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Competition and Growth in the Global Economy: Exports vs. FDI

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Competition and Growth in the Global Economy: Exports vs. FDI

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Abstract

This paper develops a two-country model of endogenous growth with step-by-step innovation and oligopolistic competition where firms serve their foreign market via exports or horizontal FDI. The process of international competition equalizes long-run growth across countries, which depends on the innovation rates of individual firms and the distribution of industries over international technological differences. A quantitative analysis of the model based on some long-run salient features of high-income countries shows that the effects of changes in trade barriers on economic growth vary with the size of barriers to FDI. Bilateral trade liberalization from high to moderate barriers yields an increase in growth from 1.79% to 2.33% when FDI barriers are high, but leaves growth unaffected when FDI barriers are low. Subsequent liberalization towards free trade decreases growth for both high and low FDI barriers because of an *excessive-competition effect.* Unilateral movements to higher or lower trade protection when trade and FDI costs are low decrease growth in both countries through an additional *relative-market-size effect.* The results highlight the importance of considering the size of barriers to both trade and FDI when analyzing the effects of trade or investment liberalization on economic growth.

Keywords: Economic growth, competition, innovation, international trade, foreign direct investment

JEL codes: F43, O31, L11, L22

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1 Introduction

How does openness to trade and multinational production among high-income countries affect competition and economic growth in those countries? This is a very important issue, particularly in the context of the current wave of protectionism, that can affect the standard of living of future generations in developed countries. To address this question, in this paper I develop and quantitatively solve a model of endogenous growth to examine the long-run effects of reducing or increasing barriers to trade and foreign direct investment (henceforth FDI) on economic growth, focusing on the effects that are mediated by changes in the competitive environment.

Most of the trade and FDI flows in the world take place among high-income countries (Markusen 2002, United Nations 2017). Moreover, despite some evidence of complementarity between trade and FDI from intra-firm trade (Lipsey and Weiss 1984, Clausing 2000), "horizontal" FDI seems to be the more prevalent form of foreign direct investment among those countries (Brainard 1997, Markusen 2002, Helpman et al. 2004, Ramondo et al. 2013). Horizontal FDI refers to investments in foreign production plants that allow a firm to serve the customers in the foreign market locally, without having to transport the product from the home country of the firm, and avoiding all the variable costs associated with exports (transportation costs, tariffs, etc.).[1]

One of the major roles of trade and FDI for countries that open their borders to foreign firms is fostering productivity growth via technological spillovers. This channel has been explored both empirically (Keller and Yeaple 2009, Branstetter 2006, Griffith et al. 2006) and theoretically (Wu 2015). In this paper I study, from a theoretical point of view, the role of trade and FDI as drivers of economic growth via another channel: the competition channel.

The role of trade in generating pro-competitive effects has been studied in depth (see, for example, Tybout 2003's survey). There is also empirical evidence of foreign competition via horizontal FDI and deregulation of entry spurring incentives for innovation and productivity growth by making competition for domestic incumbent firms tougher (Nicoletti and Scarpetta 2003, Aghion and Griffith 2005, Aghion et al. 2009, Griffith et al. 2010). But while there are several theoretical analyses of the mechanisms through which the higher competition brought about by trade can increase economic growth (see, for example, Impullitti and Licandro 2018), there is a scarcity in the theoretical literature on how globalization affects economic growth through competition when both trade and FDI are available for firms as alternative modes of accessing foreign markets.

In order to start bridging this gap in the literature, I develop and quantitatively solve a two-country model of endogenous growth with step-by-step innovation and oligopolistic competition, in which firms can sell to foreign customers either by exporting or by doing horizontal FDI. Each country is home to a representative household that supplies labor inelastically and consumes a final good to maximize lifetime utility. The final good, which can also be used for research or intermediate good production, is produced in each country by

[1] The other main form of foreign direct investment, vertical FDI, involves a fragmentation of the supply chain, locating different stages of the production process in different countries. This form of FDI is more common between high-income countries on the one hand, and emerging or low-income countries on the other.

a perfectly-competitive sector by means of a production function that combines domestic labor and a large number of domestic and foreign intermediate inputs. Both countries produce the same range of intermediate inputs, so the model abstracts from any gains from variety. Within each intermediate input industry there are two firms, one from each country, that differ in terms of productivity and that compete in prices *à la* Bertrand for both the domestic and the foreign markets. Serving the domestic market only involves production costs determined by technology. But firms face a trade-off when deciding how to serve the foreign market. They can do so via exports, bearing the variable costs associated with trade (transportation costs, tariffs, etc.), or they can do horizontal FDI, which avoids those variable costs but is subject to fixed costs related to producing and selling in the foreign country (such as the costs of maintaining production facilities or a distribution network abroad). It is the size of these barriers to trade and FDI that determines which alternative is chosen by firms to serve their foreign markets, and how competitive both the domestic and foreign markets are.

While firms can in general differ in terms of their production technologies, they can invest resources in research and development (henceforth R&D) to gradually improve that technology over time. This generates a steady-state equilibrium with a stationary distribution of intermediate good industries over international technological differences. Some industries will be characterized by firms that have the same productivity, while other industries will have one of the firms (from either country) ahead of the other in terms of productivity.

In the steady-state equilibrium, economic growth in each country is a function of the size of innovations, the innovation intensities of domestic firms, and the international distribution of industries over technology gaps. Interestingly, regardless of the size of barriers to trade and FDI, and in the absence of technological spillovers, the rates of economic growth are equalized across countries. This equalization result is explained by the process of international competition in each industry. For any given country, and any given industry, the good will be produced by either the domestic or the foreign firm. If produced by the foreign firm, the dependence of domestic growth on foreign technology is evident. But even if the good is produced by the domestic firm, foreign technology also plays a role by determining how much competitive pressure the foreign firm exerts on the domestic firm, and hence the price charged and quantity produced by the latter.

The fact that the rates of economic growth are equal across countries regardless of the size of barriers to trade and FDI does not mean that both countries will have the same relative economic size. The latter depends crucially on how high or low trade and FDI costs are, although this dependence is mediated by the endogenous distribution of industries across technological differences. While this cannot be solved for analytically, I perform numerical analyses that illustrate this property of the equilibrium.

I calibrate the model using reasonable parameter values from the endogenous growth literature to match some salient features of high-income countries such as a long-run growth rate of about 2% per year, and I perform experiments where I analyze the effect of different combinations of trade and FDI barriers on economic growth. The results of bilateral experiments, where the two countries are symmetric in terms of their barriers to both trade and FDI, show that the effect of trade liberalization on economic growth varies with the size of barriers to FDI. For example, reducing trade barriers from very high to moderate levels increases long-run economic growth from 1.79% to 2.33% if barriers to FDI are high. However, if barriers to FDI are low in the first place, changes in trade barriers in the moderate-to-high range have no effect on economic growth. This is because trade and horizontal FDI are substitute ways for firms to sell to foreign customers. If FDI barriers are sufficiently low, then no matter how low trade barriers are, FDI will be a more profitable way of competing in foreign markets.

Further reductions in trade barriers from a moderate level to free trade actually decrease economic growth, regardless of whether FDI barriers are high or low. This is because reducing market-access barriers to a very low level gives rise to what I call an *excessive-competition effect*, whereby the increase in competition brought about by the lower barriers makes firms with similar technologies innovate so much, and firms in industries with high technology gaps so little, that the equilibrium distribution of industries over technology gaps features a large proportion of industries of the second type, which lowers aggregate innovation and economic growth. The excessive-competition effect is closely related to the inverted-U effect highlighted in closed-economy endogenous growth models (see, for example, Aghion et al. 2005). The difference is that in open economies with barriers to trade or FDI, innovation incentives are determined, not just by technological differences, but also by the size of those barriers. The results of my experiments show that retaining moderate barriers to competition by foreign firms generates a more uniform distribution of innovators and higher growth than if competition is entirely based on technology, which is the case in the absence of barriers to trade and FDI.

I also perform unilateral experiments where I fix the barriers to FDI at a low (the same) level in both countries, the barriers to trade at a low level in one of the countries, and I vary trade barriers in the other country. The results of these experiments show that unilateral changes in trade barriers in that context (either towards autarky or free trade) decrease economic growth. But in this case the excessive-competition effect interacts with a *relative-market-size effect*. A unilateral change in trade barriers introduces an asymmetry between the two countries that makes one of them more competitive than the other, making the size of the two markets different through an increase in total output in the more competitive one, altering the profitability of selling in foreign markets and the innovation incentives of firms from different countries. This results in an equilibrium distribution where most industries are dominated with large technology gaps by the firms of the country with the relatively small and uncompetitive market. But since firms with higher technological advantages tend to have low innovation incentives, aggregate innovation and economic growth decrease.

This paper represents a contribution to the economic literature in several ways. To the best of my knowledge the paper provides the first endogenous growth model of step-by-step innovation that analyzes the role of both trade and FDI in determining long-run economic growth via changes in the competitive environment. Most models of competition and growth with step-by-step innovation focus on closed economies (Aghion et al. 1997, Aghion et al. 2001, Acemoglu and Akcigit 2012). Some models of endogenous growth and competition do consider open economies in the analysis, but focus on the role of trade liberalization alone (Peretto 2003, Impullitti and Licandro 2018). The results of this paper show the importance of considering both trade and FDI when analyzing the effects of globalization on economic growth. This is a first step in that direction.

The paper also contributes to the international trade and economic growth literature by focusing on the role of trade and FDI in shaping the competitive environment firms compete in. Most of the trade and growth literature (pioneered by Grossman and Helpman 1991) puts the emphasis on the role of higher openness in generating technological spillovers from either importing goods or allowing foreign firms to establish their production plants in the local market. For example, Baldwin and Robert-Nicoud (2008) show that the effect of trade liberalization on economic growth is ambiguous and varies with the nature of these technological spillovers. Sampson (2016) focuses on the effects of lower trade barriers on growth via selection-based spillovers. Wu (2015) models the trade-off between exports and horizontal FDI and how it affects economic growth by changing the quality of technological spillovers generated by either importing goods or buying them from local foreign affiliates. My paper provides a complementary role of trade and horizontal FDI by focusing on the competition channel.

Beyond its contributions to the international trade and endogenous growth literature, the paper also provides a framework to analyze the effects on regional growth of physical, regulatory, or political barriers to the movement of goods or the establishment of production facilities in different regions within a country. For example, a country whose regions are characterized by different levels of political autonomy might find that firms prefer to locate their production processes in regions where they receive a more favorable treatment and "export" their goods to less favorable regions. Another example would involve regions within a country separated by some geographical barrier (i.e., mountains, rivers, etc.) that favor location of the production process next to the customers. While exports and FDI have an inherent international dimension, the trade-off between proximity to customers, on the one hand, and exploiting economies of scale by concentrating production in one location, on the other hand, can also be analyzed within the borders of a particular country. The model developed in this paper can certainly be used to perform that kind of analysis.

The rest of the paper is organized as follows. In Section 2, I describe the model. There, I layout the assumptions and derive some analytical results. After that, in Section 3, I explain the calibration and numerical procedure to quantitatively solve the model. I also present and discuss the baseline numerical results. Section 4 is devoted to some robustness analysis with alternative model specifications. In Section 5, I provide some concluding remarks. After the references, there are two appendices. Appendix A contains all the tables and figures of the paper. In Appendix B, I provide the proofs and derivations of some analytical results.

2 The Model

2.1 Economic Environment

Time is continuous and denoted by t. There are two countries, H and F. In each country there is a representative household that supplies labor inelastically and consumes a final (consumption) good to maximize lifetime utility subject to a dynamic budget constraint. The country's final good, which can be used for consumption, research, and intermediate good production, is produced by many firms operating in a perfectly-competitive sector (hereafter the final good sector) using labor and a continuum of intermediate inputs. The range of intermediate inputs is the same for both countries. Each intermediate input is produced by a small number of firms in an oligopolistic environment. For simplicity, I assume that within each intermediate good industry there are two firms, one from each country. That is, in each industry there is an international duopoly. These two firms are heterogeneous in terms of their marginal production costs and can sell to their domestic market and to their foreign market. Selling to the latter can be done in two alternative ways: exports or horizontal FDI. In addition to produce for each market, each intermediate firm can invest resources in R&D in order to gradually improve their production technology. The model builds on the step-by-step innovation models of Aghion et al. (2001) and Acemoglu and Akcigit (2012). In what follows I discuss each of the three sectors (households, final goods, and intermediate goods) in turn.

2.2 Households

The representative household in country $i \in \{H, F\}$ chooses the entire path of consumption in order to maximize the present discounted value of lifetime utility,

$$U_i = \int_0^\infty e^{-\rho t} \ln(C_i(t))dt, \tag{1}$$

subject to the dynamic budget constraint,

$$\dot{B}_i(t) = r_i(t)B_i(t) + w_i(t)L_i - C_i(t), \tag{2}$$

where $C_i(t)$ denotes consumption of the representative household of country i at time t, $w_i(t)$ is the wage rate paid in country-i's final good sector, L_i is the size of the labor force (population) in country i, $r_i(t)$ is the country's interest rate, and $B_i(t)$ denotes asset holdings by the household. The representative household in each country holds a balanced portfolio of all the domestic firms from that country (so that profit income is nonstochastic). There is no international trade in assets, and households in both countries discount future flows of utility at the common rate of time preference ρ. The size of the labor force is assumed to be constant over time, but can be different across countries. Households supply this labor inelastically to their domestic final good sector. Workers cannot work in the intermediate good sector or migrate to the other country.

Utility maximization yields the standard Euler equation:

$$g_i^C(t) \equiv \frac{\dot{C}_i(t)}{C_i(t)} = r_i(t) - \rho \tag{3}$$

where $g_i^C(t)$ denotes the growth rate of consumption in country i at time t. This growth rate will be constant in the steady-state equilibrium derived below.

2.3 Final Goods

The final good in country i is produced by many firms in a perfectly-competitive environment by combining labor and a continuum of intermediate inputs according to a constant-returns-to-scale production function,

$$Y_i(t) = (A_i L_i)^{1-\alpha} \exp\left(\alpha \int_0^1 \ln\left(X_i(j,t)\right) dj\right), \qquad 0 < \alpha < 1, \tag{4}$$

where $Y_i(t)$ denotes final output, A_i is an efficiency parameter (constant over time), and α is the elasticity of final output with respect to intermediate inputs (or the share of expenditure on intermediate inputs). $X_i(j,t)$ denotes the quantity of intermediate good j used in the production of the final good in country i. Intermediate goods can be sourced from domestic firms, or from foreign firms by either importing the product or buying it locally from a foreign affiliate plant. The final good, in turn, can be used for consumption, research, and intermediate good production by either domestic firms or foreign firms that serve country-i's final good sector. If the latter serve country i via exports, they import the required units of country-i's final good to produce the units of their intermediate good that will be exported to country i.[2]

The representative firm from country i chooses the amount of labor and each intermediate input to maximize profits. That is, it solves the following problem:

$$\underset{L_i, [X_i(j,t)]_{j\in[0,1]}}{max} \quad Y_i(t) - w_i(t)L_i - \int_0^1 p_i(j,t)X_i(j,t)dj \tag{5}$$

subject to (4), where $p_i(j,t)$ denotes the price of intermediate good j in country i at time t. Both $p_i(j,t)$ and $X_i(j,t)$ have a unique subscript that indicates the country (i) the intermediate good is used in (regardless of the nationality of the supplier of that intermediate good). The price of the final good is normalized to 1, taking it as the numéraire.[3] The first-order conditions of the problem yield the following inverse demand functions for labor and intermediate inputs:

[2] Similarly, country-i exporters will import the final good from the foreign country to undertake the production of their intermediate good for that market. This assumption simplifies the derivation of the aggregate resource constraint for each country (see section 2.5 below).

[3] The final good can be traded internationally at no cost so that its price is equalized across countries. The analysis of barriers to trade and FDI focuses on the intermediate good sector.

$$w_i(t) = (1-\alpha)\frac{Y_i(t)}{L_i} \tag{6}$$

$$p_i(j,t) = \frac{\alpha Y_i(t)}{X_i(j,t)} \qquad j \in [0,1] \tag{7}$$

Rearranging (7) yields the demand for intermediate input j coming from country-i's final good sector:

$$X_i(j,t) = \frac{\alpha Y_i(t)}{p_i(j,t)} \qquad j \in [0,1] \tag{8}$$

Intermediate good producers for variety j take (8) as given (for both markets, $i = H$ and $i = F$) when solving their own profit-maximization problems.

2.4 Intermediate Goods

2.4.1 Technology and Costs

Each intermediate good industry is characterized by an oligopolistic environment in which two infinitely-lived firms, one from each country, compete *à la* Bertrand for their domestic and foreign markets. Within an industry, each firm produces its own variety of intermediate product, but the two varieties are assumed to be perfect substitutes. Since there is only one firm per country producing intermediate good j, firms are indexed by their country of origin $i \in \{H, F\}$.

Production by each firm is done by means of a linear technology that requires $MC_i(j,t) = 1/q_i(j,t)$ units of the final good to produce 1 unit of its intermediate good variety, where $q_i(j,t)$ denotes the productivity level of firm i producing intermediate good j. This productivity level is indexed by t because it can be improved upon if the firm invests resources in R&D and undertakes a successful innovation. Firms are heterogeneous in terms of their productivity (i.e., of their marginal production cost, $MC_i(j,t)$), defined as

$$q_i(j,t) = \lambda^{n_i(j,t)}, \tag{9}$$

where $\lambda > 1$ and $n_i(j,t) \in \mathbb{Z}_+$ denotes the number of succesful innovations undertaken by firm i up to time t. In other words, $n_i(j,t)$ indexes firm i's technology level at time t. Industries are completely characterized by the technology gap between firms H and F:

$$n(j,t) \equiv n_H(j,t) - n_F(j,t), \quad n(j,t) \in \mathbb{Z} \tag{10}$$

Industries where $n(j,t) > 0$ have firm H as the technological leader, while $n(j,t) < 0$ represents the case in which F is the technological leader. I refer to industries where $n(j,t) = 0$ as *neck-and-neck* industries and to industries with $n(j,t) \neq 0$ as *leader-and-follower* industries. This is the standard terminology in models of competition and growth in closed economies such as Aghion et al. (2001) or Acemoglu and Akcigit (2012), where the technology gap is defined as a nonnegative integer because the identities of the leader and the follower are irrelevant. In this open-economy setting, however, the existence of barriers to trade and FDI (see below) that differ across countries makes it convenient to define the technology gap as in (10).[4]

Markets are segmented. When a firm competes for its domestic market, it faces no other cost than the one from producing its own variety ($MC_i(j,t)$). When competing for its foreign market, however, each firm has two alternative options to serve that market. On the one hand, a firm can produce in its own country and export its product to the foreign market. In that case, the firm faces not only its production cost but also a variable trade cost. Trade costs are assumed to be of the "iceberg" form, with $\tau_d MC_i(j,t) \geq MC_i(j,t)$ denoting the total variable unit cost of serving country $d \in \{H, F\}$ $(i \neq d)$ via exports.[5] This trade cost can be interpreted in a broad way encompassing both transportation costs or import tariffs.[6]

On the other hand, a firm competing for its foreign market can avoid bearing the variable trade cost τ_d by engaging in horizontal FDI, that is, by setting up a production plant in the foreign country to serve the customers (the final good sector) of that country locally. However, this alternative is subject to a fixed cost $K_d(t) = \kappa_d \alpha Y_d(t)$ that depends on the size of the destination market, $\alpha Y_d(t)$, and an index of barriers to FDI in that market, $\kappa_d \in [0,1]$. The latter captures various barriers that make maintaining production facilities or an efficient distribution network abroad costly. One could think of different barriers such as language or cultural differences that make it difficult to maintain relationships with foreign workers, or overcome regulatory barriers in the destination market.[7] Although establishing and maintaining distribution channels abroad also matters for exporters, their fixed costs of doing so are normalized to zero. Thus, the fixed cost of FDI in the model captures the cost above and beyond the fixed cost faced by exporters. The index can also reflect difficulties in transferring technology from the firm's headquarters to the affiliate production plant. Here it is assumed those costs don't vary with the distance to the destination market, although as shown by Keller and Yeaple (2013), gravity is an important factor in determining technology transfer costs. As with the trade costs, FDI barriers in the model are broadly defined.

