Seller-buyer Bargaining Explained by Fixed Bargaining Costs, Risk Preferences and Value Discovery

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DANIEL	Is he here?
AL ROSE	No, he'd like you to come visit with him.
DANIEL	He's boosting his price.
AL ROSE	He said he'd like to speak with whoever is doing the buying.

— Paul Thomas Anderson, *There Will Be Blood*

This paper solves for equilibria of bargaining games with a seller and a buyer where there is no discounting between periods but players pay fixed bargaining costs for each period they bargain. In this setting, for the seller to cut prices gradually and effectively, the buyer needs to be risk averse. If players are not allowed to terminate bargaining in a finite game, the seller will raise the equilibrium prices. Allowing players to terminate bargaining causes the players to never make a deal with each other. Allowing the buyer to discover the value of the good along with bargaining termination enables the buyer to stop the seller from offering a high price and the seller to engage in price skimming by gradually lowering the price in equilibrium.

JEL classification: C78; C73; D82

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1 Introduction

This paper is on seller-buyer bargaining games with discrete time periods and incomplete information about the buyer. Only the seller makes the offers in the models and the seller may not

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know what price the buyer is willing to pay. The seller offers the bargaining price and the buyer has the option to accept it or reject it. Rejecting it might mean that the buyer can bargain in the future for a better offer from the seller.

The four key factors that are involved in the game are fixed bargaining costs, buyer's risk preference, bargaining termination and value discovery. In the literature on bargaining, a key issue is in providing the two players with incentives to come to terms in a timely manner. If the parties do not have incentives to make a deal quickly, bargaining may take arbitrarily long. To provide this incentive, this paper uses fixed bargaining costs. Existing literature usually uses discounting for this purpose.[1] However, I exclude discounting and model phenomena for which fixed bargaining cost are more realistic. Fixed bargaining costs mean that for every period a party bargains, that party incurs a fixed cost for that period. I solve for the weak Perfect Bayesian Equilibriums (wPBEs) and Perfect Bayesian Equilibriums (PBEs).[2]

For deals that take years such as the AB InBev and Modelo merger where it took AB InBev almost five years to buy Modelo, discounting may be the right tool.[3] However, for negotiations that take seconds or minutes in total, fixed bargaining costs are a better tool. When people are bargaining for 10 minutes over the price of a painting, the value of the painting or the money from the sale will not change by getting it 10 minutes later unless you need the painting urgently. Instead, what would matter is the opportunity cost of that 10 minutes. That 10 minutes could be spent on productive or enjoyable activities. Such costs are better represented by fixed bargaining costs.

An even stronger case for fixed bargaining costs instead of discounting is in bargaining where the time that the deal is made does not affect when the deal takes effect. One could be bargaining for a purchase of a sofa that can only be delivered months later. A firm and the labor union could be bargaining on October over a labor contract that will take place on January. In such cases, even a few days' delay in reaching a deal will not change when the parties will get the utility from the deal. Instead, a party may consider the cost of arranging a meeting, transportation costs and opportunity cost of time spent in the furtherance of bargaining.

During bargaining, a seller often engages in price skimming. According to Chang and Lee (2022, p. 1), price skimming refers to price discrimination where the seller first offers a high price and gradually lowers the price as time goes on. In employing such a strategy when the buyer's value is unknown to the seller, the seller's goal is to get the buyers who value the good more to buy at a higher price and to get the buyers who value the good less to buy at a lower price. With discounting, a buyer with a higher value for the good will see a greater utility loss from the delay and has a greater incentive to make an early deal. Therefore, for some series of prices, the buyer with high value will buy early at a high price and the buyer with low value will buy late at a low price.

However, when the buyer has risk-neutral or risk-seeking preferences and fixed bargaining costs but no discounting, a buyer with a higher value for the good no longer has a greater incentive for a quicker deal. This means that the seller loses the ability to make price skimming reliably work as intended. However, when a buyer has decreasing marginal utility because of

1. See Fudenberg and Tirole (1983), Cramton (1984), and Sobel and Takahashi (1983)

2. wPBEs are defined in Mas-Colell, Whinston, and Green (1995, p. 283-285) and PBEs are defined in Fudenberg and Tirole (2005, p. 331-333). Unlike the wPBE, the PBE also restricts beliefs of the equilibrium path. Unlike Mas-Colell, Whinston, and Green (1995, p. 283-285), in the PBEs of this paper, each player has a belief just at the information sets where the player acts. Also, except for the initial belief, the player's belief is updated from the last information set in which the player acted based on the new information.

3. See DePamphilis (2015, p. 50–51), Sorkin (2008) and Sanburn (2012).

risk-averse utility, a buyer with a higher value for the good sees greater utility. Thus, the buyer with a higher value has a smaller utility gain from a lower price and hence a smaller incentive to wait for a lower price. For fixed bargaining costs, reliable and effective price skimming is only possible when the buyer is risk averse.

Schweighofer-Kodritsch (2022) and Kambe (2025a, 2025b) are theoretical papers on bargaining games with fixed bargaining costs and alternating offers. Schweighofer-Kodritsch (2022) demonstrates that in equilibrium, bargaining can become a trap where a player get a worse outcome compared to that of no deal. Allowing players to terminate bargaining removes the trap. Kambe (2025a) shows that players use take-it-or-leave-it strategies in the unique equilibrium in the game with bargaining termination. Kambe (2025b) finds that the option to terminate improves the outcome for player with the higher cost when this player makes an offer. Shaked (1994) and Ponsati and Sákovics (1998) find a continuum of equilibria for bargaining games with discounting and bargaining termination. This is because variations in when a player terminates bargaining can lead to different equilibria. Experiments with bilateral offers have shown that when people are not given the option to terminate bargaining, a winning strategy is to make an opening offer favorable to its proposer (this is the seller in this paper).[4]

To see whether these results are true in my games with fixed bargaining costs where only the seller makes the offer, I compare bargaining outcomes for models where players are not allowed to terminate bargaining (the model for sections 3 and 4) or are allowed (the models for sections 5 and 6). I explain how for finite period games, the prohibition of bargaining termination lets the seller make high offers and get a greater payoff in wPBEs. However, in section 5, such wPBEs disappear because the buyer terminates bargaining when offered a high price from the seller. In section 6, I extend the models of the previous sections so that the buyer has a chance to learn the value of the good through bargaining. In this section's PBEs, the buyer can induce a low price from the seller using the buyer's ability to terminate bargaining.

Sections 3 and 4 show that a model with fixed bargaining costs that does not let the players terminate bargaining is problematic regardless of the buyer's risk preference. Fixed bargaining costs without discounting mean that players' bargaining cost can increase boundlessly if a deal continues to be not made. In a game with finite periods, this amplifies the seller's first-mover advantage. This is similar to how bargaining can become a trap in Schweighofer-Kodritsch (2022). However, in a game with infinite periods, the buyer can punish the seller for infinite periods if the seller keeps insisting on a price higher than the equilibrium price. These phenomena mean that some equilibria have extreme prices for both finite and infinite period games without discounting.

In reality, bargaining costs cannot be increased without limit because if total bargaining costs are too high, players would terminate bargaining. To account for this, in section 5, I give both players the ability to terminate bargaining. This leads to a new equilibrium. When the buyer is able to terminate bargaining but doesn't, the seller finds out that the buyer's value for the good is high. This gives the seller an incentive to raise the price. The seller's unyielding attempt to raise the price leads the players to terminate bargaining without a single offer made. Therefore, unlike Schweighofer-Kodritsch (2022) and Kambe (2025a, 2025b) none of which allow bargaining termination without a single offer made, having bargaining termination in this section does not get any player a better transaction nor any transaction.

In section 6, I add a disincentive for the seller to raise prices by modeling value discovery. The buyer can find out the value of the good through bargaining that involves asking questions about the good and getting answers. In bargaining, lower values may be harder to discover for

4. See Chertkoff and Conley (1967), Liebert et al. (1968), Galinsky and Mussweiler (2001), and Yukl (1974).

the buyer because the seller would want to hide information on lower values. If the buyer finds the value to be low, the buyer may terminate bargaining without buying the good. This possibility provides the disincentive to keep the seller from raising the price and allows bargaining over multiple periods and price skimming in PBEs to exist. Similar to Shaked (1994) and Ponsati and Sákovics (1998), a continuum of PBEs may exist due to differences in which situations have a player terminate bargaining.

2 Literature Review

This paper is related to four groups of papers. The first group has papers on bargaining with fixed bargaining costs or bargaining termination. Rubinstein (1982) solves for equilibria of the bargaining games where, unlike my paper, bargainers know each other's preference. These games include games with fixed bargaining costs and games with discounting. For the games with fixed bargaining costs, Rubinstein (1982) finds that the player with lower fixed bargaining cost gets a dominating share of the pie. In the introduction, I discussed Schweighofer-Kodritsch (2022) and Kambe (2025a, 2025b) which are on bargaining with fixed bargaining costs and alternating offers and Shaked (1994) and Ponsati and Sákovics (1998) which are on bargaining with discounting and bargaining termination.

Porter and Rosenthal (1989) solves for bargaining under the split the difference mechanism with fixed bargaining costs and finds problematic equilibria. However, unlike my paper's extensions, Porter and Rosenthal (1989) does not allow for bargaining termination. Perry (1986) find that for its bargaining game with bilateral offers and bargaining termination, with infinite periods, there is at most one offer made. While this paper is the paper similar to my paper in this group, unlike my paper, Perry (1986) solves for the case where the bargaining cost is only paid by a bargainer when this bargainer makes an offer. Cramton (1991) adds discounting to Perry (1986)'s model and finds that the equilibria can be similar to that of the discounting model. Karagözoglu and Rachmilevitch (2017) finds a wide range of equilibria for bargaining with fixed participation costs. However, participation costs are different from my bargaining costs as bargainers who do not the participation cost do not immediately drop out of bargaining.

The second group has papers that deal with bargaining using risk preferences. Roth (1985) showed that for Rubinstein (1982)'s model with discounting, a risk-averse bargainer has an disadvantage. Dickinson (2003) shows that when bargaining payoffs in case of a dispute are represented by a lottery, risk aversion causes the bargainer to get less in negotiations but risk love makes negotiation failure more likely. In contrast to these papers, Volij and Winter (2002) shows that under non-expected utility preferences, risk aversion may be beneficial for bargaining. Osborne (1985) shows that in bargaining an individual may want to take an action that identifies him as more risk averse.

The third group is for papers that have bargaining with incomplete information. Rubinstein (1985) and Harris (1985) have models where one player's discount factor is private information and random. Harris (1985) also has a model where one player's bargaining cost is private and random. Rubinstein (1985) finds that the belief about this private information has a clear connection to the equilibrium. Harris (1985) finds pooling equilibrium and separating equilibrium depending on the distribution of the discount factor or the distribution of the fixed bargaining costs. In Chang and Lee (2022)'s bargaining, both the buyer's valuation and the outside option are private information. Chang and Lee (2022) shows how the outside option is related to the seller's profit. Excluding Harris (1985), papers in the second and third group are not on bargaining costs. Harris (1985) has random bargaining costs and fixed value of the good. I have fixed bargaining costs but the value of the good may be random.

