DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 00-14

Demand Growth and Strategically Useful Idle Capacity

Jack Robles
Department of Economics, University of Colorado at Boulder
Boulder, Colorado

November 2000

Center for Economic Analysis
Department of Economics

University of Colorado at Boulder
Boulder, Colorado 80309

Demand Growth and Strategically Useful Idle Capacity

Jack Robles
Dept. of Economics,
University of Colorado
roblesj@spot.colorado.edu

First Draft March 30, 2000
Revised November 14, 2000

Thanks to Garey Ramey for helpful discussions.

Demand Growth and Strategically Useful Idle Capacity

Abstract

This paper presents a model of an incumbent firm and a potential entrant. If entry occurs, then competition proceeds through cournot quantity competition. My model, like those of (e.g.) Dixit and Ware, includes a strategic use of capacity prior to entry. However, it differs in that I consider a two period model, in which second period demand is larger than first period demand. I show that demand growth can result in a use for idle capacity. This result does not require the assumption of strategic complements, and therefore works with, e.g., linear demand.

1 Introduction

The idea that a firm might create productive capacity for the purpose of preempting a (potential) rival is hardly novel. Further, there is no lack of empirical evidence of firms maintaining a persistent stock of idle capacity.[1] However, the current body of theoretical models concerning preemptive capacity has not directly addressed the issues in Justice Hand's decision on what has become the text book case on *preemptive idle capacity,* Alcoa Aluminum.[2] In his decision, Justice Hand suggests that Alcoa did "always anticipate increases in demand for ingot and be prepared to supply them." Further, he suggests that the rational behind Alcoa's behavior was that there was "no more effective exclusion than progressively to embrace each new opportunity as it opened, and to face every newcomer with new capacity..."[3] This paper investigates Justice Hand's assertion that the maintenance of idle capacity is an effective method of entry deterence when demand growth is anticipated.

While Dixit (1980) and Bulow et. al. (1985) demonstrate that capacity can be used for entry deterence, they have tied the strategic use of *idle* capacity to cases in which the output of the two firm's are strategic complements.[4] Basu and Singh (1990) have shown

[1]For example: Esposito and Esposito (1974), Cossuta and Grillo (1986), Rosembaum (1989). More to the point, Mathis and Koscianski (1995), Shaanon (1997) and Hall (1990) find evidence that firms use idle capacity to prevent entry. However, Ghemawat (1984) and Gilbert and Lieberman (1987) find otherwise.

[2]Covered in, for example, Martin (1993, pg. 98.)

[3]Hand (1941,1947).

[4]See also: Dixit (1979), Spence (1977), and Fudenberg and Tirole (1984). For elaborations see: Barnham and Ware (1993), Eaton and Lipsey (1980, 1981), Eaton and Ware(1987), Fudenberg and

that idle capacity might arise if capacity is only one of the entry deterence instruments available to the incumbent.[5] However, these models work with only a single period, and so assume away the possibility of demand growth. Consequently, the relationship between demand growth and strategically useful idle capacity suggested by Justice Hand can not be present. In this paper, I show that in the face of growing demand, entry deterence may necessitate the maintenance of idle capacity. This result requires neither strategic complements, nor the presence of additional deterence instruments. Rather, it follows from an entrant's willingness to take early losses in order to gain a foothold in a market and make profits in later stages. Knowing the value of a foothold, the incumbent firm recognizes that deterence requires sufficient capacity to make both the current and future periods unprofitable for the potential entrant. If demand is growing, then this might require maintaining idle capacity.

Beyond the Alcoa Case, these arguments shed some light on the case of Dupont's alleged attempts to achieve and maintain market dominance in titanium dioxide. Dupont's advantage was based upon lower costs from learning by doing (see e.g. Gilbert and Harris (1981).) However, part of the accusation leveled at Dupont involved the preemption of their rival's capacity investment. In particular, Dupont built a plant in DeLisle Mississippi "despite the acknowledgment that the completed facility might

Tirole (1983), Salop (1979), Schmalensee (1981), and Spence (1979).

[5]Basu and Singh (1990) use a *Stackelberg perfect* equilibrium to capture the commitment value of the Incumbent's other instruments.

have to be held in readiness for operation ... until market conditions had sufficiently improved."[6] Hence, my analysis sheds light on at least a portion of Dupont's behavior.

The formal model is a two period game with an incumbent and a potential entrant. In both periods, firms have an opportunity to build additional capacity, after which they engage in Cournot quantity competition. In the first period, the incumbent firm sets capacity before the potential entrant may do so. However, the incumbent maintains this first mover advantage in the second period only if there is no entry in the first period. Otherwise the two firms set second period capacity simultaneously. That is, the value of a toehold is modeled as the negation of the incumbent's first mover advantage. I find that a two period model behaves in many ways the same as a one period model. However, it is possible to establish that, given sufficient growth in demand, entry deterence requires the presence of idle capacity. With linear demand, one can demonstrate the existence of cases in which entry deterence with idle capacity is a subgame perfect equilibrium.

There have been previous temporal models with capacity choice. For example, Spulber (1981) also examines a two period model. However, Spulber does not distinguish between first and second period capacity, and does not allow entry to occur in the first period. Hence, even if Spulber's model did include demand growth, it would not allow the type of behavior studied here. Gilbert and Harris (1984), Eaton and Lipsey (1980) and Reynolds (1987) all examine dynamic capacity games, but assume away

[6] Dobsons et. al. (1994, pg. 166).

the possibility of idle capacity. Eaton and Lipsey (1979) consider a growing spatial market, and show that an incumbent will expand into new markets before entry occurs.[7] Reynolds (1986) performs simulations of the American aluminum industry after the Alcoa decision, and finds that a dominant firm model (Kydland, 1977) does the best job of replicating the persistent idle capacity in that market.[8]

The remainder of the paper is organized as follows: the model is presented in Section 2, and analyzed in Section 3. Section 4 concludes. Many proofs are contained in the appendix.

