Engineering artificial gauge fields with ultracold atoms - part II

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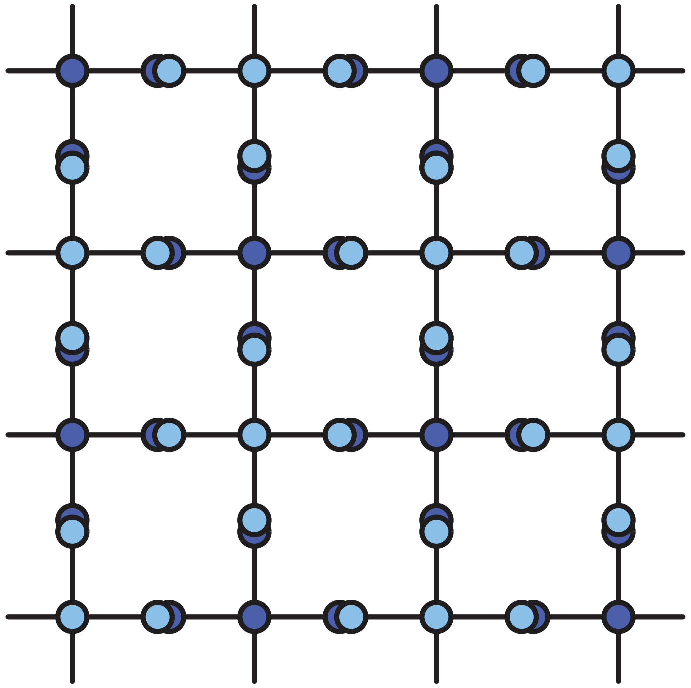

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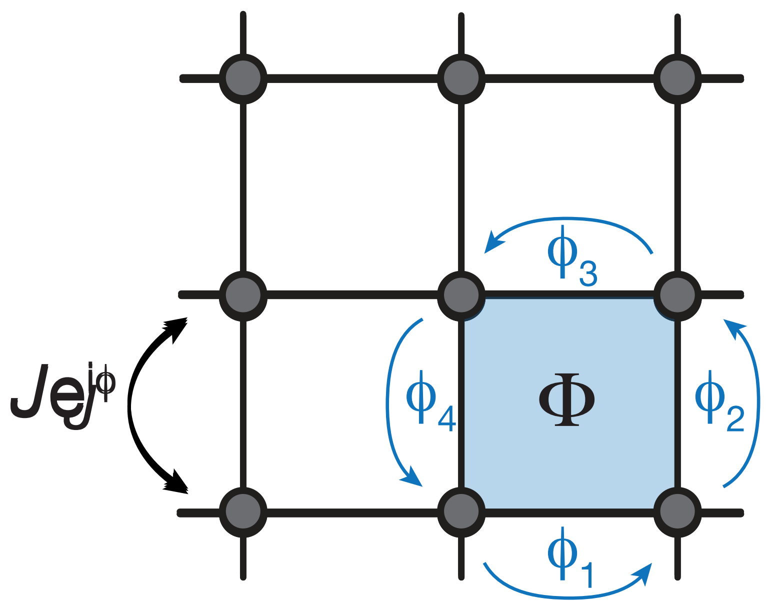
Artificial magnetic fields

Non-interacting lattice Hamiltonian:

$$\hat{H} = -\sum_{\langle i,j \rangle} J_{ij} \hat{a}_i^\dagger \hat{a}_j + \mathrm{h.c.}$$



Charged particles in magnetic field
→ acquire geometric phase

Peierls substitution: $J_{ij} \to J_{ij} \mathrm{e}^{i\phi_j}$

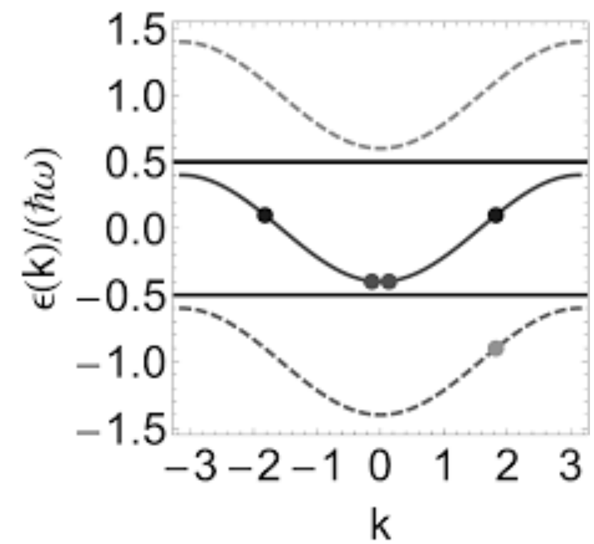
$$\phi_j = \frac{q}{\hbar} \int_{x_j}^{x_i} \mathbf{A} \mathrm{d}\mathbf{l}, \quad \mathbf{B} = \nabla \times \mathbf{A}$$

Phase around closed loop:

$$\boxed{\Phi = \sum_{\square} \phi_j = 2\pi \frac{\Phi_B}{\Phi_0}}$$

Φ_B: magn. flux
$\Phi_0 = h/q$: magn. flux quantum

Floquet engineering



Basic idea:

- Time-periodic driven Hamiltonian

$$\boxed{\hat{H}(t) = \hat{H}(t+T)}$$ T: driving cycle

- *Stroboscopic* time evolution governed by *time-independent Floquet Hamiltonian* $\hat{H}^F$

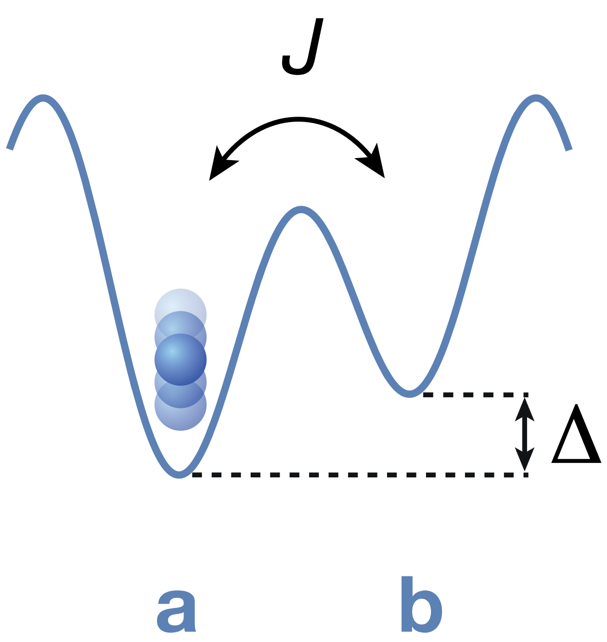
$$\boxed{\hat{U}(T,0) = \exp\left(-\frac{i}{\hbar}T\hat{H}^F\right)}$$

⇒ Engineer **Floquet Hamiltonian** with desired properties!

N. Goldman et al. PRX (2014); M. Bukov et al. Adv. in Phys. (2015); A. Eckardt, Rev. Mod. Phys. (2017)

Laser-assisted tunneling

Multi-photon processes:



- Tilted double-well potential

$$\hat{H} = -J\left(\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}\right) + \Delta\, \hat{b}^\dagger \hat{b}$$

- *Resonant* modulation at $\Delta_\nu = \nu\hbar\omega$

$$\hat{V}(t) = V_0\, \cos(\omega t + \phi)\hat{a}^\dagger \hat{a}$$

- Restored tunneling (Floquet theory)

$$\boxed{J_{\text{eff}} = J\mathcal{J}_\nu(\chi)\mathrm{e}^{i\nu\phi}} \qquad \chi = \frac{V_0}{\hbar\omega}$$

N. Goldman et al. PRX (2014); M. Bukov et al. Adv. in Phys. (2015); A. Eckardt, Rev. Mod. Phys. (2017)

Laser-assisted tunneling

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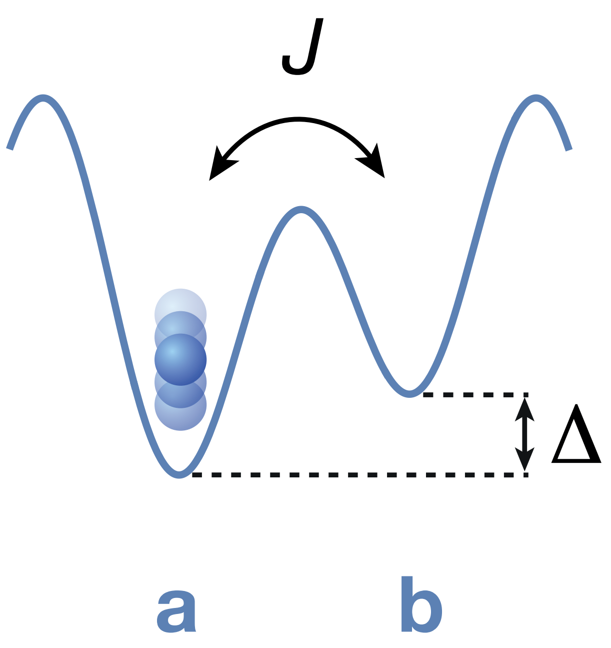