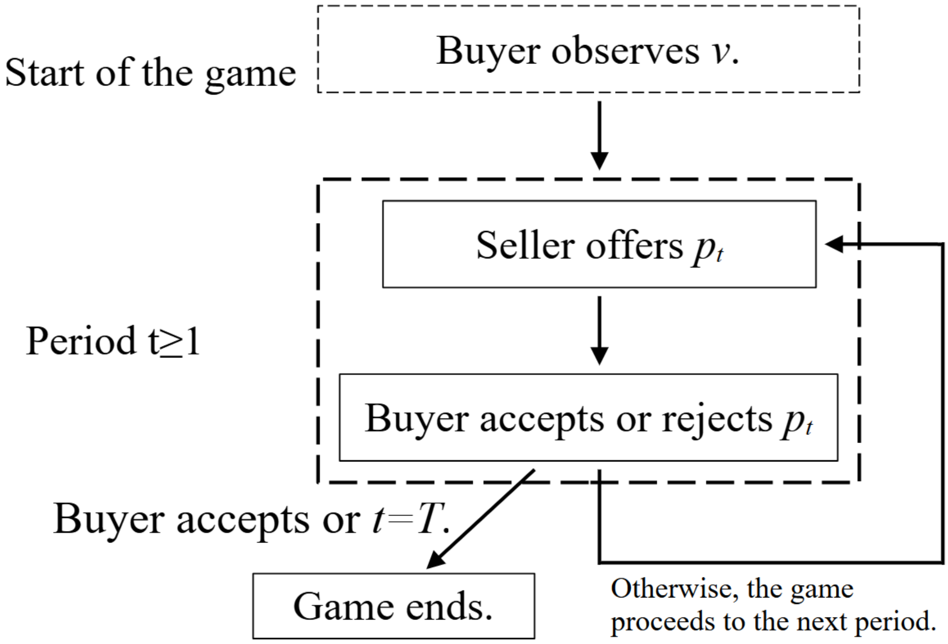

Figure 1: Game Tree for the Basic Model

The final group has to do with papers on the Coase Conjecture. Coase (1972) presents the Coase Conjecture that when a monopolist sells a durable good to patient consumers, the monopolist will sell at the market price. However, Groseclose (2024) shows that when the monopolist's discount rate approaches 1 and the consumers' does not, the Coase conjecture fails. Zhang and Chiang (2020) states price skimming can be optimal for a durable goods seller in the presence of consumers' reference price effects which mean that purchase decisions are based on the reference price formed by past prices. In Ausubel and Deneckere (1989)'s model of a durable goods monopolist, when the time between offers is close to 0, punishments for deviation become effective and a Folk Theorem establishes that seller's payoff can take on a wide range of values. This is contrary to the Coase Conjecture. My buyers are different from those in the Coase Conjecture because my buyers have to pay fixed bargaining costs for delaying purchase and are not infinitely patient.

3 Basic Model

3.1 Specification

In the basic model, there is a seller (he) and a buyer (she). Seller's production cost of the good is γ and public information. Buyer's value of the good is v. This is a random variable and buyer's private information. The image of v is $V \in R^1$. V is bounded. At the start of the game, nature decides v.

The game has $T \geq 1$ discrete periods, starting with period 1. T can be finite or infinite. When the game proceeds to period t, the seller first offers a price, $p_t \in R^1$ to the buyer. If the buyer accepts the price. the sale is made and the game ends. If the buyer rejects the price, the game continues to the next period unless $t = T$ in which case, the game ends. The game tree is in figure 1.

If the game ends with a sale, let τ be the period when the sale is made. For each period in which the players bargained, the seller payed a fixed bargaining cost of $c_S > 0$ and the buyer payed a fixed bargaining cost of $c_B > 0$.

If the sale is made, seller's payoff from the game is

$$U_S(p_\tau, \tau) = u_S(p_\tau - \gamma) - \tau c_S.$$

where $u_S(p_\tau - \gamma)$ is the seller's utility from the sale. τc_S is his total bargaining cost accumulated over τ period of bargaining. If the game ends in period T without a sale and T is finite, seller's payoff is $-Tc_S$. If the game never ends, seller's payoff is $-\infty$.[5]

If the purchase is made, buyer's payoff from the game is

$$U_B(p_\tau, \tau) = u_B(v - p_\tau) - \tau c_B.$$

Similarly to the seller, $u_B(v - p_\tau)$ is the buyer's utility from the purchase. τc_B is her total bargaining cost. If the game ends after finite periods without a sale, buyer's payoff is $-Tc_B$. If the game never ends, buyer's payoff is $-\infty$.[6]

u_S and u_B are weakly increasing utility functions on R^1. They are concave or convex. This means they can be risk averse, risk-neutral or risk-seeking functions. By changing these functions, I can change the risk preferences of the players.

3.2 Price Skimming and the Buyer's Risk Preference

Price skimming refers to a seller's price discrimination strategy of offering lower prices as time goes on. If the seller is the one who makes the offers, price skimming decides whether bargaining can take multiple offers. If the seller does not lower prices as time goes on and the buyer knows that this is the seller's strategy, the buyer would either not buy or take the first offer. Then, the seller's first offer can be thought of as a take-it-or-leave-it offer. The buyer can ignore subsequent offers and just consider whether buying at the first offer price is optimal.

Therefore, in bargaining where the buyer and the seller bargain over different prices for some time, price skimming happens. The seller's goal in price skimming is to make buyers who value the product highly buy at a high price and buyers who value the product less buy at a low price. Whether this is possible depends on the buyer's utility function.

Suppose that the seller's strategy is to offer p_t and upon rejection, to offer $p_{t+1} < p_t$. Then in period t, the buyer can buy now or alternatively delay and buy in period $t+1$. The delayed purchase gives her a payoff of $u_B(v - p_{t+1}) - (t+1)c_S$. Buying now gives her a payoff of $u_B(v - p_t) - tc_S$.The difference is

$$u_B(v - p_{t+1}) - u_B(v - p_t) - c_S. \tag{1}$$

The benefit of delayed purchase is $u_B(v - p_{t+1}) - u_B(v - p_t)$ and the cost of delayed purchase is c_S.

If the buyer's utility function is risk neutral, Take the case of $u_B(v - p_\tau) = v - p_\tau$. Then, equation 1 becomes

$$p_t - p_{t+1} - c_S.$$

In this case, for a buyer of any type, the benefit of delayed purchase is $p_t - p_{t+1} > 0$. Because the benefit and the cost are unchanging for buyers of all types, if the benefit, $p_t - p_{t+1}$, is greater

5. This means that the seller's payoff and expected payoff can be real numbers or $-\infty$.
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than the cost, c_S, all buyers want the delayed purchase. If the cost, c_S, is greater, all sellers want to buy now. If the benefit and cost are equal, all buyers are indifferent between the two options. In other words, there is no way for the seller to reliably make buyers with high value buy at a high price and buyers with low value buy at a low price using price skimming.

The situation is worse for the seller when the buyer's utility function is strictly convex so the buyer is risk seeking. The benefit of delayed purchase, $u_B(v-p_{t+1})-u_B(v-p_t)$, is increasing in v. This is easy to see for the case where u_B is differentiable since then, the marginal utility is increasing.[7]. The benefit of the delayed purchase is greater when the buyer values the good more. This means that the buyer with a higher value for the good is more inclined to wait for a lower price. Therefore, if price skimming works to price discriminate buyers of different types, the buyers with high value buy at a low price and the buyers with low value buy at a high price. This is the opposite of what the seller wants. If this happens in reality, the seller should just announce that he will sell at the high price the buyers with low value will buy for and that he will not bargain. This way, all buyers will buy at this high price.

Price skimming can only work reliably the way the seller intends it to when the buyer's utility function is strictly concave i.e. when the buyer is risk averse. Then, the benefit of delayed purchase, $u_B(v-p_{t+1})-u_B(v-p_t)$ is decreasing in v. Again, this is easy to see when u_B is differentiable since then, the marginal utility is decreasing.[8]. The benefit of delayed purchase is now greater for the buyers who value the good less. Therefore, for decreasing prices, the seller may get the buyers with high value to buy at a high price and the buyers with low value to buy at a low price. When the seller does this, the buyer with high value for the good makes the deal early because further bargaining will have decrease her payoff.

4 Results

First, I find key characteristics of wPBEs for the basic model.

Proposition 1. *Fix u_S, u_B, c_B and c_S. If u_B is continuous, u_S is increasing and u_B and u_S are unbounded from above and below, for all t and p, there exists some T' for which if $\infty > T > T'$, $P(p_t > p) = 1$ and $E(U_S) > p$ and $E(U_B) < -p$ in any wPBE.*

Roughly speaking, the above proposition means that for a finite game with $T < \infty$, as $T \to \infty$, the price for any period in a wPBE goes to ∞, the seller's expected payoff in a wPBE goes to ∞ and the buyer's expected payoof goes to $-\infty$. The logic can be explained by a simplified example. In period T, the last period, a buyer whose willingness to pay is \$1 will accept a price of 1. Now apply backward induction. If the buyer's bargaining cost is 1, in period $T-1$, the buyer will accept a price of 2 since delaying the purchase will not increase her payoff. In period $T-2$, she will accept 3 and so on. So for any period t, the more periods left in the game, the higher the price for the current period. Furthermore, as the number of periods left goes to infinity, the price for the current period will do so as well. Roughly speaking, when prices for all periods, especially the early periods, go to $-\infty$, the buyer faces a choice of whether to wait an extremely large number of periods and incur an extremely large bargaining cost or to buy earlier for an extremely high price. Therefore, the buyer's expected payoff will go to $-\infty$.

I will provide an intuitive explanation of why this increase in prices leads to an infinitely high expected payoff for the seller. Take the following alternative strategy by the seller. If the early prices can be increased to infinity, in period 1, the seller can force the buyer to wait a

7. For the general case, this result is proven in lemma 1's (1).
8. For the general case, this result is proven in lemma 1's (1).

large number of periods till the price gets lower or buy now for a high price. This is because if the buyer concludes that he prefers to not pay the bargaining cost of waiting many periods, the period 1 offer that is better than paying the bargaining cost will be accepted. With infinitely many periods, the total bargaining cost it takes to get to a period with low price can be increased to infinity. With enough periods, the seller can get any high offer accepted in period 1. Since the seller can always choose to deviate to this strategy in a wPBE, the equilibrium strategy payoff is weakly greater.

Next, the following proposition finds pure PBEs when there are infinite periods. Note that I specify the equilibria that I find to be PBEs which is a stronger condition for equilibria than wPBEs.

Proposition 2. *If $T = \infty$ and u_B has no upper bound, for any $p \in R^1$, there exists a pure strategy PBE where the sale always happens in period 1 for $p_1 = p$.*

The above proposition states that, under mild conditions, for any price including negative prices, there is a PBE where the buyer always buys and does so in the first period. So, if the proposition holds, there are infinitely many PBEs and any price can be the PBE sale price. The reason can be explicated using a simplified example in which the buyer's bargaining cost is 1. The buyer's strategy involves cutoff prices for which the buyer accepts any offer at this price or below and rejects all offers above this price. Suppose in the first period, the cutoff price is 1. If the seller offers a price greater than 1, the buyer rejects this offer and punishes the seller by lowering the cutoff price to 0 in the second period. If the seller offers a price greater than 0 in the second period, the buyer again rejects this offer and punishes the seller by lowering the cutoff price to -1 in the third period and so on. On the other hand, the seller's strategy is to always make the offer at the current cutoff price unless the last offer was at or below that period's cutoff price in which case, the last offer is repeated.

The seller's strategy is optimal because there is no way for the seller to get the buyer to accept a price above the current cutoff price. The buyer's strategy of never accepting an offer above the cutoff price is optimal because by rejecting such offers and punishing the seller in the next period, the buyer can get the seller to succumb to punishment in the next period. Since the seller will make a better offer the next period, the buyer will be sufficiently compensated for rejecting the current offer. The buyer gets no gain from rejecting offers on or above the current cutoff price because when these offers are rejected, the seller offers the same price in the next period. Rejecting those offers will not lower the price. When the game had finite periods, there was no equilibrium like this because in the last period of the game, there was no way for the buyer to punish the seller for the seller's offer in the period.

This proposition is akin to the folk theorem. The similarity is that when there is little or no discounting and infinite periods, punishments become very effective and using punishments, a wide variety of expected payoffs can be supported in equilibria as long as the players' expected payoffs are weakly greater than their expected payoffs from continued punishment.

For the basic model, the cause of equilibria with extreme prices and extremely effective punishments is that players have no ability to terminate bargaining. In reality, for bargaining with finite periods, if a seller asked \$1000 for a pencil and truthfully stated that for the next 1000 offers he will insist on unreasonably high prices before making reasonably low offers, the buyer would just stop bargaining without buying. However, without this option, the seller has large bargaining power. This explains why making an opening offer favorable to its proposer is an effective strategy in bargaining experiments without termination. The proposer has a large bargaining power.

For bargaining with infinite periods, if the price offered by the seller is too high and rejecting the offer does not decrease the price, the buyer would also just stop bargaining without buying.