Notice that while the index of barriers to FDI is constant over time, the total fixed costs of FDI do vary over time because of the market size component. One can interpret this as reflecting the difficulties in maintaining capacity, a distribution network, or transferring technology to a larger market, which are all the more difficult in the presence of the barriers captured by κ_d. While dealing with a larger market is a problem that domestic firms would presumably also have to face, here I simplify the analysis by assuming the latter don't have to incur

[4]The fact that the technology gap is defined as the difference between $n_H(j,t)$ and $n_F(j,t)$ and not the other way around is without loss of generality.

[5]That is, for 1 unit of the good to arrive at the destination market, $\tau_d \geq 1$ units have to be produced. The extra units $\tau_d - 1 \geq 0$ "melt" in transit.

[6]For welfare analysis the distinction between the two is important. While tariff revenue can be rebated to households, transportation costs cannot. Here the distinction is not relevant because the focus of the paper is on the effect of barriers to trade and FDI (broadly defined) on economic growth, not welfare analysis.

[7]Even if those regulations also affect domestic producers, they can overcome them more easily given their deeper knowledge of the domestic market.

any fixed costs when producing for the domestic final good sector (or that $\kappa_d = 0$ for domestic production). The lack of fixed costs for both domestic producers and exporters allows me to focus on the effects of the trade-off between the variable costs of trade and the fixed costs of FDI.

One more comment about the FDI costs is in order. Here I assume that, every period, firms only have to pay the FDI cost if they actually produce for the foreign market in that period. That is, the cost of FDI in the model is fixed but not sunk. This has important implications for the analysis. In particular, it makes the decision of serving the foreign market through exports or FDI a static one. This allows me to model in a simple way the effects of global competition on innovation, while still capturing the trade-off between exports and FDI.

2.4.2 Static Profit Analysis

In this subsection, I derive the static profits made by firms in industries characterized by different technology gaps. To simplify the notation, in what follows I drop the industry and time indicators, j and t. I start with industries where firm H is the technological leader. Then, I analyze the industries where firm F is the technological leader. For all leader-and-follower industries, I first analyze the domestic market of the technological leader, and then the foreign market. I conclude this subsection with the analysis of neck-and-neck industries.

H Leaders ($n > 0$)

When firm H is the technological leader, it has a lower marginal production cost than firm F. In the absence of trade and FDI costs, the logic of Bertrand competition would guarantee firm H capturing both markets. However, which firm has the lowest total variable unit cost and which firm will produce for a particular market depends on the size of trade and FDI costs. This is illustrated in Figures 1-3 in Appendix A. In the figures, the middle vertical line represents marginal production costs for both firms. Since firm H is the technological leader, MC_H is below MC_F. The other two lines represent total variable unit costs for each firm in each market.

Domestic firms' total variable unit costs coincide with marginal production costs. The same happens for foreign firms when they consider doing FDI (red circles). For exporters, however, total variable unit costs include production and trade costs (blue circles). Having the lowest marginal production cost guarantees firm H capturing its domestic market regardless of the barriers to access that market through trade or FDI. But the price it charges in that market *does* depend on the size of trade and FDI costs. If FDI costs are too high (how high will be established below), then firm H will charge $\tau_H MC_F$ (blue circle on the left part of Figures 1-3). It can't charge anything higher than that because then firm F would undercut H with exports. It won't charge anything lower because it can make higher profits by raising the price. However, if FDI costs are low enough, then the price charged by H will be somewhere between MC_F (red circle) and $\tau_H MC_F$ (blue circle). This is because a low fixed cost of FDI would allow firm F to undercut H via FDI if the latter tried to charge a price of $\tau_H MC_F$, making variable profits high enough to compensate the fixed cost.

If the costs of exporting to country F are high (Figure 1), firm H cannot win in the foreign market by exporting. It could win by doing FDI if the variable profit made in that market (an increasing function of the technology gap) is enough to compensate the fixed cost of FDI. If not, the high barriers to trade and FDI will allow firm F to capture its domestic market.

If trade costs to access market F are sufficiently low (Figure 2), then firm H has lower unit costs whether it exports or does FDI. So, even if FDI costs are very high, it will capture market F via exports. If FDI costs are low enough, it will capture it via FDI instead. If trade and FDI costs are such that firm H is indifferent between the two options, I assume it chooses to produce at home and export to the foreign country.

Finally, if trade costs are such that firm H's total unit cost of exports exactly matches MC_F (Figure 3), then firm H could at best tie with firm F by exporting. If FDI costs are too high, then I assume firm F captures its domestic market but makes zero profits (because of the threat of firm H undercutting with exports). If FDI costs are low enough, firm H captures the market by doing FDI.

With all this in mind, in what follows I establish the threshold values of trade and FDI costs in each country that delineate the different regimes described above. Tables 1 and 2 in Appendix A provide a summary of all the possible outcomes that arise from different combinations of trade and FDI costs in markets H and F, respectively. Appendix B provides derivations of all these expressions.

Market H Firm H is the winner in this market regardless of trade and FDI barriers, but the price it charges depends on the size of those barriers. If $\kappa_H \geq 1 - 1/\tau_H$ (FDI costs are high relative to trade costs), the threat of firm F exporting to market H determines the limit price charged by firm H to be $p_H = \tau_H MC_F$ (see Table 1, column 1). From (8), the quantity produced by firm H will be $X_H = \alpha Y_H / \tau_H MC_F$. Firm H's profits in this market will be

$$\Pi_{HH} = \left[1 - \frac{1}{\tau_H}\lambda^{-n}\right]\alpha Y_H, \tag{11}$$

where the first subscript in Π_{HH} indicates the nationality of the firm, and the second subscript indicates the market in which it makes the profits. Analogous notation is used for other combinations of nationality and market served. Firm F makes $\Pi_{FH} = 0$ since the market is captured by firm H. Notice that firm H's profits in (11) are increasing in market size, Y_H, the technology gap with firm F, n, and the size of the trade cost, τ_H. The higher the trade cost, the higher the price firm H can charge in its domestic market.

But there is a limit to how high trade costs can be. If they are too high, such that $\kappa_H < 1 - 1/\tau_H$ (FDI costs are low relative to trade costs), then the relevant threat of firm F undercutting in market H comes from FDI, not exports. In that case, the limit price charged by firm H would be determined by the following zero-profit condition of firm F undercutting with FDI:

$$[p_H - MC_F]\, X_H = K_H \Leftrightarrow p_H = \frac{MC_F}{1-\kappa_H} \tag{12}$$

So, if FDI costs are low relative to trade costs, firm H will charge the price given in (12). From (8), the quantity produced by firm H will be $X_H = \frac{\alpha Y_H}{MC_F}(1-\kappa_H)$. Firm H's profits in this case will be

$$\Pi_{HH} = \left[1 - (1-\kappa_H)\lambda^{-n}\right]\alpha Y_H, \tag{13}$$

and $\Pi_{FH} = 0$. It is important to note that the zero-profit condition in (12) just determines the price charged by firm H. Firm F does not actually do FDI (or exports) in market H when firm H is the technological leader. As before, (13) shows that the latter's profits (when FDI costs are low) are increasing in market size and the technology gap. But now profits depend on the size of FDI costs. The higher the protection received by FDI barriers, the higher the profit made in the domestic market, as long as those barriers are not too high, in which case profits would be given by (11).

Market F As discussed above, which firm wins the competition in market F depends on the size of trade and FDI barriers to access that market. Table 2 summarizes all the possibilities. For a given technology gap, if trade costs are sufficiently low such that $\tau_F < \lambda^n$, then firm H will have lower unit costs than firm F regardless of the alternative chosen to serve market F. Whether it engages in exports or FDI depends on which option is more profitable. Firm H will be indifferent between the two options (in which case I assume it exports) if and only if both exports and FDI yield the same profits (inclusive of fixed costs):

$$\underbrace{[p_F - \tau_F MC_H]\, X_F}_{\text{Exports}} = \underbrace{[p_F - MC_H]\, X_F - K_F}_{\text{FDI}} \Leftrightarrow \left[1 - \tau_F\lambda^{-n}\right]\alpha Y_F = \left[1 - \lambda^{-n} - \kappa_F\right]\alpha Y_F \tag{14}$$

This is equivalent to $\kappa_F = [\tau_F - 1]\lambda^{-n}$. So, if FDI costs are too high, so that $\kappa_F \geq [\tau_F - 1]\lambda^{-n}$, firm H will serve market F via exports. If $\kappa_F < [\tau_F - 1]\lambda^{-n}$, then it will serve the market with FDI. In either case, firm F makes zero profits ($\Pi_{FF} = 0$), the price charged by firm H is $p_F = MC_F$, and the quantity produced is $X_F = \alpha Y_F / MC_F$. Profits for firm H from each alternative are given in (14) and in Table 2 (columns 1 and 2). These are increasing in market size and the technology gap with firm F, and decreasing in the size of barriers to access the market.

If trade costs are high ($\tau_F \geq \lambda^n$), then firm H will never capture market F with exports because its unit costs would be at least as high as MC_F. If FDI costs are low ($\kappa_F < 1 - \lambda^{-n}$), firm H will win the competition with FDI. Price, output, and profits are in this case the same as in the previous case of high trade costs and low FDI costs (see Table 2, column 5). However, if FDI costs are such that $\kappa_F \geq 1 - \lambda^{-n}$, the high protection from both import and FDI competition will make firm F the winner in its domestic market. As in the case of Market H, the price charged by the domestic firm depends on the relative size of trade and FDI costs (see columns 3 and 4 in Table 2). For very high FDI costs ($\kappa_F \geq 1 - \frac{1}{\tau_F}$), the price charged by firm F is determined by the threat of firm H exporting. For intermediate FDI costs, $\kappa_F \in [1 - \lambda^{-n}, 1 - \frac{1}{\tau_F})$, the threat of FDI dictates what price firm F charges. The price, output and profit expressions are analogous to those in the Market H analysis, but reversing the roles of the subscripts H and F, and noticing that firm F makes higher profits when its technological disadvantage is smaller (when n gets closer to zero).

F Leaders ($n < 0$)

The analysis of industries in which firm F has the technological advantage is analogous to the cases where firm H is the leader, but with the roles of the H and F subscripts reversed. All the possible outcomes in markets F and H are given in Tables 3 and 4, respectively. As before, profits for the leader in its domestic market are increasing in the size of the market, the technology gap (a more negative n), and barriers to trade and FDI for firm H to access that market. Profits for the leader in the foreign market (when it wins the competition) depend negatively on the barriers to access that market.

Neck-and-Neck Industries ($n = 0$)

In industries where both firms have the same technology, each firm will always capture its domestic market and never the foreign market (see Figure 4). The best-case scenario in the foreign market is when barriers to trade and/or FDI are absent. In that case, I assume the domestic firm produces for that market but makes zero profits (because of the threat of the foreign firm undercutting with either trade or FDI). When there are barriers to trade *and* FDI, the price charged by the domestic firm depends on the relative sizes of those barriers. Price, output, and profit expressions for markets H and F are identical to those given in Tables 1 and 3, respectively.

2.4.3 Innovation and Dynamics

At any given point in time, firms have a certain level of productivity determined by their production technology, and I assume that technology is protected forever by a patent. But they can upgrade their technology by spending resources in R&D according to the innovation function

$$z_i(j,t) = \eta \left(\frac{R_i(j,t)}{Y_i(t)} \right) = \eta(e_i(j,t)), \tag{15}$$

where $z_i(j,t)$ denotes the Poisson flow rate of innovation of firm i in industry j at time t. As shown in (15), the innovation rate is a function of R&D expenditures $R_i(j,t)$, adjusted for an aggregate measure of the level of productivity in country i, proxied by the level of final output in that country, $Y_i(t)$. Since the latter grows over time, reaching a certain probability of innovation requires higher expenditures in R&D. This intends to capture the idea that innovation requires more resources the more advanced the existing technology in the country is. I denote the productivity-adjusted R&D expenditures by $e_i(j,t)$.

The function $\eta(\cdot)$ is twice-continuously differentiable and has the following properties: 1) $\eta(0) = 0$ (no R&D, no innovation); 2) $\eta'(e) > 0$ for $e \in [0, \bar{e})$ (higher productivity-adjusted R&D, up to a certain level, increases the probability of innovation); 3) $\eta'(e) = 0$ for $e \in [\bar{e}, \infty)$ (spending $\bar{e}$ or more doesn't increase the probability of innovation); and 4) $\eta''(e) < 0$ for $e \in [0, \bar{e})$ (diminishing returns to R&D).

From (15), the R&D expenditures required to reach a certain innovation rate are given by the function

$$R_i(j,t) = \eta^{-1}(z_i(j,t))Y_i(t) = \Gamma(z_i(j,t))Y_i(t), \tag{16}$$

where $\Gamma(\cdot) = \eta^{-1}(\cdot)$. From the properties of $\eta(\cdot)$, the function $\Gamma(\cdot)$ is characterized by the following properties: 1) $\Gamma(0) = 0$; 2) $\Gamma'(z) > 0$ for $z \in [0, \bar{z})$, where $\bar{z} \equiv \eta(\bar{e})$; 3) $\Gamma'(z) \to \infty$ as $z \to \bar{z}$; and 4) $\Gamma''(z) > 0$ for $z \in [0, \bar{z})$ (convex R&D cost).

A firm with technology level $n_i(j,t)$ at time t, upgrades to level $n_i(j,t)+1$ at time $t + \Delta t$ with probability $z_i(j,t)\Delta t + o(\Delta t)$ by spending $R_i(j,t)$ (adjusted by $Y_i(t)$), where $o(\Delta t)$ captures second-order terms such that $lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$. Thus, for an industry with technology gap $n(j,t)$ at time t, the gap at $t + \Delta t$ is given by

$$n(j, t+\Delta t) = \begin{cases} n(j,t)+1 & \text{with prob. } z_H(j,t)\Delta t + o(\Delta t) \\ n(j,t)-1 & \text{with prob. } z_F(j,t)\Delta t + o(\Delta t) \\ n(j,t) & \text{with prob. } 1 - z_H(j,t)\Delta t - z_F(j,t)\Delta t - o(\Delta t) \end{cases} \tag{17}$$

That is, within a small interval $(t, t+\Delta t)$, at most one innovation (by H or by F) can happen. If firm H innovates, the technology gap defined in (10) increases by one step (firm H has a higher technological advantage, or a lower disadvantage if $n < 0$). If firm F is the succesful innovator, n decreases (firm F catches up with firm H if $n > 0$, or increases its advantage if $n < 0$). The gap stays constant if both firms fail to make a succesful innovation.

At any point in time, there is a distribution of industries over technology gaps, with $\mu_n(t) \geq 0$ denoting the proportion of industries with technology gap n at time t, and

$$\sum_{n=-\infty}^{+\infty} \mu_n(t) = 1 \tag{18}$$

Industries flow in and out of a particular state (technology gap) n in the way described in (17).

2.5 Equilibrium

2.5.1 Definition

Since the technology gap is the only payoff-relevant variable that determines firms' choices, the focus is on Markov Perfect Equilibria (MPE). This rules out collusive behavior between the two firms, which makes the analysis simpler. From now on, I drop all the intermediate industry indices j and identify all the firm- and industry-level variables with the corresponding technology gap n. For example, at the firm level, $z_H^n(t)$ denotes the innovation rate of firm H at time t in an industry with technology gap n. At the industry level, $p_i^n(t)$ denotes the price charged by the winner of the competition for market i at time t in an industry with technology gap n.

DEFINITION (Allocation). Given the levels of trade and FDI costs (τ_H, τ_F, κ_H, κ_F), an *allocation* is defined as a list of pricing, production, and innovation decisions ($p_i^n(t)$, $X_i^n(t)$, $z_i^n(t)$) for $i \in \{H, F\}$, $n \in \mathbb{Z}$, and $t \geq 0$, a sequence of interest rates $r_i(t)$, and a distribution of industries across technology gaps $\mu_n(t)$ for all $t \geq 0$.

From the static profit analysis of section 2.4.2, the output of intermediate goods for the final sector of country i in industries with technology gap n can be written as[8]

$$X_i^n(t) = \alpha Y_i(t) q_i^n(t) \phi_i(n, \tau_i, \kappa_i), \tag{19}$$

where

$$\phi_H(n, \tau_H, \kappa_H) = \begin{cases} (1-\kappa_H)\lambda^{-n} & \text{for } n \geq 0,\ \tau_H \geq 1, \text{ and } \kappa_H \in [0, 1-\frac{1}{\tau_H}) \\ & \text{for } n < 0,\ \tau_H > \lambda^{-n}, \text{ and } \kappa_H \in \left(1-\lambda^n, 1-\frac{1}{\tau_H}\right) \\ \frac{1}{\tau_H}\lambda^{-n} & \text{for } n \geq 0,\ \tau_H \geq 1, \text{ and } \kappa_H \in [1-\frac{1}{\tau_H}, 1] \\ & \text{for } n < 0,\ \tau_H > \lambda^{-n}, \text{ and } \kappa_H \in [1-\frac{1}{\tau_H}, 1] \\ 1 & \text{for } n < 0,\ \tau_H > \lambda^{-n}, \text{ and } \kappa_H \in [0, 1-\lambda^n] \\ & \text{for } n < 0,\ \tau_H \leq \lambda^{-n}, \text{ and } \kappa_H \in [0, 1] \end{cases}, \tag{20}$$

and

$$\phi_F(n, \tau_F, \kappa_F) = \begin{cases} (1-\kappa_F)\lambda^{n} & \text{for } n \leq 0,\ \tau_F \geq 1, \text{ and } \kappa_F \in [0, 1-\frac{1}{\tau_F}) \\ & \text{for } n > 0,\ \tau_F > \lambda^{n}, \text{ and } \kappa_F \in \left(1-\lambda^{-n}, 1-\frac{1}{\tau_F}\right) \\ \frac{1}{\tau_F}\lambda^{n} & \text{for } n \leq 0,\ \tau_F \geq 1, \text{ and } \kappa_F \in [1-\frac{1}{\tau_F}, 1] \\ & \text{for } n > 0,\ \tau_F > \lambda^{n}, \text{ and } \kappa_F \in [1-\frac{1}{\tau_F}, 1] \\ 1 & \text{for } n > 0,\ \tau_F > \lambda^{n}, \text{ and } \kappa_F \in [0, 1-\lambda^{-n}] \\ & \text{for } n > 0,\ \tau_F \leq \lambda^{n}, \text{ and } \kappa_F \in [0, 1] \end{cases} \tag{21}$$

The functions $\phi_H(\cdot)$ and $\phi_F(\cdot)$ capture, in a general way, all the possible competition regimes for each market in an industry with a given technology gap n and different combinations of trade and FDI barriers (see Tables

[8] Technically, the actual productivity level of firm i in industries with technology gap n can vary across those kinds of industries. I sometimes slightly abuse the notation and denote that level of productivity by $q_i^n(t)$.

