A Partnership Between Laser Optics and Mathematics

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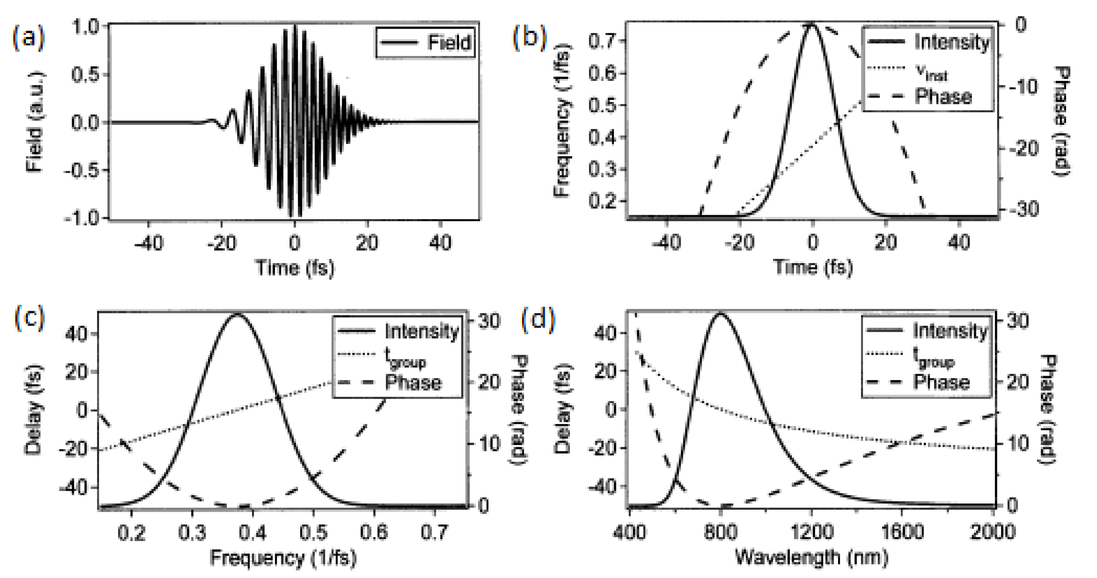
FROG: Frequency-Resolved Optical Gating and Laser Pulses

The FROG measurement technique was developed by Rick Trebino and Daniel J. Kane in 1991 to measure laser pulses

Why FROG?

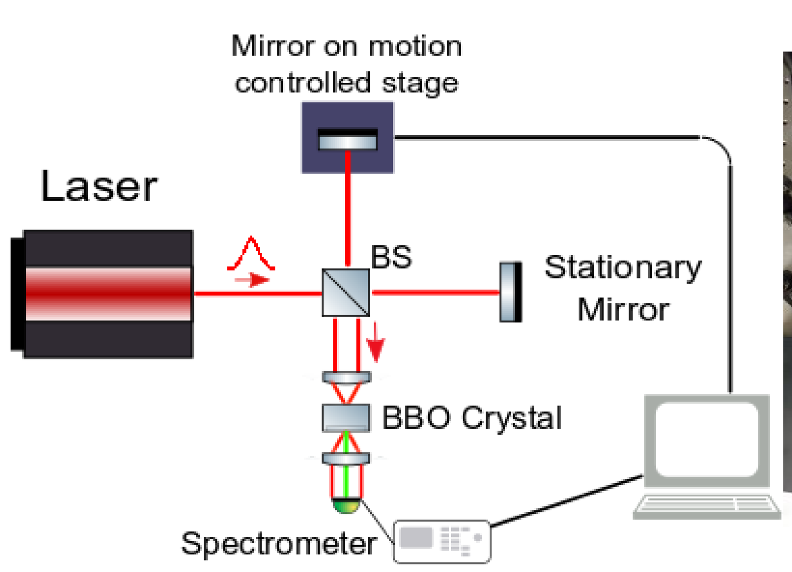
- Sophisticated method of characterizing and measuring ultrashort laser pulses on the order of femtoseconds
- Gives crucial information about the phase and intensity in both time and wavelength of the pulses
- Has significant built-in noise correction

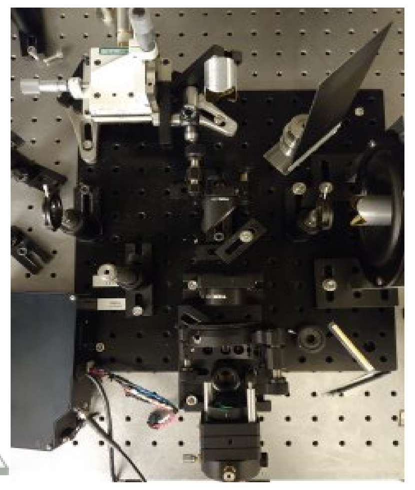


Trebino, Rick. *Frequency-Resolved Optical Gating*. (2000). Springer.