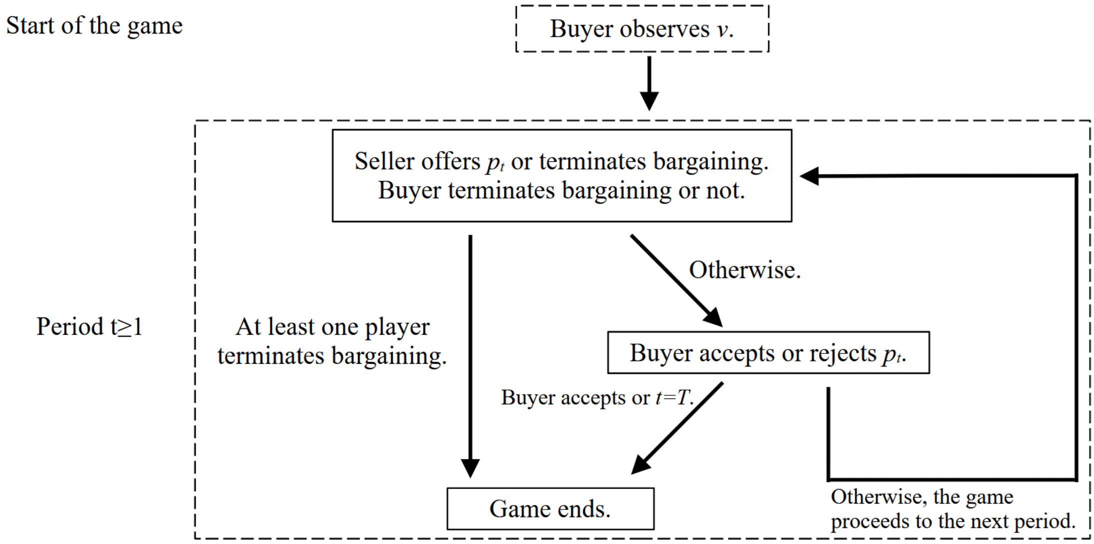

Figure 2: Game Tree for the Extended Model with Termination

For this type of bargaining, if the price that the buyer would accept is too low and every time the seller tries to make a higher offer, the buyer responds by punishing the seller, the seller would stop bargaining without a sale. Punishments are limited in their effectiveness in reality.

5 Extended Model with Termination

5.1 Specification

For this section's model, the players' decisions are different. At the beginning of a period, the seller does not just offer p_t. Instead, he either offers p_t or terminates bargaining. Simultaneously with this action by the seller, the buyer decides to terminate bargaining or not. Allowing both players to terminate bargaining in any period deals with the issue with the previous section's model. If a player does not terminate bargaining for the period, I describe this by saying that the player bargains in the period. If the player does not terminate bargaining in period 1, the description is that the player bargains.

If at least one player terminates bargaining in period t, the game ends in period t. If both players bargain in the period, the buyer sees the price and then decides whether to accept or reject it. As before, if the buyer accepts the offer or $t = T$, the game ends. Otherwise, the game continues to the next period. Figure 2 shows the game tree for this model.

The players' payoffs for when the game ends with a sale or never ends are the same as before. If the game ends without a sale, the seller's payoff is $-\tau_S c_S$ and the buyer's payoff is $-\tau_B c_B$. τ_S and τ_B are, respectively, the number of periods the buyer and the seller bargained in.

If one player terminates bargaining but the other player bargains for the period, the game still ends without a sale. However, the player who did not terminate still pays the bargaining cost for the period. In other words, if in a period, a player chooses to bargain and pays the associated costs with it but the other player quits, the player bargains in the period in vain. However, in the period, the seller cannot offer a price without bargaining and the buyer cannot see the price without bargaining. In reality, a party might not know whether the other side has quit bargaining. When a party states that it is unwilling to bargain any further, that statement might not be believed.[9] Thus, a party may attempt to bargain and pay the bargaining cost without knowing that the other side quit. For instance, if a corporate seller exerts effort to make a detailed bargaining proposal but the buyer has already quit bargaining, the seller's effort is futile. Similarly, if the buyer keeps in contact with the seller or stays at the meeting place hoping the seller will make another offer but the seller has quit, the buyer's effort is wasted.

5.2 Results

Proposition 3. *If u_S and u_B have no lower bound, u_S is increasing and u_B is continuous, in any pure strategy wPBE, players never bargain.*

To see why players never bargain, for the finite periods case, consider the following simplified example. Assume that the buyer's bargaining cost is 1. In the last period of the game or a period where both players are set to terminate bargaining in the next period, if the seller's strategy is to offer a price of 2 only buyers whose willingness to pay is 3 or greater will bargain. Others will terminate bargaining. The seller knows this. Therefore, if the buyers' willingness to pay is 3 or greater, the seller knows he can get the buyers to pay 3. So the seller has an incentive to raise the price to 3. However, if the sellers strategy is to offer a price 3, only buyers whose willingness to pay is 4 or greater will bargain and the seller can get these buyers to pay 4 and so on. This demonstrates that for any price, the seller has an incentive to raise the price. Therefore, in a wPBE, no price is optimal for the seller. Instead, the seller does not offer a price and the players do not bargain in this period.

Go to the period before. For this earlier period, players will terminate bargaining in the next period. Then, by the same logic, the players will not bargain in this earlier period either. Using backward induction, the players do not bargain in any period.

The proof for the infinite periods case can be described roughly using what prices the seller will offer. Even in a game with infinite periods, there is some lower bound for the seller's price that applies to all wPBE. This is because if the price is too low, the seller would rather terminate bargaining than offer it. Given this lower bound, in a wPBE where the players bargain, the game eventually reaches a period in which the seller will not make any meaningful price cuts. Then in this period, the buyer either bargains or terminates bargaining. If the buyer views and accepts this period's offer, that means the buyer's willingness to pay is equal to or greater than the sum of the bargaining cost and the price. Then, as in the finite periods case, the seller should raise the price right after the buyer decides to bargain for this period. This price raise makes use of the fact that the buyer cannot observe the offer without paying the bargaining cost. The seller's price before the raise is not optimal for him, which is a contradiction. Thus, in a wPBE, the players do not bargain.

In both the finite period model and the infinite period model, the result that both players do not bargain has a fundamental cause. The cause is that the seller recognizes that if the buyer does not terminate, the buyer's value is high. The seller recognizes that the buyer finds the price and the bargaining cost acceptable. Once the buyer enters bargaining for the period, the bargaining cost is sunk cost. The seller can take advantage of this by raising the price by the bargaining cost. The seller uses the bargaining cost to get the buyer to take a deal that the buyer would possibly not have taken if she knew about it from the beginning. The seller's unwavering attempt to raise the price causes the buyer to not bargain.

9. See Ma, Yang, and Savani (2019) and Chuang (2025).

6 Extended Model with Termination and Value Discovery

6.1 Specification

In this section, on top of the previous section's model with bargaining termination, I model the phenomenon where a buyer discovers the value of a good. In the new model, initially, the buyer may not know the value of the good for her when this value is relatively low. In reality, a buyer might be unsure whether the good is worth the cost. When she is unsure, she may try to obtain more information to ascertain the value of the good in bargaining. Then, bargaining would consist of more than just offering prices and would also involve asking questions about the good and getting answers. Through this process, the buyer may find that the good has certain aspects that the buyer does not like.

Obtaining such information would be arduous because the seller would hide it. This argument justifies why the relatively low values are harder to discover in the model. Also, even if the seller gives the buyer information about the good, that does not necessarily mean that the seller knows the buyer's value for the good. For instance, a seller might be reluctant to reveal why the clothing he is selling has been repaired but not know whether that matters to the buyer.

For complicated deals such as labor contract deals, the value discovery may actually involve narrowing down the details of the deal. For instance, instead of just negotiating the wage, the firm and the union may also negotiate overtime pay, sick leave, etc. As these auxiliary conditions are agreed upon through the process of negotiation, the buyer of labor, the firm, also discovers how much surplus it will see from the deal.

The first difference between this section's model and the previous section's model has to do with the buyer's value, v. For this section's model, v is no longer private information because while the seller does not observe v as before, the buyer may not observe it either. The buyer is either low or high type. The buyer is low type with probability P_L in which case, v is chosen from a uniform distribution on $[0,1]$. The buyer is high type with probability $1-P_L$ in which case, v is chosen from a uniform distribution on $[1,2]$ (The rest of this paper uses $P_L = \frac{2}{3}$). The buyer observes her type at the start of the game. The high type observes v at the start of the game but the low type does not. In period 2, if both players bargain, the buyer observes v regardless of his type. The new game tree is in figure 3.

The seller's utility function is $u_S(p_\tau - \gamma) = p_\tau - \gamma$, which is linear. The buyer's utility function is the following.

$$u_B(x) \equiv \begin{cases} \ln(x+1) & \text{if } x \geq 0 \\ -x^2+x & \text{otherwise} \end{cases}$$

This function is specifically chosen because it has nice properties that allows me to express the equilibrium prices with simple formulas. (The details on how I use this function to prove the formulas are in appendix 3.) This function has a different form for when $x < 0$ because $\lim_{x \to -1} \ln(x+1) = -\infty$. Appendix 3 proves that this function is an increasing, differentiable, strictly concave function. Recall that in subsection 3.2, I discussed why the buyer's utility had to be strictly concave for reliable and effective price skimming. If $p_\tau = v$, $u_B(p_\tau - v) = 0$. In other words, when the value of the good equals the price of purchase, the buyer's utility is 0. The game has more than 1 period. It can have infinitely many periods. p_t^* is the price that the seller offers in period t on the equilibrium path. p_t^{**} is the price that the seller offers in period t if the buyer deviated in period $t-1$ by not buying and this was the only deviation before period t.

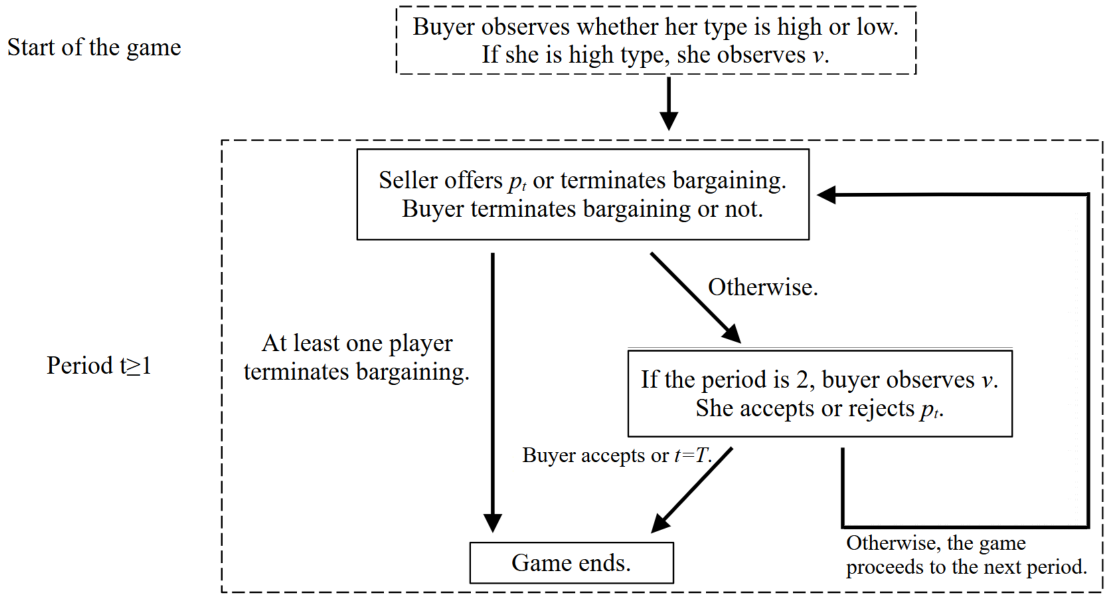

Figure 3: Game Tree for the Extended Model with Termination and Value Discovery

When the buyer's value of the good is relatively low, the buyer does not know exactly how low it is. The value could be so low that the buyer does not want to buy the good. By bargaining and talking to the seller, the buyer can extract information about the good from the seller that gives the exact value of the good. However, this process is costly. Note that if the buyer plays a pure strategy, any low type buyer who has not observed v always plays the same action in the same situation.

6.2 Results

The equilibria of the model for this section can differ in the maximum number of periods bargaining takes. Furthermore, as will be shown below, even for the same set of parameters, there can be multiple PBEs with different prices. Therefore, in this subsection, instead of solving for every PBE of the model, I present examples 1 and 2 which have representative PBEs. Example 1 demonstrates price skimming and how the buyer stops the seller from raising the prices with her ability terminate bargaining. Example 2 demonstrates how the seller can lower the period 1 price to have the buyer buy in period 1 and prevent the low type buyer from not buying after discovering her low value.

Definition 1.
The following are two settings.

1. $c_B = 0.01$ *and* $p_1 \in [0.7, 0.71]$.
2. $c_B \in [0.01, 0.02]$ *and* $p_1 = 0.7$.

The following example looks at PBEs with price skimming under settings 1 and 2.

