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Electronic Commerce and Tax Competition:
When Consumers Can Shop Across Borders and On-Line

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Abstract

The use of the Internet as a way of shopping has increased sharply over the last several years. Given that Internet sales are essentially not taxed, state governments are concerned about dwindling sales tax revenues due to increasing e-commerce sales. This paper addresses this issue by considering a tax competition model in which consumers can shop in either conventional brick-and-mortar stores or online. This model can describe the emergence of electronic commerce over the past several years. It also provides a framework in which these potential revenue losses can be examined. Equity issues related to an Internet sales tax are also highlighted by allowing incomes to vary across consumers.

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1 Introduction

As little as five years ago, the Internet was a little known phenomenon and a non-issue in terms of government policy. How quickly times change. In the United States, the Internet Tax Freedom Act establishes a moratorium on Internet taxes through October, 2006[1]. Currently, online sales of physical goods are treated like mail-order catalog sales. If a firm does not have what is known as nexus (a substantial physical presence) in the state where the customer lives, they are not required to collect sales tax on the purchase[2]. Consumers in most states are officially responsible for sending in use taxes on these purchases, however there is little or no enforcement of this[3]. Sales tax revenue, therefore, is generally not collected on these sales. As a result, state governments are concerned about dwindling sales tax revenues due to increasing tax-free electronic commerce sales. Currently, sales taxes account for on average 33% of revenues at the state level and 11% of revenues at the local level in the United States (U.S. General Accounting Office).

Electronic commerce (e-commerce) sales are defined by the U.S. Department of Commerce as "sales of goods and services over the Internet, an extranet, Electronic Data Interchange, or other online system. Payment may or may not be made online." Retail e-commerce sales were first reported by the U.S. Department of Commerce for the last quarter of 1999. The most current estimate puts e-commerce sales during the second quarter of 2000 at $5.518 billion (0.68 percent of total retail sales), an increase of 5.3 percent over the previous quarter (Census Bureau). While currently small, the growth of e-commerce sales is dramatic: one estimate puts e-commerce business-to-consumer sales at $454 billion by 2004, increasing the potential for revenue losses at the state and local level (Forrester Research).

The current debate over whether or not e-commerce sales should be taxed

[1] The original expiration date of October, 2001, was extended by Congress.

[2] For a good explanation of nexus and implications for electronic commerce, see Fox and Murray (1997) and Goolsbee and Zittrain (1999).

[3] Five states currently do not have state sales or use taxes, and 19 other states do not allow local governments to impose sales or use taxes (Cline and Neubig).

focuses on several issues. Those favoring a tax assert that the introduction of an electronic commerce tax would do irreparable harm to the growth of the Internet as consumers return to main-street shops. Their opponents cite concerns of lower state government revenues due to increasing e-commerce sales, the resulting decrease in public good provision, and issues regarding equity.

This paper examines several issues relevant to the current e-commerce tax debate. A framework is developed in which the potential revenue losses due to increasing e-commerce sales can be examined. This model can also describe the emergence of e-commerce. In addition, equity issues are explored by considering a case in which consumer incomes vary. This paper is not discussing how taxing purchases made using the Internet would affect its growth. It is, however, an important first step in trying to answer the question of whether or not electronic purchases should be taxed.

Given the current e-commerce tax debate, it seems relevant to examine this issue using a theoretical framework. And while economists have questioned both the legal and economic aspects of taxing e-commerce sales, little consensus has evolved. McLure (1997) and Fox and Murray (1997) both provide a discussion of the constitutional and economic issues at hand. However, they reach opposite conclusions with regards to how tax policy should treat e-commerce sales. McLure concludes that widespread reform of the sales tax system is needed rather than simply taxing electronic commerce per se. Fox and Murray conclude that a sales tax incorporating electronic commerce should be largely consistent with the current system. In a recent report, the General Accounting Office projects sales tax losses from Internet sales to be between 1 and 5% of projected sales tax revenues in 2003. Their report is an important first step into quantifying the effects of e-commerce, but it remains a first step. There is much uncertainty regarding these results due to a lack of empirical data and a vast array of projections for Internet growth. Both the disparity in the economic literature and the lack of empirical data provide motivation for the establishment of a theoretical framework in which a more rigorous analysis can be undertaken.

The emergence of electronic commerce over the past several years can be described within this model. The cost of computing technology has decreased substantially relative to income within the last ten years. This partly describes the emergence of e-commerce, as consumers find the equipment necessary to shop online more affordable. The Internet environment has also become more accepted by shoppers, given technological improvements in security and advancements in ease of use. These all function to lower the costs of shopping online, represented here by a single fixed cost. While there are certainly supply-side issues as well, the acceptance and willingness of consumers to shop online is a crucial element in the success of e-commerce.

Governments in each region essentially compete for consumers by setting their tax rates, taking into account that higher tax rates will drive some consumers away and therefore lower their tax base. This tax competition framework is useful because it allows an examination of the interaction between governments before and after the participation of the Internet region. The Internet region will "enter" this model as long as there are some consumers who are willing to shop there. This will occur if, for some consumers, the benefit from shopping online (the utility they receive there) outweighs the cost (the cost of accessing the Internet). It is assumed that the Internet firm is located in a separate, remote, region. This modeling choice is a natural starting place because the focus here is on sales to consumers by firms who do not have a physical presence, or nexus, in the consumer's state. Therefore, given a sufficient decrease in the fixed cost of shopping online relative to income, this framework predicts the following. The Internet region will choose to "enter" the model, moving us from a 2-region conventional-business only model to a 3-region model that sustains both conventional business and electronic commerce.

In the United States, taxes on Internet purchases are essentially zero due to nonenforcement of use taxes[4]. Internet taxes can potentially be collected based on either a origin (location of purchase) or destination (location of consumer)

[4] If consumers were honest and sent in use taxes, this would effectively institute destination taxes on "remote" Internet purchases and origin taxes on "local" purchases, i.e., conventional purchases in home and neighboring regions.

principle. Because Internet taxes are currently zero and therefore consumers can purchase online without paying taxes, this model assumes origin taxation on all purchases. The European Union has also implemented the Value Added Tax (VAT) on purchases within Member States using the origin principle, providing additional incentives for analysis of a potentially non-zero origin tax[5].

This model provides a framework in which the potential revenue losses associated with increasing e-commerce sales can be examined. Each region chooses tax rates endogenously, leading to potentially nonzero tax rates for the Internet region. Comparisons of tax rates, bases, and revenues are made for the following three cases: no Internet region participation; Internet region participation with endogenously chosen tax rate; and Internet region participation with zero online tax rate (sometimes referred to as the "status quo"). The case in which the Internet region participates with a zero tax rate will always result in lower tax rates, bases, and therefore revenues than when the Internet region participates and sets a nonzero tax rate. The general findings of this tax competition model (in which all three regions choose tax rates endogenously) are therefore only amplified if we consider the status quo in the United States (zero Internet tax). In order to examine tax revenues in each of these cases, recall that the Internet firm is located in a separate region. Therefore, there exist 3 regions in the two cases in which the Internet participates and 2 regions otherwise. One must therefore examine what happens to *total* revenue collections across all regions in order to truly pin down the effects of Internet-induced competition.

Consumers have unit demands for a single good. The focus is therefore given to *where* the purchase is made, rather than *if* the good is purchased (and how much). A Hotelling style model is constructed in which consumers are uniformly distributed along a line connecting two conventional shopping centers

[5] The European system is complicated, as there are many rules governing so-called "distance selling." If a seller exceeds a threshold level of sales to private consumers in another Member State (that threshold being set by the country in which the consumers live), they must obtain a VAT registration number in the destination country and remit VAT taxes to the destination country (at the VAT rate of the destination country). Buyers who purchase over a certain threshold must also register for VAT and pay their country's VAT tax to their government (Finnish Tax Administration). This is essentially taxation using the destination principle. As the origin principle is enforced for small purchases, this framework allows an analysis pertinent for these types of transactions.

or regions. If they choose to shop conventionally, they must travel to one of the two regions. Travel is costly in terms of time, and therefore consumers would like to find alternative means of acquiring goods. The Internet provides this alternative, although there is a fixed cost associated with Internet shopping. This fixed cost can be thought of as access costs that must be paid in order to shop online, including computer access, time and money spent learning how to use a computer, getting connected to the Internet, etc. Once paid, the fixed cost allows the online purchase to be made without travel costs: the good is shipped to them directly. This convenience is most valued by the consumers who live in remote areas, far away from either conventional shopping center. They incur the highest travel costs associated with conventional shopping and therefore are the most likely to pay the fixed cost and shop online.

An analysis of the cases with Internet shopping versus the model without e-commerce shows the following. Given a sufficient decrease in the fixed cost of online shopping, consumers in remote regions will begin to shop in the Internet region. As a result of this, the tax base in both conventional regions will decrease relative to the case in which the Internet region does not participate. The optimal tax rates are also lower when faced with competition from the Internet region[6]. This is because consumers face a choice of *where* to buy, i.e., they can shop across borders. In the model without e-commerce, each conventional region competes with its neighbor, the other conventional region. Since the Internet is non-geographic in nature (consumers don't incur travel costs to shop there), it can be thought of as a very close neighboring region. In the model with Internet commerce, the new "neighboring" region for both conventional regions becomes the Internet. The costs of shopping across the border are now much lower, leading to increased competition for consumers and lower tax rates. Since revenue is comprised of the tax rate times the tax base, lower bases and rates necessarily mean that tax revenues in both conventional regions will be lower with a competing Internet region than before. Keeping in mind that the

[6] Recall that this assumes that the Internet region is setting tax rates endogenously. The case where the Internet tax is zero will result in even lower tax rates, bases, and revenues.

number of regions changes, examination of total revenue collections across all regions is necessary. Total revenues in all regions fall with an endogenously chosen Internet tax rate. Total revenues are even lower in the case where the Internet region has a zero tax. This is clearly due to the competition between governments as a result of Internet shopping. This result suggests that the concerns of state governments are potentially justified.

President Clinton and others have focused attention on the “Digital Divide”. The idea of the digital divide is that the rich and educated have the best access to computers and the Internet. It is these people, therefore, who will thrive in today’s high-tech labor market. More importantly, the poor have limited access to computers and the Internet. If they cannot learn the computer skills needed to find employment in these high-paying, high-growth sectors, they will necessarily fall further and further behind (National Telecommunications and Information Administration). This motivates an examination of the equity issues relating to an e-commerce sales tax, which is achieved by allowing consumer incomes to vary. More high-income consumers shop online than low-income consumers. This is intuitive because they have a higher willingness to pay the fixed cost to shop online and their time is worth more to them (making conventional shopping less attractive). Therefore, if the online tax rate is lower than conventional tax rates, the average rich person will pay relatively less of their income in sales tax than the average poor person. This would make an Internet sales tax regressive in this framework. However, with further decreases in the fixed cost of online shopping, the tax in the online region increases, making a sales tax less regressive over time. The sales tax rates and bases in this model are both a function of income distribution. Higher income inequality leads to lower optimal tax rates in all regions. However, the tax bases are ambiguous with respect to income distribution. Therefore, a change in income inequality will cause an undetermined change in revenues in this simple framework.

It is important to keep in mind the assumptions that underlie this model and therefore drive the results. First, it is assumed that the Internet firm is located in a separate region. This implies that the tax revenue from online shopping flows out of the conventional regions where consumers live. This is a natural starting place, as the focus here is remote sales to consumers in which the firms have no physical presence, or nexus. Tax revenues are collected based on the origin of the transaction, rather than the destination, i.e., residential location of the consumer. Destination based taxes, in the context of this model, would eliminate competition altogether, as the incentive to shop across borders would disappear. If consumers were honest and paid use taxes, remote sales would be subject to destination taxes, whereas local sales would be subject to origin taxes[7]. Therefore, there would be competition between local governments for local consumers but no competition for distant consumers (those who shop using the Internet, mail-order, or by phone). Destination taxes would be more interesting in the case of endogenous firm location. In this model, however, firm location is fixed. Consumers in this model have all pertinent information before they decide whether or not to purchase to computer and where to shop. The policy variable chosen by each region is the tax rate.

This model is intended as an initial step into a potentially rich theoretical area relating to electronic commerce. Valuable insights into the potential revenue losses and equity issues regarding e-commerce taxes are gained. Consideration of extensions in which each assumption is systematically relaxed would be useful to examine the sensitivity of these results.

2 The Model

2.1 One-Income

This model builds on the unit demand idea and utility function structure of Friberg, Ganslandt, and Sandström (2000). Their model focuses on prices set by monopoly firms in conventional versus online markets. This model differs from their basic structure by adding a second conventional region, having spa-

[7] For truly local purchases, destination and origin taxation would be equivalent. For conventional purchases in close, neighboring regions, however, the current sales tax system is based on the origin of the purchase.

tially dispersed consumers, abstracting from the price-setting behavior of firms, assuming perfect competition instead of monopoly, and considering the tax competition that arises when regional governments choose tax rates.

