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Relative Economic Efficiency
and the Provision of Rationed Goods

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Abstract:

Public lands and rivers currently support many recreational activities for which demand seriously exceeds supply. Almost all of these recreational opportunities, such as hunting and rafting permits, are allocated either through lottery, queue, or some combination of the two. Clearly, the current allocation is economically inefficient since low- and high-value users are equally likely to receive permits. Political opposition prevents the resource manager from exclusive use of market allocations. We present a simple relative efficiency measure for evaluating the economic efficiency of alternative allocations. We also evaluate alternative allocations in which some of the available permits are distributed via auction, the remaining via lottery. The auction/lottery allocation combines the efficiency properties associated with market allocation and the desirable equity results of a lottery. In order to illustrate the simplicity and utility of our measure, we apply our method to the allocation of moose hunting permits in Maine. Using data from a 1996 moose hunting permit auction, we show that economic efficiency is greatly enhanced by auctioning less than 1% of the available permits.

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1 Introduction

Public lands and rivers in the United States currently support many recreational activities for which demand seriously exceeds the supply. Prominent examples include rafting along the Colorado River in the Grand Canyon, which now has a 14 year queue for individual rafting permits, and hunting, where in some states the number of permit applicants is 165 times the number of permits issued. Almost all of these recreational opportunities are allocated to the public either through lottery, queue, or some combination of the two. In the case of big game hunting permits, some states offer a very small number of permits, as few as a single permit, for sale through an auction. The available auction data show that many of these quantity rationed resources are *very highly* valued. For example, in a 1998 auction for a Calgary bighorn sheep permit, the winning bidder paid $405,000 US.

Economic efficiency requires that these resources flow to their most highly valued use. A properly functioning market could easily obtain an economically efficient allocation of these resources. However many citizens, even nonusers, oppose market allocation of these publicly provided goods. Typically, opponents of market allocation cite concerns over equity as their primary reason for opposition.[1] Opponents fear that given the extreme excess demand for these resources, all recreational opportunities may very well flow exclusively to high income individuals.

Clearly the current allocation of these resources is economically inefficient since low-value users are just as likely to end up with permits as high-value users. However the concern

[1]There exists opposition to market allocation even within the field of economics. Nickerson (1990) notes "markets that allocate by willingness to pay are not usable or even desirable distribution mechanisms for allocating publicly managed goods."

over equity may very well be justified. More importantly, it is easy to comprehend why so many users oppose market allocation. Under the current allocation systems, all applicants are participating in lotteries or queues for which the expected returns are positive, otherwise the applicants would not participate. One can see by the sheer excess demand numbers alone that moving to market allocation could potentially result in welfare losses, at least in expectation, for hundreds of thousands of recreationists and hunters. Thus it is perfectly rational for users to oppose market allocation and hence for us to see the political equilibrium that has resulted from greatly increased demand for these resources over the years. Economists should not expect resource managers to warmly embrace the notion of market allocation simply because it increases economic efficiency. A unilateral move to a market allocation by an individual resource manager predictably would lead to a user rebellion, a move tantamount to professional suicide by the resource manager.

While the constraints placed on allocation options by the public are real and pressing, it is in both the resources manager's and the public's interest to consider the opportunity cost of the current allocation systems. Nickerson (1990) advocates more careful analyses of outdoor regulation in order to "reduce the costs of the regulatory process both monetarily and politically." He suggests that these analyses would afford resource managers better information on the effectiveness of various policies, as well as the receptiveness of the public to various policies. In addition, selling the right to some, but not all, recreational permits may loosen an agency's financial constraint. If funds are inefficiently allocated to resource management, then additional revenues obtained through selling permits may actually increase the total number of permits available. We do not claim that this is always the case, but an individual resource manager would certainly make use of this information were it readily available. Often the information is not readily available and/or is not available in a form useful to the resource manager.

Previous work on the allocation of non-market commodities has focused primarily on the evaluation of various pricing policies and the valuation of the resource.[2] For example, Sandrey et al. (1983) analyze the demand for antlerless elk tags in Oregon and use this analysis to evaluate alternative pricing schemes. They compare various schemes on the basis of their pricing, allocative, and revenue effects. In this paper we present a new method for comparing various allocations on the basis of economic efficiency. We propose a simple relative efficiency measure for evaluating the economic efficiency of the current or a proposed allocation. The relative efficiency measure is formed using value information obtained from an auction of one or more permits. Our measure can be used to gauge the effect on economic efficiency of offering different numbers of permits through auction or lottery/queue. The resource manager can also evaluate the potential revenue that could be gained from selling one or more permits. Using the example of Maine moose hunting, we show that economic efficiency is greatly enhanced by selling a very small number of permits. Due to the simplicity of our measure, implementation only requires the use of a basic spreadsheet program. Our measure helps managers consider different allocations by providing a tool that is easy to understand and to implement.

2 Examples of Excess Demand

In this section we provide examples of recreational opportunities for which there is excess demand. Our first example, already mentioned briefly above, is the allocation of permits

[2] For a discussion of the valuation of lottery-rationed goods, such as hunting permits, see Loomis (1982), Loomis (1982), Loomis et al. (1985), Boxall (1995), and Buschena et al. (2001).

for rafting on the Colorado River in the Grand Canyon. According to the Grand Canyon Private Boaters Association, "Because of its reputation as the premier whitewater experience, no other river is more in demand...".[3] Depending upon user type, potential users enter one of two user pools, commercial or non-commercial. Permits are then allocated via queue within each user pool. Non-commercial or private permit applicants pay a $100 application fee to have their name placed on the wait list. Each year the applicant must submit a form of continuing interest in order to maintain his place in the queue. In addition to the application fees and the travel cost of the trip, those obtaining permits must pay a $100 permit fee per trip participant. Non-commercial applicants who enter the queue in 2000 can expect to wait approximately 14 years before obtaining a permit.[4] In addition to the queue there is an auxiliary system for cancellations. Each week hopeful applicants can call the agency to find out if there has been a recent cancellation. While the cancellation system favors individuals higher in the queue, there is not a secondary list for cancellations. Obtaining a cancellation permit is more a matter of luck and persistence in calling the cancellation line.

Approximately 75% of all user days are allocated to commercial outfitters, leaving only 25% for private individuals. Private individuals can also gain access to the river by signing up with a commercial outfitter. Commercial trips must be guided by a certified guide and so a private paddler is not on her own. For this reason, the serious paddler does not view private and commercial trips as perfect substitutes. For the Grand Canyon, there may be substantial efficiency and revenue gains to selling some of the commercial user days to private users.

