Engineering artificial gauge fields with cold atoms

– slides –

I. Laser-assisted tunneling (Floquet engineering)

Recap: perturbative calculation of the Floquet Hamiltonian using a high-freq. expansion

General form of Floquet's theorem:

$$\boxed{\hat{\mathcal{U}}(t_f, t_i) = \underbrace{\hat{P}(t_f)}_{=\, e^{-i\hat{K}(t_f)}} \, e^{-i/\hbar \, \hat{H}_F (t_f - t_i)} \, \hat{P}^{\dagger}(t_i)}$$

where $\hat{P}(t) = \hat{P}(t+T)$ is time periodic and usually denoted as kick operator

t_i : initial time

t_f : final time

$t_f - t_i$: arbitrary, including non-stroboscopic times

a) Commutator relations

Assume: $\boxed{\hat{H}(t) = \hat{H}_0 + \hat{V}(t)}$

time periodic

$$\rightarrow \quad \hat{V}(t) = \sum_{n=1}^{\infty} \left(\hat{V}^n e^{in\omega t} + \hat{V}^{-n} e^{-in\omega t} \right)$$

$$\hat{H}_F = \hat{H}_0 + \frac{1}{\hbar\omega} \sum_{n=1}^{\infty} \frac{1}{n} \left[\hat{V}^n, \hat{V}^{-n} \right] +$$

$$+ \frac{1}{2(\hbar\omega)^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left(\left[\left[\hat{V}^n, H_0 \right], \hat{V}^{-n} \right] + \left[\left[\hat{V}^{-n}, \hat{H}_0 \right], \hat{V}^n \right] \right)$$

$$+ \mathcal{O}\left(\frac{1}{\omega^3} \right)$$

$$\boxed{\hat{K}(t) = \sum_{n \neq 0} \frac{1}{in\hbar\omega} \hat{V} e^{in\omega t} + \mathcal{O}\left(\frac{1}{\omega^2} \right)}$$

b) Magnus expansion

useful f. description of stroboscopic long-time dynamics

$$\boxed{\hat{H}_F = \sum_{n=0}^{\infty} \hat{H}_F^{(n)}}$$

with $$\hat{H}_F^{(0)} = \frac{1}{T}\int_0^T \hat{H}(t)\,dt$$

$$\hat{H}_F^{(1)} = \frac{-i}{2\hbar T}\int_0^T\int_0^{t_2} \left[\hat{H}(t_2),\ \hat{H}(t_1)\right] dt_1\, dt_2$$

$\vdots$

+ higher orders that scale as $\frac{1}{\omega^n}$

c) Resonant driving

additional terms in the Hamiltonian $\hat{H}_0$ that diverge with $\omega \to \infty$, denoted as $\hat{H}_\omega$

$\Rightarrow$ perform a unitary transformation

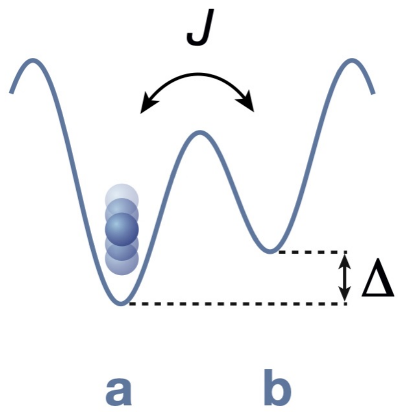
$$\hat{R}(t) = e^{i/\hbar\, \hat{H}_\omega t}$$

$$\Rightarrow \boxed{\mathcal{H}(t) = \hat{R}(t)\,\hat{H}(t)\,\hat{R}^\dagger(t) - i\hbar\,\hat{R}(t)\,\partial_t\,\hat{R}^\dagger(t)}$$

d) Example: Two-level system

energy levels $|0\rangle$ and $|1\rangle$

with large energy offset Δ, J is the coupling between the two levels



→ energy difference between the two eigenstates is

$$E_{gap} = \sqrt{\Delta^2 + 4J^2}$$

In the limit of large Δ → no coupling between the two levels

Add resonant periodic modulation : $\hbar\omega = \Delta$

$$\hat{H}(t) = J\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right) + \hbar\omega\, |1\rangle\langle 1| +$$

diverging with $\omega \to \infty$

$$+ V_0 \cos(\omega t + \varphi)\, |0\rangle\langle 0|$$

Transformation into rotating frame:

problem simplifies in rot. frame using

$$\boxed{\hat{R}(t) = e^{i\omega |1\rangle\langle 1| t + i/\hbar \int^t \hat{V}(t')dt' |0\rangle\langle 0|}}$$

with $\hat{V}(t) = V_0 \cos(\omega t + \varphi)$

$$\Rightarrow \quad \hat{R}(t) = e^{i\omega t |1\rangle\langle 1| + i \frac{V_0}{\hbar\omega} \sin(\omega t + \varphi) |0\rangle\langle 0|}$$