Example 1. *In both of the settings 1 and 2, if* $\gamma = 0.01$ *and* $c_S = 0.005$, *there exists a pure strategy PBE where the following holds.*

(1) $p_1^* > p_2^* = \frac{e^{c_B} p_1^* + 2\gamma(e^{c_B} - 1)}{4e^{c_B} - 3}$

(2) $2 < \frac{e^{cB} p_1^* - p_2^*}{e^{cB} - 1} < 3$

(3) High type buys in period 1 *if* $v \geq \frac{e^{cB} p_1^* - p_2^*}{e^{cB} - 1} - 1$ *and buys in period* 2 *if* $v < \frac{e^{cB} p_1^* - p_2^*}{e^{cB} - 1} - 1$.

(4) Low type bargains in period 2.

(5) Probability that low type buys in period 2 *is positive.*

(6) If period 3 *exists, the buyer's strategy in period* 3 *is to terminate bargaining.*

In the above example, (1) means that the equilibrium price decreases in period 2. The high type buys in period 1 if $v \geq \frac{e^{cB} p_1^* - p_2^*}{e^{cB} - 1} - 1$ and buys in period 2 if $v < \frac{e^{cB} p_1^* - p_2^*}{e^{cB} - 1} - 1$. The low type buys in period 2 or does not buy. This means that the seller engages in price skimming advantageously by lowering the price in the second period and getting the buyers with high value buy early and the buyers with low value buy late. (4) and (5) mean that the low type buyer discovers her value of the good through bargaining in period 2 and based on that, may decide to buy. In 6, I limit bargaining to two periods on the equilibrium path for simplicity.

(1) indicates that once p_1^*, the period 1 price on the equilibrium path, is given, p_2^* is known from it. However, as will be explained in detail, the value of p_1^* can vary for different PBEs even for the same parameters. Figure 4 depicts PBEs from example 1 and setting 1.[10] For the parameters here, for any $p_1 \in [0.7, 0.71]$, there exists some PBE where $p_1^* > p_2^* = \frac{e^{cB} p_1^* + 2\gamma(e^{cB} - 1)}{4e^{cB} - 3}$.

The high type buyer's strategy is optimal because of her risk aversion and value. In period 1, she knows that the price will be lower in the next period. However, whether she will wait for the lower price depends on her value. Since she is risk averse, if her value is high, her utility from purchase is already high and she sees little benefit from waiting for a lower price. Therefore, she buys in period 1. She only buys in period 2 when her value and her utility from period 1 purchase are low.

If a low type buyer buys in period 1, she does so without knowing the value of the good. This is because it takes two periods of bargaining for a low type buyer to learn her exact value of the good. Buying in period 1 means that she might get negative utility from the good if her value is actually low. Therefore, she does not want to purchase in period 1. In period 2, after she decides to bargain, she sees her value of the good. Then, she buys the good if the value is weakly greater than the price. She never buys if the value is less than the price.

In period 2, the seller does not want to increase the price further because increasing the price means losing more buyers. The low type buyer might not buy in period 2 if after bargaining in the period, she discovers that her value is unexpectedly low and her utility is negative. The seller finds the optimal price by weighing the risk of losing the buyer with low value against the benefit of a higher price. This is different from section 5's model where the seller had no cost of raising the price by a small amount from the expected price both in the last period and the period after which both players terminate bargaining. The difference explains why players bargain in the PBEs of example 1.

The seller finds it optimal to have a higher price in period 1. This is because in period 1, the high type is the only type that might buy. In period 2, the low type might buy as well and the low type requires a lower price for purchase. As figure 4's first graph shows, if p_1^* is greater, the high type buyer is more likely to delay purchase. Then, in period 2, the seller expects the buyer to have a higher value of the good. Therefore, as p_1^* increases, p_2^* increases. This is shown in figure 4's second graph.

10. Lemma 8 in appendix 3 proves the characteristics of these PBEs.

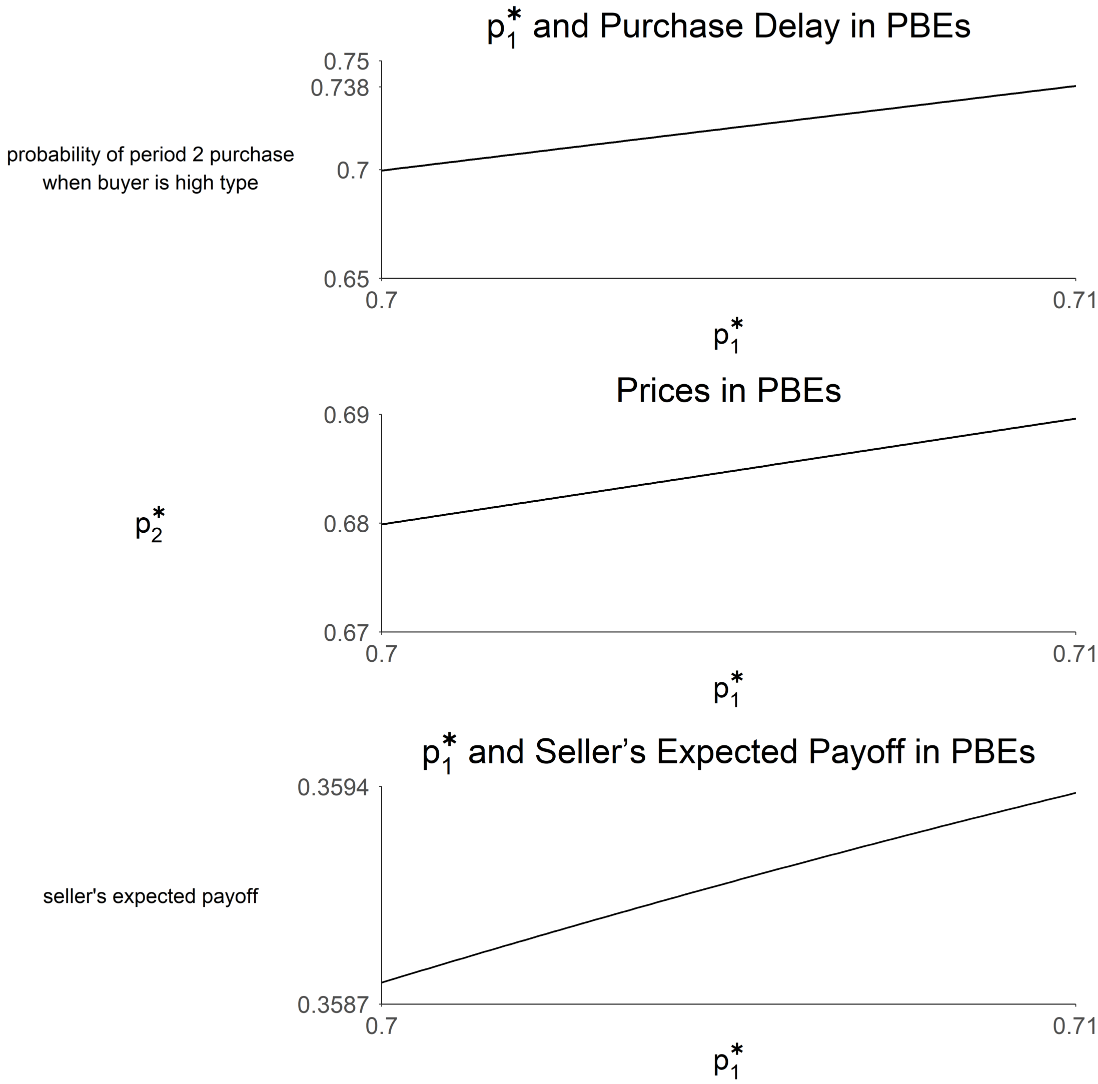

Figure 4: p_1^* and PBEs

Figure 4's third graph shows that the seller's payoff is higher when $p_1^* = 0.71$ compared to when $p_1^* = 0.7$. This raises the question of why he does not always offer $p_1 = 0.71$. The reason is that in the different PBEs with different values of $p_1^* \in [0.7, 0.71]$, the buyer's strategy is different. In the PBE with $p_1^* = 0.7$, if the seller raises the price to $p_1 > 0.7$, the buyer interprets this deviation to mean that the seller will terminate bargaining in the next period and the low type buyer terminates in the next period. In the PBE with $p_1^* = 0.71$, the same thing happens for $p_1 > 0.71$. In other words, for the PBEs with $p_1^* = 0.7$ and $p_1^* = 0.71$, there is a difference in how high p_1 has to be for the low type buyer to terminate bargaining. This demonstrates that the buyer's ability and willingness to terminate bargaining gives her bargaining power. If the seller knows that a too high p_1 will drive away the buyer, the seller does not offer such a high p_1. Low type buyer's strategy to give up on bargaining when p_1 is too high is optimal if the seller will do so as well.

Figure 5 shows PBEs from example 1 and setting 1. As the buyer's bargaining cost, c_B, increases, the high type buyer becomes more likely to buy in period 2 instead of period 1. The

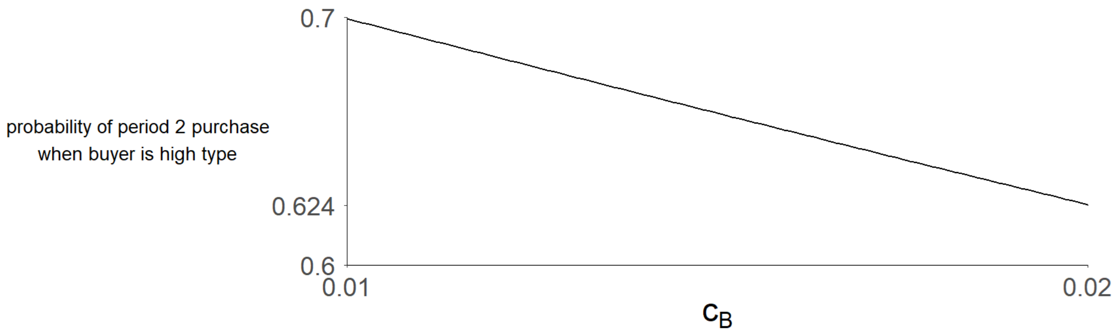

Figure 5: Buyer's Cost of Bargaining and Purchase Delay in PBEs

low type always bargains in period 2.[11]

Now, I will show an example PBE where there is no price skimming.

Example 2.

$$\gamma = 0$$

$$c_S = 0.25$$

$$c_B = 0.01$$

In this setting, there exists a PBE where $p_1^* \approx 0.35$*,* $p_2^{**} = 0.5$ *and the buyer always buys in period* 1*.* p_1^* *is the period* 1 *price on the equilibrium path.* p_2^{**} *is the price the seller offers in period* 2 *if the buyer deviated by rejecting* p_1^* *and there was no other deviation in period* 1*.*

For the PBE above, the sale is always made and it is made in period 1. The seller knows that if he bargains with the low type buyer in period 2, the low type buyer might find out that her value for the good is near 0. Then, she might decide to never buy or ask for a low price. Given the seller's high bargaining cost, he chooses to avoid this by offering a low price in period 1. This is how he has the sale always happen in period 1. Note that in the PBE of this example, the buyer never actually discovers her value because the bargaining ends before that. However, unlike section 5's model, the example's PBE still has the players bargain. This proves that the probability of the value discovery does not have to be positive for a PBE where players bargain. The potential for the discovery can be enough. This is because the potential may provide enough incentive for the seller to not raise the price.

6.3 Effective Bargaining Strategies

In this section's model, the buyer's ability to keep the seller from raising the price can come from two possibilities. The first is that if the price is too high, the buyer might think that no acceptable deal is possible and terminate bargaining. The second is that through bargaining, the buyer might find out that her value is low and reject the seller's offer.

To raise the first possibility, the buyer should create the impression that if the offer is too high, she will think that there is no hope of getting an acceptable deal and she will never buy the good. For instance, the buyer could state that she is "just looking around" and ready to leave if she cannot get a low price. To raise the second possibility, the buyer should be inquisitive.

11. Lemma 8 in appendix 3 proves the characteristics of these PBEs.

She should find out as much as she can about the good through bargaining and then stress that because of the downsides of the good that she found, she cannot buy at a high price. However, this strategy should not be employed to the extent that the seller gives up trying to sell the good.

The seller can counter these strategies to increase his expected payoff. This happens in the equilibria shown in section 6. To counter the first strategy, in making offers, the seller should not just consider the possibility that the buyer will reject the offer and the seller has to make a new offer. He should also refrain from making offers that cause the buyer to give up on bargaining and never buy. A common strategy for negotiations is to make an opening offer extremely favorable for the person making the offer.[12] Some experiments have found that in bargaining, an opening offer favorable to the proposer leads to a better deal for the proposer.[13] However, in many of these studies, subjects were not given the option to terminate the bargaining and the experiment. Even if subjects were given the option to terminate, they may not have wanted to because they were interested in the experiment or they believed it was not what the experimenters wanted. In reality, an extreme opening offer may lead the other side to believe that there is no chance of a good deal and terminate bargaining.