2 Model

The model presumes that an incumbent firm has a first mover advantage only until the entrant establishes a toehold in the industry. The timing of the model in period one is: 1) the incumbent (I) sets capacity. 2) The entrant (E) makes his entry decision, and sets capacity (if he enters,) and 3) firms in the market set output simultaneously at the intersection of their reaction functions.[9] If there is still only a single firm in the market at the beginning of the second period, then the timing in the second period mimics that in the first. However, if there was entry in period one, then in the second period firms

[7]Eaton and Lipsey suggest that there is *excess* capacity in this model. However, there is no *idle* capacity.

[8]Following the decision Kaiser and Reynolds both became significant players in the Aluminum market.

[9]Of course the reaction functions are determined by capacity choices. It is presumed that output has no consequence for later periods, and is set at the single period Nash equilibrium.

set (increases) capacity simultaneously after which they simultaneously set output.

Throughout, subscripts will denote time periods, $t = 1, 2$, and superscripts will denote either players $i = I, E$ or special outcomes. For example, q_t^i refers to firm i's output in period t. In period t, for an aggregate output Q, prices are determined by a inverse demand $P_t(Q)$. Demand in both periods is assumed to satisfy: $P_t' + P_t'' \cdot Q > 0$. Demand growth is formalized by requiring that $P_2(Q) > P_1(Q)$ and that $|P_2'| \leq |P_1'|$ for all aggregate outputs Q. This is satisfied, for example, with linear demands: $P_t = a_t - bQ$ with $a_2 > a_1$. These assumptions guarantee that reaction function are downward sloping in rival output, and shift outward between periods. Capacity, K_t^i is modeled as a commitment on marginal costs. Hence total operating costs for firm i in period t are $F + cK_t^i + c[q_t^i - K_t^i]^+$. Firms may not decrease their capacity so that $K_1^i \leq K_2^i$ for both firms. In addition to the costs above, there is a sunk cost of $\bar{F}$ which must be paid upon entry into the market. It will be convenient to denote by $\pi_t^i(q_t^I, q_t^E) = (P_t - c)q_t^i$, the firm's 'variable' profits.

Let $R_t(\cdot, K)$ be the reaction function in period t for a firm in the market with capacity K. Let $\underline{R}_t(\cdot) = R_t(\cdot, 0)$ and $\bar{R}_t(\cdot) = R_t(\cdot, \infty)$. A superscript i on any of these functions indicates that it is a firm i reaction function.

My results depend upon Ware's (1984) analysis of a single period capacity setting game, so let us suppress the time subscripts for the moment. Denote the (zero capacity) Cournot equilibrium as the point $CN = (CN^I, CN^E)$ (throughout a superscript i

denotes the projection onto q^i) and denote the point where $\bar{R}^I$ and $\underline{R}^E$ intersect as V.[10] In the Dixit (1980) model, the dominant firm sets capacity so as to make his preferred point on $\underline{R}^E$ between CN and V the Nash equilibrium of the post entry output game. Presuming that both points are feasible, he chooses between accommodating entry at the stackelberg point S and detering entry by committing to the limit output. Ware (1984) modifies Dixit's model by allowing the (potential) entrant to set capacity as well. At this point, the entrant has the commitment opportunity, and sets his capacity to choose a point on $R^I(\cdot, K^I)$ between the intersections with $\underline{R}^E$ (point U) and $\bar{R}^E$ as the equilibrium of the quantity setting game. In equilibrium, the entrant never uses this ability, but its presence constrains the incumbent's capacity choice. Specifically, consider the point $\tilde{S}$, the entrants preferred point on $\bar{R}^I$ between U and V.[11] If the incumbent sets her capacity too high, then the Entrant will prefer this point to the intersection of $R^I(\cdot, K^I)$ and $\underline{R}^E$. Let W be the point on $\underline{R}^e$ such that $\pi^E(W) = \pi^E(\tilde{S})$. Clearly the incumbent firm must have higher profits at W, than at S. Hence in the Ware (1984) one period model, the incumbent firm does not set capacity higher than W^I. Deterence is only possible if the limit output is less than W^I, and a Stackelberg leader would set her capacity to choose her preferred point on $\underline{R}^E$ between CN and W. Call this the generalized Stackelberg point, and denote it $\hat{S}$.

[10]Since the model is symmetric, $CN^I = CN^E$. The distinction between firms is made only to keep notation standard.

[11]This should perhaps be spoken of as the constrained preferred point, since it incorporates the constraint that the entrant can not commit to more output than he produces at the point U.

3 Analysis

Equilibria can be placed within three classes based upon when entry takes place. There are: *entry equilibria*, with entry in the first period; *delayed entry equilibria* with entry in the second period; and *deterence equilibria*, with no entry what so ever. The analysis of a delayed entry equilibrium adds little to Ware (1984). Further, as we shall see below, there is never idle capacity in a delayed entry equilibrium. Consequently, I focus on *entry equilibria*, and *deterence equilibria*.

The first step is to determine behavior in the second period. Second period output is set by the intersection of the reaction functions $R_2^i(\cdot, K_2^i)$. Since the game ends after the second period, no firm will build idle capacity in the second period.[12] Furthermore, since the only purpose for idle capacity in the first period is to influence the second period, no firm would build capacity in the first period beyond what is used in the second period. Hence in equilibrium, $q_2^i \geq K_2^i$ for both firms.

