Frequency-Dependent Definitions of Microstrip Characteristic Impedance

Edward F. Kuester, David C. Chang and Leonard Lewin

Electromagnetics Laboratory
Department of Electrical Engineering
University of Colorado
Boulder, CO, USA

ABSTRACT

The question of the appropriate definition of the characteristic impedance of microstrip transmission line for nonzero frequency is examined. By studying two specific excitation problems, it is found that different choices of the characteristic impedances can considerably simplify equivalent circuits of microstrip sources or junctions in particular cases. Since the choice of the impedance is ultimately arbitrary, some suggestions are made as to how a "universal" definition of this quantity can be made.

1. INTRODUCTION

In recent years, a controversy has arisen as to the "correct" definition of the characteristic impedance of microstrip transmission line when frequency dependent effects become important. Different definitions which are equivalent in the static limit generally exhibit a completely different dependence on frequency [1,2]. Since for any non-TEM mode any transmission-line representation must necessarily be an "equivalent" one, no unique definition of Z_c can be given, despite some authors' insistence to the contrary (e.g., [3]). In fact, one often chooses the characteristic impedance equal to the wave impedance of a mode (where one can be unambiguously defined, as in the case of pure TE or TM modes) or even decide to set $Z_c = 1$ (see, e.g., [4]). Neither of these choices agrees with the "transmission-line" definition of Z_c, even in the static limit, and their use is strictly dictated by convenience in a particular application.

Discussions in the literature attempting to reconcile the differences in the various definitions of $Z_c(\omega)$ or promoting one particular definition over the others have all made certain approximations or assumptions about the quasi-TEM mode which may not be reasonable in all applications [1-3], [5]. Recently [6,7], an attempt has been made to infer the characteristic impedance $Z_c(\omega)$ from a rigorous calculation of the scattering matrix at a certain microstrip discontinuity to a transmission line which supports a true TEM mode. Unfortunately, the manner in which $Z_c(\omega)$ is inferred presupposes that no equivalent circuit accounting for reactive effects near the junction need be included. Thus, while the intent of [6,7] is certainly valid, the use of $Z_c(\omega)$ as evaluated therein may actually require a very complicated equivalent circuit in order to apply to other types of discontinuity.

What few experimental results exist for $Z_c(\omega)$ are inconclusive [8,9]. As has been noted elsewhere [10], this may be due to the fact that interpretation of the measured data depends on several assumptions made, implicitly at least, as to what equivalent circuit is needed to model junction effects in the experiment. Specifically, assumptions about the phases of the reflected signals appear to have been made. Thus, these results suffer from the same ambiguities as the theoretical results of [6,7].

In this paper, we shall briefly review the various definitions of $Z_c(\omega)$ which have appeared in the literature. Using two examples of the excitation of microstrip, we will show how two of these definitions arise most naturally in applications. It will be seen that the controversy surrounding the definition of $Z_c(\omega)$ cannot be absolutely resolved, and that its choice is largely a matter of convenience. Several criteria will be presented which, in our opinion, should be considered if a "universal" definition of $Z_c(\omega)$ is to be agreed upon.

2. DEFINITIONS OF $Z_c(\omega)$

Following [1,2], we can define several possible versions of a frequency dependent characteristic impedance for the quasi-TEM mode of open microstrip (Fig. 1). The line consists of a conducting strip of width 2ℓ lying on a dielectric substrate of thickness t and relative permittivity ε_r deposited on a conducting ground plane. We will suppose that the fields of the quasi-TEM mode can be computed by some method, possibly only numerically, and are given by $\bar{E}_0(y,z)$, $\bar{H}_0(y,z)$. The propagation factor $\exp(i\omega t - ik_0\alpha_0 x)$

is suppressed, where $k_o = \omega\sqrt{\mu_o\varepsilon_o}$, and α_o is the normalized propagation constant of the mode $(\alpha_o^2 = \varepsilon_{eff})$.

We define (somewhat arbitrarily) a voltage and current associated with this mode as

$$V_y = -\int_0^t \bar{E}_o(y,z)\cdot\bar{a}_z dz \qquad (1)$$

$$I = -\int_{-\ell}^{\ell} \bar{a}_z\times[\bar{H}_o(y,t+0) - \bar{H}_o(y,t-0)]dy \qquad (2)$$

and (not arbitrarily) the power carried by the mode as

$$P = \frac{1}{2}\int_0^\infty dz \int_{-\infty}^{\infty} dy\ \bar{E}_o\times\bar{H}_o\cdot\bar{a}_z \qquad (3)$$

assuming that, under lossless conditions, $\bar{E}_o$ and $\bar{H}_o$ have been chosen to be real. Three primary definitions of $Z_c(\omega)$ suggest themselves:

$$Z_{vi}(\omega) = V_y/I \qquad (4)$$

$$Z_{pi}(\omega) = 2P/I^2 \qquad (5)$$

$$Z_{pv}(\omega) = V_y^2/2P = Z_{vi}^2/Z_{pi} \qquad (6)$$

Further, definitions (4) and (6) also depend on the path (value of y) selected for the definition of V_y in (1). The quantities $Z_{vi}(\omega)$ and $Z_{pi}(\omega)$ have usually been found to be increasing functions of frequency [13]. Little can be said about the other definitions and analyses used in [1-3,5] because they rely on a specific dispersion model which is only approximate and restricts the generality of the definitions.