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[3]This information was obtained from http://www.gcpba.org.

[4]Commercial applicants typically experience a maximum wait between one and two years.

Big game hunting permits are another example of excess and greatly increasing demand. For example, the Colorado Division of Wildlife has far many more applicants for big game hunting permits than available permits.[5] Table 1 summarizes year 2000 information for selected species.

Table 1
Colorado 2000 Hunting Permit Statistics

	Bighorn Sheep	Moose	Mountain Goat	Desert Sheep	Bear	Antelope	Elk
Applicants	10652	13085	5403	951	10113	73839	266299
Permits Issued	376	79	216	8	2969	12295	156762
Applicants/ Permits	28.33	165.63	25	118.9	3.4	6.0	1.7

The table suggests that big game permits in Colorado are very scare commodities. Colorado uses a fairly complicated queue/lottery system for allocating permits.[6] Applicants submit an application form by species in which they indicate their first and second most preferred region from the available regions. In addition to geographic considerations, regions may also differ by the number of permits issued. Permits available in each area are further divided into resident and non-resident. The number of resident permits available in a region exceeds the number of non-resident permits. Applicants are first sorted by area and then by points. Each applicant accumulates points according to the number of years wait and resident/non-resident status. Possible points range from zero to eleven. High point applicants receive first consideration. Permits are awarded by points in descending order until a point category is

[5]This information was obtained from http://www.dnr.state.co.us/wildlife/hunt.

[6]Buschena, Anderson et al. (2001) offer a more detailed discussion of the allocation of elk hunting permits in Colorado and introduce a method of inferring permit values under this system.

reached for which the number of permits remaining exceeds the number of applicants in the point category. At this point, permits are allocated by random drawing. Applicants must submit a check for permit fee. The Division of Wildlife returns the checks to all unsuccessful applicants.

Each year, the Colorado Division of Wildlife provides one bighorn sheep permit to the Foundation for North American Wild Sheep for auction.[7] Seventeen states in the U.S., British Columbia, Alberta, Tiberone, Baja, and Hualapai have at sometime provided one or more bighorn sheep permits for the auction held at the annual Foundation meetings. Permits are individually sold in ascending price auctions. A total of $15M (nominal) has been raised over the years through the bighorn sheep auctions.[8] In some states, the Foundation gets a percentage of each sale while other states, including Colorado, receive all of the auction proceeds. Table 2 provides the summary statistics for the permit auction data.[9]

Table 2: Auction Summary Statistics
Foundation for North American Wild Sheep Bighorn Sheep Permit

Number	Minimum	Quartile 1	Median	Average	Quartile 3	Maximum
208	$13,000	$35,000	$50,000	$72,494	$79,000	$405,000

As the summary statistics show, the winning auction bids for the bighorn sheep permits are large. Thirty-six of the winning bids exceed $100,000. Fifty percent are over $50,000. These large winning bids do not, however, speak to the efficiency of allocating other permits

[7]The Foundation for North American Wild Sheep (http://www.fnaws.org) is a non-profit organization whose mission is to improve the populations and habitat for wild sheep in North America.

[8]The Foundation has also raised a little over $2M from the auctioning of permits for other species.

[9]We thank Paula Karres of the Foundation for North American Wild Sheep for the data and information regarding the auction.

through lottery, queue, or lottery/queue combination. In order to analyze the economic efficiency of the allocation, we require information about the economic valuations of the larger group of resource users, not just the auction winners. The Foundation data only contains the winning bids.

The State of Maine auctions moose hunting permits annually.[10] In recent years Maine has offered five permits through its annual auction. Unlike the Foundation auction, the Maine auction uses a discriminative auction in which bidders submit their bids via mail. After the auction takes place, Maine holds a random lottery for the remaining moose hunting permits, typically about 2000 per year. The Maine auction procedure has the advantage over the Foundation auction in that it yields bid data for all those participating in the auction. In the next section, we propose a way of using this data to gauge the economic efficiency of the current or a proposed allocation.

3 Relative Efficiency Measure and Total Value Discussion

Now let us consider the problem from the perspective of the resource manager. Given that the good is not currently provided in the market and there exists excess demand at the current permit price, permits are effectively quantity rationed. The resource manager must decide how to allocate the rationed good among those who value the permits. Specifically, he must determine how to allocate ρ permits/goods, which we assume to be identical, among n

[10]Maine currently auctions its own permits. In 1996 and 1997 the Foundation for North American Wild Sheep auctioned a single permit for Maine in each respective year.

consumers where $n >> \rho$.[11] As discussed above, he must consider both issues of efficiency and equity/political feasibility when considering different allocations. As economists, we restrict our attention to the question of economic efficiency. We develop an efficiency measure which may be used by the resource manager, along with other considerations of equity/political feasibility, to determine the appropriate allocation.

In order to develop this efficiency measure, we first establish upper and lower bounds on the total value (*TV*) of any given allocation. The total value of an allocation is the sum of the monetary valuations of those individuals who receive permits under that particular arrangement. The monetary valuation is simply the maximum willingness to pay, also referred to in the economics literature as compensating surplus, for a permit. Note that total value can be calculated with precision only when we know each individual's valuation for the good, v_i where $i = 1, \ldots, n$. Without loss of generality, we refer to individuals by the descending rank of their valuations, $v_1 > v_2 > \ldots > v_n$. We use this notation in discussing the total value associated with various allocations.

Total value is maximized, *i.e.* economic efficiency is obtained, when the resource manager auctions off all ρ permits. This pure auction has the benefit of encouraging individuals to reveal information about their valuations.[12] Under fairly general conditions, perfect revelation in an auction is incentive compatible. Vickrey (1961) explores the Nash equilibrium bidding

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[11]While an interesting issue, we do not address the optimal choice of ρ. We assume that the number of permits to be allocated, ρ, is predetermined and fixed. Sandrey, Buccola et al. (1983) similarly assume a predetermined supply in their analysis of elk hunting permits..

[12] We use the term "pure auction" to indicate an auction of all ρ available permits.

strategies of risk neutral agents in single unit auction where individual valuations are drawn from a uniform distribution. He extends his model to include multiple unit auctions in which each bidder desires at most one unit of the good Vickrey (1962). Vickrey proves that perfect revelation is incentive compatible in a competitive uniform price auction.[13] Harris and Raviv (1981) formulate Nash equilibrium bid functions for multiple unit discriminative auctions in which each winning bidder pays an amount equal to her bid.