To counter the second strategy, the seller can use the following strategy. If the probability that the buyer will find out something she doesn't like about the good through bargaining and reject the buyer's offer is too high, the seller can reduce this possibility and bargaining costs with a limited-time discount. In other words, in such situations, the seller should say "If you buy this now, I will give you a discount." This is consistent with the seller's low equilibrium price in example 2. This is similar to how, in reality, limited-time discounts such as Black Friday deals may encourage consumers to buy on impulse without carefully considering the benefits and the costs of a good.

7 Conclusion

This paper explains bargaining with incomplete information about the buyer when there is no discounting but fixed bargaining costs exist. For such bargaining games, the seller can only advantageously and reliably employ price skimming when the buyer is risk averse. The paper has three models for such games. For the basic model with finite periods, as the number of periods goes to infinity, in wPBEs, prices and the seller's expected payoff go to infinity as well and the buyer's expected payoff goes to negative infinity. This explains why bargaining experiments without bargaining termination find that an opening offer favorable to its proposer is advantageous. If the game has infinite periods, for any price, there exists a PBE with that as the sale price.

For the extended model with termination, in the pure strategy wPBE, players terminate bargaining as soon as possible without a deal. In the extended model with termination and value discovery, I find PBEs with price skimming and a PBE without it. In the PBEs with price skimming, the first price can differ depending on what price causes the low type buyer to not bargain in the future. The seller does not offer a price so high that this happens. In the PBE without price skimming, the seller offers a low price in the first period so that the sale is always made and it is made in the first period. This strategy works to preclude the possibility of the buyer discovering the exact value of the good and consequently not buying the good.

12. See Volkema (1999).

13. See Chertkoff and Conley (1967), Liebert et al. (1968), Yukl (1974), and Galinsky and Mussweiler (2001). Galinsky and Mussweiler (2001) states that "the first offer, once made, appears to function as an anchor toward which final agreements are assimilated."

Appendix 1. Lemmas and proofs used in section 4

Lemma 1 is used to prove lemma 2 and propositions 1 and 3. Lemma 2 is used to prove proposition 1.

Lemma 1. *Suppose $\psi' > \psi$ and $p' > p$.*

(1) If u_B is strictly concave (strictly convex), $u_B(\psi' - p) - u_B(\psi' - p') < (>) u_B(\psi - p) - u_B(\psi - p')$.

(2) If u_B is concave (convex), $u_B(\psi' - p) - u_B(\psi' - p') \leq (\geq) u_B(\psi - p) - u_B(\psi - p')$.

Proof. Let $x = \psi - p'$, $y = \psi' - p$ and $z = p' - p$.

$$y - x = \psi' - \psi + p' - p > z = p' - p > 0$$

$y - x > z > 0$ means that if u_B is (strictly) concave,

$$\frac{y-x-z}{y-x} u_B(x) + \frac{z}{y-x} u_B(y) \leq (<) u_B(x\frac{y-x-z}{y-x} + y\frac{z}{y-x}) = u_B(x+z)$$

and

$$\frac{z}{y-x} u_B(x) + \frac{y-x-z}{y-x} u_B(y) \leq (<) u_B(x\frac{z}{y-x} + y\frac{y-x-z}{y-x}) = u_B(y-z).$$

Therefore, $u_B(x) + u_B(y) \leq (<) u_B(y-z) + u_B(x+z)$. Similarly, if u_B is (strictly) convex, $u_B(x) + u_B(y) \geq (>) u_B(y-z) + u_B(x+z)$.

$$u_B(y-z) + u_B(x+z) \gtreqqless u_B(x) + u_B(y)$$
$$\leftrightarrow$$
$$u_B(\psi' - p') + u_B(\psi - p) \gtreqqless u_B(\psi - p') + u_B(\psi' - p)$$
$$\leftrightarrow$$
$$u_B(\psi - p) - u_B(\psi - p') \gtreqqless u_B(\psi' - p) - u_B(\psi' - p')$$

■

Definition 2.
$\underline{v} \equiv \inf V$
$\bar{v} \equiv \sup V$
$\bar{V}$ is the closure of V.

Lemma 2. *If $t < t'$, u_B is continuous and $\forall v' \in \bar{V}, u_B(v' - p_t) - tc_B > u_B(v' - p) - t'c_B$, $\exists \varepsilon > 0, \forall v' \in \bar{V}, u_B(v' - p_t - \varepsilon) - tc_B > u_B(v' - p) - t'c_B$.*

Proof.

$$0 > tc_B - t'c_B$$

If $p_t < p$,

$$\forall v' \in \bar{V}, u_B(v' - p_t) - u_B(v' - p) > 0$$

If $p_t = p$,

$$\forall v' \in \bar{V}, u_B(v' - p_t) - u_B(v' - p) = 0$$

The $p_t < p$ case is proven. Suppose $p_t \geq p$. If u_B is concave, $\check{v} = \bar{v}$. If u_B is convex, $\check{v} = \underline{v}$. By lemma 1's (2), when $\varepsilon > 0$,

$$\forall v' \in \bar{V}, u_B(\check{v} - p) - u_B(\check{v} - p_t - \varepsilon) \leq u_B(v' - p) - u_B(v' - p_t - \varepsilon)$$

and

$$\forall v' \in \bar{V}, u_B(\check{v} - p_t - \varepsilon) - u_B(\check{v} - p) \geq u_B(v' - p_t - \varepsilon) - u_B(v' - p).$$

$$u_B(\check{v} - p_t) - tc_B > u_B(\check{v} - p) - t'c_B$$
$$\leftrightarrow$$
$$u_B(\check{v} - p_t) - u_B(\check{v} - p) > (t - t')c_B$$
$$\leftrightarrow$$
$$\exists \varepsilon > 0, u_B(\check{v} - p_t - \varepsilon) - u_B(\check{v} - p) > (t - t')c_B$$

For this ε,

$$\forall v' \in \bar{V}, u_B(v' - p_t - \varepsilon) - u_B(v' - p) > (t - t')c_B$$

■

Proof of Proposition 1.

The first part of this proof establishes that for any t, p and history of the game before period t, there exists some T' for which if $T > T'$, $P(p_t > p) = 1$. In period T, if $u_B(\underline{v} - p_T) > 0$ and $u_S(p_T - \gamma) < 0$, any buyer buys and seller prefers to offer p'_T for which $u_S(p'_T - \gamma) \geq 0$.

$S_L \equiv \{\underline{p} | P(p_t > \underline{p}) = 1$ for any history before period t and any wPBE where t is the last period.$\}$

The above argument establishes that $S_L \neq \emptyset$. Let $p_L \in S_L$.

Assume a wPBE.

$$S_T \equiv \{\underline{p} | P(p_T > \underline{p}) = 1 \text{ for any history before period } T\}$$

Notice that $S_L \subset S_T$. Therefore, I can set $\underline{p}_T \in S_T$ such that $\underline{p}_T \geq p_L$.

Say v can be any element of $\bar{V}$. Pick $t < T$. Suppose that for all $t' \in (t, T]$, $P(\underline{p}_t \geq \check{p}) = 1$. If, by the seller's action, $\forall v \in \bar{V}$ and $t' \in (t, T], u_B(v - p_t) - u_B(v - \underline{p}_{t'}) > (t - t')c_B$,

$$\forall v \in \bar{V}, u_B(v - p_t) - tc_B > u_B(v - \underline{p}_{t'}) - t'c_B.$$

Then, by lemma 2, seller's action is suboptimal because there exists some $\varepsilon > 0$ by which seller can raise p_t and have the buyer buy in period t. Therefore, if p_t is optimal, for some $t' \in (t, T]$,

$$\exists v \in \bar{V}, u_B(v - p_t) - u_B(v - \underline{p}_{t'}) \leq (t - t')c_B.$$

$$S^* \equiv \{p_t | \exists v \in \bar{V}, u_B(v - p_t)) \leq u_B(v - \underline{p}_{t'} + (t - t')c_B\}$$

There exists some $\check{p}$ such that if $p_t < \check{p}$, $p_t \notin S^*$ and if $p_t > \check{p}$, $p_t \in S^*$. $P(p_t \geq \check{p}) = 1$. By the continuity of u_B,

$$\exists v \in \bar{V}, u_B(v - \check{p}) - u_B(v - \underline{p}_{t'}) \leq (t - t')c_B.$$

Set $\underline{p}_t = \check{p}$. This way I can set $\underline{p}_t$ for any period $t \leq T$.

For any period $t < T$,

$$\exists v \in \bar{V}, u_B(v - \underline{p}_t) - u_B(v - \underline{p}_{t'}) \leq (t - t')c_B$$

and $\underline{p}_t > \underline{p}_{t'}$.

I will use proof by induction. Suppose u_B is concave. $\check{v}$ is a function whose codomain is $\bar{V}$.

$$\exists \check{v}_{(T,T)} \in \bar{V}, \forall v \leq \check{v}_{(T,T)}, u_B(v - \underline{p}_T) - u_B(v - \underline{p}_T) \leq (T - T)c_B$$

Now I will prove for $T'' < T$. Suppose that for all $T' \in (T'', T]$,

$$\exists \check{v}_{(T',T)} \in \bar{V}, \forall v \leq \check{v}_{(T',T)}, u_B(v - \underline{p}_{T'}) - u_B(v - \underline{p}_T) \leq (T' - T)c_B.$$

For some $T' \in (T'', T]$,

$$\exists v \in \bar{V}, u_B(v - \underline{p}_{T''}) - u_B(v - \underline{p}_{T'}) \leq (T'' - T')c_B.$$

By lemma 1's (2),

$$\exists \check{v}_{(T'',T')} \in \bar{V}, \forall v \leq \check{v}_{(T'',T')}, u_B(v - \underline{p}_{T''}) - u_B(v - \underline{p}_{T'}) \leq (T'' - T')c_B.$$

Therefore,

$$\exists \check{v}_{(T'',T)} \in \bar{V}, \forall v \leq \check{v}_{(T'',T)}, u_B(v - \underline{p}_{T''}) - u_B(v - \underline{p}_T) \leq (T'' - T)c_B. \tag{2}$$

Suppose u_B is convex, by a similar logic, for $T'' < T$,

$$\exists \check{v}_{(T'',T)} \in \bar{V}, \forall v \geq \check{v}_{(T'',T)}, u_B(v - \underline{p}_{T''}) - u_B(v - \underline{p}_T) \leq (T'' - T)c_B. \tag{3}$$

Since $\bar{V}$ is bounded and u_B is monotonic and has bounded variation on any closed and bounded interval in R^1, the first part of this proof that I stated in the beginning of the proof is proven.[14]

For the second part, pick an arbitrary p'_1. Formulas 2 and 3 prove that $\underline{p}_t$ is decreasing in t. Recall that $\underline{p}_T$ is bounded from below. Since V is bounded, $u_B(v - \underline{p}_T)$ is bounded from above and $u_B(v - p'_1) - c_B$ is bounded from below. There exists some $\bar{t}$ for which if $t \in (\bar{t}, T]$,

$$\forall v \in V, u_B(v - \underline{p}_t) - tc_B < u_B(v - p'_1) - c_B$$

If seller deviates to p'_1 and the game has $\bar{t} + 1$ or more periods, buyer prefers to buy in period 1 compared to buying in period $t > \bar{t}$.

For a sufficiently large T, $p'_1 < \underline{p}_t$ with any $t \leq \bar{t}$. Furthermore, $-Tc_B < u_B(v - p'_1) - c_B$. Therefore, if seller deviates to p'_1 and T is sufficiently large, buyer buys in period 1 with probability 1. In this case, the seller gets a payoff of $u_S(p'_1 - \gamma) - c_S$. Since the seller gets this payoff from the deviation, the expected payoff from the equilibrium strategy is weakly greater.

Pick an arbitrary p, Given p_L, there exists some period $\underline{t} \geq 1$ such that in any wPBE, if $t < \underline{t}$, $u_B(\underline{v} - p_L) - tc_B < -p$. Pick p' such that $u_B(\underline{v} - p') < -p$. For a sufficiently large T, in any wPBE, if $t \leq \underline{t}$, $P(p_t > p') = 1$. Thus, for a sufficiently large T, $E(U_B) < -p$ in any PBE. ■

Proof of Proposition 2.