1-4 in the Appendix for details). For example, $\phi_H(n, \tau_H, \kappa_H) = (1 - \kappa_H)\lambda^{-n}$ for industries where market H is captured by the domestic firm because, regardless of which firm is the technological leader, trade and FDI costs are such that firm H has the lowest unit costs of serving that market, and the threat of firm F doing FDI determines the price charged by firm H. Similarly, $\phi_H(n, \tau_H, \kappa_H) = \frac{1}{\tau_H}\lambda^{-n}$ for industries where, again, trade and FDI costs give firm H the lowest unit costs to serve its domestic market, but the price it charges is determined by the threat of firm F exporting to country H. Finally, $\phi_H(n, \tau_H, \kappa_H) = 1$ for industries in which firm F is the technological leader and trade and FDI costs are not high enough to protect firm H from foreign competition. The function $\phi_F(\cdot)$ has a similar interpretation for competition in market F. Both functions take values between zero and one, with higher values indicating higher competition.

Before providing the formal definition of equilibrium for this model, I introduce some more notation for aggregate variables. Let $R_i(t)$, $M_i(t)$, $NX_i(t)$ denote, respectively, the aggregate level of R&D expenditures, the total cost of intermediate good production for the domestic market (by domestic or foreign firms), and net exports, in country i at time t. By definition, aggregate R&D expenditures are given by:

$$R_i(t) = \sum_{n=-\infty}^{+\infty} \mu_n(t) R_i^n(t) \tag{22}$$

The total cost of intermediate good production for market i by either domestic firms or foreign firms (from $d \neq i$) is

$$\begin{aligned} M_i(t) = \sum_{n=-\infty}^{+\infty} & \mu_n(t) [\chi_i^{DOM}(n, \tau_i, \kappa_i) MC_i(t) X_i^n(t) \\ & + \chi_i^{EX}(n, \tau_i, \kappa_i) \tau_i MC_d(t) X_i^n(t) + \chi_i^{FDI}(n, \tau_i, \kappa_i) \left(MC_d(t) X_i^n(t) + K_i \right)], \end{aligned} \tag{23}$$

where, letting $\xi(i) = -1$ if $i = H$ and $\xi(i) = 1$ if $i = F$,

$$\chi_i^{DOM}(n, \tau_i, \kappa_i) = \begin{cases} 0 & \text{if } \phi_i(n, \tau_i, \kappa_i) = 1 \\ 1 & \text{otherwise} \end{cases} \tag{24}$$

$$\chi_i^{EX}(n, \tau_i, \kappa_i) = \begin{cases} 1 & \text{if } \phi_i(n, \tau_i, \kappa_i) = 1 \text{ , } \tau_i < \lambda^{\xi(i)n} \text{ , and } \kappa_i \geq [\tau_i - 1]\lambda^{-\xi(i)n} \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

$$\chi_i^{FDI}(n, \tau_i, \kappa_i) = \begin{cases} 1 & \text{if } \phi_i(n, \tau_i, \kappa_i) = 1 \text{ and } \chi_i^{EX}(n, \tau_i, \kappa_i) = 0 \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

The functions given in (24)-(26) are indicators of whether market i is served by the domestic firm, the foreign firm via exports, or the foreign firm via FDI. As can be seen from (23), the cost for domestic producers is based on technology alone, while the cost for foreign producers also involves either variable trade costs or fixed costs, depending on whether they capture market i with exports or FDI.

Net exports are given by the negative of net repatriated profits from serving the foreign market,

$$\begin{aligned} NX_i(t) = \sum_{n=-\infty}^{+\infty} \mu_n(t)[(\chi_i^{EX}(n,\tau_i,\kappa_i)\Pi_{di}^{EX} - \chi_d^{EX}(n,\tau_d,\kappa_d)\Pi_{id}^{EX}) \\ + (\chi_i^{FDI}(n,\tau_i,\kappa_i)\Pi_{di}^{FDI} - \chi_d^{FDI}(n,\tau_d,\kappa_d)\Pi_{id}^{FDI})], \end{aligned} \tag{27}$$

where

$$\Pi_{id}^{EX} = \left[1 - \tau_d \lambda^{\xi(i)n}\right] \alpha Y_d \tag{28}$$

$$\Pi_{id}^{FDI} = \left[1 - \lambda^{\xi(i)n} - \kappa_d\right] \alpha Y_d \tag{29}$$

are the profits made by firm i in market d by exporting and doing FDI, respectively (see Tables 2 and 4 in the Appendix). Since the final good is the numéraire, profits made in foreign markets are repatriated (imported) in units of the final good. Similarly, profits made by foreign firms from exports or FDI are exported back to their country in units of the final good. When the firms from country i make higher profits in market d than the profits firms from country d make in market i, country i is a net importer of the final good and country d is a net exporter. In equilibrium, the world market for the final good clears so that $NX_H = -NX_F$.

Given all these definitions, I formally define an equilibrium as follows:

DEFINITION (Equilibrium). Given the levels of trade and FDI costs (τ_H, τ_F, κ_H, κ_F), an *equilibrium* is defined as an allocation, and aggregate variables ($Y_i(t)$, $C_i(t)$, $R_i(t)$, $M_i(t)$, $NX_i(t)$), for $i \in \{H, F\}$ such that 1) $p_i^n(t)$ and $X_i^n(t)$ satisfy (7) and (19)-(21); 2) innovation rates $z_i^n(t)$, $i \in \{H, F\}$, maximize the net present discounted value of lifetime profits given the innovation decisions of rivals $z_d^n(t)$, $d \in \{H, F\}$, $d \neq i$; 3) industry proportions $\mu_n(t)$ satisfy (18); 4) the Euler equation (3) is satisfied; 5) aggregate R&D satisfies (22); 6) the cost of intermediate good production $M_i(t)$ satisfies (23); 7) final output $Y_i(t)$ satisfies (4); and 8) all markets (assets, final good, intermediate goods) clear.

Since there is no international trade in assets, the market clearing condition for assets in country i is $\dot{B}_i(t) = R_i(t)$. This means that new savings by the representative household finance the R&D expenditures of the domestic firms. In Appendix B I show that this condition, together with the representative household's budget constraint (2), the cost of intermediate good production (23), and the definitions of profits in the final and intermediate good sectors, imply the aggregate resource constraint in each country is satisfied. That is,

$$Y_i(t) = C_i(t) + R_i(t) + M_i(t) + NX_i(t) \tag{30}$$

Substituting (19) into the final output production function (4) and rearranging yields equilibrium final output in country i:

$$Y_i(t) = A_i L_i \alpha^{\frac{\alpha}{1-\alpha}} \left[Q_i(t)\right]^{\frac{\alpha}{1-\alpha}} \left[\Phi_i(t, \tau_i, \kappa_i)\right]^{\frac{\alpha}{1-\alpha}}, \tag{31}$$

where $Q_i(t)$ and $\Phi_i(t, \tau_i, \kappa_i)$ are defined such that

$$\ln\left(Q_i(t)\right) \equiv \int_0^1 \ln\left(q_i(j,t)\right) dj, \tag{32}$$

and

$$\ln\left(\Phi_i(t, \tau_i, \kappa_i)\right) \equiv \sum_{n=-\infty}^{+\infty} \mu_n(t) \ln\left(\phi_i(n, \tau_i, \kappa_i)\right) \tag{33}$$

$Q_i(t)$ is an index of the technology of all the domestic intermediate good firms, while $\Phi_i(t, \tau_i, \kappa_i)$ is a weighted average of the competition regime indices of industries at different technology gaps. In general, the latter varies over time because it depends on the proportions $\mu_n(t)$, which vary with the innovation rates of all firms as described in the previous section.

2.5.2 Steady-State Equilibrium

For the rest of the paper I focus on steady-state equilibria where aggregate variables grow at constant rates and the international distribution of industries over technology gaps is stationary, so that $\mu_n(t) = \mu_n$ is constant over time. The latter implies that the aggregate indices of competition, $\Phi_i(t, \tau_i, \kappa_i)$, are constant over time. Thus,

$$g_i^Y \equiv \frac{\dot{Y}_i}{Y_i} = \frac{\alpha}{1-\alpha} \frac{\dot{Q}_i}{Q_i}, \tag{34}$$

where g_i^Y denotes the growth rate of final output in country i. Since growth of final output depends only on the evolution of the technology index of domestic firms, $Q_i(t)$, in general the two countries could grow at different rates. The following proposition rules out that possibility.

PROPOSITION 1 (Equality of Growth Rates). Given a stationary distribution of industries across technology gaps, so that $\mu_n(t) = \mu_n$ is constant over time for all $n \in \mathbb{Z}$, the growth rate of final output (34) is equal in both countries.

Proof. See Appendix B.

The intuition behind this result comes from the process of international competition in each industry. As discussed above, the output of each intermediate good sold in a particular country depends on the price chosen by the winner of the competition in that market. If the winner is the domestic firm, the price charged will depend on the marginal production cost and hence the technology of the foreign firm. If the foreign firm is the winner, the price that firm charges will be equal to the marginal production cost of the domestic firm. But the latter can be interpreted as a function of the marginal cost of the foreign firm and the technology gap between them. Thus, the final output produced with all the intermediate inputs depends on the level of foreign technology and the distribution of industries across technology gaps. But since the latter is assumed to be stationary, final output growth depends only on the evolution of foreign technology. Since, from (34), final output growth in the foreign country depends on the evolution of that same technology index, growth in both countries must be equal. It is remarkable that this happens even in the absence of any technological spillovers in the intermediate goods sector. The next proposition establishes what the growth rate of final output equals to.

PROPOSITION 2 (Steady-State Growth Rate). Given a stationary distribution of industries across technology gaps, so that $\mu_n(t) = \mu_n$ is constant over time for all $n \in \mathbb{Z}$, the steady-state growth rate of final output in both countries is given by

$$g^Y = \frac{\alpha}{1-\alpha} \ln(\lambda) \sum_{n=-\infty}^{+\infty} \mu_n z_i^n, \qquad i \in \{H, F\} \tag{35}$$

Proof. See Appendix B.

Proposition 2 says that final output growth, which is proportional to the growth rate of the domestic (or foreign) technology index, depends on the size of innovations (λ), the innovation rates of domestic (or foreign) firms in industries with different technology gaps (z_i^n), and the distribution of industries (μ_n). Before deriving the conditions that determine the innovation rates and the steady-state distribution, I establish one more result.

PROPOSITION 3 (Equality of Interest Rates). Given a stationary distribution of industries across technology gaps, so that $\mu_n(t) = \mu_n$ is constant over time for all $n \in \mathbb{Z}$, the steady-state growth rates of consumption in each country are equal to g^Y and given by (35). Moreover, interests rates in both countries are equal and given by

$$r^* = g^Y + \rho \tag{36}$$

Proof. The result comes from the fact that aggregate R&D, the total cost of intermediate good production, and net exports, are all proportional to final output. Therefore, they all grow at the rate g^Y. From the aggregate resource constraint (30), consumption in both countries must also grow at the rate g^Y. This, combined with the Euler equation (3), implies (36). ■

Notice that interest rates are equalized across countries even in the absence of international trade in assets. This is entirely driven by the process of international competition that equalizes the growth rates of final output.

The innovation rates of firms in industries with a given technology gap n are chosen to maximize the net present discounted value of lifetime profits (net of R&D costs). The value of the firms competing in an industry with technology gap n can be written as (see Appendix B)

$$r^* V_H^n(t) - \dot{V}_H^n(t) = \max_{z_H^n(t)\geq 0} \left\{ \begin{array}{l} \Pi_{HH}^n(t) + \Pi_{HF}^n(t) - \Gamma(z_H^n(t))Y_H(t) \\ +z_H^n(t)\left[V_H^{n+1}(t) - V_H^n(t)\right] \\ +z_F^n(t)\left[V_H^{n-1}(t) - V_H^n(t)\right] \end{array} \right\} \tag{37}$$

$$r^* V_F^n(t) - \dot{V}_F^n(t) = \max_{z_F^n(t)\geq 0} \left\{ \begin{array}{l} \Pi_{FF}^n(t) + \Pi_{FH}^n(t) - \Gamma(z_F^n(t))Y_F(t) \\ +z_H^n(t)\left[V_F^{n+1}(t) - V_F^n(t)\right] \\ +z_F^n(t)\left[V_F^{n-1}(t) - V_F^n(t)\right] \end{array} \right\}, \tag{38}$$

where $V_i^n(t)$ denotes the net present discounted value of firm i in an industry with technology gap n, at time t, and $\dot{V}_i^n(t)$ denotes the change in that value. Equations (37) and (38) can be interpreted as (rearranged) arbitrage equations that equate the return of investing in an intermediate good firm (dividends plus capital gains) to the return of investing the amount $V_i^n(t)$ in a risk-free asset that yields r^*. The first line on the right-hand side of both equations gives the static profits in the domestic and foreign markets net of R&D costs. The second and third lines show the expected change in firm values depending on which firm makes a succesful innovation. If firm H innovates, which happens al the flow rate $z_H^n(t)$, the industry transitions to a state with a higher technology gap $n+1$. If firm F innovates, which happens at the flow rate $z_F^n(t)$, the gap decreases to $n-1$.

Since profits and R&D costs are proportional to final output, $V_i^n(t)$ grows over time at the steady-state rate g^Y. To find the steady-state innovation rates, I define stationary values as

$$v_i^n \equiv \frac{V_i^n(t)}{Y_i(t)} \qquad i \in \{H, F\} \tag{39}$$

These are constant over time since $V_i^n(t)$ grows at the same rate as $Y_i(t)$. With the newly defined values, and using (36), the value equations (37) and (38) can be written as (see Appendix B):

$$\rho v_H^n = \max_{z_H^n \geq 0} \left\{ \begin{array}{l} \pi_{HH}^n + \pi_{HF}^n\left(\frac{1}{\omega}\right) - \Gamma(z_H^n) \\ +z_H^n\left[v_H^{n+1} - v_H^n\right] \\ +z_F^n\left[v_H^{n-1} - v_H^n\right] \end{array} \right\} \tag{40}$$

$$\rho v_F^n = \max_{z_F^n \geq 0} \left\{ \begin{array}{c} \pi_{FF}^n + \pi_{FH}^n \omega - \Gamma(z_F^n) \\ + z_H^n \left[v_F^{n+1} - v_F^n \right] \\ + z_F^n \left[v_F^{n-1} - v_F^n \right] \end{array} \right\}, \tag{41}$$

where $\pi_{id}^n \equiv \Pi_{id}^n(t)/Y_d(t)$ denote profits per unit of final output in market d, and $\omega \equiv Y_H(t)/Y_F(t)$. Since final output grows at the same rate in both countries, ω is constant over time in steady-state. The first-order conditions of the right-hand side problems in (40) and (41) imply the following innovation rates:

$$z_H^n = \max \left\{ 0, \Gamma'^{-1}(v_H^{n+1} - v_H^n) \right\} \tag{42}$$

$$z_F^n = \max \left\{ 0, \Gamma'^{-1}(v_F^{n-1} - v_F^n) \right\} \tag{43}$$

Since $\Gamma''(z) > 0$ (convexity of the R&D cost function), the innovation rates are increasing in the incremental value of a successful innovation (higher n for firm H, lower for firm F). The *max* operator takes care of the fact that for very high technology leads, the incremental value of additional innovations gets smaller and smaller and eventually is equal to zero. In that case, leaders choose zero innovation rates.

The innovation intensities determine the entry and exit flows of industries in and out of a given state n. Since the steady-state distribution of industries over technology gaps is stationary, entry and exit flows must offset each other so that $\mu_n(t) = \mu_n$ for all t. This is shown by the following equation:

$$(z_H^n + z_F^n)\mu_n = z_H^{n-1}\mu_{n-1} + z_F^{n+1}\mu_{n+1} \qquad \forall n \in \mathbb{Z} \tag{44}$$

There is one such equation for each state (technology gap). An industry with technology gap n will flow out of that state at the flow rate $z_H^n + z_F^n$ since either firm H or firm F can make a successful innovation. Since there is a proportion of μ_n industries with technology gap n, exit flows are given by the left-hand side of (44). Entry flows into state n are given in the right-hand side of (44) and can happen from either state $n-1$ (if firm H innovates), or from state $n+1$ (if firm F innovates). Given the innovation rates from equations (42) and (43), equations (44) and (18) pin down the proportions of industries at different technology gaps. The innovation rates and the proportions provide all that is needed to calculate the steady-state growth rate g^Y. However, the static profits in the value equations (40) and (41) depend on the ratio of final outputs across countries ω, which is endogenous. In the next proposition I summarize the characterization of the steady state equilibrium.[9]

PROPOSITION 4 (Steady-state equilibrium). Given the levels of trade and FDI costs, (τ_H, τ_F, κ_H, κ_F), a *steady-state equilibrium* is an equilibrium in which 1) firms' innovation rates are given by (42)-(43) and firm

[9]Given the mathematical similarity of the model with the one in Acemoglu and Akcigit (2012), the existence of the steady-state equilibrium is guaranteed for reasonable parameter values. The reader is referred to the Appendix of Acemoglu and Akcigit (2012) for details on how such a proof can be constructed.

values satisfy (40)-(41); 2) the industry proportions μ_n are uniquely determined by equations (18) and (44) for all $n \in \mathbb{Z}$; 3) final output (and all aggregate variables) in both countries grow at the constant rate given by (35); 4) the interest rate is the same across countries and given by (36); and (5) $\omega \equiv Y_H(t)/Y_F(t)$ is constant over time.

The next section provides a numerical solution for the steady-state equilibrium of the model.

3 Quantitative Analysis

3.1 From the Model to Numerical Analysis

To solve the model of the previous section numerically, I make some adjustments that I describe in what follows. First, as in Acemoglu and Akcigit (2012), the numerical solution relies on a uniformization procedure (see Appendix B) that turns the value functions in (40)-(41) into contraction mappings. That allows me to use a value function iteration procedure to solve for the innovation rates and values of intermediate good firms. Second, I specify an exact form for the innovation and R&D cost functions, $\eta(\cdot)$ and $\Gamma(\cdot)$:

$$z = \eta(e) = \begin{cases} \theta e^{\sigma} & \text{for } e \in [0, \bar{e}) \\ \theta \bar{e}^{\sigma} & \text{for } e \in [\bar{e}, \infty) \end{cases}, \qquad \theta > 0,\ 0 < \sigma < 1 \tag{45}$$

$$e = \eta^{-1}(z) = \Gamma(z) = \left(\frac{z}{\theta}\right)^{1/\sigma} \quad \text{for } z \leq \bar{z} \equiv \eta(\bar{e}) \tag{46}$$

In (45), θ and σ are parameters that capture the efficiency and concavity (diminishing returns to productivity-adjusted R&D) of the innovation process. (46) just inverts (45) to give productivity-adjusted R&D expenditures as a function of the desired innovation intensity.

Finally, I put a limit on the technological advantage that a firm can have over its foreign competitor. That is, I fix the maximum and minimum technology gaps (as defined in the model) to be finite. In particular, and to save in computing time, I assume $n \in \{-3, -2, -1, 0, 1, 2, 3\}$. This allows me to show the main mechanisms of the model while making the computation relatively fast. In section 4 I provide alternative results with a higher maximum technology gap.

The imposition of a maximum technology gap can be interpreted in economic terms as a patent policy that relaxes the protection of old technologies, so that followers can freely adopt them, once the leaders reach a certain technological advantage. This implies that the innovation rates of leaders at the maximum technology gap will be zero, since they cannot increase their value and innovation is costly.

