Low dimensional quantum gases

T. Giamarchi

http://dqmp.unige.ch/giamarchi/

Plan of the lectures

- Equilibrium, basic notions of Tomonaga-Luttinger liquid
- More on TLL (factionalization, topology, etc.)
  Effect of perturbations (Lattice and disorder)
- Notions of transport in 1D
  From 1D to 3D; Ladders; Dimensional crossover

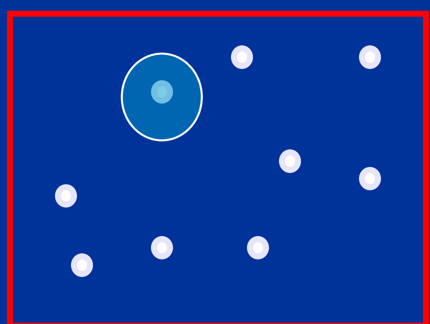
Why one dimension ?

Three urban legends about 1D

- It is a toy model to understand higher dimensional systems.
- It does not exist in nature ! This is only for theorists !
- Everything is understood there anyway !

One dimension is specially interesting

- No individual excitation can exist (only collective ones)

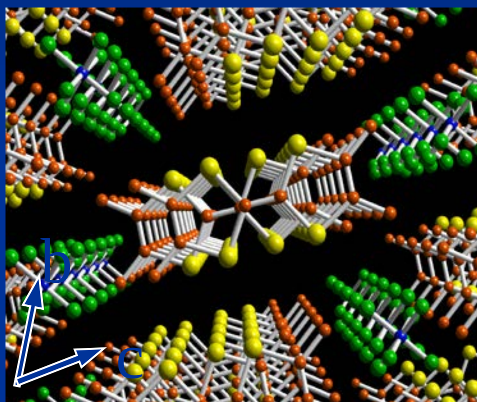


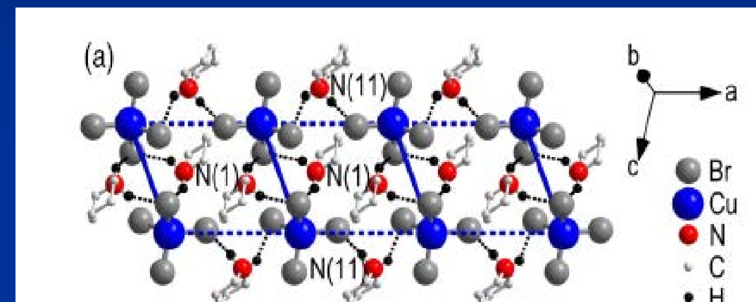
- Strong quantum fluctuations

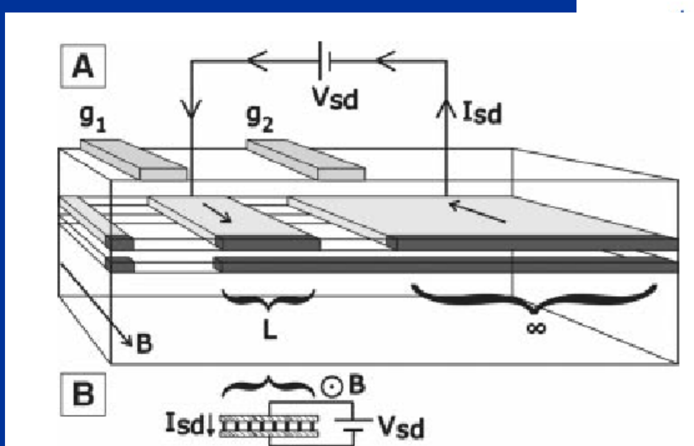
$$\psi = |\psi| e^{i\theta}$$

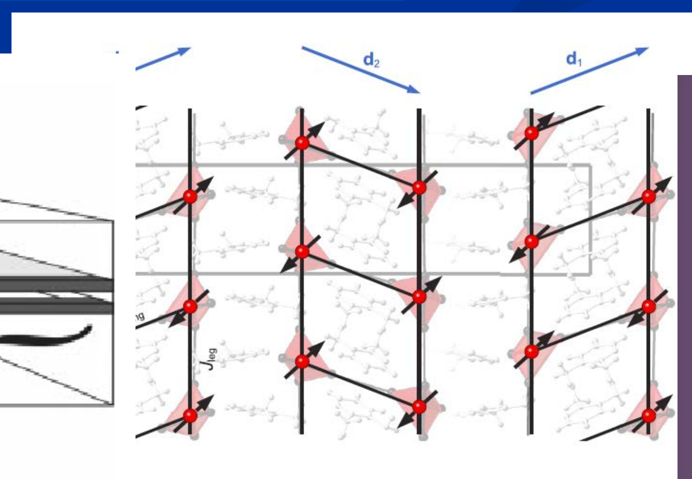
Difficult to order

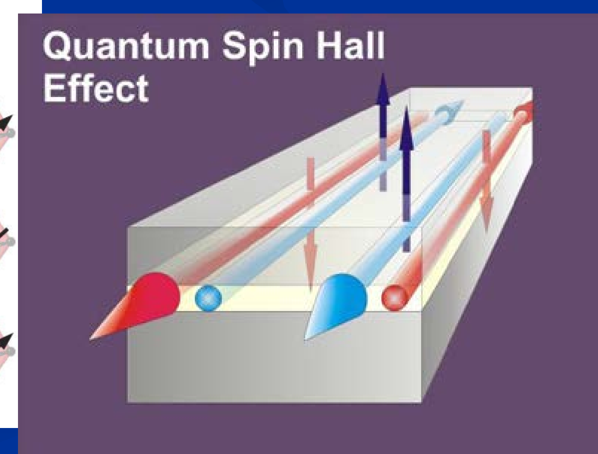
Many CM or cold atoms Systems











Drastic evolution of the 1d world

- New methods (DMRG, correlations from BA, etc.)
- New systems (cold atoms, magnetic insulators, etc.)
- New questions (strong SOC, out of equilibrium, etc)

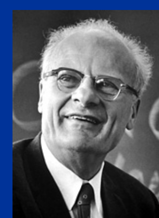
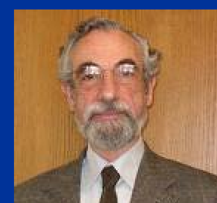
How to treat ?

- ``Standard" many body theory



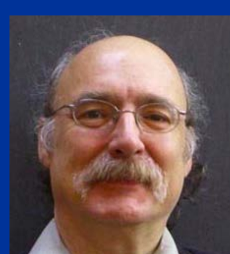


- Exact Solutions (Bethe ansatz)

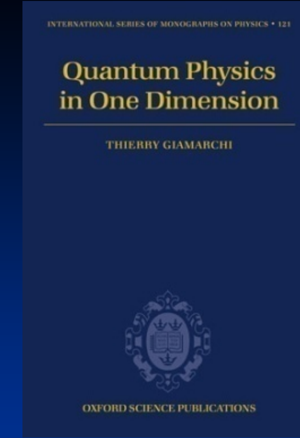
- Field theories (bosonization, CFT)





- Numerics (DMRG, MC, etc.)


References



TG, Quantum physics in one dimension, Oxford (2004)

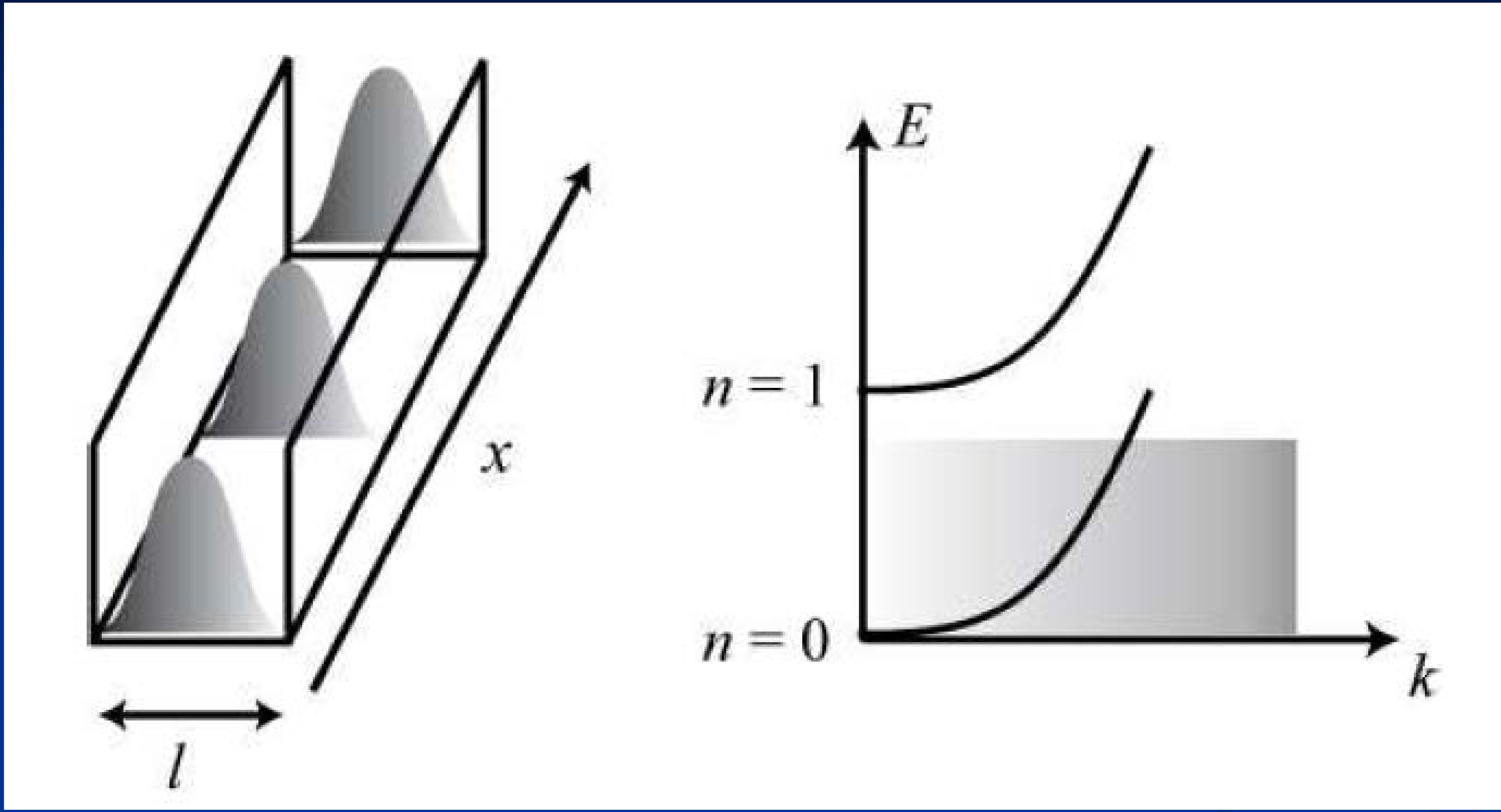
TG in ``Understanding Q. Phase Transitions”, Ed. L. Carr, CRC Press (2010)

M. Cazalilla et al.,
Rev. Mod. Phys.83 1405 (2011)

TG, Int J. Mod. Phys. B 26 1244004 (2012)

TG, C. R. Acad. Sci. 17 322 (2016)

And now we start….