A Hotelling style spatial differentiation model is developed that describes the geographic nature of two conventional regions (Hotelling 1929). An electronic region where consumers can shop online is then added. This new region is non-geographic in nature: consumers do not incur travel costs when they shop there. In order to shop online, however, consumers must pay a fixed cost associated with online access. This fixed cost can be thought of as the cost of accessing a computer, the time and money spent learning to use it, the costs of setting up internet service, etc. Consumers are uniformly distributed along a line from 0 to 1, their location denoted by the parameter λ. Consumers are identical and have unit demands for one good. Holding money yields no utility, meaning that a consumer's utility if they purchase nothing is zero. This restricts attention to the choice of *where* the purchase is made, rather than *if* the purchase is made. Each individual decides in which region to purchase one unit: either in a conventional region (R_1 or R_2) or online (R_3).

The structure of the game is as follows. Governments set tax rates. Then, consumers decide where to shop. They either access a computer and shop online or shop conventionally. The decision of whether or not to pay the fixed cost to shop online is made at the same time as the decision of where to shop. The consumers know all information regarding prices in all regions before deciding where to shop. This paper abstracts from the search process and also assumes that information is delivered without cost to the consumer. There is no uncertainty as to the price when the consumer arrives in either conventional region or logs on to their computer.

The utility received by a given consumer is characterized as follows. If they shop in region 1, they must travel there in order to make the purchase. Region 1 is located at 0, and there is a utility loss due to the time spent traveling there. This time cost increases with income, implying that time becomes more valuable at higher incomes. A further utility loss occurs because the amount t_1 must be paid to acquire the good. Similarly, the consumer may shop in region 2. In order to purchase in region 2, they must travel to 1. Again, the time cost causes a utility loss and then they face t_2 when they arrive. To shop online, the consumer faces both the fixed cost of access, T, and the price online t_3. There is, however, no travel cost associated with this purchasing option. T encompasses all costs associated with shopping online, including having the good shipped to the consumer's home.

Utility with the purchase of one unit is therefore:

$$U = \begin{cases} \alpha\Theta - \lambda\Theta - t_1 & : \quad \text{when buying in region 1} \\ \alpha\Theta - (1-\lambda)\Theta - t_2 & : \quad \text{when buying in region 2} \\ \alpha\Theta - T - t_3 & : \quad \text{when buying in region 3} \end{cases}$$

where Θ is income, λ is the location of the consumer, and t represents the price the consumer faces in each region. The parameter α is used to scale up incomes to the point where each consumer decides to make a purchase. This is because focus is given to *where* and not *if* the purchase is made. The important feature here is that there is a difference in convenience (in terms of time) between online shopping and conventional shopping. This is reflected in the utility structure by making the time cost $\lambda\Theta$ dependent on income for conventional shopping and the cost T *not* dependent on income for online shopping. This is one of many potential modeling choices. As long as there is a lower opportunity cost (in terms of time) for online shopping versus conventional shopping, this can be reflected by setting the time cost for online shopping to zero and the time cost for conventional shopping to a positive number.

The model abstracts from possible cost advantages associated with economies of scale in production that would likely be associated with online merchants. Perfect competition is assumed in all regions, implying that firms would charge marginal cost. We further assume that marginal costs are identical in all regions. For simplicity, the marginal cost is set to zero. As a result, the consumer price in each region is simply the tax rate. Firm location is also assumed fixed.

In figure 1, the bold line shows the highest possible utility for each person based on their location (denoted by λ). Consumers who live from 0 to $\underline{\lambda}$ will shop in region 1 because they can achieve the highest possible utility by doing so. The person located at $\underline{\lambda}$ will be indifferent between shopping in region 1 and shopping online. People between $\underline{\lambda}$ and $\overline{\lambda}$ find that shopping online (in region 3) yields the highest utility. The person at $\overline{\lambda}$ will be indifferent between shopping in region 2 and shopping online. Lastly, people between $\overline{\lambda}$ and 1 will shop in region 2. As expected, consumers who live close to region 1 are more likely to shop there. Individuals who live in "remote" areas, i.e. around 1/2, are more likely to shop online, given the high travel costs (and subsequent utility loss) they would incur if they shopped conventionally. Symmetry can be observed here between regions 1 and 2.

2.2 Emergence of Electronic Commerce

The emergence of e-commerce over the last several years is described by this model. People will only shop online if they can achieve higher utility by doing so. The utility of shopping online is adversely affected by T. Therefore consumers weigh the costs and benefits of shopping online versus those of conventional shopping. Consumers in remote areas are the most likely to shop electronically because they incur the highest travel costs when they shop conventionally. When $T \geq \Theta$, not even these consumers shop online because they can still achieve higher utility by traveling to a conventional market. Only when T falls sufficiently does paying the fixed cost to shop online become worthwhile.

In thinking about the model without Internet commerce, T was sufficiently high that region 3 could not capture any customers no matter how low they set their tax rate. In this case, there would be no threat of entry from region 3 and this framework predicts a conventional-business only model. Specifically, region 3 would not be able to attract any business even if their tax rate were zero. This would occur if the utility of the person living at 1/2 (the most remote consumer and the most likely to shop online) would still receive higher utility from shopping in region 1 (or 2) than shopping online even given a tax rate of zero in region 3:

$$U_1(\lambda = \frac{1}{2}, t_1^*) > U_3(\lambda = \frac{1}{2}, t_3 = 0)$$

This occurs when $T \geq \frac{3}{2}\Theta$. Caution is needed here, as there only exists a Nash equilibrium if it's the case that $T < \Theta$. There might not be an equilibrium that exists at higher values of T. Intuitively, however, for very high fixed costs of online shopping, the Internet region will not enter the model.

The model without Internet commerce (in which region 3 does not compete) is fundamentally different than saying region 3's tax rate is zero. The difference arises because of the threat of entry posed by region 3. If the tax rate is simply zero, there remains the threat (in the eyes of regions 1 and 2) that region 3 will compete and set a non-zero tax rate. With sufficiently high fixed costs, however, there is no threat of entry by region 3, no matter what the actions of the other regions.

It is easy to argue that, over the past few years, the cost of computing technology has decreased substantially with respect to income. This partly explains the emergence of e-commerce. Security improvements and advancements in ease of use have all helped to lower this fixed cost of online shopping. So, the emergence of electronic commerce can be thought of in terms of comparative statics in this model. A sufficient decrease in the fixed cost of shopping online has occured to move us from a 2-region model conventional business-only model to a 3-region model which sustains e-commerce.

2.3 Revenue Maximization

At first, it is assumed that each region's government maximizes tax revenue. The advantage of revenue maximization is the ease in which the model can be solved. This is not necessarily the best choice for the objective function. The social welfare maximizing result is examined later as an extension and it is shown that the revenue maximization problem is a special case of the social welfare maximizing case.

Given that T is sufficiently low relative to income, the Internet region has business and results are derived for the three-region model with Internet commerce in which all three regions set tax rates endogenously.

Region 1's maximization problem is:

$$\max_{t_1} \mathbf{R_1} = t_1 \int_0^{\underline{\lambda}} f(\lambda)\ d\lambda \tag{1}$$

where $\underline{\lambda}$ is the location of the consumer who is indifferent between shopping in region 1 and online. Setting the utility from shopping in region 1 equal to the utility from shopping in region 3 and solving for λ yields

$$\underline{\lambda} = \frac{T + t_3 - t_1}{\Theta} \tag{2}$$

Note that increasing t_3 will increase $\underline{\lambda}$, meaning that the number of people shopping online will decrease if region 3 increases its tax rate. If region 1 increases their tax rate, this will lower $\underline{\lambda}$ (increase the number of people shopping online and decrease those shopping in region 1). Each region takes into account that increasing their tax rate will lower their tax base, as consumers at the margin will decide to shop in the neighboring region instead.

Solving this maximization problems yields a reaction function for t_1 which is linear with respect to both the neighboring region's tax rate (where the neighbor is the region 3, the Internet) and the fixed cost of shopping online, T.

$$t_1^* = \frac{T + t_3}{2}. \tag{3}$$

The maximization problem for region 2 is similarly defined, except the limits of integration run from $\overline{\lambda}$ to 1, where $\overline{\lambda}$ is the location of the person who is indifferent between shopping in region 2 or shopping online. The symmetry between regions 1 and 2 guarantees that the reaction function for region 2 is identical to that of region 1. Both regions compete with their "neighbor", region 3.

The maximization for region 3 is:

$$\max_{t_3} \mathbf{R_3} = t_3 \int_{\underline{\lambda}}^{\overline{\lambda}} f(\lambda)\ d\lambda \tag{4}$$

and the following reaction function for t_3 results:

$$t_3^* = \frac{\Theta - 2T + t_1 + t_2}{4}. \tag{5}$$

Note that region 3 has two neighbors, and therefore the optimal tax is an increasing function of the tax rates in both neighboring regions. The higher the tax across either "border", the higher tax region 3 will set.

Solving these equations simultaneously yields the following candidate Nash equilibrium results:

$$\begin{aligned} t_1^* = t_2^* &= \frac{2T + \Theta}{6} \\ t_3^* &= \frac{\Theta - T}{3} \end{aligned} \tag{6}$$

The following conditions must hold in order for a Nash equilibrium to exist:

(i) $\Theta > 0$, income must be greater than zero

(ii) $T > 0$, the fixed cost of shopping online must be greater than zero

(iii) $T < \Theta$, the fixed cost of shopping online cannot be greater than Θ.

Also, in order to show that this candidate Nash equilibrium exists, one must examine conditions under which each region would potentially want to deviate from the above tax rates. A deviation for region 1 would involve lowering their tax rate to a point where they would capture all online shoppers. This would mean a change of tax regime because their tax base would change. Please refer to the appendix for an in-depth dicussion of the existence of the Nash equilibrium and an analysis of these potential deviations[8].

The optimal tax rates for regions 1 and 2 are increasing in T, whereas the optimal tax for region 3 is decreasing in T. Since T is the fixed cost of shopping online, this result is quite intuitive. The higher the cost of entry into the Internet region, the lower the tax rate they can charge. All tax rates (both conventional and electronic) are increasing in Θ, income.

[8] It should be noted that the equilibrium analysis in the appendix are performed using the two-income extension to the model, where the potential deviations are slightly more complicated.

Solving for the revenue base cutoff values of λ:

$$\begin{aligned}\underline{\lambda} &= \frac{T}{3\Theta} + \frac{1}{6} \\ \lambda^* &= \frac{1}{2} \\ \overline{\lambda} &= \frac{5}{6} - \frac{T}{3\Theta}\end{aligned} \tag{7}$$

The values of lambda indicate what the tax base will be in each region in equilibrium. These results are also quite intuitive. The higher the fixed cost of shopping online, the fewer people will shop there. Higher incomes will cause an increased willingness to pay the fixed cost of Internet access, leading to more shopping online and therefore a higher tax base for region 3.

The Nash equilibrium levels of revenues in each of the regions is:

$$\begin{aligned}R_1 = R_2 &= \frac{(2T+\Theta)^2}{36\Theta} \\ R_3 &= \frac{2}{9} \cdot \frac{(\Theta - T)^2}{\Theta}\end{aligned} \tag{8}$$

2.4 Comparison of Cases With- and Without- Electronic Commerce

In both the case with e-commerce and without, each region's optimal tax rate is a function of their neighboring region's tax rate. In the model with only conventional business, region 1 therefore competes with region 2 and vice versa. In the model with e-commerce, both conventional regions must now compete with a new "neighbor", the Internet (region 3). Three cases are of interest here: the case with no competing Internet region, the case in which the Internet competes and endogenously sets its tax rate, and the case in which the Internet competes but sets a zero tax rate (status quo).

Reaction functions:

Without e-commerce	*With e-commerce,* $t_3 > 0$	*With e-commerce,* $t_3 = 0$
$t_1^* = t_2^* = \frac{t_2+\Theta}{2}$	$t_1^* = t_2^* = \frac{T+t_3}{2}$	n/a
n/a	$t_3^* = \frac{\Theta-2T+t_1+t_2}{4}$	n/a

Reaction functions are not reported here for the case in which $t_3 = 0$ because these functions collapse into equilibrium values. Equilibrium tax rates are discussed shortly.

Figure 2 shows the reaction function of $t_1(t_N)$, where N denotes the neighboring region. This figure only looks at the case without Internet commerce and the case in which the Internet tax is chosen endogenously. Examination of the reaction function shows that, for a given tax rate set by the neighboring region, the optimal rate in two-region model without Internet commerce is always above the optimal rate in the three-region model with Internet commerce. The slope of the reaction function remains the same, i.e. each region will react to changes in a similar way in either model.

In order to assess whether or not the tax rates will be lower with electronic commerce than without, the Nash equilibrium tax rates for the conventional regions in all cases are examined.