Using this unitary transformation, we find the new Hamiltonian

$$\boxed{\mathcal{H}(t) = J |0\rangle\langle 1| e^{i\eta(t)} + J |1\rangle\langle 0| e^{-i\eta(t)}}$$

with $\eta(t) = -\left(\omega t - \frac{V_0}{\hbar\omega} \sin(\omega t + \varphi)\right)$

$\Rightarrow$ time-dependence now moved to coupling terms

$\Rightarrow$ $\mathcal{H}(t)$ is periodic in time without any diverging terms

$\Rightarrow$ Compute effective Floquet Hamiltonian e.g.

using the Magnus expansion

$$\hat{\mathcal{H}}_F = \frac{1}{T}\int_0^T \mathcal{H}(t)\,dt =$$

$\tau = \omega t$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left[J\,|0\rangle\langle 1|\,e^{-i\left[\tau - \frac{V_0}{\hbar\omega}\sin(\tau+\varphi)\right]} + \text{h.c.}\right] d\tau$$

$$= J\,\mathcal{J}_1\!\left(\frac{V_0}{\hbar\omega}\right) e^{i\varphi}\,|0\rangle\langle 1| + \text{h.c.}$$

where $\mathcal{J}_1(x) = \frac{1}{2\pi}\int_0^{2\pi} e^{-i(\tau - x\sin\tau)}\,d\tau$

is the first-order Bessel function of the first kind.

$\Rightarrow$ In the presence of resonant periodic modulation, the coupling between levels $|0\rangle$ and $|1\rangle$ is restored with an effective coupling strength

Multi-photon processes

with $\boxed{\Delta = \nu \cdot \hbar\omega}$, where $\nu \in \mathbb{Z}$

same derivation:

$$\boxed{\hat{\mathcal{H}}_F = J\, \mathcal{J}_\nu\!\left(\frac{V_0}{\hbar\omega}\right)\left[e^{i\nu\varphi}|0\rangle\langle 1| + e^{-i\nu\varphi}|1\rangle\langle 0|\right]}$$

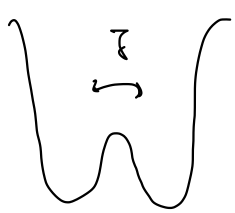
↑

Bessel function of first kind of order ν

$$\rightarrow \quad \mathcal{J}_\nu(x) = \frac{1}{2\pi}\int_0^{2\pi} e^{i(\nu\tau - x\sin\tau)}\, d\tau$$

e.g. zero order:

$\Rightarrow$ no energy difference between $|0\rangle$ and $|1\rangle$



$|0\rangle$ $|1\rangle$

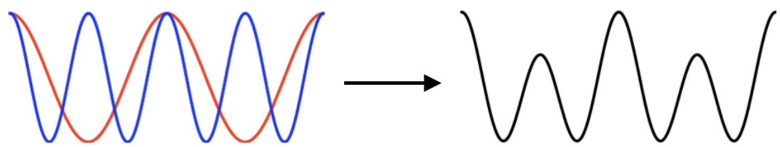
plus periodic modulation

$\rightarrow$ effective tunneling:

renormalization according to $\mathcal{J}_0(x)$ $\rightarrow$ $\boxed{J_{eff} = J\, \mathcal{J}_0\!\left(\frac{V_0}{\hbar\omega}\right)}$