- *Resonant* modulation at $\Delta_\nu = \nu\hbar\omega$

$$\hat{V}(t) = V_0\, \cos(\omega t + \phi)\hat{a}^\dagger \hat{a}$$

- Reflection properties of Bessel function:

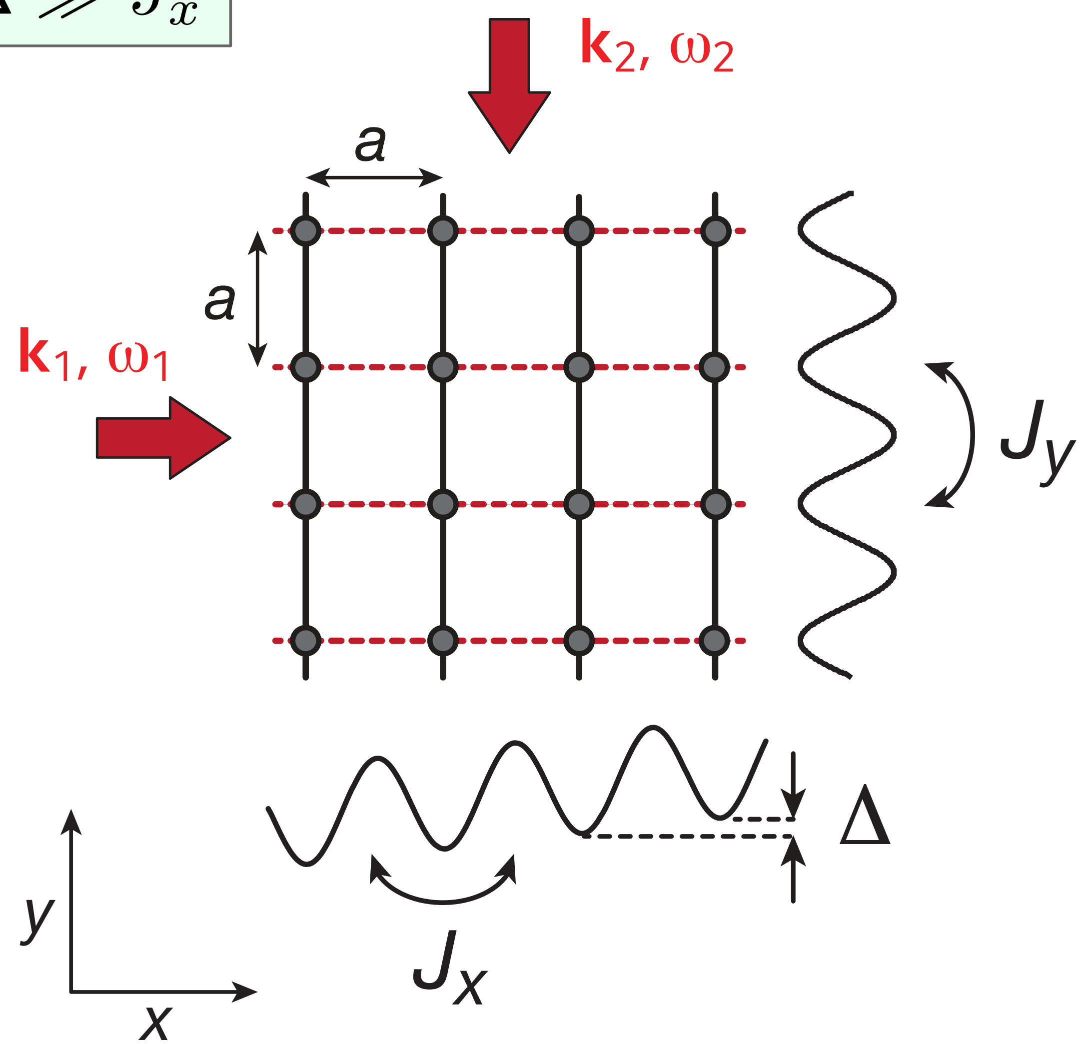
$$\boxed{\mathcal{J}_{-\nu}(\chi) = (-1)^\nu \mathcal{J}_\nu(\chi)}$$

N. Goldman et al. PRX (2014); M. Bukov et al. Adv. in Phys. (2015); A. Eckardt, Rev. Mod. Phys. (2017)

Engineering complex tunneling

Tunneling inhibited along one direction
+ bare tunneling along the orthogonal direction

$$\boxed{\Delta \gg J_x}$$



Apply resonant modulation with *far-detuned* running-wave beams:

$$\boxed{\omega_2 - \omega_1 = \Delta/\hbar}$$

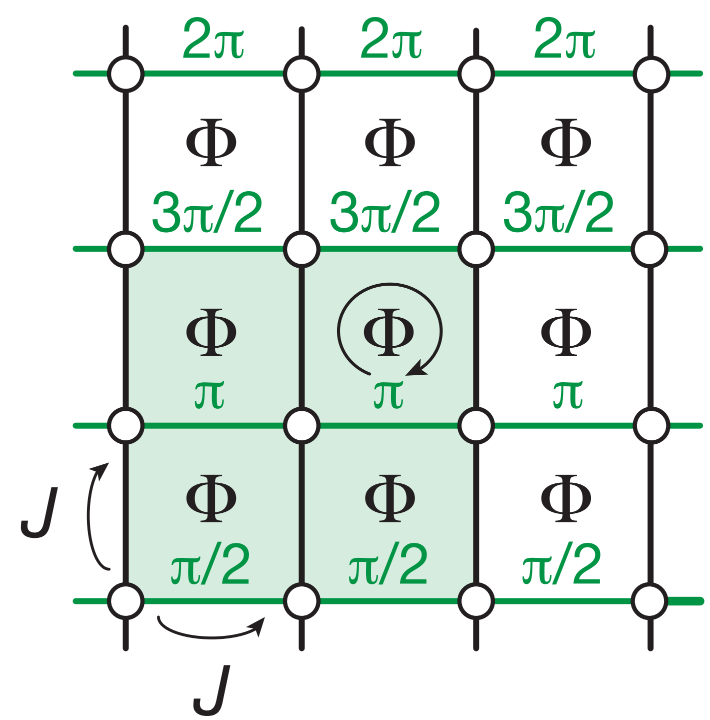
Local optical potential creates on-site modulation:

$$V(x,y,t) = V_0 \cos(\omega t + \phi(\mathbf{r}))$$

$$\boxed{\phi(\mathbf{r}) = \delta\mathbf{k}\cdot\mathbf{r}}$$

MA *et al.*, PRL (2011,2013); H. Miyake *et al.*, PRL (2013); E. M. Tai *et al.*, Nature (2017)