3.2 Calibration of Parameter Values

To perform the numerical analysis, I choose some of the parameter values based on the literature on endogenous growth. I also calibrate the efficiency parameter of the innovation function (45), θ, to target a long-run growth rate of 2%, which is the standard figure for long-run growth in high-income countries. The baseline parameter values are given in Table 5 in Appendix A.

The parameter α from the final goods production function (4) is chosen to be equal to 0.3 so that the share of labor in total final output revenue is 70%. As in Acemoglu and Akcigit (2012), I choose a value for the concavity parameter of the innovation function in the range of estimates by Kortum (1993), $\sigma = 0.3$. I also choose the households' discount rate to be $\rho = 0.05$, which together with the target growth rate of 2%, yields and interest rate of 7%. The parameter that controls the size of innovations, λ, is set equal to 1.1, similar to its value in Aghion et al. (2001). In section 4 I also explore alternative specifications in terms of other values of λ. Finally, I normalize the maximum innovation rate $\bar{z}$ to 1.

In the baseline calibration, I assume that both countries have the same trade barriers and I fix those to have a low value of 1.11. FDI barriers are also the same for both countries but I fix them at the maximum value of 1. I focus on this baseline specification to make the model comparable with other trade and growth models that focus on the competition channel but don't include FDI. As a result of that, the numerical solution in the baseline case yields a somewhat too high proportion of exporters in both countries (39%). More in line with other endogenous growth models, the solution yields a percentage of industries with technology gaps such that $| n | \leq 1$ of 21%.

I also assume that countries have the same population size and efficiency parameters in the final good production function (4) and are symmetric in every other respect. One could think of this baseline specification as an application of the model to two countries (regions) of similar sizes such as the United States and the European Union prior to its extension to Eastern Europe (EU-15). I also solve the model for alternative specifications where country H has twice as high a population as country F. In that case, the model would be a better fit to analyze the interplay between the U.S. (country H) and some other smaller country such as Japan (country F).

3.3 Baseline Experiments

3.3.1 Bilateral Experiments

I first present the results of experiments where, for the baseline calibration, I solve the model for different combinations of trade and FDI barriers assuming the two countries are symmetric in terms of those barriers and all other parameters of the model. I refer to these as bilateral experiments. For these, and the unilateral experiments that follow in the next subsection, the results should be understood as a comparison of steady states for different sizes of trade and FDI barriers. These don't include transitional dynamics of changes in those barriers. While those transitions are interesting and important, they go beyond the scope of this paper, whose focus is on long-run economic growth.

Figure 5 in Appendix A shows graphically the effect of bilateral changes in trade and FDI barriers on the common rate of economic growth. In the graph, $\tau_H = \tau_F = \tau \in [1,3]$ while $\kappa_H = \kappa_F = \kappa \in [0,1]$. Fixing FDI barriers to its highest level of 1, the graph shows that moving from autarky ($\tau = 3$) to free trade ($\tau = 1$) increases the rate of economic growth from 1.79% to 1.94%, which is a sizable increase if sustained for long periods of time. However, the growth rate reaches a maximum of 2.33% (for high FDI barriers) when $\tau = 1.33$, not in free trade. Similarly, fixing trade barriers at its maximum and allowing FDI costs to vary, we can see that moving from $\kappa = 1$ to $\kappa = 0$ also increases the growth rate from 1.79% to 1.94%. Again, the maximum growth rate is not reached for the lowest level of barriers, but for $\kappa = 0.25$, when the growth rate is again 2.33%. This suggests that when only one mode of accessing foreign markets (either exports or FDI) is available, reducing barriers to that available mode from an autarky position to free trade/FDI increases economic growth, but retaining some (relatively small) barriers yields the maximum growth rate.

What if there are no barriers to FDI in the first place? That is, suppose that FDI barriers are fixed at $\kappa = 0$, and trade barriers are reduced from autarky to free trade. In that case, the rate of economic growth remains constant at 1.94%. This is because, no matter how low the trade barriers get, the absence of barriers to FDI makes the latter the most profitable option for competing in the foreign market for technological leaders, and the most credible threat of undercutting for technological followers. The same result is achieved if there are no barriers to trade in the first place, and FDI costs are reduced from 1 to 0. Trade and horizontal FDI are alternative modes of accessing foreign markets and firms only consider the most profitable one. But it is important to consider barriers to both alternatives when analyzing the effects of reducing or increasing those barriers on economic growth.

What explains this non-monotonicity in the rate of economic growth when barriers to trade and/or FDI are reduced? As shown in equation (35), reproduced here for convenience, the steady-state growth rate depends on the distribution of industries across technology gaps and the innovation rates of firms in those industries.

$$g^Y = \frac{\alpha}{1-\alpha} \ln(\lambda) \sum_{n=-\infty}^{+\infty} \mu_n z_i^n, \qquad i \in \{H, F\}$$

Figure 6 shows the distribution of industries across technology gaps for $\kappa = 1$ and three different levels of trade barriers: autarky ($\tau = 3$), free trade ($\tau = 1$), and the growth-maximizing level of trade costs ($\tau = 1.33$). As can be seen in the figure, the distribution has a U-shaped form for all three levels of trade costs, with the highest mass for leader-and-follower industries with high technology gaps. This U-shaped pattern can be explained by looking at Figures 7 and 8. They show the innovation rates of firms H and F, respectively, at different technology gaps and for the same three levels of trade costs as in Figure 6. For a fixed level of trade costs, both firms have the highest innovation rates when they face neck-and-neck competition and low innovation rates when they have big technological advantages or disadvantages. This is the inverted-U pattern found in Schumpeterian models of endogenous growth such as Aghion et al. (2001) or Acemoglu and Akcigit (2012). Firms are discouraged to innovate when they are far away from their rivals, and have large incentives to innovate when competition is fierce (escape-competition effect).

The reason the distribution of industries has a U-shaped form is that innovation rates are very high in neck-and-neck industries and leader-and-follower industries with low technology gaps, and very low in leader-and-follower industries with high technology gaps. This makes low-gap (closer to zero) states very unstable, with industries transitioning out of them very frequently to absorbent high-gap states (closer to $n = 3$ or $n = -3$).

However, this process is much more intense in free trade than when $\tau = 3$ or $\tau = 1.33$. This is because in free trade, competition is entirely based on technological differences. When there are barriers to trade, firms with high technological advantages have even higher advantages in the domestic market (which lowers the incentives to innovate) and lower advantages (or even disadvantages) in the foreign market (which raises incentives to innovate when barriers are moderate and lowers them when barriers are too high). The balance of the effects on both the domestic and foreign markets turns out to yield lower innovation rates at technology gaps $n \geq -1$ for H firms and $n \leq 1$ for F firms. For two-step followers, raising barriers to 1.33 actually increases their innovation rates because the effect of moderate domestic protection dominates (they were not selling in the foreign market in free trade). But moving to autarky makes the domestic market less competitive for them than in free trade, so innovation rates decrease. For 3-step followers, raising barriers to either 1.33 or 3 gives higher incentives to innovate than in free trade, although barely so for autarky barriers.

How does all this explain the inverted-U pattern of economic growth in Figure 5? When trade barriers are very high (autarky), the distribution of industries and the innovation rates of firms across technology gaps are relatively uniform, but the latter are too low to generate much growth. When barriers are lowered to a moderate level, the distribution of industries is almost as uniform as in autarky, but the innovation rates are much higher, which yields higher growth. When barriers are lowered to free trade (or to very low levels), the innovation rates of firms with low or no technological advantage increase substantially, but the distribution of industries becomes concentrated on low-innovation industries, which yields lower growth than when barriers are moderate (although still higher than in autarky). I call this the *excessive-competition effect*, whereby there are a few industries that are very competitive and highly innovative, and a majority of industries where the leaders have too much market power and there are little incentives for innovation for leaders and followers. Ironically, the excessive competition in neck-and-neck industries yields an equilibrium distribution of industries where leaders have too much market power.

As noted above, the effects of reductions in trade barriers on economic growth depend on the level of FDI barriers. To be sure, I perform the same exercise as in Figures 6-8 but with the growth-maximizing level of FDI costs, $\kappa = 0.25$. The results are shown in Figures 9-11. The only difference with Figures 6-8 is that now reducing trade barriers from $\tau = 3$ to $\tau = 1.33$ is irrelevant. When $\kappa = 0.25$, reducing trade costs from autarky to a moderate level still leaves FDI as the more profitable way of serving the foreign market, so neither the incentives to innovate nor the distribution of industries is affected by that reduction in trade barriers. That explains the constant growth rate for that range of trade costs in Figure 5. When trade barriers are reduced even further, then trade becomes more profitable than FDI. But since FDI barriers were moderate in the first place, the same excessive-competition effect as in Figures 6-8 takes place and growth decreases when moving to free trade. This time autarky trade barriers yield higher growth than free trade, not because autarky is necessarily growth enhancing, but because FDI costs were low enough that trade was never an option considered by firms to access foreign markets.

3.3.2 Unilateral Experiments

In addition to the bilateral experiments of the previous section, I perform experiments in which I fix FDI barriers for both countries at the same level, trade barriers for country F at the baseline value ($\tau_F = 1.11$), and I allow trade costs to vary for country H, with $\tau_H \in [1, 3]$. I refer to these as unilateral experiments.

In the baseline unilateral experiment I assume that both countries are symmetric in terms of all parameters of the model, including FDI barriers, which I fix at the level $\kappa_H = \kappa_F = 0.1$. This value is close to the data for the OECD's FDI restrictiveness index for the United States and the high-income countries of the European Union (Kalinova et al. 2010), so this experiment can be understood as how different levels of trade barriers to access the U.S. market affect economic growth in both the U.S. and the European Union (given their relatively low values of FDI barriers).

Consistently with the results of the bilateral experiments discussed above, Figure 12 shows that departing from the moderate level of trade barriers in the baseline reduces economic growth, although only slightly so when going towards autarky. This mild effect of higher trade barriers is explained by the low level of FDI barriers in place. At the baseline, both trade and FDI barriers are identical in both countries, and trade is the preferred option by firms to access foreign markets, but barely so because of the low FDI barriers. The increase in trade barriers in country H does not need to be very high for FDI to become the preferred way for F firms to access market H. Once this happens, additional increases in trade barriers have no effect on competition and economic growth. Moving towards free trade is a different story. A reduction in trade barriers from the baseline has a substantial negative effect on economic growth.

The mechanism behind this result is more nuanced than in the bilateral experiments. In the latter, countries were symmetric in every respect. But as shown in equations (40)-(41), firms' profits in foreign markets, and their incentives to innovate, depend on the relative final output of both countries ω. This ratio, in turn, depends on the competition indices Φ_H and Φ_F given in (33). In a symmetric equilibrium, those indices are identical and $\omega = 1$, so it has no effect on profits. The same is true in the baseline specification of this unilateral experiment. But the moment barriers to trade change in country H, the competition indices become different and ω starts having an effect on profits and innovation incentives. The change in the competition indices and ω can be seen in Figure 13. Moving towards autarky decreases competition in market H and increases it in market F. As a result of that, ω decreases, and profits in foreign markets become higher for H firms and smaller for F firms. This is a *relative-market-size effect*. Figures 14-15 show that this gives higher incentives to innovate to H firms and lower incentives to F firms, which explains why in Figure 16 the distribution of industries shifts towards higher concentration on industries with H leaders. This is consistent with the decrease in competition in market H and the increase in market F.

The movement towards free trade has the opposite effects on the competition indices and ω. But these effects have a higher magnitude now, so innovation incentives are much higher now for F firms and much lower for H firms. As a result of that, the distribution of industries shifts towards a very high proportion of industries dominated by F firms. Since in free trade competition is based on technological differences, and leaders with high advantages have low incentives to innovate, economic growth is much lower than in the baseline or autarky, due to the excessive-competition effect in market H.

3.4 Discussion

Before exploring alternative specifications of the model, it is worth at this point discussing some of the main implications of the results of the baseline experiments.

The main message from those experiments is that changes in barriers to either trade or FDI have different effects on economic growth depending on the size of barriers to the alternative modes of serving foreign markets. For example, if FDI barriers are high, trade liberalization from autarky will increase growth until trade barriers reach a moderate level, but will decrease growth in the subsequent movement towards free trade. However, if FDI barriers are low in the first place, so that FDI is the most profitable option of selling in foreign markets, then the initial rounds of trade liberalization will have no effect on economic growth, and the subsequent movement towards free trade, when trade becomes more attractive than FDI, will have the same growth-decreasing effect as when barriers to FDI are high in the first place. So, when analyzing the effects of globalization through trade or investment liberalization on economic growth, it is important to consider the size of barriers to *both* trade and FDI, because firms will always choose the most profitable alternative to access foreign markets.

As far as the mechanisms behind the results are concerned, the results of the bilateral experiments in the previous section suggest that it is important to consider, not just how innovative firms are within certain industries, but how different innovation incentives can result in an equilibrium distribution of industries with low competition and innovation, which can be detrimental to growth. From a policy perspective, if the goal of a country is to maximize its long-run growth rate, having a few competitive industries with very innovative firms and many industries whose leaders have very high market power and very low innovation rates, is probably not the way to go. The results of the bilateral experiments seem to suggest this is the case when barriers to trade and/or FDI are very low, due to the excessive-competition effect. Moderate barriers to foreign competition seem to yield a more balanced distribution of industries in terms of competition and innovation incentives, which results in higher growth. Finally, maintaining high barriers to trade and FDI retains the property of having a more uniform distribution of industries, but with much lower innovation incentives and growth.

The results of the unilateral experiments with otherwise symmetric countries also suggest that moderate barriers are growth maximizing. But in this case, the relative-market-size effect has to be taken into account. A unilateral change in trade barriers, deviating from a scenario with symmetric barriers for both trade and FDI, makes one market more competitive than the other, leading to higher output in the more competitive market, and generating asymmetric incentives for innovation for firms in different countries. This biases the distribution of industries in a way that most industries are dominated by the firms of the country whose market is less competitive. For example, when country H unilaterally raises trade barriers, market F becomes more competitive than market H, increasing final output in the former, and lowering final output in the latter. The higher relative demand in market F, together with the high barriers to access market H, gives higher innovation incentives to H firms, which end up having large technological advantages in a high share of intermediate good industries. This lowers economic growth in both countries.

The excessive-competition and relative-market-size effects reinforce each other to lower economic growth when the unilateral move is towards free trade. In that case, the country that lowers its trade barriers ends up having a more competitive market relative to the other country. This lowers the incentives to innovate of the firms from the liberalizing country and increases the incentives of the firms in the other country, which end up being the technological leaders for many industries. This is the relative-market-size effect at play. But because free trade makes technological differences the only determinant of innovation incentives, the lower trade barriers introduce so much competition that the share of industries dominated by the country that did not change its trade barriers is much higher than the share captured by the other country when it moves towards autarky. Since firms with high technological leads innovate very little, growth decreases more when trade barriers are very low.

4 Alternative Specifications

In this section I perform additional experiments with alternative specifications of the model. First, I perform experiments in which country H has a higher population size than country F. Second, I make the parameter that controls the size of innovations (λ) higher or lower to see how that affects the results of the previous section. Finally, I allow for a larger range of technology gaps, so that firms with 3-step leads choose positive innovation rates. To simplify the analysis, for all the specifications I focus on bilateral experiments only. I conclude this section with a discussion of the robustness of the model to these alternative specifications.

4.1 Different Population Size Across Countries

The baseline specification assumed symmetry in terms of all parameters of the model, including population size. But the results of the baseline unilateral experiments suggest that introducing asymmetries in the model will give rise to the relative-market-size effect. I test that idea in this section.

Here I show the results of bilateral experiments in which both countries always vary their trade/FDI barriers in the same direction and magnitude. But I depart from the baseline specification by making country H twice as large as country F in terms of population. That is, $L_H = 2$ and $L_F = 1$. Figure 17 shows the effects of different combinations of trade/FDI barriers on economic growth. As in Figure 5, for high levels of FDI barriers, lowering trade barriers from autarky to a moderate level raises growth, but reducing barriers even further decreases it.

However, the growth rate plummets in those last rounds of trade liberalization compared to what happens in the baseline experiments. The reason is that, in addition to the excessive competition effect found in the baseline bilateral experiments, now the relative-market-size effect also plays a role. Since market H is now twice as large as market F, the ratio of final outputs, ω, is higher than when countries are symmetric (see Figure 18). That ratio remains constant for most of the reduction in trade barriers in both countries, since competition increases in a balanced way, but the larger relative size of market H gives much higher incentives to F firms to innovate once access to that market is liberalized. This starts giving F firms the technological lead in more and more industries, which explains why the competition index increases in H and decreases in F, causing a spike in the ω ratio. The F firms continue gaining the leadership in more industries as trade is liberalized further, but at a lower rate, so competition increases again in country F and ω goes back to the ratio determined by the relative population size. In free trade, since the majority of industries are dominated by F firms with 3-step leads (see Figure 19), and innovation is low for those firms, growth decreases substantially.

4.2 Size of Innovations

The steady-state rate of economic growth depends not only on the distribution of industries across technology gaps and the innovation rates of firms in different industries, but on the size of the innovations they make, λ. In the baseline specification, I assumed $\lambda = 1.1$. In this section I explore how the results change when $\lambda = 1.05$ or $\lambda = 1.15$. It is expected that the magnitudes of the growth rates at different combinations of trade and FDI costs will change for different innovation sizes. The question is if the non-monotonic pattern present in Figure 5 changes as well.

Figures 20 and 21 show the results for $\lambda = 1.05$ and $\lambda = 1.15$, respectively. Indeed, the same qualitative pattern of economic growth rates for different sizes of barriers to trade and FDI remains for different innovation sizes. As expected, growth rates are much higher when the size of innovations is large, and much lower when the size of innovations is low. But now growth is maximized at different levels of trade and FDI costs than in the baseline ($\tau = 1.33$ and $\kappa = 0.25$). For $\lambda = 1.05$, the highest growth rate is achieved when $\tau = 1.17$ and

$\kappa = 0.14$, whereas for $\lambda = 1.15$, growth is maximized when $\tau = 1.5$ and $\kappa = 0.33$. From the static profit analysis of Section 2.4.2 (see Tables 1-4), whether trade and FDI costs are considered to be high or low depends on λ. For example, a given level of trade costs that was considered low in the baseline specification can be too high when $\lambda = 1.05$. Thus, for foreign competition to be excessive, trade or FDI barriers have to become much lower than in the baseline. The opposite happens when $\lambda = 1.15$.

This can be seen in Figures 22-23. They show industry distributions for $\kappa = 1$ and the three values of trade costs used in Figure 6. That is, for $\tau = 1$, $\tau = 3$, and the growth-maximizing value of trade costs in the baseline specification, $\tau = 1.33$. Figure 22 illustrates the case of $\lambda = 1.05$. As suggested above, the moderate level of trade costs that maximizes growth in the baseline is now too high, so the distribution of industries does not change much when moving from autarky to $\tau = 1.33$. For competition to become excessive in this case, trade costs have to go below $\tau = 1.17$. In free trade, the distribution becomes U-shaped for the reasons discussed earlier.

Figure 23 shows what happens when $\lambda = 1.15$. Now the moderate level of trade costs in the baseline is actually too low (lower than the growth-maximizing level of 1.5), so the excessive-competition effect is at play, and the distribution of industries already has a clear U shape at $\tau = 1.33$. Moving to free trade reinforces that pattern. But even though the distribution of industries becomes less uniform at higher levels of trade costs than for $\lambda = 1.05$, the latter case is characterized by higher shares of industries with high technology gaps (for H or F firms), which explains the lower growth rates in that case.