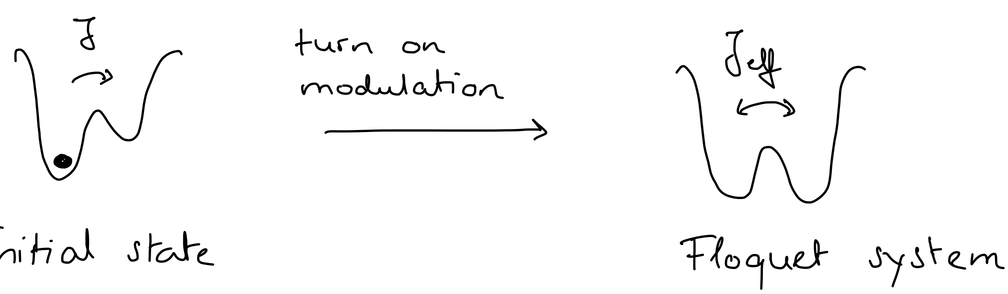
e) Measurement

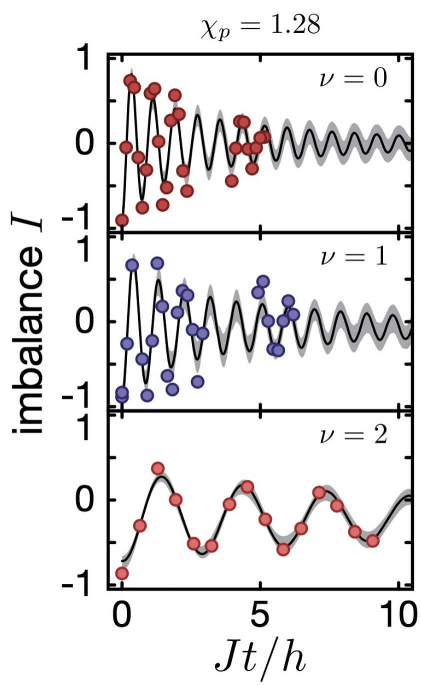
optical superlattices (3D array of isolated double wells)



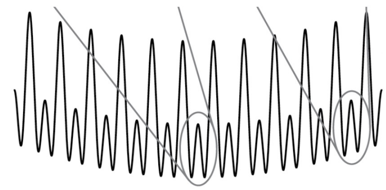
→ Calibration of effective tunnel coupling

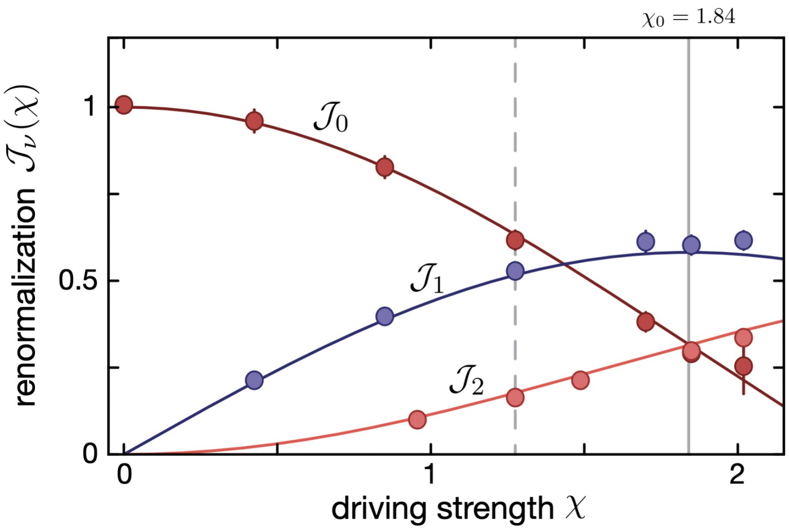
→ Prepare initial state, where atoms are localized on the lattice site with lower energy





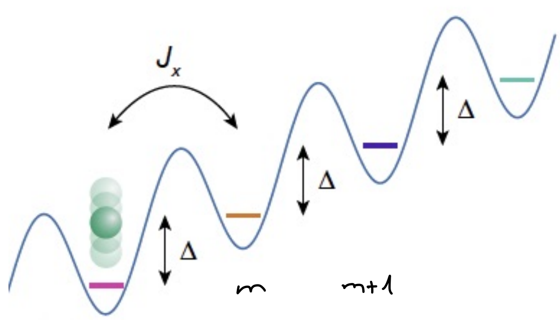
C. Schweizer et al. Nat Phys. 15, 1168 (2019) dephasing is due to external harmonic confinement





see also:
C. Sias et al., Phys. Rev. Lett. 100, 040404 (2008)
S. Mukherjee et al., New J Phys. 17, 115002 (2015)
F. Meinert et al., Phys. Rev. Lett. 116, 205301 (2016)