Hofstadter model



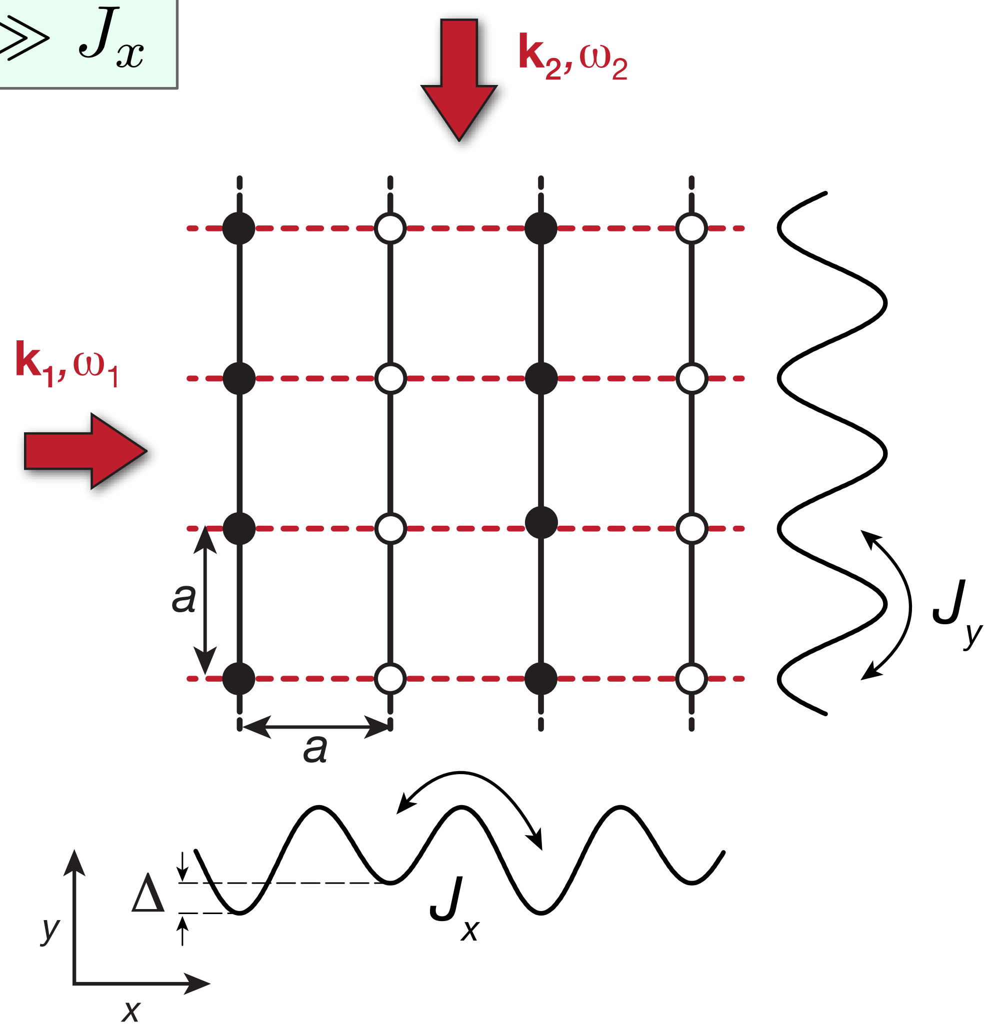
$$\hat{H} = -J\sum_{m,n}\left(\mathrm{e}^{in\Phi}\hat{a}^\dagger_{m+1,n}\hat{a}_{m,n} + \hat{a}^\dagger_{m,n+1}\hat{a}_{m,n} + \mathrm{h.c.}\right)$$

P.G. Harper, Proc. Phys. Soc., Sect.A 68, 874 (1955);
M.Y. Azbel, Zh. Eksp. Teor. Fiz. 46, 929 (1964); D.R. Hofstadter, Phys. Rev. B14, 2239 (1976)

Question

What is the flux configuration for this modulation scheme?

$$\boxed{\Delta \gg J_x}$$



Resonant modulation with *far-detuned* running-wave beams:

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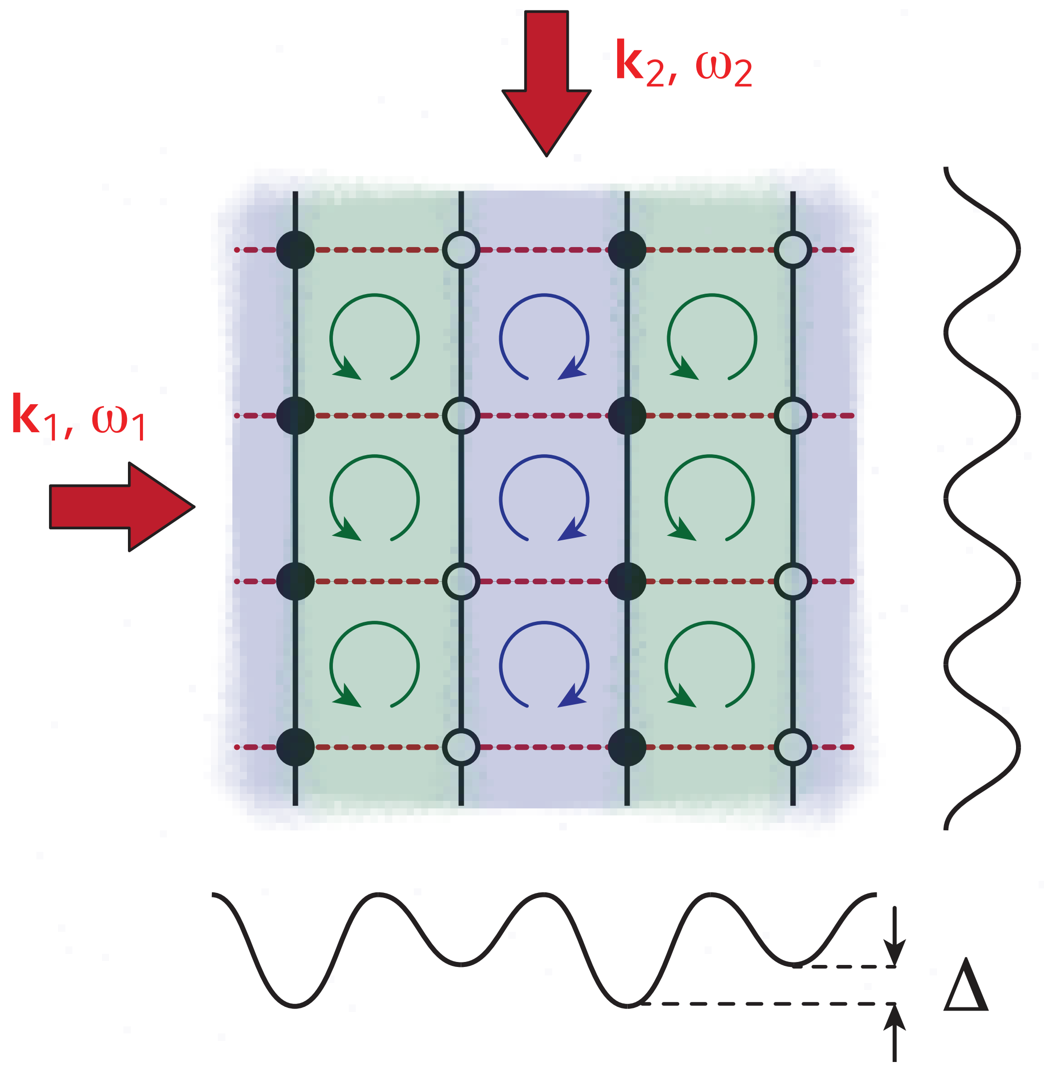
Answer

Staggered flux configuration (zero net magnetic field)

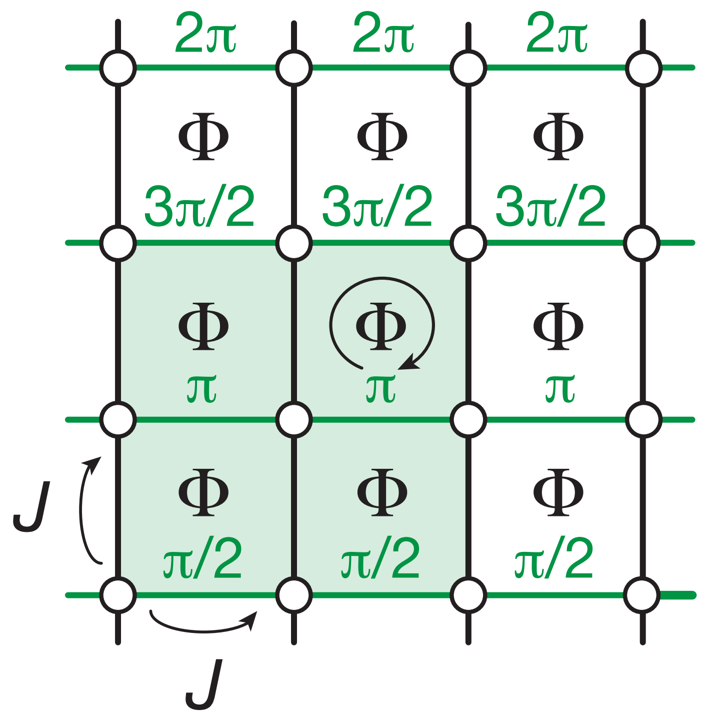


Sign of Peierls phases depends on sign of energy offset:

$$\Delta \Longleftrightarrow -\Delta$$
$$Ke^{i\phi_{m,n}} \Longleftrightarrow -Ke^{-i\phi_{m,n}}$$

Hofstadter model

$$\hat{H} = -J \sum_{m,n} \left(e^{in\Phi} \hat{a}^{\dagger}_{m+1,n} \hat{a}_{m,n} + \hat{a}^{\dagger}_{m,n+1} \hat{a}_{m,n} + \text{h.c.} \right)$$



Discrete translational symmetry
no longer determined by lattice vectors

- Magnetic translation operators!
- Increased magnetic unit cell (flux 2π)

P.G. Harper, Proc. Phys. Soc., Sect.A 68, 874 (1955);
M.Y. Azbel, Zh. Eksp. Teor. Fiz. 46, 929 (1964); D.R. Hofstadter, Phys. Rev. B14, 2239 (1976)

How to probe topology?

