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# Bargaining on Behalf of Others and How to Fix It For Political Gridlocks and Labor Strikes

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These days, when the ruler listens to men's words, he delights in their eloquence and does not bother to inquire if they are apt, and when he embarks upon some undertaking, he is thrilled by the report of what is to be accomplished and does not demand to see actual results.

— Han Feizi

This paper models bargaining where each side has a principal and replaceable agents. Such bargaining occurs when congressional representatives are bargaining for legislation on behalf of voters or when a firm and a union are bargaining for a labor contract. If agents who bargain have incomplete information and are unsure about what offers will be accepted, their principal may replace agents who do not bargain aggressively, thinking that the agents may have high bargaining costs. To avoid replacement, agents may act excessively aggressively to signal that they have lower bargaining costs, even if they believe that aggression is harmful to their principal. The social cost of bargaining can be lowered by forcing agents to bargain in arduous conditions or by taxing both sides for bargaining time. This tax can replace labor strikes.

JEL classification: C78; C73; D72; J52; D81; D82

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### 1 Introduction

Legislative bargaining and labor contract bargaining are commonly done by representatives on behalf of their parties, constituencies or organizations. In a representative democracy, congressional representatives bargain bills for the people of the nation who elect them. For labor contract bargaining, labor union leaders bargain for the members of their union. On the firm's side, the firm may delegate bargaining to a negotiator. Even if the head of the firm bargains personally, the head is answerable to a board of directors and shareholders. In such situations, those who bargain may be under pressure to get a good deal for their side or to negotiate for

a long time for a good deal.<sup>1</sup> If they fail to bargain accordingly, they may lose their jobs as representatives.

When agents on both sides face the pressure and the threat of removal, it can be difficult to reach a deal in short time. For legislative bargaining, this leads to political gridlock, which can take many forms. Representatives in congress may engage in filibusters, by which they delay or block votes on a bill using procedural tactics such as continuing to speak.<sup>2</sup> The filibuster has been subject to criticisms of blocking beneficial laws.<sup>3</sup>

Political gridlock includes fights over government shutdowns and debt ceilings. In the U.S. a government shutdown happens when Congress does not fund the government. In a shutdown, numerous government employees are furloughed and numerous government services are paused. Many government shutdowns have happened in the U.S.<sup>4</sup> The longest government shutdown started in 2018 and lasted 35 days. According to the Congressional Budget Office, the cost of this shutdown was about \$3 billion in real GDP lost.<sup>5</sup>

If the U.S. Congress does not raise the debt ceiling which limits how much the government can borrow and, moreover, the government cannot borrow more money and cannot pay its obligations, a government default occurs.<sup>6</sup> Treasury Secretary Janet Yellen has stated that a government default "would produce an economic and financial catastrophe".<sup>7</sup> Even if a government default does not occur, debt ceiling fights are costly. The Bipartisan Policy Center estimated that the 2011 debt ceiling fight would cost \$18.9 billion in increased borrowing costs for the government over 10 years.<sup>8</sup>

Labor strikes mean that workers do not provide their labor to produce goods or services. According to the U.S. Bureau of Labor Statistics, there were 31 major work stoppages in 2024. (These are strikes or lockouts. In lockouts, management stops the employees from working. Typically, this is during a labor dispute.) These stoppages resulted in around 3.4 billion days of idleness in total, in which workers did not work.<sup>9</sup>

Political gridlocks and labor strikes are not just costly to the representatives, the firms and the workers involved. As government shutdown and debt ceiling fights show, delays or failures in passing bills that would benefit people can lead to suffering for the public. Labor strikes also hurt the consumers and other firms that do business with the firms under the strike. In other words, in these contexts, there can be substantial external and social costs of delays and failures of deals.

In this paper, I model the phenomena of bargaining on behalf of others and costly delays in deals using an infinitely repeated veto bargaining game with principals on the two sides. The

- 1. See Nam, Kim, and Yu (2009).
- 2. See Elving (2022), McCarthy (2021), and Fortin (2021).
- 3. See McCarthy (2021) and Klein (2020).
- 4. See Scholtes and Emma (2023), Hussein (2023), Schaul and Uhrmacher (2024), and Lagatta (2025)
- 5. See Emma and Scholtes (2019) and "The Effects of the Partial Shutdown Ending in January 2019" (2019).
- 6. Technically, when the government is impeded by the debt ceiling from borrowing additional money, the government may be able to avoid a default. Millhiser (2023) and Carp (2023) discusses workarounds that would allow the government to pay its obligations.
- 7. See Hussein (2023), Kolinovsky (2024), and "Yellen: U.S. default would be economic and financial "catastrophe"" (2023).
- 8. See Brown (2012), Gale (2013), "Debt Limit: Analysis of 2011-2012 Actions Taken and Effect of Delayed Increase on Borrowing Costs" (2012), and "Yellen: U.S. default would be economic and financial "catastrophe" (2023).
- 9. See "Work Stoppages: Concepts" (2019) and "Major Work Stoppages (Annual) News Release" (2025).

principals delegate bargaining to agents and can remove agents. One side is called the proposer side and the other is called the vetoer side. The agent on the proposer side makes offers and the agent on the vetoer side has the power to accept or reject them.

This game has multiform equilibria. Of these, I solve for Perfect Bayesian Equilibriums (PBEs) where at least some agents on the proposer side are not aggressive or aggressive because of the threat of removal. Aggression means that the agent is willing to risk large bargaining costs to get a good deal. Thus, in the PBEs I solve for, without the threat of removal, the agents may not bargain hard because of their bargaining costs.

I focus on the strategies of the principal and the agents on the proposer side. In the PBEs where the proposer-agents are not threated with removal, these agents bargain in the same way and their principal sees no point in removing agents. In the PBEs where the proposer-agents face the threat of removal, their principal removes agents who do not exhibit sufficient aggression because the principal wants agents who bargain hard. Agents with lower bargaining costs are counted on to bargain more aggressively than agents with high bargaining costs do. Since nonaggression will lead to removal, the agents may respond to the principal's strategy by acting aggressively even in situations where aggression is harmful to the principal. Such actions may signal to the principal that the agent is likely to have low bargaining costs. However, such aggression can cause unnecessary delays in deals and excessive bargaining costs.

There are two main ways to deal with the problem of excessive bargaining costs. Both may raise the bargaining cost per unit time. This will lead the agents to reach a deal quickly and lower the social cost of bargaining. First, the agents can be forced to bargain in quarantined meetings under arduous conditions until a deal is reached. Alternatively, in case of a political gridlock, a wealth tax can be imposed on the representatives. For labor disputes, instead of allowing strikes or lockouts, firms and unions should be given the power to make both sides pay strike taxes. Under this tax, for each day of a labor dispute, the workers would be taxed for their daily wages and the firm would be taxed for its estimated daily loss of profits from a strike.

Section 2 discusses related papers on strikes, delegation and bargaining in the literature. Section 3 specifies the model and what aggression means. Section 4 features the Adaptive and Nonadaptive Equilibrium in which the proposer-principal never replaces his agents regardless of the agent's actions. Section 5 presents the Replacement and Imitation Equilibrium in which the proposer-principal's strategy is to replace agents exhibiting a lack of aggression. Section 6 explains how in the PBEs of section 5, the proposer-agents' wages can cause them to display aggression even when aggression will likely fail in bargaining. Section 7 introduces the extended model where players have bargaining costs per unit time and demonstrates how raising the bargaining cost per unit time for agents can decrease social cost. Section 8 discusses how in reality, signaling causes excessive bargaining costs and suggests remedies for lowering social costs. Section 9 concludes. All proofs are in the appendix.

10. PBEs are defined in Fudenberg and Tirole (2005, p. 331-333). However, this definition is for games that are different from mine in that every player chooses a move at every information set and players are not signaled by nature. Unlike this definition, I have PBEs in which a player's beliefs are updated from the last information set the player acted in to the current information set where the player acts. If there is no last information set, they are updated from the beginning of the game. The belief update about a player is not influenced by other players' actions. Furthermore, instead of requiring that given the same history, any two players have the same belief about a player who is not either of them, I require that any two players have the same belief at an information set they both act in about a player who is not either of them if they observed the same set of moves and the same set of signals from nature till now.

## 2 Literature Review

This paper is related to three types of papers. First, the paper is related to the study of labor strikes. Hicks (1963, pp. 144–147) implies the "Hicks Paradox". Kennan (1986, p. 1091) explains the meaning of this paradox. If strikes and their outcomes of strikes are predictable, predicted strikes should not happen because the parties can agree to this outcome before the strike and avoid the costs of the strike. Ashenfelter and Johnson (1969) explains the occurrence of strikes using different interests of union leaders and members. If members ask for unrealistically high wage increases, the leaders may incur a strike to avoid the accusation that the leaders have sold out to management and to keep their power. Kennan (1980) finds when the cost of the strike is greater, the strike is more likely to end early.

On the problem of strikes, many solutions are suggested in the literature. As a solution, Hicks (1963, p. 147) offers conciliation by frequent meetings of firms and labor union leaders that will help them understand each other. Reder and Neumann (1980) states that a detailed bargaining protocol for labor contract negotiations can prevent strikes. Hameed (1971) discusses using government intervention to stop strikes.

Like my paper, previous papers have also discussed non-stoppage strikes and/or strike taxes as solutions to labor strikes. According to Sosnick (1964), in a non-stoppage strike, workers continue to work but workers and the firm pay strike taxes. Sosnick (1964) suggested the total amounts of penalty should be the same for both sides. Levmore (2025) also stated the total amounts of penalty could be the same for both sides but thought that forbidding strikes would be politically unacceptable. Levmore (2025) would tax them for the external costs of a strike. Stokes (1969) suggested that the union and the firm could also agree to a penalty scheme for the next labor dispute when they agree on a labor contract and the penalty could be given to a charity. Marceau and Musgrave (1949) thought that the strike taxes should approximate the potential losses of a traditional strike. McGalmont (1962) points out the problem that it is difficult to estimate the firm's profit loss for a potential traditional strike.

The second type of papers is on political agency models and political games. Rogoff (1990) has a signaling model where voters do not fully know whether a politician is competent. A competent incumbent politician sets taxes too low and government spending too high before elections. This signals the voters that the politician is competent. In Banks and Sundaram (1993). the (median) voter does not know the candidates' cost. Acting in the voter's interests is costly. Incumbents with lower cost act more in the voter's interest. Cameron and Gibson (2020) imputes political gridlocks to the desire of agents to signal the principals that the agents are trustworthy.

Groseclose and McCarty (2001)'s model is not a principal-agent model where the principal takes an action that directly affects the agent's utility. The model's players are the electorate, the congress and the president. The congress and the president bargain over a bill but the bill does not affect the electorate's utility. A gridlock can occur over the bill when the president cares about the electorate's approval. Similar to Groseclose and McCarty (2001) and Cameron and Gibson (2020), I explain political gridlocks as a result of representatives' efforts to win the minds of voters.

The third type involves bargaining. My paper solves for a game where the principals on both sides delegate bargaining to their agents and there is incomplete information about the agents' bargaining costs. Rubinstein (1982) studied bargaining with fixed bargaining costs and found equilibria where bargaining concludes in one period. Hwang (2025) also studied fixed bargaining costs but found that when offers are one-sided, finite period games are advantageous to the side that makes offers. Cramton (1984) found that when only the seller makes offers and

both players have incomplete information about the other players's valuation, increasing costs of delay benefits the seller and hurst the buyer. Because of this, the seller is expected to delay offers and the buyer prefers shorter waits for offers.

Other papers have studied models where agents bargain on behalf of principals or a principal bargains with an agent. In McCall (1990), when a negotiator represents members of the labor union, union members may benefit by having the negotiator reach a wage settlement with arbitration some of the time. Segendorff (1998) when nations delegate agents to bargain over a public good, delegation makes at least one of the nations worse off compared to mutual self-representation. Nations may choose agents who are biased against the public good. In Kartik, Kleiner, and Van Weelden (2021), the proposer offers a set of options to a vetoer. The paper interprets this to mean that the proposer can delegate the policy or action to a vetoer. It shows that the proposer typically offers the vetoer multiple options to choose from instead of offering just one policy or action.

Cai (2000) analyzes bargaining games where only one side delegates bargaining to an agent. An agent has a bargaining cost. An agent with a lower bargaining cost can intentionally delay a deal to signal the agent's low cost to the principal in order to keep the agent's job even though this delay does not result in a good deal. Among the papers on bargaining with delegation, my paper is most similar to Cai (2000). However, my paper has delegation on two sides and the delay in deals explained using agents' uncertainty about what deal is possible.

## 3 Model

#### 3.1 Players

There is a proposer-principal and a vetoer-principal. There are infinite proposer-candidates (numbered  $1, 2, \cdots$ ) and infinite vetoer-candidates (numbered  $1, 2, \cdots$ ). There are also infinite periods starting with period t = 1. In each period, one proposer-candidate has the role of the proposer-agent and one vetoer-candidate has the role of the vetoer-agent. Any player on the proposer side is referred to as "he" and any player on the vetoer side is referred to as "she".

#### 3.2 Periods and Stages

At the start of the game, each candidate receives his or her type from nature. Afterwards, the proposer-candidates receive signals which will be described in the Signals and Beliefs subsection. For both proposer-agents and vetoer-agents, at the start of period t, if it is period 1 or the agent in the previous period has been replaced, agent whose number is t is brought in from the agents' side. Once a candidate becomes the agent, the candidate continues to be the agent until the agent is replaced. Once agents are replaced, they are unable to take on the role again.

11. However, in Cai (2000), the different bargaining costs of the agents actually imply their different abilities to get a comparatively good deal in the same situation. My model does not imply this. For Cai (2000)'s signal using delays to work in reality, in legislative bargaining, a representative who engages in a filibuster and/or causes gridlock and fails to get a good deal need be better at getting good deals than one who does not engage in a filibuster or cause a gridlock. In labor contract bargaining, the union leader who conducts a strike and fails to get a good deal need be better at getting good deals than to a leader that does not conduct one. Furthermore, the principal need know this.

In each period, the game goes through two stages.<sup>12</sup> In other words, stages are nested inside periods. In period t, at the beginning of stage  $s \in \{1,2\}$ , the proposer-agent offers a proposal or an action,  $a_{t,s} \in R^1$ , to the vetoer-agent. If the vetoer-agent accepts this, the deal is made for the period. In stage 1, if the vetoer-agent rejects the offer, the game continues to the next stage. In stage 2, if she rejects the offer, there is no deal for the period.  $a_t$  is the deal made for period t.  $s_t$  is the stage at which the deal is made for period t.<sup>13</sup>

Once the deal is made or fails for the period, the game reaches the end of period t and the principals decide whether to remove and replace their agents. Afterwards, remaining agents who have not been removed are automatically retired and replaced with probability  $1 - \gamma$  for  $\gamma \in (0,1)$ . Retirements happen independently. Then, the game continues to the next period. <sup>14</sup>

## 3.3 Signals and Beliefs

The game has incomplete information. Each proposer-candidate receives a signal for each of the vetoer-candidates from nature. The signals are random variables whose images are  $\{\sigma_L, \sigma_H\}$ .  $\sigma_L$  ( $\sigma_H$ ) is interpreted as the low (high) signal that the vetoer-candidate is likely low (high) type—low (high) type indicates the vetoer-candidate's low (high) bargaining cost. A signal is independent of any other random variable excluding the type of the vetoer-candidate for whom the signal is sent. Probability of  $\sigma_L$  is the same for all vetoer-candidates of the same type. The signals mean that the proposer-candidates and the proposer-agents are better informed than the proposer-principal. The vetoer-candidates do not receive signals about proposer-candidates because modeling such signals is not important. <sup>15</sup>

 $P_{L|L}$  is the conditional probability that the vetoer-candidate is low type when  $\sigma_L$  is sent about her to the proposer-candidates.  $P_{H|L}$  is the conditional probability that the vetoer-candidate is high type when  $\sigma_L$  is sent about her to the proposer-candidates.  $P_{L|H}$  and  $P_{H|H}$  are defined in the same way.  $1 > P_{L|L} > P_{L|H}$  and  $1 > P_{H|H} > P_{H|L}$ . If any player believes that the vetoer-candidate is low type with probability  $P_{L|L}$  and high type with probability  $P_{H|L}$ , the player is said to believe  $b_L$  about the vetoer-candidate. If any player believes that the vetoer-candidate is low type with probability  $P_{L|H}$  and high type with probability  $P_{L|H}$ , the player is said to believe  $b_H$  about the vetoer-candidate. Depending on the vetoer-candidates' moves in the game, players' beliefs about the vetoer-candidates' can change. A player who believes that the vetoer-candidate is low (high) type with probability 1 is said to believe  $b_A$  ( $b_Y$ ).