f, Extension to extended 1D lattices



- on-site potential Δ_m
- periodic modulation

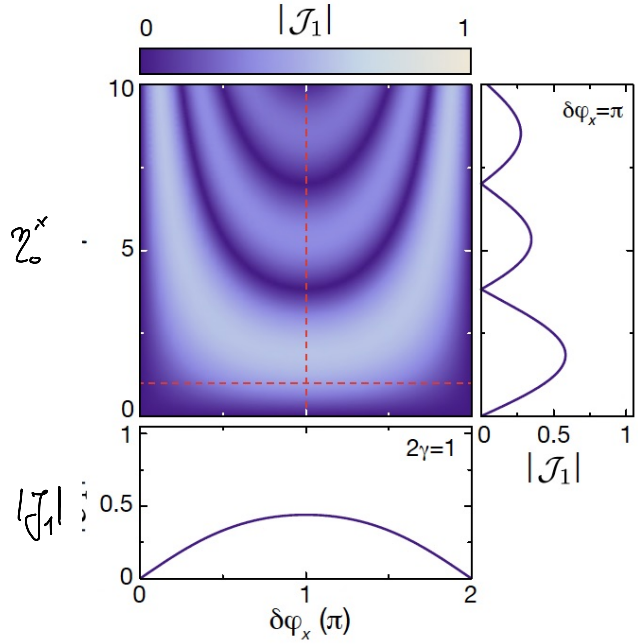
$$V_0 \sin(\omega t + \varphi_m)$$

Floquet Hamiltonian with effective tunnel coupling

$$J_{eff} = J_x \, J_1(\eta_0^x) \, e^{i\phi_m}$$

with $\eta_0^x = \frac{2V_0}{\hbar\omega} \sin\left(\frac{\varphi_{m+1} - \varphi_m}{2}\right)$, which defines the argument of the Bessel function

$\phi_m = \frac{\varphi_{m+1} + \varphi_m}{2}$, defines the phase that appears in the eff. complex tunneling element



$\Rightarrow$ one-photon processes define effective tunneling and phase

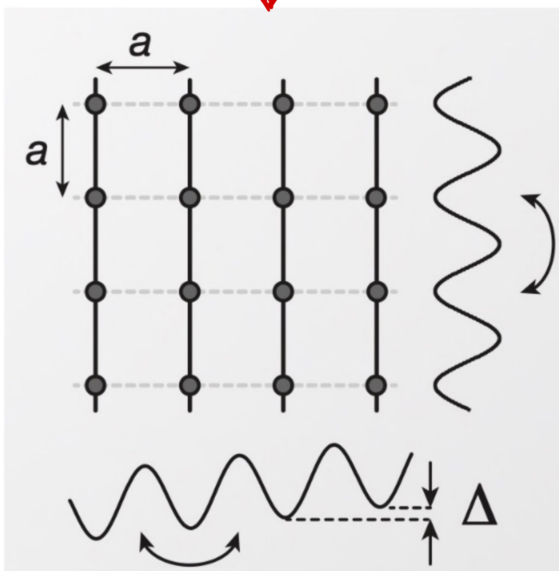
BUT : effective flux is defined by phase variation perp. to the laser-assisted tunneling direction

g, Artificial gauge fields in 2D

uniform flux : Hofstadter model

$\vec{k}_2, \omega_2$

$\vec{k}_1, \omega_1$



J_y

J_x

→ tunneling inhibited along x for $\Delta \gg J_x$

Add periodic modulation:

Interference between additional laser beams with wave vectors $\vec{k}_1, \vec{k}_2$ and frequencies ω_1, ω_2

$$V(\vec{r},t) = V_0 \cos(\omega t + \phi(\vec{r})), \quad \phi(\vec{r}) = \delta\vec{k}\cdot\vec{r}$$

$$\delta\vec{k} = \vec{k}_2 - \vec{k}_1$$

Resonance condition:

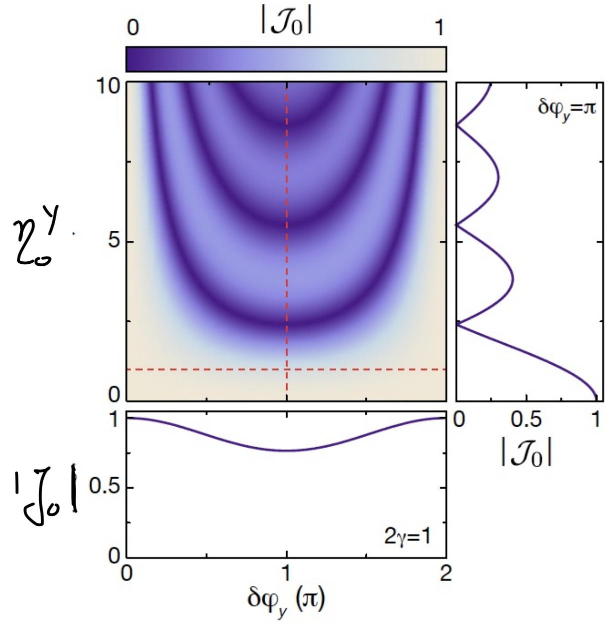
$$\boxed{\omega_2 - \omega_1 = \Delta/\hbar}$$

effective Floquet Hamiltonian with renorm. tunnelings

- along x: $J_{eff}^x = J_x \mathcal{J}_1(\eta_0^x)\, e^{i\phi_{m,n}}$
- along y: $J_{eff}^y = J_y \mathcal{J}_0(\eta_0^y)$