Probing band topology

Chern number:

$$\boxed{\nu_\mu = \frac{1}{2\pi}\int_{\mathrm{BZ}} \Omega_\mu \mathrm{d}^2 q}$$

$|u_\mu(\mathbf{q})\rangle$: periodic Bloch function,

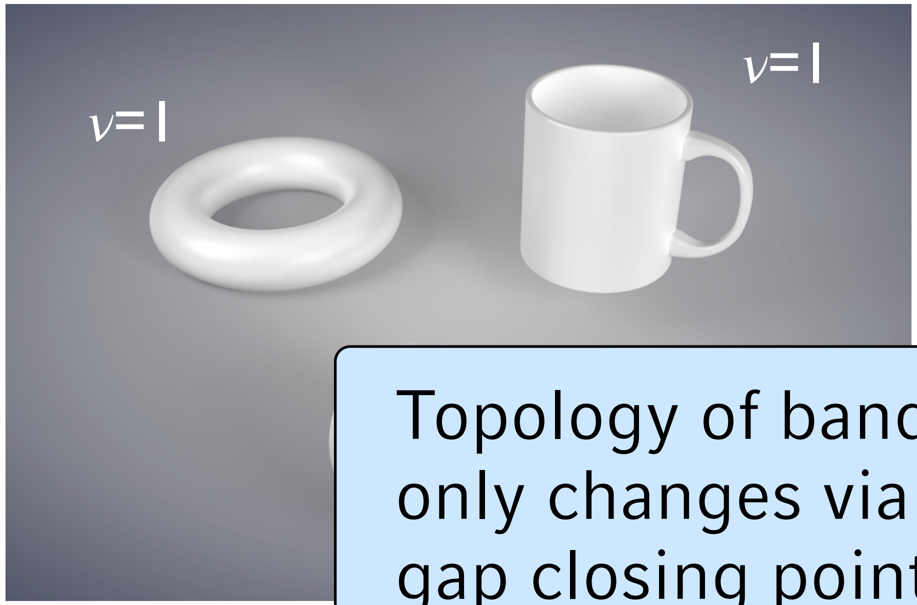
μ: band index

Berry curvature: $\Omega_\mu = i\left(\langle \partial_{q_x} u_\mu | \partial_{q_y} u_\mu \rangle - \langle \partial_{q_y} u_\mu | \partial_{q_x} u_\mu \rangle\right)$

Gauss-Bonnet theorem:

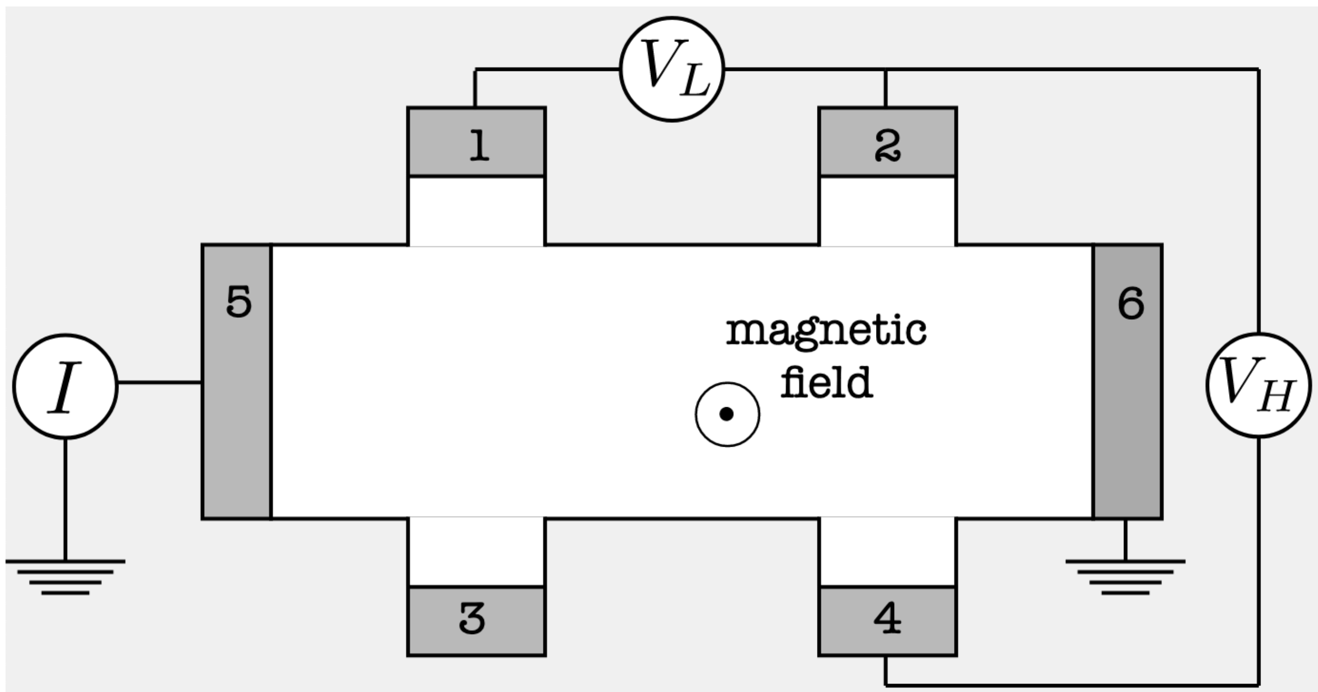
$$\boxed{\frac{1}{2\pi}\int_{\mathcal{M}} \Omega \, dS = 2(1-\nu)}$$

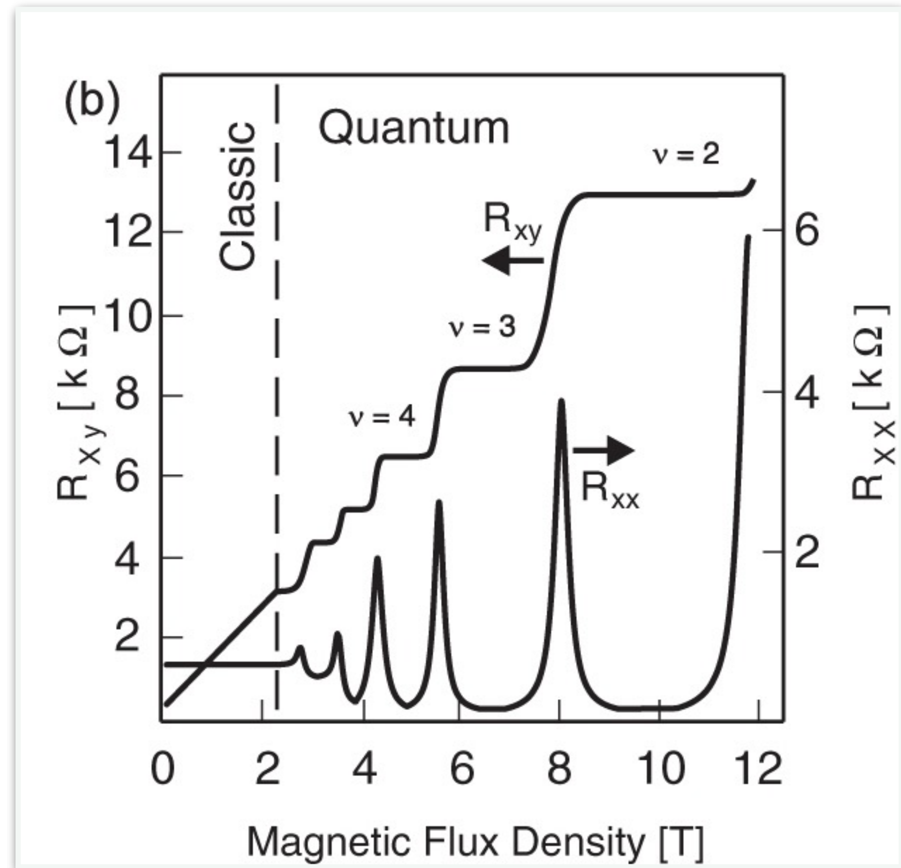
local curvature



Topology of bands only changes via gap closing points!

