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General Equilibrium Macroeconomic Models and Superior Information

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Abstract

We present and assess a procedure to evaluate dynamic, stochastic, general equilibrium macroeconomic models when agents in the economy use an information set superior to that used by researchers.

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1. Introduction

Real business cycle (RBC) studies typically evaluate the ability of general equilibrium artificial economies to account for stylized facts observed in the actual economy. The evaluation first consists of constructing an artificial economy by specifying the market features (e.g. preferences, technology, and market clearing conditions) and a law of motion for its forcing variables. Then, statistics generated from a calibrated version of this economy are confronted to those found in the actual economy. The artificial economy is refuted when generated statistics fail to match those from the data.

This evaluation is a joint test of the market features and the law of motion. Arguably, statistics computed from the artificial economy may not match those from the data simply because the law of motion is misspecified. In most RBC studies, the standard law of motion is a vector autoregression (VAR) that involves only forcing variables. This presumes that the relevant information set used by economic agents incorporates exclusively the history of forcing variables. It seems most likely, however, that the law of motion in the actual economy includes other exogenous variables. That is, the relevant information set is superior: it includes the history of both forcing and other exogenous variables. Unfortunately, omitting these other exogenous variables may lead to serious mismeasurements of statistics derived from the artificial economy.

Campbell and Deaton (1989) and Campbell and Shiller (1987) postulate that this superior information can be captured from an augmented law of motion that includes both forcing and endogenous variables. This relies on the intuition that agents reveal the relevant information set through their own behavior, which is summarized by endogenous variables. The augmented law is attractive because it only requires knowledge of forcing and endogenous variables, and not of those other exogenous variables.

In this note, we assess the use of the augmented law of motion in the evaluation of general equilibrium macroeconomic models. To do so, we first define an actual economy from a simple model that includes both forcing and other exogenous variables. We then construct two artificial economies, one with the standard law of motion and one with the augmented law. We prevent the use of the true law of motion by imposing that the other exogenous variables are observable only by agents. We show that the augmented law of motion encompasses the standard law and that it fully accounts for the superior information set used by agents. Finally, we numerically illustrate that the statistics (typically used in RBC studies) induced with the augmented law of motion are numerically and statistically closer to the data than the statistics generated with the standard law of motion.

2. The Actual Economy

For illustrative purposes, we define the actual economy as a simple version of the Lucas (1978) asset pricing model. The market features are:

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t,$$

$$c_t + p_t s_{t+1} = (p_t + y_t) s_t, \tag{2}$$

$$s_t = 1 \quad \text{and} \quad c_t = y_t, \tag{3}$$

where c_t denotes consumption, p_t is the asset price, s_t is the number of shares, y_t is the dividend, and $0 < \beta < 1$. Equation (1) displays the preferences of a risk-neutral representative agent, (2) his budget constraint, and (3) the market clearing conditions.

The true law of motion is given by the stationary bivariate VAR:

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} u_{yt} \\ u_{zt} \end{pmatrix}$$

or

$$W_t = \Pi W_{t-1} + U_t, \tag{4}$$

where y_t is the forcing variable, z_t is a composite of other exogenous variables, and U_t is a vector of zero-mean innovations with covariance matrix $\Omega = E\{U_t U_t'\}$.

The asset pricing implications of the market features are summarized in the Euler equation $p_t = \beta E_t\{p_{t+1}\} + \beta E_t\{y_{t+1}\}$. Solving this equation forward, we obtain the agent's forward-looking decision rule:

$$p_t = \sum_{j=1}^{\infty} \beta^j E_t\{y_{t+j}\}, \tag{5}$$

where $\lim_{T\to\infty} \beta^T E_t\{p_{t+T}\} = 0$. The agent constructs his expectations in (5) from the law of motion (4) such that:

$$p_t = e_1 \beta \Pi [I - \beta \Pi]^{-1} W_t = \Theta W_t = \theta_1 y_t + \theta_2 z_t,$$

where $e_1 = (1 \quad 0)$ and I is the identity matrix. Note that (6) explicitly involves the exogenous variable z_t ($\theta_2 \neq 0$), as long as the agent uses the exogenous variable to improve his forecasts of the future forcing variable ($\pi_{12} \neq 0$).