$\Rightarrow$ complex tunneling along x, real tunneling along y

$\Rightarrow$ effective magnetic flux $\Phi = \phi_{m,n} - \phi_{m,n+1}$

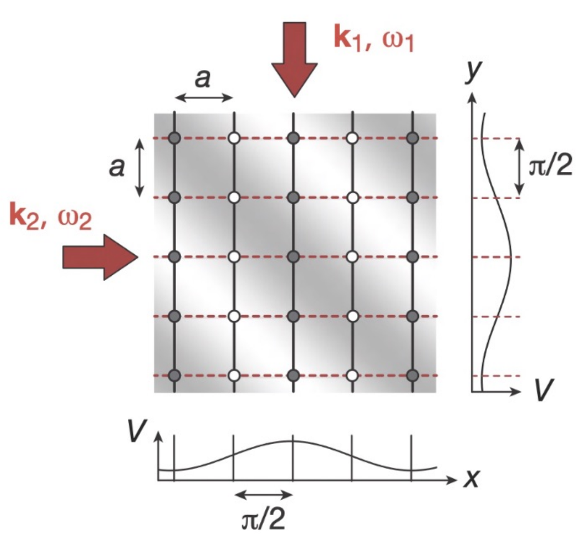


example:

$\Phi = \pi/2$

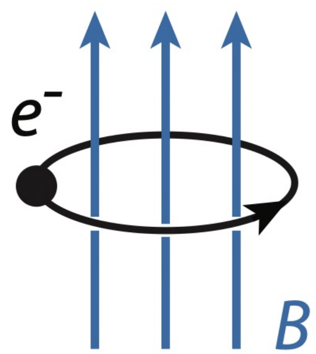
flux is fully tunable

$\boxed{\Phi = \delta k_y a}$

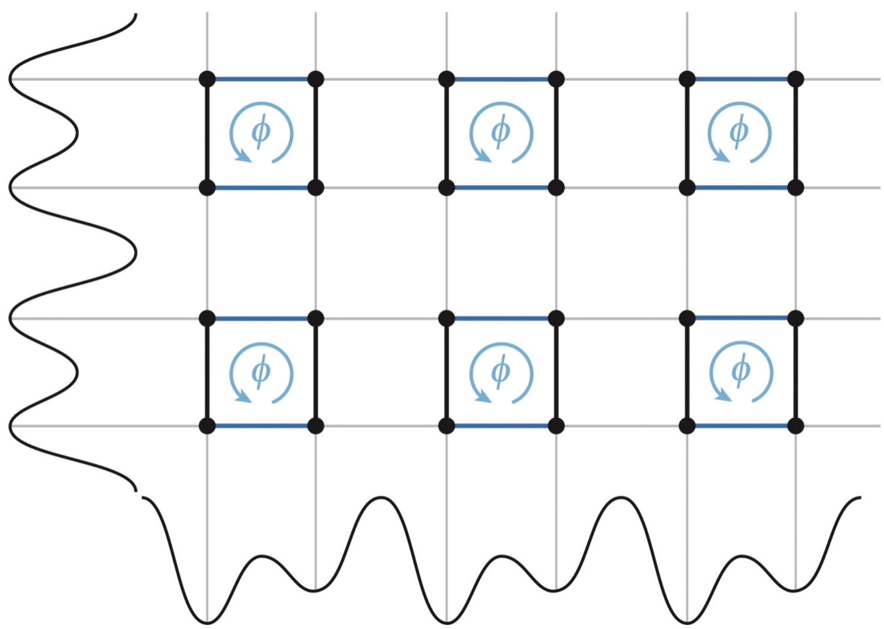


h, Measuring the effective flux

Cyclotron motion



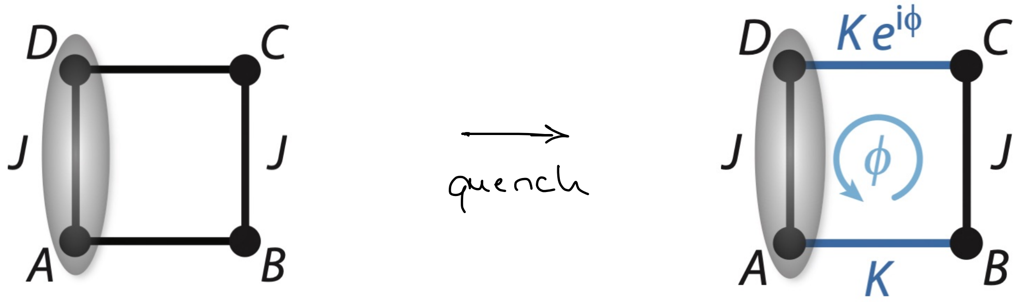
⇒ measure real-space motion in superlattices