3. EXCITATION BY A SLOT VOLTAGE GENERATOR

Consider the microstrip whose cross-section is shown in Fig. 1. A voltage V_o is impressed across an infinitesimal gap in the strip at $x=0$. The formulation is similar to that of [11] and [12], with the exception that the longitudinal electric field no longer vanishes on the strip, but satisfies

$$E_x(x,y,z=t) = -V_o\delta(x); \quad |y| \le \ell, \quad -\infty < x < \infty \qquad (7)$$

Defining the Fourier transform with respect to x as

$$\left.\begin{aligned} f(x) &= \int_{-\infty}^{\infty} \tilde{f}(\alpha)e^{-ik_o\alpha x}d\alpha \\ \tilde{f}(\alpha) &= \frac{k_o}{2\pi}\int_{-\infty}^{\infty} f(x)e^{ik_o\alpha x}dx \end{aligned}\right\} \qquad (8)$$

we can obtain an integral equation for the charge density $\tilde{\rho}(y) = \tilde{\rho}(y;\alpha)$ on the strip:

$$\int_{-\ell}^{\ell} G_e(y-y')\tilde{\rho}(y')dy' = \frac{i\varepsilon_o\alpha}{\alpha^2-1}V_o + A\cosh\sqrt{\alpha^2-1}\,k_o y \qquad |y|\le\ell \qquad (9)$$

where A is a constant to be determined later and G_e is defined in [11] or [12]. The integral equation for the current distribution $\tilde{J}_x(y) = \tilde{J}_x(y;\alpha)$ is

$$\int_{-\ell}^{\ell} G_m(y-y')\tilde{J}_x(y')dy' = \frac{iV_o}{\eta_o(\alpha^2-1)} + \alpha c_o[A\cosh\sqrt{\alpha^2-1}\,k_o y + \int_{-\ell}^{\ell} M(y-y')\tilde{\rho}(y')dy'] \qquad |y|\le\ell \quad (10)$$

where $\eta_o = (\mu_o/\varepsilon_o)^{\frac{1}{2}}$, $c_o = (\mu_o\varepsilon_o)^{-\frac{1}{2}}$, and G_m and M are defined in [11] or [12]. The charge and current satisfy a conservation equation which can be considered as fixing the constant A:

$$\alpha\int_{-\ell}^{\ell}\tilde{J}_x(y)dy = c_o\int_{-\ell}^{\ell}\tilde{\rho}(y)dy$$

Using the "one-term moment method" of [12], we assume that

$$J_x(x,y) \approx J_o(x)J_x^{(o)}(y)$$

where $J_x^{(o)}(y)$ is the static current distribution on the strip, and find that

$$J_o(x) = \frac{iV_o}{2\pi\eta_o}\int_{-\infty}^{\infty}\frac{e^{-ik_o\alpha|x|}}{D(\alpha)}d\alpha \qquad (11)$$

where

$$D(\alpha) = \alpha^2/C(\alpha) - L(\alpha) \qquad (12)$$

and $C(\alpha)$ and $L(\alpha)$ are defined in [12] as Sommerfeld-type integrals.

Deforming the contour of integration in (11) into the lower half-plane around the singularities of the integrand splits $J_o(x)$ into a component due to the quasi-TEM mode and a component due to higher-order (radiation modes):

$$J_o(x) = \frac{V_o}{\eta_o D'(\alpha_o)}e^{-ik_o\alpha_o|x|} + \frac{iV_o}{2\pi\eta_o}\int_{C_1+C_2}\frac{e^{-ik_o\alpha|x|}}{D(\alpha)}d\alpha \qquad (13)$$

Here α_o is the pole corresponding to the quasi-TEM mode, while C_1 and C_2 are contours enclosing the branch cuts of $D(\alpha)$ in the lower half of the complex α-plane. Although the integral in (13) is infinite at $x=0$ (because of the infinite capacitance associated with a delta-function generator), this could be avoided by modifying the derivation for a finite-gap generator. Setting $x=0\pm$ in (13), we find that the most natural interpretation of the result in this case is that of Fig. 2. The characteristic impedance $Z_c(\omega)$, from the first term of (13), is to be

$$Z_c(\omega) = \frac{\eta_o}{2}D'(\alpha_o) \qquad (14)$$

whereas the admittance Y_r due to the radiation modes is

$$Y_r \rightarrow \frac{i}{2\pi\eta_o}\int_{C_1+C_2}\frac{e^{-ik_o\alpha|x|}}{D(\alpha)}d\alpha\ , \quad \text{as}\quad x\to 0 \qquad (15)$$

The significant point here is that the presence of Y_r has no effect on the rest of the equivalent circuit, and in particular, upon the definition of $Z_c(\omega)$. In addition, definition (14) can be shown to be equivalent to $Z_{pi}(\omega)$ given by (5), within the approximations of the foregoing analysis. For this particular means of exciation, therefore, $Z_{pi}(\omega)$ seems to offer the most convenience in terms of simplifying the equivalent circuit for the source.