Let Ω be the intersection of the second period reaction function if capacity is left unchanged between periods. For example, $R_2^I(\Omega^E, K_1^I) = \Omega^I$. Proposition 1 gathers and formalizes the above observations.

Proposition 1 *In any equilibrium, $K_1^i \leq \Omega^i \leq W_2^I$.*

In Proposition 1, W_2^I is simply a number, which serves as a bound on both firm's

[12]This follows from exactly the same logic which lies behind Dixit's (1980) results.

capacity. In the current analysis, it is not a priori clear which firm will set the largest capacity. In fact, as Proposition 8 (Appendix) demonstrates, there are a continuum of second period equilibrium continuations following first period entry. In particular, consider the outcome if the incumbent (resp. entrant) has a first mover advantage in the second period. If both firms anticipate this outcome, then they are both choosing a best response. Hence, an outcome in which either of the firms has the ability to commit to his second period capacity is a equilibrium when the firms choose simultaneously. Further there is a full range of equilibria 'in between' these two cases which might be thought to correspond to intermediate distributions of commitment power. I make the following assumption to avoid multiplicity of equilibria.

Assumption E In an entry equilibrium:1) $q_2^i = \Omega^i$ for $i = I, E$,

2) $K_1^E \geq q_1^E$.

Part 2 of Assumption E is innocuous, and merely serves to make the statement of Propositions easier.[13] Under Assumption E1, output in both periods of an entry equilibrium is determined by the first period capacity choice. If both firms have $K_1^i \leq CN_2^i$, then CN_2 is the second period output. If one firm has $K_1^i > CN_2^i$, then that firm's first period capacity (and the other firm's reaction function) determine second period capacity. That is, second period output is chosen as if there were no second period capacity decision.[14] This reflects the idea that capacity is a commitment device,

[13]See Proposition 9 in the Appendix.
[14]The ability to set capacity in the second still plays a role, in that it limits the incumbent's ability

and once entry occurs, the incumbent has lost his first mover advantage, and hence his commitment advantage. In Section 4, I argue that Assumption E Part 1 rules out implausible equilibria, and captures the desirable aspects of a forward induction argument (as in Bagwell and Ramey, 1996.)

Since adding another period to the game has not changed the fundamental role of capacity, some aspects of equilibria should remain qualitatively unchanged. Capacity should only be built if it has commitment value, in either the first or second period. The incumbent's first mover advantage should, in equilibrium, leave the entrant without a desire to use his capacity for commitment. That is, the Entrant, should he enter in the first period, should build only capacity he will use in the first period. And finally, the incumbent should, at a minimum be able to guarantee himself the modified Stackelberg outcome, $\hat{S}_1$, in the first period.

Proposition 2 *Presume Assumption E holds. In an entry equilibrium, the entrant's first period capacity is equal to his first period output, and the incumbent's first period capacity is equal to her output in either the first or second period.*

In a deterence equilibrium, if the incumbent's first period capacity is greater than her first period output, then her capacity is greater than her second period Cournot output.

In any equilibrium, the incumbent's first period output is greater than or equal to the

to set a high capacity in the first period. That is, Assumption E1 is not meant to negate Proposition 1, but only to select amongst the possible equilibria.

minimum of her first period monopoly and first period generalized Stackelberg output.

Proposition 2 implies that idle capacity is not a possibility in a *delayed entry equilibrium.* Let $\underline{M}_t = (\underline{R}_t^I(0), 0)$. Clearly the first term, $\underline{M}_t^I$, is the monopoly output. Let $\bar{M}_t = (\bar{R}_t^I(0), 0)$. $\bar{M}_t^I$ is the monopoly output for a firm with no marginal costs. Consider an outcome with delayed entry and idle first period capacity. If capacity is left idle in the first period, then $K_1^I > \bar{M}_1^I$. Since the entrant wishes to enter in the second period, but not in the first, it must be the case that $\pi_1^E(\tilde{S}_1) \leq F$. Hence $\pi_1^E(W_1) \equiv \pi_1^E(\tilde{S}_1) \leq F$. Now consider an outcome which is the same, except that $K_1^I = \max\{W_1^I, \underline{M}_1^I\} < \bar{M}_1^I$. The entrant's incentive to enter in the second period is unchanged, and so we may expect the same second period profits for both firms. However, the incumbent no longer has to pay the first period costs of maintaining idle capacity, and is producing closer to the profit maximizing output, $\underline{M}_1^I$, in the first period. Clearly the incumbent's profits are higher, and the original outcome was not an equilibrium.

Proposition 3 *There is never idle capacity in a delayed entry equilibrium.*

In cases where idle first period capacity is possible, that possibility must depend upon the amount of demand growth.

Assumption G *There is sufficient growth in demand that commitment in the second period requires idle capacity in the first:* $CN_2^I > \bar{M}_1^I$.

By the definition of $\bar{M}_1^I$, idle capacity in the first period of a deterence equilibrium occurs when $K_1^I > \bar{M}_1^I$. On the other hand, by Proposition 2, we know that for the incumbent's first period capacity to have a consequence in the second period, it must be greater than CN_2^I. Hence Assumption G is the requirement that commitment in the second period requires idle capacity in the first period.[15] This simplifies the analysis of both deterence equilibria, and Stackelberg leadership.