For the analysis that follows, we assume that the chosen auction format is competitive uniform price so that bidders reveal their valuations. Then, under a pure auction, the ρ individuals with the highest bids, and valuations by incentive compatibility, receive the permits. As discussed earlier, this economic approach is likely to receive considerable opposition from consumers.

In practice, the resource manager is unable, due to political pressures, to auction off all of the available permits. In other words, the pure auction is politically infeasible from the perspective of the resource manager. The often fierce opposition to market-based allocations on the part of some resource users prevents the resource manager from exclusive use of these allocations. Therefore, we assume that he will never choose to auction off all ρ permits. Instead, we assume that the resource manager may auction off only a subset of the total number of permits available. Specifically, he may auction off at most $\rho^{/}$ permits where $\rho^{/} \ll \rho$. Concerns for equity and revenue generation will affect the choice of $\rho^{/}$. We suspect that, due to political opposition, the resource manager can realistically allocate only a very small fraction of permits

[13] A multiple unit auction in which the price paid by all winners is equal to the first bid rejected is commonly referred to as a competitive uniform price auction.

via market-based methods.

Because the resource manager is constrained in the number of permits he may auction off, he is unable to achieve economic efficiency as obtained with a pure auction. We can calculate a *feasible* upper bound on the total value of any allocation.[14] Let the *feasible pure auction* be an auction in which all ρ' permits are allocated via auction. The total value of the feasible pure auction (FPA) allocation is given by:

(1)
$$TV_{FPA} = \sum_{i=1}^{\rho'} v_i$$

The feasible pure auction case serves as a meaningful benchmark for evaluating other potential allocations because it serves as an upper bound on the total value for any other allocation.

We can construct a measure of the relative efficiency of a given allocation of the ρ' permits, *m*, by examining the total value of the feasible pure auction and the total value of the alternative allocation, TV_m, in ratio form. The relative efficiency of allocation *m* is given by the following expression:

(2)
$$e_m = \frac{TV_m}{TV_{FPA}} = \frac{TV_m}{\sum_{i=1}^{\rho'} v_i}$$

Our measure falls in the unit interval, $0 < e_m \leq 1$, and measures the percentage of maximum

[14] Note that the true upper bound is the total value when the resource manager auctions off all ρ permits. Because the manager is restricted, in practice, from choosing this allocation, the true pure auction has limited value as a benchmark. Also, in order to calculate the total value of this allocation, we must have valuation information on at least the ρ highest valued users. As will be illustrated in the next section, this information requirement is rarely satisfied.

surplus obtained by allocation *m*, hence the term relative economic efficiency. The relative efficiency measure equals one when *m* is the feasible pure auction , but will be less than one for all other allocations.

Now we develop an expression for the relative efficiency of allocating all ρ' permits via lottery. This serves as a useful benchmark since it places a lower bound on the relative efficiency of any allocation *m* from any combination of auction and lottery. Note that when all ρ' permits are allocated via lottery, individuals are not given the opportunity to reveal any information about their valuations for the good. While the true total value of the lottery is unknown, we can characterize the form of total surplus of the lottery in expected value terms. Under the lottery, each individual receives a permit with probability $1/n$, where n is the number of lottery entrants. The expected total value of the lottery is simply the mean valuation from the population of users, μ_n, multiplied by the number of permits.

(3)
$$E\,[\,TV_L\,] = \frac{\rho'}{n}\sum_{i=1}^{n} v_i = \rho'\,\mu_n$$

This calculation requires value information on all n lottery entrants. As will be shown in the next section, the resource manager often has valuation information for only a subset of the lottery entrants. Suppose then, that the resource manager only has information on the highest n' bidders. Using the mean valuation of the available sample, $\mu_{n'}$, he can calculate an estimate of the expected total value of the lottery:

(4)
$$T\hat{V}_L = \frac{\rho'}{n'}\sum_{i=1}^{n'} v_i = \rho'\,\mu_{n'}$$

Provided $\mu_{n'} > \mu_n$, $T\hat{V}_L$ will overestimate the true total value of the lottery. Under the realistic assumption that not all individuals have identical valuations for the permits, $TV_{FPA} > T\hat{V}_L$. Using the sample bid information, we form an estimate of the expected relative efficiency of the lottery format given by:

(5)
$$\hat{e}_L = \frac{\frac{\rho'}{n'}\sum_{i=1}^{n'} v_i}{\sum_{i=1}^{\rho'} v_i} = \frac{\rho' \mu_{n'}}{TV_{FPA}}$$

The expectation of the relative efficiency of the lottery is random only in the numerator; the denominator is always given by the sum of the ρ' largest values. Similarly, the estimate of expected relative efficiency of the feasible pure auction is always 1. If we consider allocation by a combination of auction and lottery, then the expected relative efficiency of any combination allocation is bounded above by one and below by $\hat{e}_L$.

3.1 The Combination Auction/Lottery as an Information Source

From the perspective of the resource manager, both the feasible pure auction and pure lottery have advantages and disadvantages. The feasible pure auction has the advantage of maximizing total value but may have political costs. The feasible pure auction also acts as an information source of values. The pure lottery, on the other hand, is desirable in terms of equity but can potentially yield a very economically inefficient outcome if the distribution of individual values is right-skewed. In addition, the pure lottery fails to reveal information about individual valuations. In order to capture some of the respective benefits of both pure allocation methods, we consider an alternative scheme that combines the two mechanisms. The auction allows some of the permits to go to the highest valued uses while still providing a lottery for some and perhaps most users. Furthermore, the auction will yield information about individual values which can be used by the resource manager to gauge economic efficiency.

Fortunately, the resource manager is not restricted to choosing either the pure lottery or feasible pure auction allocations. Specifically, consider an allocation mechanism in which *j* permits are auctioned off while the remaining $k = \rho' - j$ permits are allocated via lottery. We consider two cases of the proposed mechanism, an auction/lottery combination in which the auction occurs before the lottery and a lottery/auction combination which reverses the order. Before turning to these two cases, note that our alternative allocation mechanism has different incentive properties than the feasible pure auction format discussed earlier. Once individuals have the opportunity to "win" a permit in either the auction or lottery (but not both), they may no longer have an incentive to submit bids equal to their valuations. The bid value will be conditioned on the fact that they still may be able to win in the lottery, which will result in the bid submitted by each individual being less than her valuation. While it is important to acknowledge this result in order to be technically above board, we believe the difference is likely to be small when *n* is large.[15] On account of our intuition, and more importantly for tractability, we continue to assume that each individual submits an auction bid equal to her valuation.