Equilibrium tax rates:

Without e-commerce	*With e-commerce,* $t_3 > 0$	*With e-commerce,* $t_3 = 0$
$t_1^* = t_2^* = \Theta$	$t_1^* = t_2^* = \frac{2T+\Theta}{6}$	$t_1^* = t_2^* = \frac{T}{2}$
n/a	$t_3^* = \frac{\Theta-T}{3}$	$t_3 = 0$

Several comparisons are of interest here. First, for the conventional tax rates in the model with endogenous (i.e. $t_3 > 0$) Internet taxes to be lower than the same rates in the model without e-commerce, $\frac{2T+\Theta}{6} < \Theta$. This implies that $T < \frac{5}{2}\Theta$. Since, for an equilibrium to exist, it is necessary that $T < \Theta$, this will be true in all cases in which the equilibrium exists. Second, for the conventional tax rates in the case where $t_3 = 0$ to be less than the endogenous Internet tax case, it must be the case that $\frac{T}{2} < \frac{2T+\Theta}{6}$. This implies that $T < \Theta$, which is the same as the equilibrium condition. Therefore, for any equilibrium that exists, the following ranking of tax rates will exist: t_1(no Internet)$>$ t_1(Internet with $t_3 > 0$)$>$ t_1(Internet with $t_3 = 0$). Therefore, the general results of this model in which t_3 is endogenously set will be reinforced if we consider the status quo in which $t_3 = 0$.

The tax rate set by conventional governments is only part of the concern of state tax officials. Figure 3 shows what happens to the tax base of region 1 (a similar statement can be made about the tax base of region 2) both with and without Internet commerce. As long as an equilibrium exists in which all three regions are active, the base will be lower for either conventional region with e-commerce than before. Lower bases and rates will necessarily lead to lower revenues.

Revenues by Region and Total Revenue Across All Regions:

	Without e-commerce	*With e-commerce, $t_3 > 0$*	*With e-commerce, $t_3 = 0$*
Region 1	$\frac{\Theta}{2}$	$\frac{(2T+\Theta)^2}{36\Theta}$	$\frac{T^2}{4\Theta}$
Internet	n/a	$\frac{2}{9}\frac{(\Theta-T)^2}{\Theta}$	0
Total Revenue	Θ	$\frac{2(2T+\Theta)^2}{36\Theta} + \frac{2}{9}\frac{(\Theta-T)^2}{\Theta}$	$\frac{T^2}{2\Theta}$

Since the addition of the Internet increases the number of regions, one must also consider what happens to total tax revenues collected across all regions. Symmetry between regions 1 and 2 guarantees that all tax rates, bases, and revenues will be identical for those regions. With conventional shopping only, regions 1 and 2 both set their tax rate equal to Θ. Since they each tax half the people, their tax bases are both equal to $\frac{1}{2}$ and total revenues equal Θ. The easiest comparison is between the cases without Internet commerce and the case with Internet commerce when $t_3 = 0$. Setting $t_3 = 0$ means that region 3 doesn't collect any tax revenue, and allows an examination of the competitive tax rate effect at work in this model. In the case where the Internet competes but sets a zero tax rate, region 1's tax rate is $\frac{T}{2}$ and it's base is $\underline{\lambda} = \frac{T-t_1}{\Theta}$. This yields tax revenue for region 1 of $\frac{T^2}{4\Theta}$. Since region 2's revenue is the same, total revenue is $\frac{T^2}{2\Theta}$. Revenues with Internet commerce (with $t_3 = 0$) will be lower if $T^2 < 2\Theta^2$. This will be the case if $T < \sqrt{2}\Theta$. For an equilibrium to exist, it must be the case that $T < \Theta$. Therefore, total revenues across all regions will be lower with a competing Internet region if the online tax is zero. The case in which the Internet region sets a tax rate endogenously is more complicated. The total tax revenue will be lower with the Internet region than without the Internet region if $\Theta > \frac{2(2T+\Theta)^2}{36\Theta} + \frac{2}{9}\frac{(\Theta-T)^2}{\Theta}$. This implies that $13\Theta^2 > 8T^2 - 4\Theta T$. To get a better understanding of revenues in this case, consider a numerical example.

2.5 A Numerical Example

In order to get a general feeling for the results of this model, consider the values $T = \frac{1}{2}$ and $\Theta = 1$. The three cases of interest are shown in the following table:

	Without e-commerce	*With e-commerce, $t_3 > 0$*	*With e-commerce, $t_3 = 0$*
Eq'm Tax Rates	$t_1 = 1$	$t_1 = \frac{1}{3}$	$t_1 = \frac{1}{4}$
	$t_2 = 1$	$t_2 = \frac{1}{3}$	$t_1 = \frac{1}{4}$
	n/a	$t_3 = \frac{1}{6}$	$t_3 = 0$
Tax Base	R1: $0 \longrightarrow \frac{1}{2}$	R1: $0 \longrightarrow \frac{1}{3}$	R1: $0 \longrightarrow \frac{1}{4}$
	R2: $\frac{1}{2} \longrightarrow 1$	R2: $\frac{2}{3} \longrightarrow 1$	R2: $\frac{3}{4} \longrightarrow 1$
	n/a	R3: $\frac{1}{3} \longrightarrow \frac{2}{3}$	R3: $\frac{1}{4} \longrightarrow \frac{3}{4}$
Regional Revenues	R1: $\frac{1}{2} = 0.5$	R1: $\frac{1}{9} = 0.1111$	R1: $\frac{1}{16} = 0.0625$
	R2: $\frac{1}{2} = 0.5$	R1: $\frac{1}{9} = 0.1111$	R1: $\frac{1}{16} = 0.0625$
	n/a	R3: $\frac{1}{18} = 0.0555$	R3: 0
Total Revenues	1	$\frac{5}{18} = 0.277$	$\frac{1}{8} = 0.125$

The tax rates set in conventional regions fall from 1 to $\frac{1}{3}$ as a result of a competing Internet region. In the case with a zero online tax, t_1 is even lower: $\frac{1}{4}$. Tax bases are also lower in the case with a competing Internet region, falling from $\frac{1}{2}$ before Internet commerce to $\frac{1}{3}$ with an endogenous Internet tax to $\frac{1}{4}$ with a zero Internet tax. Total revenues (across all regions) are also reported. Without Internet commerce, total tax revenues are $\Theta = 1$. Revenues are lower with a competing Internet region, although the competitive rate effect is large (revenues fall from 1 to 0.277). Consideration of the status quo shows an even greater competitive effect, with total revenues at a very low 0.125, one-eighth the level before Internet commerce. It should be noted that this numerical example does not give general results that can be extended to the rest of the model. In general, the ranking of the revenues can be done but the magnitude of the revenues in relationship to each other cannot be specified.

3 Social Welfare Maximization

One might argue that revenue maximization would be an unlikely objective function for the government in any region. It is shown that the revenue maximizating Nash values found earlier are a special case of the social welfare maximizing results derived here. The revenue maximization problem can therefore approximate the more complicated social welfare problem since all qualitative results will be identical.

Suppose that the conventional regions 1 and 2 aim to maximize the social welfare of the consumers who live there. Consumers between 0 and 1/2 live in region 1, and consumers between 1/2 and 1 live in region 2. The non-geographic nature of the Internet precludes consumers from living there. Because there are no consumers who would benefit from the provision of a public good, it is assumed that region 3's government continues to maximize tax revenue. This makes the Internet similar to a tax haven, a region that typically has unusually low tax rates and a very small population. Thinking about why tax rates in tax havens are so low, the demand for public goods is relatively low because there are so few people living there. The possibility of region three maximizing social welfare is left as an extension.

Social welfare is defined as simply the sum of individual utilities for people who live in a specific region. Continuing with the assumption that $\underline{\lambda} < \overline{\lambda}$, i.e., that there is Internet shopping, the maximization problems for regions 1 and 2 become the following:

$$\max_{t_1} \mathbf{SW_1} = \int_0^{\underline{\lambda}} [(\Theta - \lambda\Theta - t_1) + G(R)]\ d\lambda + \int_{\underline{\lambda}}^{1/2} [(\Theta - T - t_3) + G(R)]\ d\lambda \tag{9}$$

$$\max_{t_2} \mathbf{SW_2} = \int_{\overline{\lambda}}^{1} \{[\Theta - (1-\lambda)\Theta - t_2] + G(R)\}\ d\lambda + \int_{1/2}^{\overline{\lambda}} [(\Theta - T - t_3) + G(R)]\ d\lambda \tag{10}$$

For region 1, the first integral is the utility from private good consumption plus public good consumption of region 1 residents who shop in region 1. The second integral for region 1 includes the utility of region 1 residents who shop online. In this framework, region 1 still cares about their utility, despite the fact that their tax revenues are paid to region 3's government.

Each government uses revenues to finance a public good, which add to utility through $G(R)$. The public good adds to utility in a linear and separable way. For simplicity, the marginal benefit of the public good is assumed to be constant. This allows for a closed-form solution to the social welfare problem and a straightforward comparison between the social welfare maximizing equilibrium tax rates and those derived earlier. This implies that $\frac{\partial G}{\partial R} = k$, where $k > 2$. For a discussion of why $k > 2$, please refer to the appendix. Since the status quo where $t_3 = 0$ causes the reaction functions of both conventional regions to collapse, attention here is given to the case where t_3 is endogenously chosen. It should be kept in mind that the results in which $t_3 = 0$ are similar to the case in which t_3 is endogenously chosen.

Reaction functions:

Revenue Maximization	*Social Welfare Maximization*
$t_1^* = \frac{T + t_3}{2}$	$t_1^* = \frac{(T + t_3)}{2}\frac{(k-2)}{(k-1)}$
$t_2^* = \frac{T + t_3}{2}$	$t_2^* = \frac{(T + t_3)}{2}\frac{(k-2)}{(k-1)}$
$t_3^* = \frac{\Theta - 2T + t_1 + t_2}{4}$	$t_3^* = \frac{\Theta - 2T + t_1 + t_2}{4}$

The reaction functions in the social welfare case are a function of k, the marginal benefit of the public good. As k increases, the consumers care more about the level of the public good, i.e., the public good is valued in utility more like a private good. The maximization of tax revenue is equivalent to the social welfare maximization when $k \to \infty$. As k approaches infinity, it is as if they have been given a lump sum tax rebate, which could be used for private goods consumption.

Nash values:

Revenue Maximization	*Social Welfare Maximization*
$t_1^* = \frac{2T + \Theta}{6}$	$t_1^* = \frac{(2T + \Theta)}{2}\frac{(k-2)}{(3k-2)}$
$t_2^* = \frac{2T + \Theta}{6}$	$t_2^* = \frac{(2T + \Theta)}{2}\frac{(k-2)}{(3k-2)}$
$t_3^* = \frac{\Theta - T}{3}$	$t_3^* = \frac{(k-1)\Theta - kT}{3k-2}$

In the limit, as $k \to \infty$, the social welfare maximizing solutions for the reaction functions and Nash values approach the revenue maximizing values. For the Nash values above, applying l'Hôpital's rule[9], one has

[9] the infinity over infinity case

$$\lim_{k\to\infty} \frac{(2T+\Theta)}{2}\frac{(k-2)}{(3k-2)} = \frac{(2T+\Theta)}{6}$$
$$\lim_{k\to\infty} \frac{(k-1)\Theta - kT}{3k-2} = \frac{\Theta - T}{3} \tag{11}$$

Examination of figure 4 also shows that, as k approaches infinity, the social welfare maximizing reaction functions of the conventional regions approach the reaction functions from the revenue maximization setup. Figure 4 also shows that the reaction functions for the social welfare case are always below those for the revenue maximization case. This implies that all quantitative results derived in the revenue maximization setting hold for the social welfare maximizing setting. The more tractable revenue maximization model therefore approximates the social welfare problem in this case where utility is linear and separable and the marginal benefit of the public good is constant. This is particularly useful when an extension to two incomes level is considered.

4 Two-Income Extension

4.1 General Set-Up

In order to shed light on the equity issues involved with an e-commerce sales tax, it is necessary to construct an environment in which consumers' incomes differ. In this framework, more high-income consumers shop online than low-income consumers. This is intuitive because they have a higher willingness to pay the fixed cost associated with online shopping and their time is worth more (making conventional shopping less attractive). The conditions under which the online tax rate is lower than conventional tax rates are explored. In this case, taxes paid by the average high-income consumer are lower than those paid by the average low-income consumer, making an Internet sales tax regressive in this framework. It is also found that the optimal tax rates are a function of the distribution of income. Both conventional and Internet tax rates are negatively related to income inequality, i.e. the closer incomes are to each other, the higher are tax rates everywhere. Tax bases in all regions are ambiguously affected by income inequality.

Consumers are identical except for their level of income, which is either high, Θ_H or low, Θ_L, which is exogenous and fixed[10]. As before, consumers are uniformly distributed along a line from 0 to 1, and each individual's location is denoted by λ. Each consumer has unitary demand for the good and they decide in which region to purchase the good given relative tax-inclusive prices, travel costs, etc. Utility from holding money is zero, guaranteeing that each consumer will buy one unit. Each region maximizes tax revenue, although it is known from the previous section that the qualitative results will be consistent with the social welfare maximization problem. Utility of each type of consumer that shops in each of the regions is given below:

Utility with the purchase of one unit is therefore:

$$U^I = \begin{cases} \alpha\Theta_I - \lambda\Theta_I - t_1 & : \quad \text{when buying in region 1} \\ \alpha\Theta_I - (1-\lambda)\Theta_I - t_2 & : \quad \text{when buying in region 2} \\ \alpha\Theta_I - T - t_3 & : \quad \text{when buying in region 3} \end{cases}$$

where $I \in L, H$. Referring to figure 5, the bold regions denote the highest level of utility for each type of consumer. Visual inspection shows that there will be more high-income consumers shopping online than low-income consumers.

