* **9**, 263–296.

Hansen, L.P. and T.J. Sargent, 1980, Formulating and estimating dynamic linear rational expectations models, *Journal of Economic Dynamics and Control* **2**, 7–46.

King, R.G., C.I. Plosser, and S. Rebelo, 1988, Production, growth and business cycles: I. The basic neoclassical model, *Journal of Monetary Economics* **21**, 195–232.

King, R.G., C.I. Plosser, and S. Rebelo, 1987, Production, growth and business cycles: Technical appendix, mimeo University of Rochester.

King, R.G. and M.W. Watson, 1996, Money, prices, interest rates and the business cycle, *Review of Economics and Statistics* **78**, 35–53.

Normandin, M., 1999, Budget deficit persistence and the twin deficits hypothesis, *Journal of International Economics* **49**, 171-193.

Rotemberg, J.J. and M. Woodford, 1996, Real-business-cycle models and the forecastable movements in output, hours, and consumption, *American Economic Review* **86**, 71–89.

Shiller, R., 1972, Rational expectations and the structure of interest rates, Ph.D. dissertation (Massachusetts Institute of Technology, Cambridge).

Table 1. Volatility of Growth Rates

	$\sigma_{\Delta\ln(Y)}$	$\sigma_{\Delta\ln(C)}$	$\sigma_{\Delta\ln(I)}$	$\frac{\sigma_{\Delta\ln(C)}}{\sigma_{\Delta\ln(Y)}}$	$\frac{\sigma_{\Delta\ln(I)}}{\sigma_{\Delta\ln(Y)}}$
U.S. Data	0.856	0.512	2.294	0.598	2.679
Basic	0.787	0.488	1.763	0.620	2.241
	(0.000)	(0.229)	(0.000)	(0.290)	(0.000)
Augmented	0.825	0.538	3.010	0.652	3.649
	(0.813)	(0.879)	(0.463)	(0.735)	(0.260)

Note: σ_x denotes the standard deviation of x_t in percentages, where x_t=$\Delta\ln(Y_t)$, $\Delta\ln(C_t)$, and $\Delta\ln(I_t)$. $\Delta\ln(Y_t)$, $\Delta\ln(C_t)$, and $\Delta\ln(I_t)$ are the growth rates of per capita output, per capita consumption, and per capita investment. Numbers in parentheses are p-values associated with a $\chi^2(1)$ statistic of the test that the ratio of predicted to observed volatility is unity. This statistic uses the variance of the ratio, which is computed as $D'\Xi D$ — where D is the vector of numerical derivatives of the ratio with respect to the parameters of the appropriate laws of motion, and Ξ is the covariance matrix of these parameters.

Table 2. Cross-Correlation of Growth Rates

	corr $[\Delta \ln(C_t), \Delta \ln(Y_{t+k})]$						
k	-4	-2	-1	0	1	2	4
U.S. Data	0.087	0.296	0.344	0.764	0.395	0.275	0.128
Basic	0.008	0.061	0.064	0.813	0.090	0.003	0.004
	(0.000)	(0.000)	(0.000)	(0.437)	(0.000)	(0.000)	(0.000)
Augmented	0.099	0.146	0.141	0.747	0.609	0.196	0.018
	(0.970)	(0.609)	(0.489)	(0.870)	(0.045)	(0.768)	(0.746)

	corr $[\Delta \ln(I_t), \Delta \ln(Y_{t+k})]$						
k	-4	-2	-1	0	1	2	4
U.S. Data	0.139	0.173	0.330	0.827	0.318	0.197	0.123
Basic	-0.006	-0.042	0.282	0.928	0.126	0.000	0.001
	(0.000)	(0.000)	(0.005)	(0.000)	(0.000)	(0.000)	(0.000)
Augmented	0.000	0.159	0.585	0.804	0.448	0.130	0.014
	(0.007)	(0.862)	(0.132)	(0.915)	(0.278)	(0.353)	(0.007)