4.3 Higher Range of Technology Gaps

In the baseline specification I assumed that the maximum technology lead any firm could achieve is 3. While this assumption was made mostly to save in computing time, in this section I study how the baseline results change when that maximum technology gap is higher. For simplicity, I assume the new maximum gap is 4. As in the baseline, firms with the highest leads will not innovate since the incremental value of an innovation is zero and innovating is costly.

Figure 24 shows how trade and FDI barriers affect growth in this specification. As in the baseline, growth increases when moving from autarky to moderate trade barriers (for high barriers to FDI), although growth rates in both cases are higher (2.02% in autarky, 2.55% for trade costs of 1.44). Unlike in the baseline, however, free trade yields slightly lower growth (1.9%) than in autarky. This occurs because now firms with 3-step leads, having the possibility of increasing their advantage, have higher values. That encourages firms at lower technology gaps to innovate to achieve such a great position (see Figures 26-27). But this makes the excessive-competition effect to be more intense and to happen at higher levels of trade costs (growth is maximized at $\tau = 1.44$ instead of at $\tau = 1.33$), which yields industry distributions where the U shape is more pronounced than in the baseline (see Figure 25), and hence lower economic growth for low levels of trade barriers.

4.4 Discussion

The baseline experiments showed the importance of considering both trade and FDI barriers when analyzing the effects of trade or FDI liberalization on economic growth. The main mechanisms at work were the excessive-competition and relative-market-size effects. The numerical analysis under alternative specifications in terms of asymmetries in population size, higher or lower size of innovations, and a higher range of technology gaps, suggests that those mechanisms are relatively robust to these alternative specifications of the model. However, while the qualitative patterns seem to hold well, the quantitative effects of different trade and FDI barriers on economic growth are somewhat sensitive to these specifications.

While giving precise quantitative answers is important to understand the effects of globalization, the goal of this paper is not to provide such precise measures of the effects of trade and FDI barriers on economic growth, but to call attention to the fact that models that only allow for trade as the only form of accessing foreign markets can provide a wrong assessment of the effects of trade liberalization on economic growth that take place via the competition channel. As shown in the previous sections, those effects can be very different depending on the size of barriers to FDI.

The analysis also points out the importance of measuring the size of trade and FDI barriers in each country, to make a better assessment of policies directed at changing those barriers with the goal of making economic growth as high as possible. This is especially relevant nowadays that there seems to be a resurgence of protectionism in high-income countries.

5 Concluding Remarks

In this paper I have developed a model of endogenous growth to assess the role of trade and horizontal FDI among high-income countries in shaping long-run growth, with a focus on the effects of trade and FDI barriers on the degree of competition in each market. The model highlights the importance of considering both modes of accessing foreign markets when analyzing trade or investment liberalization policies.

When barriers to FDI are very high, bilateral movements towards free trade yield higher growth than autarky, but moderate barriers to trade are growth maximizing. The decrease in growth from a situation with moderate barriers to free trade is explained by an excessive-competition effect whereby very high innovation rates in neck-and-neck industries and low innovation rates in industries of the leader-and-follower type yield an equilibrium distribution of industries with a high mass of low-innovation, leader-and-follower industries. Moderate barriers generate a more uniform distribution of innovators that maximizes aggregate innovation and economic growth.

Since trade and horizontal FDI are substitute modes of accessing the foreign market, low barriers to FDI make trade liberalization from autarky to a moderate level of trade barriers irrelevant in terms of its effects on economic growth. Subsequent reductions in trade barriers that make exporting the preferred way of selling to foreign customers reduce growth because of the excessive-competition effect.

Unilateral changes in trade barriers in similar countries, or bilateral changes in countries of different size, give rise to a relative-market-size effect that makes countries asymmetric in terms of the degree of competition and shifts the distribution of industries so that the firms from one country become technological leaders with high advantages over their rivals for most products. Since these kinds of firms have lower incentives for innovation, economic growth tends to decrease as a result of the unilateral change in barriers.

While these qualitative patterns are consistent across different specifications, the model's quantitative results are somewthat sensitive to different parameter values. This suggests it is important to have good, structural measures or estimates of the elements captured by those parameters, such as the size of innovations by different firms, to give an accurate assessment of the quantitative effects of globalization on economic growth. This model just provides a first step in the analysis of trade and FDI barriers and their effects on economic growth via changes in competition.

The model also makes a few assumptions that make the analysis more tractable. For example, the fixed costs of FDI are assumed to be non-sunk, which makes the profit analysis static. Relaxing that assumption would give richer interactions between the exports-versus-FDI trade-off and innovation decisions. This is an interesting avenue for further research.

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Appendix A: Figures and Tables

Model Figures and Tables

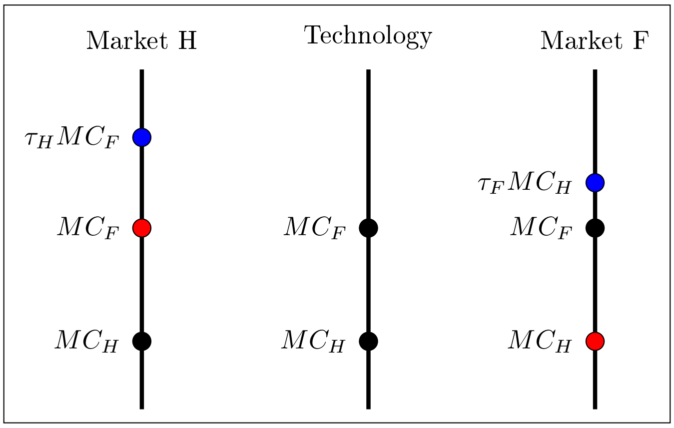

Figure 1: H Leader, High τ_F

Notes: The vertical line in the center represents marginal costs for firms H and F in a given industry. The other two lines represent total variable unit costs in each market. The black circles represent marginal production costs. The blue circles represent total unit costs from exporting. The red circles represent total variable unit costs from doing FDI. In this case, the trade costs to access market F are so high that the total unit costs of exporting for firm H are higher than the marginal production costs of firm F.

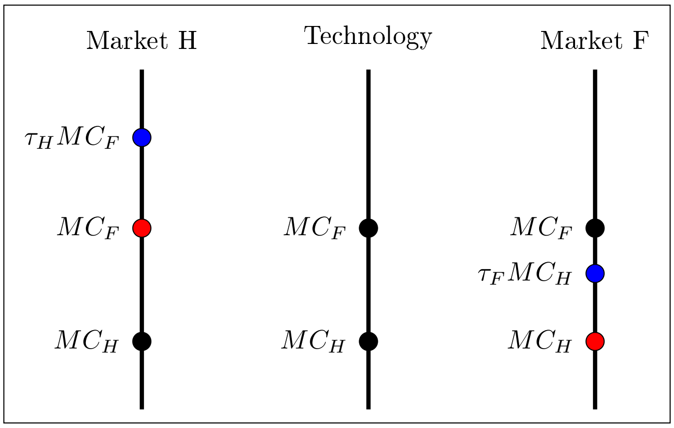

Figure 2: H Leader, Low τ_F

Notes: The vertical line in the center represents marginal costs for firms H and F in a given industry. The other two lines represent total variable unit costs in each market. The black circles represent marginal production costs. The blue circles represent total unit costs from exporting. The red circles represent total variable unit costs from doing FDI. In this case, the trade costs to access market F are low enough that the total unit costs of exporting for firm H are lower than the marginal production costs of firm F.

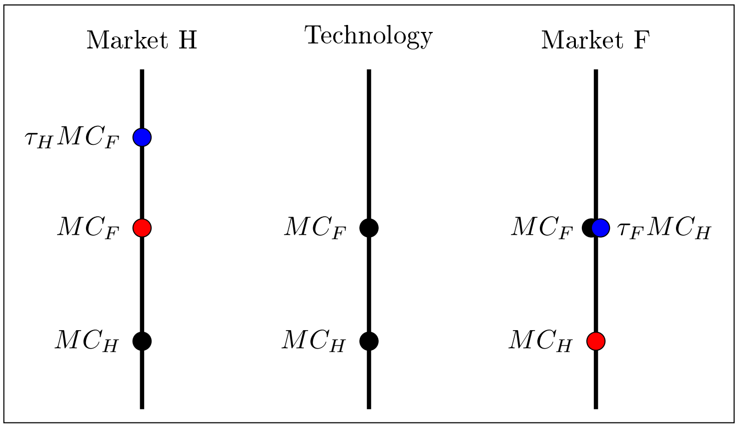

Figure 3: H Leader, Intermediate τ_F

Notes: The vertical line in the center represents marginal costs for firms H and F in a given industry. The other two lines represent total variable unit costs in each market. The black circles represent marginal production costs. The blue circles represent total unit costs from exporting. The red circles represent total variable unit costs from doing FDI. In this case, the trade costs to access market F are at a level such that the total unit costs of exporting for firm H are identical to the marginal production costs of firm F.

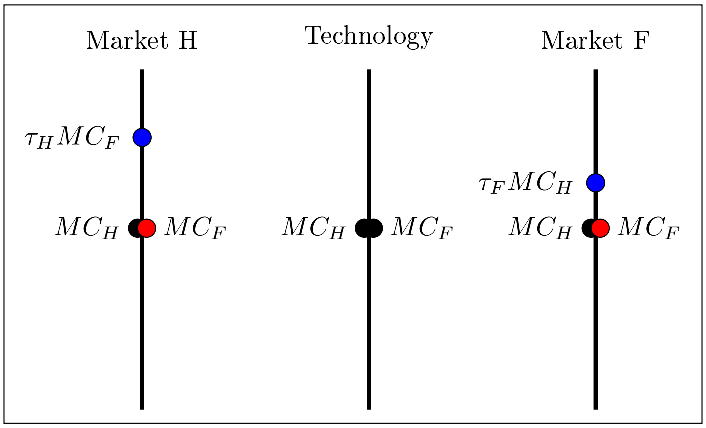

Figure 4: Neck-and-Neck Industry. High τ_H, Low τ_F

Notes: The vertical line in the center represents marginal costs for firms H and F in a given industry. The other two lines represent total variable unit costs in each market. The black circles represent marginal production costs. The blue circles represent total unit costs from exporting. The red circles represent total variable unit costs from doing FDI. In this case, firms have identical marginal production costs, and the trade costs to access market F are lower than those to access market H.

Table 1: H Leader, Market H

	High FDI Cost $\kappa_H \geq 1 - 1/\tau_H$	Low FDI Cost $\kappa_H < 1 - 1/\tau_H$
Winner	Firm H	Firm H
p_H	$\tau_H MC_F$	$\frac{MC_F}{1-\kappa_H}$
X_H	$\frac{\alpha Y_H}{\tau_H MC_F}$	$\frac{\alpha Y_H}{MC_F}(1-\kappa_H)$
Π_{HH}	$\left[1 - \frac{1}{\tau_H}\lambda^{-n}\right] \alpha Y_H$	$[1 - (1-\kappa_H)\lambda^{-n}]\, \alpha Y_H$
Π_{FH}	0	0

Notes: The table represents the different competition regimes that can exist in market H when firm H is the technological leader ($n > 0$), for different combinations of trade and FDI costs to access that market. For each combination, the table specifies the winner of the competition, the price charged and the output produced by the winner, and the profits made by each firm in that market.

Table 2: H Leader, Market F

	Low Trade Cost $\tau_F < \lambda^n$		High Trade Cost $\tau_F \geq \lambda^n$		
	High FDI Cost $\kappa_F \geq [\tau_F - 1]\lambda^{-n}$	Low FDI Cost $\kappa_F < [\tau_F - 1]\lambda^{-n}$	High FDI Cost $\kappa_F \geq 1 - \frac{1}{\tau_F}$	Medium FDI Cost $\kappa_F \in [1 - \lambda^{-n}, 1 - \frac{1}{\tau_F})$	Low FDI Cost $\kappa_F < 1 - \lambda^{-n}$
Winner	Firm H (Exports)	Firm H (FDI)	Firm F	Firm F	Firm H (FDI)
p_F	MC_F	MC_F	$\tau_F MC_H$	$\frac{MC_H}{(1-\kappa_F)}$	MC_F
X_F	$\frac{\alpha Y_F}{MC_F}$	$\frac{\alpha Y_F}{MC_F}$	$\frac{\alpha Y_F}{\tau_F MC_H}$	$\frac{\alpha Y_F}{MC_H}(1-\kappa_F)$	$\frac{\alpha Y_F}{MC_F}$
Π_{HF}	$\left[1 - \tau_F\lambda^{-n}\right] \alpha Y_F$	$\left[1 - \lambda^{-n} - \kappa_F\right] \alpha Y_F$	0	0	$\left[1 - \lambda^{-n} - \kappa_F\right] \alpha Y_F$
Π_{FF}	0	0	$\left[1 - \frac{1}{\tau_F}\lambda^n\right] \alpha Y_F$	$[1 - (1-\kappa_F)\lambda^n]\, \alpha Y_F$	0

Notes: The table represents the different competition regimes that can exist in market F when firm H is the technological leader ($n > 0$), for different combinations of trade and FDI costs to access that market. For each combination, the table specifies the winner of the competition, the price charged and the output produced by the winner, and the profits made by each firm in that market.

Table 3: F Leader, Market F

	High FDI Cost $\kappa_F \geq 1 - 1/\tau_F$	Low FDI Cost $\kappa_F < 1 - 1/\tau_F$
Winner	Firm F	Firm F
p_F	$\tau_F MC_H$	$\frac{MC_H}{1-\kappa_F}$
X_F	$\frac{\alpha Y_F}{\tau_F MC_H}$	$\frac{\alpha Y_F}{MC_H}(1-\kappa_F)$
π_{FF}	$\left[1 - \frac{1}{\tau_F}\lambda^n\right]\alpha Y_F$	$[1-(1-\kappa_F)\lambda^n]\,\alpha Y_F$
π_{HF}	0	0

Notes: The table represents the different competition regimes that can exist in market F when firm F is the technological leader ($n < 0$), for different combinations of trade and FDI costs to access that market. For each combination, the table specifies the winner of the competition, the price charged and the output produced by the winner, and the profits made by each firm in that market.

Table 4: F Leader, Market H

	Low Trade Cost $\tau_H < \lambda^{-n}$		High Trade Cost $\tau_H \geq \lambda^{-n}$		
	High FDI Cost $\kappa_H \geq [\tau_H - 1]\lambda^n$	Low FDI Cost $\kappa_H < [\tau_H - 1]\lambda^n$	High FDI Cost $\kappa_H \geq 1 - \frac{1}{\tau_H}$	Medium FDI Cost $\kappa_H \in [1-\lambda^n, 1-\frac{1}{\tau_H})$	Low FDI Cost $\kappa_H < 1 - \lambda^n$
Winner	Firm F (Exports)	Firm F (FDI)	Firm H	Firm H	Firm F (FDI)
p_H	MC_H	MC_H	$\tau_H MC_F$	$\frac{MC_F}{(1-\kappa_H)}$	MC_H
X_H	$\frac{\alpha Y_H}{MC_H}$	$\frac{\alpha Y_H}{MC_H}$	$\frac{\alpha Y_H}{\tau_H MC_F}$	$\frac{\alpha Y_H}{MC_F}(1-\kappa_H)$	$\frac{\alpha Y_H}{MC_H}$
π_{FH}	$[1-\tau_H\lambda^n]\,\alpha Y_H$	$[1-\lambda^n-\kappa_H]\,\alpha Y_H$	0	0	$[1-\lambda^n-\kappa_H]\,\alpha Y_H$
π_{HH}	0	0	$\left[1-\frac{1}{\tau_H}\lambda^{-n}\right]\alpha Y_H$	$[1-(1-\kappa_H)\lambda^{-n}]\,\alpha Y_H$	0

Notes: The table represents the different competition regimes that can exist in market H when firm F is the technological leader ($n < 0$), for different combinations of trade and FDI costs to access that market. For each combination, the table specifies the winner of the competition, the price charged and the output produced by the winner, and the profits made by each firm in that market.

Numerical Analysis

Table 5: Baseline Parameter values

Parameter	Value	Parameter	Value
α	0.3	λ	1.1
ρ	0.05	σ	0.3
A_H	1	θ	2.07
A_F	1	$\bar{z}$	1
L_H	1	$\tau_H = \tau_F$	1.11
L_F	1	$\kappa_H = \kappa_F$	1

Notes: The table provides the parameter values used in the baseline experiments. See the main text for an explanation of each value.

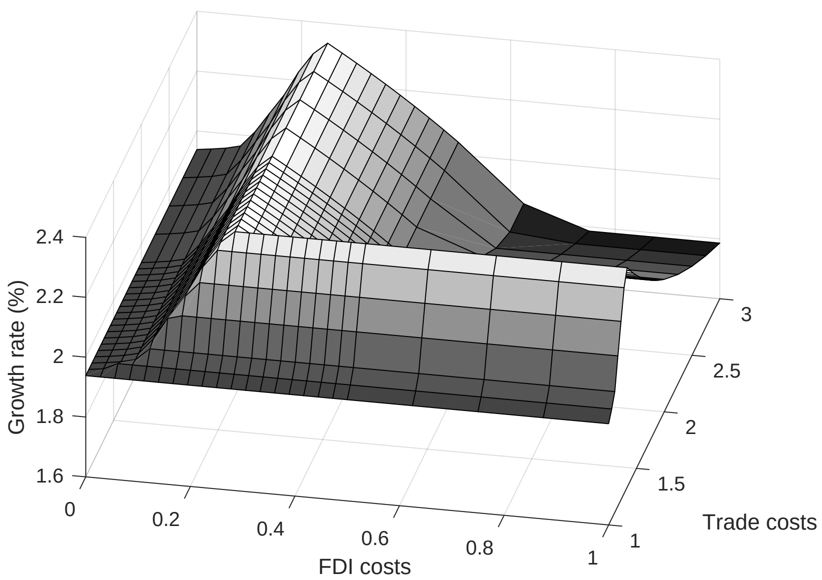

Figure 5: Trade/FDI Costs and Economic Growth: Bilateral

Notes: The figure represents the rate of economic growth in both countries for different combinations of trade and FDI costs in the baseline specification. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model.

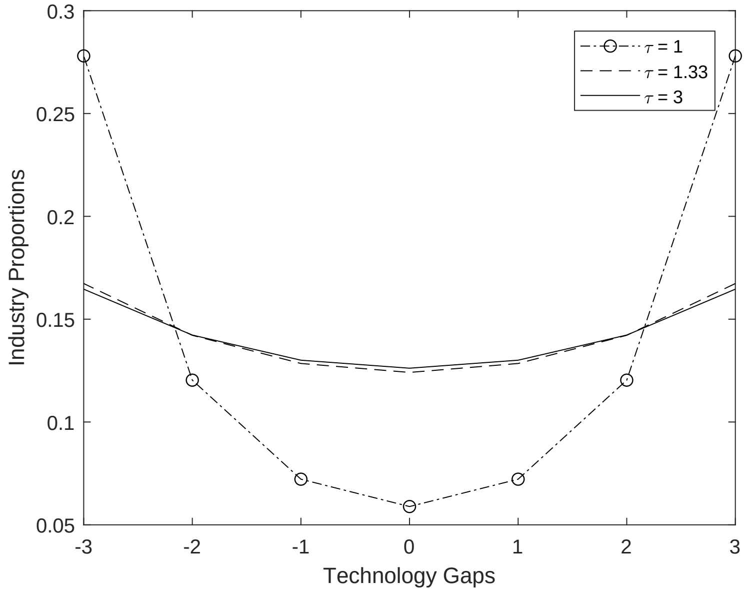

Figure 6: Trade Costs and Industry Distributions: Bilateral ($\kappa = 1$)

Notes: The figure represents the proportions of industries at different technology gaps in the baseline bilateral specification for $\kappa_H = \kappa_F = 1$ and three different levels of trade costs. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model.