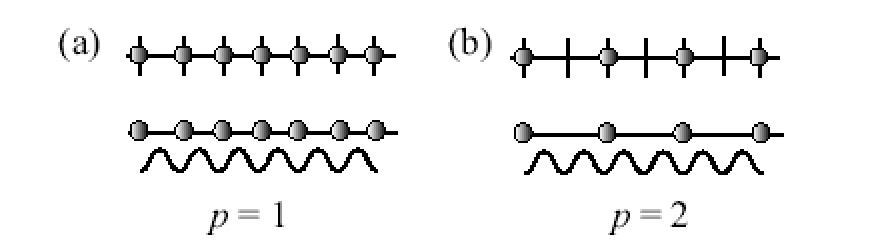
x
l
E
n = 1
n = 0
k

Typical problem (e.g. Bosons)

- Continuum:

$$H = \int dx \frac{(\nabla\psi)^\dagger(\nabla\psi)}{2M} + \frac{1}{2}\int dx\, dx'\, V(x-x')\rho(x)\rho(x') - \mu \int dx\, \rho(x)$$

- Lattice:



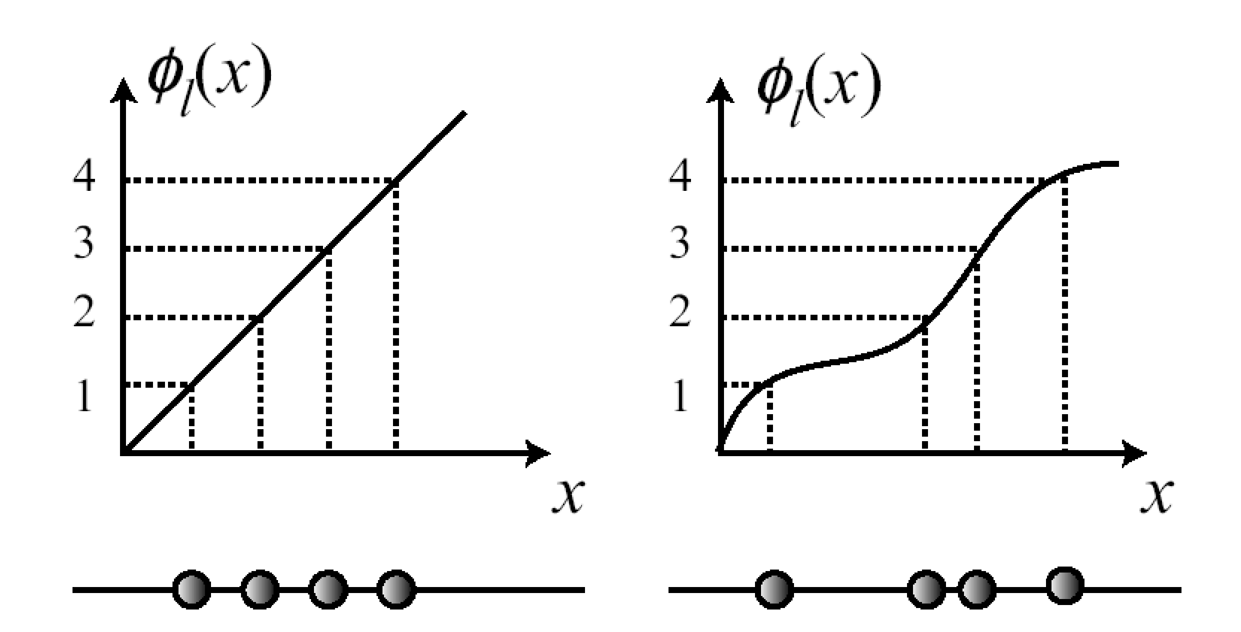
$$H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + U \sum_i n_i(n_i - 1) - \mu \sum_i n_i$$

Luttinger liquid physics

Labelling the particles

$$\rho(x) = \sum_i \delta(x - x_i)$$
$$= \sum_n |\nabla \phi_l(x)| \delta(\phi_l(x) - 2\pi n)$$

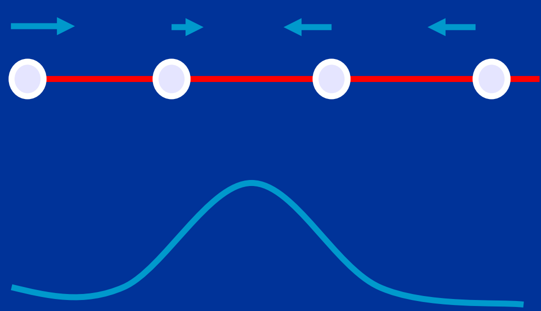
1D: unique way of labelling

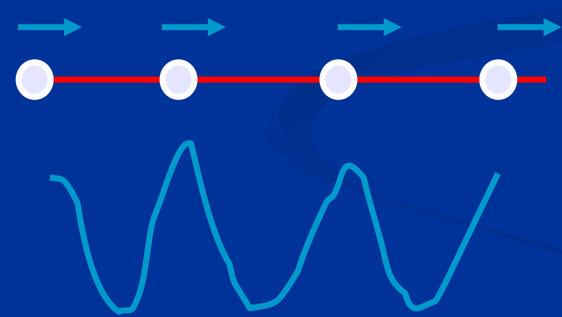


$$\phi_l(x) = 2\pi\rho_0 x - 2\phi(x)$$

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right]\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

$\phi(x)$ varies slowly





CDW

$q \sim 0$

$q \sim 2\pi\rho_0$

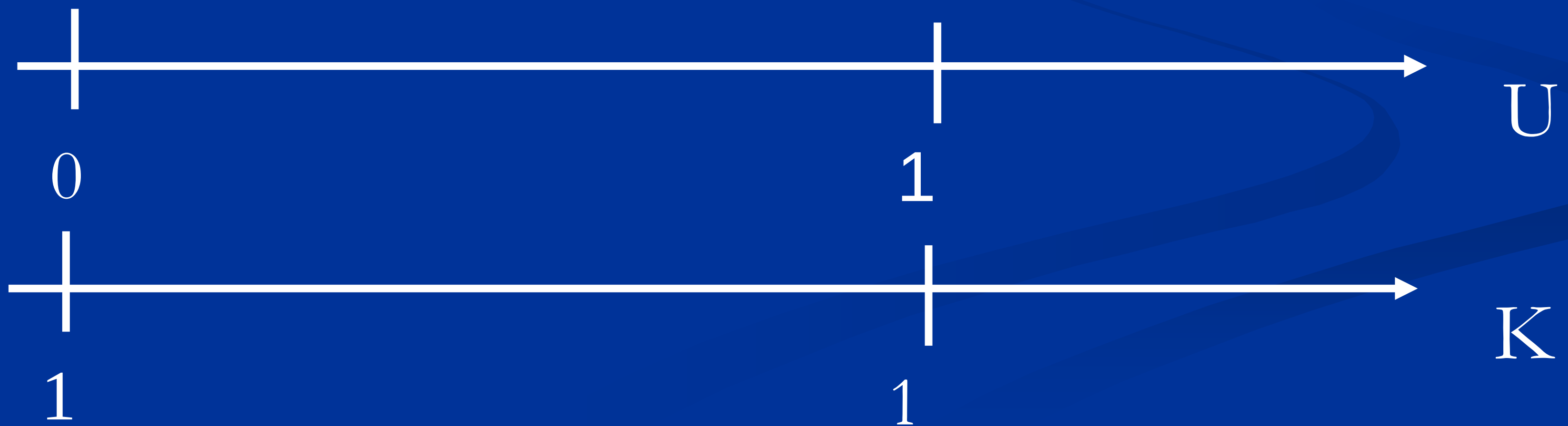
$$\psi^{\dagger}(x) = [\rho(x)]^{1/2} e^{-i\theta(x)}$$

θ: superfluid phase

$$[\frac{1}{\pi}\nabla\phi(x), \theta(x')] = -i\delta(x - x')$$

Quantum fluctuations

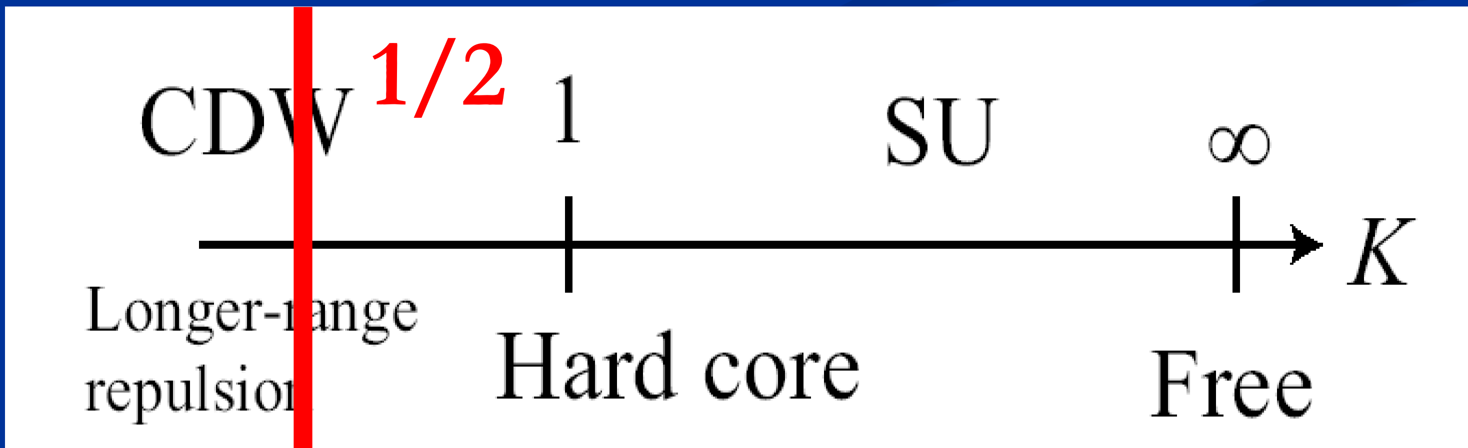
$$H = \frac{\hbar}{2\pi}\int dx[\frac{uK}{\hbar^2}(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2]$$

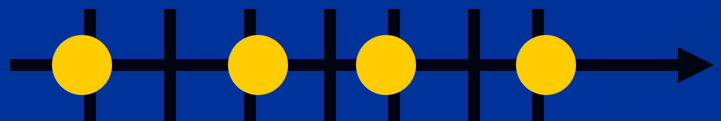


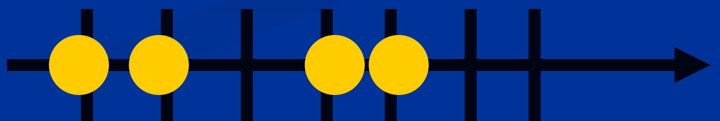
Correlations

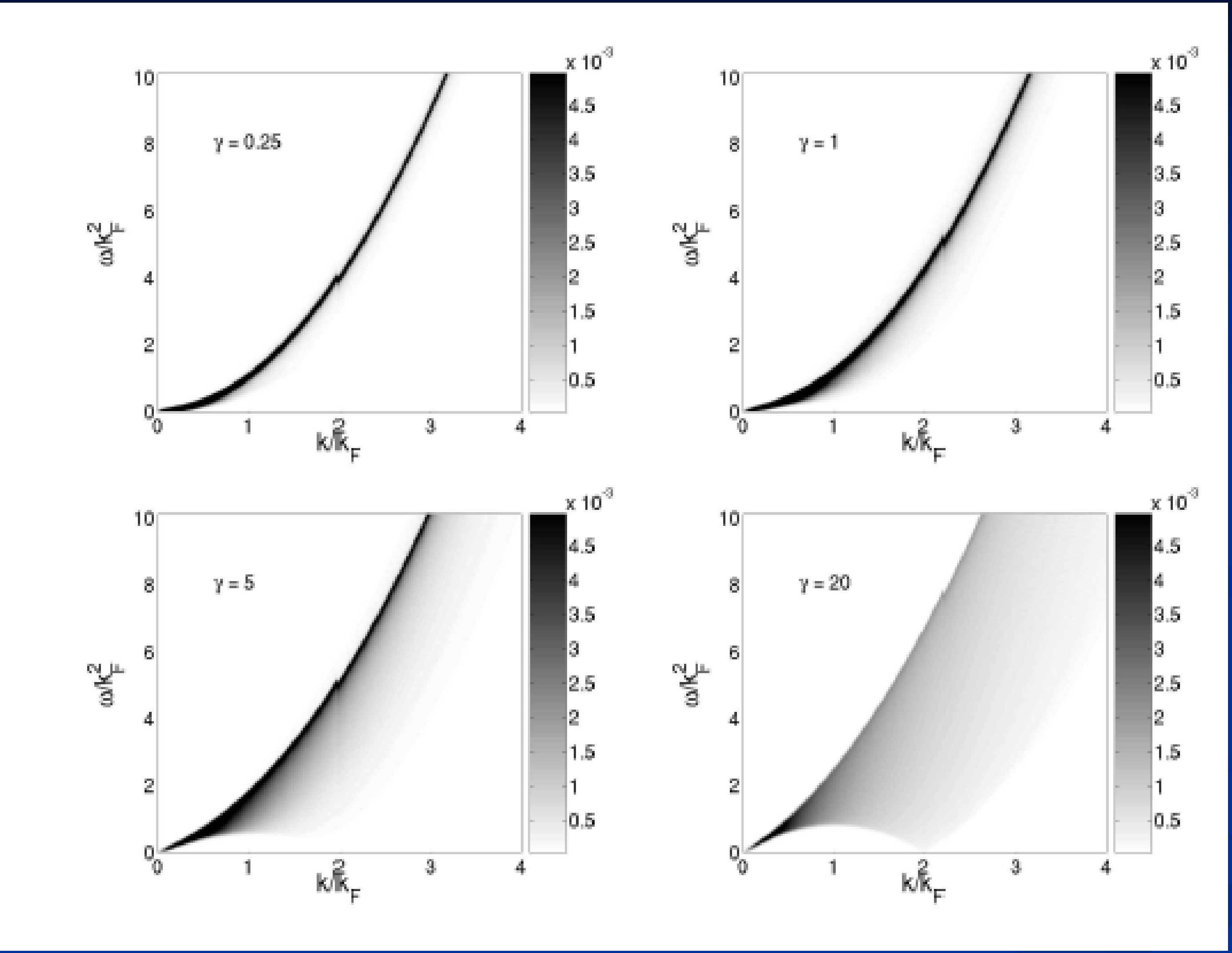
$$\langle \psi(r)\psi^\dagger(0)\rangle = A_1\left(\frac{\alpha}{r}\right)^{\frac{1}{2K}} + \cdots$$

$$\langle \rho(r)\rho(0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(y_\alpha^2 + x^2)^2} + A_3\cos(2\pi\rho_0 x)\left(\frac{1}{r}\right)^{2K} + \cdots$$