3.2 Case 1: Auction/Lottery

Suppose the resource manager decides to allocate permits in two stages. In the first stage, *j* permits are auctioned off to the *j* highest bidders, each of whom pays a price equal to the first

[15]We believe this because even with a large valuation, the expected return from the lottery is fairly small when *n* is large.

bid rejected. In the second stage, the remaining $k = \rho' - j$ permits are distributed via lottery to the remaining $n - j$ individuals. In this two-stage mechanism, the resource manager chooses j, the number of permits to be allocated via auction. Recall that the total number of permits available for allocation via auction, ρ', is fixed so that permits not allotted in the auction are distributed via lottery.

The resource manager's choice of j depends on his preferences for equity and efficiency. In order to determine how a change in j affects the relative efficiency of the mechanism, we must first derive an expression for the expected total value of the auction/lottery combination. Letting j represent the number of permits auctioned, the expected total value of the auction/lottery combination is given as follows.

(6) $$E\,[TV_{A/L}] = (\rho' - j)\left(\frac{1}{n - j}\right)\sum_{i=j+1}^{n} v_i + \sum_{i=1}^{j} v_i = (\rho' - j)\mu_{n-j} + \sum_{i=1}^{j} v_j$$

The second term in this expression is the total value of the auction portion of this two-stage mechanism. The first term is the expected total value of the lottery stage after the auction stage has proceeded. The expected value for each permit in the lottery stage, μ_{n-j}, is the mean of the population once the j highest values have been removed. By construction, we must have $\mu_n > \mu_{n-j}$.[16] We can think of the expected total value measure as being formed after the bids are submitted. The resource manager knows the bids, and therefore the auction winners, but does

[16]When n is large and j is small, the mean of the entire population, μ_n should be fairly close to μ_{n-j}.

not yet know the identities of the lottery winners.[17] If everyone entered the auction in the first stage, there would be individual valuations for all users and so the resource manager could calculate the expected total value for the combination as well as the expected relative efficiency as a function of the number of permits auctioned.[18]

(7)

$$E\ [\ e(j)\ ] = \frac{(\rho' - j)\mu_{n-j} + \sum_{i=1}^{j} v_j}{\sum_{i=1}^{\rho'} v_i}$$

As discussed earlier, the resource manager rarely has value information for all n individuals. As in the application presented later, resource managers often have access to bid values for only a subset of lottery entrants. Provided this is the case, he can calculate an estimate of $E[e(j)]$ and use this estimate to compare different allocations. Suppose the resource manager has bids for only the top $n' < n$ lottery entrants. In other words, only a portion of the lottery entrants enter the auction. In this case, μ_n , the population mean is unknown. The resource manager can, however, use the information contained in the sample of n' bidders to examine the efficiency of different allocations. Let $\mu_{n'}$ represent the mean bid of the auction entrants and $\mu_{n'-j}$ be the auction sample mean when the top j bidders are removed. Using the auction sample bid data, we can estimate the total value of the allocation as follows:

(8)

$$\hat{TV}_{A/L} = \frac{\rho' - j}{n' - j} \sum_{i=j+1}^{n'} v_i + \sum_{i=1}^{j} v_i = (\rho' - j)\ \mu_{n'-j} + \sum_{i=1}^{j} v_i$$

[17] The total value of the auction/lottery combination can also be thought of as an expected total value where the expectation is taken after the auction stage.

[18] If everyone submitted a bid, the resource manager would know the exact total value since there would be information on all users. In considering alternative allocations, the expected value would be a more useful predictor since we know that it minimizes predicted mean square error.

The estimate of relative efficiency is then given by:

(9)

$$\hat{e}(j) = \frac{(\rho' - j)\ \mu_{n'-j} + \sum_{i=1}^{j} v_i}{\sum_{i=1}^{\rho'} v_i}$$

Without bid information on all *n* lottery entrants, we are unable to calculate the actual expected efficiency measure but we can obtain an estimate by using the bid data contained in the auction sample. Fortunately, we can determine the direction and magnitude of the bias for each of our estimates.[19] Our estimate slightly overestimates the total value and relative efficiency measures. Using the auction bids only, we actually underestimate the marginal efficiency of the allocation.[20] This result is encouraging given that the marginal efficiency is the most useful measure in determining the appropriate allocation.

The combination auction/lottery allows the resource manager to gauge the relative economic efficiency of various choices of *j* and perhaps more importantly allows the resource manager to consider the consequences of changing *j*. The resource manager can combine this information and his concerns for equity/political feasibility to determine the preferred allocation. In section 4, we present an example of a resource characterized by excess demand, moose hunting permits in Maine. Using bid data from the hunting permit auction, we calculate and compare the relative efficiency measures for potential allocation schemes and for different choices of *j*.

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[19] See the appendix for the derivations.

[20] These results require that $\mu_{n'-j} \approx \mu_{n'-(j-1)}$.

3.3 Case 2: Lottery/Auction

Consider a variation of the two-step allocation method where the lottery takes place in the first stage and the auction in the second. Since the denominator of the relative efficiency measure is constant, we will limit discussion to the actual total value of this mechanism. Note that this differs from the discussion of the auction/lottery mechanism where we developed an expected total value measure. Continue to assume that bidder *i* submits a bid equal to his valuation. In order to derive an expression for the total value of the lottery/auction combination, we first examine the total value added in each stage individually. In the first stage, the resource manager allocates $k = \rho' - j$ permits via lottery. Letting *W* represent the set of *k* lottery winners, the realized total value from the first stage is given as follows.[21]

(10) $$TV_{L/A}^{S1} = \sum_{i \in W} v_i$$

Now consider the total value from the second, auction, stage. The total value of the second stage depends upon the valuations of those individuals who receive permits in the first stage. Recall that each bidder may win at most one permit. Therefore, bidders who win permits in the lottery phase are ineligible to participate in the auction phase. This implies that only $n - k = n - (\rho' - j)$ bidders are eligible to win permits in the auction stage. Now we reorder the bids and reassign a new index such that of the remaining $n-(\rho'-j)$ bidders, $v_1' > v_2' > \ldots > v_{n-(\rho'-j)}'$. Given this notation, the total value of the second stage is given by:

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[21]Note that this is the actual total value of the first stage, not the expected total value as before. The (expected) total value of the first stage is given by $E\ [TV_{L/A}^{S1}] = (\rho' - j)\ \mu_n$.