# 3.4 Bargaining Costs, Types and Payoffs

The principals and the candidates have bargaining costs. The proposer-principal's bargaining cost is  $c_{PP}$ . The vetoer-principal's bargaining cost is  $c_{VP}$ . The candidates' bargaining costs are random variables that are independent of other candidates' bargaining costs. These costs are

- 12. Hwang (2025) shows that in a finite bargaining game with bargaining costs where one player makes all the offers having a large number of offers leads to PBEs where the player making the offers has a great advantage. To avoid this, I limit offers to two per deal.
  - 13. If no deal is made,  $a_t$  and  $s_t$  do not exist.
- 14. Without retirements, once the vetoer-principal finds an agent who does not take a bad deal for her, the principal may keep the agent forever. Then, the vetoer's side may always get good deals and get them in stage 1 forever. This makes the problem of costly delays disappear. In general, if the bargaining situation never changes, the parties could keep agreeing on the same deal forever.
- 15. This is because the vetoer-agents can make decisions based on the offers she gets and the vetoer-principal's strategy without having information about the proposer-candidates. However, the proposer-agents can use the signals they receive to decide which offer to make.

their types. Proposer-candidate i's type is  $C_{PAi}$  and these are independent and identically distributed. Vetoer-candidate i's type is  $C_{VAi}$  and these are independent and identically distributed. Candidates' types are private information.

A proposer-candidate's type is realized to be  $c_{PL}$  or  $c_{PH}$  with positive probability for both realizations. Vetoer-candidate's type is realized to be  $c_{VL}$  with probability  $P_L$  and  $c_{VH}$  with probability  $P_H$ . A proposer-candidate with type  $c_{PL}$  or  $c_{PH}$  is called low type or high type respectively. I use these terms in the same way for the vetoer-candidates.  $0 \le c_{PL} < c_{PH}$ .  $0 \le c_{VL} < c_{VH}$  and  $c_{VP} \le c_{VH}$ . Is the type realization for the proposer-agent who works in period t.  $c_{V}(t)$  is the type realization for the vetoer-agent who works in period t.

Players' payoffs are the discounted sum of their period-utilities, which I refer to as utilities. Candidates get 0 utility in periods for which they are not agents. The discount factor for the principals is  $\delta \in (0,1)$ . For the candidates, it is 1. This is because for the candidates,  $1-\gamma$ , the probability of retirement performs the role of the discounting factor.

proposer-principal's utility for period 
$$t \equiv \begin{cases} r - a_t - s_t c_{PP} & \text{if deal is made in period } t \\ -2c_{PP} & \text{otherwise} \end{cases}$$

proposer-agent's utility for period 
$$t \equiv \begin{cases} r - a_t - s_t c_P(t) + w_P & \text{if deal is made in period } t \\ -2c_P(t) + w_P & \text{otherwise} \end{cases}$$
 (1)

, where  $r \ge 0$  is a constant.  $-2c_{PP}$  and  $-2c_{P}(t)$  are used in the utilities because if no deal is made that means the players wasted two stages in bargaining.  $w_P$  is the proposer-candidate's wage from his job as the agent and is bounded from below.

$$w_P \ge -r - c_{VL} + c_{PH} \tag{2}$$

The agents on both sides have wages with lower bounds. This means the agents have incentives to keep their jobs.<sup>17</sup> Without such a wage, agents may not want the job and try to deliberately get themselves replaced. The cost of paying of wages need not be in the principals' utilities because the wages are paid in every period regardless of what happens. The differences in utilities above are the existence of the agent's wage and that the principal's bargaining cost per stage is  $c_{PP}$  and the agent's bargaining cost per stage is  $c_P(t)$ . The wage and the different bargaining costs can cause the agent to not bargain in the principal's best interest. The utility definitions mean that the proposer-principal and the proposer-agents prefer smaller  $a_t$ 's.

vetoer-principal's utility for period 
$$t \equiv \begin{cases} a_t - s_t c_{VP} & \text{if deal is made in period } t \\ -2c_{VP} & \text{otherwise} \end{cases}$$

vetoer-agent's period-utility for period 
$$t \equiv \begin{cases} a_t - s_t c_V(t) + w_V & \text{if deal is made period } t \\ -2c_V(t) + w_V & \text{otherwise} \end{cases}$$
 (3)

16.  $c_{VP}$  has an upper bound because if the vetoer-principal's bargaining cost is too high, she wants to get a quick deal even if it is unfavorable for her. If this leads her to not remove agents who take unfavorable deals, she does not function as a supervisor.

17. In reality, the "wages" on both sides are the agents' total benefits from the job. The benefits include actual wages, fringe benefits, job satisfaction and job prestige.

A deal of  $a_t = 0$  in stage 2 gives the vetoer-principal and the vetoer-agent same utilities as a no deal.  $w_V$  is the vetoer-agent's wage. <sup>18</sup>

$$w_V \in [c_{VL} + c_{VH}, \frac{1 - \gamma}{\gamma}(c_{VH} - c_{VL}) + c_{VH} + c_{VL})$$
(4)

Like in the proposer side, the difference in the utilities is in the wages and the bargaining costs per stage and the difference may cause the agent to not bargain in the principal's best interest. The utilities mean that the vetoer-principal and the vetoer-agents prefer larger  $a_t$ 's.

#### 3.5 Aggression in Bargaining

This subsection provides an intuitive explanation of aggressive and non-aggressive bargaining strategies. Consider a vetoer-agent who does not care about removal by her principal. Because of equation 3, in stage 2 the vetoer-agent accepts an offer of 0. For period t, compare the utility from this deal to the utility from a deal at  $a_{t,1} = -c_V(t)$  using the following equation. (Recall that  $a_{t,1} \in R^1$ .)

$$0 - 2c_V(t) + w_V = -c_V(t) - c_V(t) + w_V$$
(5)

Since the utilities are equal, she accepts  $-c_V(t)$  in stage 1. In other words, she will take a weakly worse offer in stage 1 because it saves her the bargaining cost of stage 2. If proposeragent knows her type in period t, he offers  $-c_V(t)$ . Thus, for any period t, if the proposer-agent knows the vetoer-agent's type, until she is replaced, he continues to offer  $-c_V(t)$  in stage 1 for period t and later periods. This means if the vetoer-agent is revealed to be high type to the proposer-agent, the vetoer-principal has an incentive to replace the agent. If the agent is not replaced, the deal will continue to be  $-c_{VH}$ . Also, the vetoer-principal has an incentive to keep an agent revealed to be low type. By keeping this agent, the deal will continue to be  $-c_{VL}$ .

I define constants,  $\bar{a}$ ,  $\check{a}$  and  $\underline{a}$ , to represent potential offers that the proposer-agent might make.

$$\bar{a} \equiv -c_{VL}$$

$$\check{a} \equiv \frac{\gamma}{1 - \gamma} (w_V - c_{VL} - c_{VH}) - c_{VH}$$

$$\underline{a} \equiv -c_{VH}$$
(6)

I impose the condition that  $P_H \check{a} - P_L c_{VP} \leq \bar{a}$  to ensure the vetoer-principal wants a deal of  $\bar{a}$  in the first stage. By expression 4,  $\underline{a} \leq \check{a} < \bar{a}$ .

Suppose a vetoer-agent is offered  $\check{a} < \bar{a}$  in stage 1 of some period t. For now, the proposer-agent does not know the vetoer-agent's type. If the vetoer-agent rejects  $\check{a}$ , from then, the proposer-agent will believe that she is low type. He will offer her 0 in the ensuing stage 2 and offer her  $\bar{a} = -c_{VL}$  in stage 1 for future periods in which she works. If the vetoer-agent accepts  $\check{a}$ , from then, the proposer-agent will believe that she is high type and the vetoer-principal will replace the vetoer-agent.

$$a_{t,1} \gtrapprox \check{a}$$
 $\leftrightarrow$ 

18.  $w_V$  has an upper bound because if it is too high, it is too easy to get the vetoer-agent to do what the vetoer-principal wants.

$$a_{t,1} \stackrel{\geq}{=} \frac{\gamma}{1 - \gamma} (w_V - c_{VL} - c_{VH}) - c_{VH} \tag{8}$$

$$a_{t,1} - c_{VH} + w_V \ge 0 - 2c_{VH} + w_V + \sum_{i=1}^{\infty} \gamma^i (\bar{a} - c_{VH} + w_V)$$
 (9)

This means a high type vetoer-agent will accept  $\check{a}$  in stage 1 of period t because the payoffs starting from period t are equal for accepting and rejecting to be believed as the low type. Note that  $\check{a} \geq -c_{VH}$  because  $w_V \geq c_{VL} + c_{VH}$ . In equation 6,  $\frac{\gamma}{1-\gamma}(w_V - c_{VL} - c_{VH}) = \sum_{i=1}^{\infty} \gamma^i (\bar{a} - c_{VH} + w_V) \geq 0$  is the redress from the proposer-agent to the vetoer-agent for accepting  $\check{a}$  and getting replaced.

Expressions 7~9 mean that when  $a_{t,1} = \check{a}$ , I have the following.

The above means that while the high type vetoer-agent accepts  $\check{a}$ , the low type vetoer-agent prefers to reject it and get  $-c_{VL}$  offered in the remaining periods for her. If the offer is  $\bar{a}$ , low type vetoer-agent will accept it. By expression 3, when she accepts  $\bar{a}$ , her utility in the period is the same as the utility from a no deal. In her remaining periods, she can also get the same utility as that from a no deal. So the high type vetoer-agent accepts  $\check{a}$  in any period if the proposer-agent believes  $b_L$  or  $b_H$  and in any period, the proposer-agent with either of these beliefs can offer  $\check{a}$  in stage 1 to figure out whether the vetoer-agent is high type.

Given the vetoer-agents' strategy, in stage 1, the proposer-agent who does not know the vetoer-agent's type faces the problem of whether he should offer  $\check{a}$  or  $\bar{a}$ . If the proposer-agents' strategy is to offer  $\check{a}$  or  $\bar{a}$  when the vetoer-agent type is unknown, any vetoer-agent will accept  $\bar{a}$  when her type is unknown.  $\check{a}$  gives a better deal for him when accepted but will only be accepted by the low type vetoer-agents.  $\bar{a}$  is a worse offer for him but will be accepted by any vetoer-agent. Because of this, offering  $\check{a}$  is more aggressive than offering  $\bar{a}$ .

If a proposer-agent offers  $\check{a}$  in stage 1, I state that he acts aggressively. If he offers  $\bar{a}$  in stage 1, I state that he acts non-aggressively. If his strategy is to offer  $\check{a}$  in stage 1 as long as he believes  $b_L$  or  $b_H$  about the vetoer-agent, I state that he bargains aggressively. If his strategy is to offer  $\bar{a}$  in stage 1 as long as he believes  $b_L$  or  $b_H$  about the vetoer-agent, I state that he bargains non-aggressively. In this context, "acting" applies to a single move but "bargaining" applies to the strategy. I also use statements such as "the proposer-agent whose strategy is to offer  $\check{a}$  in stage 1 as long as he believes  $b_L$  about the vetoer-agent bargains aggressively for  $b_L$ ".

# 4 PBEs with Proposer-agent Autonomy

The two types of PBEs in this section, the Adaptive Equilibrium and the Nonadaptive Equilibrium have proposer-agent autonomy. The autonomy means that the proposer-agents are free to bargain as they want. This is because the proposer-principal never replaces his agents even if the agents deviate from their equilibrium strategies.

The following condition states the vetoer side's strategy for the four main types of PBEs of this paper, the Adaptive Equilibrium, Nonadaptive, Replacement and Imitation Equilibrium.

The vetoer-agents' strategy is defined using cut-off points. A vetoer-agent with a cut-off point of a in a stage rejects any offer less than a and accepts any offer of a or a greater amount in that stage for any period that she works in.

#### Condition 1.

- (i) The vetoer-principal's strategy is to remove her agent in period t if her agent accepted  $a_{t',1} < \bar{a}$  for some  $t' \le t$  and not replace her agent otherwise.
- (ii) Low type vetoer-agents' cutoff point in stage 1 is  $\bar{a}$ .
- (iii) High type vetoer-agents' cutoff point in stage 1 is  $\check{a}$  when the proposer-agent believes  $b_{\Lambda}$ ,  $b_L$  or  $b_H$  about her and  $\underline{a}$  when the proposer-agent believes  $b_{\Upsilon}$  about her.<sup>19</sup>
- (iv) Low type proposer agents' cutoff point in stage 2 is 0.

Recall that for any player,  $b_{\Lambda}$  ( $b_{\Upsilon}$ ) is the player's belief that the vetoer-agent is likely low (high) type.  $b_L$  ( $b_H$ ) is the belief that corresponds to signal  $\sigma_L$  ( $\sigma_H$ ).  $\sigma_L$  ( $\sigma_H$ ) signals that the vetoer-agent is likely low (high) type and likely has low (high) cost.

 $\check{a} < \bar{a}$  means that when the proposer-agent believes  $b_L$  or  $b_H$ , the high type vetoer-agent will take a lower deal in stage 1 compared to the low type vetoer-agent. Once the agent is revealed to be high type, unless the agent is replaced, the proposer-agent can continue to get a deal of  $\underline{a} = -c_{VH}$  from her in stage 1. This is why the vetoer-principal replaces vetoer-agents who accept  $a_{t',1} < \bar{a}$ .

## 4.1 Adaptive Equilibrium

An adaptive equilibrium is a PBE where if any proposer-agent's belief about the vetoer-agent is  $b_L$ , he acts non-aggressively and if any proposer-agent's belief about the vetoer-agent is  $b_H$ , he acts aggressively. So all proposer-agents use the signal in the same manner to decide whether to act aggressively. Their strategy is justified because if the belief of  $b_L$  not  $b_H$  is true, the vetoeragent is more likely to be low type and aggression is more likely to fail. So the proposer-agents use the signals effectively to determine whether to act aggressively. Tables  $1\sim4$  specify whether proposer-agents act aggressively for different situations in the four types of PBEs of this paper.

**Definition 1.** Adaptive Equilibrium is a PBE satisfying the following.

- (i) The proposer-principal's strategy is to never replace his agents.
- (ii) Any proposer-agent bargains aggressively for  $b_H$  and non-aggressively for  $b_L$ .
- (iii) Condition 1 holds.

**Proposition 1.** An Adaptive Equilibrium exists if the following holds

(i) 
$$c_{VL} + (1 - \gamma^2)(P_{H|L}\check{a} + P_{L|L}c_{PH}) + \gamma(1 - \gamma)(P_{H}c_{VH} - P_{L}c_{PH}) \ge \gamma((1 - \gamma P_{L|L})(-P_{H}\check{a} - P_{L}c_{PH}) + \gamma P_{L|L}c_{VL})$$

(ii) 
$$c_{VL} \leq -P_{H|H}\check{a} - P_{L|H}c_{PH}$$

19. The vetoer-agent may not always know the proposer-agent's exact belief about her. However, the vetoer-agent knows that the proposer-agent observes her moves which can affect his belief about her. Therefore, she may know that the belief is either  $b_L$  or  $b_H$  and so on.