Transport measurements



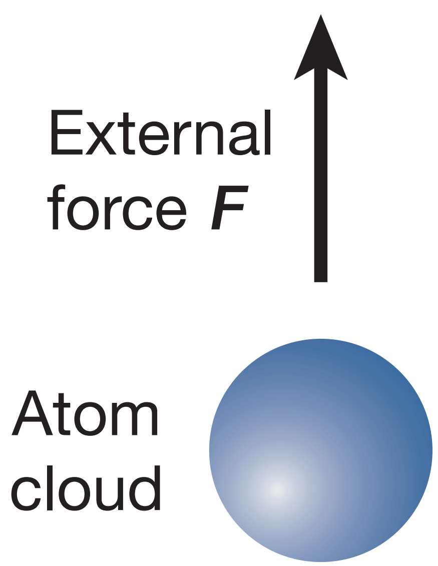


How does this work with ultracold atoms?

K. Klitzing, Rev. Mod. Phys. (1986)

Hall deflection with bosonic atoms

Semiclassical dynamics:



Atoms undergo Bloch oscillations captured by **band velocity**

$$\boxed{\mathbf{v}_{\mu}^{\text{band}} = \frac{1}{\hbar}\partial_{\mathbf{k}} E_{\mu}}$$

μ: band index

Atoms experience an **anomalous transv.** velocity prop. to Berry curvature $\boldsymbol{\Omega}_{\mu}$

$$\boxed{v_{\mu}^{x}(\mathbf{k}) = -\frac{F}{\hbar}\Omega_{\mu}(\mathbf{k})}$$

Karplus & Luttinger, Phys. Rev. (1954); D. Xiao *et al.*, Rev. Mod. Phys. (2010)
A. Dauphin & N. Goldman, PRL (2013); H. Price & N. Cooper, PRA (2012)

Hall deflection with bosonic atoms

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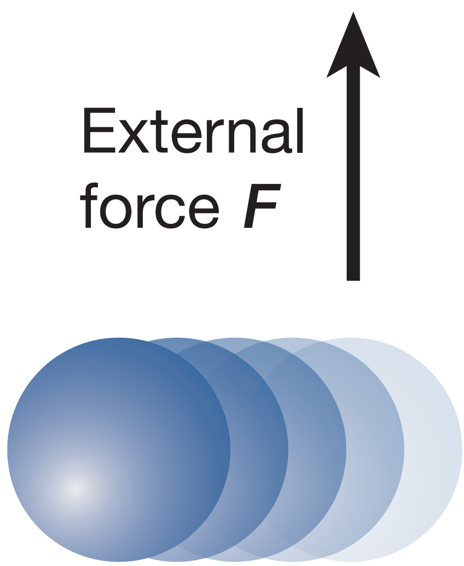

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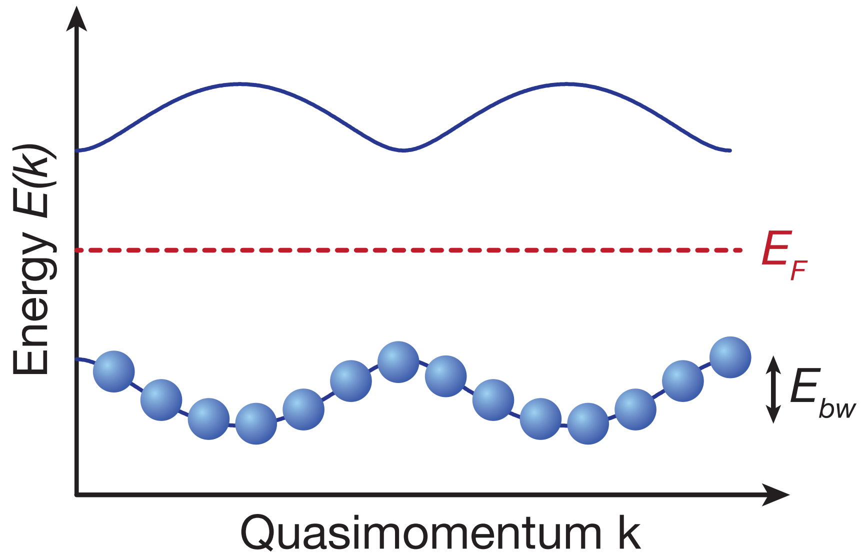
Hall deflection with bosonic atoms

Deflection of the cloud for filled band:



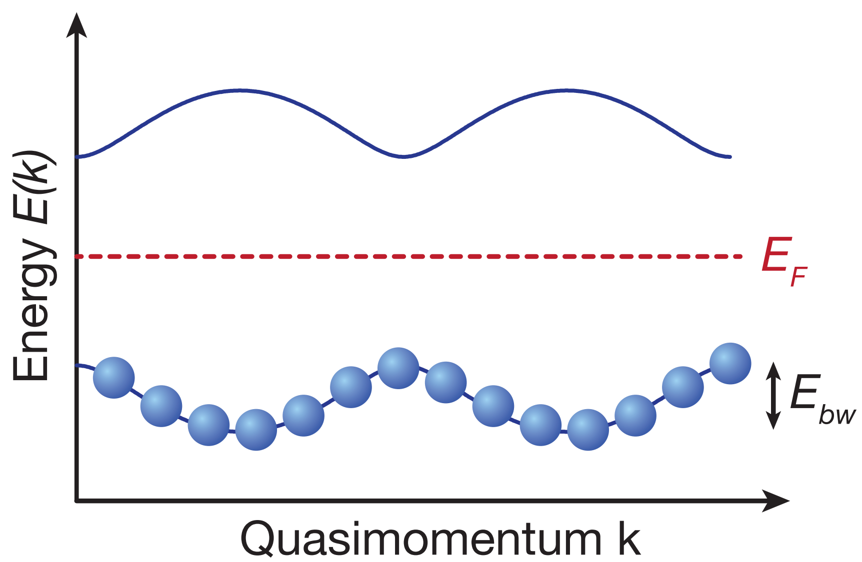
Contribution from band velocity vanishes

$$\int \partial E_\mu / \partial k_{x,y} \mathrm{d}^2\mathrm{k} = 0$$

- **Fermions**: Set Fermi energy within a spectral gap
- **Bosons**: Incoherent distribution homogeneously populating the band ($k_B T \gg E_{\mathrm{bw}}$)

Hall deflection with bosonic atoms

Deflection of the cloud for filled band:



Transverse center-of-mass motion (μth band):

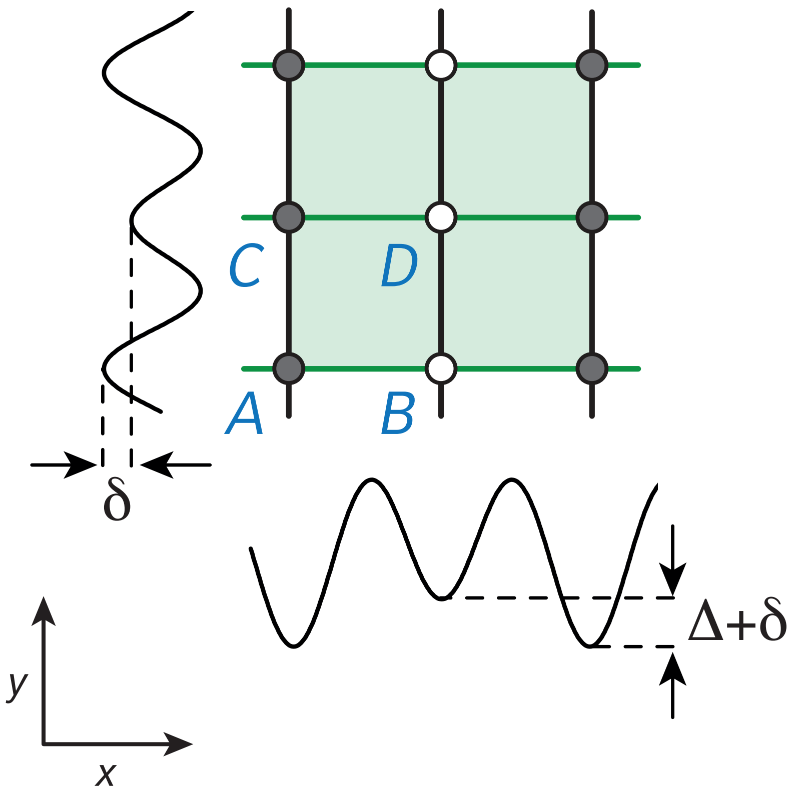
$$\boxed{x_\mu(t) = -\frac{4a^2F}{h}\,\nu_\mu\,t}$$