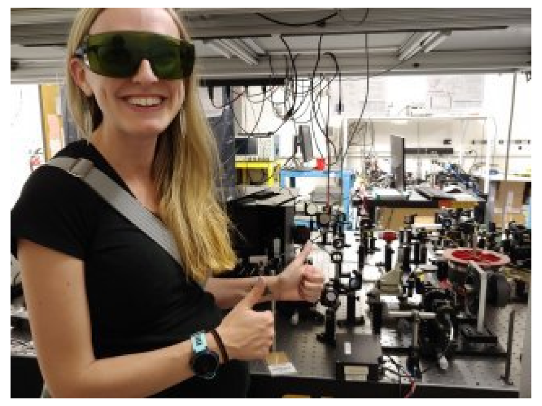
(a) An example of a single laser pulse with a changing frequency over time (called chirp)
(b) The intensity and phase over time (temporal characterization)
(c) The intensity and phase with respect to frequency
(d) The intensity and phase with respect to wavelength (spectral characterization)

Setup





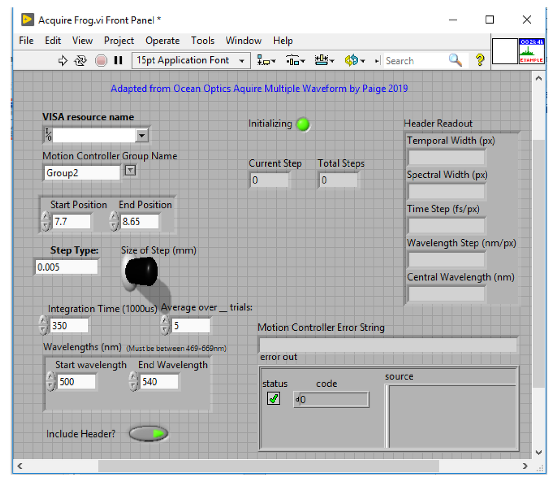
1) The incoming pulse is split with a beam splitter (BS)
2) One copy of the pulse is shifted "in time" by changing the distance traveled along one arm
3) Pulses are recombined and overlapped by focusing the beams through a BBO non-linear crystal to capture *Second Harmonic Generation* (SHG) of the combined pulses
4) This SHG signal, shown in green, is captured by a spectrometer as the moving arm distance is changed in small steps, creating a **FROG trace**

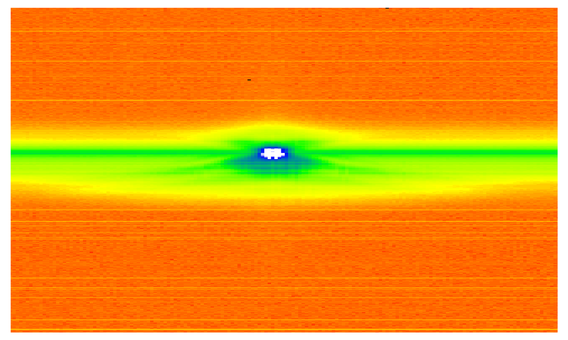


Right: Paige Casabona with FROG

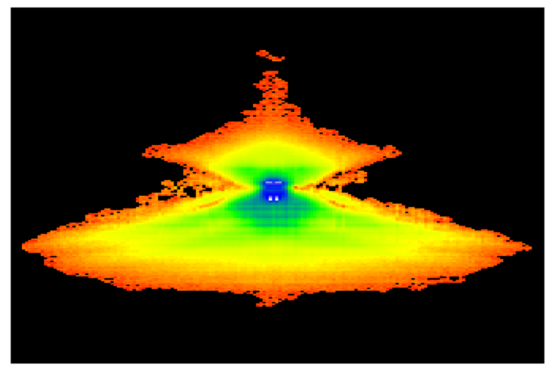
FROG Results

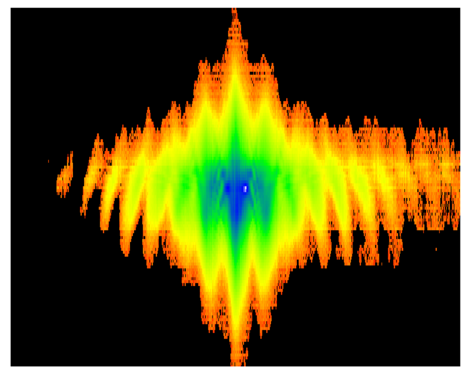
FROG Trace collection was done with a custom LabVIEW program to interface with a Newport single axis motion controller and Ocean Optics spectrometer.