As the actual economy, equations (4) and (6) are the data generating process that yield the data $W_{dt} = (y_{dt} \quad z_{dt})'$ and p_{dt} for given values of β, Π, and Ω. In what follows, however, we assume that z_{dt} is observable only by the agent. In this context, it is useful to summarize the data generating process of the actual economy as:

$$X_t = \Gamma X_{t-1} + V_t, \tag{7}$$

where $X_t = \ y_t \quad p_t\,)' = \Upsilon W_t$, $\Gamma = \Upsilon \Pi \Upsilon^{-1}$, $V_t = \Upsilon U_t$, and $\Upsilon = (\,e_1' \quad \Theta'\,)'$

3. The Artificial Economy

We define the artificial economy by assuming that it correctly specifies the market features (1), (2), (3), and the calibrated value of β. This specification yields the same forward-looking decision rule (5). We then construct expectations in (5) from a law of motion estimated from the data. The unavailability of z_{dt}, however, prevents the estimation of the true law of motion (4). Instead, we estimate two distinct laws of motion. The first is standard and only uses data for the forcing variable y_{dt}. The second augments the standard law by also including data for the endogenous variable p_{dt}.

3.1. The Standard Law of Motion

As in most RBC studies, this law of motion involves only the forcing variable:

$$y_{dt} = \gamma y_{dt-1} + v_{yt}. \tag{8}$$

We construct the expectations in (5) from estimates of (8) to obtain:

$$p_{st} = \left(\frac{\beta\gamma}{1-\beta\gamma}\right) y_{dt} = \lambda y_{dt}, \tag{9}$$

where p_{st} is the artificial price series generated under the standard law of motion. Thus, the artificial data generating process under the standard law of motion is:

$$X_{st} = \Gamma_s X_{st-1} + V_{st}, \tag{10}$$

where $X_{st} = (\,y_{dt} \quad p_{st}\,)' = \Upsilon_s y_{dt}$, $\Gamma_s = (\,e_1'\gamma \quad e_1'\lambda\gamma\,)'$, $V_{st} = \Upsilon_s v_{yt}$, and $\Upsilon_s = (\,1 \quad \lambda\,)'$. The artificial price series p_{st} coincides with p_{dt} only if $\lambda = \theta_1$ and $\theta_2 = 0$. However, if $\pi_{12} \neq 0$ such that $\theta_2 \neq 0$, then the standard law (8) misspecifies the true law (4) and the artificial series p_{st} may severely deviate from the data.

3.2. The Augmented Law of Motion

This law of motion augments the standard law by including the endogenous variable in the following stationary VAR:

$$\begin{pmatrix} y_{dt} \\ p_{dt} \end{pmatrix} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix} \begin{pmatrix} y_{dt-1} \\ p_{dt-1} \end{pmatrix} + \begin{pmatrix} v_{yt} \\ v_{pt} \end{pmatrix}$$

or

$$X_{dt} = \Gamma X_{dt-1} + V_t,$$

where V_t is a vector of zero-mean innovations.

First, note that the augmented law of motion encompasses the standard law when $\gamma_{12} = 0$ and the covariance matrix of V_t is singular. Second, note that the augmented law is of the same form as the data generating process (7), and thus must fully accounts for the superior information set. This occurs because the asset price completely summarizes the agents' relevant information. More accurately, the feedback from lagged prices to current dividends ($\gamma_{12} \neq 0$) reflects the notion that the agent exploits the information contained in the other exogenous variable to improve his forecasts of future dividends ($\pi_{12} \neq 0$). Consequently, this feedback does not indicate that the asset price causes dividends in an economic sense, but rather that it is a surrogate for the other exogenous variable that Granger-causes dividends. Interestingly, the use of this surrogate avoids the difficult task of identifying the other exogenous variable.