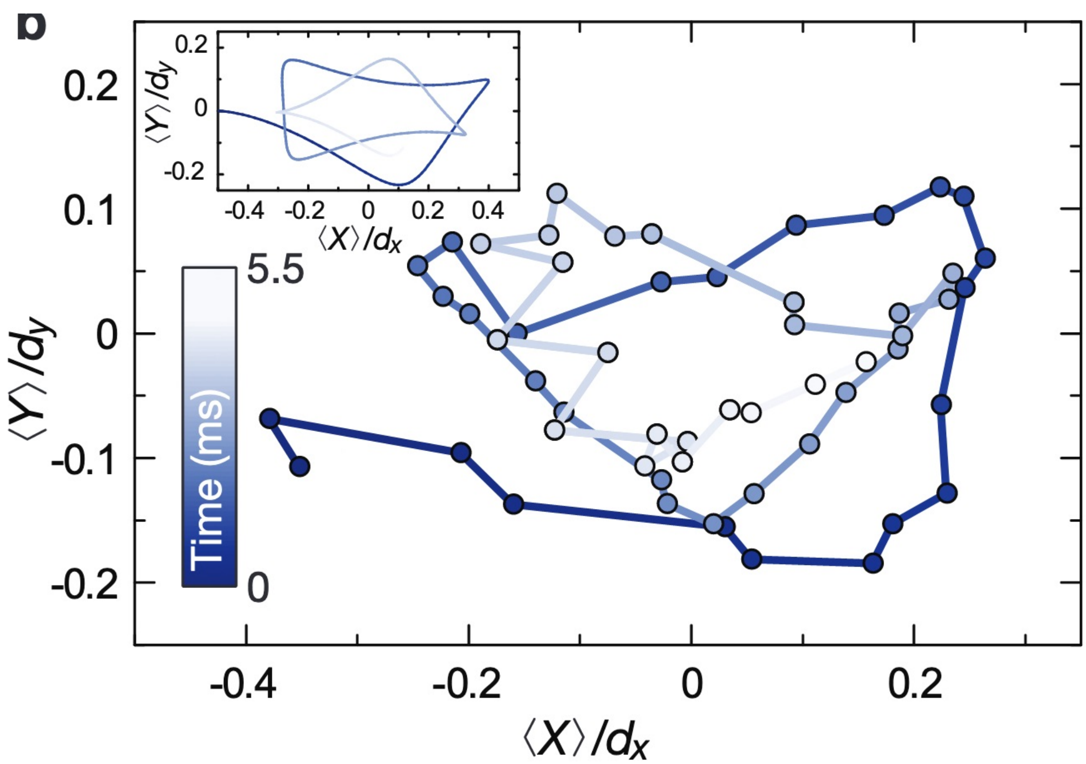


M.A. et al., Phys. Rev. Lett. 107, 255301 (2011)

Experimental protocol:

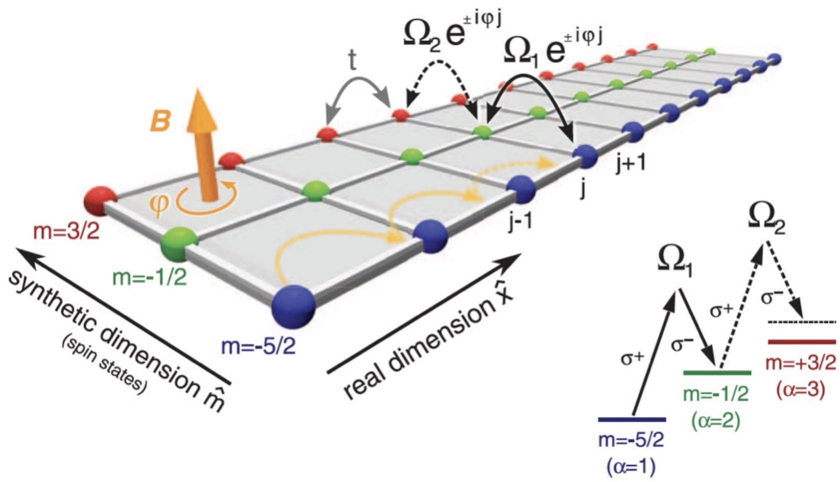
Initial state:





$\to$ extract an effective flux $\underline{\Phi = 0.73(5) \times \frac{\pi}{2}}$

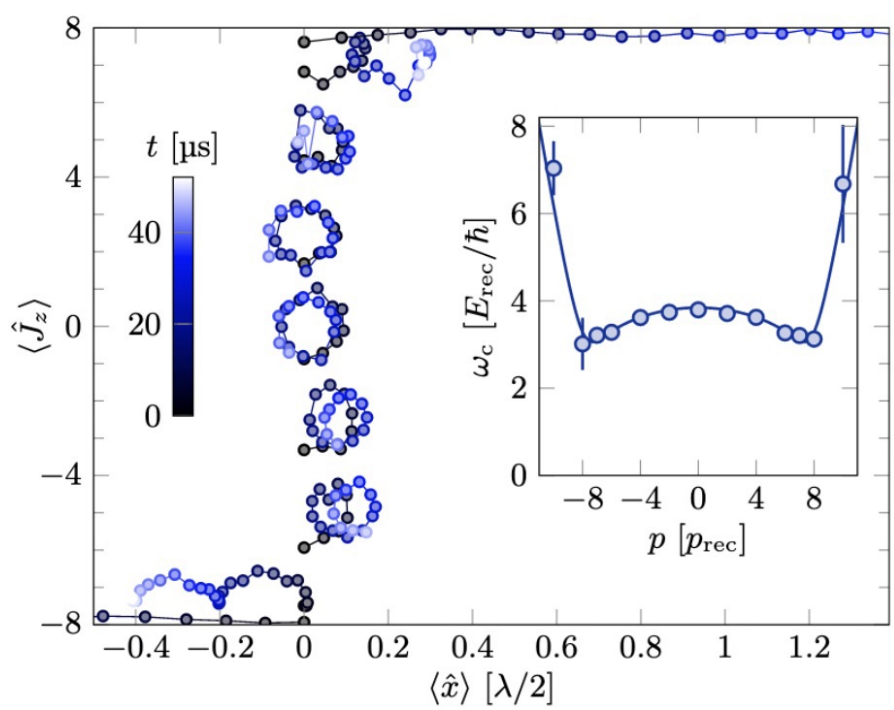
i) <u>Synthetic dimensions</u>



M. Mancini et al., Science (2015), B. K. Stuhl et al., Science (2015)

→ sharp edges : skipping orbits

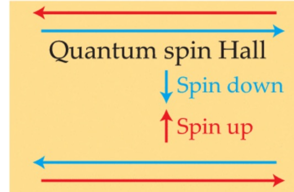
Realization with ^{162}Dy

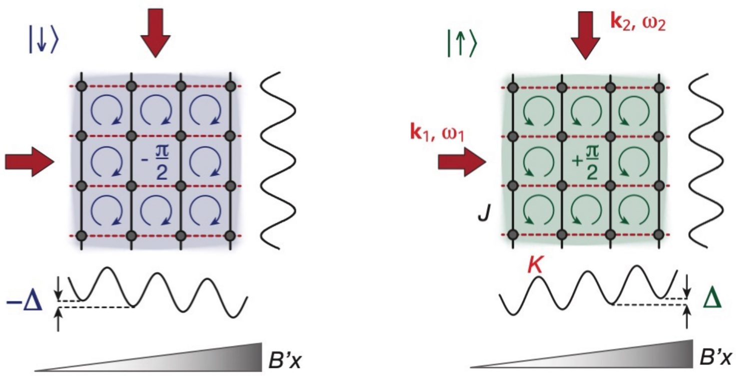


T. Chalopin et al., Nature Physics 16, 1017-1021 (2020)

j) Spin-Hall effect

M.A. et al., PRL 111, 185301 (2013)