Note: corr$[x_t, \Delta \ln(Y_{t+k})]$ denotes the correlation between x_t and lags k of $\Delta \ln(Y_t)$, where $x_t = \Delta \ln(C_t)$ and $\Delta \ln(I_t)$. $\Delta \ln(Y_t)$, $\Delta \ln(C_t)$, and $\Delta \ln(I_t)$ are the growth rates of per capita output, per capita consumption, and per capita investment. Numbers in parentheses are p-values associated with a $\chi^2(1)$ statistic of the test that the ratio of predicted to observed correlation is unity. This statistic uses the variance of the ratio, which is computed as $D'\Xi D$ — where D is the vector of numerical derivatives of the ratio with respect to the parameters of the appropriate laws of motion, and Ξ is the covariance matrix of these parameters.

Table 3. Volatility of Cyclical Components

	$\sigma_{\ln(Y)^c}$	$\sigma_{\ln(C)^c}$	$\sigma_{\ln(I)^c}$	$\frac{\sigma_{\ln(C)^c}}{\sigma_{\ln(Y)^c}}$	$\frac{\sigma_{\ln(I)^c}}{\sigma_{\ln(Y)^c}}$
U.S. Data	0.133	0.290	0.426	2.178	3.201
Basic	0.013	0.016	0.037	1.310	2.994
	(0.000)	(0.000)	(0.000)	(0.236)	(0.901)
Augmented	0.124	0.175	0.320	1.413	2.591
	(0.973)	(0.808)	(0.980)	(0.304)	(0.984)

Note: σ_x denotes the standard deviation of x_t, where $x_t = \ln(Y_t)^c$, $\ln(C_t)^c$, and $\ln(I_t)^c$. $\ln(Y_t)^c$, $\ln(C_t)^c$, and $\ln(I_t)^c$ are the cyclical components of the logarithms of per capita output, per capita consumption, and per capita investment. Numbers in parentheses are p-values associated with a $\chi^2(1)$ statistic of the test that the ratio of predicted to observed volatility is unity. This statistic uses the variance of the ratio, which is computed as $D'\Xi D$ — where D is the vector of numerical derivatives of the ratio with respect to the parameters of the appropriate laws of motion, and Ξ is the covariance matrix of these parameters.

Table 4. Cross-Correlation of Cyclical Components

	corr $[\ln(C_t)^c, \ln(Y_{t+k})^c]$						
k	-4	-2	-1	0	1	2	4
U.S. Data	0.787	0.671	0.675	0.679	0.666	0.654	0.638
Basic	-0.285	-0.129	-0.136	-0.101	-0.157	-0.151	-0.117
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Augmented	0.934	0.948	0.960	0.970	0.949	0.929	0.905
	(0.563)	(0.254)	(0.249)	(0.248)	(0.264)	(0.286)	(0.354)
	corr $[\ln(I_t)^c, \ln(Y_{t+k})^c]$						
k	-4	-2	-1	0	1	2	4
U.S. Data	0.118	0.146	0.159	0.173	0.179	0.185	0.366
Basic	0.169	0.037	0.042	0.023	0.070	0.067	0.558
	(0.913)	(0.690)	(0.679)	(0.624)	(0.685)	(0.652)	(0.745)
Augmented	0.189	0.205	0.215	0.226	0.231	0.237	0.248
	(0.983)	(0.989)	(0.990)	(0.991)	(0.992)	(0.992)	(0.990)

Note: corr$[x_t, \ln(Y_{t+k})^c]$ denotes the correlation between x_t and lags k of $\ln(Y_t)^c$, where $x_t = \ln(C_t)^c$ and $\ln(I_t)^c$. $\ln(Y_t)^c$, $\ln(C_t)^c$, and $\ln(I_t)^c$ are the cyclical components of the logarithms of per capita output, per capita consumption, and per capita investment. Numbers in parentheses are p-values associated with a $\chi^2(1)$ statistic of the test that the ratio of predicted to observed correlation is unity. This statistic uses the variance of the ratio, which is computed as $D'\Xi D$ — where D is the vector of numerical derivatives of the ratio with respect to the parameters of the appropriate laws of motion, and Ξ is the covariance matrix of these parameters.