Because the incumbent's first period capacity determines the outcome in both periods, when acting as a Stackelberg leader, he trades off more profitable commitment today with less profitable commitment tomorrow and vice versa. There are two extreme cases to consider. A type 1 Stackelberg leader chooses capacity to commit in the first period, but gives up the ability to commit in the second. That is, she chooses $K_1^I = q_1^I \leq \min\{CN_2^I, W_1^I\}$. Clearly this leads to an equilibrium with outputs of $\hat{S}_1$ in the first period and CN in the second. A type 2 Stackelberg leader commits in the second period, and 'over commits' in the first. That is she chooses $K_1^I = q_2^I > \max\{CN_2^I, W_1^I\}$. In this case, the incumbent has idle first period capacity and the first period output is $\tilde{S}_1$. In the second period the output is set at $(K_1^I, \underline{R}_2^E(K_1^I))$. Under Assumption G, these are the only types of accommodating behavior which can arise in equilibrium.[16] However, if there is demand growth, but Assumption G does not hold, then there is an intermediate type of Stackelberg leader, who both uses all of her capacity in the first

[15] Obviously one might look for a somewhat weaker assumption, which allows for cases in which idle capacity is a possibility, and cases in which commitment occurs without idle capacity. Since any capacity greater than W_2^I has no commitment power in either period, $W_2^I > \bar{M}_1^I$ is clearly the weakest possible growth assumption.

[16] By Proposition 2. In fact, a somewhat weaker condition, $W_1^I \leq CN_2^I$ suffices. This condition, however, is not relevant when considering deterence equilibria.

period, and commit to an output in the second. In this case, the incumbent chooses K_1^I such that $\hat{S}_1^I \leq q_1^I = K_1^I = q_2^I \leq \hat{S}_2^I$.[17] Entrant outputs are at $\underline{R}_t^E(K_1^I)$.

We are now ready to turn to the paper's central issue, under what conditions can idle capacity occur in equilibrium. Throughout what follows, Assumptions G is maintained. The following five conditions must be satisfied: 1) It is possible to deter first period entry, but 2) only if the incumbent maintains idle capacity. 3) It is possible to deter entry in the second period. 4) The incumbent prefers entry deterence to being a Stackelberg leader, and 5) given that entry has not occurred in the first period, the incumbent prefers to deter it in the second period as well. The first three of these conditions are statements about the Entrant's payoffs in different situations. They might be restated as 1') $\pi_1^E(\tilde{S}_1) + \pi_2^E(W_2) \leq \bar{F} + 2F$, 2') $\pi_1^E(W_1) + \pi_2^E(CN_2) \geq \bar{F} + 2F$, and and 3') $\pi_2^E(W_2) \leq \bar{F} + 2F$. These conditions can be translated to $\pi_1^E(\tilde{S}_1) + \pi_2^E(W_2) \leq \bar{F} + 2F \leq \pi_1^E(W_1) + \pi_2^E(CN_2)$ and $F \leq [\pi_2^E(CN_2) - \pi_2^E(W_2)] + \pi_1^E(W_1)$. Since $\pi_1^E(W_1) \equiv \pi_1^E(\tilde{S}_1)$, $\pi_2^E(W_2) < \pi_2^E(CN_2)$ and $0 < [\pi_2^E(CN_2) - \pi_2^E(W_2)] + \pi_1^E(W_1)$, one can choose $\bar{F}$, and F such that conditions 1,2 and 3 hold. This yields:

Proposition 4 *Under Assumption G, one can find find values for F and $\bar{F}$ such that a deterence equilibrium requires idle first period capacity.*

Observe that Proposition 4 is merely a statement that there are circumstances under which, if the incumbent wishes to deter entry, then he must maintain idle capacity. To

[17] If $S_t^I < W_t^I$ in both periods, then it follows that $S_1^I < q_1^I = K_1^I = q_2^I < S_2^I$.

demonstrate that such equilibria actually exist, one must show that the entrant prefers deterence through idle capacity over being a Stackelberg leader. Because it is not so straight forward to compare the incumbent's payoffs in different circumstances, some further structure must be imposed. For the remainder of the paper, linear demand is assumed.

Assumption L Demand is linear: $P_t = a_t - b(q_t^E + q_t^I)$ with $a_2 > a_1 > c$.

There still remains the problem that the payoffs for detering entry depend crucially upon fixed costs. Hence in comparing payoffs, it is convenient to fix upon a particular case. Specifically, let us presume for now that $K_1^I = \underline{M}_2^I$ is sufficient to deter entry in both periods. The task of finding values for F and $\bar{F}$ which justify this presumption will be addressed later. The first benefit from Assumption L is the ability to rule out type 2 Stackelberg leadership.

Proposition 5 *Let Assumptions E and L hold. If there exists* $K_1^I \leq \min\{\underline{M}_2^I, W_2^I\}$ *which is sufficient to deter entry in both periods, then type 2 Stackelberg leadership never occurs in equilibrium.*

The intuition of Proposition 5 is that either the first period or the second period is in some sense more important. If the first period is more important, then the incumbent prefers type 1 Stackelberg leadership to type 2. If the second period is more important, then the incumbent prefers to deter entry, because by presumption, entry deterence is not difficult. It now remains to show that there are cases in which the incumbent prefers deterence to type 1 Stackelberg leadership, for which the following assumption is useful.

Assumption D $(a_1)^2 + 2a_1c - (c)^2 \geq (a_2 - c)(5c - a_2)$.

Sufficient conditions for Assumption D to hold are: $\frac{a_1}{c} \geq 1.5$ or $\frac{a_2}{c} \geq 4.5$. Proposition 6 informs us that Assumption D is an algebraic statement that the second period is more important than the first period, so that the incumbent prefers deterence, when it is relatively easy, to type 1 Stackelberg leadership.