Response is proportional to Chern number of μth band:

$$\nu_\mu = (1/2\pi)\int_{\mathrm{FBZ}} \Omega_\mu \mathrm{d}^2\mathrm{k}$$

Hall deflection with bosonic atoms

Experimental preparation protocol

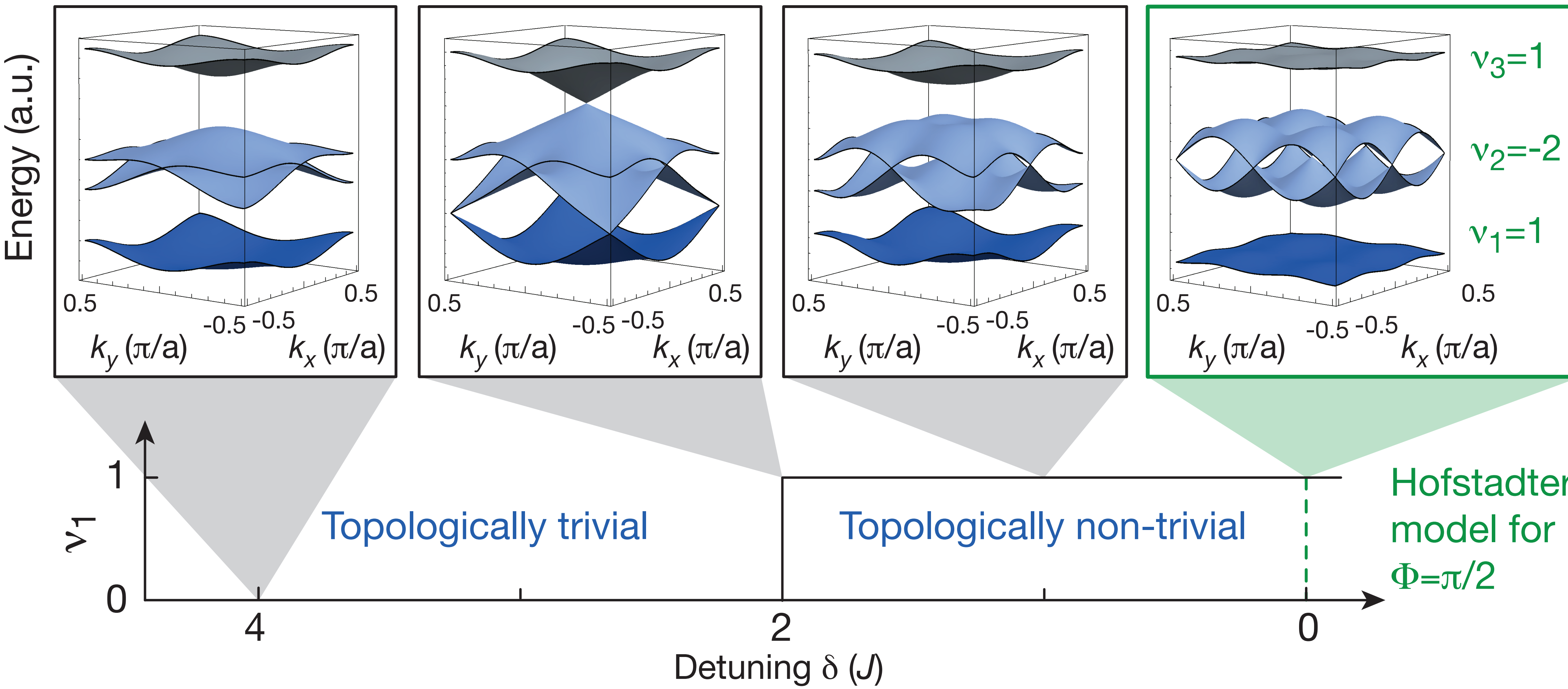


Challenge:
lowest tight-binding band
splits in the **magnetic subbands**

Solution:
preserve number of bands during
loading sequence

Hall deflection with bosonic atoms

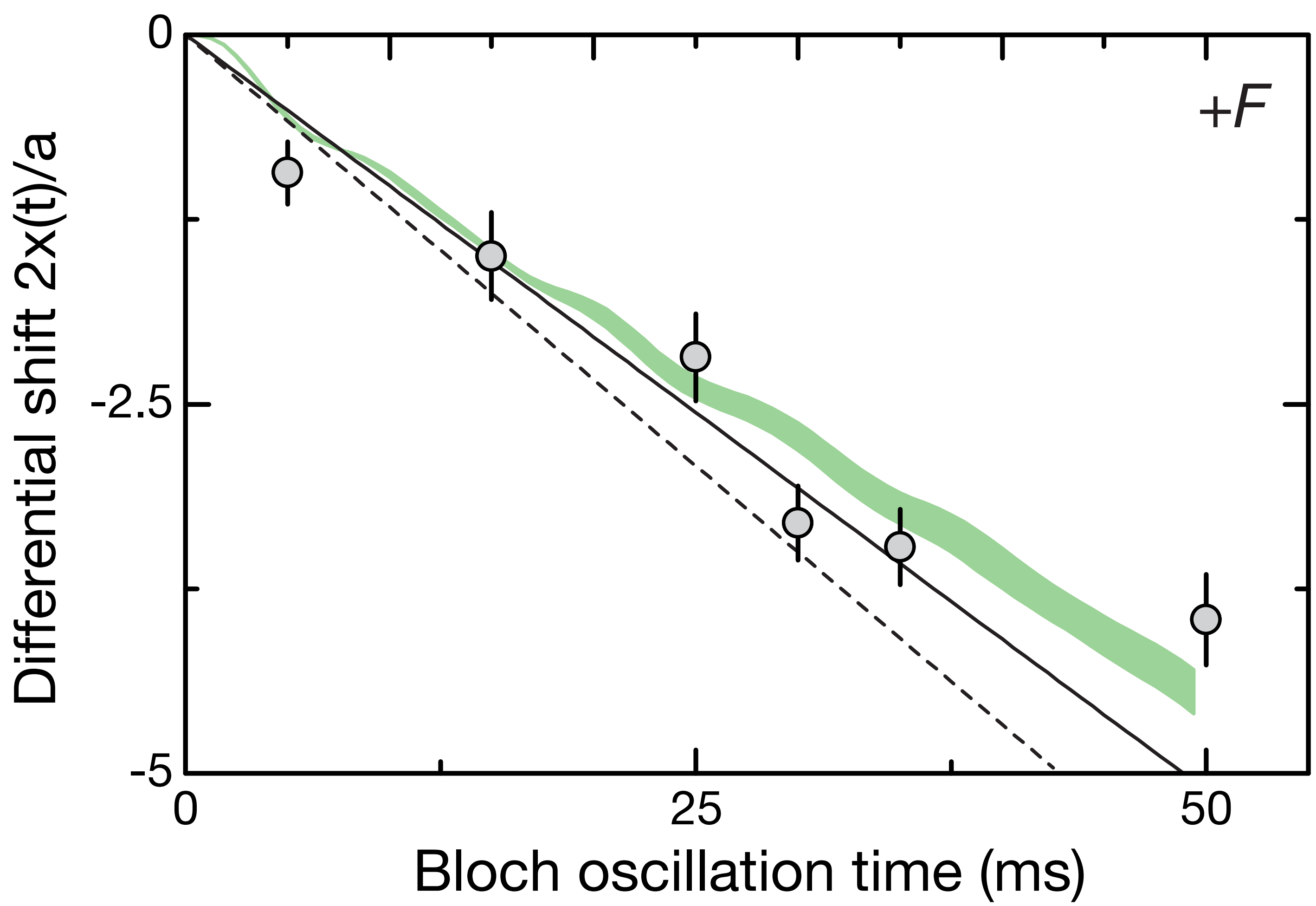
Experimental preparation protocol



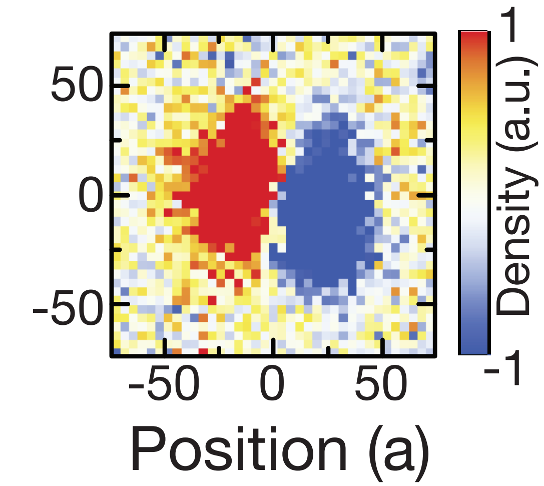
Topological phase transition (gap-closing point) at $\delta = 2J$