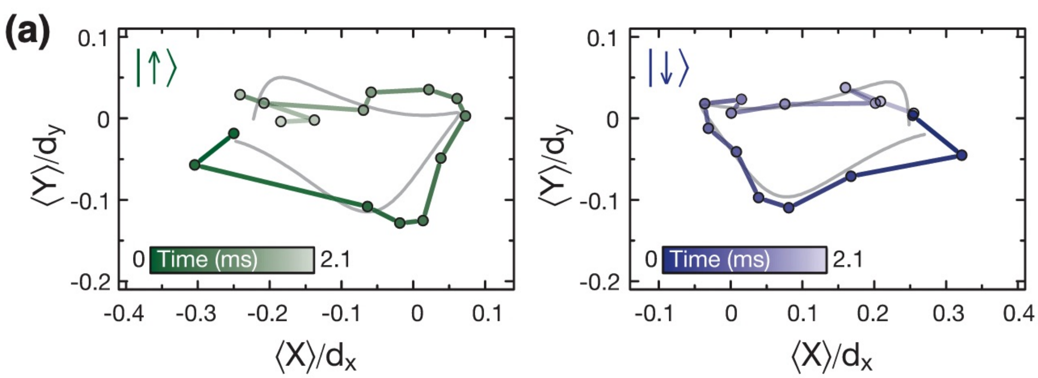


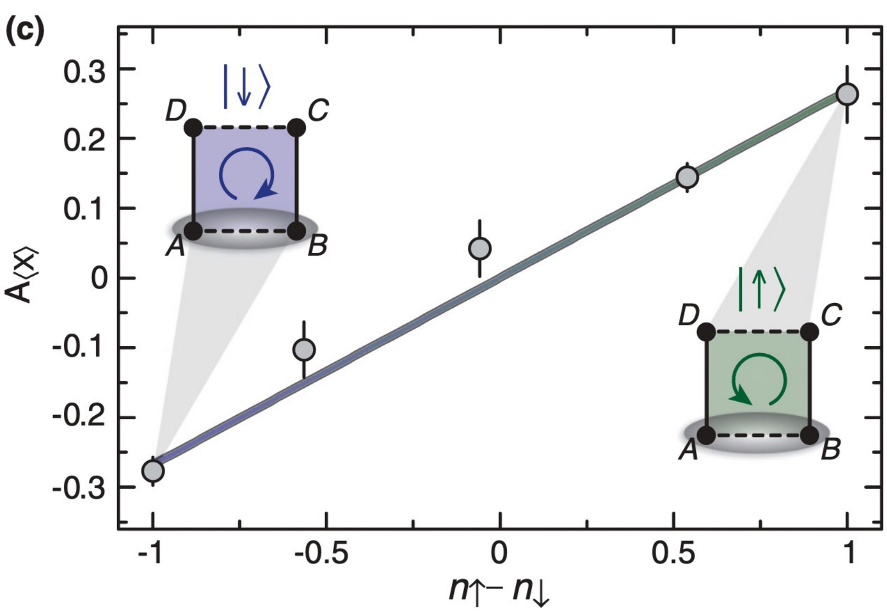


$|\downarrow\rangle = |F=2, m_F = 1\rangle$

$|\uparrow\rangle = |F=1, m_F = -1\rangle$

$|\uparrow\rangle$ and $|\downarrow\rangle$ have opposite magn. moments



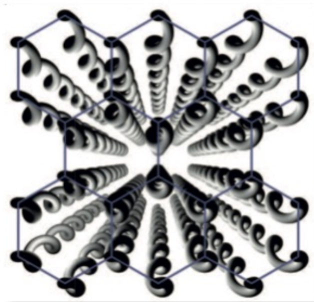


free-space: M. C. Beeler *et al.*, Nature **498**, 201 (2013)

solid state devices: Y. K. Kato *et al.*, Science **306**, 1910 (2004); J. Wunderlich *et al.*, PRL **94**, 047204 (2005)

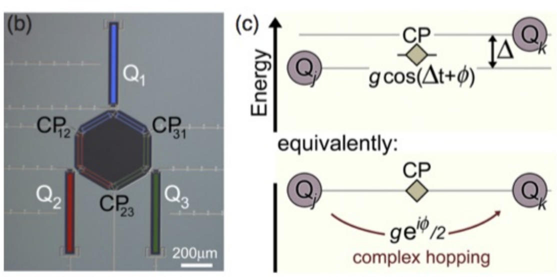
Other synthetic systems

Coupled waveguide arrays



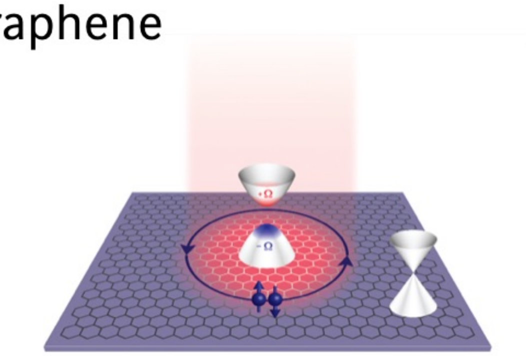
M. C. Rechtsman et al., Nature (2013)

Superconducting circuits



P. Roushan et al., Nat. Phys. (2017)

Graphene



J. W. McIver et al., Nat. Phys. (2020)

Review: M.A., Comptes Rendue Phys. (2018)