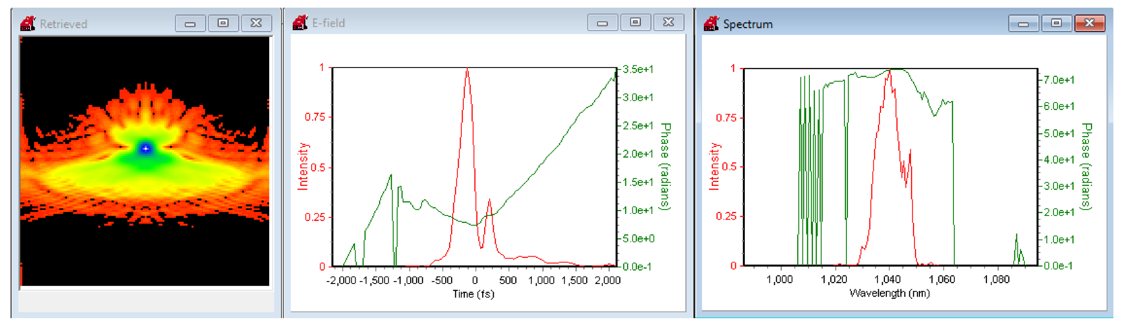


Above: A "raw" FROG trace. Time axis is horizontal, wavelength vertical

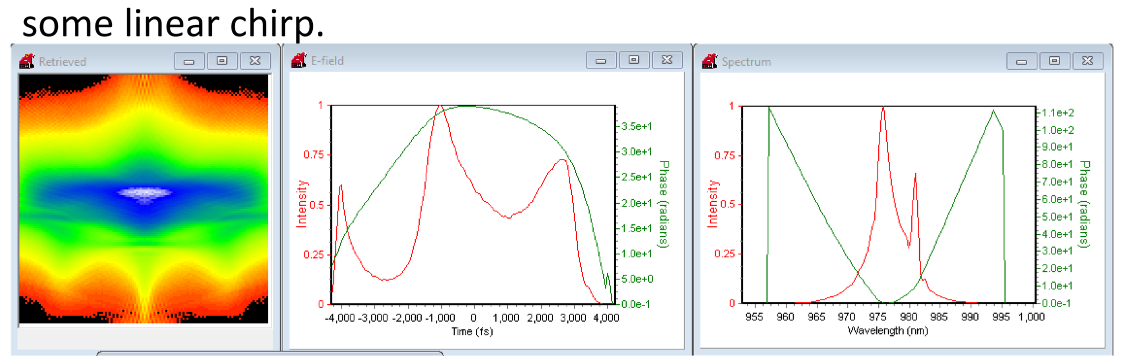




Two examples of FROG traces from different laser pulses after background noise removal and cleaning. These processed images are then fed into an algorithm to procedurally extract the time and wavelength intensity and phase information. Results for each are shown below.



"Retrieved" Pulse from first image above. Temporal width 247.3 fs, Spectral width 13.83 nm. Evidence of third order effects and some linear chirp.



"Retrieved" Pulse from second image above. Temporal width 7064 fs, Spectral width 6.769 nm. Evidence of multiple pulses as well as self phase modulation

Future Uses of FROG

Dr. Gopinath's research group will be able to use the FROG to characterize new laser pulses and refine pulse generation by identifying things like wanted or unwanted chirp. Pulse applications are very sensitive to phase changes the FROG can assist in nonlinear optics applications such as:

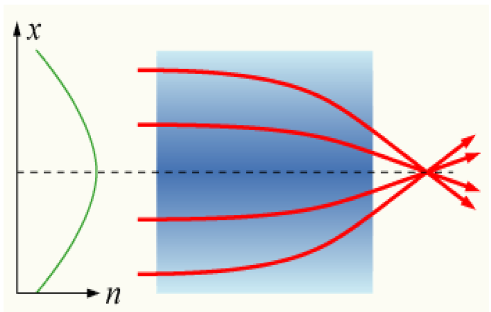
- **Multiphoton microscopy** – for imaging living tissues
- **Spectroscopy** – for studying dynamics on very short time scales
- **Frequency conversion** – to generate visible or near ultraviolet pulses from infrared laser sources

Calculus 3: Teaching Path Integrals through Optics Applications

Goal: Path integrals of scalar valued functions is a standard topic in Calculus 3 that is notoriously difficult to understand at a concrete level. We identified this topic and *optical path length* as a concrete example to help calculus students understand this abstract idea.