Proposition 6 $\pi_1^I(\bar{M}_1) + \pi_2^I(\underline{M}_2) - c(\underline{M}_2 - \bar{M}_1) \geq \pi_1^I(\hat{S}_1) + \pi_2^I(CN_2)$ *if and only if Assumption D holds.*

Observe that Proposition 6 is a statement that the incumbent would be willing to hold the second period monopoly capacity, $\underline{M}_2^I$, in the first period to deter entry. Hence, while Assumption D is 'tight' for Proposition 6, there are clearly cases in which Assumption D does not hold, but the incumbent is nonetheless willing to hold idle capacity. Likewise, if Assumption D holds with a strict inequality, then the incumbent would be willing to hold capacity greater than $\underline{M}_2^I$ to deter entry. However, this gives us an easy case to check for parameter values such that entry deterence is both possible, and desired by the incumbent. It remains to show that there are values of F and $\bar{F}$, and a choice of first period capacity, which satisfy all of the conditions. That is, let us fix $\Omega = (K^L, \underline{R}_2^E(K^L))$ for come capacity level K^L. Our task is completed by finding F and $\bar{F}$ such that there exists K^L, with $CN_2^I < K^L \leq \hat{S}_2^I$, satisfying the following conditions. $\pi_1^E(W_1) + \pi_2^E(\Omega) = \bar{F} + 2F < \pi_1^E(W_1) + \pi_2^E(CN_2)$ assures that entry deterence is possible in the first period, if and only if the incumbent maintains at least capacity K^L. Since $K^L > CN_2^I$, and Assumption G holds, this involves idle capacity. $\pi_2^E(\hat{S}_2) \leq \bar{F} + F$ assures that deterence in the second period is feasible, and never requires more than the monopoly level of capacity. Hence, delayed entry equilibria are ruled out. Now observe that since $\pi_2^E(\Omega) < \pi_2^E(CN_2)$, it is always possible to assure these conditions.[18] By Assumption D, $\pi_1^I(\bar{M}_1) + \pi_2^I(\underline{M}_2) - c(K^L - \bar{M}_1) \geq \pi_1^I(\bar{M}_1) + \pi_2^I(\underline{M}_2) - c(\underline{M}_2 - \bar{M}_1) \geq \pi_1^I(\hat{S}_1) + \pi_2^I(CN_2)$. This guarantees that the incumbent prefers bearing the cost of idle capacity to sharing the market.

Proposition 7 *Let Assumptions G, L, and D hold. There are values for the fixed and sunk costs such that there is a unique equilibrium satisfying Assumption E. This equilibrium is a deterence equilibrium in which the incumbent maintains idle first period capacity.*

4 Conclusion

We have found that for an incumbent to deter entry when there is demand growth can require the maintenance of idle capacity. The model was chosen to be as simple as possible, and could be generalized. One simple extension would be the addition of physical depreciation. Since it is only the post depreciation capacity that has any commitment value, depreciation would, all other things being equal, increase the capacity require-

[18] By essentially the same arguments as Proposition 4.

ments for entry deterence. Consequently, physical depreciation should serve much the same role as demand growth in making idle capacity strategically meaningful.

Less trivial would be to extend the model to a longer (possibly infinite) sequence of periods. Since a foothold is valuable in all future periods, an incumbent firm would have to consider demand in all future periods when choosing capacity. Reynolds (1987) has analyzed such a model for a duopoly market. He finds that concern over future periods increases the capacity which firms hold. This seems to indicate that results similar to those contained herein could be found in an infinite horizon model. However, Reynolds' analysis depends upon firm payoffs being quadratic in capacity, which disallows the possibility of idle capacity.[19]

It remains to consider the possibility of selecting equilibria through some method other than assumption E. One might follow Basu and Singh (1990) and select an equilibrium by presuming that the incumbent has means for committing other than capacity. In this case, one would select the continuation in the second period most favorable to the incumbent. This approach would have the draw back that entry might not occur, because the entrant correctly anticipates that the continuation following entry involves very large second period capacity by the incumbent. In essence, following Basu and Singh (1990) involves giving the incumbent a first mover advantage in both periods, which remove any value within the model from achieving a toehold.

[19]Reynolds analyses a differential game within a continuous time framework.

Another possibility would be to select equilibria through a forward induction argument (as in Bagwell and Ramey, 1996.) In this case, because there are multiple equilibrium continuations in the second period, the entrant has a second mover advantage. That is, forward induction requires that if entry occurs in the first period, then the entrant must be planning a continuation in the second period that would yield him positive profits. Clearly for entry deterence to be forward induction rational requires that there be no continuations which yield the entrant positive profits. Since the entrant's most preferred continuation is when the incumbent does not increase her capacity, the requirements for deterence would remain unchanged. However, the entrant's second mover advantage might have a dramatic consequence in entry equilibria. For example, it might be the only continuations which give the entrant positive profits are those in which the entrant acts as a Stackelberg leader in the second period. This would clearly make a toehold in the industry worth more than a simple negation of the incumbent's first mover advantage. On the other hand, because there are many cases in which many equilibria would survive forward induction, the question of selection would remain.