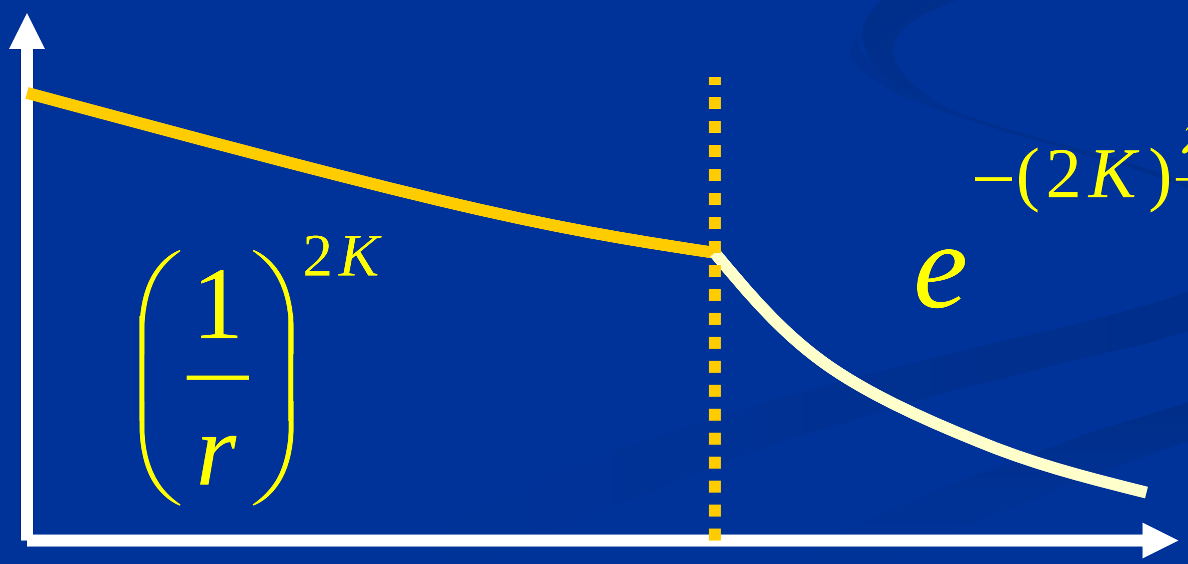
S(q,!) J.S. Caux et al PRA 74 031605 (2006)

Finite temperature

Conformal theory

β

χ



$$\left(\frac{1}{r}\right)^{2K}$$

$$e^{-(2K)\frac{\pi x}{\beta}} = e^{-x/\xi_\beta}$$

Other 1D systems

Spins

Use boson or fermions mapping

$$S^{+} = (-1)^{i} e^{i\theta} + e^{i\theta} \cos(2\phi)$$

$$S^{z} = \frac{-1}{\pi} \nabla \phi + (\text{-}1)^{i} \cos(2\phi)$$

Powerlaw correlation functions

$$\langle S^z(x,0) S^z(0,0) \rangle = C_1 \frac{1}{x^2} + C_2 (-1)^x \left(\frac{1}{x}\right)^{2K}$$

$$\langle S^+(x,0) S^-(0,0) \rangle = C_3 \left(\frac{1}{x}\right)^{2K+\frac{1}{2K}} + C_4 (-1)^x \left(\frac{1}{x}\right)^{\frac{1}{2K}}$$

Non universal exponents K(h,J)

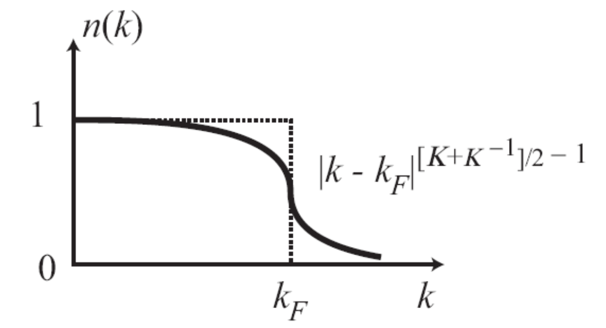
Fermions

$$\psi_F^\dagger(x) = \psi_B^\dagger(x) e^{i\frac{1}{2}\phi_l(x)}$$

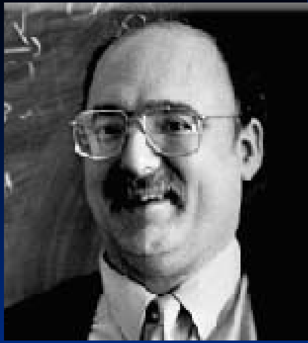
$$\psi_F^\dagger(x) = [\rho_0 - \frac{1}{\pi}\nabla\phi(x)]^{1/2} \sum_p e^{i(2p+1)(\pi\rho_0 x - \phi(x))} e^{-i\theta(x)}$$

Right ($+k_F$) and left ($-k_F$) particles

$$\langle\rho(x,\tau)\rho(0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(x^2+y_\alpha^2)^2} + \rho_0^2 A_2 \cos(2\pi\rho_0 x)\left(\frac{\alpha}{r}\right)^{2K} + \rho_0^2 A_4 \cos(4\pi\rho_0 x)\left(\frac{\alpha}{r}\right)^{8K}$$



Luttinger liquid concept



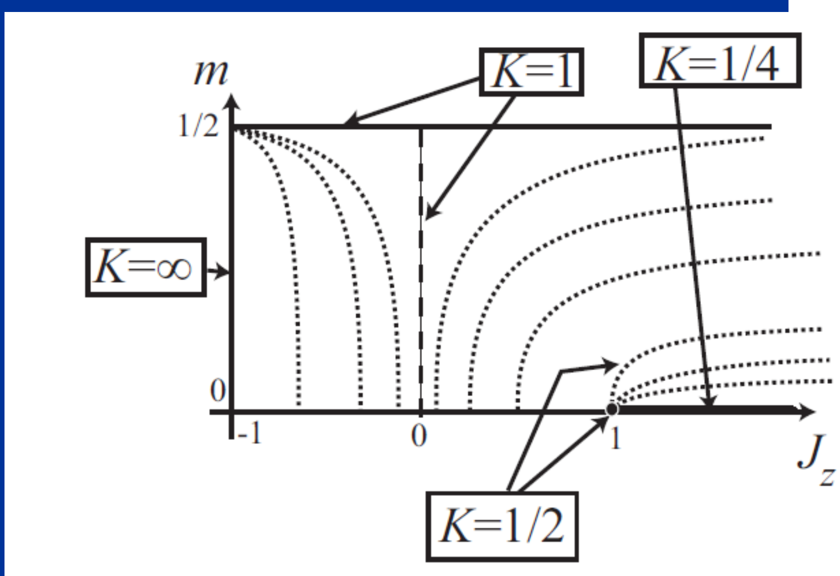
- How much is perturbative ?
- Nothing (Haldane):
  provided the correct u,K are used
- Low energy properties: Luttinger liquid
(fermions, bosons, spins…)

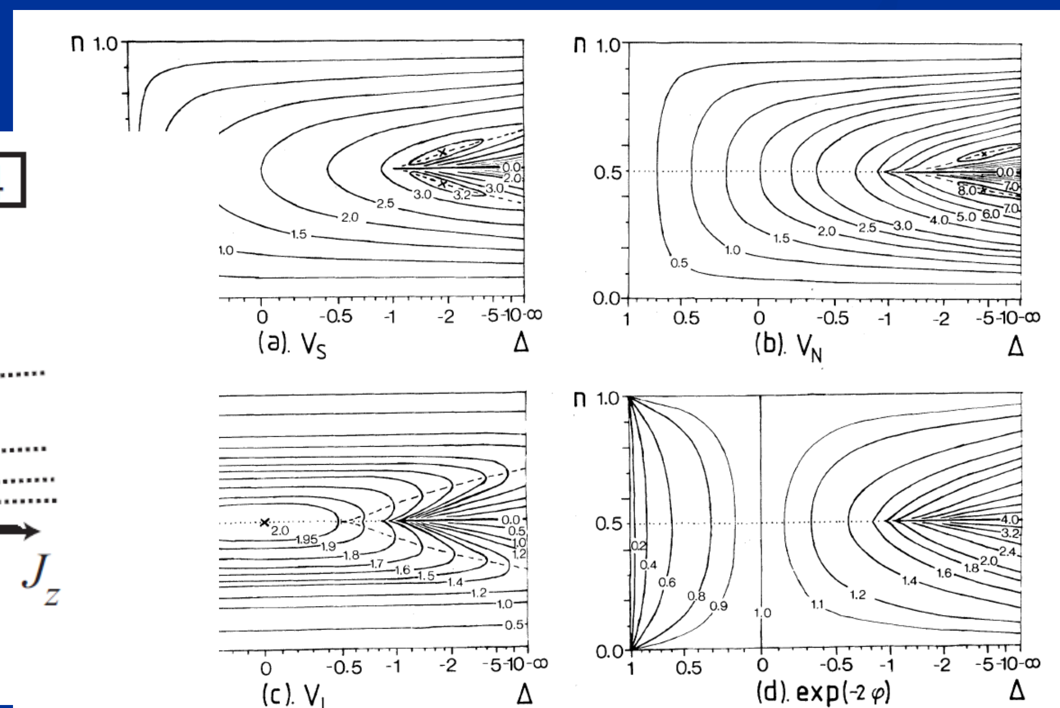
Luttinger parameters

General Relation of Correlation Exponents and Spectral Properties of One-Dimensional Fermi Systems: Application to the Anisotropic $S = \frac{1}{2}$ Heisenberg Chain

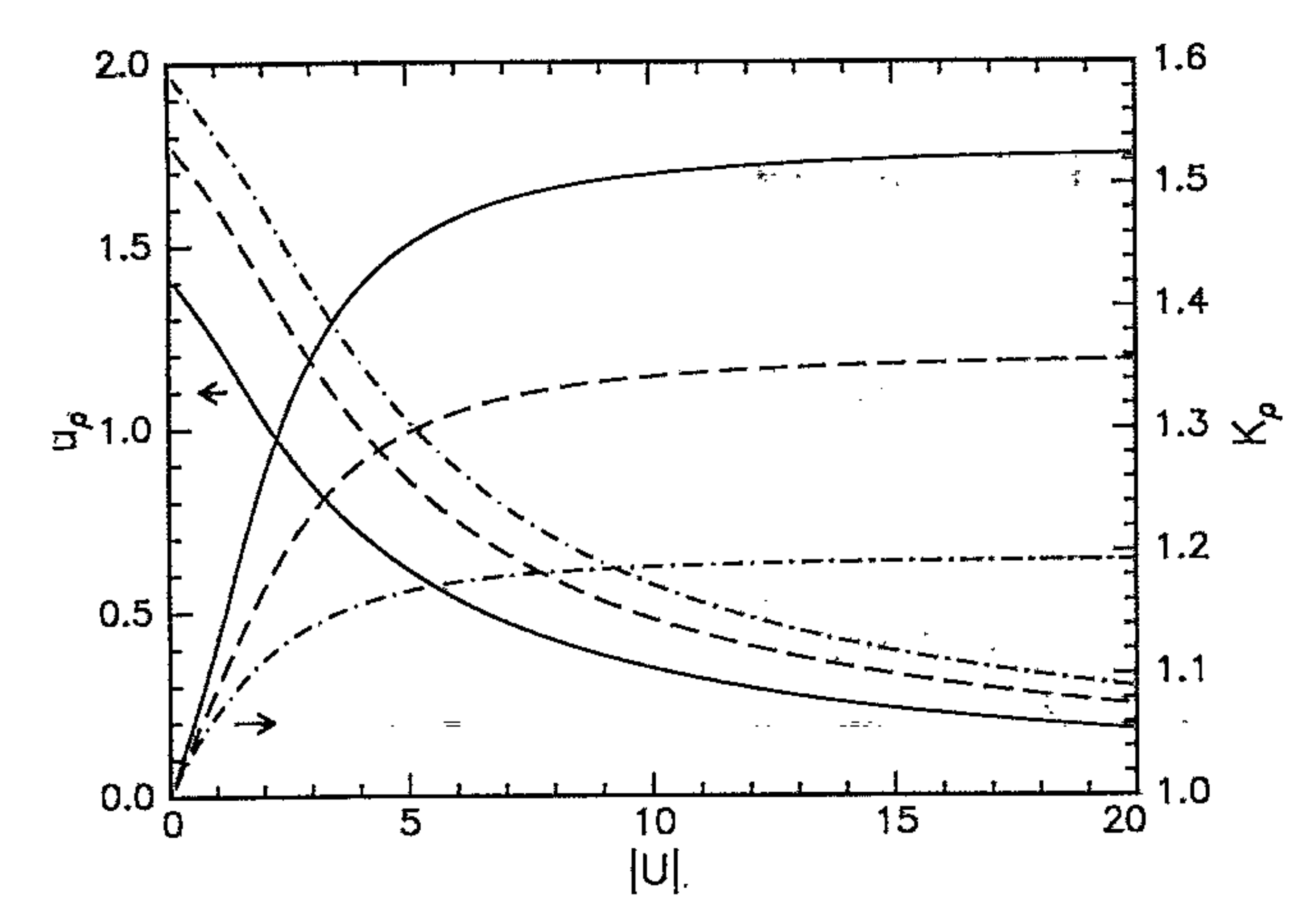
F. D. M. Haldane

Haldane, F. D. M. (1980). *Phys. Rev. Lett.*, **45**, 1358.