14. See Apostol (1985, p. 94,128).

Pick $p \in R^1$. Since u_B is weakly increasing, u_B is bounded on $[\underline{v} - p, \bar{v} - p]$. There exists some $\bar{u}$ for which $\forall v \in [\underline{v}, \bar{v}]$ and $v' \in [\underline{v}, \bar{v}], u_B(v-p) - u_B(v'-p) \leq \bar{u}$. There exists some $p' \in R^1$ such that $c_B + \bar{u} \leq u_B(\underline{v} - p') - u_B(\bar{v} - p)$.

$$\forall v \in V, c_B + \bar{u} \leq u_B(v - p') - u_B(\bar{v} - p) \tag{4}$$

$$\forall v \in V, -\bar{u} \leq u_B(\bar{v} - p) - u_B(v - p) \tag{5}$$

Combine formulas 4 and 5.

$$\forall v \in V, c_B \leq u_B(v - p') - u_B(v - p) \tag{6}$$

For any $p \in R^1$, there exists some $p' \in R^1$ satisfying formula 6 holds.

Start again from an arbitrary p. For the rest of this proof, I will use $p_0 = p$ and the function $\acute{p}(p_{i-1})$ for $i \geq 2$ where

$$\forall v \in V, c_B \leq u_B(v - \acute{p}(p_{i-1})) - u_B(v - p_{i-1}).$$

I also define sequence π_i to denote the state of the game. $\pi_1 = 0$. Consider $i \geq 2$. If $p_{i-1} > p_{i-2}$, $\pi_i = 1$. If $p_{i-1} \leq \acute{p}(p_{i-2})$, $\pi_i = 0$. Otherwise, $\pi_i = \pi_{i-1}$.

Suppose the seller's strategy is the following. For $i \geq 1$, if $\pi_1 = 0$, $p_i = p_{i-1}$ and if $\pi_i = 1$, $p_i = \acute{p}(p_{i-1})$. This means $p_1 = p$.

The buyer's strategy is defined using ζ_t. $\zeta_t = p''$ means that the buyer's cutoff price for period t is p''. In other words, in period t, buyer accepts $p_t \leq p''$ and rejects $p_t > p''$.

For $i \geq 1$, if $\pi_i = 0$, $\zeta_i = p_{i-1}$ and if $\pi_i = 1$, $\zeta_i = \acute{p}(p_{i-1})$. This means $\zeta_1 = p$. By this strategy, buyer always chooses $\zeta_i \leq \zeta_{i-1}$.

I will prove that the strategies are optimal. Consider the periods when it is the seller's turn to act. In period $i \geq 1$, if $\pi_i = 0$, seller knows that the buyer will never accept any price above p_{i-1}. In period $i \geq 1$, if $\pi_i = 1$, seller knows that buyer will never accept any price above $\acute{p}(p_{i-1})$.

Consider the periods when it is the buyer's turn to act. If in period i, $\pi_i = 0$ and $p_i \leq p_{i-1}$, buyer knows that seller will never offer a price lower than p_i. If $p_i > p_{i-1}$ instead, buyer knows that if she rejects p_i, she can weakly increase her payoff by accepting p_{i+1}.

If in period i, $\pi = 1$ and $p_i \leq \acute{p}(p_{i-1})$, buyer knows that seller will never offer a price lower than p_i. If $\pi = 1$ and $p_i > \acute{p}(p_{i-1})$ instead, buyer knows that if she rejects p_i, she can weakly increase her payoff by accepting p_{i+1}. ∎

Appendix 2. Lemma and proofs used in subsection 5.2

The following lemma is used to prove proposition 3.

Lemma 3. *Suppose that u_S has no lower bound and is given. The set of all prices for which it is the seller's strategy to offer the price for some history, in some pure strategy wPBE that satisfy the following 2 conditions has a lower bound.*

- *$T=\infty$*
- *The seller's strategy is to always bargain.*

Proof. I will call this set, S. For a pure strategy wPBE, suppose that the game has infinite periods and seller always bargains.

There exists some p such that $u_s(p-\gamma) < 0$. If by the seller's strategy, there is some period t where $p_t < p$ and by the buyer type v's strategy, she buys in a later period, there exists some period $t' > t$ where $p_t' < p_t$ and the buyer type v buys. If $p_t < p$ and the sale is made in period t or later or never made, seller's utility is non-positive or he does not get utility.

This means that if by the players' strategies, there is some period t where $p_t < p$, seller prefers to terminate bargaining in period t. Therefore, S is bounded from below. ■

Proof of Proposition 3.

Suppose that $t = T < \infty$ or in period t, both player's strategy is to always terminate bargaining in the next period no matter what happened in period t and before. In period t, according to the buyer's strategy, if $u_B(v-p_t)-c_B < 0$, the she terminates bargaining and if $u_B(v-p_t)-c_B > 0$, she buys. If the seller terminates bargaining in this period, the buyer terminates bargaining as well.

I now use proof by contradiction. Consider the distribution of v according to the seller's belief in this period when the seller does not terminate bargaining in this period. If the seller believes $P(u_B(v-p_t)-c_B \geq 0) = 0$, the seller prefers to terminate bargaining in this period.

If the seller believes $P(u_B(v-p_t)-c_B \geq 0) > 0$, by the least-upper-bound property, the infimum exists for $\{v|u_B(v-p_t)-c_B \geq 0\}$. Call this infimum $\underline{v}'$. Since u_B is continuous, $u_B(\underline{v}'-p_t)-c_B < 0$ cannot be true.

$$u_B(\underline{v}'-p_t)-c_B \geq 0$$

In this period, the buyer does not bargain if $v < \underline{v}'$ because this means $u_B(v-p_t)-c_B < 0$. Since u_B is continuous, there exists some $\varepsilon > 0$ for which if $v \geq \underline{v}'$, $u_B(v-(p_t+\varepsilon)) > 0$. The seller prefers to raise the price to $p_t+\varepsilon$ in which case, by the buyer's original strategy, she will buy if her strategy is to bargain in this period.

Therefore, in any pure strategy wPBE with $T < \infty$, both players terminate bargaining in period t. Induction means that the players terminate bargaining in any period no matter what happened before that period.

Next, I will prove for $T = \infty$. In period $t+1$, if the seller's strategy is to terminate bargaining in period $t+1$ and regardless of what p_t was played, the buyer also terminates bargaining in period $t+1$. Induction using the logic above means that in period t and any earlier period, players will terminate bargaining.

Pick $t > 0$. Consider the game in period t. For $t' > t$, $h(t')$ is a sequence of prices the buyer played starting from period t and ending at $t'-1$ that satisfy the condition that in period t', the seller's strategy is to bargain. If no such $h(t')$ exists, the case is proven.

If u_B is concave, pick $\check{v}$ as a lower bound of V. If not and u_B is convex, pick $\check{v}$ as an upper bound of V. Let $p_{h(t'),t'}$ be the $p_{t'}$ that the seller plays in period $t' > t$ given $h(t')$.

In period t, suppose that $\forall t' > t$ and $h(t') : u_B(\check{v}-p_{h(t'),t'})-t'c_B < u_B(\check{v}-p_t)-tc_B - \frac{t'-t}{2}c_B$. If $t' > t$ and $p_{h(t'),t'} \geq p_t$,

$$v \in V : u_B(v-p_{h(t'),t'})-t'c_B < u_B(v-p_t)-tc_B - \frac{t'-t}{2}c_B. \tag{7}$$

If $t' > t$ and $p_{h(t'),t'} < p_t$, by lemma 1's (2), formula 7 holds as well.

$$\forall t' > t,\ h(t') \text{ and } v \in V : u_B(v-p_{h(t'),t'})-t'c_B < u_B(v-p_t)-tc_B - \frac{t'-t}{2}c_B$$

$$\forall t' > t \text{ and } v \in V : u_B(v-p_{t'})-t'c_B < u_B(v-p_t)-tc_B$$

Then, if $u_B(v - p_t) - c_B < 0$, the buyer's strategy in this period is to terminate bargaining. If the seller believes that $P(u_B(v - p_t) - c_B \geq 0) = 0$, the seller prefers to terminate bargaining.

Consider the case where the seller believes that $P(u_B(v - p_t) - c_B \geq 0) > 0$. By the least-upper-bound property, $\underline{v}'$ exists. If $v < \underline{v}'$, the buyer would terminate bargaining in this period. Since u_B is continuous, if $u_B(\underline{v}' - p_t) - c_B \geq 0$, there exists some $\varepsilon > 0$ for which the following 3 inequalities hold.

$$u_B(\underline{v}' - (p_t + \varepsilon)) > 0$$

$$\forall v \geq \underline{v}' : u_B(v - (p_t + \varepsilon)) > 0$$

$$\forall t' > t, h(t') \text{ and } v \in V : u_B(v - p_{h(t'),t'}) - t'c_B < u_B(v - (p_t + \varepsilon)) - tc_B$$

The seller prefers to raise the price to $p_t + \varepsilon$ in which case, the buyer will buy if her strategy is to bargain in this period. Therefore, for some $t' > t$ and $h(t')$, $u_B(\check{v} - p_{h(t'),t'}) - \frac{t'}{2}c_B \geq u_B(\check{v} - p_t) - \frac{t}{2}c_B$.

Using induction, if for all $t' > t$, some $h(t')$ can be found, for any $\bar{t} > t$, there exists some $t'' > \bar{t}$ and $h(t'')$ where $u_B(\check{v} - p_{h(t''),t''}) - \frac{t''}{2}c_B \geq u_B(\check{v} - p_t) - \frac{t}{2}c_B$ and $u_B(\check{v} - p_{h(t''),t''}) - u_B(\check{v} - p_t) \geq \frac{t''-t}{2}c_B$. However, this violates lemma 3. Therefore, there is no pure strategy wPBE where the seller does not terminate bargaining in any period. ■

Appendix 3. Lemmas and proofs used in section 6

Lemma 4 is used to prove example 1. Lemma 5 is used to prove example 1. Lemma 6 is used to prove lemma 8 and example 1. Lemma 7 is used to prove examples 1 and 2. Lemma 8 is used to proven lemma 9. Lemma 9 is used to prove example 1.

Note that the following proposition is for the u_B defined in section 6.1.

Proposition 4. *u_B is increasing, differentiable and strictly concave.*

Proof. Consider $\ln(x+1)$ for $x \geq 0$.

$$\frac{d\ln(x+1)}{dx} = \frac{1}{x+1} > 0 \tag{8}$$

$$\frac{d^2\ln(x+1)}{dx^2} = -\frac{1}{(x+1)^2} < 0 \tag{9}$$

Consider $-x^2 + x$ for $x \leq 0$.

$$\frac{d - x^2 + x}{dx} = -2x + 1 > 0 \tag{10}$$

$$\frac{d^2 - x^2 + x}{dx^2} = -2 < 0 \tag{11}$$

When $x = 0$, $\ln(x+1) = -x^2 + x = 0$. Also, for this case, equations 8 and 10 are equal. u_B is differentiable. Since the derivative is positive, u_B is increasing.

Formulas 8, 9, 10 and 11 show that $\frac{du_B(x)}{dx}$ is decreasing and that $u_B(x)$ is strictly concave.[15] ■

15. See Avriel et al. (2010, p. 22–23)

Lemma 4. *If $p_1 \in [0.5, 1)$, the low type's expected utility from buying period 1 is negative.*

Proof. When $p_1 \in [0,1]$, the low type's expected utility from buying in period 1 is the following.

$$\int_{p_1+1}^{2} \ln(v_u - p_1)\, dv_u + \int_{1}^{p_1+1} -(v_u - p_1 - 1)^2 + (v_u - p_1 - 1)\, dv_u \tag{12}$$

$$= (2-p_1)\ln(2-p_1) - (2-p_1) - (-(p_1+1-p_1)) + (0+0) - (\frac{p_1^3}{3} + \frac{p_1^2}{2}) \tag{13}$$

$$= (2-p_1)\ln(2-p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2} \tag{14}$$

I take the derivative of the last line for $p_1 \in [0,1]$.

$$\frac{d(2-p_1)\ln(2-p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2}}{dp_1} = -\ln(2-p_1) - p_1^2 - p_1 \tag{15}$$

By equation 15, when $p_1 \in [0.5, 1)$, the low type's expected utility is decreasing in p_1. If $p_1 = 0.5$,

$$(2-p_1)\ln(2-p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2} < 0.$$

Therefore, when $p_1 \in [0.5, 1)$, the low type's expected utility from buying in period 1 is negative. ■

Definition 3.

$$v_u = v + 1$$

v_u is a random variable like v.