4. EXCITATION BY A PIN CURRENT SOURCE

Of course, the slot voltage source might not be expected to provide a very realistic picture of the type of excitation used in practical microstrip configurations, particularly for wide microstrip or microstrip patch antennas. Perhaps more typical would be a "pin current source," a line current $\bar{J} = I_p\delta(x)\delta(y-y_o)$ connected between the ground plane $z=0$ and the strip at $z=t$. The analysis of the current excited on the strip is quite similar to that for the slot voltage source, but the tangential electric field at the strip, produced by the induced strip currents, must now cancel that produced by the pin. Reciprocity arguments similar to those employed in [14] can be used to simplify the manipulations. From the strip current, a voltage $V_{yo}(x)$ can be defined (similar to (1)) at any point along the strip. The results of this analysis (which will not be reporduced here) suggest that the equivalent circuit of this pin generator should be that of Fig. 3, where the characteristic impedance $Z_c(\omega)$ is now most conveniently chosen as $Z_{pv}(\omega)$ from (6). Once again, the simplicity of this choice is reflected in the fact that radiation effects, lumped into Z_p in Fig. 3, have absolutely no effect on the definition of $Z_c(\omega)$.

There is a certain nonuniqueness associated with the choice of y_o, since the voltage V_{yo} may well depend rather strongly on this position if ω is sufficiently large.

This situation is, however, similar to an ambiguity encountered in defining a voltage for the TE_{10} mode of a rectangular waveguide [15]. The transverse variation of this field in a guide of width a causes the "voltage" to vary as $\sin(\pi x/a)$. When shunt discontinuities or excitations are encountered which are not centered in the guide, the referral of the voltage to its value at the center can be accomplished in the equivalent circuit by simply including an ideal transformer whose turns ratio is $\sin^2(\pi x/a)$. A similar conclusion can be made in microstrip problems, although the voltage transformation may no longer take so simple a form as $\sin^2(\pi x/a)$ in this case.

5. DISCUSSION

These two simple examples have shown that a specific choice for $Z_c(\omega)$ can lead to a particularly simple equivalent circuit for certain types of sources, but will generally produce a very complicated equivalent circuit in other situations. It seems to us that the same thing can be said regarding equivalent circuits of discontinuities. The authors of [6,7] have made a choice of $Z_c(\omega)$ based on the requirement that a particular discontinuity have a trivial equivalent circuit at the junction. This, of course, will in general not lead to similarly simple equivalent circuits for other junctions; indeed, very different kinds of discontinuity may require a full T- or Π- network to represent the junction effects even though the junction itself appears fairly simple.

We can make a number of speculations about the effects of discontinuities. Purely shunt discontinuities, such as shorting strips or pins to the ground plane, etc., are probably best accommodated by the Z_{pv} definition. On the other hand, discontinuities which are primarily in series, such as longitudinal gaps, are probably best described using Z_{pi}. We must await full, accurate, frequency-dependent solutions of these and other canonical problems in order to see whether these speculations are justified. It may turn out that, for example, the additional complication in the equivalent circuit of a "series" discontinuity when Z_{pv} is used is not too serious. If a "compromise" candidate for a "best" or "universal" definition of $Z_c(\omega)$ can be agreed upon which can conveniently be applied to a sufficiently wide range of microstrip problems, this will have to be determined on the basis of the solutions of canonical problems. If no definition can be found which has a sufficiently broad usefulness, one may have to bear all three definitions in mind, and change between them as circumstances dictate.

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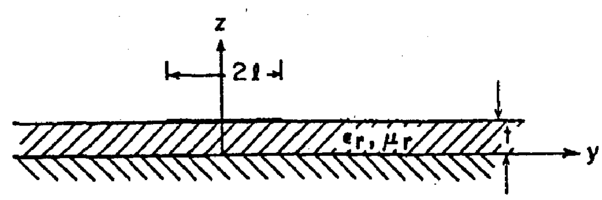

Fig. 1

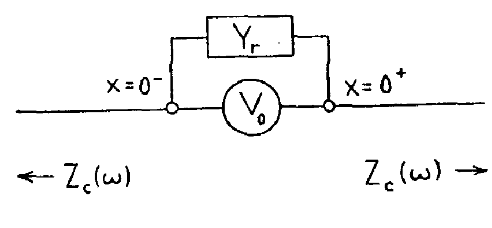

Fig. 2

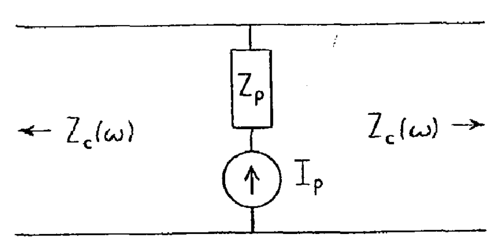

Fig. 3