Attractive Hubbard model

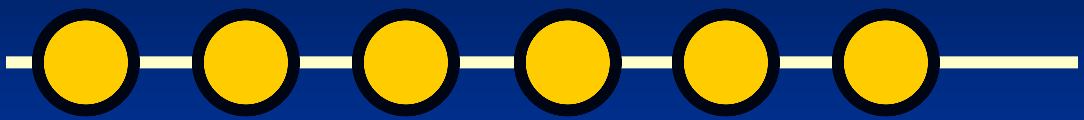


TG + B. S. Shastry PRB 51 10915 (1995)

``Quantitative'' theory possible

- Trick: use thermodynamics and BA or numerics
- Compressibility: u/K
- Response to a twist in boundary: u K
- Specific heat : T/u
- Etc.

Tonks limit



U = 1 : spinless fermions

Not for n(k) : $\psi_F \neq \psi_B$

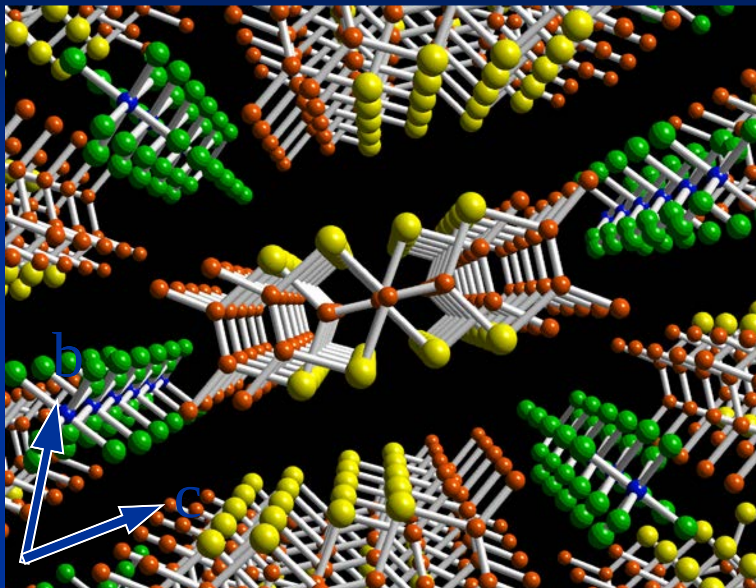
Free fermions: $$\langle \rho(x)\rho(0)\rangle \propto \cos(2k_F x)\left(\frac{1}{x}\right)^2$$

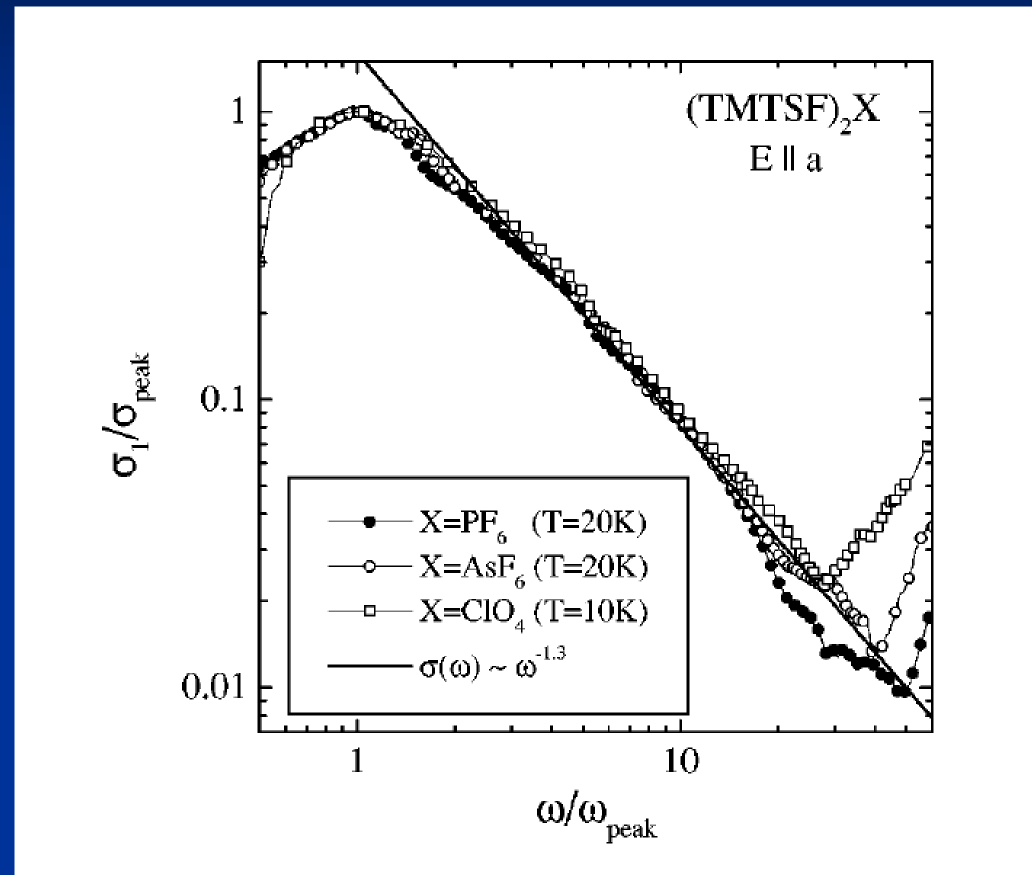
K=1

Note: $$\langle \psi_B(x)\psi_B(0)^\dagger \rangle \propto \left(\frac{1}{x}\right)^{1/2}$$

Tests of Luttinger liquids

Organic conductors





$$\sigma(\omega) \sim \omega^{-\nu}$$

TG PRB (91) :
Physica B 230 (1996)

A. Schwartz et al. PRB 58 1261 (1998)

First observation of LL/powerlaw !!

Cold gases

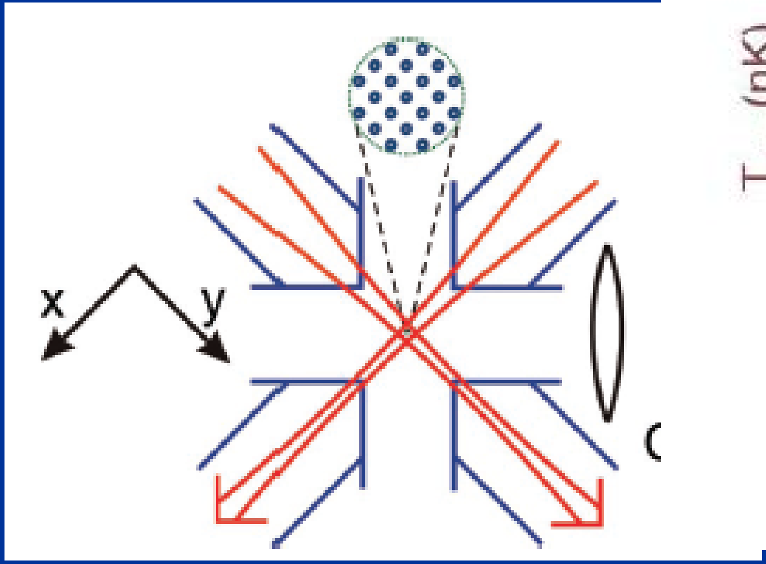
Bosons (continuum)

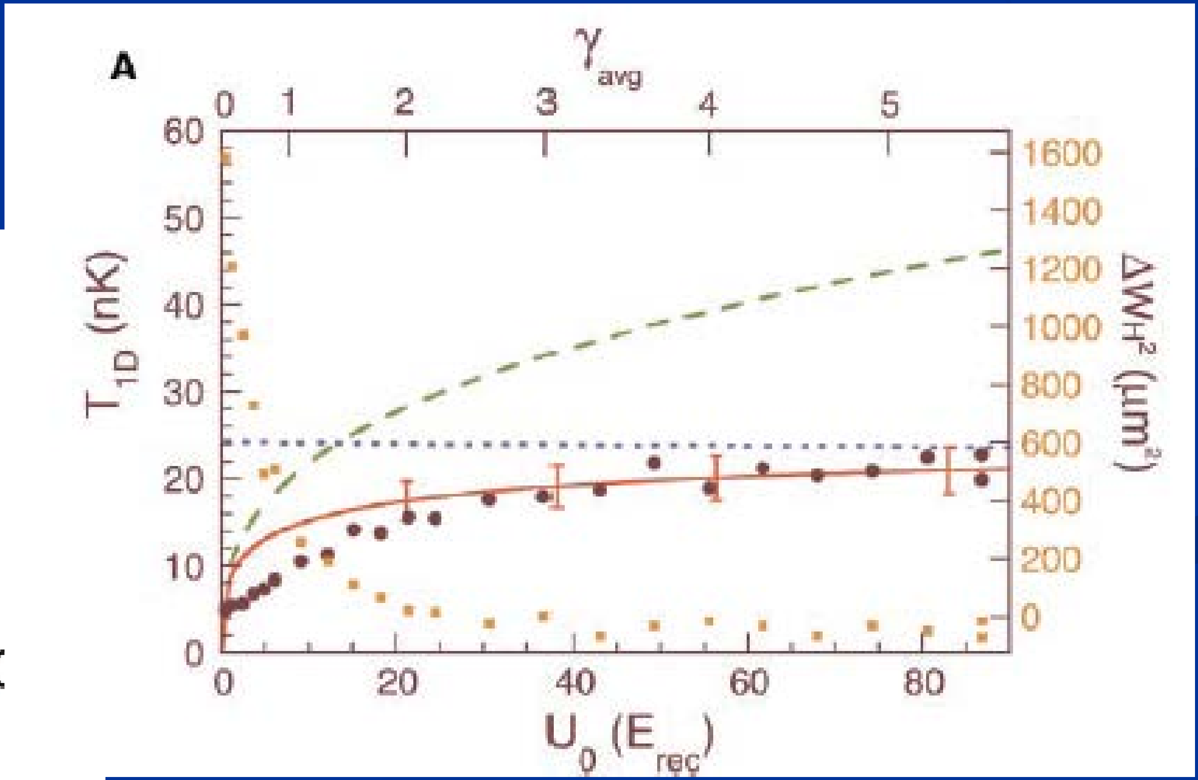
Observation of a One-Dimensional Tonks-Girardeau Gas