Gradient Index (GRIN) Optics

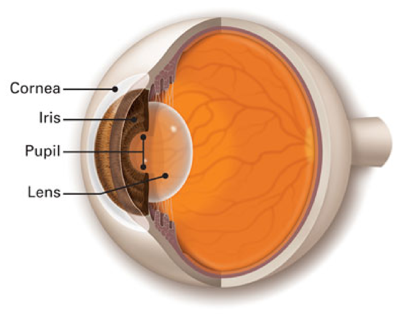
GRIN optics have a varying index of refraction at different positions within the optical medium that influences the path that light takes through the medium.



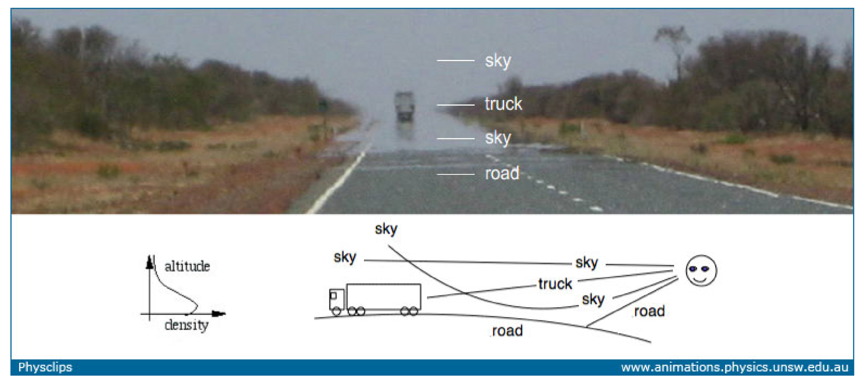
An example of a GRIN lens. The light bends towards the region with higher index of refraction, focusing the light as a typical spherical lens might, but it has flat surfaces

https://commons.wikimedia.org/wiki/File:Grin-lens.png

Some examples of GRIN lens effects in nature:



Human eye. https://www.aao.org/eye-health/anatomy/lens-9



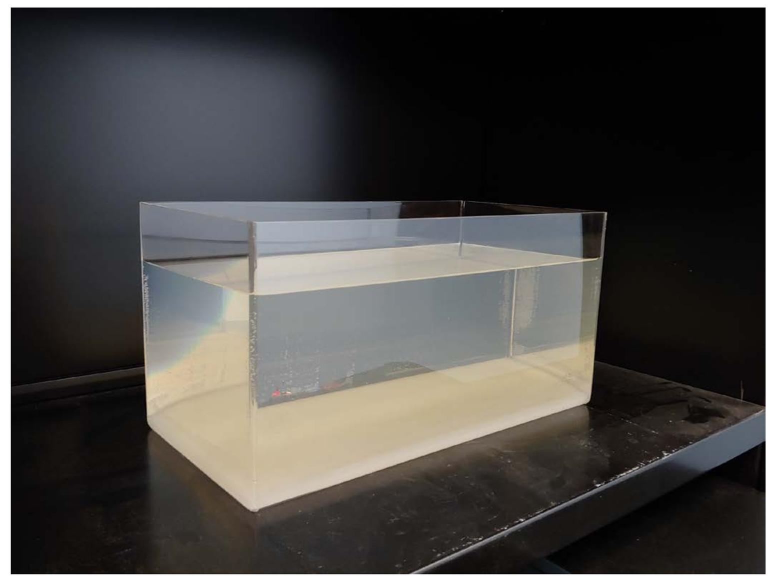
Mirage http://www.animations.physics.unsw.edu.au/jw/light/mirages-green-flash-sky-colours.htm