Lemma 5. *If $v_u > p_1$, $v_u > p_2$, $\ln(v_u - p_1) \gtreqless \ln(v_u - p_2) - c_B$ is equivalent to $\frac{e^{c_B}p_1 - p_2}{e^{c_B}-1} \lesseqgtr v_u$.*

Proof.

$$\ln(v_u - p_1) \gtreqless \ln(v_u - p_2) - c_B$$

$$\frac{v_u - p_1}{v_u - p_2} \gtreqless e^{-c_B}$$

$$\frac{v_u - p_2}{v_u - p_1} \lesseqgtr e^{c_B}$$

$$v_u - p_2 \lesseqgtr e^{c_B} v_u - e^{c_B} p_1$$

$$e^{c_B} p_1 - p_2 \lesseqgtr e^{c_B} v_u - v_u$$

$$e^{c_B} p_1 - p_2 \lesseqgtr v_u(e^{c_B} - 1)$$

$$\frac{e^{c_B} p_1 - p_2}{e^{c_B} - 1} \lesseqgtr v_u$$

■

Lemma 6.

$$\frac{e^{c_B} p_1 - p_2}{e^{c_B} - 1} - 4p_2 + 2\gamma = 0 \leftrightarrow p_2 = \frac{e^{c_B} p_1 + 2\gamma(e^{c_B} - 1)}{4e^{c_B} - 3}$$

Proof.

$$\frac{e^{c_B}p_1 - p_2}{e^{c_B}-1} - 4p_2 + 2\gamma = 0.$$

$$(-\frac{1}{e^{c_B}-1} - 4)p_2 = -\frac{e^{c_B}p_1}{e^{c_B}-1} - 2\gamma$$

$$(-1-4e^{c_B}+4)p_2 = -e^{c_B}p_1 - 2\gamma(e^{c_B}-1)$$

$$p_2 = \frac{-e^{c_B}p_1 - 2\gamma(e^{c_B}-1)}{-4e^{c_B}+3}$$

$$p_2 = \frac{e^{c_B}p_1 + 2\gamma(e^{c_B}-1)}{4e^{c_B}-3}$$

■

Lemma 7.

1. *Under $p_1 \in [0,1]$, there exists a unique $p_1 \approx 0.45$ for which*

$$(2-p_1)\ln(2-p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2} = 0.$$

2. *If $c_B \in [0.01, 0.02]$ and $\gamma \in [0, 0.01]$, under $p_1 \in [0,1]$, there exists a unique $p_1 \leq 0.365$ for which*

$$(2-p_1)\ln(2-p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2} = \frac{3-\gamma}{2}\ln(\frac{3-\gamma}{2}) + \frac{\gamma-1}{2} - c_B.$$

This unique $p_1 \leq 0.365$ is increasing in c_B and γ.

Proof.

$$\frac{d\frac{3-\gamma}{2}\ln(\frac{3-\gamma}{2}) + \frac{\gamma-1}{2} - c_B}{d\gamma} = -\frac{1}{2}\ln(\frac{3-\gamma}{2}) - \frac{1}{2}(\frac{3-\gamma}{2})\frac{1}{\frac{3-\gamma}{2}} + \frac{1}{2} = -\frac{1}{2}\ln\frac{3-\gamma}{2} < 0$$

$\frac{3-\gamma}{2}\ln(\frac{3-\gamma}{2}) + \frac{\gamma-1}{2} - c_B$ is decreasing in c_B and γ.

If $p_1 \in [0,1]$, by equation 15, $(2-p_1)\ln(2-p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2}$ is decreasing in p_1.

If $p_1 = 0$,

$$(2-p_1)\ln(2-p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2} = 2\ln(2) - 1 > 0.38$$

If $p_1 = 1$,

$$(2-p_1)\ln(2-p_1) - 1 + p_1 - \frac{p_1^3}{3} - \frac{p_1^2}{2} < 0.$$

By the intermediate value theorem, the unique p_1 of 7.1 exists. Similarly, since $c_B \in [0.01, 0.02]$ means $1.5\ln(1.5) - 0.5 - c_B \in [0.08, 0.1]$, the unique p_1 of 7.2 exists.

Formula 15 means that as c_B or γ increases, the unique p_1 of 7.2 increases. ■

Lemma 8. *If $p_2 = \frac{e^{c_B}p_1 + 2\gamma(e^{c_B}-1)}{4e^{c_B}-3}$, $\frac{e^{c_B}p_1 - p_2}{e^{c_B}-1}$ and p_2 are increasing in p_1. If $p_1 > \gamma$ as well, $\frac{e^{c_B}p_1 - p_2}{e^{c_B}-1}$ and p_2 are decreasing in c_B.*

Proof. Lemma 6 means that

$$\frac{e^{c_B}p_1 - p_2}{e^{c_B}-1} = 4p_2 - 2\gamma.$$

$p_2 = \frac{e^{c_B}p_1+2\gamma(e^{c_B}-1)}{4e^{c_B}-3}$ is increasing in p_1.

$$\begin{aligned}
&p_1 > \gamma\\
&3p_1 > 2\gamma\\
&-3p_1 < -2\gamma\\
&2\gamma(4e^{c_B}-3) - 2\gamma = 4(2\gamma(e^{c_B}-1))\\
&p_1 \times 4e^{c_B} + 2\gamma(4e^{c_B}-3) - 2\gamma = 4(e^{c_B}p_1 + 2\gamma(e^{c_B}-1))\\
&(p_1+2\gamma)(4e^{c_B}-3) = p_1 \times 4e^{c_B} - 3p_1 + 2\gamma(4e^{c_B}-3)\\
&\qquad < p_1 \times 4e^{c_B} + 2\gamma(4e^{c_B}-3) - 2\gamma\\
&\qquad = 4(e^{c_B}p_1 + 2\gamma(e^{c_B}-1))\\
&\frac{dp_2}{de^{c_B}} = \frac{(p_1+2\gamma)(4e^{c_B}-3) - 4(e^{c_B}p_1+2\gamma(e^{c_B}-1))}{(4e^{c_B}-3)^2} < 0
\end{aligned}$$

p_2 is decreasing in c_B. ■

Lemma 9. *In settings 1 and 2, let p_2 be $\frac{e^{c_B}p_1+2\gamma(e^{c_B}-1)}{4e^{c_B}-3}$. p_2 is increasing in p_1 and γ and decreasing in c_B. $p_1 > p_2 > 0.6$.*

Proof. By lemma 8, p_2 is increasing in p_1 and decreasing in c_B.

$$\begin{aligned}
&p_2 = \frac{e^{c_B}p_1 + 2\gamma(e^{c_B}-1)}{4e^{c_B}-3}\\
&2\gamma < 3p_1\\
&2\gamma(e^{c_B}-1) < (3e^{c_B}-3)p_1\\
&\frac{2\gamma(e^{c_B}-1)}{4e^{c_B}-3} < \frac{3e^{c_B}-3}{4e^{c_B}-3}p_1\\
&p_2 = \frac{e^{c_B}p_1 + 2\gamma(e^{c_B}-1)}{4e^{c_B}-3} < p_1 \qquad (16)\\
&p_2 = \frac{e^{c_B}p_1 + 2\gamma(e^{c_B}-1)}{4e^{c_B}-3} \geq \frac{0.7}{4e^{c_B}-3} > 0.6
\end{aligned}$$

■

Proof of Example 1.

First, I will describe the equilibrium strategies. Both players' strategies are to bargain. For period 3 and any period after, if the period exists, the players' strategy is to terminate bargaining.

In period 1, seller offers the equilibrium p_1. If the buyer rejects this, in period 2, seller offers the equilibrium p_2. Suppose the seller deviates to a different price in period 1 and the game continues to period 2. Let p be a function of γ and c_B. The output of this function is given by lemma 7.2. Let p' be given by 7.1. If $p_1 \in [p(\gamma, c_B), p']$, the seller believes that the buyer is low type and offers $p_2 = \frac{\gamma+1}{2}$. If $p_1 \notin [p(\gamma, c_B), p']$, the seller terminates bargaining in period 2.

The buyer's strategy for period 1 is the following. If p_1 is not a deviation, high type buyer buys if $v_u \geq \frac{e^{c_B} p_1 - p_2}{e^{c_B} - 1}$.

If p_1 is a deviation, then, the high type buyer buys for this offer if and only if $v \geq p_1$. In period 1, the low type buyer buys if and only if $p_1 < p(\gamma, c_B)$.

In period 2, the buyer bargains if p_1 is not a deviation or $p_1 \in [p(\gamma, c_B), p']$. In period 2, when the buyer sees the price, the buyer buys if $v \geq p_2$.

Since both players' terminate bargaining in period 3 and any period after if the period exists, the players' strategies to do so are optimal. If p_1 was a deviation and $p_1 < p(\gamma, c_B)$, both players terminate bargaining in period 2. Therefore, the strategies to do so are optimal.

I will prove that the seller's strategy is optimal. If the seller deviates to $p_1 = 1$, his expected utility is $\frac{1}{3}(1-\gamma) \geq 0.3$. The seller prefers to bargain. If p_1 was the equilibrium p_2 or $p_1 \in [p(\gamma, c_B), p']$, in period 2, his expected utility from $p_2 = 0.5$ is at least $0.5(0.5-\gamma)$. Therefore, in these cases, the seller prefers to bargain in period 2.

Consider the optimality of p_2. By lemma 6, for the equilibrium p_2,

$$\frac{e^{c_B} p_1 - p_2}{e^{c_B} - 1} = 4p_2 - 2\gamma.$$

By lemma 9,

$$2 < \frac{e^{c_B} p_1 - p_2}{e^{c_B} - 1} < 3.$$

$$1 < \frac{e^{c_B} p_1 - p_2}{e^{c_B} - 1} - 1 < 2.$$

Let $\check{v}$ be this $\frac{e^{c_B} p_1 - p_2}{e^{c_B} - 1} - 1$. If p_1 is not a deviation, $p_2 < \gamma$ and $p_2 > \check{v}$ are not optimal. If the seller sets $p_2 \in [1, \check{v}]$, seller's expected utility in period 2 is $\frac{1}{3} p_2(\check{v} - p_2)$.

$$\frac{1}{3} p_2(\check{v} - p_2) = \frac{1}{3}(-p_2^2 + \check{v} p_2) = \frac{1}{3}(-(p_2 - \frac{\check{v}}{2})^2 - (\frac{\check{v}}{2})^2)$$

From $p_2 \in [1, \check{v}]$, $p_2 = 1$ is optimal.

If the seller sets $p_2 \in [\gamma, 1]$, his expected utility in period 2 is $(p_2 - \gamma)(\frac{1}{3}(\check{v} - 1) + \frac{2}{3}(1 - p_2))$. The first order condition is

$$\frac{1}{3}(\check{v} - 1) + \frac{2}{3}(1 - p_2) + (p_2 - \gamma) \times (-\frac{2}{3}) = 0. \tag{17}$$

If $p_2 = 1$,

$$\frac{1}{3}(\check{v} - 1) - \frac{2}{3}(1 - \gamma) < 0.$$

The second order condition is

$$-\frac{2}{3} - \frac{2}{3} < 0.$$

When $p_2 = \gamma$, the expected utility is 0. The seller prefers $p_2 = 0.5$. $p_2 \in \{\gamma, 1\}$ is not optimal.

Equation 17 is equivalent to

$$(\check{v} - 1) - 4p_2 + 2\gamma + 2 = 0.$$

When both expected and chosen p_2 are $\frac{e^{c_B}p_1+2\gamma(e^{c_B}-1)}{4e^{c_B}-3}$, Lemma 6 proves that $(\check{v}-1)-4p_2+2\gamma+2=0$. $p_2=\frac{e^{c_B}p_1+2\gamma(e^{c_B}-1)}{4e^{c_B}-3}$ is optimal.

Consider the case where $p_1\in[p(\gamma,c_B),p')$. Then, in period 2, seller believes that the buyer is low type. When the buyer is low type, the seller prefers $p_2=0.5$ to $p_2<0$ or $p_2>1$. Furthermore, when $p_2\in[0,1]$, the seller's expected utility in this period is $(p_2-\gamma)(1-p_2)$.

$$(p_2-\gamma)(1-p_2)=-p_2^2+(\gamma+1)p_2-\gamma=-(p_2-\frac{\gamma+1}{2})^2+(\frac{\gamma+1}{2})^2-\gamma.$$

Therefore, in this case, $p_2=\frac{\gamma+1}{2}$ is optimal.