Toshiya Kinoshita, Trevor Wenger, David S. Weiss*

SCIENCE VOL 305 20 AUGUST 2004

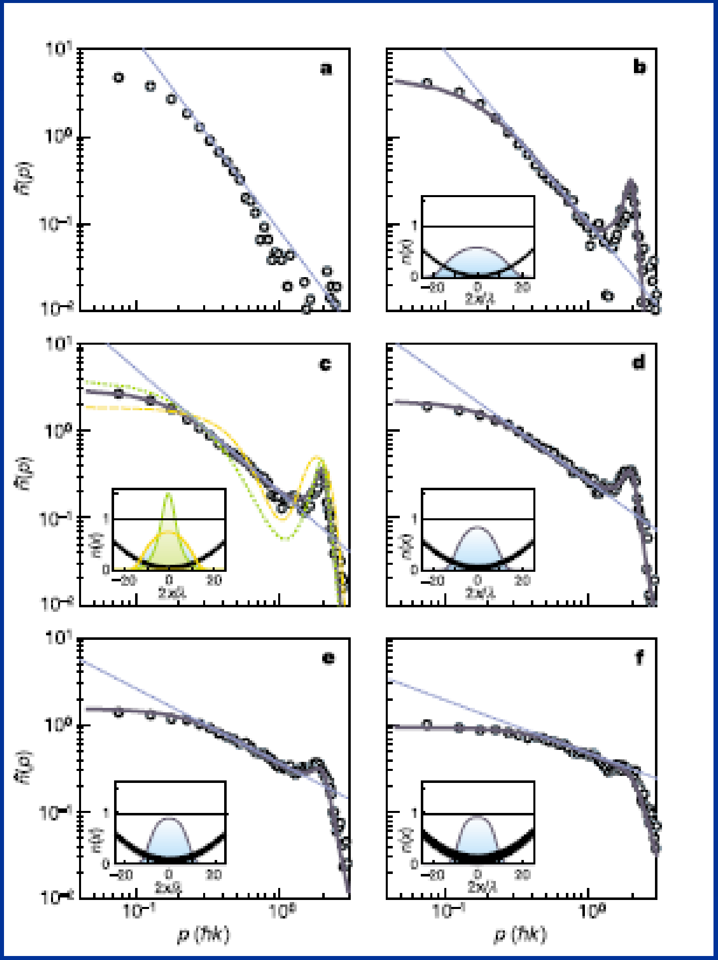
1125

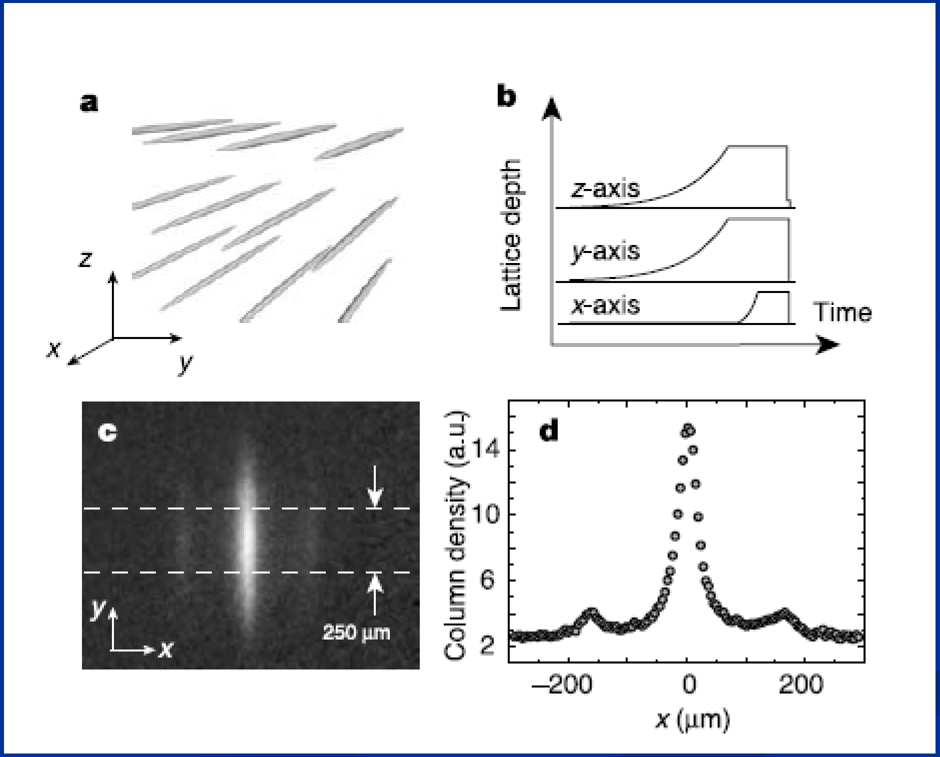




Optical lattices (dilute)

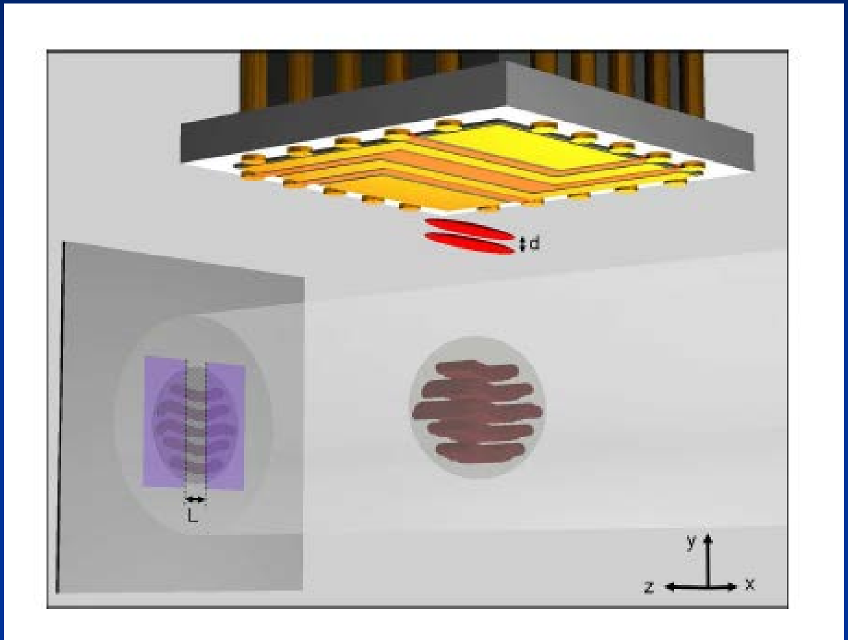
B. Paredes et al., Nature 429 277 (2004)

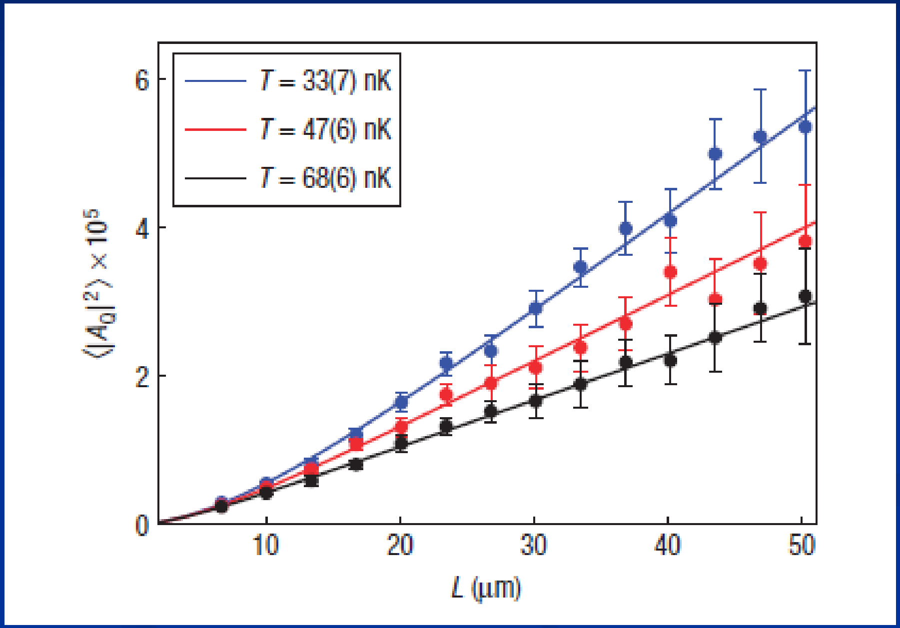




$$n(k) = \int dx e^{ikx} \langle \psi^\dagger(x)\psi(0) \rangle$$

Atom chips

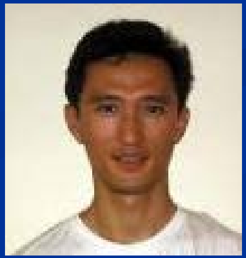




$$\int_0^L dr \langle \psi(r) \psi^\dagger(0) \rangle$$

S. Hofferberth et al. Nat. Phys 4 489 (2008)

K large (42)



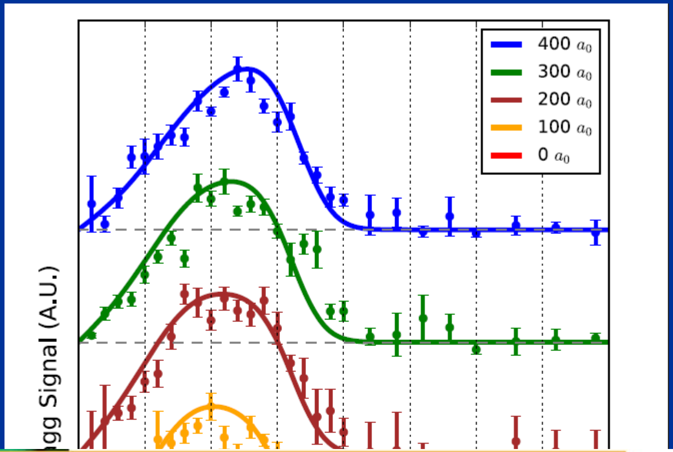
Charge velocity

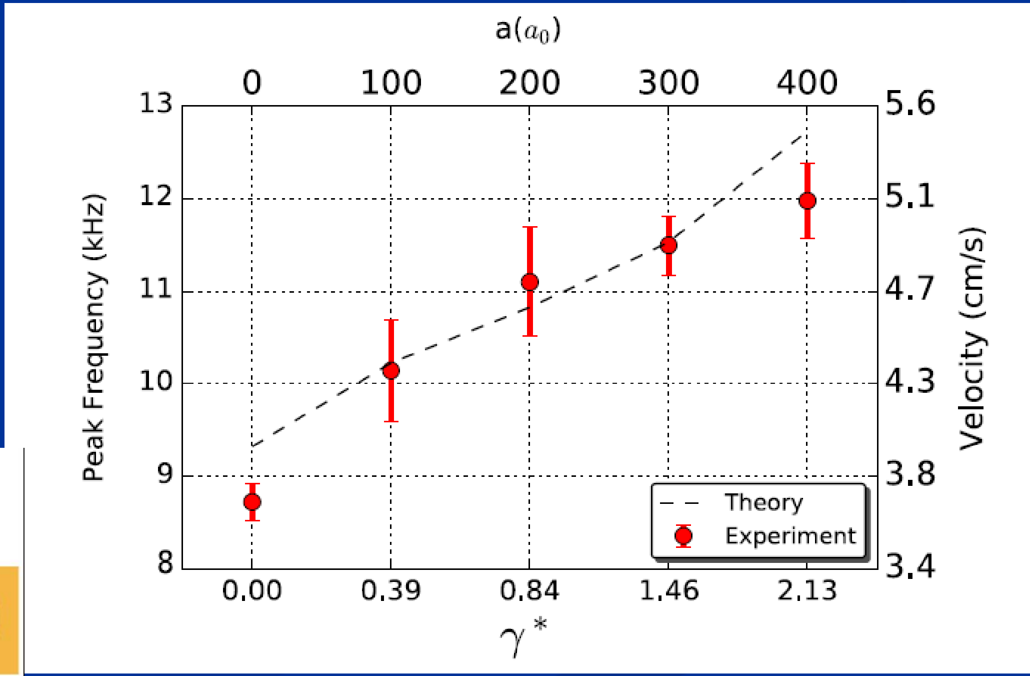
PHYSICAL REVIEW LETTERS **121,** 103001 (2018)

Editors' Suggestion

Measurement of the Dynamical Structure Factor of a 1D Interacting Fermi Gas

T. L. Yang,[1] P. Grišins,[2] Y. T. Chang,[1] Z. H. Zhao,[1] C. Y. Shih,[1] T. Giamarchi,[2] and R. G. Hulet[1]





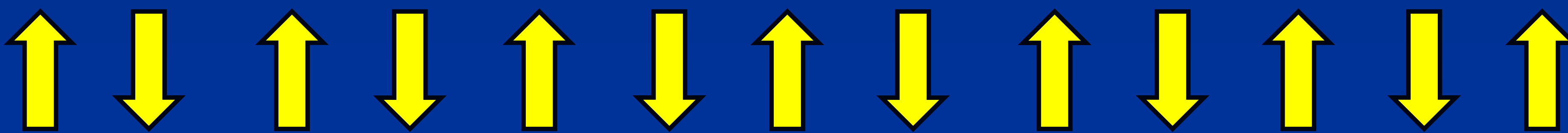
ULTRACOLD ATOMS CONFIRM 55-YEAR-OLD PHYSICS THEORY

https://www.futurity.org/one-dimensional-electrons-physics-1858622/

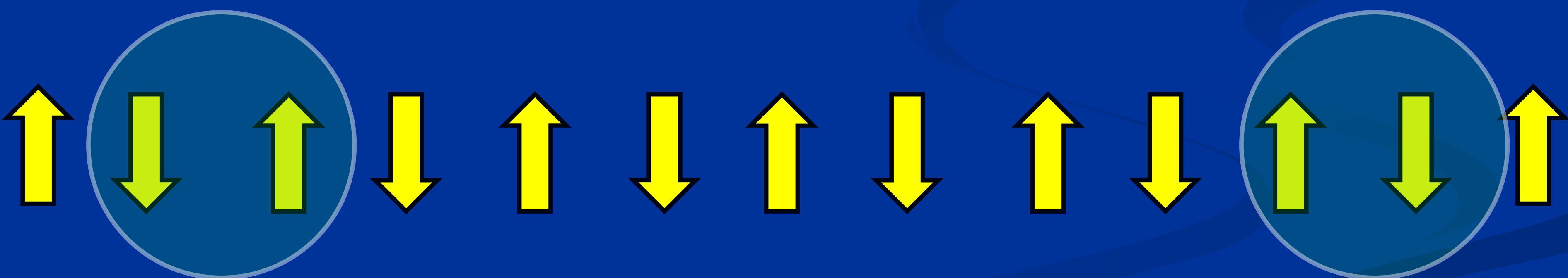
Other important 1D properties
Fractionalization of excitations