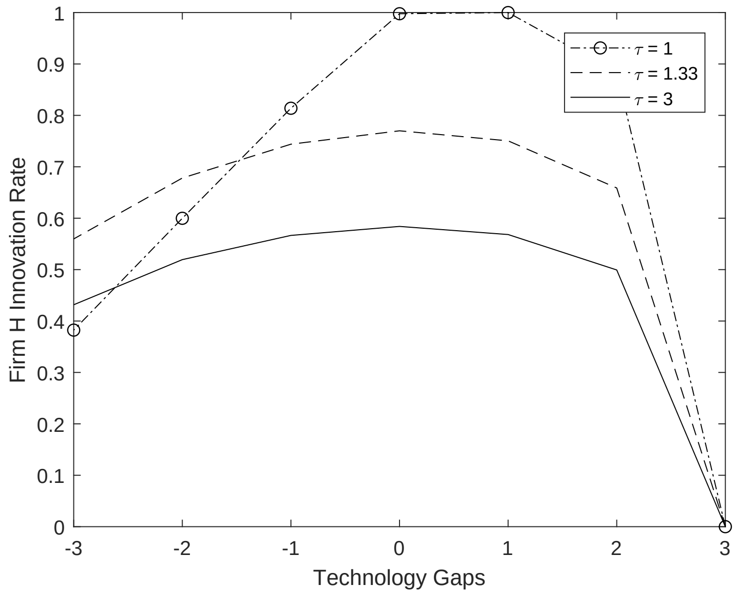

Figure 7: Trade Costs and Firm H's Innovation Rates: Bilateral ($\kappa = 1$)

Notes: The figure represents the innovation rates of firms from country H in industries at different technology gaps in the baseline bilateral specification for $\kappa_H = \kappa_F = 1$ and three different levels of trade costs. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model.

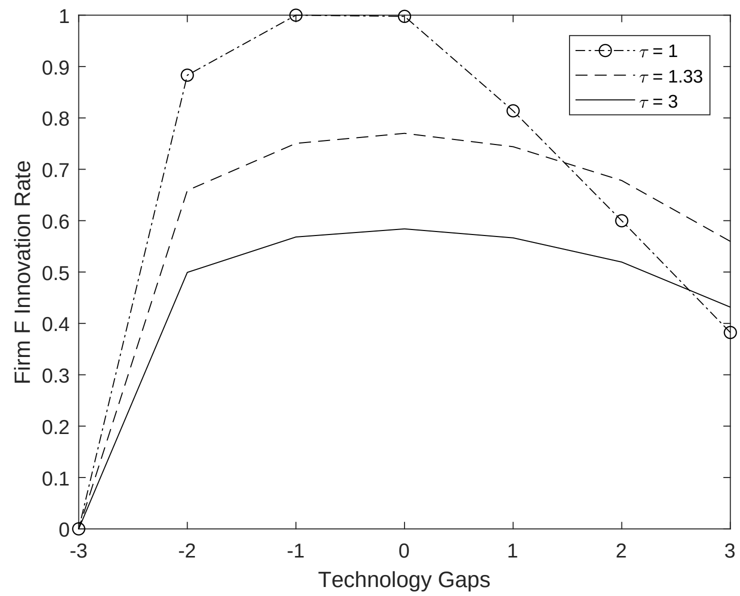

Figure 8: Trade Costs and Firm F's Innovation Rates: Bilateral ($\kappa = 1$)

Notes: The figure represents the innovation rates of firms from country F in industries at different technology gaps in the baseline bilateral specification for $\kappa_H = \kappa_F = 1$ and three different levels of trade costs. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model.

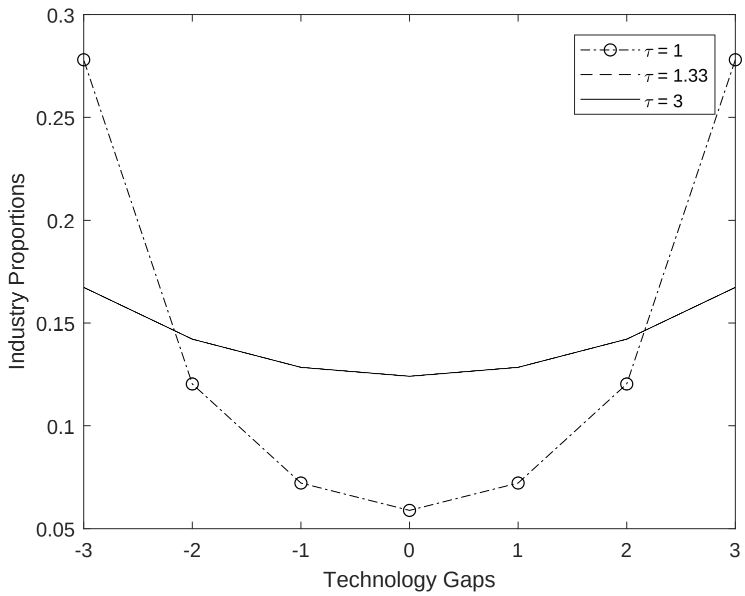

Figure 9: Trade Costs and Industry Distributions: Bilateral ($\kappa = 0.25$)

Notes: The figure represents the proportions of industries at different technology gaps in the baseline bilateral specification for $\kappa_H = \kappa_F = 0.25$ and three different levels of trade costs. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model.

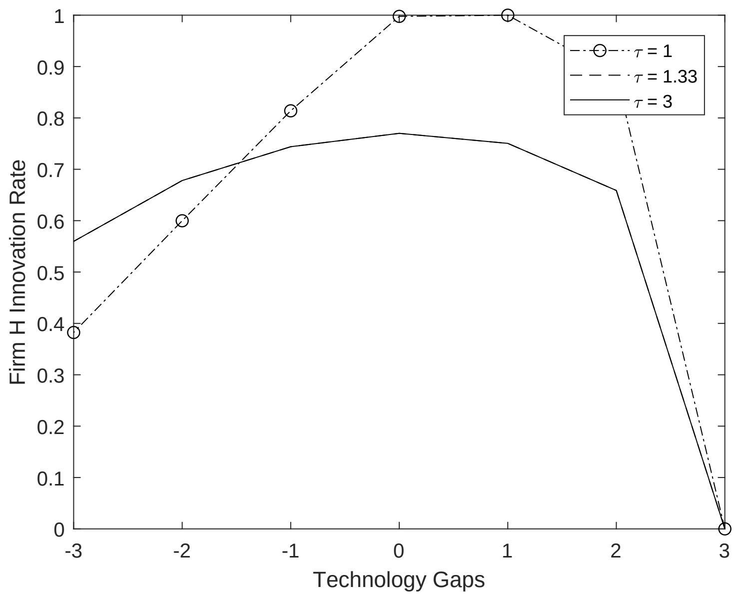

Figure 10: Trade Costs and Firm H's Innovation Rates: Bilateral ($\kappa = 0.25$)

Notes: The figure represents the innovation rates of firms from country H in industries at different technology gaps in the baseline bilateral specification for $\kappa_H = \kappa_F = 0.25$ and three different levels of trade costs. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model.

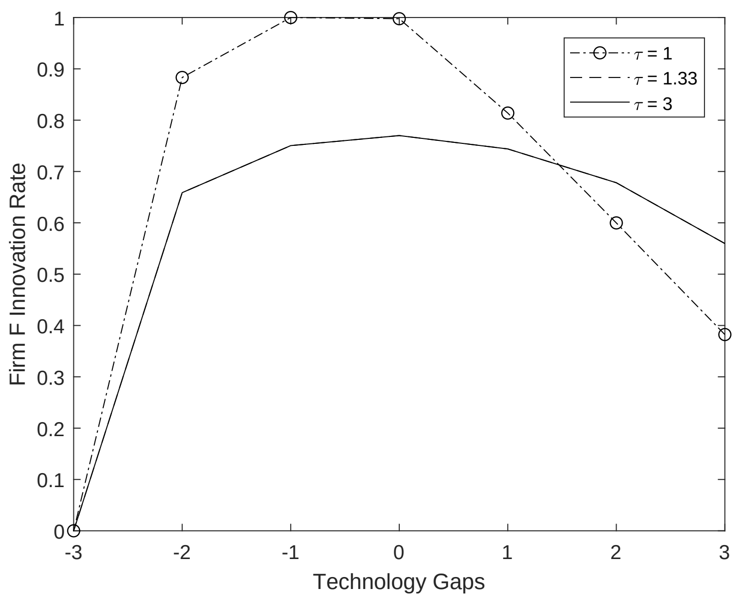

Figure 11: Trade Costs and Firm F's Innovation Rates: Bilateral ($\kappa = 0.25$)

Notes: The figure represents the innovation rates of firms from country F in industries at different technology gaps in the baseline bilateral specification for $\kappa_H = \kappa_F = 0.25$ and three different levels of trade costs. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model.

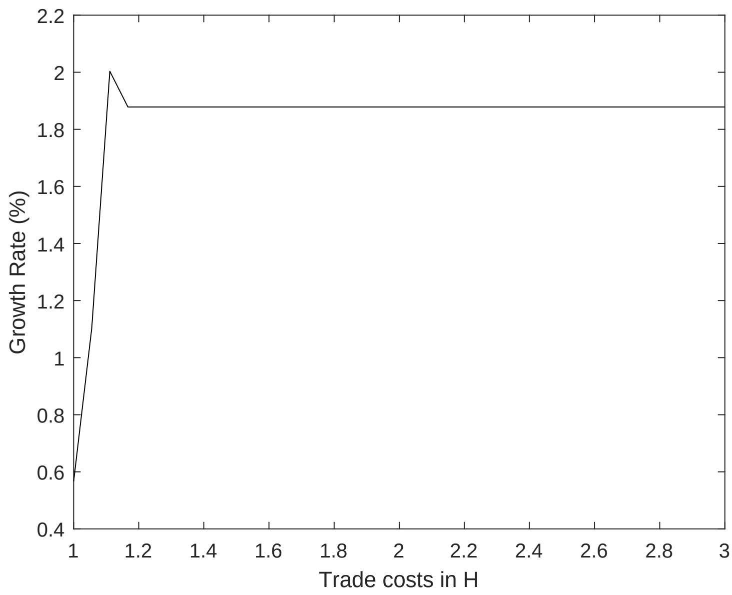

Figure 12: Trade Costs in H and Economic Growth: Unilateral

Notes: The figure represents the rate of economic growth in both countries for different values of trade costs in country H. Trade costs in country F are fixed at the baseline level of 1.11. FDI costs are fixed (in both countries) at a level of 0.1. The two countries are symmetric in every other parameter of the model (baseline values).

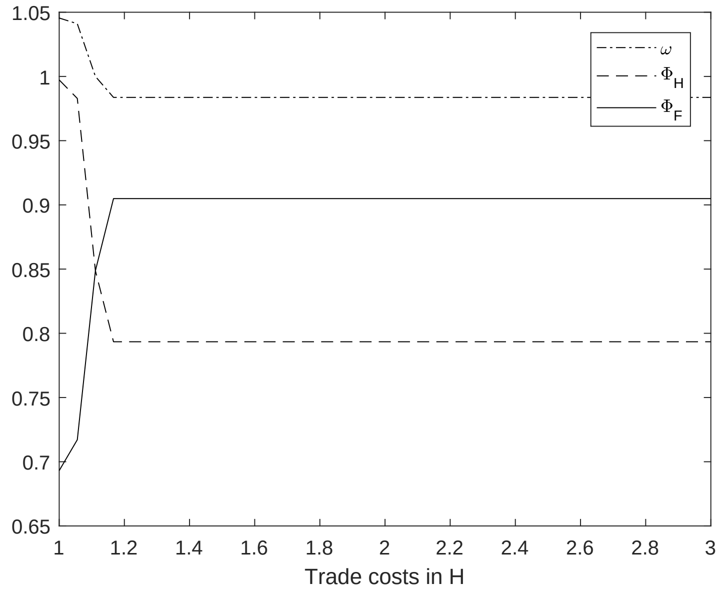

Figure 13: Trade Costs in H, Omega Ratio, and Competition Indices: Unilateral

Notes: The figure represents the ratio of final outputs $\omega = Y_H/Y_F$ and aggregate competition indices in both countries for different values of trade costs in country H. Trade costs in country F are fixed at the baseline level of 1.11. FDI costs are fixed (in both countries) at a level of 0.1. The two countries are symmetric in every other parameter of the model (baseline values).

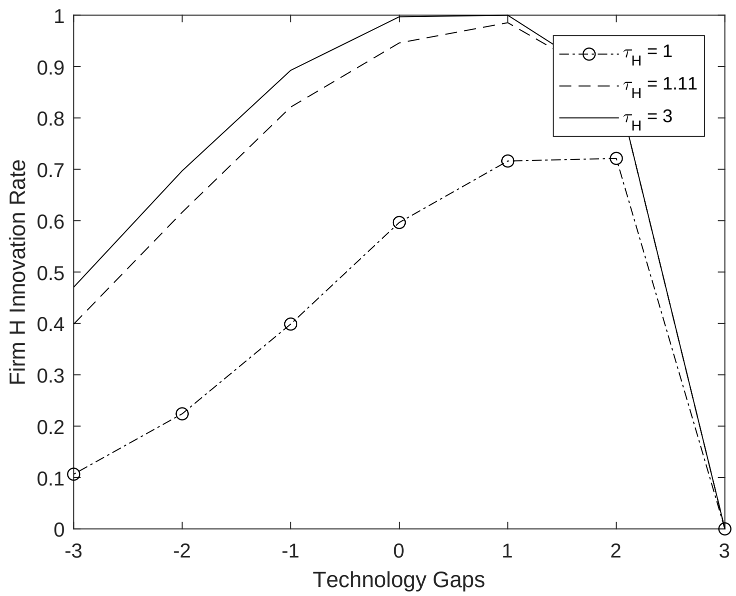

Figure 14: Trade Costs in H and Firm H's Innovation Rates: Unilateral

Notes: The figure represents the innovation rates of firms from country H in industries at different technology gaps for different values of trade costs in country H. Trade costs in country F are fixed at the baseline level of 1.11. FDI costs are fixed (in both countries) at a level of 0.1. The two countries are symmetric in every other parameter of the model (baseline values).

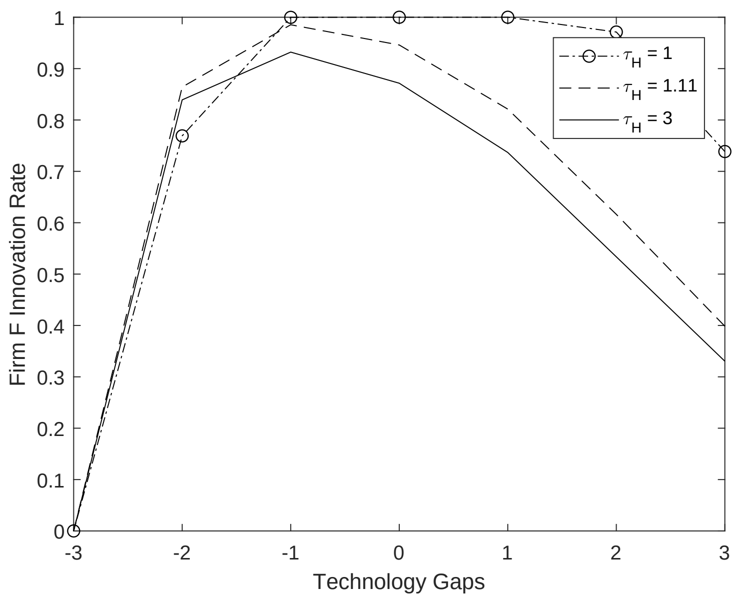

Figure 15: Trade Costs in H and Firm F's Innovation Rates: Unilateral

Notes: The figure represents the innovation rates of firms from country F in industries at different technology gaps for different values of trade costs in country H. Trade costs in country F are fixed at the baseline level of 1.11. FDI costs are fixed (in both countries) at a level of 0.1. The two countries are symmetric in every other parameter of the model (baseline values).

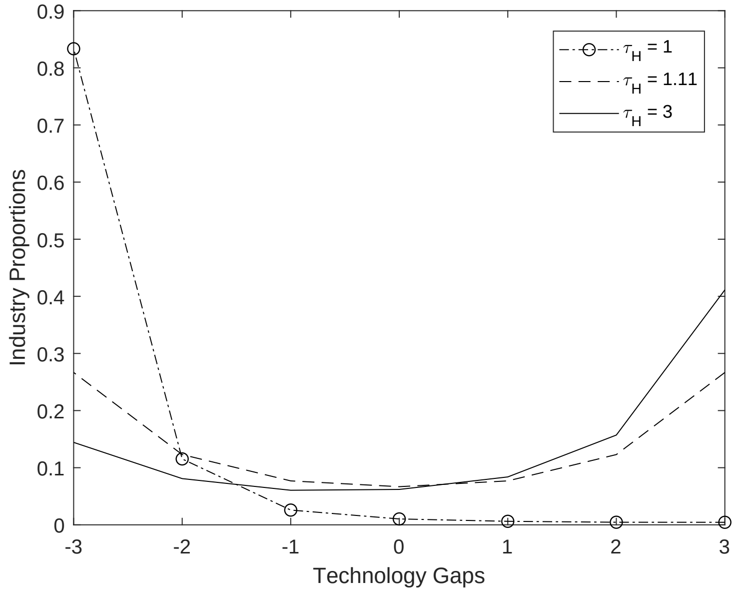

Figure 16: Trade Costs in H and Industry Distributions: Unilateral

Notes: The figure represents the proportions of industries at different technology gaps for different values of trade costs in country H. Trade costs in country F are fixed at the baseline level of 1.11. FDI costs are fixed (in both countries) at a level of 0.1. The two countries are symmetric in every other parameter of the model (baseline values).

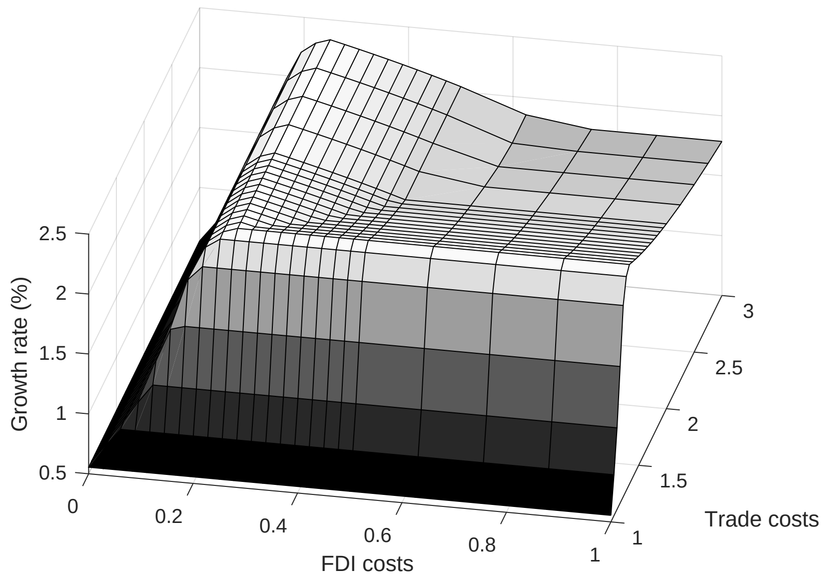

Figure 17: Trade/FDI Costs and Economic Growth: Bilateral ($L_H = 2$)

Notes: The figure represents the rate of economic growth in both countries for different combinations of trade and FDI costs. The two countries are symmetric in terms of trade and FDI costs, but country H has twice as much population as country F. The two countries are symmetric in terms of all other parameters (baseline values).