		Belief about th	ne vetoer-agent	
		$b_L$	$b_H$	
Type of proposer-agent	Low type	No	Yes	
	High type	No	Yes	

Table 1: Whether the proposer-agent acts aggressively in an Adaptive Equilibrium

		Belief about the vetoer-agent	
		$b_L$	$b_H$
Type of proposer-agent	Low type	No	No
	High type	No	No

Table 2: Whether the proposer-agent acts aggressively in a Nonadaptive Equilibrium

		Belief about the vetoer-agent	
		$b_L$	$b_H$
Type of proposer-agent	Low type	Yes	Yes
	High type	No	No

Table 3: Whether the proposer-agent acts aggressively in a Replacement Equilibrium

		Belief about the vetoer-agent	
		$b_L$	$b_H$
Type of proposer-agent	Low type	Yes	Yes
	High type	Yes	Yes

Table 4: Whether the proposer-agent acts aggressively in an Imitation Equilibrium on the equilibrium path

From the above proposition, (i) and (ii) are conditions for the proposer-agent. (i) means that the proposer-agent weakly prefers nonaggression to aggression when he believes  $b_L$  about the vetoer-agent. (ii) means that the proposer-agent weakly prefers aggression to nonaggression when he believes  $b_H$  about the vetoer-agent. On (ii), if the proposer-agent who believes  $b_L$  or  $b_H$  acts non-aggressively, he can get a deal of  $-a_t = -\bar{a} = c_{VL}$ . Equation 1 shows that the proposer-agent wants a larger  $-a_t$ . If he acts aggressively and the vetoer-agent is high type, he gets a deal of  $-a_t = -\check{a} > -\bar{a}$ . If he acts aggressively and the vetoer-agent is low type, he gets a deal of  $-a_t = 0$  in stage 2 and pays an additional bargaining cost. The additional cost is  $c_{PL}$  for the low type proposer-agent and  $c_{PH}$  for the high type proposer-agent.  $P_{L|L}$ ,  $P_{H|L}$ ,  $P_{L|H}$  and  $P_{H|H}$  are the conditional probability of the vetoer-agent's type for the signals.

(ii) means that for a proposer-agent of any type, if he believes  $b_H$  about the vetoer-agent, the expected utility from aggression is weakly greater than the expected utility from nonaggression. He can get a better deal by aggression if the vetoer-agent is high type. However, if not, he ends up paying an additional bargaining cost. Since all proposer-agent act the same for two relevant beliefs of  $b_L$  and  $b_H$ , the proposer-principal has no incentive to replace his agent.

## 4.2 Nonadaptive Equilibrium

A Nonadaptive Equilibrium is a PBE where all proposer-agents act non-aggressively when the agent believes  $b_L$  or  $b_H$  about the vetoer-agent. No proposer-agent is willing to take the risk of aggression and higher bargaining cost for him even when the agent believes  $b_H$  not  $b_L$  and aggression is comparatively more likely to succeed.

**Definition 2.** A Nonadaptive Equilibrium is a PBE satisfying the following.

- (i) The proposer-principal's strategy is to never replace his agents.
- (ii) Any proposer-agent bargains non-aggressively.
- (iii) Condition 1 holds.

In a Nonadaptive Equilibrium, on the equilibrium path, the principal-agent's beliefs about the vetoer-agents is  $b_L$  ( $b_H$ ) if the signal for the vetoer-agent is low (high). Furthermore, the principal-agent always offers  $\bar{a}$  in stage 1 and this is always accepted.

**Proposition 2.** An Nonadaptive Equilibrium exists if  $c_{VL} \ge -P_{H|H}\check{a} - P_{L|H}c_{PL}$ .

Consider a proposer-agent who believes  $b_H$  about the vetoer-agent. The proposer-agent gets  $-a_t = -\bar{a} = c_{VL}$  if he acts non-aggressively. If the proposer-agent acts aggressively, and the vetoer-agent is high type, he gets  $-a_t = -\check{a}$ . If the proposer-agent acts aggressively and the vetoer-agent is low type, he gets  $-a_t = 0$  in stage 2 and pays an additional bargaining cost (of  $c_{PL}$  if he is low type) for getting a deal later in stage 2. For a belief of  $b_H$ ,  $P_{H|H}$  and  $P_{L|H}$  are, respectively, the conditional probabilities that the vetoer-agent is high and low type. When the belief is  $b_H$  not  $b_L$ , the proposer-agent believes that aggression is more likely to succeed. Thus,  $c_{VL} \geq -P_{H|H}\check{a} - P_{L|H}c_{PL}$  means that when a proposer-agent believes  $b_L$  or  $b_H$  about the vetoer-agent, her expected utility is greater for nonaggression compared to aggression and she weakly prefers nonaggression. Given the large  $P_{L|H}$  and small of  $P_{H|H}$ , all proposer-agents think aggressive bargaining will not work.

The Nonadaptive Equilibrium represents situations where no representative is willing to resort to political gridlock to get a favorable deal or the labor union leader is unwilling to strike to get a higher wage. The representative or the leader knows that doing so will likely fail. To avoid the costly delays in deals, he sticks to making deals favorable for the other side. In the Adaptive and Nonadaptive Equilibria, there is a consensus among proposer-agents on the best course of action given the signals and the proposer-agents bargain in the same manner. Since the proposer-principal is unable to get an agent who bargains in a different manner, the proposer-agents have autonomy to bargain how they want.<sup>20</sup>

# 5 PBEs with Proposer-agent Subjection

This section explains two types of PBEs, the Replacement Equilibrium and the Imitation Equilibrium. In both types, the high type proposer-agent is more reluctant to act aggressively and the strategies differ for the low and high type proposer-agent. (In the Imitation Equilibrium, the high type is less aggressive off the equilibrium path.) Both Replacement and Imitation Equilibria satisfy the following condition.

20. Readers may point out that while there are no incentives for the proposer-principals to replace the agents, there is no incentive to keep the agents either. However, in reality, principals may feel attachment to agents who worked for them or agents with more experience may be better at their jobs. Both factors would weigh against replacing agents.

#### Condition 2.

- (i) The proposer-principal's strategy is to replace any proposer-agent if the proposer-agent did not act aggressively for  $b_L$  or  $b_H$  in some previous period or this period. Otherwise, proposer-agents are not replaced.
- (ii) The low type proposer-agent's strategy is to bargain aggressively.
- (iii) Condition 1 holds.

 $b_L(b_H)$  is the belief that corresponds to the low (high) signal which indicates that the vetoeragent is likely low (high) type and likely has low (high) bargaining cost. In both types of PBEs, the proposer-principal replaces agents who do not act aggressively for  $b_L$  or  $b_H$ . This leads the low type proposer-agent to bargain aggressively.

## 5.1 Replacement Equilibrium

**Definition 3.** A Replacement Equilibrium is a PBE satisfying the following.

- (i) Condition 2 holds.
- (ii) The high type proposer-agent's strategy is to bargain non-aggressively.
- (ii) means the high type proposer-agents are replaced for nonaggression. The signal for a vetoer-agent is  $\sigma_L$  or  $\sigma_H$ . This means a proposer-agent believes  $b_L$  or  $b_H$  about her. Thus, the high type proposer-agent acts non-aggressively against her and doing so gets him promptly replaced. On the other hand, the low type proposer-agents are not replaced.

**Proposition 3.** The following is a sufficient condition for the existence of an Replacement Equilibrium. (i) is also a necessary condition.

$$(i) c_{VL} \le -P_H \check{a} - P_L c_{PP} \tag{10}$$

(ii) 
$$c_{VL} \leq -P_{H|L}\check{a} - P_{L|L}c_{PL} + \gamma(r + c_{VL} - c_{PL} + w_P)$$

(iii) 
$$c_{VL} \ge -P_{H|H}\check{a} - P_{L|H}c_{PH} + \frac{\gamma}{1-\gamma}(r + c_{VL} - c_{PH} + w_P)$$

- (i) is a condition on the proposer-principal's preference. The proposer-principal does not receive the signals. Therefore, for a new vetoer-agent, he believes that she is low type with probability  $P_L$  and high type with probability  $P_H$ . If the proposer-agent acts non-aggressively against this vetoer-agent, he gets  $-a_t = -\bar{a} = c_{VL}$ . If the proposer-agent acts aggressively against her and she is high type, he gets  $-a_t = -\check{a} > -\bar{a}$ . However, if she is low type, he gets  $-a_t = 0$  in stage 2 and pays an additional bargaining cost of  $c_{PP}$ .
- When (i) holds, the proposer-principal, from his perspective, wants or tolerates aggression. The proposer-principal deems a proposer-agent who does not act aggressively for  $b_L$  or  $b_H$  to be high type. If this agent keeps his job in the future, the agent will still not act aggressively for  $b_L$  or  $b_H$ . Thus, the proposer-principal will replace this agent following definition 2.(i). If the new proposer-agent is low type, the new agent will be more aggressive.
- (ii) means that the proposer-agent's wage,  $w_P$  is high enough that the low type proposer-agent finds it optimal to bargain aggressively and not get removed. (iii) means that  $w_P$  is low enough that the high type proposer-agent finds it optimal to bargain non-aggressively even if it gets him removed. While  $w_P$  is the same for both types,  $c_{PL} \le c_{PH}$  means that the cost of failed aggression, the additional bargaining cost, is weakly higher for the high type.

## 5.2 Imitation Equilibrium

**Definition 4.** An Imitation Equilibrium is a PBE satisfying the following.

- (i) Condition 2 holds.
- (ii) The high type proposer-agent's strategy is to act non-aggressively for  $b_L$  and  $b_H$  if in some previous period, he did not act aggressively for  $b_L$  or  $b_H$ .
- (iii) The high type proposer-agent's strategy is to act aggressively for  $b_L$  and  $b_H$  otherwise.

Following condition 2.(i), in an Imitation Equilibrium, the proposer-principal replaces proposer-agents who do not exhibit aggression. Under the threat of replacement, both types of proposer-agent bargain aggressively on the equilibrium path. Hence, the high type proposer-agent imitates the behavior of the low type proposer-agent and on the equilibrium path, agents are never replaced.

**Proposition 4.** The following is a sufficient condition for the existence of an Imitation Equilibrium. (i) is also a necessary condition.

(i) 
$$c_{VL} \leq -P_H \check{a} - P_L c_{PP}$$

(ii) 
$$c_{VL} \le -P_{H|L}\check{a} - P_{L|L}c_{PH} + \gamma(r + c_{VL} - c_{PH} + w_P)$$

(iii) 
$$c_{VL} \ge -P_{H|H}\check{a} - P_{L|H}c_{PH}$$

- (i) and proposition 3's (i) are the same.
- (ii) means that the proposer's wage,  $w_P$  is high enough that any proposer-agent finds it optimal to aggressively for  $b_L$  and  $b_H$  on the equilibrium path. On this path, not doing so will get him removed and cost him future wages. If the high type deviated to not acting aggressively for  $b_L$  or  $b_H$  in some previous period, he will be removed at the end of the period and acting aggressively will not let him earn wages in the future. In this case, (iii) means he finds it optimal to follow definition 4.(ii) and act non-aggressively for  $b_L$  and  $b_H$  in the current period.

# 6 Effects of Signals and Inefficiency

**Proposition 5.** Given all the other parameters of the model, there exists some  $\varepsilon > 0$  such that if  $1 - P(L|L) < \varepsilon$  and  $1 - P(H|H) < \varepsilon$ , an Adaptive Equilibrium exists but Nonadaptive, Replacement and Imitation Equilibria do not exist.

The above proposition means that very accurate signals lead to an Adaptive Equilibrium. If the signal about the vetoer-agent is very accurate, the proposer-agents' beliefs will also be very accurate and any proposer-agent can use the beliefs to make the lowest offer vetoer-agent will take most of the time. This allows an Adaptive Equilibrium to exist because the proposer-agent can bargain aggressively when his belief about the vetoer-agent is given by the  $\sigma_L$  and non-aggressively when his belief is given by the  $\sigma_H$ . In this Adaptive Equilibrium, a deal will be very likely in the first stage because aggression will rarely fail. Then, filibusters, government shutdowns and labor strikes will rarely happen as they result from failures of quick deals. The proposition demonstrates that if the proposer-agent gets the best deal possible for his principal most of the time by using his very accurate information and his principal knows this, the principal lets the agent do what the agent wants and does not remove agents. Here, the principal and the agents trust each other.

**Condition 3.** Proposer-agent 1 follows the equilibrium strategy except that in period 1 he acts aggressively for  $b_H$  and non-aggressively for  $b_L$  when he is low type.

Note that the above condition means the low type proposer-agent is less aggressive compared to bargaining aggressively.

**Proposition 6.** Fix all parameters of the model except  $c_{PP}$ . If  $c_{VL} < -P_H \check{a}$ , there exists some  $\underline{C}$  such that the following holds.

- $c_{VL} < -P_H \check{a} P_L C$
- For any  $c_{PP} \ge \underline{C}$ , in any Replacement or Imitation Equilibria, if proposer-agent 1 deviates to a strategy that follows condition 3, the proposer-principal's expected payoff increases.

The above proposition demonstrates how Replacement and Imitation Equilibria can be inefficient for the proposer-principal. Recall that when aggression succeeds,  $-a_t = -\check{a}$ .  $c_{VL} < -P_H\check{a}$  means that the benefit of aggression is high.  $c_{VL} < -P_H\check{a} - P_L\underline{C}$  means that when  $c_{PP} = \underline{C}$ , the expected utility from aggression is greater than the expected utility from nonaggression for the proposer-principal when the proposer-principal's belief about the vetoer-agent is given by the type distribution of the vetoer-agent. Then, the proposer-principal wants aggressive bargaining.

If aggression fails, the proposer-principal's bargaining cost for the period increases by  $c_{PP}$ , his bargaining cost per stage. When  $c_{PP} \ge \underline{C}$ , the proposer-principal has a greater cost from aggression failure. If the signal for the vetoer-agent is low, based on the signal, the vetoer-agent is likely low type and aggression is likely to fail. Thus, if the cost of aggression failure is high and the vetoer-agent is likely low type, the expected utility from nonaggression may be greater and nonaggression can be actually better for the proposer-principal. This is demonstrated by the fact that the deviation of the low type proposer-agent, which involves acting non-aggressively in period 1 when the vetoer-agent is likely low type, increases the proposer-principals expected payoff.

The dilemma for the proposer-principal in the equilibria is that if he keeps agents who act non-aggressively, he will miss chances to get a good deal with aggression. However, since he does not receive the signals his agents receive, he cannot selectively remove agents based on whether they acted non-aggressively in cases where the vetoer-agent was likely to be low type. Hence, if he removes them, his agents respond by acting aggressively even when aggression will likely fail and is harmful to the proposer-principal. In the equilibria, even when agents are able to figure out whether bargaining longer will result in a good deal with some accuracy, the agents act aggressively even in situations when they know aggression will likely fail and is harmful to the principal.

When proposer-agents' wages are sufficiently high so that an Imitation equilibrium exists, both types of proposer-agents will bargain aggressively on the equilibrium path. However, the reason that the high type acts aggressively even in situations where aggression will likely fail is not because aggression maximizes the proposer-principal's payoff. (It might not.) Instead, it is because nonaggression will lead to removal and loss of the high wage. Hence, the proposer-principal's attempt to get low type agents who will bargain aggressively and get rid of high type agents may actually land the proposer-principal with a high type agent who imitates the low type agent to get a high wage.