(11) $$TV_{L/A}^{S2} = \sum_{i=1}^{j} v_i^{/}$$

Summing the total values of both stages, we get an expression for the total value of the lottery/auction combination.

(12) $$TV_{L/A} = \sum_{i \in W} v_i + \sum_{i=1}^{j} v_i^{/}$$

This measure has limited practical use for the resource manager wishing to evaluate alternative allocations since it requires that he know which bidders will win the lottery and thus be eliminated from the auction.[22] We can, however, approximate the total value of the lottery/auction with the estimate of expected total value of the auction/lottery developed earlier. In large samples, when *j* is small, the total values of the auction/lottery and lottery/auction are approximately equal. The two mechanisms may differ considerably along the lines of equity/political feasibility, a consideration that the resource manager can ponder when choosing the most appropriate allocation.[23] If he chooses either combination mechanism, he must also choose *j*, the number of permits to be allocated via auction. In the next section, we use sample

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[22]One can derive an expression for the expected total surplus from the lottery/auction combination. This expression is complicated because there are *n* choose *j* distinct outcomes from the lottery, occurring with equal probability. With heterogeneous values, each outcome has its own realized total value. For this reason we do not present expected total surplus or expected efficiency.

[23]When *n* is large relative to *j*, most individuals should prefer the auction/lottery since most will not gain from the auction, regardless if held before or after the lottery. Allowing the auction to come first increases the probability of being chosen in the lottery from $1/n$ to $1/(n\text{-}j)$ and so most should prefer the auction/lottery over the lottery/auction. Only individuals $i = 1, 2, \ldots \rho^{/}$ can be positively affected by the auction in the lottery/auction combination. Individual ρ could win in the auction only if lottery winners all had higher valuations than v_ρ

bid data from a 1995 Maine moose hunting permit auction in order to examine the relative economic efficiency as well as the effect of a change in *j* on the relative efficiency measure.

4. Maine Moose Hunting

The annual allocation of moose hunting permits in the state of Maine allows us to apply the concepts presented above to a resource for which there exists significant excess demand. In 1995, the state of Maine allocated 1400 moose hunting permits among approximately 70,000 applicants. Rather than allocating all of the permits through either a pure lottery or pure auction, Maine chose a mixed allocation similar to those discussed earlier. Five permits were distributed in a discriminative auction, while the remaining permits were allocated via lottery. In 1995, 124 individuals submitted bids in the auction for five permits. The mean bid submitted was $1956.62. Table 3 displays the highest 25 bids.

Table 3: Highest $ Bids 1995 Maine Moose Permit Auction

13501.00	5150.00	3796.00	3156.00	3012.00
8000.00	5102.00	3769.59	3135.00	3001.00
7500.00	5100.00	3510.99	3112.00	2777.77
6102.00	4501.50	3469.42	3027.00	2750.00
6000.00	4053.00	3252.00	3026.00	2700.00

We use the 124 bids from this auction to examine the efficiency of different choices of *j* within the auction/lottery allocation. The data limits our analysis in two respects. First, the auction format was discriminative, which differs from the incentive compatible auction (competitive uniform price) discussed in section 3. Since our application is for illustrative purposes, we assume that bidders in the Maine auction submitted bids equal to their valuations.

Second, a significant portion of lottery entrants failed to enter the auction. We suspect that individuals who entered both the auction and lottery have high values relative to those who entered only the lottery. Since the expected total value from the lottery portion of the combination is derived from the auction data, we will overestimate the total value from the lottery stage which manifests itself in an overestimate of relative efficiency.

Assume that the resource manager has decided to allocate a maximum of 14 permits, or 1% of the total number of permits, via auction. This suggests that, for our example, $\rho' = 14$. Having chosen the maximum number of permits to be auctioned off, the resource manager's interest lies in the total value of various allocations. He can also examine the marginal efficiency of various choices of *j*, the number of permits auctioned, in order to make a more informed allocation decision.

Before examining the relative efficiency of the auction/lottery combination, however, we develop bounds on the efficiency measure. The total value of the feasible pure auction is equal to the sum of the top 14 bids, TV_{FPA} = \$79,555.50. The expected total value of a pure lottery is given as follows. Our estimate of μ_n is the mean of the auction sample, $\mu_{n'}$, which equals \$1956.62. Thus our estimate of the expected total surplus from the data is $T\hat{V}_L$ = $14\mu_{n'}$ = \$27,392.68. Therefore, we expect the relative efficiency measure of an alternative allocation *m*, $\hat{e}_m$ to satisfy the following condition:

(13) $$\hat{e}_L \approx 0.3443 \leq \hat{e}_m \leq 1$$

Consider the expected efficiency measure for the auction/lottery mechanism for different choices of *j*. We can think about the pure lottery and feasible pure auction as special cases of the combination mechanism with $j = 0$ and $j = 14$ respectively. We can calculate the efficiency gains, as measured through expected relative efficiency, of moving along this spectrum from the pure lottery format to the feasible pure auction format. Column 3 of Table 4 presents relative efficiency measures of the auction/lottery mechanism for different choices of *j*. Figure 1 also shows the relationship between *j* and $\hat{e}_{A/L}$.

Table 4: Relative Efficiency and Revenue Generated for $\rho^{/} = 14$

Permits Auctioned	TV(A/L)	Relative Efficiency	Marginal Efficiency	Revenue Generated ($)	Marginal Revenue ($)
0	27392.72	0.3443		$0.00	
1	37716.96	0.4741	0.1298	$8,000.00	$8,000.00
2	43250.53	0.5437	0.0696	$15,000.00	$7,000.00
3	48421.02	0.6086	0.0650	$18,306.00	$3,306.00
4	52396.18	0.6586	0.0500	$24,000.00	$5,694.00
5	56343.87	0.7082	0.0496	$25,750.00	$1,750.00
6	59566.10	0.7487	0.0405	$30,612.00	$4,862.00
7	62798.28	0.7894	0.0406	$35,700.00	$5,088.00
8	66084.29	0.8307	0.0413	$36,012.00	$312.00
9	68854.96	0.8655	0.0348	$36,477.00	$465.00
10	71241.49	0.8955	0.0300	$37,960.00	$1,483.00
11	73420.08	0.9229	0.0274	$41,465.49	$3,505.49
12	75611.63	0.9504	0.0275	$42,131.88	$666.39
13	77586.40	0.9752	0.0248	$45,102.46	$2,970.58
14	79555.50	1.0000	0.0248	$45,528.00	$425.54

INSERT FIGURE 1

We can also examine the relationship between *j* and the marginal efficiency of the mechanism. The resource manager is likely to find the marginal efficiency measures most informative when evaluating different choices of *j*. Table 4 and Figure 2 display this relationship.