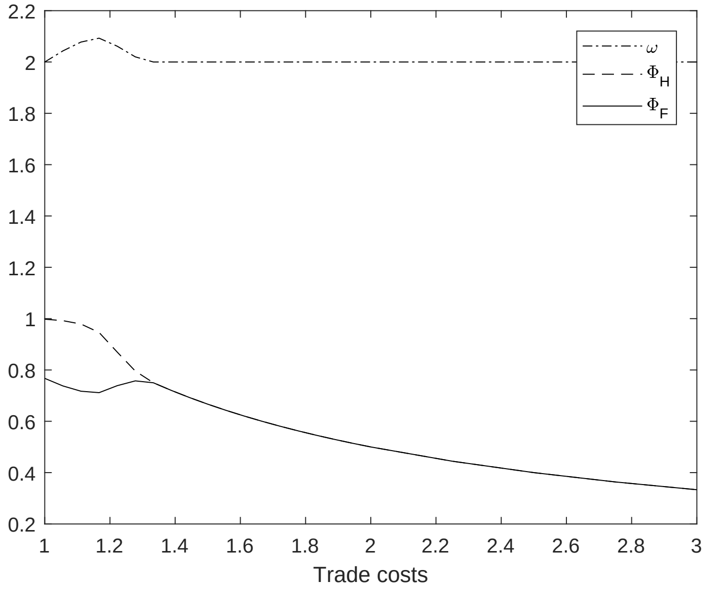

Figure 18: Trade Costs, Omega Ratio, and Competition Indices: Bilateral ($L_H = 2$)

Notes: The figure represents the ratio of final outputs $\omega = Y_H/Y_F$ and aggregate competition indices in both countries for $\kappa_H = \kappa_F = 1$ and three different levels of trade costs. The two countries are symmetric in terms of trade and FDI costs, but country H has twice as much population as country F. The two countries are symmetric in terms of all other parameters (baseline values).

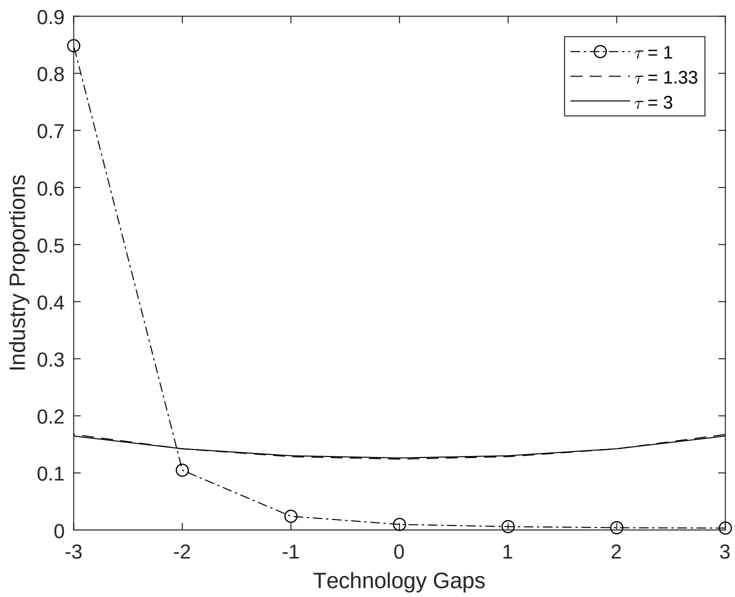

Figure 19: Trade Costs and Industry Proportions: Bilateral ($L_H = 2$)

Notes: The figure represents the proportions of industries at different technology gaps for $\kappa_H = \kappa_F = 1$ and three different levels of trade costs. The two countries are symmetric in terms of trade and FDI costs, but country H has twice as much population as country F. The two countries are symmetric in terms of all other parameters (baseline values).

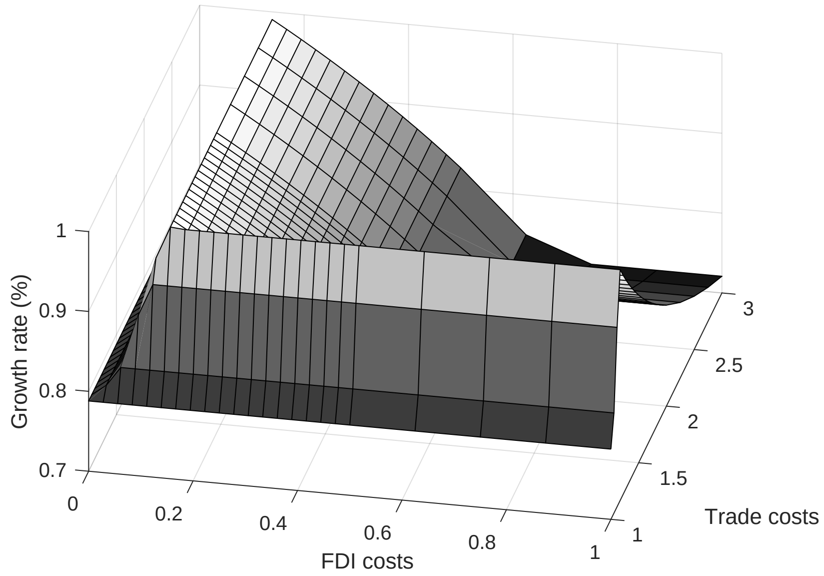

Figure 20: Trade/FDI Costs and Economic Growth: Bilateral ($\lambda = 1.05$)

Notes: The figure represents the rate of economic growth in both countries for different combinations of trade and FDI costs. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model (baseline values), but the size of innovations is set to $\lambda = 1.05$.

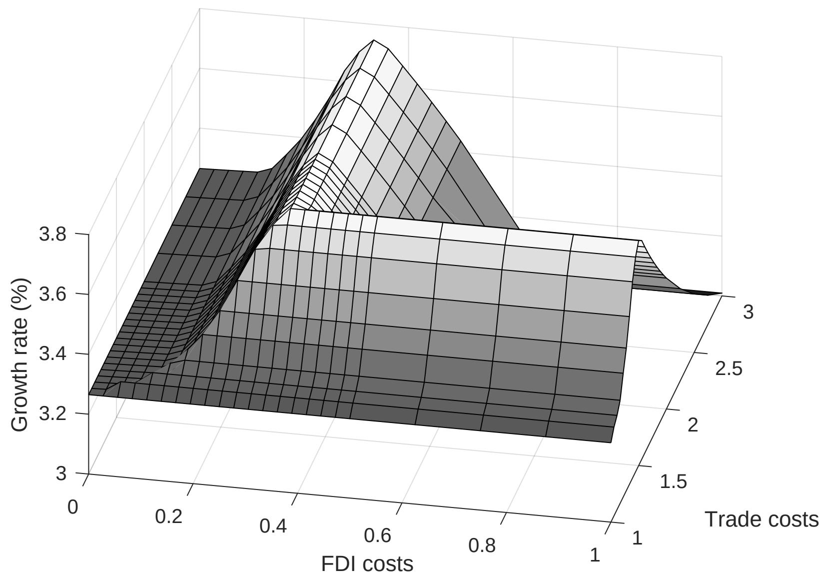

Figure 21: Trade/FDI Costs and Economic Growth: Bilateral ($\lambda = 1.15$)

Notes: The figure represents the rate of economic growth in both countries for different combinations of trade and FDI costs. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model (baseline values), but the size of innovations is set to $\lambda = 1.15$.

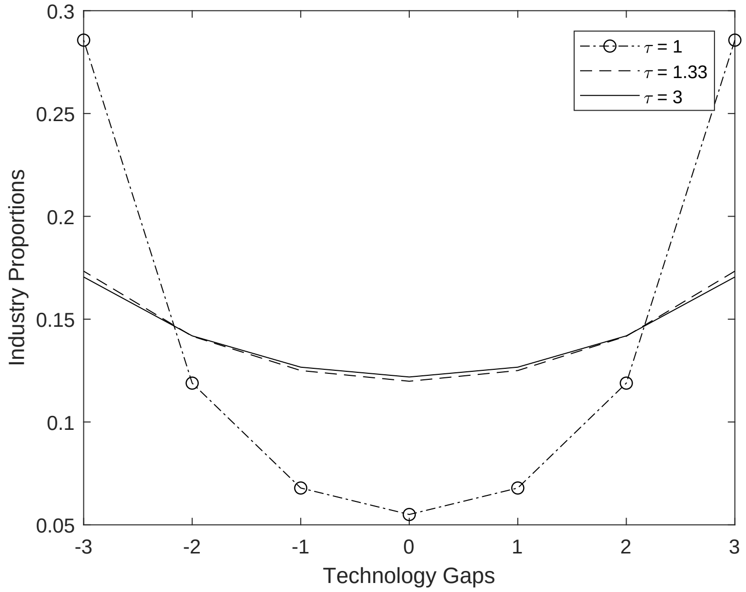

Figure 22: Trade Costs and Industry Distributions: Bilateral ($\lambda = 1.05$)

Notes: The figure represents the proportions of industries at different technology gaps for $\kappa_H = \kappa_F = 1$ and three different levels of trade costs. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model (baseline values), but the size of innovations is set to $\lambda = 1.05$.

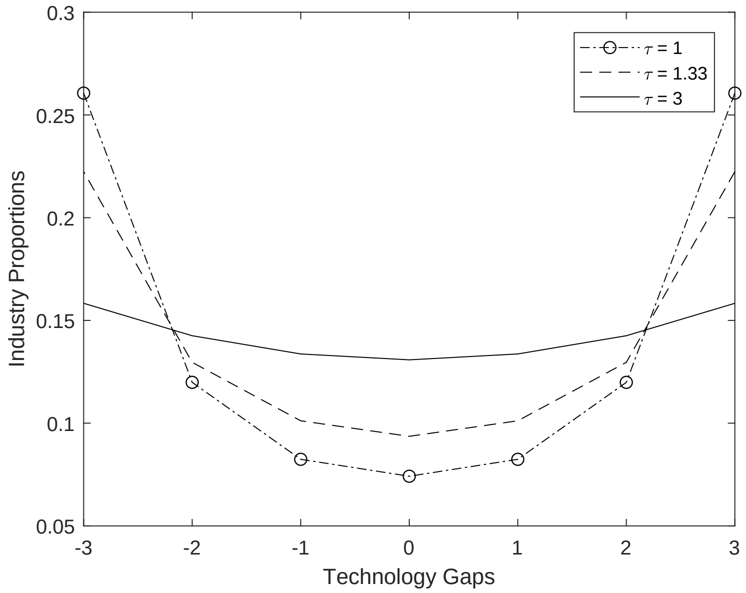

Figure 23: Trade Costs and Industry Distributions: Bilateral ($\lambda = 1.15$)

Notes: The figure represents the proportions of industries at different technology gaps for $\kappa_H = \kappa_F = 1$ and three different levels of trade costs. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model (baseline values), but the size of innovations is set to $\lambda = 1.15$.

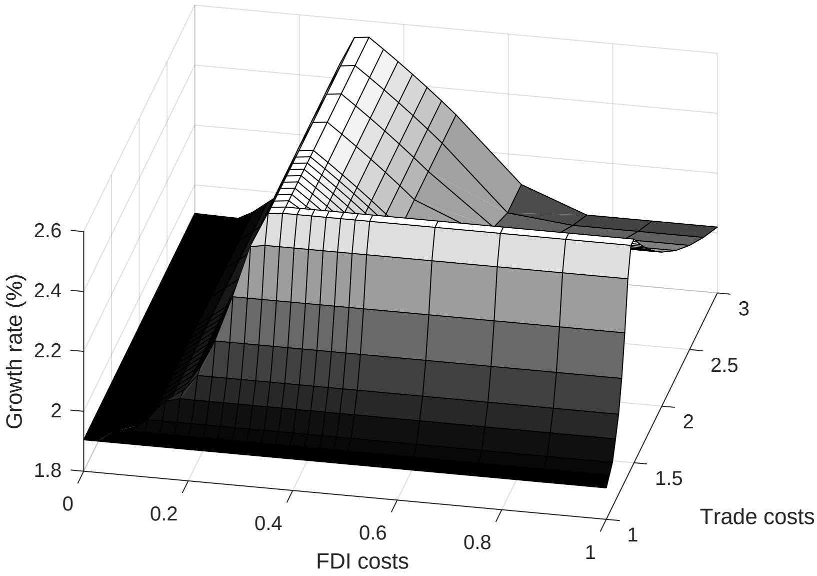

Figure 24: Trade/FDI Costs and Economic Growth: Bilateral (max $n = 4$)

Notes: The figure represents the rate of economic growth in both countries for different combinations of trade and FDI costs. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model (baseline values), but the maximum technology gap is set to 4.

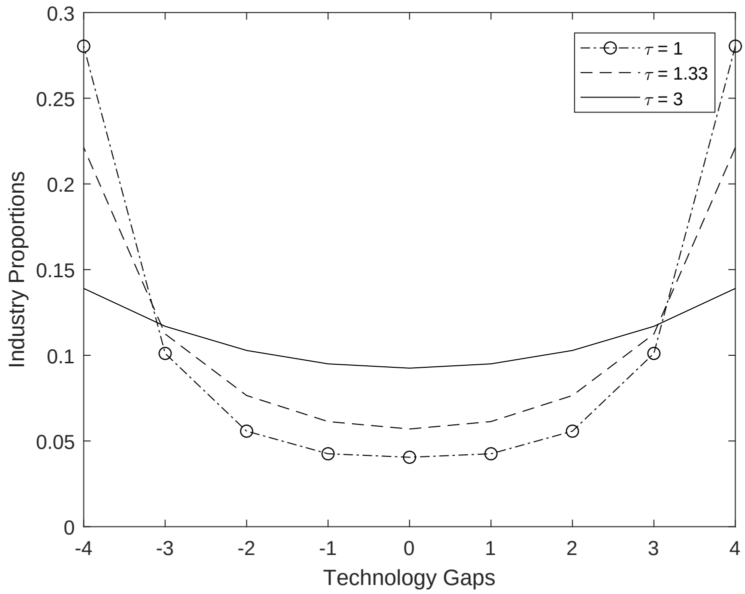

Figure 25: Trade Costs and Industry Distributions: Bilateral (max $n = 4$)

Notes: The figure represents the proportions of industries at different technology gaps for $\kappa_H = \kappa_F = 1$ and three different levels of trade costs. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model (baseline values), but the maximum technology gap is set to 4.

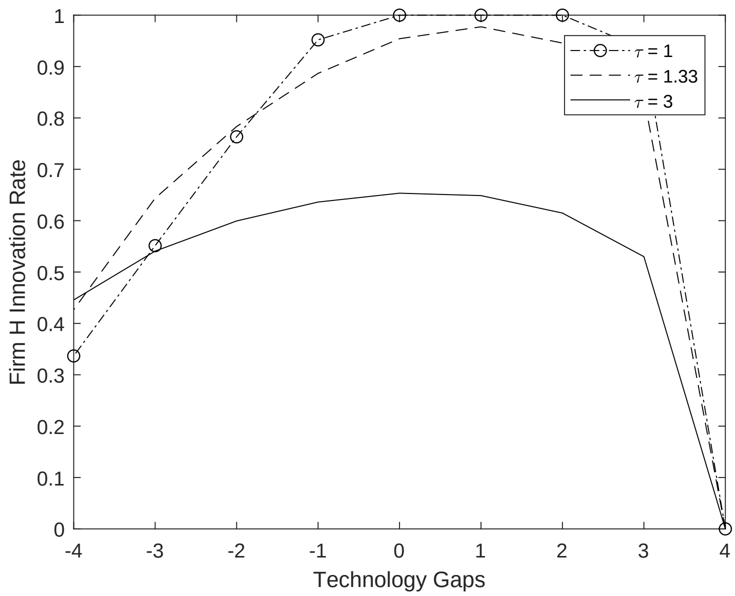

Figure 26: Trade Costs and Firm H's Innovation Rates: Bilateral (max $n = 4$)

Notes: The figure represents the innovation rates of firms from country H in industries at different technology gaps for $\kappa_H = \kappa_F = 1$ and three different levels of trade costs. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model (baseline values), but the maximum technology gap is set to 4.

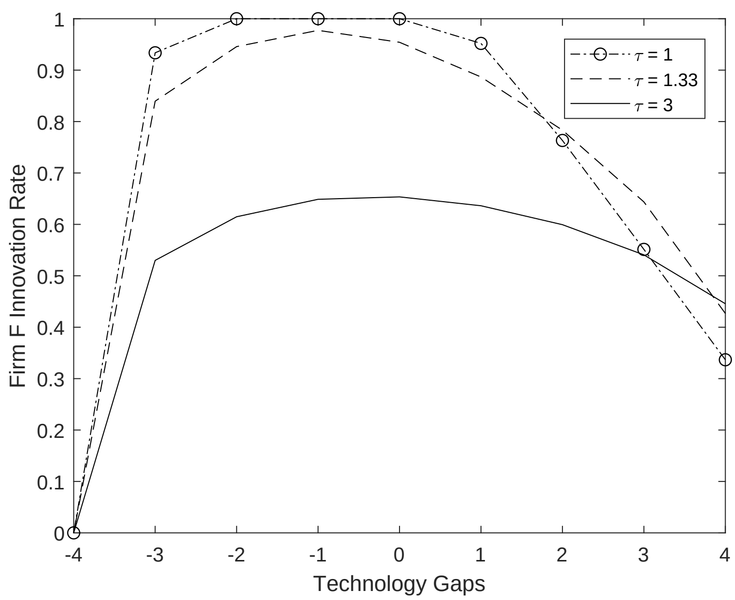

Figure 27: Trade Costs and Firm F's Innovation Rates: Bilateral (max $n = 4$)

Notes: The figure represents the innovation rates of firms from country F in industries at different technology gaps for $\kappa_H = \kappa_F = 1$ and three different levels of trade costs. The two countries are symmetric in terms of trade and FDI costs, and every other parameter of the model (baseline values), but the maximum technology gap is set to 4.

Appendix B: Proofs and Derivations

Static Profit Analysis

In this section I provide derivations of all the expressions of prices, output, and profits in each market given in Tables 1-4.