This extension requires a slight modification to the revenue functions. The tax base will be comprised of both high- and low-income shoppers, with each income group having a different cut-off value of λ. The objective function for region 1 is therefore:

$$\max_{t_1} \ \mathbf{R_1} \ = t_1 \int_0^{\underline{\lambda}_L} f(\lambda_L)\ d\lambda_L + t_1 \int_0^{\underline{\lambda}_H} f(\lambda_H)\ d\lambda_H \tag{12}$$

where $f(\lambda_L) = f(\lambda_H) = 1$, $\underline{\lambda}_L$ is the location of the low-income resident who is indifferent between shopping in region 1 and online, and $\underline{\lambda}_H$ is the high-income resident who is indifferent between shopping in region 1 and online. In figure 5, the tax base for region 1 is comprised of two bases: the low-income base is the horizontal distance from 0 to $\underline{\lambda}_L$; the high-income base from 0 to $\underline{\lambda}_H$.

[10] where $\Theta_H > \Theta_L$

Using the utility functions described above, setting the utility in region 1 equal the utility in region 2 allows the calculation of the location of λ who is indifferent between shopping region 1 and online. This yields the revenue-base cutoff values of $\underline{\lambda}$:

$$\begin{aligned}\underline{\lambda}_L &= \frac{1}{\Theta_L}\ (T + t_3 - t_1)\\ \underline{\lambda}_H &= \frac{1}{\Theta_H}\ (T + t_3 - t_1)\end{aligned} \tag{13}$$

As in the one-income case, increases in the tax rate for region 1 will lead to a decrease in their tax base (as more people shop online). This is true for both high- and low-income consumers. Similarly, increases in region 3's tax rate or T will cause fewer people to shop online. Increases in either high or low incomes will cause an increase in the willingness to pay T and therefore more online shopping.

Region 2's objective function is similarly defined:

$$\max_{t_2}\ \mathbf{R_2} = t_2 \int_{\overline{\lambda}_L}^{1} f(\lambda_L)\ d\lambda_L + t_2 \int_{\overline{\lambda}_H}^{1} f(\lambda_H)\ d\lambda_H \tag{14}$$

In figure 5, the tax base for region 2 is given by the horizontal distance $\overline{\lambda}_L$ to 1 for the low-income base and $\overline{\lambda}_H$ to 1 for the high-income base.

Again, using the utility functions described above, setting utility in region 2 equal to utility in region 3 yields high- and low-income values for $\overline{\lambda}$. These consumers are indifferent between shopping in region 2 and online:

$$\begin{aligned}\overline{\lambda}_L &= \frac{\Theta_L - t_3 + t_2 - T}{\Theta_L}\\ \overline{\lambda}_H &= \frac{\Theta_H - t_3 + t_2 - T}{\Theta_H}\end{aligned} \tag{15}$$

Region 3, the online region, has the following optimization problem:

$$\max_{t_3}\ \mathbf{R_3} = t_3 \int_{\underline{\lambda}_L}^{\overline{\lambda}_L} f(\lambda_L)\ d\lambda_L + t_3 \int_{\underline{\lambda}_H}^{\overline{\lambda}_H} f(\lambda_H)\ d\lambda_H \tag{16}$$

where the limits of integration are determined by the tax base cutoff levels of λ calculated above. Again referring to figure 5, the tax base for region 3 is given by the horizontal distance $\underline{\lambda}_L$ to $\overline{\lambda}_L$ for the low-income base and $\underline{\lambda}_L$ to $\overline{\lambda}_H$ for the high-income base.

Each region then maximizes revenues, taking into account that an increase in their tax rate will lower their tax base, i.e. change $\overline{\lambda}$ and/or $\underline{\lambda}$ for both the high- and low-income shopper. Solving each of these maximization problems yields the following first order conditions:

$$\begin{aligned} t_1^* &= \frac{T + t_3}{2} \\ t_2^* &= \frac{T + t_3}{2} \\ t_3^* = \frac{\Theta_P}{2\Theta_S} &+ \frac{t_1 + t_2 - 2T}{4} \end{aligned} \tag{17}$$

where $\Theta_P = \Theta_L * \Theta_H$ (product of the Θs) and $\Theta_S = \Theta_L + \Theta_H$ (sum of the Θs). These represent the reaction functions for each region. It can easily be shown that, setting $\Theta_L = \Theta_H$ causes these solutions to collapse back into the results found in the one-income version of the model[11].

An equilibrium will exist if the following conditions are met:

$$3(\Theta_L - \Theta_H) + \frac{\Theta_P}{\Theta_S} < T < \frac{3\Theta_L}{2} - \frac{\Theta_P}{\Theta_S}. \tag{18}$$

It must also be true that any possible deviation for any region would not increase their revenue. For example, region 1 could deviate by lowering their tax rate enough to capture all low-income online shoppers. This would change their tax base, alter their revenue function and therefore their objective function. Please refer to the appendix for a discussion of the existence of the Nash equilibrium and possible deviations. It should be noted that, despite an extensive number of conditions that must be met, it is easy to find values for Θ_L, Θ_H, and T for which this Nash equilibrium holds[12].

[11] It should be noted, however, that the number of consumers has doubled, causing tax revenues to double.
[12] Θ_L=1, $\Theta_H = 2$, and $T = 1/2$.

Solving the set of first order conditions above in equation 17 yields the following Nash equilibrium tax rates:

$$\begin{aligned} t_1^{A*} = t_2^{A*} &= \frac{\Theta_P}{3\Theta_S} + \frac{T}{3} \\ t_3^{A*} &= \frac{2\Theta_P}{3\Theta_S} - \frac{T}{3} \end{aligned} \tag{19}$$

where superscript "A" denotes that this is regime A, the candidate Nash equilibrium. Other potential regimes are explored in the deviation analysis mentioned above (located in the appendix). Symmetry between the conventional regions continues to guarantee that the tax rates are equal in regions 1 and 2. Of interest here is both the optimal tax rates in the electronic region versus the conventional regions and the fact that the optimal tax is a function of the distribution of income, Θ_P, for a given Θ_S.

4.2 Online Tax Rate versus the Conventional Tax Rates

Since more rich consumers shop online than low-income consumers, the taxes paid by consumers in both the online and conventional markets are of interest. The Nash equilibrium tax rates are given by equation 19. The online tax rate will be lower when $t_3^{A*} = \frac{2\Theta_P}{3\Theta_S} - \frac{T}{3} < t_1^{A*} = \frac{\Theta_P}{3\Theta_S} + \frac{T}{3}$. This occurs when $\frac{\Theta_P}{2\Theta_S} < T$.

Figure 6 shows the optimal tax rates in both the conventional and online regions as a function of the fixed cost of shopping online. With a sufficiently high fixed cost of shopping online relative to income, the Internet region will not "enter" the model. This occurs to the right of the point where $T = \frac{2\Theta_P}{\Theta_S}$[13]. Starting from this situation, a decrease in T holding incomes constant is necessary before the electronic region will enter and a three-region model with sustainable e-commerce exists. Region 3 will participate but set a zero tax rate when $T = \frac{2\Theta_P}{\Theta_S}$. A further decrease in T will cause region 3 to set a relatively low tax rate in order to continue to lure customers from across the borders. As the fixed

[13] Recall the discussion regarding the difference between region 3's tax rate between zero and there being no threat of entry from region 3.

cost of shopping online continues to decline, region 3 sees additional increases in their tax base and therefore the incentive to set a low tax rate declines.

In the case where $\Theta_L = 1$, $\Theta_H = 2$, and $T = 1/2$, this implies that $t_3^{A*} < t_1^{A*}$ because $1/3 < 1/2$. Holding fixed all incomes, a lower T will cause an increase in t_3^{A*}. With T sufficiently low, i.e. $= 1/3$[14], the online tax rate would equal the conventional tax rate. Further decreases in T would cause the online tax rate to exceed the conventional tax rate.

Given that more high-income consumers shop online than low-income, the tax that the average high-income person pays is a smaller percentage of their income than that of the average low-income person. In general, also, there is concern about the "tax break" going to the rich because currently the online tax is zero. This can be represented in this model by the case where $T = \frac{2\Theta_P}{\Theta_S}$. Over time, technological progress would further decrease T relative to income, leading to increases in the optimal tax of the Internet region. Therefore, while an Internet tax may be regressive at first, future technological advances that continue to lower the cost of computing will change the nature of this tax over time.

4.3 Optimal Taxes, Revenues, and Income Distribution

Examination of equation 19 shows that, for a given level of total income in the economy, Θ_S, a change in Θ_P will change the equilibrium tax rates in all three regions. In order to perform comparative statics, define $\Theta_L = (\Theta_S - \Theta_H)$. Increasing inequality is therefore represented by increasing Θ_H, which in this context would lower Θ_L sufficiently to leave Θ_S unchanged. Consider the case in which $\Theta_L = 1$, $\Theta_H = 2$, and $T = \frac{1}{2}$[15].

For a given Θ_S, Θ_P will be maximized the closer Θ_L and Θ_H[16]. So, increasing income inequality can be represented by a decrease in Θ_P. Comparative statics show that increasing/decreasing Θ_P leads to an increase/decrease in all regions'

[14] In general when $T < \frac{\Theta_P}{2\Theta_S}$

[15] Recall that these values are consistent with the existence of the candidate Nash equilibrium

[16] e.g. taking Θ_S=3, compare $\Theta_P^1 = 2$ when $\Theta_L = 1$ and $\Theta_H = 2$ to $\Theta_P^2 = 2.24$ when $\Theta_L = 1.4$ and $\Theta_H = 1.6$

tax rates, and therefore higher income inequality leads to lower tax rates everywhere. Equivalently, the more similar high- and low-incomes, the higher tax rates in all regions. Figure 7 shows the reaction functions from equation 17 . Increasing Θ_P shifts the reaction function of region 3 down, leading to a decrease in the equilibrium tax rates in all regions.

An understanding of tax revenues as a function of income equality changes requires examination of how the high- and low-income tax bases change in each region. Increasing inequality leads to an increase in Θ_H and a decrease in Θ_L. Equations 13 and 15 give the revenue cut-off levels of λ for both the high and low-income consumers as a function of tax rates. For example,

$$\frac{\partial \underline{\lambda}_L}{\partial \Theta_H} = \frac{T + t_3 - t_1}{\Theta_L^2} \tag{20}$$

This implies that the change in the tax base will depend on the relative price of goods in both regions 1 and 3. $T + t_3$ is the final user cost of purchasing 1 unit in region 3, as this includes the fixed cost and the final payment for the good.

Examination of equation 19 shows that the final cost of the good online will always be twice as high as the cost of the good in either conventional region. That is,

$$T + t_3 = T + \frac{2\Theta_P}{3\Theta_S} - \frac{T}{3} = 2\left(\frac{\Theta_P}{3\Theta_S} + \frac{T}{3}\right) = 2t_1 = 2t_2 \tag{21}$$

Therefore, each region will see the following effects from an increase in income inequality:

Region	*Low-Income Base*	*High-Income Base*
Regions 1 and 2	Increase	Decrease
Region 3	Decrease	Increase

Increases in income inequality, therefore, lead to ambiguous changes in tax revenue in each region even in this simple setting. A more complete investigation of the effects of income inequality would be useful in a setting where income is endogenously determined.

Examination of the revenues in each region also shows the ambiguity with respect to Θ_P and therefore income distribution.

$$R_1 = R_2 = \frac{T^2\Theta_S}{9\Theta_P} + \frac{\Theta_P}{9\Theta_S} + \frac{2T}{9}$$
$$R_3 = \frac{2}{9} \cdot \frac{(2\Theta_P - \Theta_S T)^2}{\Theta_P \Theta_S} \tag{22}$$

Setting $\Theta_L = \Theta_H$, this collapses almost completely to the one-income case shown in equation 8. In the original model, $R_1 = \frac{(2T+\Theta)^2}{36\Theta}$. In this case, revenue would be twice that much, seeing as though the number of residents in the model has doubled.

5 Conclusions and Extensions

This model can describe the emergence of e-commerce through a decrease in the fixed cost of shopping online relative to income. Given a sufficient decrease in this cost, the electronic region can "enter" and attract a tax base if they set relatively low tax rates. As the fixed cost of shopping online continues to decrease, increased usage of the Internet as a way of shopping is expected.

This model provides a framework in which the potential sales tax revenue losses of state governments due to increasing e-commerce sales can be explored. Concerns over these revenue losses may, indeed, be justified. Conventional regions see both lower tax bases and tax rates when the Internet region competes. This necessarily leads to lower tax revenues for conventional regions with sustained e-commerce than before. The model concentrates on the case where the Internet firm chooses tax rates endogenously. Since currently Internet purchases are not taxed, this is equivalent to the case in which the Internet tax is zero. It is shown that the results of the model are only amplified in the case where the Internet tax is zero. Total tax revenues across all regions fall with Internet commerce, illustrating the increased competition among governments due to e-commerce.

Study of the social welfare maximization formulation shows that the revenue maximizing solution is simply a special case of the social welfare formulation.