## 7 Extended Model

The extended model demonstrates how the social cost of bargaining can be reduced by shortening bargaining time. In this model, each stage of bargaining takes a fixed amount of time and the

bargaining time per stage is represented by  $\tau > 0$ . The principals' and the agents' bargaining costs per stage are their bargaining costs per unit time multiplied by  $\tau$ .  $c_{PP}$  is the proposer-principal's bargaining cost per unit time.  $\bar{c}_{PP}$  is the proposer-principal's bargaining cost per stage are denoted likewise for other players. (Recall that  $c_{VP}$  is the vetoer-principal's bargaining cost per stage.  $c_{PL}$  and  $c_{PH}$  are the bargaining cost per stage for the low and high type proposer-agents respectively.  $c_{VL}$  and  $c_{VH}$  are the bargaining cost per stage for the low and high type vetoer-agents respectively.)

```
c_{PP} = 	au ar{c}_{PP}
c_{VP} = 	au ar{c}_{VP}
c_{PL} = 	au ar{c}_{PL}
c_{PH} = 	au ar{c}_{PH}
c_{VL} = 	au ar{c}_{VL}
c_{VH} = 	au ar{c}_{VH}
```

For this model, I impose  $\bar{c}_{PP} \ge 0$  and  $P_H \check{a} \le \bar{a}$ .  $P_H \check{a} \le \bar{a}$  guarantees that the vetoer-principal wants nonaggression in the context of the following proposition.

**Proposition 7.** Suppose that a PBE belonging to one of the four types of PBEs exists and that the PBE satisfies the sufficient condition for its type in propositions  $1 \sim 4$ . Change  $\bar{c}_{PP}$ ,  $\bar{c}_{VP}$ ,  $\bar{c}_{PL}$ ,  $\bar{c}_{PH}$ ,  $\bar{c}_{VL}$ ,  $\bar{c}_{VH}$  and  $\tau$  so that  $\bar{c}'_{PP} = y\bar{c}_{PP}$ ,  $\bar{c}'_{VP} = y\bar{c}_{VP}$ ,  $\bar{c}'_{PL} = x\bar{c}_{PL}$ ,  $\bar{c}'_{PH} = x\bar{c}_{PH}$ ,  $\bar{c}'_{VL} = x\bar{c}_{VL}$ ,  $\bar{c}'_{VH} = x\bar{c}_{VH}$  and  $\tau' = \frac{\tau}{x}$  for some x and y that  $0 < y \le x$ . Then, same PBE still exists.

The above proposition shows that for the four types of PBEs, the same PBE may exist when the bargaining costs per unit time of all agents are multiplied by some x > 1 but the bargaining costs per unit time of all principals are unchanged (y = 1 in the proposition). For such PBEs, a key implication of the proposition is that when the bargaining costs per unit time of all agents are increased, bargaining time per stage decreases to  $\frac{\tau}{x}$ . Intuitively, increases in the bargaining costs per unit time mean that for the agents, the cost of waiting to make a new offer increases and the cost of waiting for a new offer increases as well. Therefore, the agents are incentivized to come to an agreement faster and bargaining time per stage decreases as a result.

Because bargaining time per stage decreases, the principals' bargaining costs per stage decreases and their expected payoffs increase. Since bargaining time per stage is multiplied by  $\frac{1}{x}$ , the bargaining costs per stage remain the same for all the agents. Because agents still pay the same bargaining costs per stage, the principals still know how hard the agents tried to get a good deal and agent removal does not increase. If there is an external cost of bargaining, the external cost would decrease as well because bargaining time per stage is reduced. Therefore, in these cases, increasing the bargaining costs per unit time of all agents reduces the social cost of bargaining.

## 8 Discussion

## 8.1 Cause of Excessive Bargaining Costs

In bargaining with delegation, the principal might be uncertain of what the best deal they can get is and not trust the agents to get the best deal for them. Agents signal that they will bargain hard for their principal through aggression. Principals may demand that the agents risk higher

bargaining costs for a better deal and threaten those that do not with removal. This can lead to political gridlock and labor strikes and is described by the Replacement and Imitation equilibria.

In reality, some filibusters are conducted despite their likely failure. Such filibusters are explained as a message to the public and/or promotion of the representative or the party conducting the filibuster.<sup>21</sup> Similarly, a strike can promote a leader of a labor union. Walter Reuther was United Automobile Workers director of General Motors Department for the General Motors Strike of 1945-1946. His leadership in the strike was one of the reasons he was elected the president of the United Automobile Workers Union in 1946.<sup>22</sup>

#### 8.2 Reduction of Social Costs

As mentioned in the introduction, political gridlock and labor strikes can entail substantial external costs. In these contexts, as proposition 7 shows, the bargaining costs per unit time should be increased to induce a faster deal and reduce the costs of the principals and external and social costs. For this solution, the increase in agents' bargaining cost per unit time can make the agents internalize the external costs of bargaining when the agents also have to pay the costs of a deal delay suffered by external parties. An alternative solution to the problem of social costs in case of labor disputes is transition to non-stoppage strikes and impose bargaining costs by taxes instead of by lost wages and profits. These taxes do not generate lost production.

#### **8.2.1 Quarantined Meeting Solutions**

On how to increase bargaining cost for agents, I look at the example of the conclave, which is the procedure of voting in a locked room for papal elections. After the election of Pope Gregory X took nearly three years, Pope Gregory X instituted new rules for papal elections. The cardinals who vote would be secluded in a conclave and they could not communicate with the outside world without the agreement of all. After five days of the conclave, they would only be given the provisions of bread, water and a little wine. Although the rule went through suspensions, a version of the rule remains in effect in the present.<sup>23</sup>

Quarantined meetings similar to the conclave that are rough for the participants can also work for congressional bargaining and labor union negotiations. In these meetings, there will be no entertainment or amenities. Outside communication is prohibited unless it is a statement to the public or is necessary for negotiation. Provisions should be strictly restricted. Meetings are only dissolved when a deal is reached. If agents bargain in such meetings and their hardship is widely reported, agents can convince their principals they are working hard and will continue to do so for the principals. Furthermore, the arduousness of the process will lead to a swift deal, which may reduce social cost.

Thus, quarantined meetings can solve political gridlock in cases where the gridlock is harmful. For instance, suppose a bill is blocked because of a filibuster, a presidential veto or disagreement between assemblies and the number of representatives in agreement needs to be higher for the bill to become law. If the goal is to dissolve the gridlock, representatives and the president should have the power to call an arduous quarantined meeting that cannot be broken up until this required number of representatives agree to pass a bill or to pass no bill.

- 21. See Cohodas (1993, pp. 294–298), Ibrahim (2025), "Ted Cruz's Obamacare All-Nighter Ends After 21 Hours" (2013), Saenz (2013), McCarthy (2013), O'Brien (2013), Choi and Jeon (2025), Kim (2025), Lee and An (2025), and Lee (2025).
  - 22. See Cormier and Eaton (1970, pp. 197, 233-234), Howe and Widick (1949) and Minchin (2024).
  - 23. See Baumgartner (2003, pp. 2,37–45) and McElwee and Balmer (2025).

Given the possibility of abuse, even if filibusters are necessary to prevent the tyranny of the majority, they should not be too easy. Otherwise, congress can become dysfunctional.<sup>24</sup> Historically, a senator filibustering a law was required to stand up and talk. Currently, filibustering a law does not require the senator to stand up and talk.<sup>25</sup> In 2021, President Joe Biden said that he supports bringing back the requirements that one needs to stand and talk during a filibuster.<sup>26</sup> More reforms can be made on top of these requirements. For instance, if 60% is required to break a filibuster, Congress can require that the 40% in support of the filibuster must remain in the chamber and stand up (with breaks for exhaustion). Making the filibuster burdensome with such rules will prevent the filibuster from being used indiscriminately. It will also lead to filibusters where representatives and their actions are more prominently reported in the media. Then, representatives will be able to better signal how hard they are working to their constituencies.

If government shutdowns and debt ceilings are not removed by laws that preclude them,<sup>27</sup> Congress can at least make the quarantined meetings mandatory for government shutdown and debt ceiling fights. To reduce or remove the social cost of a government default or shutdown, representatives can be forced into a quarantined meeting 10 days before the government shutdown and debt ceiling deadlines. Dissolving this meeting requires funding the government or raising the debt ceiling.

During the 2023 Writers Guild of America strike, for almost a month, the firms and the union did not talk to each other. After the two sides met each other again and held five days of "marathon talks", the strike that went on for nearly five months finally ended.<sup>28</sup> While not talking to the other side may have been a successful strategy that signaled reluctance to take an unfavorable deal, it entailed a large social cost in lost production.<sup>29</sup> Such phenomena can be prevented by a law requiring the parties to bargain in a quarantined meeting in case of a strike.

#### 8.2.2 Tax Solutions

Tax solutions involve taxing the agents and possibly the organizations they represent for deal delays. Taxing representatives for political gridlock reduces external and social cost by hastening deals. It also reduces social cost by having the government partially recover the cost of the delay.

To convince voters that their representatives are working hard for them and solve gridlocks in congress, congress can institute a disagreement tax. This is a wealth tax on the representatives for disagreement. For instance, suppose a bill fails to become law because of a filibuster, a presidential veto or disagreement between the assemblies and more representatives need to agree for the bill to become law. Then, the representatives should have the power to impose a tax for a fixed percentage of the personal wealth for every representative for each day until a bill is passed or until representatives of the required number agree to not pass a bill. The increased bargaining cost from this tax induces a quick deal.

This tax should also apply to the president in case of a presidential veto. For debt ceiling fights, representatives and the president can be taxed for half or more of their wealth in case of

- 24. See Klein (2020), McCarthy (2021), and Kapur (2024).
- 25. See Wawro and Schickler (2006, p. 261), Ornstein (2003), Yuan (2021), and Fortin (2021).
- 26. See "TRANSCRIPT: ABC News' George Stephanopoulos interviews President Joe Biden" (2021).
- 27. See Everett (2023) and Romm (2023)
- 28. See Maddaus (2023a, 2023b), "An end in sight? Striking writers and Hollywood studios spend second full day in negotiations" (2023), Maas and Littleton (2023), Anguiano (2023), and Wilkinson and Stewart (2023).
  - 29. See Franken (2023) and Boyle (2023).

a government default. In reality, this tax need be combined with strict restrictions on donations, payments, transfers, gifts and jobs for representatives. A wealth tax is used because representatives may differ in wealth and other types of taxes such as an income or a fixed tax can fail to adequately affect all agents to similar degrees. For instance, a billionaire representative may be fine with spending \$100 every day to block a law but a poor representative may not be.

For labor disputes, non-stoppage strikes should legally replace traditional strikes. In non-stoppage strikes, workers keep working but the workers and the firm pay strike taxes until a labor contract is made.<sup>30</sup> For instance, for every day of a dispute, the workers can be taxed for their daily wages and the firm can be taxed for estimated lost daily profit. (This loss can be estimated from number of workers on strike, their wages, previous profits, etc.) For labor strikes, taxing just the agents is problematic because the agent chosen to by the firm is not someone who campaigned for the job and this agent might prefer replacement compared to losing months' of wages. Furthermore, in reality, if only agents are taxed, the union or the firm might irrationally reject the deal and insist on a better deal. This solution has the added benefit that it does not cause lost production.

Marceau and Musgrave (1949) and McGalmont (1962) state that for a non-stoppage strike, it can be hard to estimate how much money the firm should pay. In reality, the difficulty in the estimating the cost to the firm from a strike can be different depending on types of workers on strike. For instance, it would be easier to estimate the cost from manufacturers than the cost from researchers. Then, a strike tax could be applied to firms for just the manufacturers. Researchers could stop working in a labor dispute. Also, there could be a maximum number of days for strikes. Once the maximum is exceeded, a non-stoppage strike starts. The tax for this non-stoppage should be calculated based on the actual losses the firm suffered during the shortened strike as this would be easier to calculate. For firms where strikes happened recently, it may be possible to calculate the tax based on previous strikes without a new strike.

One benefit of the tax solutions is that they can be implemented by incremental reforms. For instance, if an 1% wealth tax for each day seems too drastic and raises concern of unforeseen problems, the tax can be 0.01% for the first reform. Then, if the results of the first reform are acceptable, the tax can be gradually increased to reduce political gridlocks.

## 9 Conclusion

In the models, agents on both sides bargain in place of their principals. The proposer-agents receives signals about the vetoer-candidates' types. When these signals are highly accurate, Adaptive Equilibria exist where proposer-agents usually get the best deal they can get using the signals. Since his agents act adequately, the proposer-principal does not replace his agents. A Nonadaptive Equilibrium exists if no proposer-agent finds aggression worthwhile. In these PBEs, the proposer-principal does not replace his agents. In the Replacement and Imitation Equilibrium, the proposer-principal wants aggression from his agents. In the Replacement Equilibrium, high type proposer agents bargain non-aggressively and are replaced. However, in the Imitation Equilibrium, on the equilibrium path, all proposer-agents bargain aggressively to avoid replacement and no agents are replaced. In the Replacement and Imitation Equilibria, the proposer-agent may end up acting aggressively even when he thinks it's unlikely to work and is not in the best interest of his principal. This is to avoid replacement.

30. Sosnick (1964, pp. 78–79), Stokes (1969, p. 81) and Levmore (2025, p. 353) discuss how penalties for labor disputes can be implemented legally.

To improve social welfare, social cost of bargaining should be lowered. This may be achieved by increasing the bargaining cost per unit time for the agents as it provides an incentive for a quick deal and lets agents signal low bargaining costs swiftly. For legislative bargaining, filibusters can be made more arduous to conduct. A representative or the president could call a quarantined meeting to increase the bargaining costs of representatives. Alternatively, representatives could be taxed for a percentage of their wealth depending on how long the bargaining takes. Quarantine meetings can be mandated for labor contract negotiations. Another option is to tax the firm for estimated monthly loss and tax the union members for their monthly wages for every day of strikes.

# **Appendix 1. Lemmas and Proofs for Section 4**

**Definition 5.** Let  $b_D$  be the belief that the vetoer-agent is low type with probability  $P_L$  and high type with probability  $P_H$ .

In proving the Adaptive, Nonadaptive, Replacement and Imitation Equilibrium, I use the following condition for beliefs about the candidates.

**Condition 4.** This condition specifies what the two principals and the proposer-agent believe about the vetoer-agents. If the vetoer-agent accepted  $a_{t,1} < \bar{a}$  in some period t, the players' belief is  $b_{\Upsilon}$ . If not and the vetoer-agent rejected  $a_{t,1} \in [\check{a},\bar{a}]$  in some period t, the players' belief is  $b_{\Lambda}$ . Otherwise, both principals believe  $b_D$  and the proposer-agent believes  $b_L$  if signal for the vetoer-agent was  $\sigma_L$  and believes  $b_H$  if the signal was  $\sigma_H$ .

The above condition means that in general, principals' and proposer-agents' beliefs about the vetoer-agents are initially given by the distribution of the vetoer-candidates' types. Later, if  $a_{t,1} < \bar{a}$  is accepted, the beliefs can change to  $b_{\Upsilon}$  (high type) as only high type vetoer-agents would accept  $a_{t,1} < \bar{a}$  when their types are unknown. Also, if  $a_{t,1} \in [\check{a},\bar{a})$  is rejected, the beliefs can changed to  $b_{\Lambda}$  (low type) as only low type vetoer-agents would reject  $a_{t,1} \in [\check{a},\bar{a})$ .

**Condition 5.** For any t, any proposer-agent, his strategy is to do the following whenever he can.

- (i)  $a_{t,1} = \bar{a}$  when he believes  $b_{\Lambda}$  about the vetoer-agent.
- (ii)  $a_{t,1} = \underline{a}$  when he believes  $b_{\Upsilon}$  about the vetoer-agent.
- (*iii*)  $a_{t,2} = 0$ .