H leaders, Market H (Table 1)

Firm H wins in this market, but the price it charges is determined by whether firm F could undercut with exports or FDI. The two regions of trade/FDI costs in Table 1 are determined by the zero-profit condition in (12):

$$\begin{aligned}
[p_H - MC_F]\, X_H = K_H &\iff [p_H - MC_F]\, \frac{\alpha Y_H}{p_H} = \kappa_H \alpha Y_H \\
&\iff \left[1 - \frac{MC_F}{p_H}\right] = \kappa_H \\
&\iff p_H = \frac{MC_F}{1-\kappa_H},
\end{aligned}$$

where the first line uses equation (8) and the definition of FDI costs. This will be the price charged by firm H if and only if it is smaller than the unit costs of exporting, $\tau_H MC_F$. That is, if and only if

$$\begin{aligned}
p_H = \frac{MC_F}{1-\kappa_H} < \tau_H MC_F &\iff \frac{1}{1-\kappa_H} < \tau_H \\
&\iff \kappa_H < 1 - \frac{1}{\tau_H}
\end{aligned}$$

Otherwise, the price is determined by the threat of exports and equal to $\tau_H MC_F$. Output in each case is determined by substituting the relevant price in (8). Profits for firm H when the threat comes from exports are:

$$\begin{aligned}
\Pi_{HH} = [p_H - MC_H]\, X_H = [p_H - MC_H]\, \frac{\alpha Y_H}{p_H} \\
&= \left[1 - \frac{MC_H}{p_H}\right] \alpha Y_H \\
&= \left[1 - \frac{MC_H}{\tau_H MC_F}\right] \alpha Y_H \\
&= \left[1 - \frac{1}{\tau_H}\lambda^{-n}\right] \alpha Y_H,
\end{aligned}$$

where the last line makes use of the definition of marginal costs as the reciprocal of technology, equation (9), and the definition of the technology gap in (10). Similarly, profits for firm H when the threat comes from FDI are:

$$\begin{aligned}
\Pi_{HH} = [p_H - MC_H]\, X_H &= [p_H - MC_H]\, \frac{\alpha Y_H}{p_H} \\
&= \left[1 - \frac{MC_H}{p_H}\right] \alpha Y_H \\
&= \left[1 - (1-\kappa_H)\frac{MC_H}{MC_F}\right] \alpha Y_H \\
&= \left[1 - (1-\kappa_H)\lambda^{-n}\right] \alpha Y_H
\end{aligned}$$

H leaders, Market F (Table 2)

First, trade costs to access this market are considered low if the unit costs of exporting for firm H are smaller than the marginal production cost of firm F. That is, trade costs are low if

$$\tau_F MC_H < MC_F \Longleftrightarrow \tau_F < \frac{MC_F}{MC_H} = \lambda^n$$

Trade costs are considered high otherwise. If trade costs are low, firm H wins the competition with either trade or FDI. In any case it will charge $p_F = MC_F$ and produce $X_F = \alpha Y_F / MC_F$. But it will serve the market via exports if and only if that is more profitable than doing FDI. That is, if and only if

$$\begin{aligned}
[p_F - \tau_F MC_H]\, X_F \geq [p_F - MC_H]\, X_F - K_F &\Longleftrightarrow [p_F - \tau_F MC_H]\, \frac{\alpha Y_F}{p_F} \geq [p_F - MC_H]\, \frac{\alpha Y_F}{p_F} - \kappa_F \alpha Y_F \\
&\Longleftrightarrow \left[1 - \frac{\tau_F MC_H}{MC_F}\right] \alpha Y_F \geq \left[1 - \frac{MC_H}{MC_F} - \kappa_F\right] \alpha Y_F \\
&\Longleftrightarrow \left[1 - \tau_F \lambda^{-n}\right] \alpha Y_F \geq \left[1 - \lambda^{-n} - \kappa_F\right] \alpha Y_F \\
&\Longleftrightarrow 1 - \tau_F \lambda^{-n} \geq 1 - \lambda^{-n} - \kappa_F \\
&\Longleftrightarrow \kappa_F \geq [\tau_F - 1]\, \lambda^{-n}
\end{aligned}$$

Firm H will do FDI otherwise. The expressions for profits from exports and FDI in Table 2 are given in the third line of the above chain of implications. If trade costs are high, firm H can only win with FDI. This happens if and only if the profits from FDI are positive:

$$\begin{aligned}
[p_F - MC_H]\, X_F - K_F > 0 &\Longleftrightarrow \left[1 - \lambda^{-n} - \kappa_F\right] \alpha Y_F > 0 \\
&\Longleftrightarrow 1 - \lambda^{-n} - \kappa_F > 0 \\
&\Longleftrightarrow \kappa_F < 1 - \lambda^{-n}
\end{aligned}$$

Otherwise, firm F captures its domestic market. In that case, the price it charges depends on the threat by firm H of undercutting with exports or FDI. This is analogous to the analysis of market H above with the roles of the H and F subscripts reversed.

F leaders (Tables 3 and 4)

The derivations for all the expressions in Tables 3 and 4 mimic the ones for those of Tables 1 and 2, with the roles of H and F reversed.

Aggregate Resource Constraint

In this section I show that the aggregate resource constrain (30) is satisfied in equilibrium. To simplify the notation, I omit the time indices. First, since the final good sector is perfectly competitive, the representative firm makes zero profits. From (5),

$$\begin{aligned} Y_i &= w_i L_i + \int_0^1 p_i(j) X_i(j) dj \\ &= w_i L_i + \sum_{n=-\infty}^{+\infty} \mu_n p_i^n X_i^n \end{aligned}$$

Since industries can be dominated by domestic or foreign firms, and revenue (profits plus total costs) is determined by the costs of trade and FDI, the second term on the right-hand side can be written as

$$\begin{aligned} \sum_{n=-\infty}^{+\infty} \mu_n p_i^n X_i^n = &\sum_{n=-\infty}^{+\infty} \mu_n [\chi_i^{DOM} (\Pi_{ii}^n + MC_i^n X_i^n) \\ &+ \chi_i^{EX} ((\Pi_{di}^n)^{EX} + \tau_i MC_d^n X_i^n) \\ &+ \chi_i^{FDI} ((\Pi_{di}^n)^{FDI} + MC_d^n X_i^n + K_i)] \\ = &\sum_{n=-\infty}^{+\infty} \mu_n [\chi_i^{DOM} \Pi_{ii}^n + \chi_i^{EX} (\Pi_{di}^n)^{EX} + \chi_i^{FDI} (\Pi_{di}^n)^{FDI}] + M_i, \end{aligned}$$

using the definitions of the total cost of intermediate goods production M_i in (23) and the indicator functions χ_i^m, $m \in \{DOM, EX, FDI\}$ in (24)-(26). I omit the arguments of those indicator functions for simplicity. Combining the last two expressions and solving for labor income yields:

$$w_i L_i = Y_i - M_i - \sum_{n=-\infty}^{+\infty} \mu_n [\chi_i^{DOM} \Pi_{ii}^n + \chi_i^{EX} (\Pi_{di}^n)^{EX} + \chi_i^{FDI} (\Pi_{di}^n)^{FDI}]$$

Since the representative household effectively owns all the domestic firms, asset income $r_i B_i$ is equal to the total profits made by those firms in both markets:

$$r_i B_i = \sum_{n=-\infty}^{+\infty} \mu_n [\chi_i^{DOM} \Pi_{ii}^n + \chi_d^{EX} (\Pi_{id}^n)^{EX} + \chi_d^{FDI} (\Pi_{id}^n)^{FDI}]$$

Substituting the expressions for labor and asset income, together with the market clearing condition for assets, $\dot{B}_i = R_i$, into the budget constraint of the representative household (2) yields:

$$\begin{aligned} R_i &= Y_i - M_i - \sum_{n=-\infty}^{+\infty} \mu_n [\chi_i^{EX} (\Pi_{di}^n)^{EX} - \chi_d^{EX} (\Pi_{id}^n)^{EX} + \chi_i^{FDI} (\Pi_{di}^n)^{FDI} - \chi_d^{FDI} (\Pi_{id}^n)^{FDI}] - C_i \\ &= Y_i - M_i - NX_i - C_i, \end{aligned}$$

where the second equality makes use of the definition of net exports in (27). Rearranging the last equation yields the aggregate resource constraint.

Derivation of Equation (31)

Substituting (19) into the final output production function yields

$$\begin{aligned} Y_i(t) &= (A_i L_i)^{1-\alpha} exp\left(\alpha \int_0^1 \ln\left(X_i(j,t)\right) dj\right) \\ &= (A_i L_i)^{1-\alpha} exp\left(\alpha \int_0^1 \ln\left(\alpha Y_i(t) q_i(j,t) \phi_i(n(j,t), \tau_i, \kappa_i)\right) dj\right) \\ &= (A_i L_i)^{1-\alpha} exp(\alpha \int_0^1 \ln\left(\alpha\right) dj + \alpha \int_0^1 \ln\left(Y_i(t)\right) dj \\ &+ \int_0^1 \ln\left(q_i(j,t)\right) dj + \int_0^1 \ln\left(\phi_i(n(j,t), \tau_i, \kappa_i)\right) dj \end{aligned}$$

Using the definitions in (32)-(33) for the technology and competition indices, the fact that α and $Y_i(t)$ don't depend on j, the fact that the exponential and logarithmic functions are inverses of each other, and solving for $Y_i(t)$ yields (31).

Proof of Proposition 1 (Equality of Growth rates)

From (34) it is clear that output grows at the same rate in both countries if and only if the technology indices $Q_H(t)$ and $Q_F(t)$ grow at the same rate. Here I show this is the case. The index in country F can be written as follows:

$$
\begin{aligned}
\ln (Q_F(t)) &\equiv \int_0^1 \ln (q_F(j,t))\, dj \\
&= \int_0^1 \ln \left(q_H(j,t)\lambda^{-n(j,t)} \right) dj \\
&= \int_0^1 \ln (q_H(j,t))\, dj - \ln (\lambda) \int_0^1 n(j,t) dj \\
&= \ln (Q_H(t)) - \ln (\lambda) \sum_{n=-\infty}^{+\infty} n\mu_n(t)
\end{aligned}
$$

Rearranging yields

$$
\ln \left(\frac{Q_H(t)}{Q_F(t)} \right) = \ln (\lambda) \sum_{n=-\infty}^{+\infty} n\mu_n(t)
$$

If the distribution of industries over technology gaps is stationary, then the right-hand side of the previous equation is constant over time. That implies the two technology indices, and final output in both countries, must grow at the same rate. ■

Proof of Proposition 2 (Steady-State Growth Rate)

In steady state, growth in both countries depends on the evolution of the technology index $Q_H(t)$ (or $Q_F(t)$). For each industry with a technology gap of n, firm H upgrades its technology $q_H^n(t)$ to $q_H^n(t+\Delta t) = \lambda q_H^n(t)$ with probability $z_H^n \Delta t + o(\Delta t)$, and fails to do so with probability $1 - z_H^n \Delta t - o(\Delta t)$. Thus,

$$
\ln (Q_H(t+\Delta t)) = \ln (Q_H(t)) + \sum_{n=-\infty}^{+\infty} \mu_n(t) \left(z_H^n \Delta t + o(\Delta t) \right) \ln (\lambda)
$$

Subtracting $\ln (Q_H(t))$ from both sides, dividing by Δt, and taking the limit as $\Delta t \to 0$, yields the growth rate of $Q_H(t)$:

$$
g_H^Q \equiv \frac{d\ln (Q_H(t))}{dt} = \ln (\lambda) \sum_{n=-\infty}^{+\infty} \mu_n z_H^n
$$

Similar reasoning shows that

$$
g_F^Q \equiv \frac{d\ln (Q_F(t))}{dt} = \ln (\lambda) \sum_{n=-\infty}^{+\infty} \mu_n z_F^n
$$

Substituting in (34) yields the steady-state growth rate in (35). ■

Derivation of equations (37)-(38) and (40)-(41)

The steady-state value of firm H at time t in an industry with technology gap n is:

$$V_H^n(t) = \max_{z_H^n(t)\geq 0}\left\{\begin{array}{l}[\Pi_{HH}^n(t)+\Pi_{HF}^n(t)-\Gamma(z_H^n(t))Y_H(t)]\,\Delta t+o(\Delta t)\\ +exp\{-r^*\Delta t\}\{[z_H^n(t)\Delta t+o(\Delta t)]\,V_H^{n+1}(t+\Delta t)\\ +[z_F^n(t)\Delta t+o(\Delta t)]\,V_H^{n-1}(t+\Delta t)\\ +[1-z_H^n(t)\Delta t-z_F^n(t)\Delta t-o(\Delta t)]\,V_H^n(t+\Delta t)\}\end{array}\right\}$$

Subtracting $V_H^n(t)$ from both sides, adding and subtracting $exp\{-r^*\Delta t\}V_H^n(t)$ on the right-hand side, combining terms, and dividing everything by Δt yields

$$0 = \max_{z_H^n(t)\geq 0}\left\{\begin{array}{l}\Pi_{HH}^n(t)+\Pi_{HF}^n(t)-\Gamma(z_H^n(t))Y_H(t)+\frac{o(\Delta t)}{\Delta t}\\ +exp\{-r^*\Delta t\}\{\left[z_H^n(t)+\frac{o(\Delta t)}{\Delta t}\right]\left[V_H^{n+1}(t+\Delta t)-V_H^n(t+\Delta t)\right]\\ +\left[z_F^n(t)+\frac{o(\Delta t)}{\Delta t}\right]\left[V_H^{n-1}(t+\Delta t)-V_H^n(t+\Delta t)\right]\\ +\frac{V_H^n(t+\Delta t)-V_H^n(t)}{\Delta t}\}+V_H^n(t)\left[\frac{exp\{-r^*\Delta t\}-1}{\Delta t}\right]\end{array}\right\}$$

Taking the limit as $\Delta t\to 0$ the latter equation becomes:

$$0 = \max_{z_H^n(t)\geq 0}\left\{\begin{array}{l}\Pi_{HH}^n(t)+\Pi_{HF}^n(t)-\Gamma(z_H^n(t))Y_H(t)\\ +z_H^n(t)\left[V_H^{n+1}(t)-V_H^n(t)\right]\\ +z_F^n(t)\left[V_H^{n-1}(t)-V_H^n(t)\right]\\ +\dot{V}_H^n(t)-r^*V_H^n(t)\end{array}\right\}$$

Rearranging yields equation (37). The last term in the previous equation comes from the fact that

$$lim_{\Delta t\to 0}\left[\frac{exp\{-r^*\Delta t\}-1}{\Delta t}\right] = lim_{\Delta t\to 0}\left[\frac{exp\{-r^*\Delta t\}-exp\{-r^*\cdot 0\}}{\Delta t}\right] = \frac{d}{dt}exp\{-r^*t\}\mid_{t=0} = -r^*$$

Equation (38) can be derived analogously. To get equation (40), divide both sides of (37) by $Y_H(t)$ and use (36) to get:

$$(g^Y+\rho)\frac{V_H^n(t)}{Y_H(t)}-\frac{\dot{V}_H^n(t)}{V_H^n(t)}\frac{V_H^n(t)}{Y_H(t)} = \max_{z_H^n(t)\geq 0}\left\{\begin{array}{l}\frac{\Pi_{HH}^n(t)}{Y_H(t)}+\frac{\Pi_{HF}^n(t)}{Y_F(t)}\frac{Y_F(t)}{Y_H(t)}-\Gamma(z_H^n(t))\\ +z_H^n(t)\left[\frac{V_H^{n+1}(t)}{Y_H(t)}-\frac{V_H^n(t)}{Y_H(t)}\right]\\ +z_F^n(t)\left[\frac{V_H^{n-1}(t)}{Y_H(t)}-\frac{V_H^n(t)}{Y_H(t)}\right]\end{array}\right\}$$

Using the fact that $V_i^n(t)$ grows over time at the steady-state rate g^Y, the definitions of stationarized values, profits per unit of final output in the destination market, and the ratio of final outputs ω, yields equation (40). Similarly, dividing both sides of (38) by $Y_F(t)$ and using (36) we get:

$$(g^Y+\rho)\frac{V_F^n(t)}{Y_F(t)}-\frac{\dot{V}_F^n(t)}{V_F^n(t)}\frac{V_F^n(t)}{Y_F(t)}=\max_{z_F^n(t)\geq 0}\left\{\begin{array}{l}\frac{\Pi_{FF}^n(t)}{Y_F(t)}+\frac{\Pi_{FH}^n(t)}{Y_H(t)}\frac{Y_H(t)}{Y_F(t)}-\Gamma(z_F^n(t))\\ +z_H^n(t)\left[\frac{V_F^{n+1}(t)}{Y_F(t)}-\frac{V_F^n(t)}{Y_F(t)}\right]\\ +z_F^n(t)\left[\frac{V_F^{n-1}(t)}{Y_F(t)}-\frac{V_F^n(t)}{Y_F(t)}\right]\end{array}\right\}$$

Again, using the fact that $V_i^n(t)$ grows at the steady-state rate g^Y, the definitions of stationarized values, profits per unit of final output in the destination market, and the ratio of final outputs ω, yields equation (41).

Numerical Analysis

In this section I describe the uniformization procedure used to adjust the model for the numerical analysis. This is an adaptation of the procedure in Acemoglu and Akcigit (2012), which in turn is based on Ross (1996, pp. 282-284). The goal is to turn the dynamic optimization problem of intermediate good firms into a contraction mapping so that a value function iteration procedure can be used to find a solution in the numerical analysis.

In the model, an intermediate good industry at a certain technology gap n can transition out of that state with probabilities that depend on the innovation flow rates of each firm,

$$P_{n,n+1}=\frac{z_H^n}{z_H^n+z_F^n},\qquad P_{n,n-1}=\frac{z_F^n}{z_H^n+z_F^n},$$

where $P_{n,n+1}$ and $P_{n,n-1}$ are the probabilities of moving from state n to states $n+1$ and $n-1$, respectively. The uniformization procedure adds a fictitious transition from a state into itself. Since either firm can make a successful innovation, the transition rate out of state n is given by $\psi_n = z_H^n + z_F^n$. From the innovation function (15), firms flow rates of innovation are bounded above by $\bar{z} < \infty$. Thus, the transition rate ψ_n is bounded above by $\psi \equiv 2\bar{z} < \infty$. The procedure defines new transition probabilities (including the fictitious one),

$$\tilde{P}_{n,n+1}=\frac{\psi_n}{\psi}P_{n,n+1}=\frac{z_H^n}{2\bar{z}}$$

$$\tilde{P}_{n,n-1}=\frac{\psi_n}{\psi}P_{n,n-1}=\frac{z_F^n}{2\bar{z}}$$

$$\tilde{P}_{n,n}=1-\frac{\psi_n}{\psi}=1-\frac{z_H^n+z_F^n}{2\bar{z}},$$

and an effective discount factor,

$$\gamma\equiv\frac{\psi}{\rho+\psi}=\frac{2\bar{z}}{\rho+2\bar{z}}<1,$$

that, together with an adjustment of the stationarized profits (net of R&D costs),

$$\hat{\pi}_H = \frac{\pi_{HH}^n + \pi_{HF}^n(1/\omega^*) - \Gamma(z_H^n)}{\rho + 2\bar{z}}$$

$$\hat{\pi}_F = \frac{\pi_{FF}^n + \pi_{FH}^n \omega^* - \Gamma(z_F^n)}{\rho + 2\bar{z}},$$

allows to write the dynamic optimization problems in (40)-(41) as a contraction mapping:

$$v_i^n = \max_{z_i^n} \left\{ \hat{\pi}_i + \gamma \sum_{n'=n-1}^{n+1} \tilde{P}_{n,n} v_i^{n'} \right\} \qquad \forall n \in \mathbb{Z}$$

Once this adjustment is made, the numerical procedure to obtain the results of Sections 3 and 4 consists of the following steps:

1. Choose values for the parameters of the model. In particular set values for the trade and FDI costs in each country.

2. Guess a value of $\omega \equiv Y_H/Y_F$. A good initial guess is $A_H L_H / A_F L_F$. That takes into account potential asymmetries between the two countries and speeds up the process.

3. Calculate profits based on the values of trade and FDI costs, which define the conpetition regimes (see Tables 1-4).

4. Adjust the calculated profits as described in the uniformization procedure above.

5. Apply a value function iteration procedure to the contraction mapping defined above. Within each iteration of the value function, apply a best-response procedure to find the optimal innovation rates of each firm given what their rival chooses.

6. Once the innovation rates are obtained, calculate the industry proportions at different technology gaps using equations (18) and (44).

7. Use the proportions to calculate the competition indices and a new value of ω. If the new value differs from the guess in more than the set tolerance, update the guess with the calculated value and go back to step 2 until convergence is achieved.

8. After convergence of the ω fixed-point procedure, calculate the rate of economic growth given in (35), and store the results for the given values of the trade and FDI costs.

9. Repeat the entire procedure for new values of trade and FDI costs.