This is advantageous because the assumption of revenue maximization is not necessarily desirable. However, revenue maximization is the easiest optimization problem to solve. As k increases, the reaction functions and optimal tax rates approach the revenue maximizing results. Qualitative predictions of the revenue maximizing model are therefore consistent and the social welfare problem can be approximated in this way. This is particularly useful when the two-income extension is constructed.

In studying the two-income case, valuable insights into the equity issues surrounding an e-commerce sales tax are uncovered. More high-income consumers shop online than low-income. This has implications for equity if the tax rate online is below that found in conventional markets. This is currently the case, implying that an e-commerce sales tax would be regressive. However, with further technological improvements leading to increases in the optimal online tax rate, the regressive nature of an electronic commerce tax would diminish over time. The Nash equilibrium tax rates in the two-income model are a function of income inequality. More income inequality will lead to lower equilibrium tax rates in this simple setting. However, the effects on the tax bases (and therefore tax revenues) in each region are ambiguous with respect to income distribution. This suggests that further investigation into the effects of income distribution would be useful, especially in a setting with endogenously determined income.

It is important to consider the underlying assumptions that drive the results in this model. The implications of relaxing each of these assumptions provide natural suggestions for extensions to this paper.

First, it is assumed that the Internet firm is located in a separate region. This implies that the tax revenue from online shopping flows out of the conventional regions where the consumers live. This is a natural starting place because the focus here is on remote sales to consumers in states in which the seller does not have a physical presence, or nexus. An alternate formulation with the Internet firm located in one of the two conventional regions would have two effects. It would both funnel revenues back into a region with consumers and it would potentially change the strategic behavior of both regions. This extension could help to support the results found here regarding tax rates, bases, and revenues.

Tax revenues are collected based on the location of sale (origin taxes) rather than the location of the consumer (destination taxes). Destination based taxes in this model would eliminate the competition altogether, as the incentive to shop across borders would disappear. The current tax system is basically origin taxes for local conventional purchases (both at home and in close conventional neighboring regions) and destination taxes (use taxes, if enforced) for remote purchases. A model exploring these issues would be useful in assessing the effects of use tax enforcement.

Firm location is fixed. E-commerce firms have highly mobile capital, meaning that they can relocate with relatively little cost. Intuitively, allowing for endogenous firm location would cause competition for firms if governments compete using tax breaks, rebates, subsidies, or other incentives. This could lead to further decreases in revenues, as governments potentially try to attract businesses using tax subsidies.

Continuing to think about endogenous firm location, destination taxes would decrease the incentives for firms to move to low-tax regions, but would vastly complicate the taxes firms would need to collect. This framework would not only aid in the discussion of tax revenues, it could also help to analyze the question of whether taxing e-commerce sales harm the growth of the Internet. These added compliance costs would potentially make e-commerce less profitable to the point where firms would exit the market.

The consumers in this model have all pertinent information before they decide whether or not to access the Internet and decide where to shop. An extension could include either costly search or uncertainty regarding conventional / online prices.

The social welfare maximization formulation assumes that no consumers live in the Internet region because of its nongeographic nature. An extension allowing consumers to live in that region would be interesting to gain a better understanding of the model's sensitivity to that assumption.

The strategic variable used in this model is the tax rate. An alternate formulation could use the public good as the policy instrument. Wildasin (1988) shows that the equilibrium in which the public good is used as the strategic variable is more 'rivalrous' than that where the choice variable is the tax rate. This alternate formulation would suggest that using the public good as the choice variable would lead to more competition in this setting.

This is a static model. In order to fully understand the effects of taxing e-commerce sales, one must consider a dynamic framework in which taxing a sector lowers the returns to investment in that sector. This is perhaps the most logical framework in which to examine whether or not taxing sales on the Internet would harm its growth.

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Figure 1: One-Income Model with Three Regions