5 Appendix

Proof of Proposition 1

If $K_1^i > \Omega^i$ then $K_2^i \geq K_1^i > \Omega^i \geq q_2^i$ since the only thing that can move second period output away from Ω is if firm $j \neq i$ increases capacity which will weakly decrease firm

i's equilibrium output. Also the only way that $\Omega^i > W_2^I$ is if $K_1^i > W_2^I$, which implies $K_2^i > W_2^I$ and $K_2^j = \tilde{S}_2^E$ and firm i will be left with $K_2^i > q_2^i$. Since we know that $K_2^i \leq q_2^i$ the Proposition follows. ♣

Proposition 8 *Presume entry in period one, and fix period one capacities such that they satisfy Proposition 1. A choice of second period capacities and outputs are a second period equilibrium if and only if* $K_1^i \leq K_2^i \leq W_2^I$ $(i = I, E)$ *and one of the following conditions holds.*

1) $(K_2^I \geq K_2^E)$ $q_2^I = K_2^I = S_2^I$ *and* $K_2^E \leq q_2^E = \hat{S}_2^E$

2) $(K_2^I \geq K_2^E)$ $CN_2^I \leq q_2^I = K_2^I \leq S_2^I$ *and* $K_2^E = q_2^E = \underline{R}_2^E(K_2^I)$

3) $(K_2^I \leq K_2^E)$ $q_2^E = K_2^E = S_2^I$ *and* $K_2^I \leq q_2^I = \hat{S}_2^I$

4) $(K_2^I \leq K_2^E)$ $CN_2^E \leq q_2^E = K_2^E \leq S_2^I$ *and* $K_2^I = q_2^I = \underline{R}_2^I(K_2^E)$

Proof: Assume without loss of generality that $K_2^I \geq K_2^E$. In both cases (1) and (2) both firms produce the optimal output given the capacity choices. In both cases, the incumbent is choosing a capacity which results in output as close to $\hat{S}_2$ as possible. Hence she does not wish to change her capacity. In both cases, the entrant is producing the optimal output given the incumbent's choice of capacity, and has capacity less than or equal to his output. Hence he has no incentive to change. Presume that the conditions do not hold. If $K_2^E > \underline{R}_2^E(K_2^I)$, then (since $K_2^I \leq W_2^I$) the entrant wants to reduce his capacity. If $K_2^E < \underline{R}_2^E(K_2^I)$ and $K_2^I < S_2^I$, then the incumbent wants to increase his capacity. Hence a violation of both (1) and (2) is not an equilibrium. ♣

Proposition 9 *Let Assumption E1 hold. If there is an entry equilibrium in which $q_1^E > K_1^E$, then there is another equilibrium in which all outputs are unchanged, firms receive the same profits, and $q_1^E = K_1^E$.*

Proof: That $K_1^E < q_1^E$ implies that first period output is on $\underline{R}_1^E$ and that $q_1^E \leq CN_1^E$. Hence increasing K_1^E to q_1^E will not change the intersection of first period reaction functions, nor will it result in $K_1^E > CN_2^E$. Hence, the output in neither period will change. Since capacity is only a commitment to pay costs that must be paid if production takes place, and outputs have not changed, profits remain the same. ♣

The following four lemmas are for the purpose of proving Proposition 2.

Lemma 5.1 *Presume that Assumption E Part 1 holds. In an entry equilibrium, both firms set first period capacity less than or equal to first period output, or equal to second period output.*

In a deterence equilibrium, the incumbent sets her capacity less than or equal to her first period output, or strictly greater than the second period Cournot output.

Proof: We already know that capacity is not set above second period output, so it remain to rule out a choice of capacity greater than first period output, but less than second period output. In this case, there must be some benefit from this in the second period. In an entry equilibrium, if capacity is less than the the second period output, then by Assumption E Part 1, it has no consequence on the second period. Clearly this implies that capacity less than the second period Cournot has no consequence in the second period. ♣

Lemma 5.2 *Let Assumption E Part 1 hold. In an entry equilibrium, if* $CN_2^I \leq K_1^I \leq W_2^I$*, then the Entrant sets his capacity less than or equal to his first period output.*

Proof: If $CN_2^I \leq K_1^I \leq W_2^I$ then we know that the entrant gets no benefit from capacity in the second period, because his optimal second period output is (holding $K_2^I = K_1^I$) $\underline{R}_2(K_1^I)$ which he will receive even if $K_1^E = 0$ (Assumption E1) Hence he sets $K_1^E \leq q_1^E$.♣

The point of the following two Lemmas is to rule out the case where the dominant firm wishes to act like a Stackelberg follower in the second period, and hence chooses a low capacity to induce the entrant to chooses a 'leader' capacity. Hence in these proofs there is a possibility that the dominant firm sets his capacity at say $\tilde{S}_1^E$.

Lemma 5.3 *Let Assumption E Part 1 hold. Presume that there is an entry equilibrium in which the incumbent chooses* $K_1^I < CN_2^I$*, but the entrant chooses* $K_1^E > CN_2^I$*.*

1) If $K_1^E \leq W_1^I$ *then* $K_1^I \leq \underline{R}_1(K_1^E)$ *or* $K_1^I = \underline{R}_2(K_1^E) > \underline{R}_1(K_1^E)$*.*

2) If $K_1^E > W_1^I$ *then* $K_1^I = \tilde{S}_1^E$ *or* $K_1^I = \underline{R}_2(K_1^E) \geq \tilde{S}_1^E$

Proof: Let $\bar{K}$ denote $\underline{R}_1(K_1^E)$ for case 1 and $\tilde{S}_1^E$ for case 2. Observe, that from Lemma 5.2 we know that $\bar{K}$ would be the optimal response by the incumbent if the entrant moved first and chose the capacity suggested in one of the cases. If the incumbent has to choose some $\underline{K} < \bar{K}$ in order to get the entrant to choose K_1^E, then there is no equilibrium in which the entrant chooses K_1^E, because $\bar{K}$ and an entrant capacity lower than K_1^E is preferred (by the incumbent) to $\underline{K}$ and K_1^E which is in turn preferred to $\bar{K}$ and K_1^E. Now consider the possible choice of $K > \bar{K}$ in period 1 by the incumbent. The only reason to make such a choice would be to take away the entrant's incentive to set an even higher capacity, and this is done as cheaply as possible at $K_1^I = \underline{R}_2(K_1^E)$. ♣