Figure 1. Dynamic Responses of Growth Rates: Basic

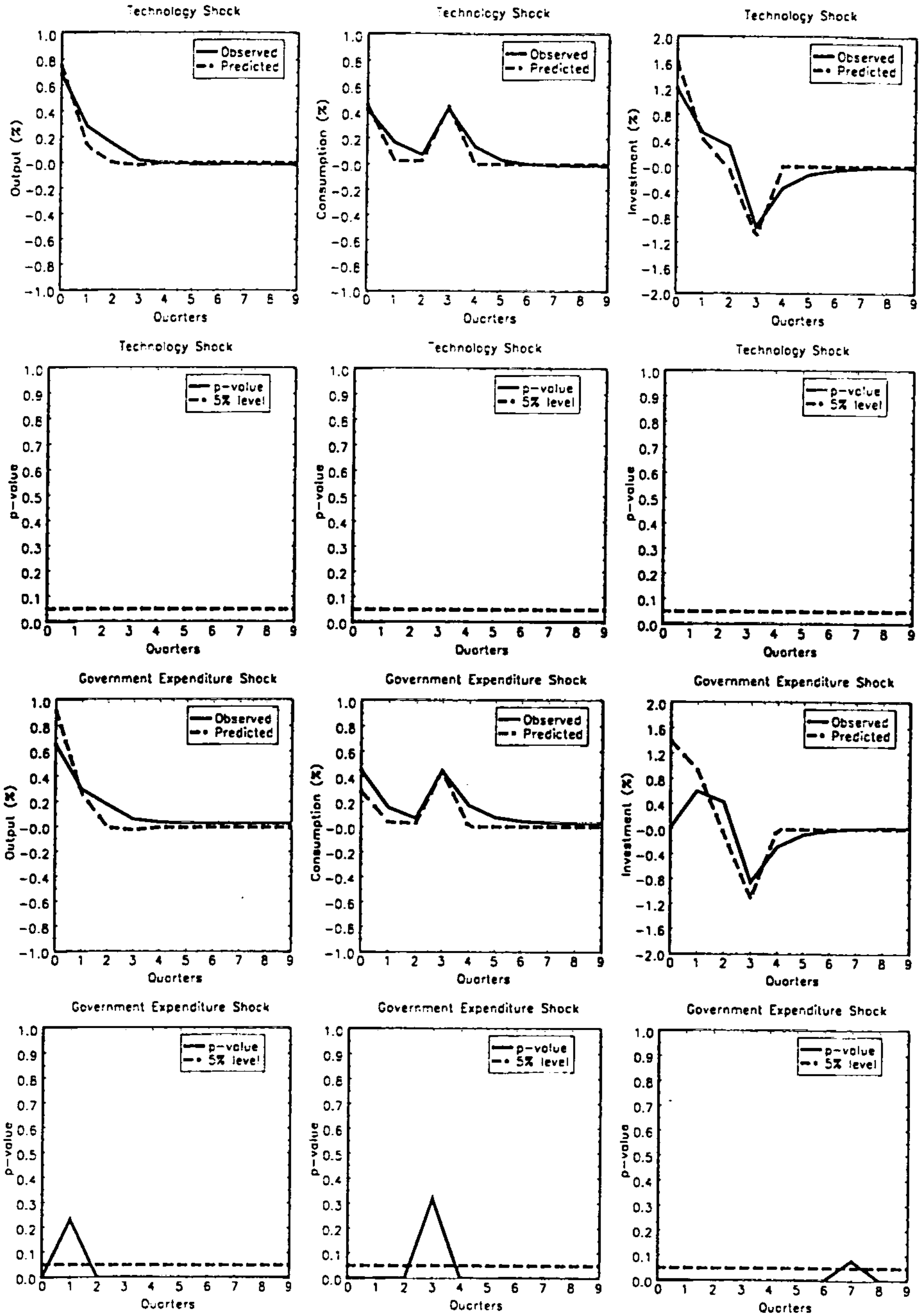

Figure 2. Dynamic Responses