Fractionalization of excitations

$$H = J\sum_i \vec{S}_i \cdot \vec{S}_{i+1}$$

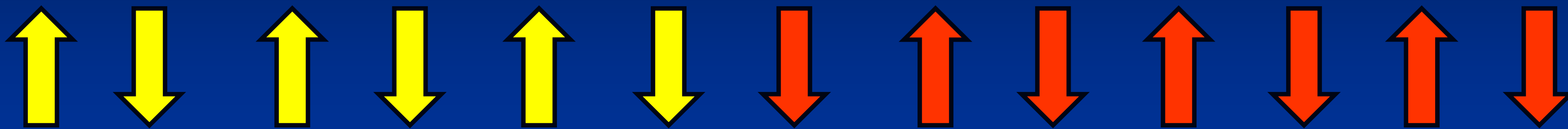


$\Delta S = -1$ $E = \epsilon(q)$ Magnon



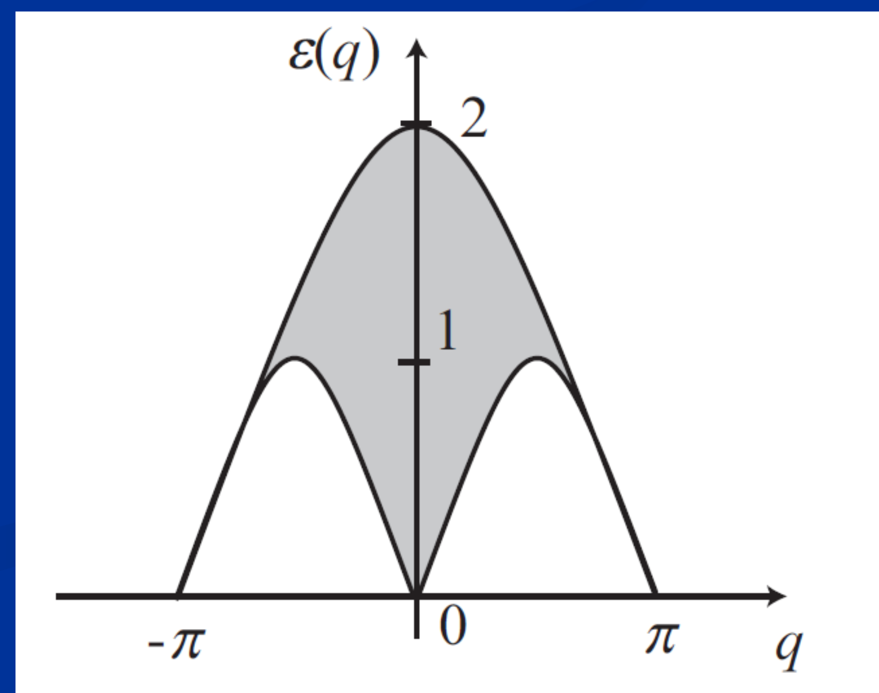
$\Delta S = -1/2$ Spinons $\Delta S = -1/2$

Magnons and spinons: 1 = ½ + ½



- Hidden (topological) order parameters

- Continuum of excitations



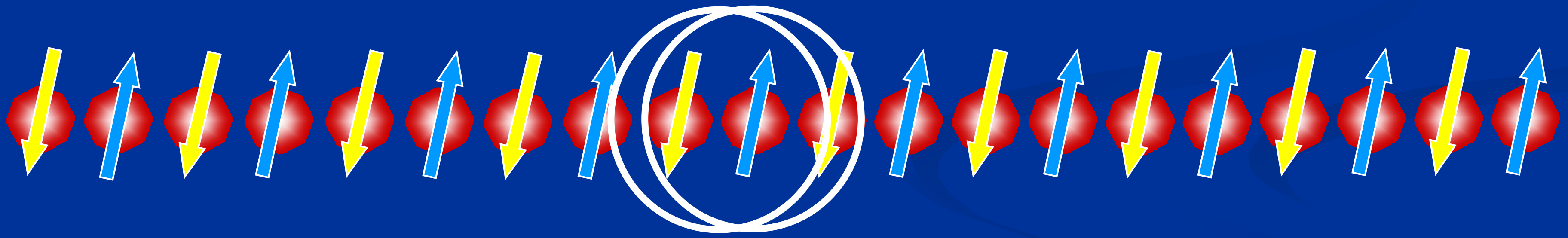
$$E(k) = \cos(k_1) + \cos(k_2)$$

$$k = k_1 + k_2$$

Deconstruction of the electron: spin-charge separation

Spin

Charge



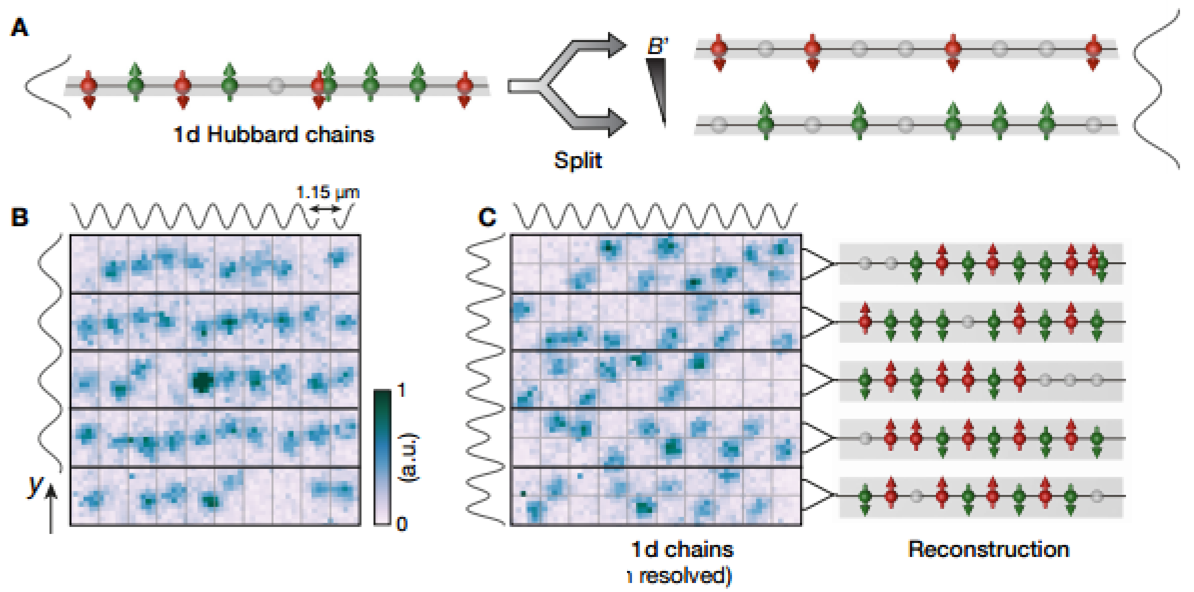
Spinon

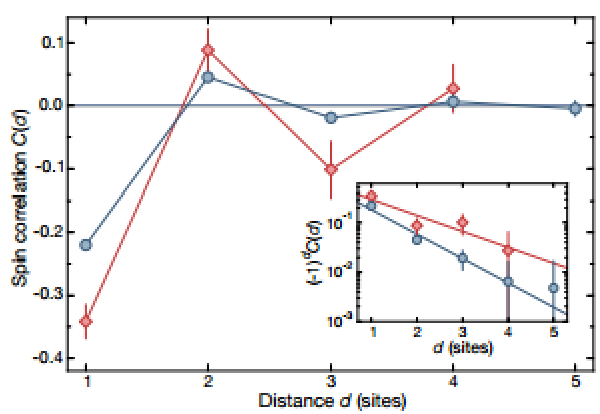
Holon

Spin and Charge Resolved Quantum Gas Microscopy of Antiferromagnetic Order in Hubbard Chains

Martin Boll[1*], Timon A. Hilker[1*], Guillaume Salomon[1*], Ahmed Omran[1], Immanuel Bloch[1,2], and Christian Gross[1†]

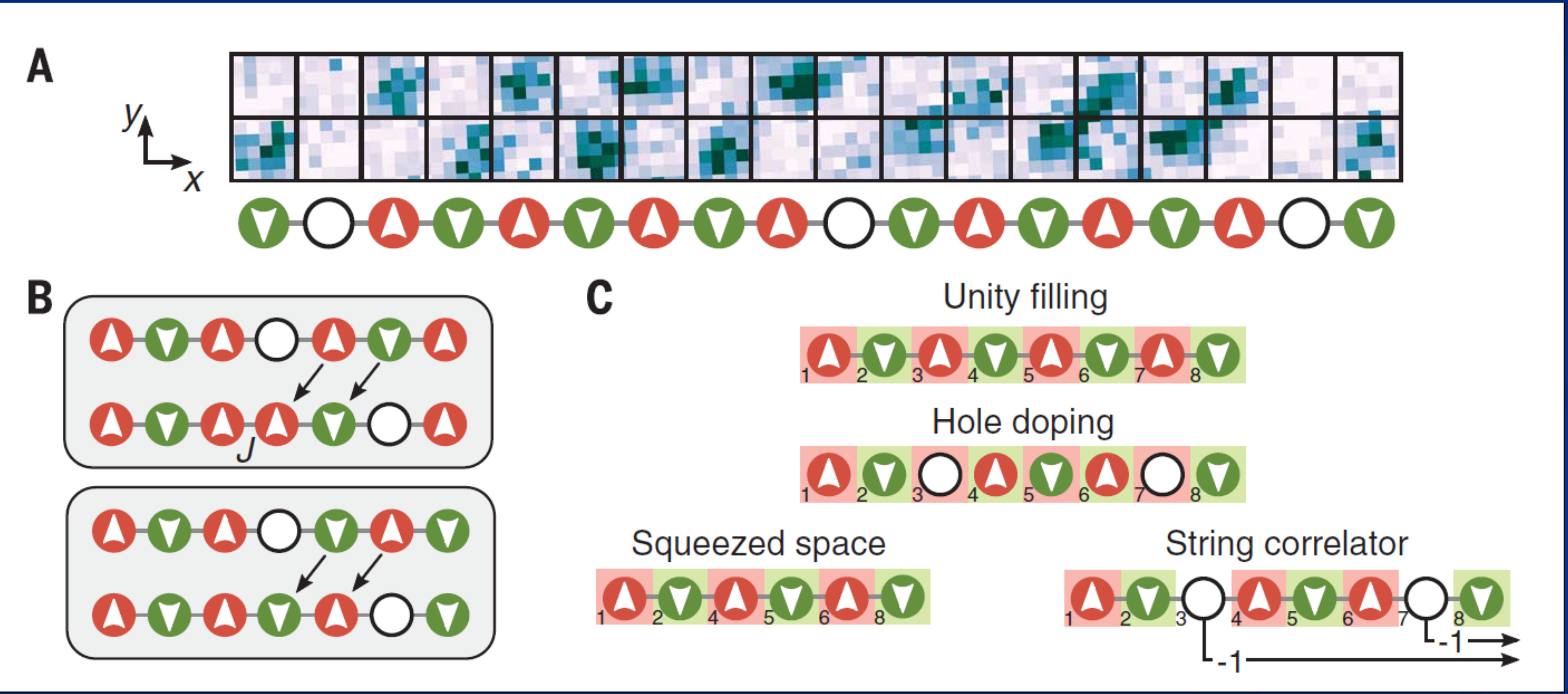
arXiv:1605.05661v2





The inset shows the decay of the rectified spin correlations $(-1)^d C(d)$ in a logarithmic plot together with an exponential fit $C(d) \propto \exp(-d/\xi)$, which reveals an average correlation length of $\xi = 0.9(1)$ sites and $\xi = 1.4(4)$ sites for the lowest entropy tube. All error bars represent one s.e.m.