Lemma 5.4 *If the dominant firm chooses $K_1^I \leq CN_2^I$ in an entry equilibrium, then the entrant chooses $K_1^E \leq CN_2^I$.*

Proof: I assume that $K_1^I \leq CN_2^I < K_1^E$, and derive a contradiction. By Lemma 5.3 it is evident that if the incumbent chose the entrant's equilibrium capacity, then a reversal of outputs would result. Therefore the incumbent must be making higher net profits than the entrant. In addition, the entrant could simply choose to mimic the incumbent's capacity choice, so that this must yield lower profits than the entrant's equilibrium choice. However, note that in this case, the entrant would make at least as much profits in the first period, and strictly more profits in the second period than the incumbent is making in equilibrium. Therefore the entrant is making higher net profits than the incumbent is, by which contradiction the Lemma is proven. ♣

Proof of Proposition 2.

Given Lemma 5.1, there remain two things to show. By putting Lemmas 5.1 and 5.4 together, we see that a first period entrant sets $K_1^E \leq q_1^E$. With Assumption E2,

$K_1^E = q_1^E$. It remains now to show only that $q_1^I \geq \min\{\hat{S}_1^I, \underline{M}_1^I\}$. This is now immediate, since if there is no first period entry, then the incumbent will wish to produce at least his monopoly output, $\underline{M}_1^I$, and if there is entry, then he will wish to produce at least $\hat{S}_1^I$ in both periods, even if she could commit period by period. ♣

Proof of Proposition 5

The first step is to quantify the profits from Type 2 Stackelberg leadership. Observe, that capacity is set purely on it value in the second period, but has to be carried for two periods. Hence it is set to maximize: $(a_2 - 2c - b(K + \frac{a_2 - c - bK}{2b}))K$ with solution $K^* = \frac{a_2 - 3c}{2b}$. Clearly for type two behavior to be valid it must be the case that $K^* > q_2^{CN}$ which requires $a_2 > 7c$. Presuming that $a_2 > 7c$, and recognizing that the first period outcome must be $\tilde{S}_1$, then net profits from type 2 Stackelberg leadership are no more than $\frac{(a_1+2c)^2}{16b} - \frac{a_2 c - 3c^2}{2b} + \frac{(a_2+c)(a_2-3c)}{8b}$. We need now to compare these net profits with $\pi_1^I(\bar{M}_1) + \pi_2^I(\underline{M}_2) - c(\underline{M}_2 - \bar{M}_1) = \frac{a_1(a_1-2c)}{4b} + \frac{(a_2-c)^2}{4b} - \frac{(a_2-a_1-c)c}{2b}$.

Algebra reveals that type 2 Stackelberg are the smaller of the two if $a_1(3a_1 - 4c) > -2((a_2)^2 - 4a_2 c - 5c^2)$ which itself follows with $a_1 > c$ and $a_2 > 7c$. ♣

Proof of Proposition 6

$\pi_1^I(\bar{M}_1) + \pi_2^I(\underline{M}_2) - c(\underline{M}_2 - \bar{M}_1) \geq \pi_1^I(\hat{S}_1) + \pi_2^I(CN_2) \Leftrightarrow \frac{a_1(a_1-2c)}{4b} + \frac{(a_2-c)^2}{4b} - \frac{(a_2-a_1-c)c}{2b} \geq \frac{(a_1-c)^2}{8b} + \frac{(a_2-c)^2}{9b} \Leftrightarrow (a_1)^2 + 2a_1 c - (c)^2 \geq (a_2 - c)(5c - a_2)$. ♣

6 Linear Appendix

I provide here algebraic expressions for the values defined in section 2. Since these are based upon the one period model, I drop the time subscript. When I mention points in q^I, q^E space, the q^I value is listed first.

$$
\begin{aligned}
CN &= (\frac{a-c}{3b}\frac{a-c}{3b}) && (1)\\
V &= (\frac{a+c}{3b}, \frac{a-2c}{3b}) && (2)\\
U &= (\frac{a}{3b}, \frac{a}{3b}) && (3)\\
S &= (\frac{a-c}{2b}, \frac{a-c}{4b}) && (4)\\
\tilde{S} &= \begin{cases} (\frac{a+2c}{4b}, \frac{a-2c}{2b}) & \text{if } a \leq 6c \\ U & \text{if } a \geq 6c \end{cases} && (5)\\
W^I &= \begin{cases} \frac{a-c}{b} - \frac{a-2c}{b\sqrt{2}}) & \text{if } a \leq 6c \\ \frac{a-c}{b} - \frac{2\sqrt{a(a-3c)}}{3b} & \text{if } a \geq 6c \end{cases} && (6)\\
W^E &= \begin{cases} \frac{a-2c}{2b\sqrt{2}} & \text{if } a \leq 6c \\ \frac{\sqrt{a(a-3c)}}{3b} & \text{if } a \geq 6c \end{cases} && (7)\\
\hat{S} &= (\min\{S^I, W^I\}, \max\{S^E, W^E\}) && (8)\\
\pi^E(\tilde{S}) = \pi^E(W) &= \begin{cases} \frac{(a-2c)^2}{8b} & \text{if } a \leq 6c \\ \frac{a(a-3c)}{9b} & \text{if } a \geq 6c \end{cases} && (9)
\end{aligned}
$$

$$\pi^I(\tilde{S}) = \; = \begin{cases} \frac{a^2-4c^2}{16b} & \text{if } a \leq 6c \\ \frac{a(a-3c)}{9b} & \text{if } a \geq 6c \end{cases} \tag{10}$$

$$\pi^I(W) = \; = \begin{cases} \frac{(a-2c)(a+c\sqrt{2})}{8b} \cdot (2\sqrt{2}-2) & \text{if } a \leq 6c \\ \frac{(a-c)\sqrt{a(a-3c}}{3b} - \frac{2a(a-3c)}{9b} & \text{if } a \geq 6c \end{cases} \tag{11}$$