of Growth Rates: Augmented

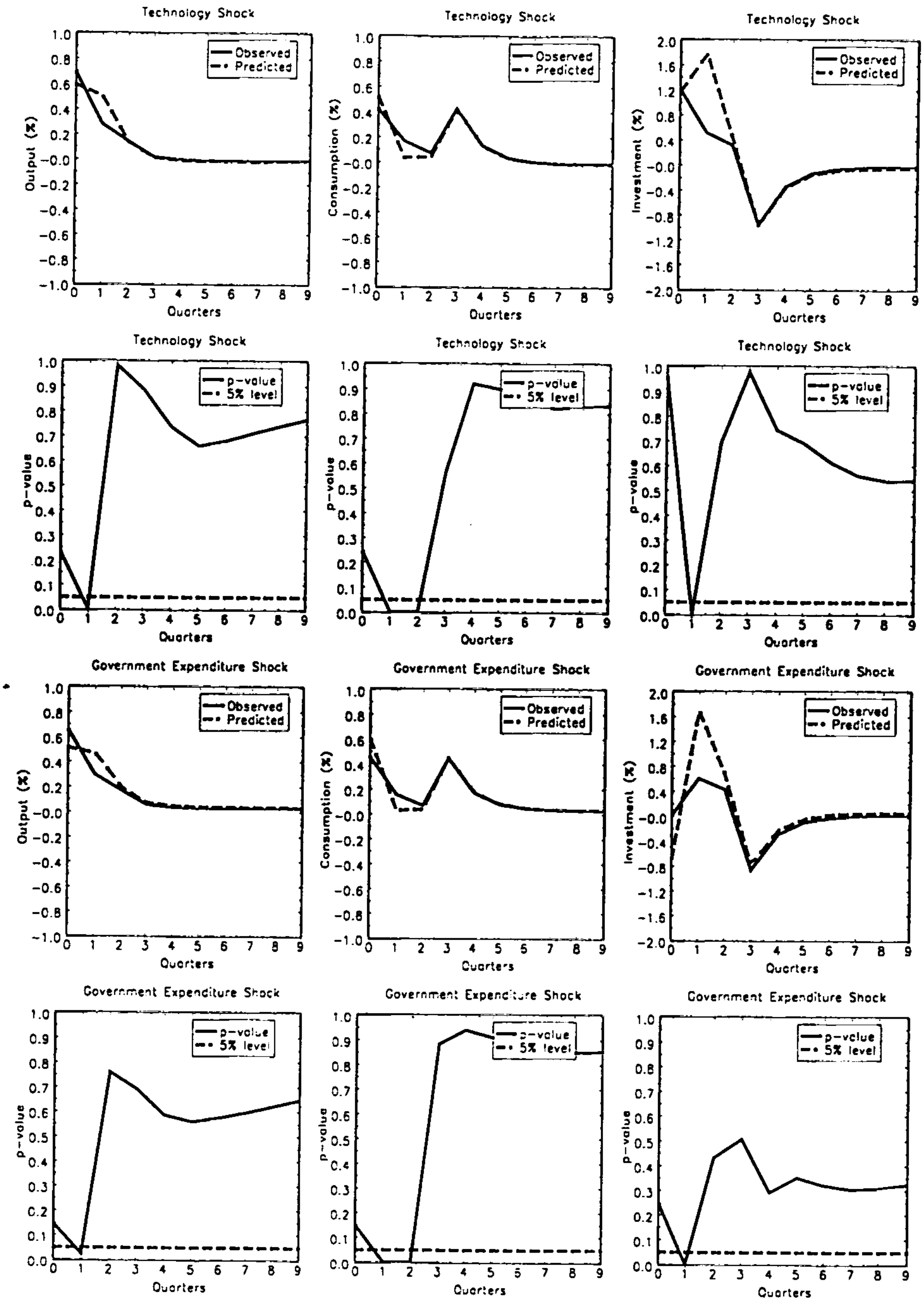

Figure 3. Dynamic Responses of Cyclical Components: Basic