We construct the expectations in (5) from estimates of (11) to obtain:

$$p_{at} = e_1 \beta\Gamma \left(I - \beta\Gamma\right)^{-1} X_{dt} = \Lambda X_{dt} = \lambda_1 y_{dt} + \lambda_2 p_{dt},$$

where p_{at} is the artificial price series generated under the augmented law of motion. The artificial data generating process under the augmented law of motion is:

$$X_{at} = \Gamma_a X_{at-1} + V_{at},$$

where $X_{at} = (\, y_{dt} \quad p_{at}\,)' = \Upsilon_a X_{dt}$, $\Gamma_a = \Upsilon_a \Gamma \Upsilon_a^{-1}$, $V_{at} = \Upsilon_a V_t$, and $\Upsilon_a = (\, e_1' \quad \Lambda'\,)'$. The price series p_{at} coincides with p_{dt} only if $\lambda_1 = 0$ and $\lambda_2 = 1$. Fortunately, these restrictions hold in the actual economy, because $\Gamma = \Upsilon\Pi\Upsilon^{-1}$ such that $\Lambda = e_1\beta\Gamma(I - \beta\Gamma)^{-1} = e_1\beta\Upsilon\Pi\Upsilon^{-1}(I - \beta\Upsilon\Pi\Upsilon^{-1})^{-1} = \Theta\Upsilon^{-1} = e_2$, where $e_1\Upsilon = e_1$ and $e_2 = (\, 0 \quad 1\,)$.

3.3. A Numerical Comparison

We numerically compare the use of the standard and augmented laws of motion in the evaluation of our actual economy. For this purpose, we generate y_{dt} and p_{dt} for 200 quarters from the data generating process (7). From these data, we calculate the true statistics (typically used in RBC studies): the standard deviation and the first-order autocorrelation of the asset price, and the contemporaneous correlation with dividends. Using the ordinary least squared estimates of the standard and augmented laws, we simulate the price series p_{st} and p_{at} for 200 quarters from the artificial generating processes (10) and (13). From these series, we compute the generated statistics. Finally, we calculate the p-value of a

$\chi^2(1)$-distributed test that a generated statistic is identical to the true statistic. Note that these p-values involve the variances of the generated statistics, which are computed from a Monte Carlo experiment with 10 000 replications.

Table 1 reports the statistics and p-values for different parametrizations of the actual economy. We impose that $\beta = 0.99$ and that $U_t \sim N(0, \Omega)$ where $\Omega = I$. Forcing $\pi_{21} = 0$, we study instances where the true law of motion exhibits oscillating ($\pi_{11} = \pi_{22} = -0.5$), smooth ($\pi_{11} = \pi_{22} = 0.5$), and nonpersistent ($\pi_{11} = \pi_{22} = 0.0$) dynamics, given that $\pi_{11} = \pi_{22}$ corresponds to the (identical) eigenvalues of (4). Panel A displays cases where the agent does not possess superior information ($\pi_{12} = 0.0$) and Panel B cases with superior information ($\pi_{12} = (-0.5, 0.5)$). Finally, the case where $\pi_{11} = \pi_{22} = \pi_{12} = 0$ is not reported, because the forcing variable then becomes an unpredictable innovation such that $p_{dt} = 0$.

Panel A documents that, in the absence of superior information, all statistics induced by the standard and augmented laws of motion are numerically and statistically very close to the true values (differences are attributable to the uncertainty associated with estimates of the laws). Statistics computed with the standard law are always insignificantly different from those generated by the data, because the standard law coincides with the true law. More importantly, the statistics and p-values computed with the augmented law are always identical to those obtained with the standard law. This accords with the notion that the augmented law encompasses the standard law. It also suggests that the power of the tests is unaffected by the use of the augmented law, even when the standard law corresponds to the true law.