**Condition 6.** Any vetoer-agent has a cutoff point of 0 in stage 2.

**Lemma 1.** Under conditions 4 and 5, in all four types of PBEs of this paper, condition 6 is optimal.

*Proof.* In all four types, condition 1 holds. By condition 1.(i), whether the vetoer-agent accepts an offer or not in stage 2 does not affect whether she is replaced. By definitions  $1\sim4$ , it does not affect whether the proposer-agents are replaced. Also, by condition 4, it does not affect the beliefs of the proposer-agents.

By condition 4, given the actions the vetoer-agent chose, the belief about the vetoer-agent is same for any proposer-agent and it is in the set of  $\{b_L, b_H, b_\Lambda, b_\Upsilon\}$ . For this set, definitions 1, 2

and 4 and condition 5 specifies the offer from any proposer-agent in the Adaptive, Nonadaptive and Replacement Equilibrium.

In the Imitation Equilibrium, condition 5 specifies the offer from any proposer-agent for when the proposer-agent believes  $b_{\Lambda}$  or  $b_{\Upsilon}$  about the vetoer-agent. Condition 5.iii specifies the stage 2 offer for the proposer-agent. If the proposer-agent did not act aggressively for  $b_L$  or  $b_H$  in some period, the agent will be replaced at the end of the current period and future proposer-agents will act aggressively for  $b_L$  and  $b_H$ . If the proposer-agent always acted aggressively for  $b_L$  or  $b_H$ , the agent will continue to do so until she is replaced.

Since accepting or rejecting the stage 2 offer does not affect future offers or utilities, by formula 3, in any period, it is optimal for the vetoer-agent to have a cutoff point of 0 in stage 2.

**Lemma 2.** Under conditions 4 and 6, in all four types of PBEs of this paper, condition 5 is optimal.

*Proof.* By definitions  $1\sim4$ , when the proposer-agent believes  $b_{\Lambda}$  or  $b_{\Upsilon}$ , the stage 1 offer does not affect whether the proposer-agent is replaced. Consider the case where the proposer-agent offers  $a_{t,1}$  in period t. By conditions 1.(i) and 4, If the proposer-agent believes  $b_{\Upsilon}$  about the vetoer-agent, the vetoer-principal will replace her at the end of the period. If the proposer-agent believes  $b_{\Lambda}$  about the vetoer-agent, the vetoer-principal will not replace her at the end of the period. Therefore, in these cases,  $a_{t,1}$  does not affect whether the vetoer-agent is replaced or not. Also, by definitions  $1\sim4$ ,  $a_{t,2}$  does not affect whether the vetoer-agent is replaced or not.

By conditions 1 and 4, once the proposer-agent believes  $b_{\Lambda}$  or  $b_{\Upsilon}$  about the vetoer-agent, his belief about the agent will not change if this belief is correct. Then, by condition 1, the cutoff point will not change for the vetoer-agent.

By condition 4, given the actions a vetoer-agent chose, the belief about the vetoer-agent is in the set of  $\{b_L, b_H, b_\Lambda, b_\Upsilon\}$ . Then, by condition 1, for future vetoer-agents, their cutoff points and replacement are not affected by the current period offers. Also, by condition 4, the current period offers will not affect beliefs about future vetoer-agents.

Because of the argument till now, when the proposer-agent believes  $b_{\Lambda}$  or  $b_{\Upsilon}$  about the vetoer-agent, according to his belief, the current stage 1 offer does not affect future utility. Also, according to his belief, the current stage 2 offer does not affect future utility. Then, condition 5 is optimal because it maximizes current utility.

**Lemma 3.** Under conditions 4 and 5, in all four types of PBEs of this paper, conditions 1.(ii) and 1.(iii) are optimal.

*Proof.* Consider a vetoer-agent in stage 1 of period t. By condition 4, the belief about this agent is  $b_{\Upsilon}$  if and only if this agent accepted  $a_{t',1} \leq \bar{a}$  in some period t'. Given this belief, condition 1.(i) means she will be replaced at the end of period t. She will not get any utility in the future periods. Thus, it is optimal for her to maximize the utility for the current period. By condition 5 and equation 5, the cutoff points maximizes the utility.

By condition 4, given the actions a vetoer-agent chose, the belief about the vetoer-agent is in the set of  $\{b_L, b_H, b_\Lambda, b_\Upsilon\}$ . Suppose the belief is not  $b_\Upsilon$ . Then, by condition 4 whether  $a_{t,1} \geq \bar{a}$  is accepted does not affect the beliefs of the principals and the proposer-agent about the vetoer-agent. Therefore, by definitions  $1{\sim}4$  and condition 5, future offers are not affected. Furthermore, by definitions  $1{\sim}4$  and condition 1.(i), agent replacements are not affected. Thus, maximizing the utility for period t is optimal and the vetoer-agent should accept  $a_{t,1} \geq \bar{a}$ .

If the vetoer-agent accepts  $a_{t,1} < \bar{a}$ , she is replaced at the end of the period and by condition 1.(i), if she rejects, she is not replaced at the end of the period. For the low type vetoer-agent,

$$a_{t,1} - c_{VL} + w_V < 0 - 2c_{VL} + w_V$$

She prefers to reject  $a_{t,1} < \bar{a}$  and accept  $a_{t,2} = 0$  because due to condition 5, in future periods that she works in, she can get a utility of  $0 - 2c_{VL} + w_V > 0$  by accepting the second period offer of 0.

For the high type vetoer-agent, if she rejects  $a_{t,1} < \bar{a}$  and accepts  $a_{t,2} = 0$ , by condition 4 and 5, in future periods that she works in, she can get a utilities of  $\bar{a} - c_{VH} + w_V = w_V - c_{VL} - c_{VH} \ge 0$ . Lemma 1 shows that in stage 2, accepting  $a_{t,2} = 0$  is optimal. By definitions  $1 \sim 4$  and condition 5,  $\bar{a} - c_{VH} + w_V$  is the best utility she can get in future periods. Formulas  $7 \sim 9$  mean that she weakly prefers to reject  $a_{t,1} < \check{a}$  and to accept  $a_{t,1} > \check{a}$ . A cutoff point of  $\check{a}$  is optimal.

#### **Definition 6.**

- $\check{b}_D$  is the belief that proposer-agent is low type with probability  $P(C_{VA1} = c_{VL})$  and high type with probability  $P(C_{VA1} = c_{VH})$ .
- $\check{b}_{\Lambda}$  is the belief that proposer-agent is low type.
- $\check{b}_{\Upsilon}$  is the belief that proposer-agent is high type.

Note that  $\check{b}_D$  follows the distribution of the proposer-agent's type.

**Condition 7.** This condition specifies both principals' belief about the principal-agent in Replacement and Imitation Equilibria. Suppose a principal-agent made an offer in some period, to a vetoer-agent while believing  $b_L$  or  $b_H$  about the vetoer-agent. If in any such period t, the principal-agent acted aggressively, the principals believe  $\check{b}_{\Lambda}$  in a Replacement Equilibrium and  $\check{b}_D$  in a Imitation Equilibrium. If in some such period t, the principal-agent did not act aggressively, principals believe  $\check{b}_{\Gamma}$ . Otherwise, the principals believe  $\check{b}_D$ .

**Lemma 4.** Under conditions  $4\sim7$ , in all four types of PBEs of this paper, condition 1.(i) is optimal.

*Proof.* By condition 4, given the actions the vetoer-agent chose, the belief about the vetoer-agent is same for any proposer-agent and it is in the set of  $\{b_L, b_H, b_\Lambda, b_\Upsilon\}$ . By the condition, the principals' beliefs about the vetoer-agent are in the set of  $\{b_\Lambda, b_\Upsilon, b_D\}$ . Then, by definitions  $1{\sim}4$  and condition 5, the proposer-agent's first stage offer is in the set of  $\{\underline{a}, \check{a}, \bar{a}\}$ . By condition 5, the second stage offer is 0.

If the first stage offer is  $\check{a}$  and it is rejected, the second stage offer of 0 will be accepted according to condition 6. Accepting the second stage offer instead of the first one has an additional bargaining cost of  $c_{VP}$  for the vetoer-principal. By combining  $\underline{a} \leq \check{a}$ ,  $P_H \check{a} - P_L c_{VP} \leq \bar{a}$  and  $c_{VP} \leq c_{VH}$ , I get

$$a \leq P_H \check{a} - P_L c_{VP} \leq \bar{a}$$
.

Take the expected deal under condition 1 when the first period offer is  $\check{a}$ , the proposer-agent believes  $b_L$  or  $b_H$  about the vetoer-agent and the vetoer-agent type is distributed by  $b_D$ . The vetoer-principal weakly prefers a deal of  $\bar{a}$  in the first stage to this expected deal and this expected deal to a deal of  $\underline{a}$  in the first stage.

For the Adaptive, Nonadaptive and Replacement Equilibrium, the first stage offer is determined by the proposer-agent's type and his belief about the vetoer-agent. In the Imitation Equilibrium, this offer is determined from the same factors and whether the proposer-agent acted non-aggressively for  $b_L$  or  $b_H$  in some previous period.

Because of this, from the vetoer-principal's perspective, the game can be seen as one that has finite states. In any period, the vetoer-principal is in one state. Each state specifies the

vetoer-principal's utility in the state given the last period state and the vetoer-principal's move in the last period. Each state also specifies probabilities for which state the game will be in the next period given the vetoer-principal's move in the current period. Because of this, for any two sets of a history and signals of the game if her move for each state is the same, her expected future payoffs are the same.<sup>31</sup>

Suppose the game is in period t. Consider functions that assign a feasible move to each state. Let  $\Phi$  be the maximum expected future payoff of such functions. To apply proof by contradiction, suppose there exists some pure substrategy,  $\zeta$  such that the payoff from the strategy is greater than  $\Phi$ . Since utility has an upper bound, there exists some period T > t for which the discounted sum of expected future utilities from  $\zeta$  till period T is greater than  $\Phi$ .

Now, construct a function the following way, For each state, there exists a finite set of substrategies that  $\zeta$  can play if the state is reached before period T. If the set is empty, assign an arbitrary feasible move to the state. Otherwise, pick a substrategy from the set that maximizes the expected future payoff and assign the first move of this substrategy to the state. The expected future payoff from the constructed function is greater than  $\Phi$ . Therefore,  $\zeta$  does not exist. Thus, by finding the optimal function, I can find the optimal strategy.

For Adaptive, Nonadaptive and Imitation Equilibrium, the game can be represented with three states. Each state corresponds to a vetoer-principal's belief about her agent. So a state is represented by an element of  $\{b_{\Lambda}, b_{\Gamma}, b_{D}\}$ . For the Replacement Equilibrium, a state is represented by a combination of vetoer-principal's belief about the proposer-agent and her agent. By condition 7, vetoer-principal's belief about the proposer-agent is in the set of  $\{\check{b}_{\Lambda}, \check{b}_{\Gamma}, \check{b}_{D}\}$ .

In all four types of PBEs, if the vetoer-principal believes  $b_D$ , replacing her agent does not affect the state nor the utility in the next period. Not replacing the agent is optimal. In an Adaptive, Nonadaptive or Imitation Equilibrium, if the belief is  $b_{\Lambda}$ , the vetoer-principal will get a deal of  $\bar{a}$  until the state changes. Therefore, it is optimal to not replace her agent. If the belief is  $b_{\Upsilon}$ , the vetoer-principal will get a deal of  $\underline{a}$  until the state changes. Therefore, it is optimal to replace her agent.

Consider the Replacement Equilibrium using condition 5. If the belief about the vetoeragent is  $b_{\Upsilon}$  and the agent is not replaced at the end of the period, in the next period, the deal is  $\underline{a}$ . If the beliefs are  $\check{b}_D$  and  $b_{\Upsilon}$ , the vetoer-principal will get a deal of  $\underline{a}$  until the state changes. By condition 7, unless her agent is replaced, the state will not change. Replacing her agent is uniquely optimal. If the beliefs are  $\check{b}_{\Upsilon}$  and  $b_{\Upsilon}$  and the agent is not replaced, the beliefs will be  $\check{b}_D$  and  $b_{\Upsilon}$  in the next period and  $\check{b}_D$  in the period after that. If the agent is replaced, the beliefs will be  $\check{b}_D$  and  $b_D$  in the next period. Replacing her agent is optimal.

If the beliefs are  $\check{b}_{\Lambda}$  and  $b_{\Upsilon}$  and the agent is not replaced in any period as long as the game is in this state, at the end of each period, there is a probability of  $1-\gamma$  that the beliefs will be  $\check{b}_D$  and  $b_{\Upsilon}$  in the next period. After this happens, the beliefs will change to  $\check{b}_D$  and  $b_D$ . If the vetoer-principal starts from beliefs of  $\check{b}_{\Lambda}$  and  $b_{\Upsilon}$  and replaces agents as long as she believes  $\check{b}_{\Lambda}$ , there is a probability of  $1-\gamma$  that the beliefs will be  $\check{b}_D$  and  $b_D$  in the next period. Thus, if the beliefs are  $\check{b}_{\Lambda}$  and  $b_{\Upsilon}$  having the same agent in the next period does not uniquely maximize utility. Replacing her agent is optimal.

If the belief about the vetoer-agent is  $b_{\Lambda}$  and vetoer-agent is not replaced at the end of the period, in the next period, the deal is  $\bar{a}$ . If the beliefs are  $\check{b}_D$  and  $b_{\Lambda}$  and the agent is not replaced, next period, the beliefs are the same. Not replacing the agent is optimal. If the beliefs are  $\check{b}_{\Upsilon}$  and  $b_{\Lambda}$  and the agent is not replaced, next period, beliefs will be  $\check{b}_D$  and  $b_{\Lambda}$  and eventually the vetoeragent will be replaced with probability 1. Then, the new vetoer-agent will face a proposer-agent

<sup>31.</sup> For a more detailed discussion of how games can be represented using states, see Fudenberg and Tirole (2005, p. 501)

distributed by  $\check{b}_D$ . If the beliefs are  $\check{b}_{\Upsilon}$  and  $b_{\Lambda}$  and the agent is replaced, in the next period, the vetoer-agent will be new and face a proposer-agent distributed following  $\check{b}_D$ . Not replacing the agent is optimal.

If the beliefs are  $\check{b}_{\Lambda}$  and  $b_{\Lambda}$ , in the next period, the vetoer-agent will face a proposer-agent distributed by  $\check{b}_{\Lambda}$  with probability  $\gamma$  and  $\check{b}_{D}$  with probability  $1-\gamma$ . For the  $\check{b}_{D}$  case, if the vetoeragent is unchanged, the belief will be  $\check{b}_{D}$  and  $b_{\Lambda}$ .  $\check{b}_{D}$  and  $b_{\Lambda}$  will change if the vetoer-agent is replaced. Then, the new vetoer-agent will face a proposer-agent distributed by  $\check{b}_{D}$ . For the  $\check{b}_{\Lambda}$  case, if the vetoer-agent is unchanged, the belief will be  $\check{b}_{\Lambda}$  and  $b_{\Lambda}$ . Afterwards, the vetoer-principal can replace the agent. Therefore, it is optimal to not replace the agent when the beliefs are  $\check{b}_{\Lambda}$  and  $b_{\Lambda}$ .

The following condition gives the principals' beliefs on Adaptive and Nonadaptive equilibria in the proofs.

**Lemma 5.** Under condition 6, for all four types of PBEs, condition 4 satisfies the requirements for both principals' and the proposer-agents' belief about the vetoer-agents.

*Proof.* Definitions 1~4 mean that condition 1 holds in all four types of PBEs. Consider the beliefs about the vetoer-agents. Condition 4 means the proposer-agent's belief is in the set of  $\{b_L, b_H, b_\Lambda, b_\Upsilon\}$ .