INSERT FIGURE 2

For choices of *j* between one and eight, the marginal efficiency curve is fairly steep, suggesting large increases in efficiency from auctioning off an additional permit. After this point, however, the curve begins to flatten. For *j* larger than eight, the efficiency gains of auctioning off an additional permit are relatively small while the consequences in terms of equity may be quite large. In this example, the resource manager, who must trade off equity and efficiency, would likely choose to auction off less than eight permits.

By construction, our measure depends upon the maximum number of permits the resource manager decides he can reasonably expect to allocate via auction. While we do not view this as a shortcoming of the measure, a discussion of the sensitivity of the measure to changes in ρ' is informative. In order to examine this issue, we present the marginal efficiency of various choices of j for $\rho' = 28$. The results of these calculations are presented in Figure 3. As expected, the marginal efficiency of a given choice of *j* is dependent upon ρ'. The shape of the marginal efficiency curve is, however, similar for both levels of ρ'. This is reassuring to the extent that, for both choices of ρ', 14 or 28, the marginal efficiency curve flattens out around $j = 8$. Before making the final allocation decision, the resource manager may wish to perform a similar sensitivity test to confirm the conclusions derived from the measure at the true ρ'.

INSERT FIGURE 3

5. Discussion

In order to evaluate the efficiency of alternative allocations using our relative efficiency measure, the resource manager will need to gather the requisite data. For either the lottery/auction or the auction/lottery, the most efficient way of collecting data is to have a single form for submitting both the lottery application and bid for the auction.

Consider, for example, an online registration program for hunting permits run by a state agency. Once the agency has invested the resources necessary to successfully run the registration system, the marginal cost of implementing the auction are minimal. The agency need only add an additional section on the online registration form that allows the hunter to enter his bid for a permit. Having a combination lottery and auction form would ensure getting valuation data from all interested parties, the ideal situation. Online submission has the added advantage that data is simultaneously entered into a data base. A simple PC set up as a server could handle applications for a reasonably large permit program.[24] Once the resource manager has obtained the bid information, he need only implement a simple spreadsheet program in order to proceed with the analysis.

In addition to ease of implementation, the auction portion of the allocation generates additional revenue, which reduces the agency's operating costs and therefore its dependence on tax revenues. The last two columns of Table 4 displays the revenues generated from the permit auction of the Maine moose hunting permits. Our calculations assume that the price paid by all

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[24]The current Colorado lottery system is costly. Applicants mail in an application form and check for the amount of the permit. After permits are allocated, the checks of the unsuccessful applicants are returned by mail. Tens of thousands of unsuccessful applicants pay postage turning in their forms and then the state pays the return postage.

auction winners is equal to the first bid rejected. The final column of Table 4 displays the marginal revenue generated for different choices of j. While revenue generation is unlikely to be his main concern, the resource manager will certainly value access to revenue information when choosing an allocation.[25]

The resource manager, when determining how to regulate access to the resource, must consider both economic efficiency and equity/political feasibility associated with potential allocations. His job is complicated further in the presence of excess demand for the activity. The method presented here, when combined with information about equity and revenue generation, aides the resource manager in making a more informed allocation decision. We provide a method for determining the relative efficiency gain (loss) associated with various allocations. Our method facilitates examination of the opportunity cost of the current allocation system, an exercise beneficial to both the resource manager and the public.

[25] Sandrey, Buccola et al. (1983) suggest that a decreased emphasis on low cost allocation policies may allow state fish and wildlife departments to become more self-supporting, and therefore less reliant on state and federal support.

6. Appendix

Let hats represent our estimates of the respective measures. First, consider the total value measure. If we had bid information for all *n* lottery entrants, then the total value of the auction/lottery combination would be given by:

(14) $$TV_{A/L} = \frac{\rho' - j}{n - j} \sum_{i=j+1}^{n} v_i + \sum_{i=1}^{j} v_i \equiv (\rho' - j)\ \mu_{n-j} + \sum_{i=1}^{j} v_i$$

Our estimate of the total value is:

(15) $$\hat{TV}_{A/L} = \frac{\rho' - j}{n' - j} \sum_{i=j+1}^{n'} v_i + \sum_{i=1}^{j} v_i \equiv (\rho' - j)\ \mu_{n'-j} + \sum_{i=1}^{j} v_i$$

Notice that the second term of each expression is the value added from the auction stage. The expressions differ only in the mean valuation used in the lottery stage of the allocation. We expect the mean value of the auction entrants (minus the top *j* bidders), $\mu_{n'-j}$, to exceed the mean value of the lottery entrants (minus the top *j* bidders), μ_{n-j}. This implies that our estimate overstates the total value of the allocation by $(\rho' - j)\ (\mu_{n'-j} - \mu_{n-j})$. Note that the measure correctly predicts the value added from the auction stage of the allocation. Our estimate overstates only the value added from the lottery stage.

Now consider the relative efficiency measure. The efficiency measure and estimate are given by:

(16) $$e(j) = \frac{\frac{\rho' - j}{n - j} \sum_{i=j+1}^{n} v_i + \sum_{i=1}^{j} v_i}{\sum_{i=1}^{\rho'} v_i} = \frac{(\rho' - j)\ \mu_{n-j} + \sum_{i=1}^{j} v_i}{\sum_{i=1}^{\rho'} v_i}$$

and

(17) $$\hat{e}(j) = \frac{\frac{\rho' - j}{n' - j} \sum_{i=j+1}^{n'} v_i + \sum_{i=1}^{j} v_i}{\sum_{i=1}^{\rho'} v_i} = \frac{(\rho' - j)\ \mu_{n'-j} + \sum_{i=1}^{j} v_i}{\sum_{i=1}^{\rho'} v_i}$$

We can difference the two expressions in order to determine the bias:

(18) $$e(j) - \hat{e}(j) = \frac{(\rho' - j)(\mu_{n-j} - \mu_{n'-j})}{\sum_{i=1}^{\rho'} v_i} < 0 \;\; since \;\; \mu_{n-j} < \mu_{n'-j}$$

Therefore, in using the auction data mean, we slightly overestimate the actual relative efficiency of the allocation. Note that the bias gets arbitrarily close to zero as $\mu_{n'-j}$ approaches μ_{n-j} .