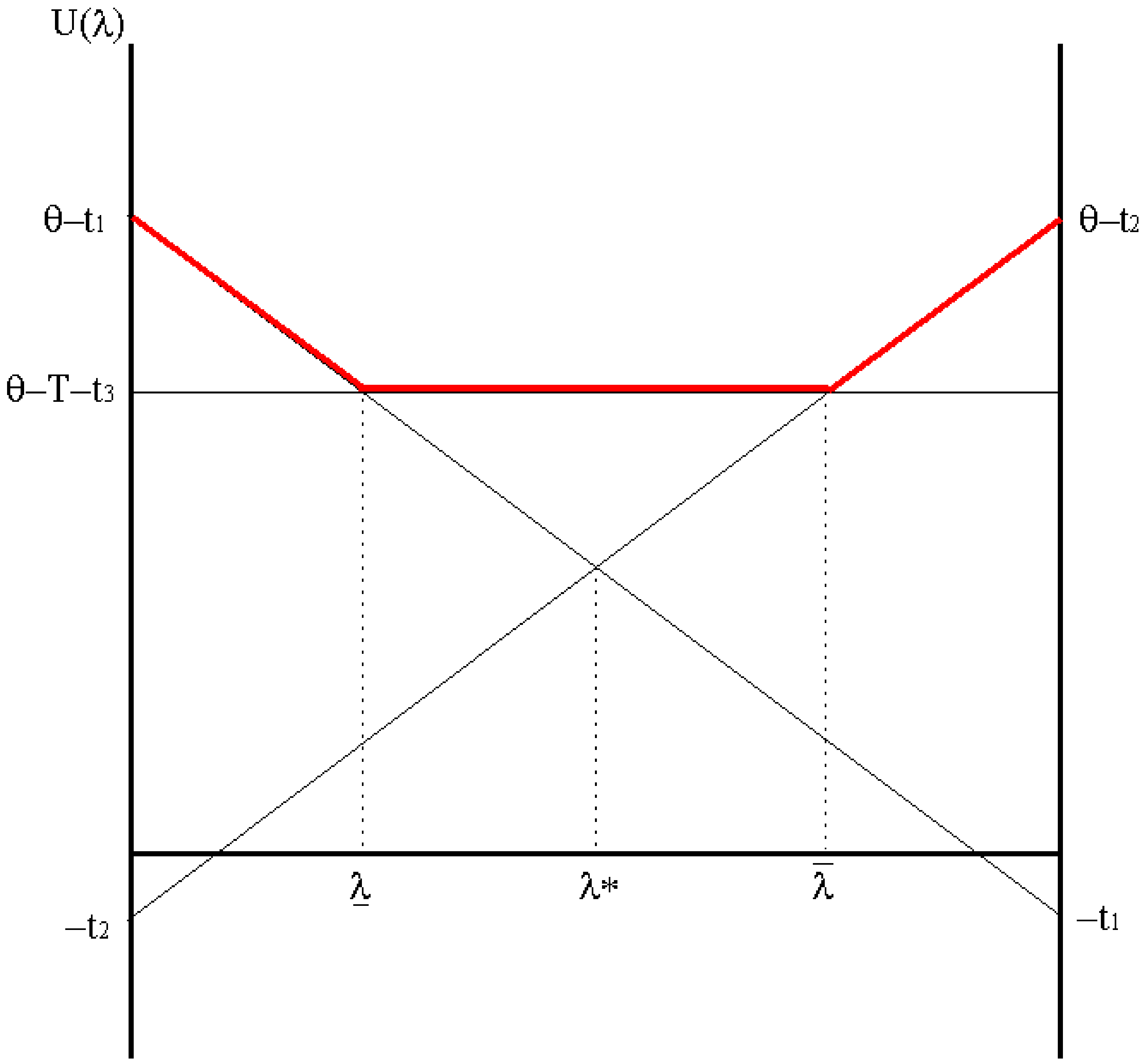


Figure 2: Conventional Tax Rates With and Without E-Commerce

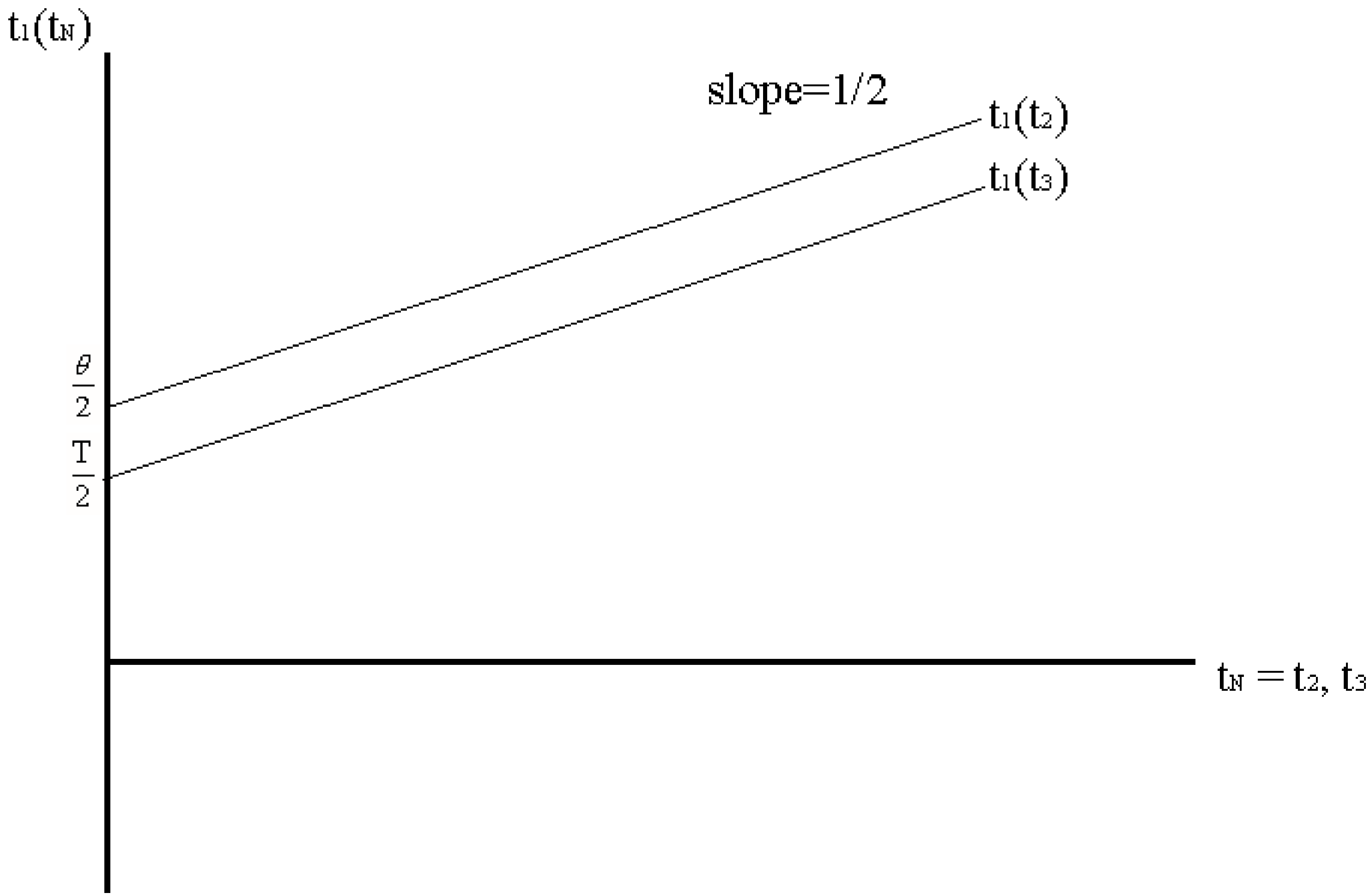


Figure 3: Conventional Tax Bases With and Without E-Commerce

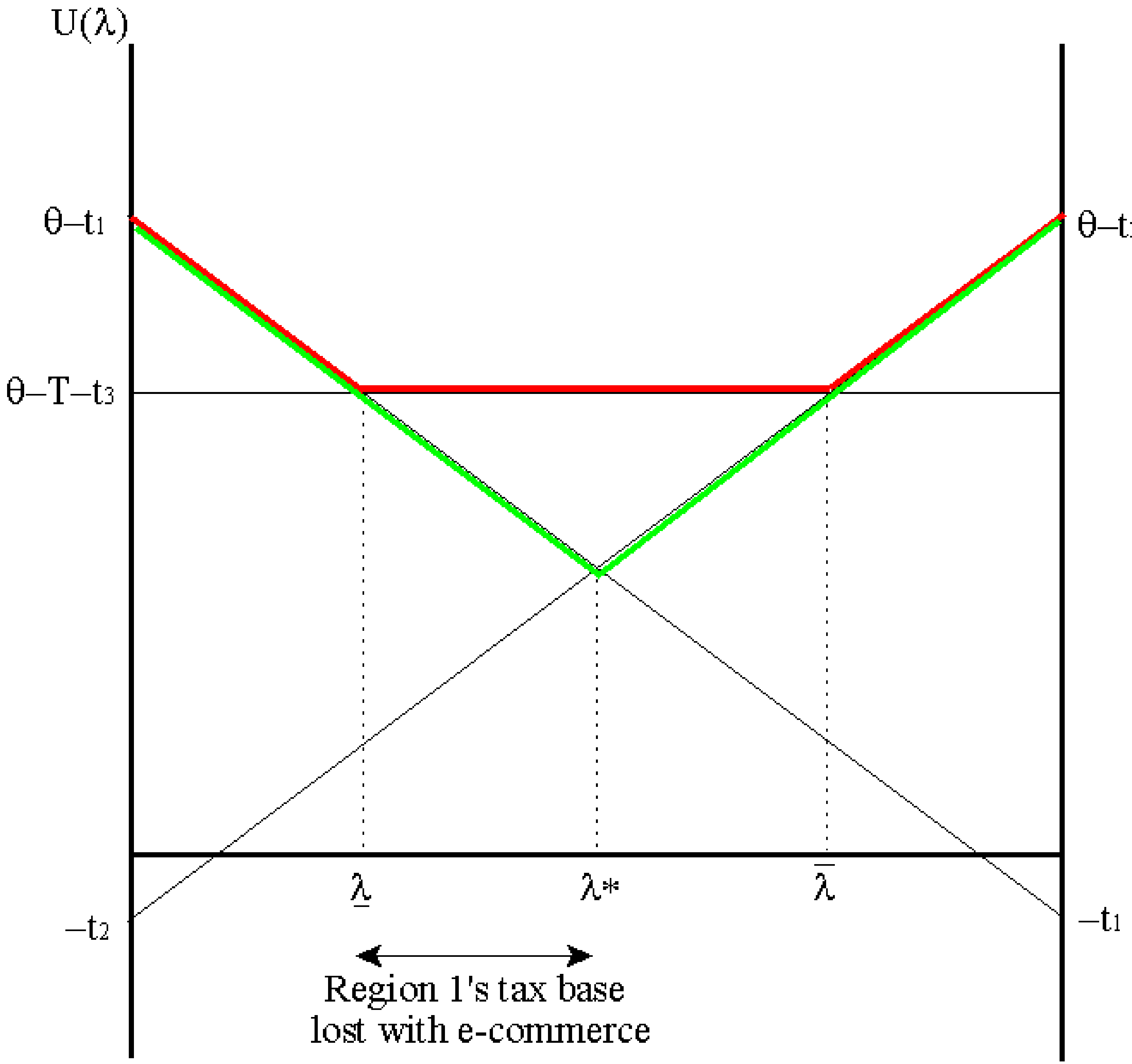


Figure 4: Revenue Maximization versus Social Welfare Maximization

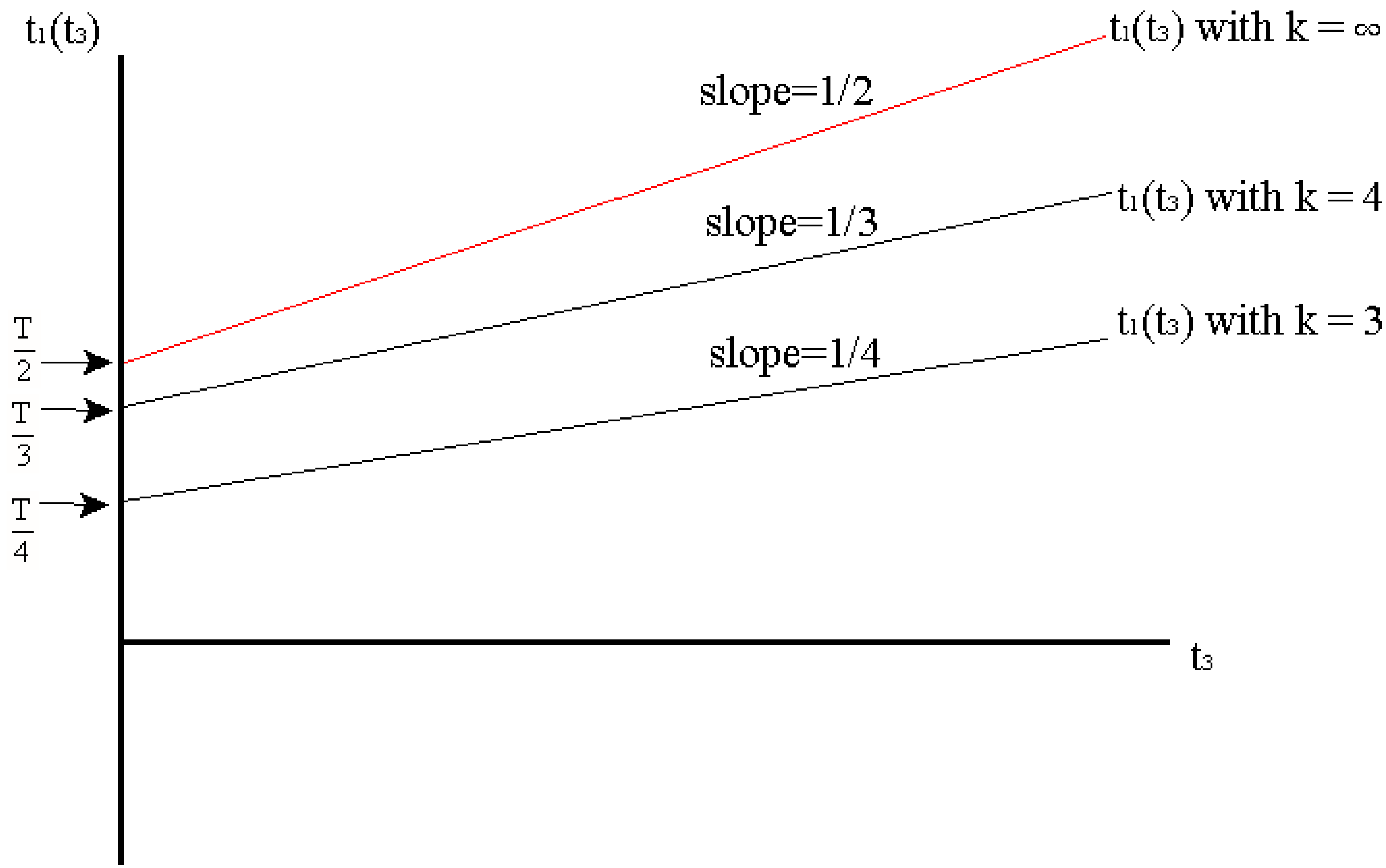


Figure 5: Two-Income Model with Three Regions

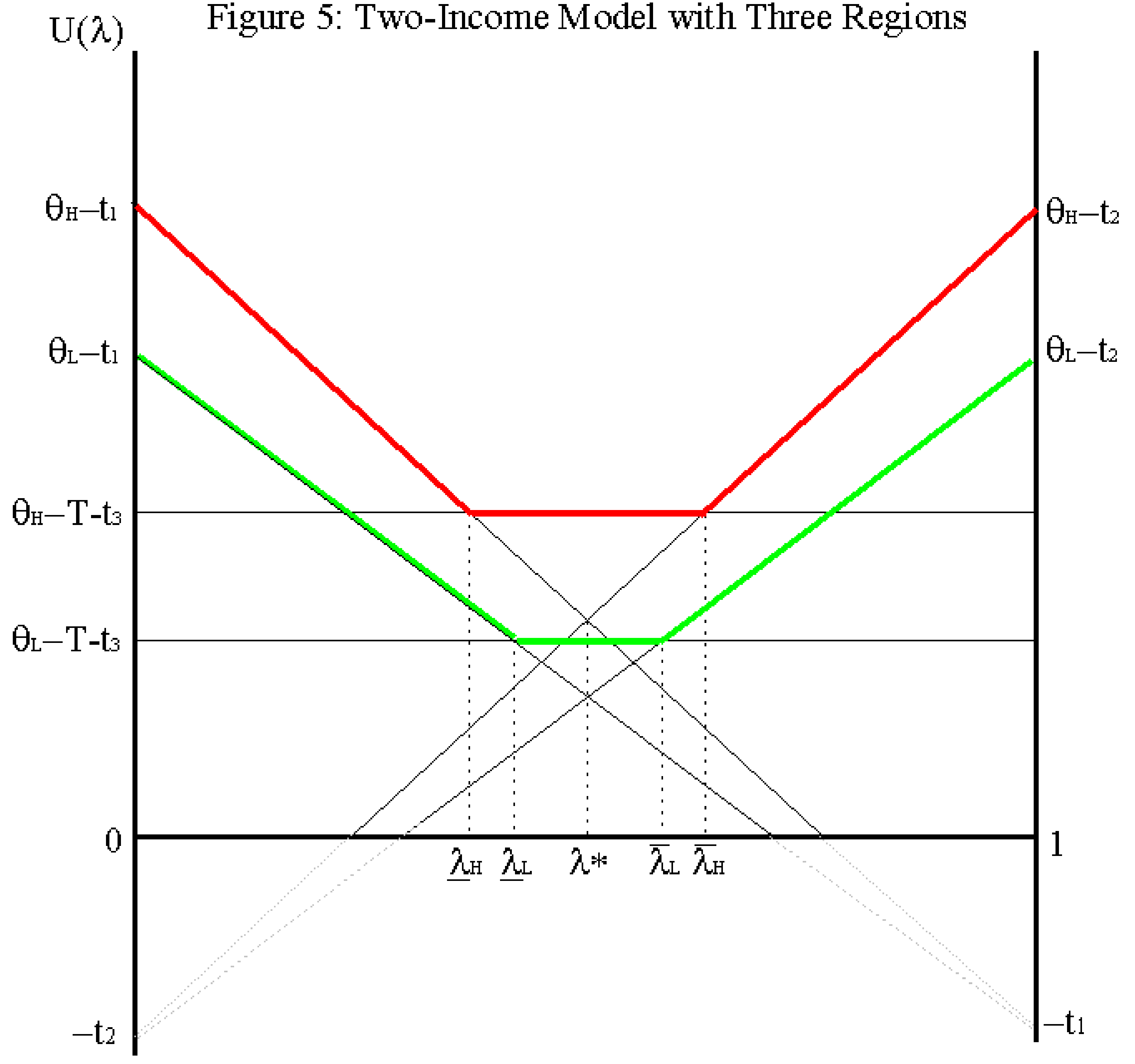


Figure 6: Tax Rates as a Function of the Fixed Cost of Shopping Online

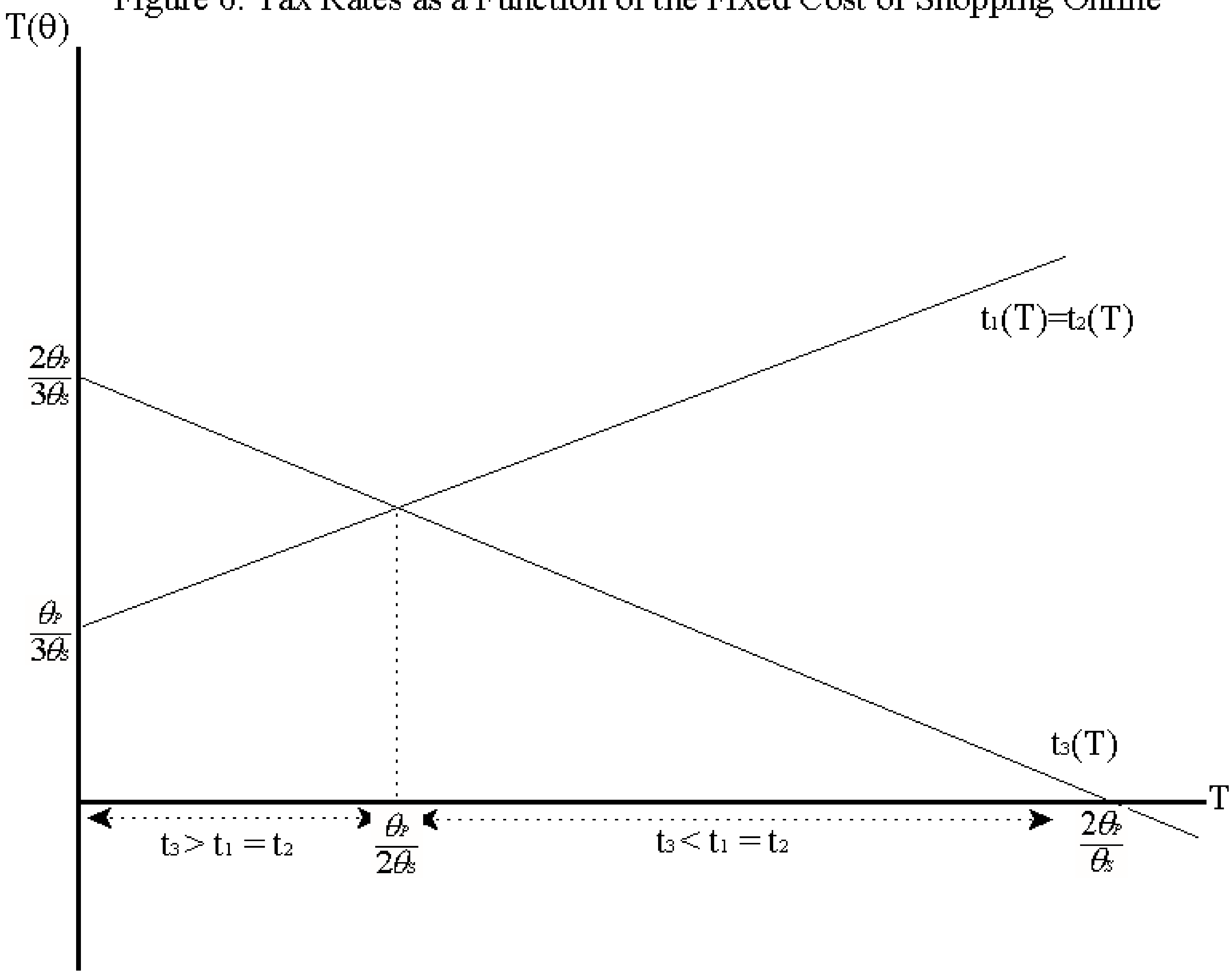


Figure 7: Reaction Functions in Two-Income Model

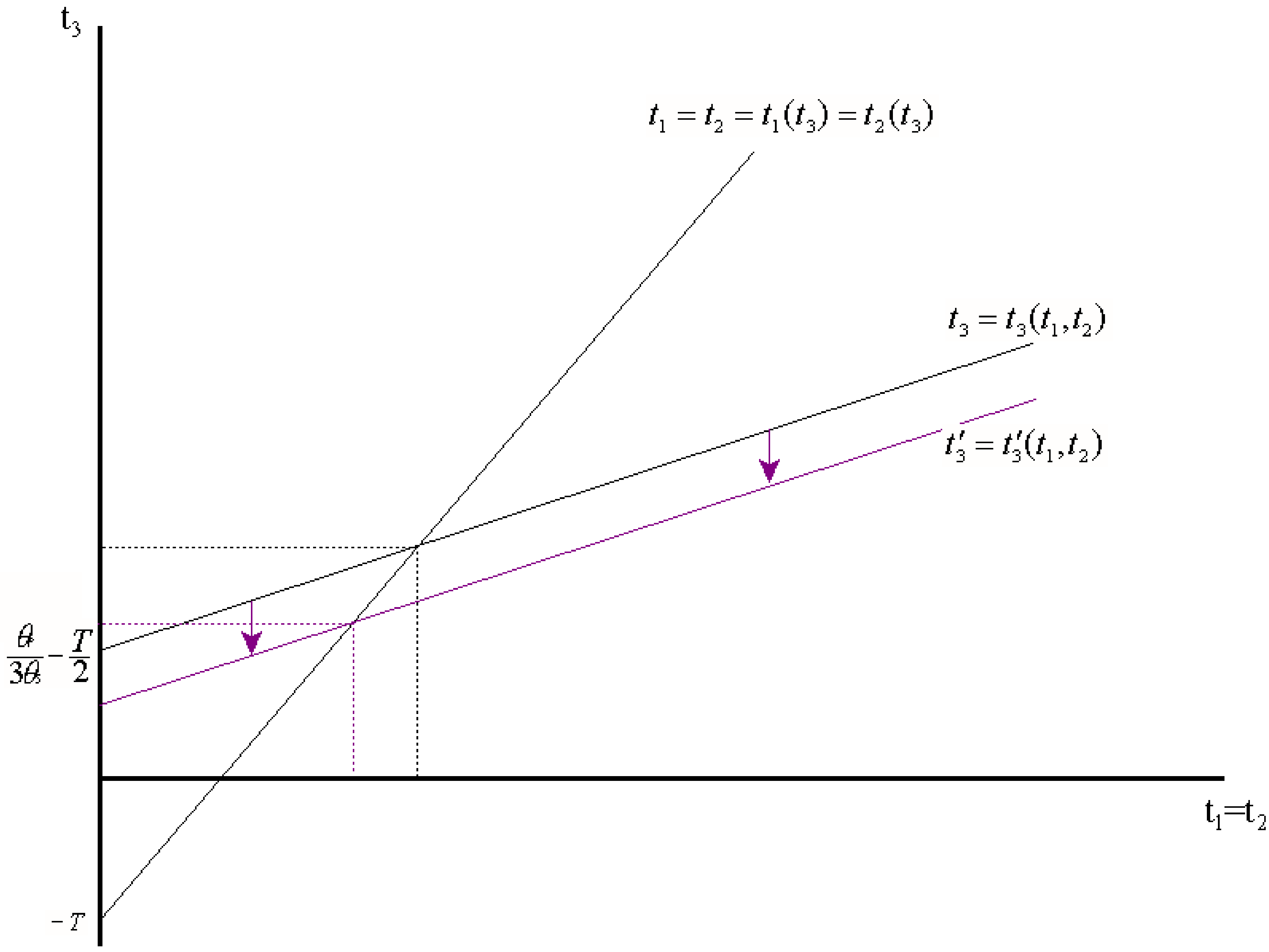


Figure 8: Deviation into Regime B for Region 1

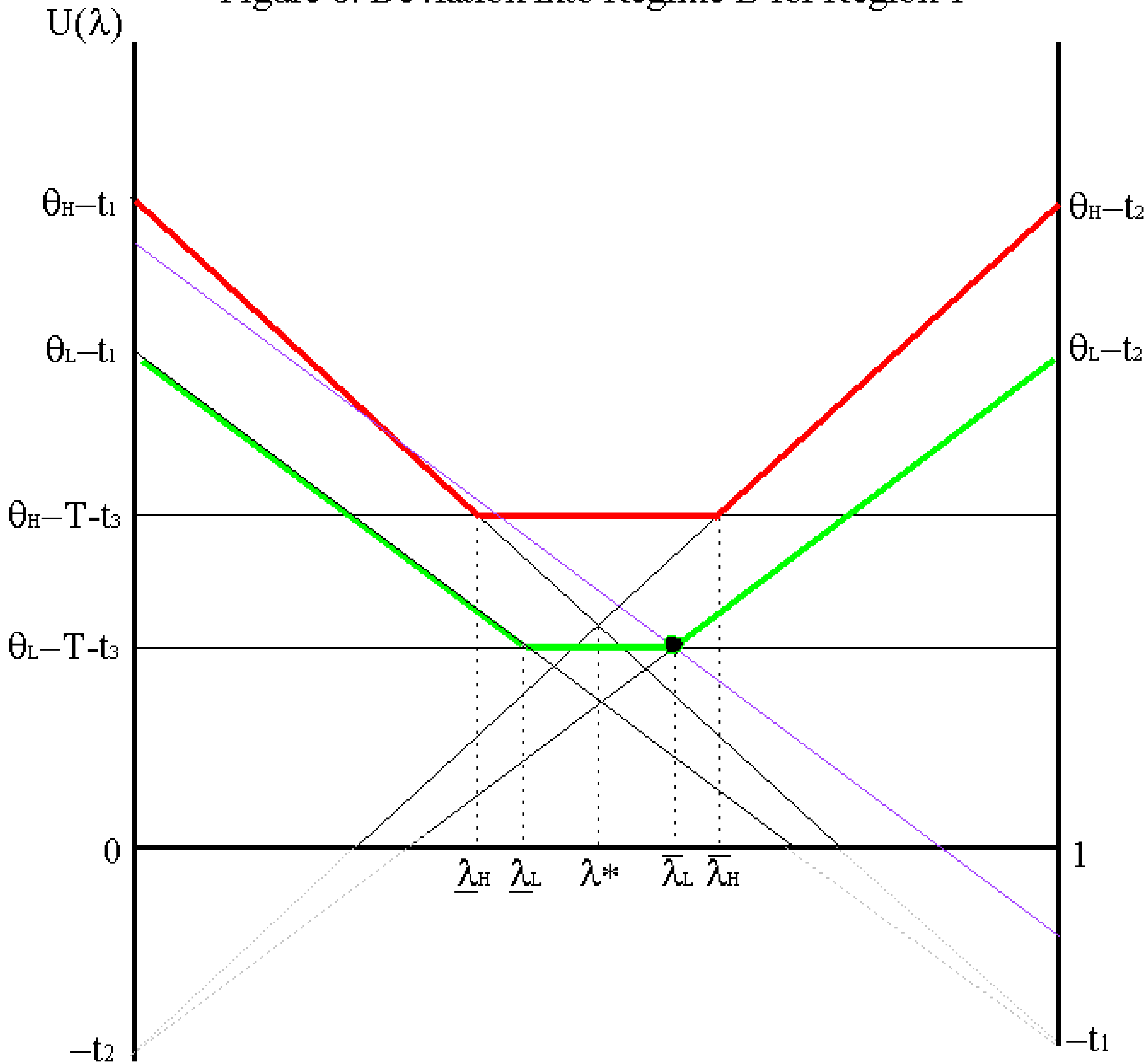
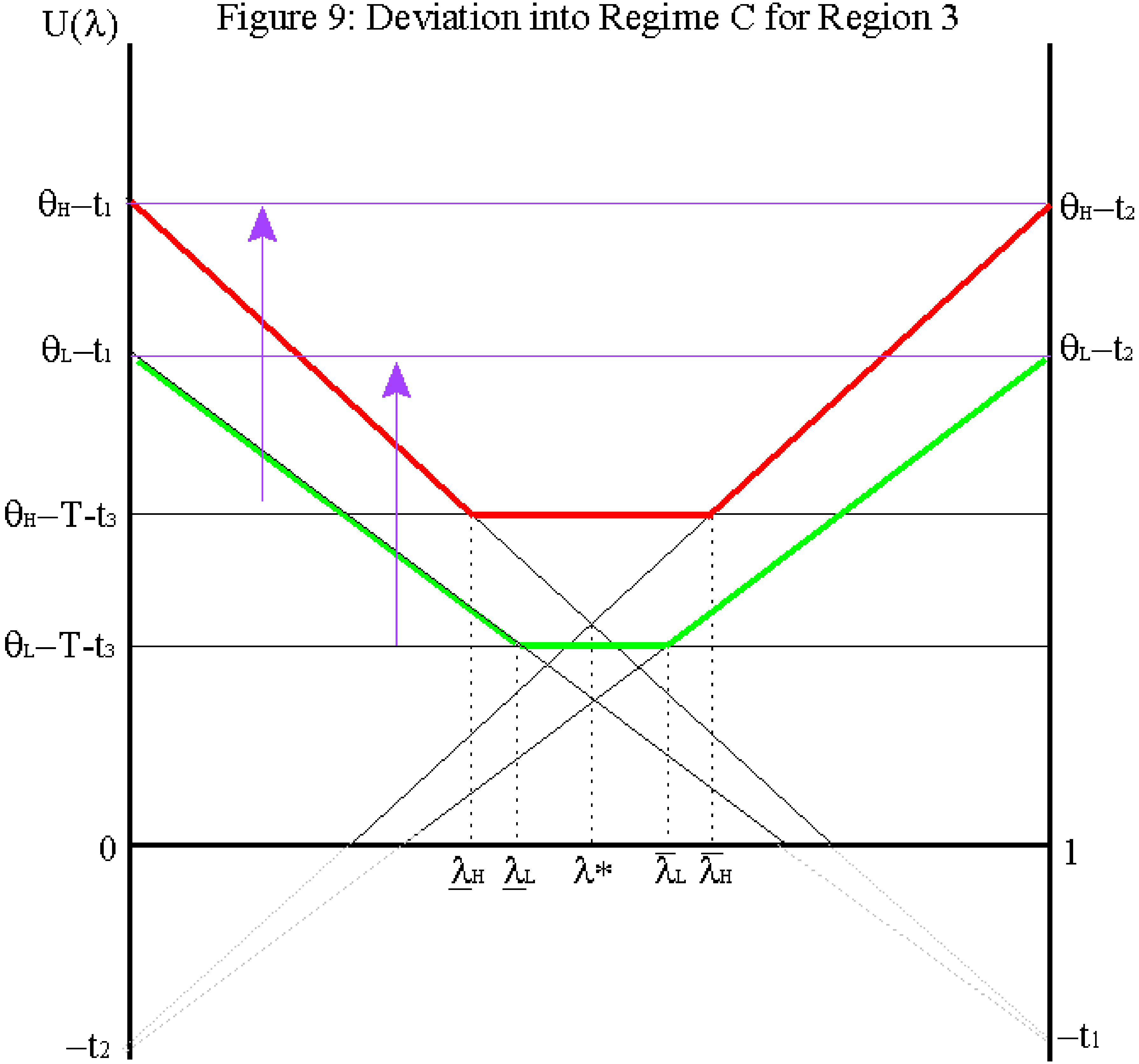


Figure 9: Deviation into Regime C for Region 3



Appendix

A Existence of Nash Equilibrium

In order for this Nash equilibrium to exist, certain conditions must hold for the exogenous parameters. Specifically, all equilibrium tax rates must be non-negative, all λs must be positive and ordered correctly, all graphical intercepts must be as pictured in figure 5, and some consumers must shop online.

(i) All tax rates non-negative: $t_1 \geq 0 \implies \frac{-\Theta_P}{\Theta_S} \leq T$: holds for any positive values of T, Θ_L, and Θ_H[17]. $t_2 \geq 0$: symmetric with t_1. $t_3 \geq 0 \implies T \leq \frac{2\Theta_P}{\Theta_S}$.

(ii) All λs positive and ordered correctly... In order for this to be the case, $0 < \underline{\lambda}_H < \underline{\lambda}_L < \lambda^*_L = \lambda^*_H < \overline{\lambda}_L < \overline{\lambda}_H < 1$.