If in stage 1, the proposer-agent faces a new vetoer-agent, he believes  $b_L$  ( $b_H$ ) about the vetoer-agent if the signal for the vetoer-agent was  $\sigma_L$  ( $\sigma_H$ ). At the end of period t, if a principal had no prior belief about the vetoer-agent or believed  $b_D$  about the vetoer-agent and  $a_{t,1} \geq \bar{a}$ , by condition 1, the principal can believe  $b_D$  about the vetoer-agent. If under the same circumstances,  $a_{t,1} < \check{a}$  and it is rejected, the belief can be  $b_D$  as well.

By condition 1, if the proposer-agent believes  $b_L$  or  $b_H$  in stage 1 and his offer for this stage is  $\bar{a}$  or more, he can believe the same in stage 2. If in the same circumstances, his offer is less than  $\check{a}$  and it is rejected, he can believe the same in both stages as well. By condition 6, the proposer-agent can keep his belief at the second stage to the next period's first stage.

If the belief of a principal or the proposer-agent is  $b_{\Upsilon}$ , the player can keep the belief in all future points of the game. Suppose the belief of a principal or the proposer-agent is  $b_{\Lambda}$  in some point of the game. Later in the game, if the vetoer-agent always rejected first period offers lower than  $\bar{a}$  from the earlier point of the game, the player can keep the belief at the later point.

Suppose the belief of a principal or the proposer-agent is  $b_{\Lambda}$ ,  $b_{L}$ ,  $b_{D}$  or  $b_{H}$  at some point of the game. If the next time, the vetoer-agent considers an first period offer, the offer is less than  $\bar{a}$  and it is accepted, by condition 1, the belief can change to  $b_{\Upsilon}$ . Suppose the belief is  $b_{L}$ ,  $b_{D}$  or  $b_{H}$ . If the offer is in  $[\check{a},\bar{a}]$  and rejected, the belief can be set to  $b_{\Lambda}$ .

**Condition 8.** Both principals always believe  $\check{b}_D$  about the proposer-agent.

**Lemma 6.** Under conditions  $4\sim6$ , for Adaptive and Nonadaptive Equilibria, (i) condition 8 satisfies the requirements for both principals' beliefs about the proposer-agents and (ii) the proposer-principal finds it optimal to never replace his agents.

*Proof.* Consider the proposer-agent's belief about the vetoer-agent. Condition 4 means that this belief is always in the set of  $\{b_L, b_H, b_\Lambda, b_\Upsilon\}$  and specified from the vetoer-agent's decisions for the offers she got and the signal about her. For each element of this set, definitions 1 and 2 and condition 5 specifies the first and second stage offers. These specified offers apply to any proposer-agent. Then, by definitions 1 and 2 and condition 6, replacing proposer-agents will not change expected deals, (ii) is proven.

In period t, suppose a principal has no prior belief about the proposer-agent or believed  $\check{b}_D$  about the proposer-agent in the previous period. Further suppose that the proposer-agent's belief was  $b_\Lambda$  or  $b_\Upsilon$  in the first stage. If the proposer-agent did not deviate in the first stage, the principal's belief can be  $\check{b}_D$ . If the proposer-agent did deviate in the first stage, the principal's belief can also be  $\check{b}_D$ .

Suppose instead that the proposer-agent's belief was  $b_L$  or  $b_H$  in the first stage. If the proposer-agent plays the move for  $b_L$  or  $b_H$  in the first stage, the principal's belief can be  $\check{b}_D$ . If the proposer-agent does not plays the move for  $b_L$  or  $b_H$  in the first stage, the principal's belief can also be  $\check{b}_D$ .

**Lemma 7.** Suppose conditions 1,4 and 6 hold and the proposer-principal's strategy is to never replace his agents.

- (i) If proposition 1's i and ii hold and the proposer-agent believes  $b_L(b_H)$  about the vetoeragent, the proposer-agent finds it optimal to act non-aggressively (aggressively).
- (ii) If  $c_{VL} \ge -P_{H|H}\check{a} P_{L|H}c_{PL}$ , the proposer-agent finds it optimal to bargain non-aggressively.

*Proof.* Consider the proposer-agent's belief about the vetoer-agent. Condition 4 means that this belief is always in the set of  $\{b_L, b_H, b_\Lambda, b_\Upsilon\}$  and specified from the vetoer-agent's decisions for the offers she got and the signal about her. For this proof, if the belief is  $b_L$ ,  $b_H$ ,  $b_\Lambda$  or  $b_\Upsilon$  in stage 1, I state that the game is in state L, H,  $\Lambda$  or  $\Upsilon$  respectively. If the proposer-agent is replaced at the end of a period, the game is in state  $\Omega$  for the next period.

Because of lemma 2, I can assume that the proposer-agent's strategy is to offer 0 in stage 2. Consider the first stage offers. In states L and H, an optimal offer must be in the set of  $\{\check{a},\bar{a}\}$ . In state  $\Lambda$ , offering  $\bar{a}$  is optimal. In state  $\Upsilon$ , offering  $\underline{a}$  is optimal. I need only consider pure strategies where the offers satisfy the above conditions.

For each state, the candidate offers for optimal offers are known. For any state and any of its candidate offers, the expected utility and the probabilities for the next period state are known.

Excluding the case where the proposer-agent retires, I can state the following. In states L and H, if the proposer-agent acts non-aggressively, the next state is L or H. If the proposer-agent acts aggressively, the next state is L or H if the first period offer is accepted and  $\Lambda$  if the first period offer is rejected. If the game is in state  $\Lambda$ , it will go to state L or H in the future will probability 1.

Let  $\dot{a}$  be a 2-tuple that represents the first stage offers for the states.  $\dot{a}_1$  and  $\dot{a}_2$  are respectively the first and second elements. Both are in the set of  $\{\check{a},\bar{a}\}$ . If  $\dot{a}=(x,y)$ , The first period offers for states L and H are x and y.

By the least-upper-bound property, a supremum exists for the set of expected payoff of strategies that satisfy  $a_{1,1} = \check{a}$  when first state of the game is L. The same is true for  $a_{1,1} = \bar{a}$ . If the supremum are equal and the supremum is attained for some set, for a set that attains the supremum, pick its move  $(\check{a} \text{ or } \bar{a})$  as  $\dot{a}_1$ . Otherwise,  $\dot{a}_1$  is the move for the set with the greater supremum.  $\dot{a}_2$  is selected the same way using H as the first state of the game.

Pick an arbitrary strategy, Suppose the game is in stage 1 of period t for some history and it is the proposer-agent's turn to act. If  $a_{t,1} \neq \dot{a}_1$  for state L or  $a_{t,2} \neq \dot{a}_2$  for state H, replace the substrategy so that  $a_{t,1} = \dot{a}_1$  for state L and  $a_{t,2} = \dot{a}_2$  for state H. Repeat for all subsequent periods and possible histories from then going from the earliest to the latest. Then the new strategy has a weakly greater future expected payoff. A strategy which plays  $\dot{a}_1$  whenever the game reaches state L and  $\dot{a}_2$  whenever the game reaches state H has a weakly greater future expected payoff. Therefore, the  $\dot{a}$  I constructed is optimal. If the expected payoff from  $\dot{a}$  is the maximum expected payoff for any  $\dot{a}$ ,  $\dot{a}$  represents an optimal strategy.

$$\begin{split} r - \bar{a} - c_{PL} &\geq P_{H|H}(r - \check{a} - c_{PL}) + P_{L|H}(r - 0 - 2c_{PL}) \\ \leftrightarrow \\ - \bar{a} &\geq P_{H|H}(-\check{a}) + P_{L|H}(-c_{PL}) \\ \leftrightarrow \\ c_{VL} &\geq -P_{H|H}\check{a} - P_{L|H}c_{PL} \\ \\ \text{Apply } P_{L|L} &> P_{L|H} \text{ and } P_{H|H} &> P_{H|L}. \\ c_{VL} &\geq -P_{H|H}\check{a} - P_{L|H}c_{PL} \\ \rightarrow \\ c_{VL} &\geq -P_{H|L}\check{a} - P_{L|L}c_{PL} \\ \leftrightarrow \\ r - \bar{a} - c_{PL} &\geq P_{H|L}(r - \check{a} - c_{PL}) + P_{L|L}(r - 0 - 2c_{PL}) \end{split}$$

Therefore, if  $c_{VL} \ge -P_{H|H}\check{a} - P_{L|H}c_{PL}$ , for both states 1 and 2, the expected utility from aggression is weakly smaller than the expected utility from nonaggression for both the low type and the high type proposer-agent. This means  $\dot{a} = (\bar{a}, \bar{a})$  is optimal. (ii) is proven.

For any  $\dot{a}$ , let  $\Phi_i$  be the expected future payoff from state i. If  $\Phi_{\Lambda} > P_H \Phi_H + P_L \Phi_L$  for a proposer-agent of some type,  $\dot{a}$  is not optimal. This is because in state  $\Lambda$ , the expected deal is  $\bar{a}$  and in states L and H, the same deal can be gotten.

Suppose 
$$c_{VL} \leq -P_{H|H}\check{a} - P_{L|H}c_{PH}$$
.

$$\begin{split} c_{VL} &\leq -P_{H|H} \check{a} - P_{L|H} c_{PH} \\ &\leftrightarrow \\ r - \bar{a} - c_{PH} &\leq P_{H|H} (r - \check{a} - c_{PH}) + P_{L|H} (r - 0 - 2c_{PH}) \end{split}$$

Therefore, in state H, expected utility from aggression is weakly greater than expected utility from nonaggression for both the low type and the high type proposer-agent. Then, the proposer-agent weakly prefers  $\dot{a} = (\bar{a}, \check{a})$  to  $\dot{a} = (\bar{a}, \bar{a})$ .

Suppose  $\Phi_{\Lambda} = P_H \Phi_H + P_L \Phi_L$  for some proposer-agent with bargaining cost, c for some  $\dot{a}$ . Suppose further that  $c_{VL} < -P_{H|H} \check{a} - P_{L|H} c$ . In state H, the proposer-agent can act aggressively and act non-aggressively for any state after this one. For state H, This is better than acting non-aggressively in any period. Any  $\dot{a}$  with  $\Phi_{\Lambda} = P_H \Phi_H + P_L \Phi_L$  is not optimal.

If 
$$c_{VL} = -P_{H|H}\check{a} - P_{L|H}c$$
, I have the following.

$$\begin{split} c_{VL} &= -P_{H|H} \check{a} - P_{L|H} c \\ \rightarrow \\ c_{VL} &> -P_{H|L} \check{a} - P_{L|L} c \\ \rightarrow \\ r - \bar{a} - c &> P_{H|L} (r - \check{a} - c) + P_{L|L} (r - 0 - 2c) \end{split}$$

This means that  $\dot{a} = (\bar{a}, \bar{a})$  is optimal. Then,  $\dot{a} = (\bar{a}, \check{a})$  is also optimal.

Consider 
$$\dot{a} = (\check{a}, \bar{a})$$
. Combine  $c_{PL} > 0 > \check{a}$  with  $P_{L|L} > P_{L|H}$  and  $P_{H|H} > P_{H|L}$ .

$$\begin{split} &P_{H|H}(c_{PL}) + P_{L|H}(\check{a}) > P_{H|L}(c_{PL}) + P_{L|L}\check{a} \\ & \leftrightarrow \\ &P_{H|H}(r+c_{PL}) + P_{L|H}(r+\check{a}) > P_{H|L}(r+c_{PL}) + P_{L|L}(r+\check{a}) \\ & \leftrightarrow \\ &P_{H|H}(r-\check{a}-c_{PL}) + P_{L|H}(r-0-2c_{PL}) > P_{H|L}(r-\check{a}-c_{PL}) + P_{L|L}(r-0-2c_{PL}) \end{split}$$

The expected utility from aggression is greater in state H compared to state L. A first stage offer of  $\check{a}$  has a weakly greater probability of being accepted in state H compared to state L. In states L and H, if the proposer-agent acts aggressively, next period, state is L, H or  $\Lambda$ . For this case, the probability that the next state is  $\Lambda$  is weakly greater when the current state is L.  $\Phi_{\Lambda} \leq P_H \Phi_H + P_L \Phi_L$  means that the expected payoff is weakly greater for  $\dot{a} = (\check{a}, \check{a})$  than  $\dot{a} = (\check{a}, \bar{a})$ .

Consider  $\dot{a}=(\check{a},\check{a})$  and a deviation from it where if the game is state L in period t,  $a_{t,1}=\bar{a}$  and  $a_{t+1,1}=\check{a}$ . Suppose  $\Phi_{\Lambda} < P_H \Phi_H + P_L \Phi_L$  for the proposer-agent. The following probabilities are calculated for the case where the game is in state L in period t and the proposer-agent is not replaced at the end of the period.

For  $\dot{a}$ , the probability that the state is L or H and the vetoer-agent is new in period t+1 is  $1-\gamma P_{L|L}$ . Then, the proposer-agent for period t+1 acts aggressively. The probability that the state is  $\Lambda$  in period t+1 is  $\gamma P_{L|L}$ . Then, the proposer-agent for period t+1 acts non-aggressively.

For the deviation, the probability that the state is L and the vetoer-agent is the same in period t+1 is  $\gamma$ . Then, the probability that the vetoer-agent is the same in period t+1 and  $a_{t+1,1}=\check{a}$  is accepted is  $\gamma P_{H|L}$ . For the rejection, the probability is  $\gamma(1-P_{L|L})$ . The probability that the state is L or H and the vetoer-agent is new in period t+1 is  $1-\gamma$ .

For  $\dot{a}$  and the deviation, in period t+2, the proposer-agent is new or the state for the period is  $\Lambda$ . For the probability  $\gamma P_{L|L}$  cases, the distribution of states in period t+2 is the same. For the remaining cases, the probability that the state is  $\Lambda$  in period t+2 is greater for  $\dot{a}$  because the deviation has the aforementioned rejection and when this rejection happens the vetoer-agent is new in period t+2 with probability 1. For  $\dot{a}$ , the probability of state  $\Lambda$  in period t+2 is greater.

Compare the expected utilities from  $\dot{a}$  to those from the deviation.

$$\begin{split} &r - \bar{a} - c_{PL} + \gamma (\gamma (P_{H|L}(r - \check{a} - c_{PL}) + P_{L|L}(r - 0 - 2c_{PL})) \\ &+ (1 - \gamma) (P_{H}(r - \underline{a} - c_{PL}) + P_{L}(r - 0 - 2c_{PL}))) \\ &\geq \\ &P_{H|L}(r - \check{a} - c_{PL}) + P_{L|L}(r - 0 - 2c_{PL}) \\ &+ \gamma ((1 - \gamma P_{L|L}) (P_{H}(r - \check{a} - c_{PL}) + P_{L}(r - 0 - 2c_{PL})) + \gamma P_{L|L}(r - \bar{a} - c_{PL})) \end{split}$$

I simplify the above formula.

$$\begin{split} & - \bar{a} + \gamma (\gamma (P_{H|L}(-\check{a}) + P_{L|L}(-0 - c_{PL})) + (1 - \gamma)(P_{H}(-\underline{a}) + P_{L}(-c_{PL}))) \geq \\ & P_{H|L}(-\check{a}) + P_{L|L}(-c_{PL}) + \gamma ((1 - \gamma P_{L|L})(P_{H}(-\check{a}) + P_{L}(-c_{PL})) + \gamma P_{L|L}(-\bar{a})) \end{split}$$

Apply  $-\underline{a} = c_{VH}$  and  $-\bar{a} = c_{VL}$ .

$$c_{VL} + \gamma(\gamma(-P_{H|L}\check{a} - P_{L|L}c_{PL}) + (1 - \gamma)(P_{H}c_{VH} - P_{L}c_{PL})) \ge -P_{H|L}\check{a} - P_{L|L}c_{PL} + \gamma((1 - \gamma P_{L|L})(-P_{H}\check{a} - P_{L}c_{PL}) + \gamma P_{L|L}c_{VL})$$

I simplify further.

$$c_{VL} + (1 - \gamma^2)(P_{H|L}\check{a} + P_{L|L}c_{PH}) + \gamma(1 - \gamma)(P_Hc_{VH} - P_Lc_{PH}) \ge \gamma((1 - \gamma P_{L|L})(-P_H\check{a} - P_Lc_{PH}) + \gamma P_{L|L}c_{VL})$$

The above implies the following.

$$\begin{aligned} &c_{VL} + (1-\gamma^2)(P_{H|L}\check{a} + P_{L|L}c_{PL}) + \gamma(1-\gamma)(P_{H}c_{VH} - P_{L}c_{PL}) \geq \\ &\gamma((1-\gamma P_{L|L})(-P_{H}\check{a} - P_{L}c_{PL}) + \gamma P_{L|L}c_{VL}) \end{aligned}$$

Therefore  $\dot{a} = (\check{a}, \check{a})$  is not optimal.  $\dot{a} = (\bar{a}, \check{a})$  is optimal. (i) is proven.