Finally, we can determine the bias of our marginal efficiency estimate as follows. The marginal efficiency of a given choice of *j* is given by:

(19) $$me(j) = e(j) - e(j-1) = \frac{[(\rho' - j)\mu_{n-j} + \sum_{i=1}^{j} v_i] - [(\rho' - (j-1))\mu_{n-(j-1)} + \sum_{i=1}^{j-1} v_i]}{\sum_{i=1}^{\rho'} v_i}$$

If we assume $\mu_{n-(j-1)} \approx \mu_{n-j}$, then the expression simplifies to:

(20) $$me(j) \doteq \frac{v_j - \mu_{n-j}}{\sum_{i=1}^{\rho'} v_i}$$

The expression for our estimate of marginal efficiency similarly reduces to the following:

(21) $$\hat{me}(j) \doteq \frac{v_j - \mu_{n'-j}}{\sum_{i=1}^{\rho'} v_i}$$

Differencing the two expressions,

(22) $$me(j) - \hat{me}(j) = \frac{\mu_{n'-j} - \mu_{n-j}}{\sum_{i=1}^{\rho'} v_i} > 0 \;\; since \;\; \mu_{n'-j} > \mu_{n-j}.$$

This suggests that using only the auction data, we slightly underestimate the marginal efficiency of a given choice of *j*.