Panel B reveals that, in the presence of superior information, most of the statistics derived with the augmented law of motion are numerically and statistically closer to the data than the statistics generated with the standard law. Statistics computed from the augmented law are always insignificantly different from those generated by the data (at all conventional levels), because the augmented law appropriately accounts for the superior information. In contrast, the statistics obtained with the standard law are often statistically different from the true statistics, because the standard law does not capture the superior information and, thus, misspecifies the true law. Finally, note that the p-values associated with the augmented law are generally larger when $\pi_{11} = \pi_{22} = (-0.5, 0.5)$ than when $\pi_{11} = \pi_{22} = 0.0$, which suggests that it is easier to capture the superior information when the true law of motion exhibits persistent dynamics.

4. Conclusion

The standard procedure to evaluate general equilibrium macroeconomic models is a joint test of the specification of the market features and of its law of motion for forcing variables. Arguably, statistics computed from macroeconomic models may not match those from the data simply because the law of motion is misspecified.

In this note, we present and assess an evaluation procedure that accounts for possible misspecifications of the law of motion. This procedure involves augmenting the law by including endogenous variables. Numerical examples reveal that statistics generated using the augmented law of motion greatly outperform those generated by the standard law of motion.

This procedure can be applied to the study of a wide variety of general equilibrium macroeconomic models. For example, the approximation method described in King, Plosser, and Rebelo (1988, 1987) generates forward-looking decision rules that can be solved using augmented laws of motion.

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Table 1. A Numerical Comparison

Panel A: No Superior Information

		$\pi_{12} = 0.0$		
$\pi_{11} = \pi_{22}$	Statistic	True	Standard	Augmented
	σ_p	0.38	0.37 (0.94)	0.37 (0.94)
	$\rho(p_t, p_{t-1})$	-0.49	-0.49 (1.00)	-0.49 (1.00)
	$\rho(p_t, y_t)$	-1.00	-1.00 (1.00)	-1.00 (1.00)
	σ_p	1.03	1.06 (0.92)	1.06 (0.92)
	$\rho(p_t, p_{t-1})$	0.51	0.51 (1.00)	0.51 (1.00)
	$\rho(p_t, y_t)$	1.00	1.00 (1.00)	1.00 (1.00)

Panel B: Superior Information

		$\pi_{12} =$ -0.5			$\pi_{12} = 0.5$		
$\pi_{11} = \pi_{22}$	Statistic	True	Standard	Augmented	True	Standard	Augmented
	σ_p	0.59	0.53 (0.37)	0.59 (0.99)	0.57	0.49 (0.20)	0.54 (0.70)
	$\rho(p_t, p_{t-1})$	-0.75	-0.64 (0.05)	-0.75 (1.00)	-0.69	-0.56 (0.02)	-0.69 (1.00)
	$\rho(p_t, y_t)$	-0.90	-1.00 (0.00)	-0.90 (0.91)	-0.90	-1.00 (0.00)	-0.90 (1.00)
	σ_p	0.49	0.06 (0.00)	0.51 (0.80)	0.53	0.02 (0.00)	0.57 (0.71)
	$\rho(p_t, p_{t-1})$	0.04	-0.06 (0.16)	-0.05 (0.36)	-0.03	-0.01 (0.86)	0.02 (0.61)
	$\rho(p_t, y_t)$	0.00	-1.00 (0.32)	-0.26 (0.13)	-0.06	-1.00 (0.35)	0.03 (0.59)
	σ_p	2.77	2.26 (0.41)	2.53 (0.74)	2.89	1.76 (0.07)	2.58 (0.67)
	$\rho(p_t, p_{t-1})$	0.59	0.58 (0.94)	0.62 (0.52)	0.64	0.56 (0.15)	0.63 (0.95)
	$\rho(p_t, y_t)$	0.73	1.00 (0.00)	0.82 (0.22)	0.64	1.00 (0.00)	0.63 (0.89)

Note: Entries are the following statistics: the standard deviation and the first-order autocorrelation of the asset price (σ_p and $\rho(p_t, p_{t-1})$), and the contemporaneous correlation with dividends ($\rho(p_t, y_t)$). True refers to statistics computed from the actual economy with the true law of motion. Standard and Augmented refer to statistics generated from the artificial economy with the standard or the augmented law of motion. Numbers in parenthesis are the p-values associated with the test that a statistic generated from the artificial economy is identical to the true statistic.