Hall deflection with bosonic atoms

Differential transv. displacement: $x(+\Phi, t) - x(-\Phi, t) = 2x(t)$



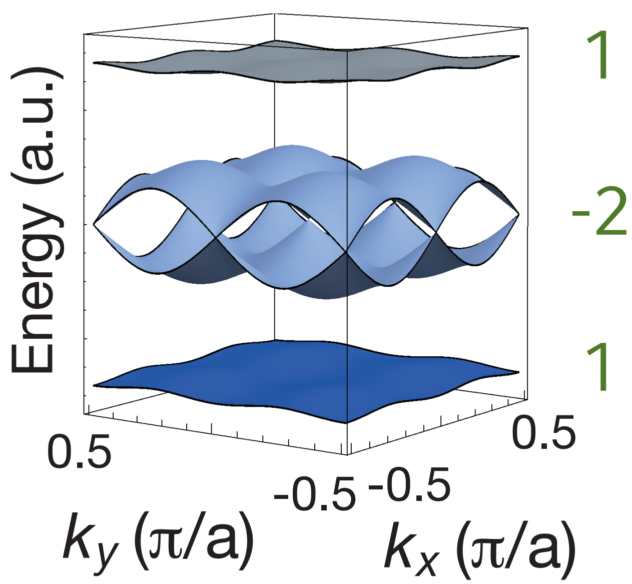
Difference image for $\pm\Phi$



Observed deflection reduced by factor $\gamma \simeq 0.37$

Hall deflection with bosonic atoms

Higher-band populations:



- If only the lowest band is filled:

$$x(t) = -\frac{4a^2 F}{h}\,\nu_1 t$$

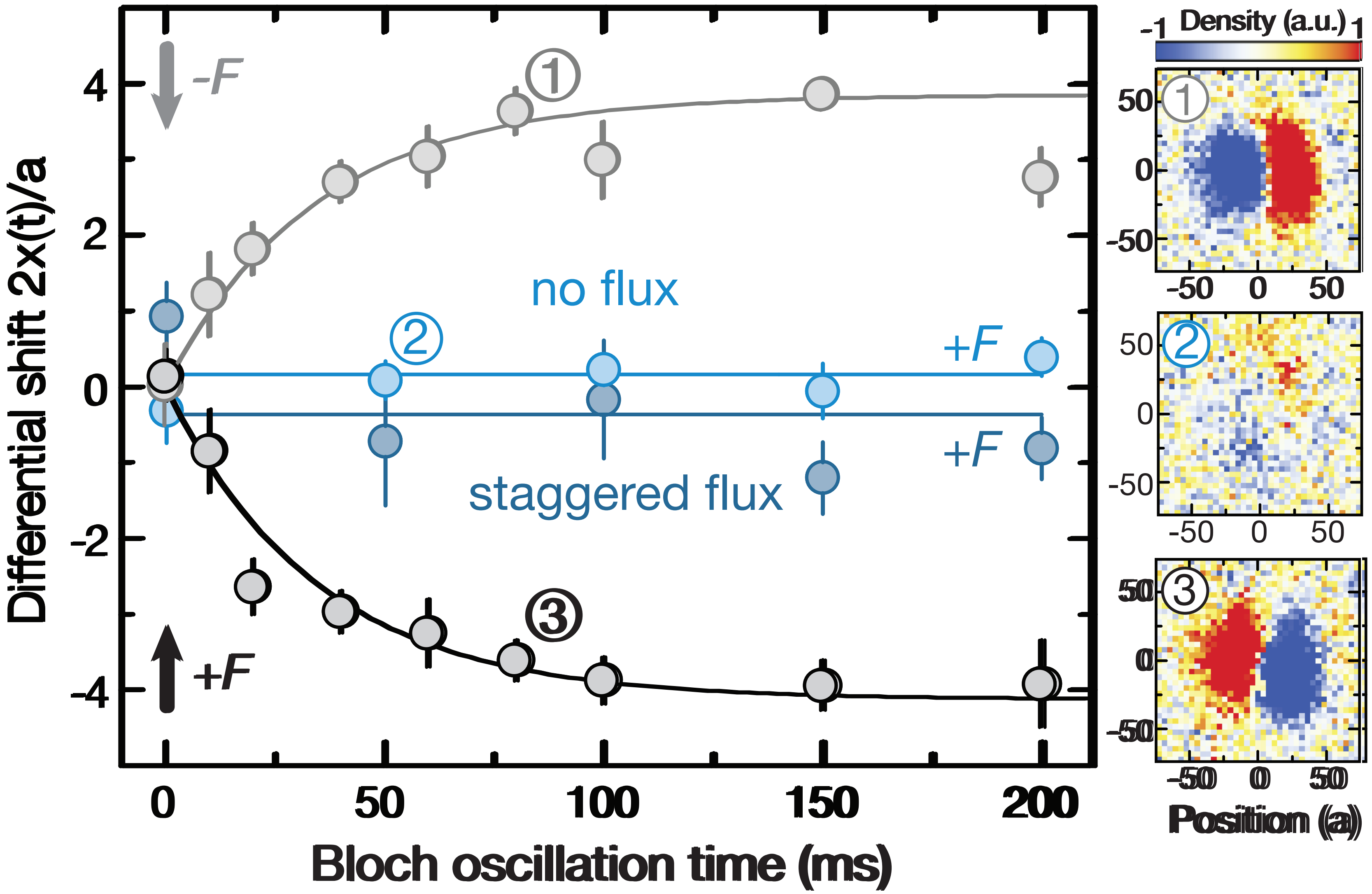
linear evolution prop. to Chern number

- Higher-bands: $x(t) \rightarrow \gamma x(t)$ with $\gamma = \eta_1 - \eta_2 + \eta_3$,