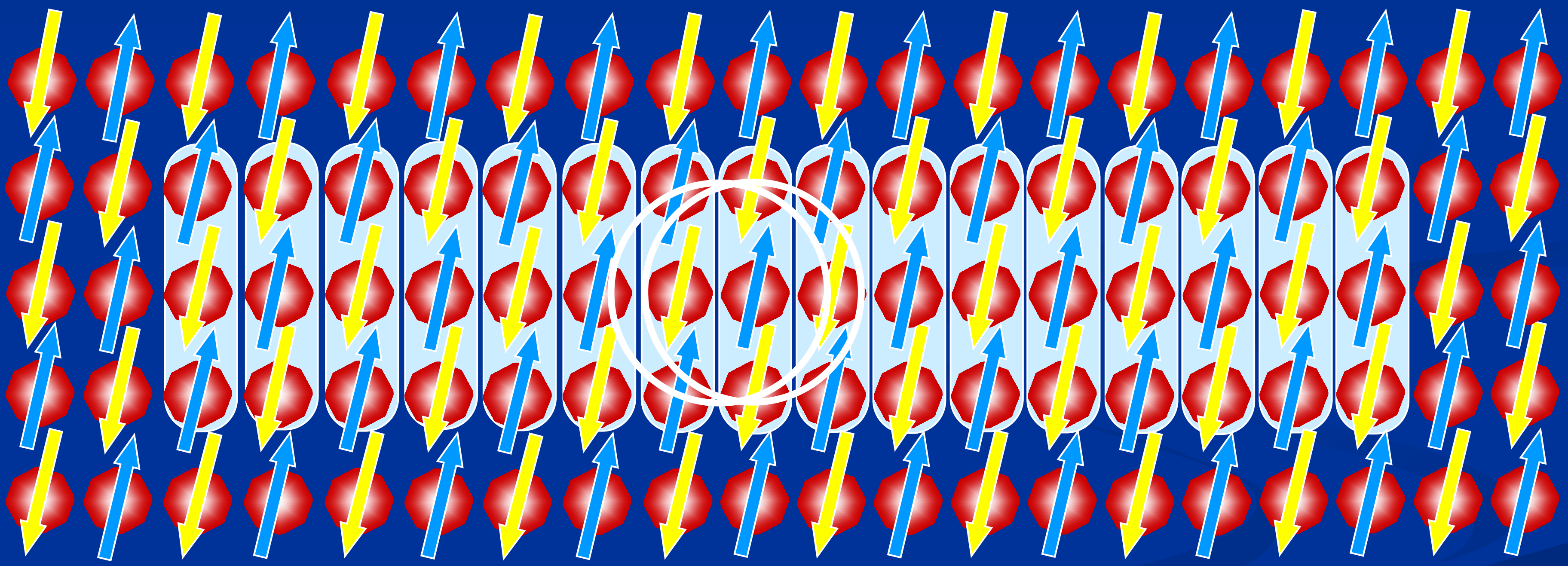
Doped Hubbard model



Hilker *et al.*, *Science* **357**, 484–487 (2017)

Spin-Charge Separation higher D ?

Spin Charge

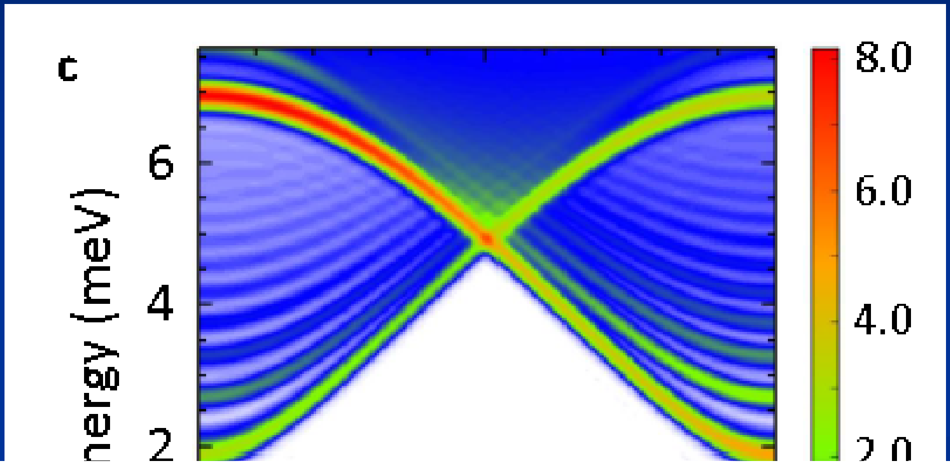


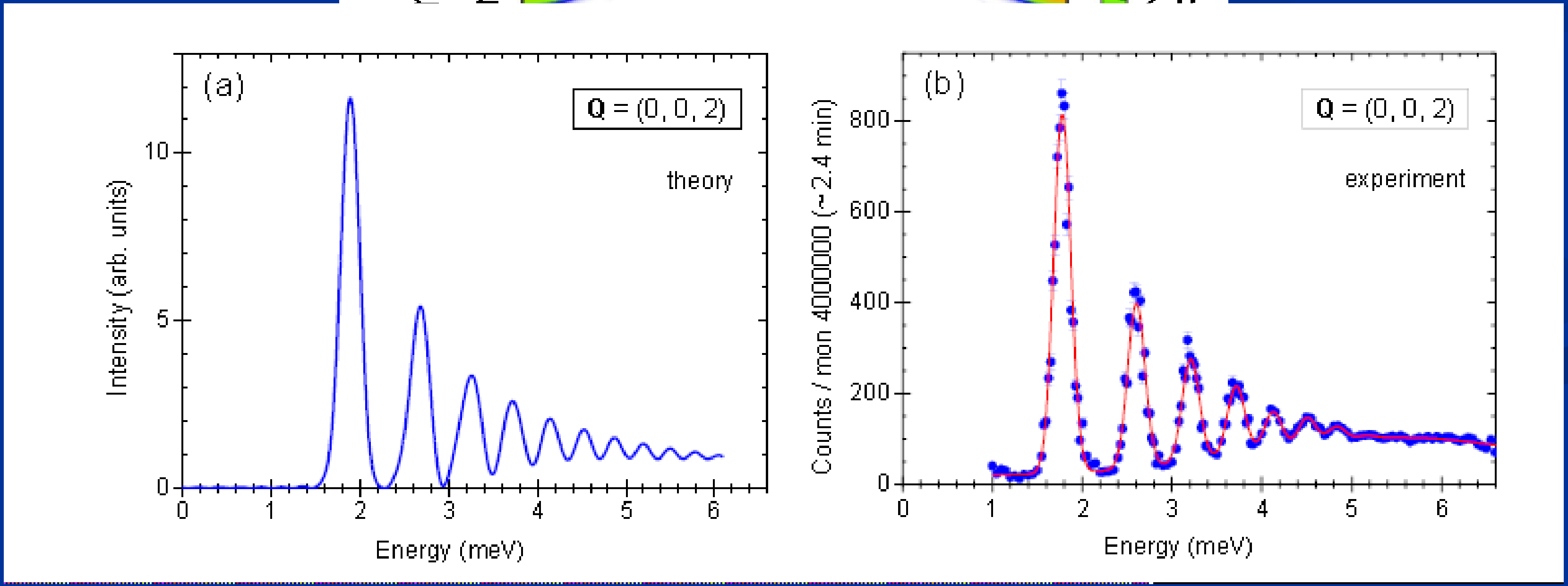
Energy increases with spin-charge separation

Confinement of spin-charge: « quasiparticle »

Confinement of spinons BaCoVO

Q. Faure, S. Takayoshi, et al. Nat. Phys 14, 716 (2018)



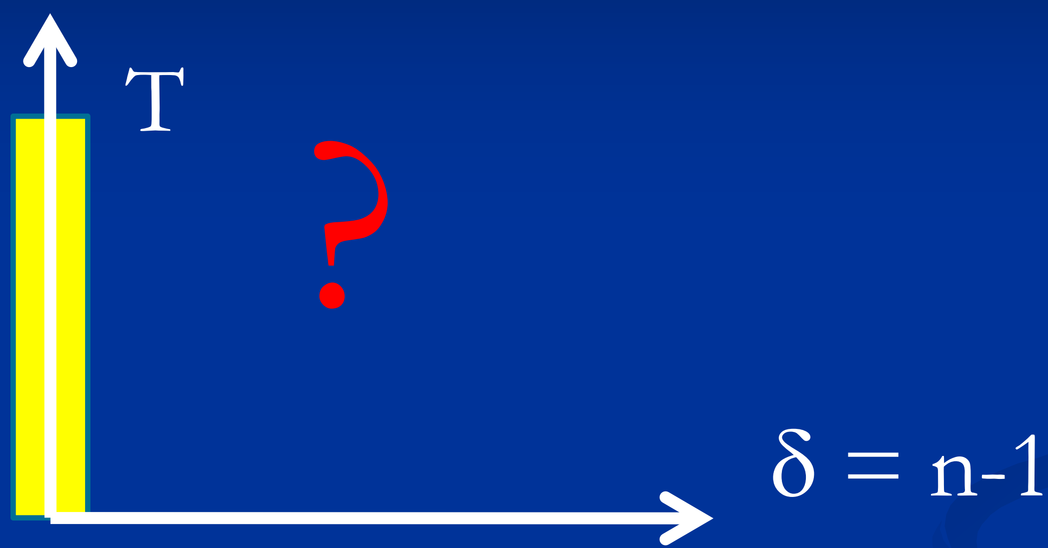


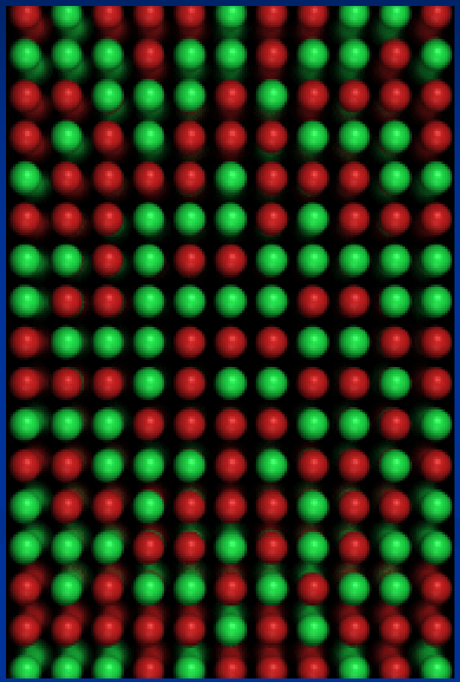
Take home message

- Good theoretical methods to deal with the case of a ``simple” equilibrium 1d systems (analytics and numerics)
- Stepping stone to go beyond: many exciting questions and problems (out of equilibrium, disorder, many chains, etc.)
- Controled experimental realizations in condensed matter and cold atoms