Consider the optimality of p_1. Let p_2' be the p_2 the buyer expects after seeing p_1. If the seller does not deviate in period 1 and plays $p_2=p_1$ in period 2, his expected payoff is

$$\begin{aligned}
&\frac{1}{3}(3-\frac{e^{c_B}p_1-p_2'}{e^{c_B}-1})(p_1-\gamma-c_S)+\\
&\frac{1}{3}(\frac{e^{c_B}p_1-p_2'}{e^{c_B}-1}-2)(p_2-\gamma-2c_S)+\frac{2}{3}(1-p_2)(p_2-\gamma)-\frac{2}{3}\times 2c_S>\\
&\frac{1}{3}(3-\frac{e^{c_B}p_1-p_2'}{e^{c_B}-1})(p_1-\gamma-2c_S)+\\
&\frac{1}{3}(\frac{e^{c_B}p_1-p_2'}{e^{c_B}-1}-2)(p_1-\gamma-2c_S)+\frac{2}{3}(1-p_2)(p_2-\gamma)-\frac{2}{3}\times 2c_S=\\
&\frac{1}{3}(1)(p_1-\gamma)+\frac{2}{3}(1-p_2)(p_2-\gamma)-2c_S=\\
&\frac{1}{3}(p_1-\gamma)+\frac{2}{3}(1-p_1)(p_1-\gamma)-2c_S=\\
&\frac{3-2p_1}{3}(p_1-\gamma)-2c_S.
\end{aligned}$$

$$\frac{d(3-2p_1)(p_1-\gamma)}{dp_1}=-2(p_1-\gamma)+(3-2p_1)=3+2\gamma-4p_1>0$$

$$\frac{3-2p_1}{3}(p_1-\gamma)-2c_S\geq\frac{3-2\times 0.7}{3}(0.7)-0.005-c_S>0.365-c_S$$

If the seller deviates to $p_1<p(\gamma,c_B)$ in this period, his payoff is less than $p(\gamma,c_B)-c_S$.

$$p(\gamma,c_B)-c_S\leq 0.365-c_S$$

If the seller deviates to $p_1\in[p(\gamma,c_B),p']$ in this period, the price of sale does not exceed $\frac{\gamma+1}{2}$ in periods 1 and 2. The expected payoff is less than $\frac{2}{3}\times\frac{\gamma+1}{2}-c_S$.

$$\frac{2}{3}\times\frac{\gamma+1}{2}-c_S\leq 0.34-c_S$$

If p_1 is a deviation and $p_1>p'$ in this period, seller only sells to the high types. $p_1<1$ and $p_1>2$ are not optimal. If $p_1\in[1,2]$, his expected utility is $\frac{1}{3}(p_1-\gamma)(2-p_1)-c_S$.

$$\begin{aligned}
\frac{1}{3}(p_1-\gamma)(2-p_1)-c_S&=\frac{1}{3}(-p_1^2+(\gamma+2)p_1-2\gamma)-c_S\\
&=\frac{1}{3}(-(p_1-\frac{\gamma+2}{2})^2+(\frac{\gamma+2}{2})^2-2\gamma)-c_S\\
&=\frac{1}{3}(-(p_1-\frac{\gamma+2}{2})^2+\frac{(\gamma-2)^2}{4})-c_S\leq\frac{1}{3}-c_S
\end{aligned}$$

Now, I will prove that the buyer's strategy is optimal. Suppose p_1 is a deviation, $p_1 \notin [p(\gamma, c_B), p']$ and the buyer is deciding whether to accept it. Since the seller will terminate bargaining if the game proceeds to the next period, it is optimal for the buyer to buy when her expected utility is 0 or greater. Therefore, it is optimal for the high type to buy if and only if $v \geq p_1$.

If the buyer is low type, expected utility is positive for $p_1 \leq 0$ but negative for $p_1 \geq 1$. When $p_1 \in [0,1]$, low type's expected utility of purchase is given by formulas 12∼14 and formula 15 means that the expected utility is decreasing in p_1. By lemma 7, it is optimal for the low type accept $p_1 < p(\gamma, c_B)$.

Suppose that $p_1 \in [p(\gamma, c_B), p']$ and the buyer is deciding whether to accept it. If the buyer rejects, she can get $p_2 = \frac{\gamma+1}{2}$. However, this is a higher price and the high type prefers to buy now.

Consider the low type buyer's expected utility from buying in period 2 when $p_1 \geq p(\gamma, c_B)$ and $p_2 \in [0,1]$.

$$\begin{aligned}\int_{p_2+1}^{2} \ln(v_u - p_2)\, dv_u &= (2-p_2)\ln(2-p_2) - (2-p_2) - (-(p_2+1-p_2)) \\ &= (2-p_2)\ln(2-p_2) - 1 + p_2\end{aligned} \tag{18}$$

If $p_2 = \frac{\gamma+1}{2}$,

$$(2-p_2)\ln(2-p_2) - 1 + p_2 = \frac{3-\gamma}{2}\ln\frac{3-\gamma}{2} + \frac{\gamma-1}{2}$$

If $p_2 < 1$,

$$\begin{aligned}\frac{d(2-p_2)\ln(2-p_2) - 1 + p_2}{dp_2} &= -\ln(2-p_2) - (2-p_2)\frac{1}{2-p_2} + 1 \\ &= -\ln(2-p_2) < 0.\end{aligned} \tag{19}$$

For the low type, when $p_1 \in [p(\gamma, c_B), p']$ rejecting p_1 and bargaining for p_2 is better than accepting p_1.

Consider p_2 on the equilibrium path. By lemma 9, p_2 is increasing in p_1 and γ and decreasing in c_B. If $p_1 = 0.71$, $\gamma = 0.01$ and $c_B = 0.01$, $p_2 < 0.7$. If $p_2 = 0.7$, by formula 18,

$$\int_{p_2+1}^{2} \ln(v_u - p_2)\, dv_u > 0.04$$

Therefore, by formula 19, in all three settings, low type's expected equilibrium payoff is positive. In all three settings, any high type's payoff from buying in period 2 is positive if the seller plays the equilibrium p_2. It is optimal for the low type and the high type to bargain. By lemma 4, it is optimal for the low type to not buy for the equilibrium price in period 1 and bargain in period 2. In period 2, it is optimal for the high type to bargain. Furthermore, by equation 19, after seeing $p_1 \in [p(\gamma, c_B), p']$, it is optimal for the buyer to bargain in period 2.

Consider the high type's strategy when she sees that p_1 is not a deviation. Lemmas 5 and 9 mean if $v_u \geq \frac{e^{c_B} p_1 - p_2}{e^{c_B} - 1}$, she weakly prefers buying in period 1 to buying in period 2.

In period 2, it is optimal for the buyer who saw the offer to accept it if $v \geq p_2$. ■

Proof of Example 2.

The seller's strategy is to bargain. In period 1, the buyer offers p_1 defined by 7.2 which is approximately 0.35. Let p_1 defined by lemma 7.1 be p_1'. In period 2, if $p_1 < p_1'$, seller believes that the buyer is low type. In this period, if $p_1 < p_1'$, seller offers $p_2 = 0.5$ and if $p_1 \geq p_1'$, he terminates bargaining. He terminates bargaining in period 3 and any subsequent period if the period exists.

The buyer's strategy is also to bargain. In period 2, if $p_1 < p_1'$, the buyer bargains and if $p_1 \geq p_1'$, she terminates bargaining. She terminates bargaining in period 3 and any subsequent period if the period exists.

I will describe the high type buyer's strategy first. In period 1, the high type buys as long as $v \geq p_1$. In period 2, the high type who saw the price buys if $v \geq p_2$.

Next, if p_1 is lower than or equal to the equilibrium p_1, the low type buys in period 1. If p_1 is greater, the low type does not buy.

The players' strategies to terminate bargaining in period 3 if the period exists are optimal because if the period exists, the other player terminates bargaining in the period. The player's strategies to terminate bargaining in period 2 when $p_1 \geq p_1'$ are optimal for a similar reason.

I will prove that the seller's strategy is optimal. Consider period 2 for $p_1 < p_1'$. Here, seller prefers $p_2 = 0$ to $p_2 < 0$ and weakly prefers $p_2 = 1$ to $p_2 > 1$. If $p_2 \in [0,1]$, seller's expected utility is

$$p_2(1-p_2) = -(p_2-0.5)^2 + 0.25.$$

Since the seller can get an expected utility of 0.25, the seller weakly prefers to bargain in this period. p_2 is optimal.

Consider period 1. In the equilibrium, seller sells in this period and gets a utility of approximately 0.35. The seller prefers to bargain. Seller can make the sale with the equilibrium price or a lower one. If the price is lower, the seller's utility is less. If the price is higher, the seller can only sell to the high type in this period. The seller's expected payoff when $p_1 \in [1,2]$ is

$$\frac{1}{3}p_1(2-p_1) = \frac{1}{3}(-(p_2-1)^2+1) \leq \frac{1}{3}.$$

In the next period, seller's expected utility is 0.25 or less. The seller prefers to play the equilibrium strategy compared to deviating to a higher price.

I will now prove that the buyer's strategy is optimal. In period 2, after seeing the price, buying when $v \geq p_2$ is optimal for the buyer. Given this, in period 2, when $p_1 \geq p_1'$, by bargaining the low type buyer can get a utility of $\int_{0.5}^{1} \ln(v-0.5+1)\,dv$ for $p_2 = 0.5$.

$$\begin{aligned}\int_{0.5}^{1} \ln(v-0.5+1)\,dv &= \int_{1}^{1.5} \ln(v)\,dv = 1.5\ln(1.5) - 1.5 - (\ln(1)-1) \\ &= 1.5\ln(1.5) - 0.5 \approx 0.11 > 2c_B > 0\end{aligned} \tag{20}$$

In period 2, when $p_1 \geq p_1'$, it is optimal for the low type buyer to bargain. For $p_2 = 0.5$, the high type's utility from the purchase is at least $\ln(2-0.5)$.

$$\ln(2-0.5) \approx 0.41 > 2c_B \tag{21}$$

In period 2, when $p_1 \geq p_1'$, it is optimal for the high type to bargain as well.

Consider period 1. For the equilibrium p_1, a high type buyer's utility from purchase is at least $\ln 2 - p_1$. By formula 21, he prefers to bargain. If the seller offers $p_1 < p_1'$, by formula 21, the high type's utility from the purchase is positive. If he buys in the next period instead, his utility will be smaller. If the seller offers $p_1 \geq p_1'$, the seller will not offer a price in period 2 and after seeing p_1, buying when $v \geq p_1$ is optimal.

If $p_1 > 1$ and low type buyer accepts p_1, her expected utility is negative. The low type buyer's optimal strategy in this case is to terminate bargaining in period 2. If $p_1 \leq 0$ and the low type buyer accepts p_1, her expected utility is

$$\int_1^2 \ln(v_u - p_1)\, dv_u = \int_{1-p_1}^{2-p_1} \ln(v_u)\, dv_u$$
$$= (2-p_1)\ln(2-p_1) - (2-p_1) - ((1-p_1)\ln(1-p_1) - (1-p_1))$$
$$= (2-p_1)\ln(2-p_1) - (1-p_1)\ln(1-p_1) - 1$$

If $p_1 = 0$,

$$(2-p_1)\ln(2-p_1) - (1-p_1)\ln(1-p_1) - 1 \approx 0.39.$$

If $p_1 \leq 0$,

$$\frac{d(2-p_1)\ln(2-p_1) - (1-p_1)\ln(1-p_1) - 1}{dp_1}$$
$$= -\ln(2-p_1) + (2-p_1) \times -\frac{1}{2-p_1} + \ln(1-p_1) - (1-p_1) \times -\frac{1}{1-p_1}$$
$$= \ln(1-p_1) - \ln(2-p_1) < 0$$

Therefore, when the low type buyer sees $p_1 \leq 0$, accepting it is optimal.

Suppose $p_1 \in [0,1]$. If the low type buyer accepts p_1, her expected utility is given by formulas 12∼14. For the equilibrium p_1, lemma 7.2 establishes that in period 1, the low type buyer weakly prefers accepting it to bargaining in period 2. Formula 20 means that the low type prefers to bargain. Formulas 15 means that the low type buyer prefers accepting a lower p_1. Suppose that p_1 is higher. If $p_1 < p_1'$, low type prefers to reject p_1 and buy in period 2. If $p_1 \geq p_1'$, by lemma 7.1, low type weakly prefers terminating bargaining in period 2 to buying. ■

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