$0 < \underline{\lambda}_H \implies t > -\frac{\Theta_P}{\Theta_S}$: holds by construction.

$\underline{\lambda}_H < \underline{\lambda}_L \implies \frac{T(\Theta_L - \Theta_H}{\Theta_P} < \frac{\Theta_H - \Theta_L}{\Theta_S}$': holds because $\Theta_H > \Theta_L$.

$\underline{\lambda}_L < \lambda^*_L \implies T < \frac{3\Theta_L}{2} - \frac{\Theta_P}{\Theta_S}$: collapses with several other assumptions.

$\lambda^*_L = \lambda^*_H \implies 1/2 = 1/2$.

$\lambda^*_H < \overline{\lambda}_L \implies T < \frac{3\Theta_L}{2} - \frac{\Theta_P}{\Theta_S}$: same as condition above.

$\overline{\lambda}_L < \overline{\lambda} \implies \frac{(\Theta_L - \Theta_H)T}{\Theta_P} < \frac{(\Theta_H - \Theta_L}{\Theta_S}$: this holds by construction.

$\overline{\lambda}_H < 1 \implies -\frac{\Theta_P}{\Theta_S} < T$: this also holds by construction.

(iii) All intercepts in figure 5 must be ordered correctly.

$\Theta_H - t_1 > \Theta_L - t_1$: holds by construction.

$\Theta_H - t_2 > \Theta_L - t_2$: holds by construction.

$\Theta_L - T - t_3 < \Theta_H - T - t_3$': holds by construction.

(iv) Some people shop online.

$\Theta_L - T - t_3 > U(\Theta^*_L)$: this means that the utility from shopping in region 3 must be higher for at least one individual with low income. The most likely person to shop online is the person who would otherwise be indifferent between shopping in region 1 or 2. This person is Θ^*_L. Therefore, their utility must be higher online, guaranteeing online participation from low-income shoppers.

$\Theta_H - T - t_3 > U(\Theta^*_H)$: similar to above for high-income online participation.

These conditions collapse into the following:

$$3(\Theta_L - \Theta_H) + \frac{\Theta_P}{\Theta_S} < T < \frac{3\Theta_L}{2} - \frac{\Theta_P}{\Theta_S}. \tag{23}$$

In addition to these conditions, examination of possible deviations that could make any of the regions better off must be undertaken.

[17] recall that $\Theta_P = \Theta_L \Theta_H$ and that $\Theta_S = \Theta_L + \Theta_H$.

B Deviation for Region 1 - Regime "B"

Superscript "A" is used to denote the original candidate Nash equilibrium values. The first possible deviation for region 1 involves lowering t_1 sufficiently as to capture the low-income online shoppers.

See figure 8 for a graphical interpretation of the shift into regime "B". Lowering t_1 corresponds to shifting out the utility curves of both the low- and high-income individuals. Region 1 can lower t_1 to a point where they capture all of the low-income online shoppers - shown by the point on the graph. For any tax rates lower than this (pushing out further the t_1 line) the tax base for region 1 is the following: $R_1^B : 0 \rightarrow \lambda_L^*$ and $0 \rightarrow \underline{\lambda}_H$.

Note that this is different than the original tax base, which was given by: $R_1^A : 0 \rightarrow \underline{\lambda}_L$ and $0 \rightarrow \underline{\lambda}_H$.

The tax rate such that region 1 shifts into regime B occurs is denoted $\bar{t}$. This is the value of t_1 where $\overline{\lambda}_L = \underline{\lambda}_L$:

$$\bar{t}_1 = 2T + 2t_3 - t_2 - \Theta_L$$

$$\bar{t}_1 = T + \frac{\Theta_P}{\Theta_S} - \Theta_L \tag{24}$$

where the first equation denotes this cutoff $\bar{t}_1$ as a function of tax rates and the second denotes $\bar{t}_1$ as a function of exogenous parameters.

The maximization problem for region 1 in regime B is:

$$\max_{t_1} \mathbf{R}_1^B = t_1 \int_0^{\lambda_L^*} f(\lambda_L)\ d\lambda_L + t_1 \int_0^{\underline{\lambda}_H} f(\lambda_H)\ d\lambda_H \qquad s.t.\ t_1 \leq \bar{t}_1 \tag{25}$$

This collapses to:

$$R_1^B = t_1[\lambda_L^* + \underline{\lambda}_H]$$

One can then solve for λ_L^* and $\underline{\lambda}_H$ as functions of t_1 and substitute in:

$$\lambda_L^* = \frac{1}{2} + \frac{(\Theta_P + T)}{6\Theta_L\Theta_S} - \frac{t_1}{2\Theta_L} \tag{26}$$

$$\underline{\lambda}_H = \frac{T}{\Theta_H} + \frac{2\Theta_P}{2\Theta_H\Theta_S} - \frac{T}{3\Theta_H} - \frac{t_1}{\Theta_H} \tag{27}$$

Checking second order conditions to see that the revenue function described above is strictly concave: $\frac{\partial^2 R_1^B}{t_1^2} = -2\frac{(\Theta_H + 2\Theta_L)}{2\Theta_P}$ (always negative).

At the regime cutoff tax rate value $\bar{t}_1$, the values of the revenue functions are equal. Knowing this, there are two possible cases to examine: 1) If $\frac{\partial R_1^B}{\partial t_1} > 0$, the regime B revenue function is increasing at $\bar{t}_1$. Given the continuity and concavity of both functions, the maximum value of t_1 within regime B will occur at $\bar{t}_1$ and that this value cannot be a global maximum.

2) If $\frac{\partial R_1^B}{\partial t_1} < 0$, the revenue function in regime B is decreasing at the point where it intersects $\bar{t}_1$. In this case, the local maximum for regime B could possibly be a global max. One can calculate R_1^{B*}, the maximum of the regime B revenue function.

This yields:

$$t_1^{B*} = \frac{\Theta_P}{\Theta_H + 2\Theta_L}[\frac{1}{2} + \frac{\Theta_P + T}{6\Theta_L\Theta_S} - \frac{2T}{3\Theta_H} + \frac{2\Theta_P}{3\Theta_H}\Theta_S] \tag{28}$$

Comparing R_1^{B*} with R_1^{A*} from the original candidate Nash equilibrium, one can then develop conditions under which regime A's solution will continue to be a global maximum.

Intuitively, the tax bases change in each regime, making the revenue functions different. One must then examine the derivatives of each function with respect to the tax rate at each regime cutoff tax rate (in this case, $\bar{t}_1$). Then, check the local maximum value of t against t_1^{A*}, the candidate Nash global maximum. A large list of conditions results, each of which must hold in order for the candidate Nash to remain the global maximum.

C Deviation for Region 1 - Regime "C"

Regime C, where region 1 lowers t_1 such that they capture all the low- and high-income online shoppers.

In solving for $\bar{\bar{t}}_1$, note that this is where $\underline{\lambda}_H = \overline{\lambda}_H$. Solving these for t_1 yields:

$$\bar{\bar{t}}_1 = 2T + 2t_3 - t_2 - \Theta_H \tag{29}$$

Substituting in the Nash values for t_2 and t_3 and solving for t_1 yields:

$$\bar{\bar{t}}_1 = T + \frac{\Theta_P}{\Theta_S} - \Theta_H \tag{30}$$

The revenue maximization problem within Regime C is:

$$\max_{t_3} \mathbf{R}_1^C = t_1 \int_0^{\lambda_L^*} f(\lambda_L)\, d\lambda_L + t_1 \int_0^{\lambda_H^*} f(\lambda_H)\, d\lambda_H$$
$$s.t.\ t_1 \leq \bar{\bar{t}}_1 \tag{31}$$

Following the same procedure as with regime B, substitute in for λ_L^* and λ_H^* (where both are expressed as a function of t_1). Again, at the regime cutoff tax rate value $\bar{\bar{t}}_1$, the values of the R_1^B and R_1^C are equal. Knowing, this there are two possible cases to be examined:

1) If $\frac{\partial R_1^C}{\partial t_1} > 0$, the regime C revenue function is increasing at $\bar{\bar{t}}_1$. Given the continuity and concavity of both functions, the maximum value of t_1 within regime C will occur at $\bar{\bar{t}}_1$ and that this value cannot be a global maximum.

2) If $\frac{\partial R_1^C}{\partial t_1} < 0$, the revenue function in regime B is decreasing at the point where it intersects $\bar{\bar{t}}_1$. In this case, the local maximum for regime C could possibly be a global max. One can calculate R_1^{C*}, the maximum of the regime C revenue function and then compare this value to the original Nash equilibrium value for t_1. Conditions under which the original candidate Nash equilibrium continues to be a global maximum can then be calculated.

As region 2 is symmetric with region 1, no explicit discussion of the possible deviations for region 2 are presented here.

D Deviation for Region 3 - Regime "D"

Region 3 can deviate by lowering their tax rate and capturing business from both of their neighboring regions. Because of the geometric nature of the tax rates and bases, lowering t_3 enough to capture the low-income consumers in both region 1 and 2 would also be sufficient to capture the high-income consumers as well.

Examination of figure 9 shows why this is the case. As region 3 decreases their tax rate, the horizontal utility line for those people in region 3 increases. In order to capture all the business of the low-income consumers, they must raise this horizontal line (decrease t_3) to a point where the utility from shopping online is greater than or equal to their utility from shopping in a conventional market. In the case of the low-income consumer, this implies $\Theta_L - T - t_3 = \Theta_L - t_1$ (the borderline case).

Note that in solving the high income case, $\Theta_H - T - t_3 = \Theta_H - t_1$, the Θs drop out. Intuitively, the difference between the low-income conventional market utility (at $\lambda = 0$) and e-commerce utility is exactly the same as the difference between the high-income conventional market utility (again, at $\lambda = 0$). Therefore, lowering the tax to $\bar{t}_3$ will capture both high- and low-income consumers.

In this respect, the analysis is much more simple than before. There is only deviation to be described:

$$\bar{t}_3 = \frac{\Theta_P}{3\Theta_S} - \frac{2T}{3}$$

For tax rates lower than this amount, all consumers will shop online. Using the same procedure as before, one can examine under which conditions $\bar{t}_3 <$

t_3^{A*}, where t_3^{A*} refers to the candidate Nash equilibrium value of t_3 described originally.

$\bar{t}_3 < t_3^{A*}$ when $0 < \frac{\Theta_P}{3\Theta_S} + \frac{T}{3}$, which will always be the case because the right-hand side is always positive.

Revenue in regime D can be written as

$$\mathbf{R_3^D} = t_3 \int_0^1 f(\lambda_L) d\lambda_L + t_3 \int_0^1 f(\lambda_H) d\lambda_H \tag{32}$$

where $f(\lambda_L) = f(\lambda_H) = 1$.

This simplifies to $R_3^D = 2t_3$, and since $\frac{\partial R_3^D}{\partial t_3} = 2$, $\bar{t}_3$ will be the local maximum for regime D. Knowing the revenue function is continuous and strictly concave, t_3^{A*} is the global maximum for R_3.

It is easy to check to see if $R_3^A(t_3^{A*}) > R_3^D(\bar{t}_3)$: $R_3^A(t_3^{A*}) = \frac{2}{9}\frac{(2\Theta_P - \Theta_S T)^2}{\Theta_P \Theta_S}$ and $R_3^D(\bar{t}_3) = 2\bar{t}_3$ implies that $5\Theta_P^2 + 4\Theta_P\Theta_S T + 2\Theta_S^2 T^2 > 0$. This will hold for all Θ and T ¿ 0, which ours are. Therefore, it is guaranteed that the candidate Nash equilibrium will also be the global maximum.

Intuitively, within regime D, the marginal revenue (of increasing the tax rate) is 2. This is because everyone is shopping in their region. In that case, they would set the highest tax possible, subject to the constraint that all people would still shop there.

However, the continuity of the revenue function then tells us that the maximimum value of R_3^D cannot be a global maximum, because R_3^D is increasing at $\bar{\bar{t}}_3$.

It should be noted that, despite the lengthy list of conditions that arise, it is relatively easy to find values for T, Θ_L, and Θ_H for which this Nash equilibrium exists. $T = \frac{1}{2}$, $\Theta_L = 1$, $\Theta_H = 2$ are consistent with this Nash Equilibrium.

E Social Welfare: The Valuation of the Public Good

The marginal benefit of the public good is assumed to be constant in order to calculate a closed-form solution for the social welfare maximizing reaction function and equilibrium tax rates. This allows easy comparison with the revenue maximizing result. The revenue maximization problem approximates the results for the social welfare maximization problem, but only under certain conditions. In order for the qualitative results given by the revenue maximization problem to mirror those of the social welfare problem, the revenue maximization reaction functions must be defined and they must lie entirely above or below the reaction functions from the social welfare problem. Please refer to the Reaction function and Nash equilibrium value tables on pages 19 and 20. In order for these functions to be defined: $k \neq 2$ or the reaction function would be equal to zero for all values; $k \neq 1$ or they are not defined. Also, the Nash equilibrium results must obey $k \neq 2$ and $3k - 2 \neq 0$. $k > 2$ guarantees that all of these conditions hold and that the social welfare maximizing reaction lie everywhere below those from the revenue maximization problem.