7. Figures

Figure 1 Relative Efficiency of Auction/Lottery Combination

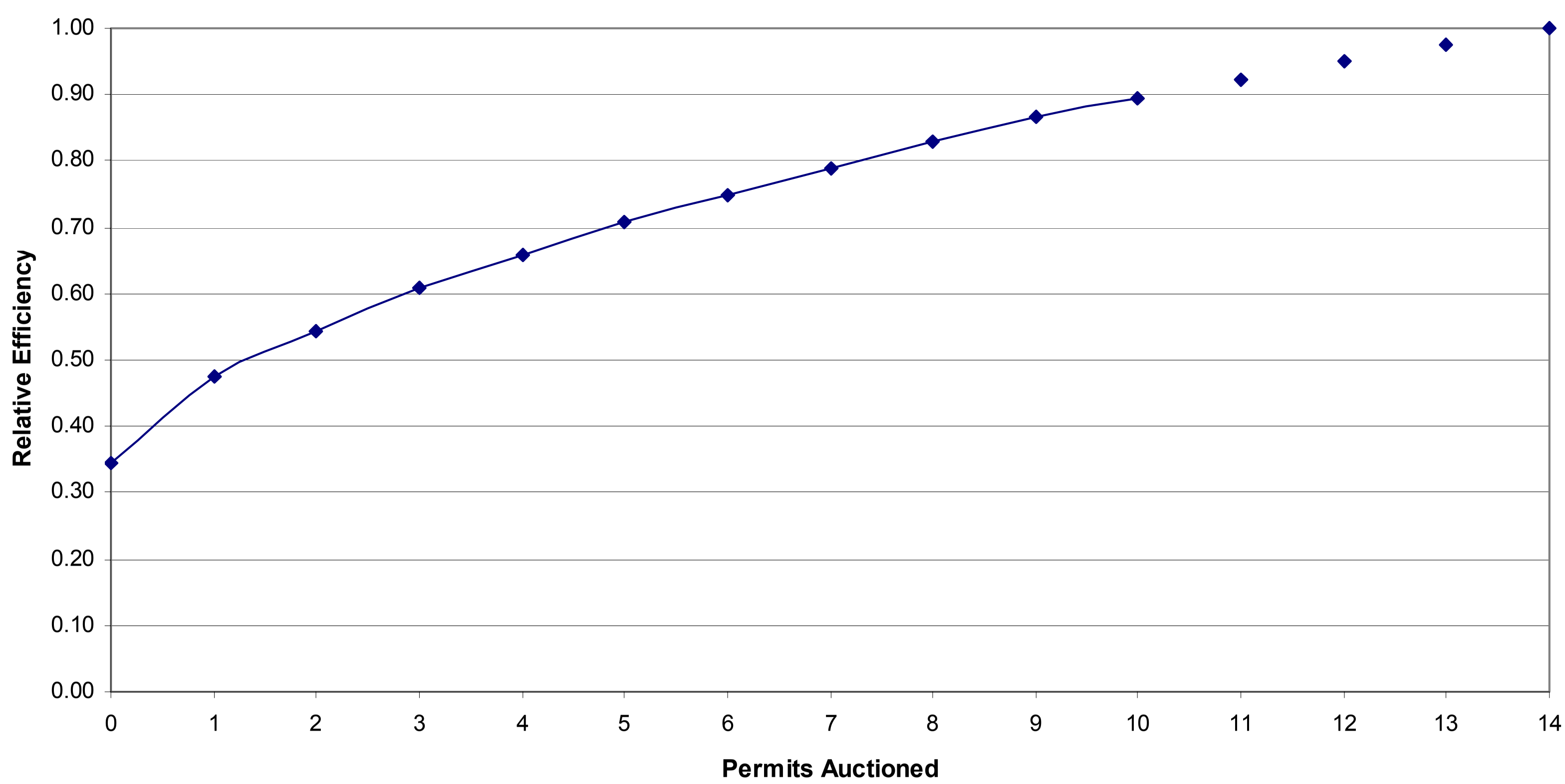


Figure 2 Marginal Efficiency of Auction/Lottery Combintation

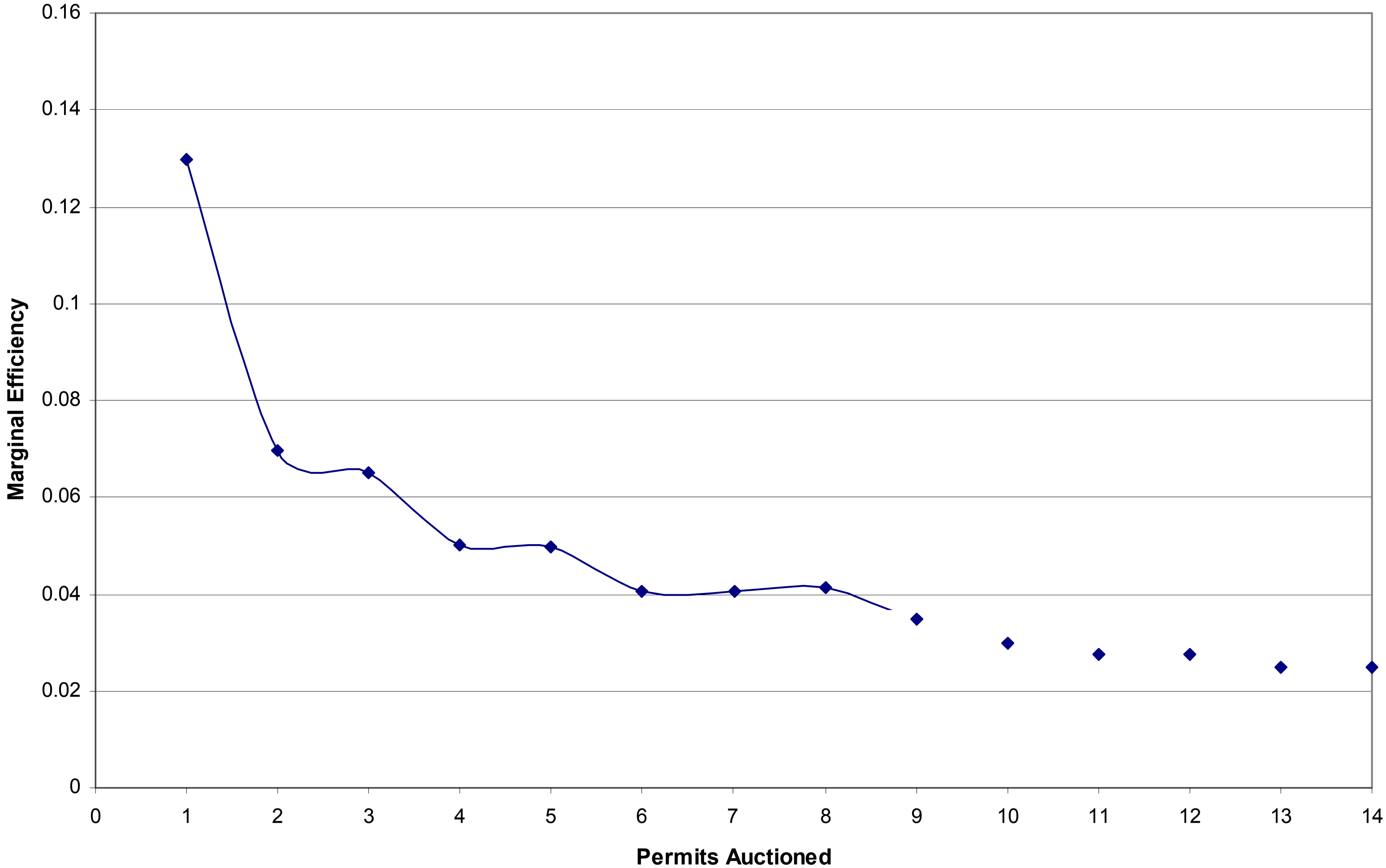
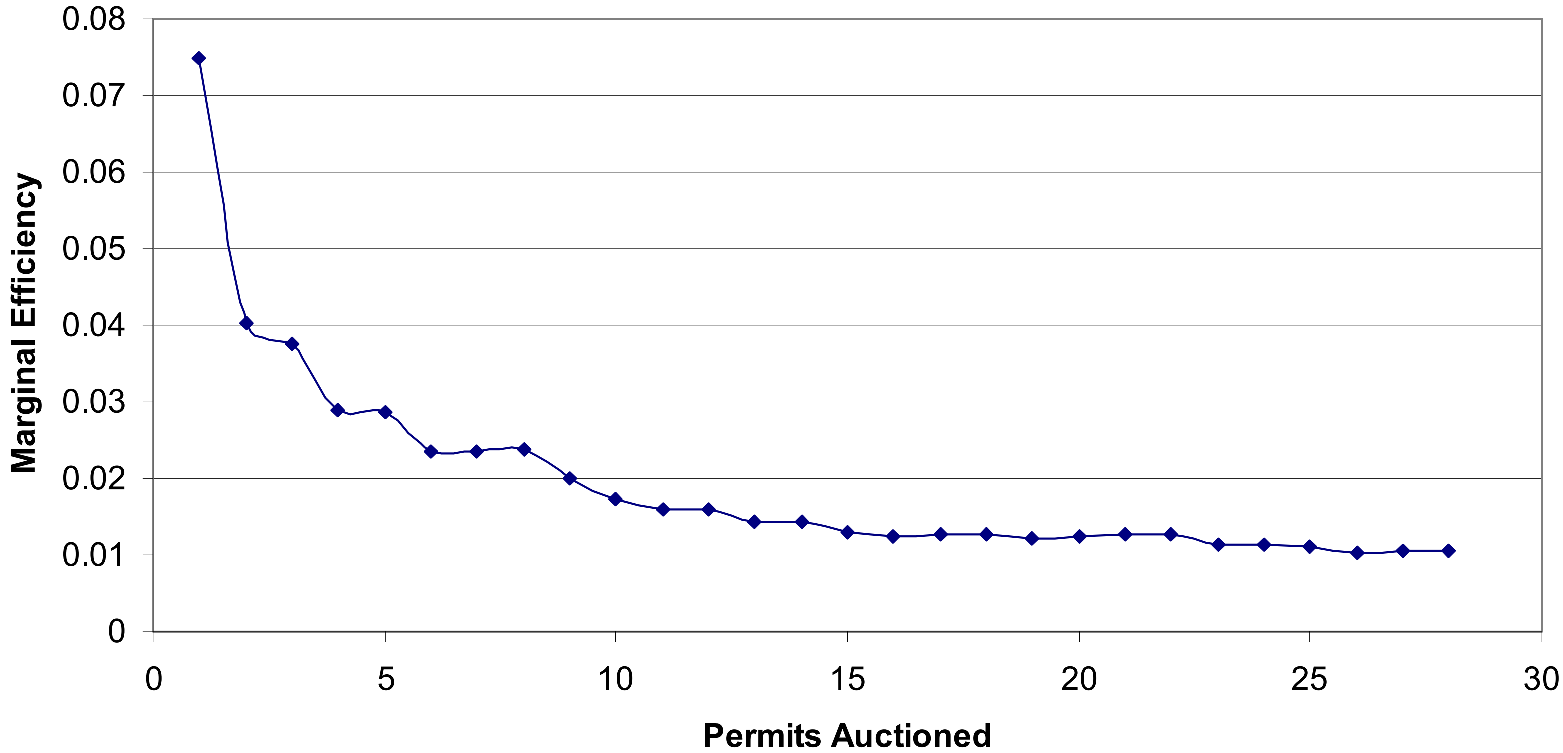


Figure 3 Marginal Effiency, 28 Feasible Permits



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