#### **Proof of Proposition 1.**

For the Adaptive Equilibria of this proof, the strategies are given by definition 1 and conditions 5 and 6 and the beliefs are given by conditions 4 and 8. Lemma 5 and lemma 6's (i) prove that the beliefs satisfy the requirements for a PBE. Lemmas  $1\sim4$  and 7 and lemma 6's (ii) prove that the strategies are optimal.

#### **Proof of Proposition 2.**

For the Nonadaptive Equilibria of this proof, the strategies are given by definition 2 and conditions 5 and 6 and the beliefs are given by conditions 4 and 8. Lemma 5 and lemma 6's (i) prove that the beliefs satisfy the requirements for a PBE. Lemmas  $1\sim4$  and 7 and lemma 6's (ii) prove that the strategies are optimal.

# **Appendix 2.** Lemmas and Proofs for Section 5

**Lemma 8.** Under conditions 4 and 5, for Replacement and Imitation Equilibria, condition 7 satisfies the requirements for both principals' beliefs about the proposer-agents.

*Proof.* Consider the proposer-agent's belief about the vetoer-agent. Condition 4 means that this belief is always in the set of  $\{b_L, b_H, b_\Lambda, b_\Upsilon\}$  and specified from the vetoer-agent's decisions for the offers she got and the signal about her. For each element of this set, definitions 3 and 4 and condition 5 specifies the first and second stage offers of the low type proposer-agent. For the Replacement Equilibrium, the first and second state offers of the high type proposer-agent are also specified. For the Imitation Equilibrium, definition 3 specifies the first and second state offers of the high type proposer-agent given his previous decisions for the offers he got.

In period t, if the proposer-agent's belief was  $b_{\Lambda}$  or  $b_{\Upsilon}$  in the first stage, condition 5 specifies the first and second period offers for the beliefs. Therefore, if there a principal has no prior belief for the proposer-agent, the principal can believe  $\check{b}_D$  and if the principal had a belief for the proposer-agent in the last period, the principal can keep the belief. If a principal believes  $\check{b}_{\Lambda}$  or  $\check{b}_{\Upsilon}$  about a proposer-agent, the principal can have the same belief about the proposer agent until the agent is replaced.

Suppose that for period t, a principal believed  $\check{b}_D$  or  $\check{b}_\Lambda$  about the proposer-agent in period t-1 or there is no prior belief about the proposer-agent. Further suppose that the proposer-agent's belief was  $b_L$  or  $b_H$  in stage 1 of period t. In a Replacement equilibrium, if  $a_{t,1} = \check{a}$ , the principal can believe  $\check{b}_\Lambda$  about the proposer-agent and if  $a_{t,1} \neq \check{a}$ , the principal can believe  $\check{b}_\Upsilon$  in this period.

In an Imitation Equilibrium, consider a proposer-agent who before period t, always acted aggressively when he believed  $b_L$  or  $b_H$  about the vetoer-agent or never made an offer to a vetoer-agent while believing  $b_L$  or  $b_H$  about her. If  $a_{t,1} = \check{a}$ , the principal can believe  $\check{b}_D$  about this proposer-agent. Now consider a proposer-agent who offered  $a_{t,1} \neq \check{a}$ , for him, the proposer-principal can believe  $\check{b}_{\Gamma}$ .

**Lemma 9.** Under weak inequality 10 and conditions  $4\sim7$ , in a Replacement or Imitation Equilibrium, the proposer-principal finds condition 2.i optimal.

*Proof.* Consider game from proposer-principal's perspective. Condition 4 means that the proposer-principal's belief about the vetoer-agent is always in the set of  $\{b_{\Lambda}, b_{D}, b_{\Upsilon}\}$ . For this proof, if the belief is  $b_{\Lambda}$ , I say that the game is in state 1 and if not, I say that the game is in state 2. By condition 7, the proposer-principal's belief about the proposer-agent is in the set of  $\{\check{b}_{\Lambda}, \check{b}_{D}, \check{b}_{\Upsilon}\}$  in a Replacement Equilibrium and in the set of  $\{\check{b}_{D}, \check{b}_{\Upsilon}\}$  in a Imitation Equilibrium. For this proof, I call a proposer-agent about whom the proposer-principal believes  $\check{b}_{\Lambda}$  in an Replacement Equilibrium or  $\check{b}_{D}$  in an Imitation Equilibrium a hawk and a proposer-agent about whom the proposer-principal believes  $\check{b}_{\Upsilon}$  a dove.

If the game is in state 1, by condition 5, until the vetoer-agent is replaced, the deal will be made in stage 1 for  $\bar{a}$  and this replacement only happens by retirement. If the game in state 2 or the vetoer-agent is replaced at the end of the period, in stage 1 of the next period, the proposer-agent will face a vetoer-agent about whom he believes  $b_L$  or  $b_H$ . Then, by definitions 3 and 4, the first stage offer from a hawk will be  $\check{a}$ . If this is accepted, the proposer-principal's belief will be  $b_{\Gamma}$  and the game will be in state 2. If this is rejected, the the proposer-principal's belief will be  $b_{\Lambda}$  and the game will be in state 1. Also, by conditions 5 and 6, if the game reaches stage 2, a deal of 0 will happen. The first stage offer from a dove will be  $\bar{a}$ . This is always accepted and will not lead the game to state 2.

If the deal is made in stage 1 for  $\bar{a}$ , the proposer-principal gets  $r - \bar{a} - c_{PP}$ . If the first stage offer is  $\check{a}$  and the proposer-principal believes  $b_D$  about the vetoer-agent, his expected payoff is  $P_H(r - \check{a} - c_{PP}) + P_L(r - 0 - 2c_{PP})$ .

$$\begin{split} r - \bar{a} - c_{PP} &\leq P_H(r - \check{a} - c_{PP}) + P_L(r - 0 - 2c_{PP}) \\ \leftrightarrow \\ - \bar{a} &\leq P_H(-\check{a}) + P_L(-c_{PP}) \\ \leftrightarrow \\ c_{VL} &< -P_H\check{a} - P_Lc_{PP} \end{split}$$

Then by weak inequality 10, the proposer-principal weakly prefers offering  $\check{a}$  to  $\bar{a}$  in stage 1 when he believes  $b_D$  about the vetoer-agent.

If the game is in state 1, the type of the proposer-agent does not matter for the future deals until the vetoer-agent is replaced. If the game is in state 2, the game remains in this state until an first stage offer of  $\check{a}$  is made. Then, the game moves to state 1 if the offer is rejected. If it is accepted, the state does not change. Therefore, a strategy by which the first stage offer of  $\check{a}$  is made sooner whenever the game is in stage 2 or the game is in stage 1 and the vetoer-agent is replaced gives a weakly greater expected payoff.

In an Imitation Equilibrium, every new agent that comes after replacement will be a hawk. Therefore, if the proposer-agent is a dove, replacing the proposer-agent means that all future proposer-agents will be hawks. The proposer-principal finds it optimal to replace only doves.

In a Replacement Equilibrium, in state 2 and when the vetoer-agent is replaced at the end of the period, low types offer  $\check{a}$  in the first stage of next period. Suppose the belief about the

proposer-agent is  $\check{b}_{\Lambda}$  or  $\check{b}_{D}$ . By condition 7, if the next time he makes a offer to a vetoer-agent while believing the vetoer-agent to be  $b_{L}$  or  $b_{H}$ , he acts aggressively, the belief will be  $\check{b}_{\Lambda}$ .

To get a low type for when the game reaches state 2 or the vetoer-agent is replaced, the proposer-principal should replace doves. If the belief about the proposer-agent is  $\check{b}_D$ , the belief is the same for any proposer-candidate that did not work as an agent. In this case, not replacing the agent is optimal.

**Definition 7.**  $\Phi_A$  is the payoff for the proposer-principal from getting a utility of  $P_H(r - \check{a} - c_{PP}) + P_L(r - 0 - 2c_{PP})$  for every period.  $\Phi_N$  is the payoff for the proposer-principal from a deal of  $a_{t,1} = \bar{a}$  for every period t.

**Lemma 10.** In any Replacement or Imitation Equilibrium,  $c_{VL} \leq -P_H \check{a} - P_L c_{PP}$ .

*Proof.* I use proof by contradiction and start by assuming  $c_{VL} > -P_H \check{a} - P_L c_{PP}$ . Suppose that Replacement or Imitation Equilibrium exists. Excluding the proposer-principal, every player plays the equilibrium strategy. For a new vetoer-agent, the proposer-agent believes  $b_L$  or  $b_H$ . The proposer-agent's first stage offer is  $\check{a}$  or  $\bar{a}$ . If the first stage offer is  $\bar{a}$ , the proposer-agent's belief is the same in stage 1 of the next period.

If the offer is  $\check{a}$  and it is rejected, the proposer-agent's belief in stage 1 of the next period is  $b_{\Lambda}$ . If the offer is  $\check{a}$  and accepted, the vetoer-agent is removed at the end of the period. If the proposer-agent's belief in stage 1 is  $b_{\Lambda}$ , his offer will be  $\bar{a}$  and the stage 1 belief will be the same in future periods.

If the vetoer-agent was offered  $\bar{a}$  in stage 1 for every period in which she worked, the proposer-principal believes  $b_D$  about this agent. If the vetoer-agent was offered  $\check{a}$  in stage 1 for some period in which she worked, the proposer-principal does not believe  $b_D$  about this agent.

The above explanation means that the proposer-principal's expected payoffs from the Replacement and Imitation Equilibrium are fixed. Let  $\Phi_I^*$  be the proposer-principal's expected payoff from the Imitation Equilibrium. If, at the end of a period t, the proposer-principal believes  $b_D$  about the vetoer-agent and the proposer-agent is replaced, by his belief, the discounted future expected payoff is  $\delta^t \Phi_I^*$  in a Imitation Equilibrium.

In a Replacement Equilibrium, let  $\delta^{t-1}\Phi_L^*$  be the discounted future expected payoff at stage t when the vetoer-agent is new and the proposer-agent is low type and makes offer.  $\Phi_L^*$  is fixed. In this case, the future stage 1 offers will be  $\check{a}$  or  $\check{a}$  with positive probability for both.

Suppose that in a Replacement Equilibrium, at period 2, the high type proposer-agent offered  $\bar{a}$  in stage 1 while believing  $b_L$  or  $b_H$  about the vetoer-agent. At the end of the period, the proposer-principal can deviate to a strategy where he never replaces the proposer-agent as long as he continues to believe that the proposer-agent is high type from period 2 to the current period but follows the equilibrium strategy otherwise. When the proposer-agent retires, in the next period, the new proposer-agent is low or high type. Until a low type proposer-agent takes the job, the deals are made in stage 1 at  $\bar{a}$ . When a low type proposer-agent first takes the job in period t, the discounted future expected payoff is  $\delta^{t-1}\Phi_L^*$ .

$$\begin{split} c_{VL} &> -P_H \check{a} - P_L c_{PP} \\ &\rightarrow \\ r - \bar{a} - c_{PP} &> P_H (r - \check{a} - c_{PP}) + P_L (r - 0 - 2c_{PP}) \end{split}$$

At the end of period 1, if the low type proposer-agent offered  $\check{a}$  in stage 1 and it was accepted. The proposer-principal prefers to remove the proposer-agent and increase the probability that the next proposer-agent is high type. This is because the proposer-principal's expected payoff

is greater when the next proposer-agent is high type and the proposer-principal follows the aforementioned deviation.

Suppose, in period 1 of an Imitation Equilibrium, the high type proposer-agent deviates and acts non-aggressively. Then, this agent is removed at the end of the period. If his principal also deviates, removes the agent only when the agent still works at the end of period 2 and plays the equilibrium strategy otherwise, the future expected payoffs are the same when the agent is retired at the end of period 1. If the agent is not retired at the end of period 1, the agent will work one more period when the principal deviate.

$$c_{VL} > -P_H \check{a} - P_L c_{PP}$$

$$\rightarrow$$

$$r - \bar{a} - c_{PP} > P_H (r - \check{a} - c_{PP}) + P_L (r - 0 - 2c_{PP})$$

$$\rightarrow$$

$$\Phi_I^* < \Phi_N$$

$$r - \bar{a} - c_{PP} + \delta \Phi_N = \Phi_N$$

$$\rightarrow$$

$$r - \bar{a} - c_{PP} + \delta \Phi_I^* > \Phi_I^*$$

The principal prefers to deviate.

#### **Proof of Proposition 3.**

For the Replacement Equilibria of this proof, the strategies are given by definition 3 and conditions 5 and 6 and the beliefs are given by conditions 4 and 7. Lemma 5 and 8 prove that the beliefs satisfy the requirements for a PBE. Lemmas  $1\sim4$  and 9 prove that the strategies are optimal for the principals and the vetoer-agents and that condition 5 is optimal. Lemma 10 proves that (i) is necessary.