Fermat's Principle and Optical Path

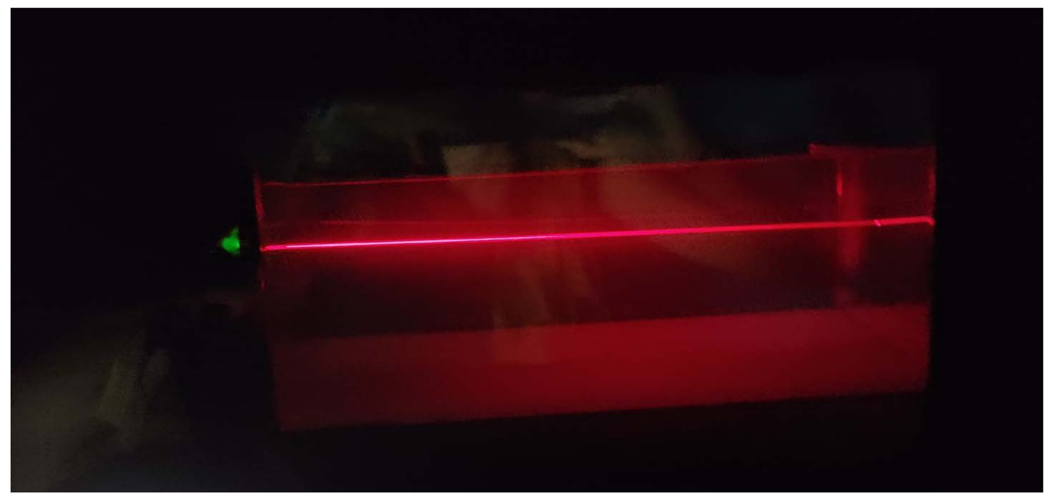
Light does not always travel in a straight line. In fact it takes the *fastest* path from point A to point B, not necessarily the shortest path. This is Fermat's Principle and involves minimizing the time to travel a certain path, based on the **optical path length**

$$\int_C n(x,y,z)\,ds$$

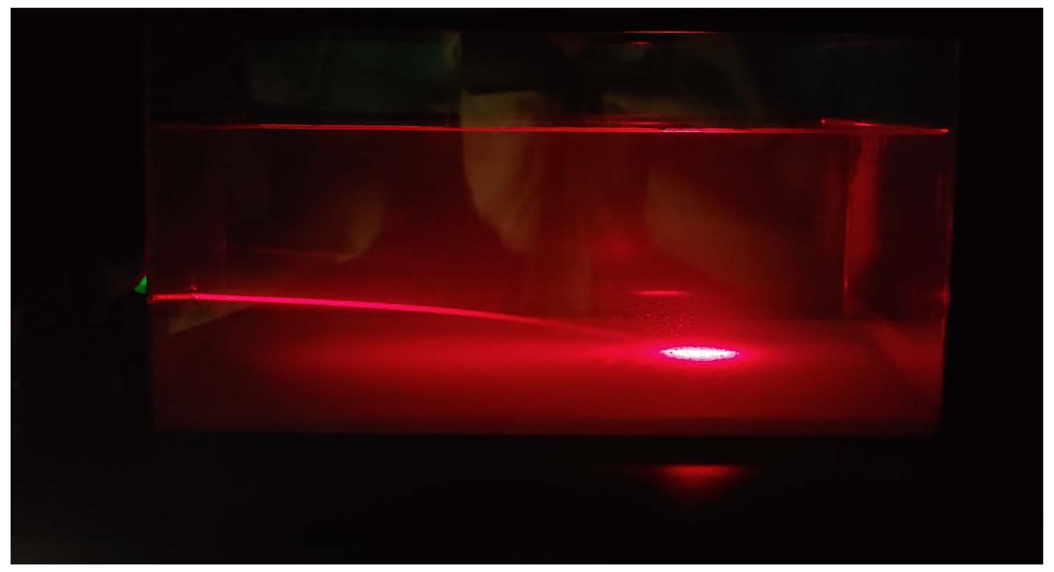
We can see this effect by creating a medium with varied index of refraction, n, (a GRIN lens). The visuals below come from a gradient of sugar dissolved in water with a laser going through the tank.



(Left) A large quantity of sugar is dissolved from the bottom of an acrylic tank of water. The density (and thus the index of refraction) is much higher at the bottom than at the top



A laser pointer travels straight through near the top of the water where the index of refraction is relatively constant



Near the bottom of the tank, a distinct curve is seen in the light path due to the gradient of the index of refraction

Connections to Calculus

Calculus 3 students will connect this demo and the behavior of light in GRIN lenses to calculus with three linked activities

- Using optimization to derive Snell's Law of diffraction
- Using principles of integration (infinite sums of infinitely small "steps") to determine optical path length through a GRIN medium
- Connecting the components of a scalar line integral with physical quantities (path length, velocity of light, time traveled)
- Computing multiple path integrals to show that the minimal optical path through a GRIN lens is not a straight line

Acknowledgements

I could not have done any of this work without the great mentorship and guidance from Dr. Juliet Gopinath and Lange Simmons, plus the rest of the ultrafast optics research group at CU. I came into the summer research project knowing almost nothing about lasers and have developed a huge amount of understanding that will inform both my teaching and understanding of light.

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