η_i : rel. band populations

Derivation relies on particle-hole symmetry & $\sum_\mu \nu_\mu = 0$

Hall deflection with bosonic atoms

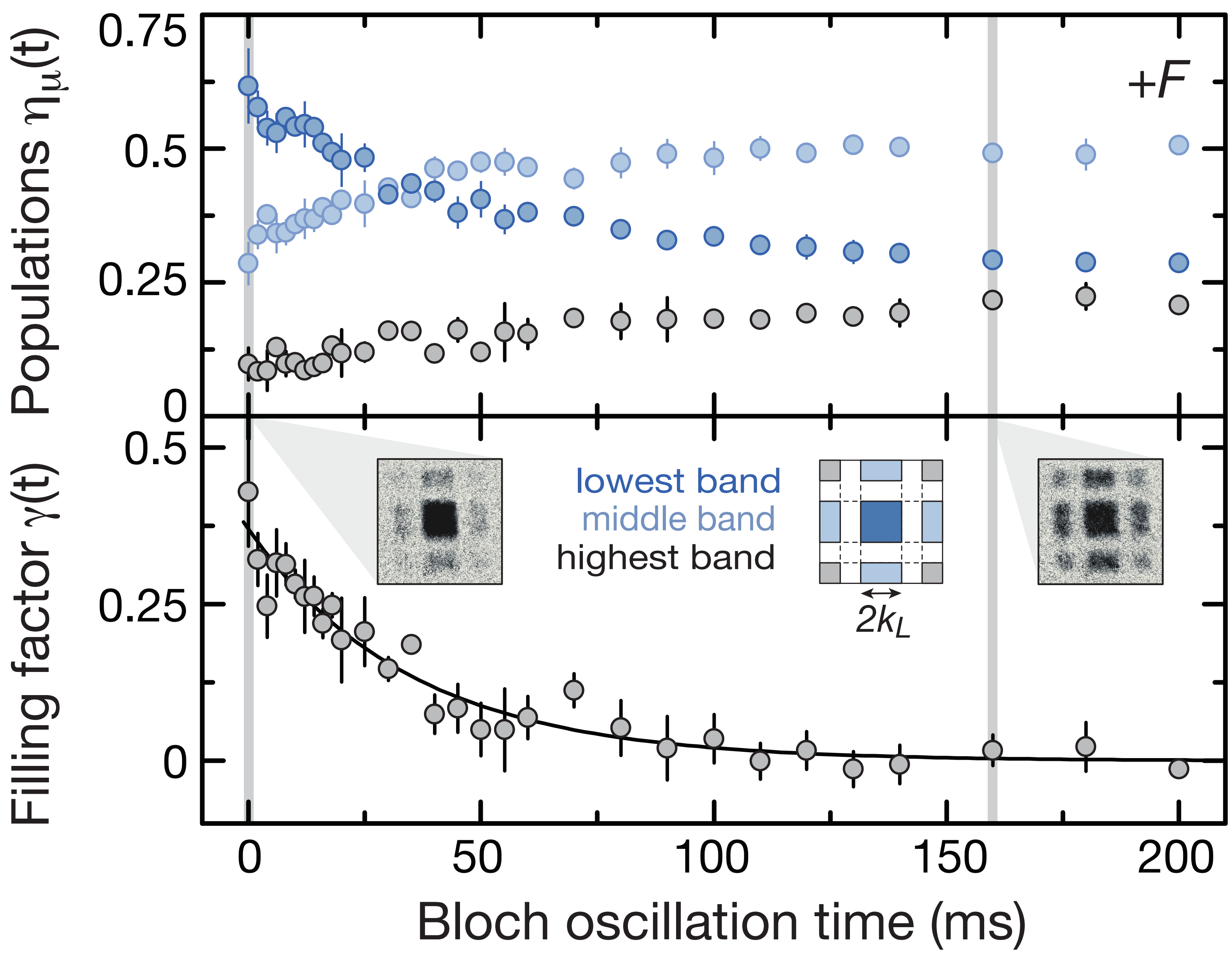


Saturation for longer evolution times

MA *et al.*, Nature Physics 11, 162 (2015)

Hall deflection with bosonic atoms

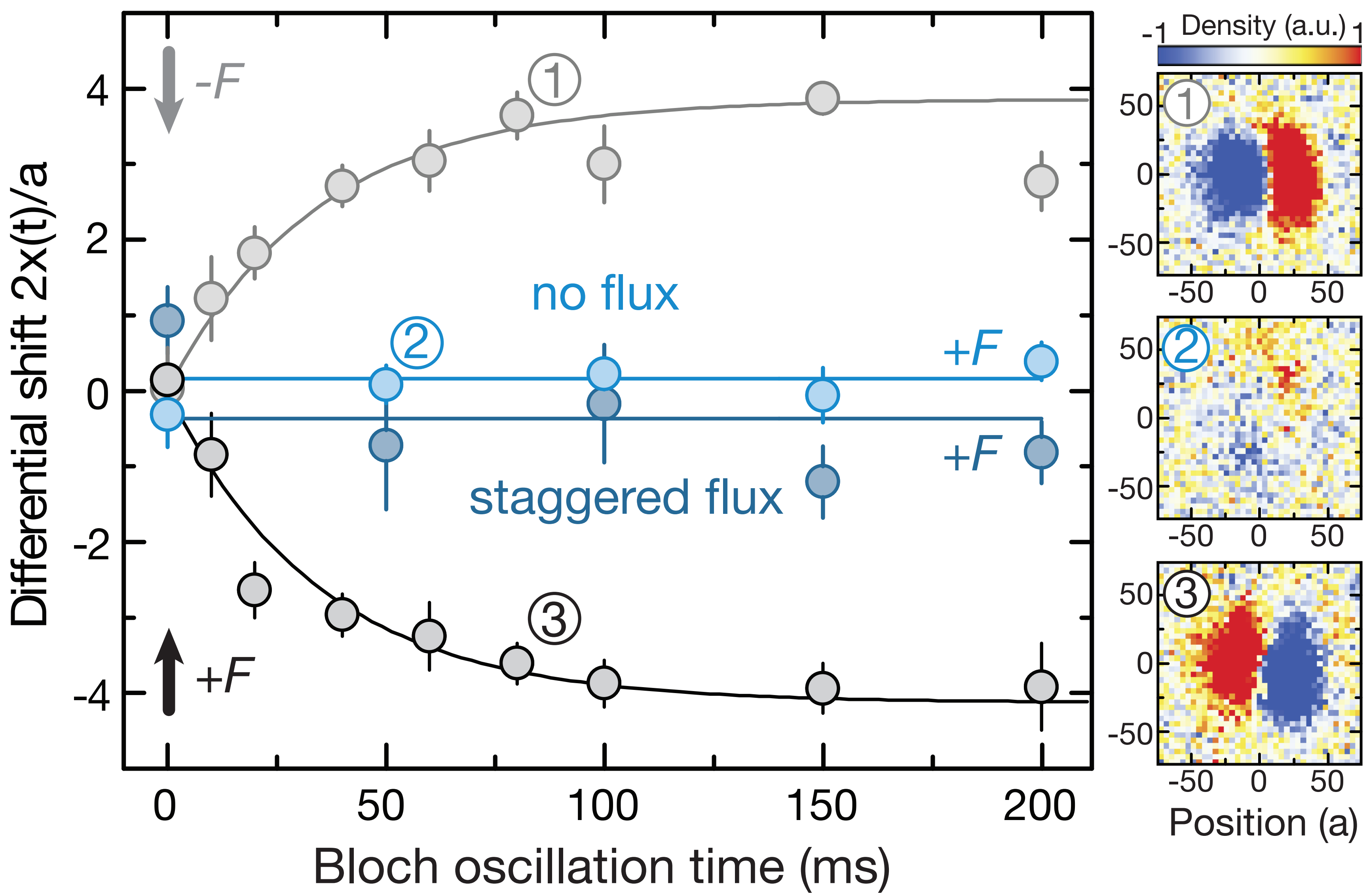
Measured band populations:



T. Bilitewski *et al.*, PRA 91, 033601 & PRA 91, 063611 (2015), S. Choudhury et al., PRA 91, 023624 (2015)

Hall deflection with bosonic atoms

Combination of both: $x(t) = -\frac{4Fa^2}{h}\nu_1 \int_0^t \gamma(t')\mathrm{d}t'$



$$\nu_{\mathrm{exp}} = 0.99(5)$$

MA *et al.*, Nature Physics 11, 162 (2015)
see also: G. Jotzu et al., Nature (2014)

Probing band topology

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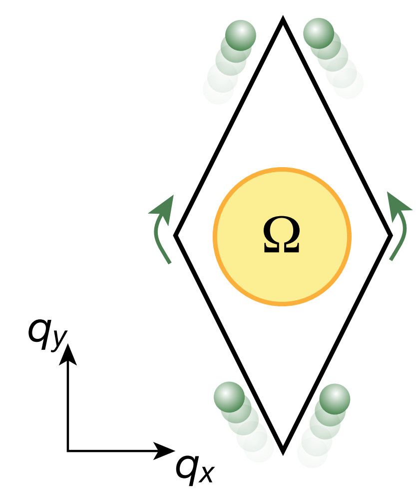
$$\mathcal{C}^{\mu} = \frac{1}{2\pi}\int_{\mathrm{BZ}} \Omega^{\mu} \mathrm{d}^2 q$$

$|u_{\mu}(\mathbf{q})\rangle$: periodic Bloch function,

μ: band index

- Interferometric measurements / transport:

  M. Atala *et al.*, Nat. Phys. (2013); L. Duca *et al.*, Science (2015)
  G. Jotzu et al., Nature (2014); M. A. et al., Nature Phys. (2015)

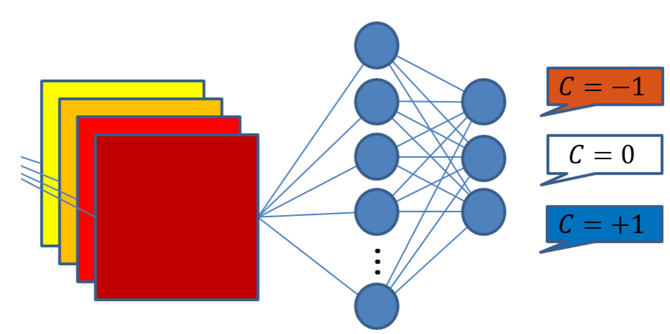


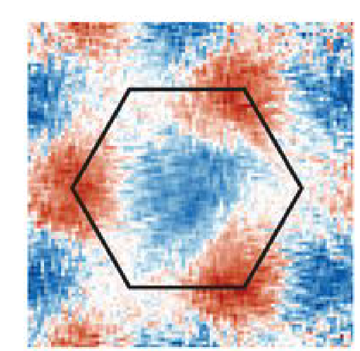
- State tomography:

  N. Fläschner *et al.*, Science (2016); T. Li *et al.*, Science (2016)

- Linking number / circular dichroism / machine learning:

  Tarnowski *et al.*, Nat. Comm. (2019);
  L. Asteria *et al.*, Nat. Phys. (2019);
  B. Rem *et al.*, Nat. Phys. (2019)





see also: experiments in synth. dimensions