Formula 2 means that in any period, any proposer-agent can get non-negative utility by offering  $\bar{a}$ . I will prove that the low type proposer-agent's strategy is optimal. For the case where he believes  $b_L$  about the vetoer-agent, compare his utility from nonaggression to his discounted utilities from aggression in the current period and acting non-aggressively next period.

$$\begin{split} r - \bar{a} - c_{PL} + w_P &\leq P_{H|L}(r - \check{a} - c_{PL}) + P_{L|L}(r - 0 - 2c_{PL}) + w_P + \gamma(r - \bar{a} - c_{PL} + w_P) \\ \leftrightarrow \\ - \bar{a} &\leq P_{H|L}(-\check{a}) + P_{L|L}(-c_{PL}) + \gamma(r - \bar{a} - c_{PL} + w_P) \\ \leftrightarrow \\ c_{VL} &\leq -P_{H|L}\check{a} - P_{L|L}c_{PL} + \gamma(r + c_{VL} - c_{PL} + w_P) \end{split}$$

Consider the case where he believes  $b_H$  instead by using  $P_{L|L} > P_{L|H}$  and  $P_{H|H} > P_{H|L}$ .

$$c_{VL} \leq -P_{H|H}\check{a} - P_{L|H}c_{PL} + \gamma(r + c_{VL} - c_{PL} + w_P)$$

Next, I will prove that the high type proposer-agent's strategy is optimal. For the case where he believes  $b_L$  about the vetoer-agent, compare his utility from nonaggression to the sum of discounted utilities for when he acts aggressively in the current period and non-aggressively after that and he is not removed.

$$\begin{split} r - \bar{a} - c_{PH} + w_{P} &\geq \\ P_{H|H}(r - \check{a} - c_{PH}) + P_{L|H}(r - 0 - 2c_{PH}) + w_{P} + \sum_{i=1}^{\infty} \gamma^{n} (r - \bar{a} - c_{PH} + w_{P}) \\ &\leftrightarrow \\ r - \bar{a} - c_{PH} + w_{P} &\geq \\ P_{H|H}(r - \check{a} - c_{PH}) + P_{L|H}(r - 0 - 2c_{PH}) + w_{P} + \frac{\gamma}{1 - \gamma} (r - \bar{a} - c_{PH} + w_{P}) \\ &\leftrightarrow \\ - \bar{a} &\geq P_{H|H}(-\check{a}) + P_{L|H}(-c_{PH}) + \frac{\gamma}{1 - \gamma} (r - \bar{a} - c_{PH} + w_{P}) \\ &\leftrightarrow \\ c_{VL} &\geq -P_{H|H}\check{a} - P_{L|H}c_{PH} + \frac{\gamma}{1 - \gamma} (r + c_{VL} - c_{PH} + w_{P}) \end{split}$$

Consider the case where he believes  $b_H$  instead by using  $P_{L|L} > P_{L|H}$  and  $P_{H|H} > P_{H|L}$ .

$$c_{VL} \ge -P_{H|L}\check{a} - P_{L|L}c_{PH} + \frac{\gamma}{1-\gamma}(r + c_{VL} - c_{PH} + w_P)$$

From the results till now and formula 2, I have the following two weak inequalities.

$$r - \bar{a} - c_{PH} + w_P \ge P_{H|H}(r - \check{a} - c_{PH}) + P_{L|H}(r - 0 - 2c_{PH}) + w_P$$
$$r - \bar{a} - c_{PH} + w_P \ge P_{H|L}(r - \check{a} - c_{PH}) + P_{L|L}(r - 0 - 2c_{PH}) + w_P$$

Therefore, when the belief about the vetoer-agent is  $b_L$  or  $b_H$ , the expected utility from a first stage offer of  $\bar{a}$  is weakly greater compared that of  $\check{a}$ . By formula 2, the expected utility from the offer of  $\bar{a}$  is weakly greater than 0. Because of the vetoer side's strategies, the expected payoff from the next period that is gotten by aggression in the current period is weakly less than  $\sum_{i=1}^{\infty} \gamma^n (r - \bar{a} - c_{PH} + w_P)$ . Therefore, nonaggression in the current period is optimal.

#### **Proof of Proposition 4.**

For the Imitation Equilibria of this proof, the strategies are given by definition 4 and conditions 5 and 6 and the beliefs are given by conditions 4 and 7. Lemma 5 and 8 prove that the beliefs satisfy the requirements for a PBE. Lemmas  $1\sim4$  and 9 prove that the strategies are optimal for the principals and the vetoer-agents and that condition 5 is optimal. Lemma 10 proves that (i) is necessary.

I will prove the optimality of the strategies for the low type proposer-agent and the high type proposer-agent who always acted aggressively for  $b_L$  and  $b_H$  or did not make an offer for  $b_L$  and  $b_L$ . By formula 2,  $r - \bar{a} - c_{PH} + w_P \ge 0$  and  $r - \bar{a} - c_{PL} + w_P \ge 0$ . For the case where the proposer-agent believes  $b_L$ , compare the expected payoffs of the high type proposer-agent starting from the current period for nonaggression and aggression. By the vetoer-agents' strategy, from the next period, the proposer-agent can get a first period offer of  $\bar{a}$  accepted.

$$\begin{split} r - \bar{a} - c_{PH} + w_P &\leq P_{H|L}(r - \check{a} - c_{PH}) + P_{L|L}(r - 0 - 2c_{PH}) + w_P + \gamma(r - \bar{a} - c_{PH} + w_P) \\ \leftrightarrow \\ - \bar{a} &\leq P_{H|L}(-\check{a}) + P_{L|L}(-c_{PH}) + \gamma(r - \bar{a} - c_{PH} + w_P) \\ \leftrightarrow \\ c_{VL} &\leq -P_{H|L}\check{a} - P_{L|L}c_{PH} + \gamma(r + c_{VL} - c_{PH} + w_P) \end{split}$$

Consider the case where he believes  $b_H$  instead by using  $P_{L|L} > P_{L|H}$  and  $P_{H|H} > P_{H|L}$ .

$$c_{VL} \leq -P_{H|H}\check{a} - P_{L|H}c_{PH} + \gamma(r + c_{VL} - c_{PH} + w_P)$$

I now have the following two weak inequalities.

$$c_{VL} \le -P_{H|L}\check{a} - P_{L|L}c_{PL} + \gamma(r + c_{VL} - c_{PL} + w_P)$$
$$c_{VL} \le -P_{H|H}\check{a} - P_{L|H}c_{PL} + \gamma(r + c_{VL} - c_{PL} + w_P)$$

For both  $b_L$  and  $b_H$ , the proposer-agent finds it optimal to act aggressively.

Next, I will prove that optimality of the high type proposer-agent's strategy for the case where he did not act aggressively for  $b_L$  or  $b_H$  in some period and he makes a first period offer while believing  $b_L$  or  $b_H$ . Since he will be replaced at the end of the period, I need only compare the expected utilities of the current period for nonaggression and aggression. Consider the case where the proposer-agent believes  $b_H$ .

$$\begin{split} r - \bar{a} - c_{PH} + w_P &\geq P_{H|H}(r - \check{a} - c_{PH}) + P_{L|H}(r - 0 - 2c_{PH}) + w_P \\ \leftrightarrow \\ - \bar{a} &\geq P_{H|H}(-\check{a}) + P_{L|H}(-c_{PH}) \\ \leftrightarrow \\ c_{VL} &\geq -P_{H|H}\check{a} - P_{L|H}c_{PH} \end{split}$$

Consider the  $b_L$  case by using  $P_{L|L} > P_{L|H}$  and  $P_{H|H} > P_{H|L}$ .

$$c_{VL} \ge -P_{H|L}\check{a} - P_{L|L}c_{PH}$$

# **Appendix 3. Proofs for Section 6**

#### **Proof of Proposition 5.**

From 
$$-c_{VH} = \underline{a} \le \check{a} \to \check{a} + c_{VH} \ge 0$$
, I get 
$$(1 - \gamma^2)c_{VL} + (1 - \gamma^2)c_{PH} > -\gamma(1 - \gamma)P_H(\check{a} + c_{VH}).$$
 
$$(1 - \gamma^2)c_{VL} + (1 - \gamma^2)c_{PH} + \gamma(1 - \gamma)(P_Hc_{VH}) > -\gamma(1 - \gamma)P_H\check{a}$$
 
$$c_{VL} + (1 - \gamma^2)c_{PH} + \gamma(1 - \gamma)(P_Hc_{VH}) > \gamma((1 - \gamma) \times -P_H\check{a} + \gamma c_{VL})$$
 If  $P(L|L) = 1$  and  $P(H|H) = 1$ , 
$$c_{VL} + (1 - \gamma^2)(P_{H|L}\check{a} + P_{L|L}c_{PH}) + \gamma(1 - \gamma)(P_Hc_{VH} - P_Lc_{PH}) >$$
 
$$\gamma((1 - \gamma P_{L|L})(-P_H\check{a} - P_Lc_{PH}) + \gamma P_{L|L}c_{VL}).$$

Comparing this with proposition 1's (i) proves that proposition 1's (i) is satisfied when P(L|L) and P(H|H) are close to 1.

$$\check{a} < \bar{a} = -c_{VI}$$

Therefore, if I violate the assumptions of the model and set P(L|L) = 1 and P(H|H) = 1,

$$c_{VL} < -P_{H|H}\check{a} - P_{L|H}c_{PH} \tag{11}$$

Comparing this with proposition 1's (ii) proves that proposition 1's (ii) is satisfied when P(L|L) and P(H|H) are close to 1. An Adaptive Equilibrium exists when P(L|L) and P(H|H) are close to 1.

In a Nonadaptive, Replacement or Imitation Equilibrium, the vetoer-agent will accept any offer greater than 0 in stage 2. Compare the expected utilities from aggression and nonaggression when the high type proposer-agent believes  $b_H$  about the vetoer-agent and the vetoeragent's cutoff point in stage 2 is 0.

$$\begin{split} r - \bar{a} - c_{PH} + w_P &< P_{H|H}(r - \check{a} - c_{PH}) + P_{L|H}(r - 0 - 2c_{PH}) + w_P \\ \leftrightarrow \\ - \bar{a} &< P_{H|H}(-\check{a}) + P_{L|H}(-c_{PH}) \\ \leftrightarrow \\ c_{VL} &< -P_{H|H}\check{a} - P_{L|H}c_{PH} \end{split}$$

If I violate the assumption of the model and set P(L|L) = 1 and P(H|H) = 1, inequality 11 holds. If P(L|L) and P(H|H) are close to 1, the expected utility from aggression is greater.

Formula 2 means that by the vetoer-agents' strategy, the high type can get a non-negative payoff in any period t that he works in with  $a_{t,1} = \bar{a}$ . Then, in a Nonadaptive, Replacement or Imitation Equilibrium, there exists some information set at which high type proposer-agent prefers to deviate to a strategy where he is aggressive in the current period and offers  $\bar{a}$  in the first stage of future periods.

#### **Proof of Proposition 6.**

Suppose there is a Replacement or Imitation PBE. From this equilibrium, proposer-agent 1 may or may not deviate to condition 3.

For a new vetoer-agent, the proposer-agent's belief is  $b_L$  or  $b_H$  in stage 1. If the proposer-agent's believes  $b_L$  or  $b_H$  about the vetoer-agent in stage 1, the offer is  $\check{a}$  or  $\bar{a}$ . By condition 1,  $\bar{a}$  is always accepted. Then, in the next period, the proposer-agent believes  $b_L$  or  $b_H$  about the vetoer-agent in stage 1. If the offer is  $\check{a}$  and it is accepted, the vetoer-agent is removed. Then, in the next period, the proposer-agent believes  $b_L$  or  $b_H$  about the new vetoer-agent in stage 1.

If the offer is  $\check{a}$  and it is rejected, the proposer-agent believes  $b_{\Lambda}$  in stage 2, the proposer-agent makes an offer of 0 and this is accepted. Then, the proposer-agent's belief about the same player will be  $b_{\Lambda}$  in stage 1 of the next period. If the proposer-agent believes  $b_{\Lambda}$  about the vetoer-agent in stage 1 and this belief is correct, the proposer-agent offers  $\bar{a}$  which is accepted. In the next period, the proposer-agent has the same belief about the same player.

Therefore, the first stage offer is always  $\check{a}$  or  $\bar{a}$ . All agents accept  $\bar{a}$ . Once the vetoer-agent is offered  $\check{a}$  in stage 1, the offer is never repeated to the same agent. Therefore, when the vetoer-agent is offered  $\check{a}$ , the proposer-principal's expected utility is  $P_H(r-\check{a}-c_{PP})+P_L(r-0-2c_{PP})$ . When the vetoer-agent is offered  $\bar{a}$ , the expected utility is  $r-\bar{a}-c_{PP}$ .

Let  $\Phi^*$  be the equilibrium expected payoff. Let  $\Phi'$  be the expected payoff from the proposer agent 1 deviation to condition 3.  $\Phi^*$  is between  $\Phi_A$  and  $\Phi_N$  inclusively.

$$\underline{u} \equiv -0.5\check{a}(P_H - P_{H|L}) > 0$$

Here, the inequality is by  $P_{H|H} > P_{H|L}$ . If  $c_{VL} + P_H \check{a} + \underline{u} \le 0$ , pick  $\underline{C}_1$  such that  $c_{VL} = -P_H \check{a} - P_L \underline{C}_1 - \underline{u}$ . If  $c_{VL} + P_H \check{a} + \underline{u} > 0$ , pick  $\underline{C}_1 = 0$ . This means  $\underline{C}_1 \ge 0$ . If  $c_{PP} \ge \underline{C}_1$ , the following holds

$$\begin{split} c_{VL} &\geq -P_H \check{a} - P_L c_{PP} - \underline{u} \geq -P_{H|L} \check{a} - P_{L|L} c_{PP} + \underline{u} \\ \\ c_{VL} &\geq -P_{H|L} \check{a} - P_{L|L} c_{PP} + \underline{u} \\ \\ &\rightarrow \\ r - \bar{a} - c_{PP} \geq P_{H|L} (r - \check{a} - c_{PP}) + P_{L|L} (r - 0 - 2c_{PP}) + \underline{u} \\ \\ min\{\Phi_A, \Phi_N\} + P(\text{probability that the low signal is sent for vetoer-agent 1})u \leq \Phi' \end{split}$$

In any Replacement or Imitation Equilibrium, by lemma 10,  $c_{VL} \leq -P_H \check{a} - P_L c_{PP}$ . Pick  $\varepsilon > 0$  for this PBE. Pick  $\underline{C}_2$  such that  $-\varepsilon < c_{VL} + P_H \check{a} + P_L \underline{C}_2$  and  $c_{VL} < -P_H \check{a} - P_L \underline{C}_2$ . If  $c_{PP} \geq \underline{C}_2$ ,

$$-\varepsilon < c_{VL} + P_H \check{a} + P_L c_{PP} \le 0$$

$$\rightarrow$$

$$-\varepsilon < r - \bar{a} - c_{PP} - P_H (r - \check{a} - c_{PP}) - P_L (r - 0 - 2c_{PP}) \le 0$$

In any Replacement or Imitation Equilibrium, there exists some  $\underline{C}_3$  such that  $c_{VL} < -P_H \check{a} - P_L \underline{C}_3$  and if  $c_{PP} \ge \underline{C}_3$ ,  $|\Phi_A - \Phi_N| \le 0.5\underline{u}$ .

$$\underline{C} = max\{\underline{C}_1,\underline{C}_3\}$$

# **Appendix 4.** Proof for Section 7

#### **Proof of Proposition 7.**

Start from the original PBE. Consider the case where x = y.

$$c_{PP} = \frac{\tau}{x} x \bar{c}_{PP} = \tau' \bar{c}'_{PP}$$

Similar equalities hold for  $c_{VP}$ ,  $c_{PL}$ ,  $c_{PH}$ ,  $c_{VL}$  and  $c_{VH}$ . Since the bargaining costs per stages are the same, the same PBE exists.

Consider the case where y < x. From the new PBE where x = y, reduce y so that  $y \in (0,x)$ .  $c_{PP}'' = \frac{\tau}{x} y \bar{c}_{PP}$  and  $c_{VP}'' = \frac{\tau}{x} y \bar{c}_{VP}$  are the bargaining costs per stage after the reduction of y.  $c_{PL}$ ,  $c_{PH}$ ,  $c_{VL}$  and  $c_{VH}$  do not change after this reduction.

$$c_{PP} = \frac{\tau}{x} x \bar{c}_{PP} \ge \frac{\tau}{x} y \bar{c}_{PP} = c_{PP}^{"} \tag{12}$$

If  $c_{VP} < 0$ ,

$$c_{VP} = \frac{\tau}{x} x \bar{c}_{VP} < \frac{\tau}{x} y \bar{c}_{VP} = c_{VP}'' < 0.$$

Then,  $c_{VP} \le c_{VH}$  and  $P_H \check{a} - P_L c_{VP} \le \bar{a}$  still hold when y is reduced. (Recall that these conditions are required by the model.) If  $c_{VP} \ge 0$ ,

$$c_{VP} = \frac{\tau}{r} x \bar{c}_{VP} \ge \frac{\tau}{r} y \bar{c}_{VP} = c_{VP}'' \ge 0.$$

Then,  $c_{VP}$  is weakly smaller. This means  $c_{VP} \le c_{VH}$  still holds. When  $c_{VP} \ge 0$ ,  $P_H \check{a} - P_L c_{VP} \le \bar{a}$ . If weak inequality 10 was satisfied originally, by formula 12, it is still satisfied after the reduction of y. The new PBE still exists after y is reduced.

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