In the linear model, $R_t(\cdot, K)$ is defined as follows.

$$R_t(q, K) = \begin{cases} \frac{a_t-bq}{2b} & \text{if } q < \frac{a_t-2bK}{b} \\ \frac{a_t-bq-c}{2b} & \text{if } q > \frac{a_t-2bK-c}{b} \\ K & \text{otherwise} \end{cases} \tag{12}$$

Below I nail down some stuff. For $W^I \geq \underline{M}$ requires that $6c \geq a$ and that $\frac{a}{c} \leq \frac{2\sqrt{2}-1}{\sqrt{2}-1} \approx 4.5$.

References

[1] Kyle Bagwell and Garey Ramey. Capacity, entry and forward induction. *Rand J. of Economics*, 27:660–680, 1996.

[2] Brad Barham and Roger Ware. A sequential entry model with strategic use of excess capacity. *Canadian J. Econ.*, 26:286–298, 1993.

[3] Kaushik Basu and Nirvikar Singh. Entry-deterence in stackelberg perfect equilibria. *Int. Econ. Review*, 31:61–71, 1990.

[4] Jeremey Bulow, John Geanakoplos, and Paul Klemperer. Holding idle capacity to deter entry. *Economic Journal*, 95:178–182, 1985.

[5] D. Cossutta and M. Grillo. Excess capacity, sunk costs and collusion: a noncooperative bargaining game. *Int. J. Industrial Organization*, 4:251–270, 1989.

[6] Avinash Dixit. A model of duopoly suggesting a theory of entry barriers. *Bell J. of Economics*, 10:20–32, 1979.

[7] Avinash Dixit. The role of investment in entry deterence. *Economic Journal*, 90:95–106, 1980.

[8] Douglas C. Dobson, William G. Shepherd, and Robert D. Stoner. Strategic Capacity Preemption: DuPont (Titnium Dioxide) (1980). in *The Antitrust Revolution,*

The Role of Economics, edited by John E. Kwoka, jr, and Lawrence J. White. New York, NY.: Harper Collins College Publishers, 1994.

[9] B. Curtis Eaton and Richard G. Lipsey. The theory of market preemption: The persistence of excess capacity and monopoly in growing spatial markets. *Economica*, 1979:95–106, 1979.

[10] B. Curtis Eaton and Richard G. Lipsey. Exit barriers are entry barriers: the durability of capital as a barrier to entry. *Bell J. of Economics*, 10:721–729, 1980.

[11] F. Esposito and L. Esposito. Excess capacity and market structure. *Rev. Econ. Stat.*, 56:188–194, 1974.

[12] Drew Fudenberg and Jean Tirole. Capital as a commitment: Strategic investment to deter mobility. *J. Econ. Theory*, 31:227–250, 1983.

[13] Drew Fudenberg and Jean Tirole. The fat-cat effect, the puppy-dog ploy, and the lean and hungry look. *Amer. Econ. Review*, 74:361–366, 1984.

[14] Richard Gilbert and Richard Harris. Investment decisions with economies of scale and learning. *Amer. Econ. Review*, 71:172–177, 1981.

[15] Richard Gilbert and Richard Harris. Competition with lumpy investment. *Rand J. of Economics*, 15:197–212, 1984.

[16] Learned Hand. U.S. *vs.* Alcoa Aluminum Company of America *et.al.*. 44 F. Supp. 97(1941), 148 F. 2d 416(1945).

[17] Finn Kydland. Equilibrium solutions in dynamic dominant-player models. *J. Econ. Theory*, 15:307–324, 1977.

[18] Stephen Martin. *Industrial Economics: Economic Analysis and Public Policy.* Prentice Hall, Englewood Cliffs, NJ 07632, 1993.

[19] R. Masson and J Shannan. Excess capacity and limit pricing: an empricial test. *Economica*, 53:365–378, 1982.

[20] S.S. Reynolds. Strategic capital investment in the american aluminum industry. *J. Industrial Economics*, 34:?, 1986.

[21] D. Rosenbaum. An emprical test of the effect of excess capacity in price setting, capacity constrained supergames. *Int. J. Industrial Organization*, 7:231–241, 1989.

[22] Steven C. Salop. Strategic entry deterence. *Amer. Econ. Review*, 69:335–338, 1979.

[23] Richard Schmalensee. Economies of scale and barriers to entry. *J. Polit. Econ.*, 89:1228–1238, 1981.

[24] A. Michael Spence. Entry, capacity, investment and oligopolistic pricing. *Bell J. of Economics*, ?:534–544, 0000.

[25] A. Michael Spence. Investment strategy and growth in demand in a new market. *Bell J. of Economics*, 10:1–19, 1979.

[26] Roger Ware. Sunk costs and strategic commitment: a proposed three-stage equilibrium. *Economic Journal*, 94:370–378, 1984.