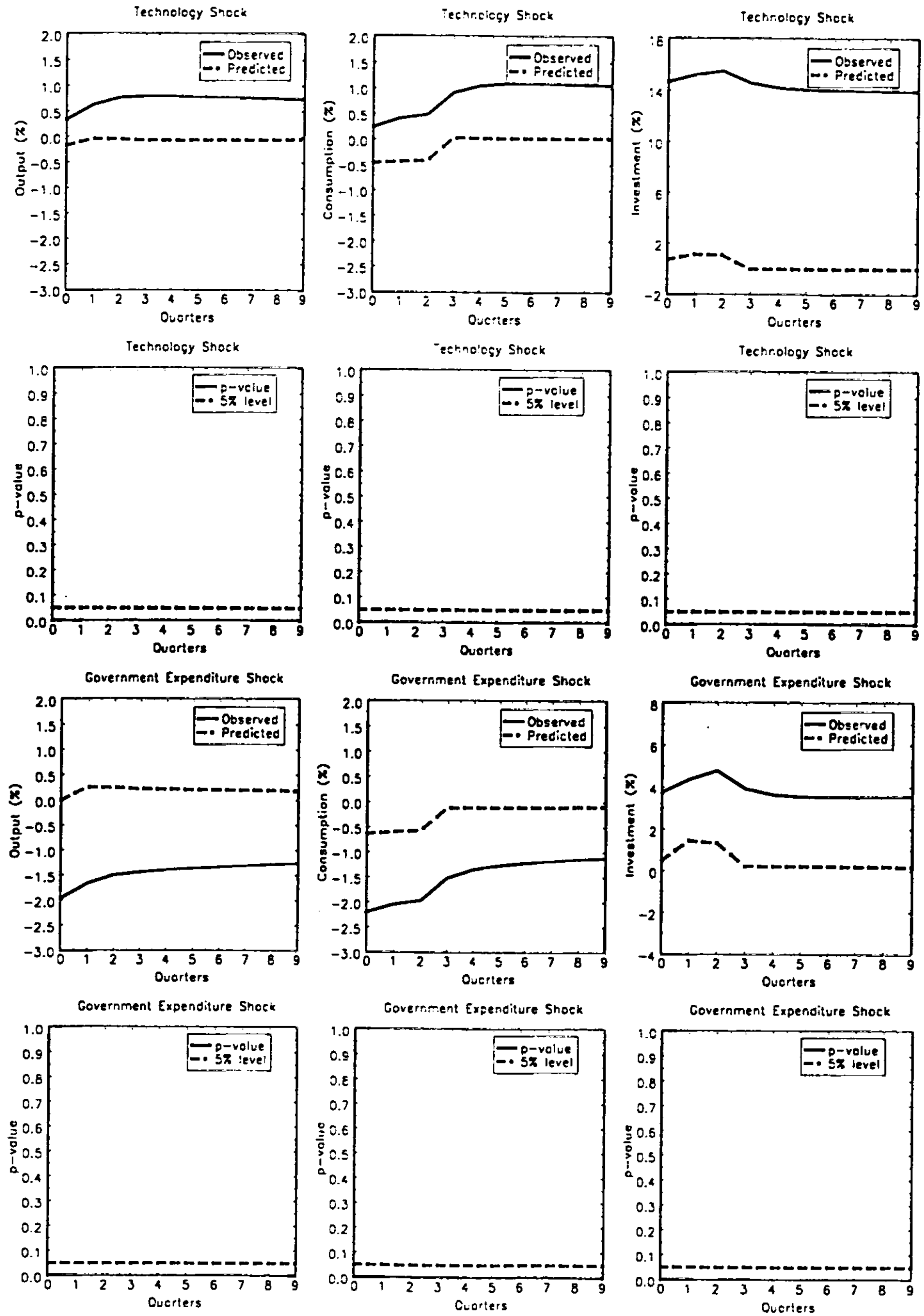

Figure 4. Dynamic Responses of Cyclical Components: Augmented