Effect of lattices: Mott transition

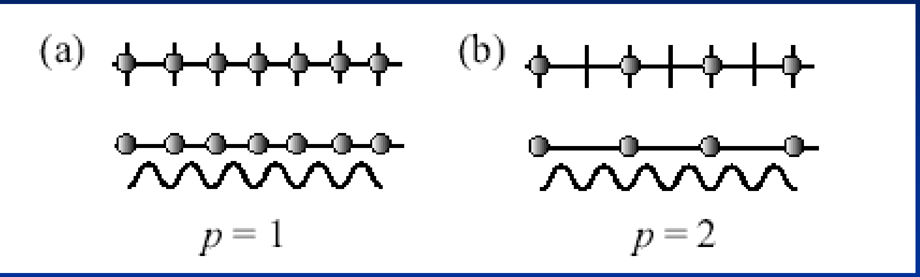
Mott transition





- Mott insulator (n=1)
- T < T_N : antiferromagnetic phase

Periodic lattice



$$H = \int dx V_0 \cos(Qx)\rho(x)$$

$$H = \int dx V_0 \cos(Qx)\rho_0 e^{i(2\pi\rho_0 x - 2\phi(x))}$$

- Incommensurate: $Q \neq 2\pi\rho_0$

$$H = \int dx \cos(2\phi(x) + \delta x)$$

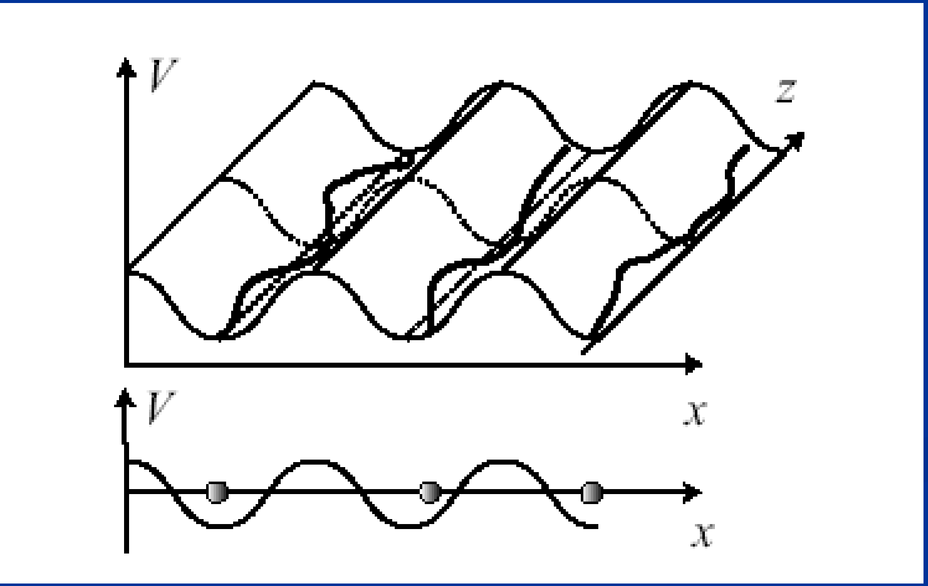
- Commensutate: $Q = 2\pi\rho_0$

$$H = \int dx \cos(2\phi(x))$$

Competition

$$S_0 = \int \frac{dx d\tau}{2\pi K}[\tfrac{1}{u}(\partial_\tau \varphi(x,\tau))^2 + u(\partial_x \varphi(x,\tau))^2]$$

$$S_L = -V_0 \rho_0 \int dx d\tau \cos(2\varphi(x))$$



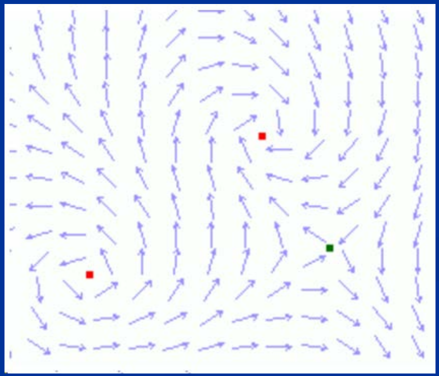
Beresinskii-Kosterlitz-Thouless transition at K=2

String order parameter

BKT transition

$$H = -J\sum_{\langle i,j\rangle}\vec{S}_i\cdot\vec{S}_j = -J\sum_{\langle i,j\rangle}\cos(\theta_i-\theta_j)$$

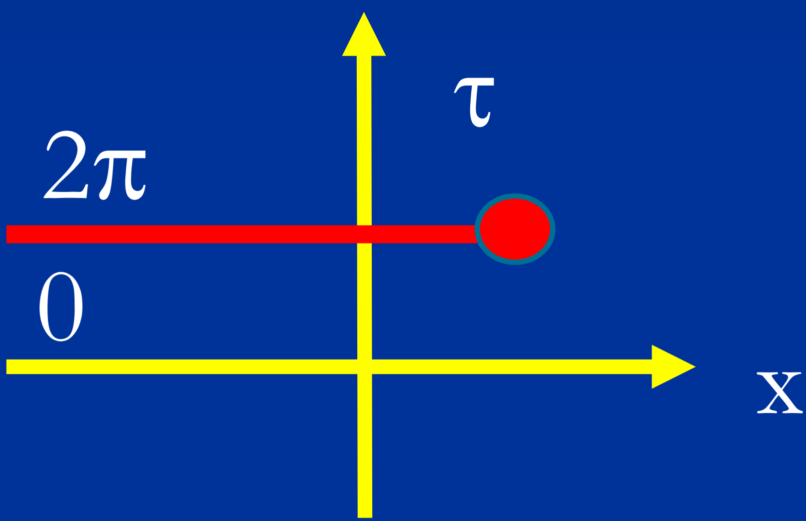
- BKT: remarkable transision going outside the paradigm of Landau’s phase transitions
- A transition wihout an order parameter
- Topological Vortex excitations



Vortex operator

$$e^{iaP}\,|\,x\rangle = |\mathrm{x+a}\rangle$$

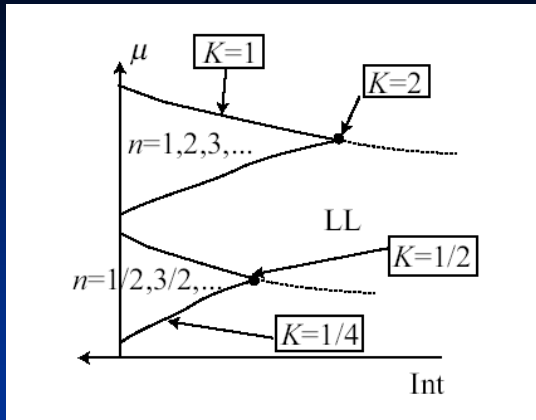
$$\phi(x,\tau) = \pi\int_{-\infty}^{x} dx'\,\Pi_{\theta}(x',\tau)$$

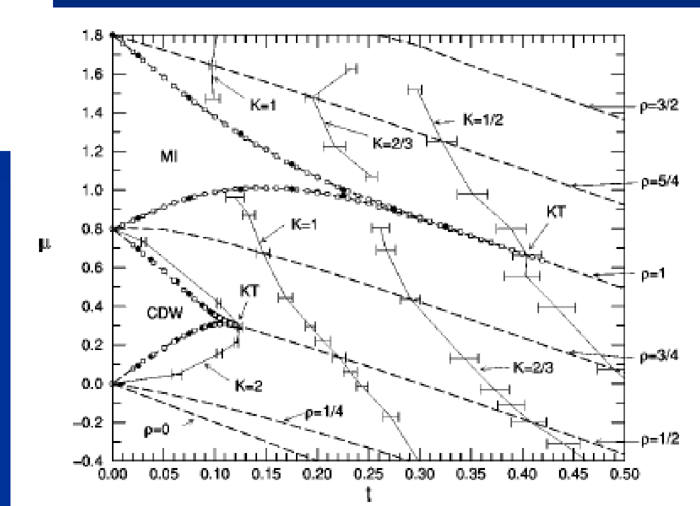


$$\cos(2\phi(x_1,\tau_1))$$

- Vortex operator for θ
- K : inverse temperature
- g : vortex fugacity

$$S = \frac{K}{2\pi}\int dx\,d\tau\left[\frac{1}{u}(\partial_{\tau}\theta) + u(\partial_{x}\theta)^{2}\right] - g\int dx\cos(2\phi)$$





Mott insulator:

ϕ is locked

Density is fixed

TG, Physica B 230 975(97): arXiv/0605472 (Salerno lectures);

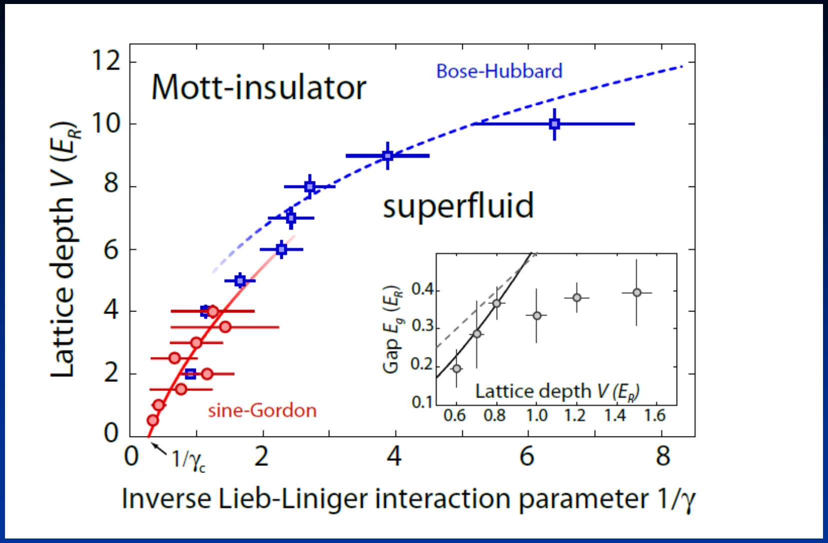
Oxford (2004);

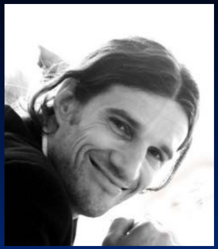
M. Cazalilla et al., Rev. Mod. Phys.83 1405 (2011)

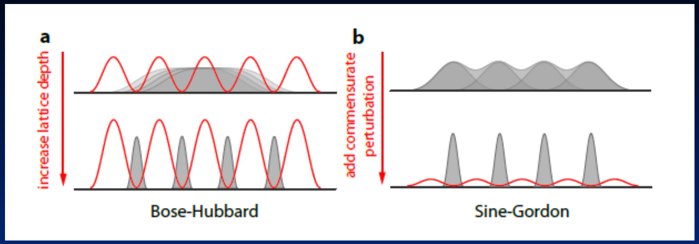
T. Kuhner et al. PRB 61 12474 (2000)

Gap in the excitation spectrum

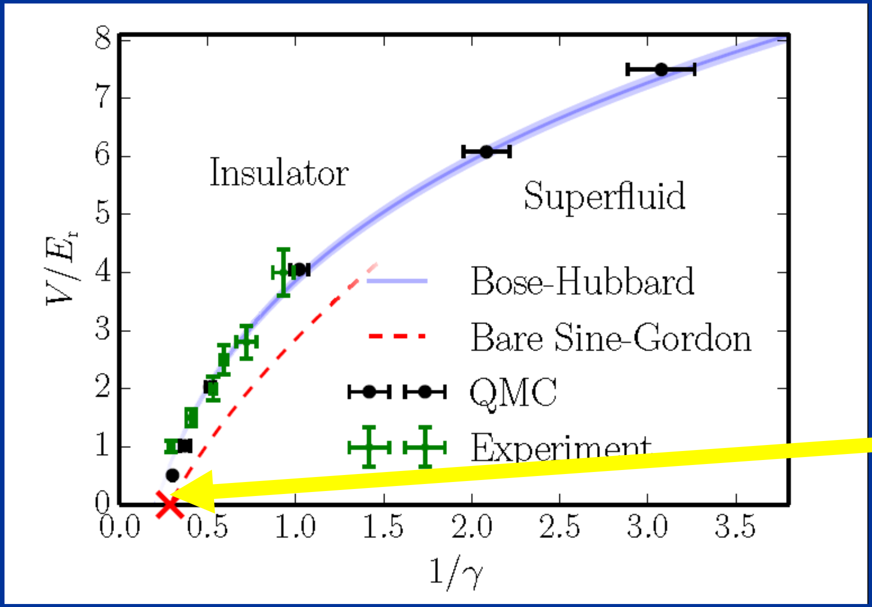
$$G(r) \propto e^{-r/\xi}$$







E. Haller et al. Nature 466 597 (2010)



Renormalized
Sine-Gordon

Shows:

K* = 2

G. Boeris et al. PRA 93 93, 011601(R) (2016)

Non local (topological) order

$$\rho(x) \sim \nabla\phi(x)$$

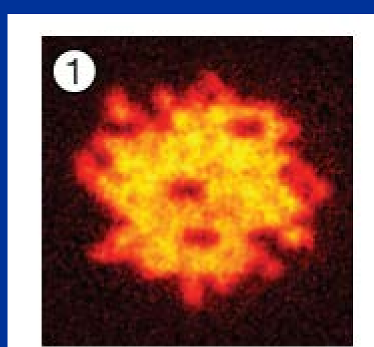
$$\mathcal{O}_{\mathrm{P}}^2 = \lim_{l\to\infty} \mathcal{O}_{\mathrm{P}}^2(l) = \lim_{l\to\infty}\left\langle \prod_{k\le j\le k+l} e^{i\pi\delta\hat{n}_j}\right\rangle$$

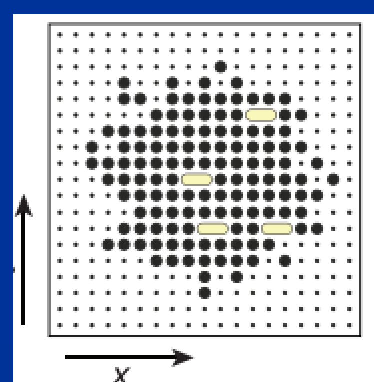
E. Berg, E. Dalla Torre, T. Giamarchi, E. Altman, *Phys. Rev. B* **77**, 245119 (2008).

Observation of Correlated Particle-Hole Pairs and String Order in Low-Dimensional Mott Insulators

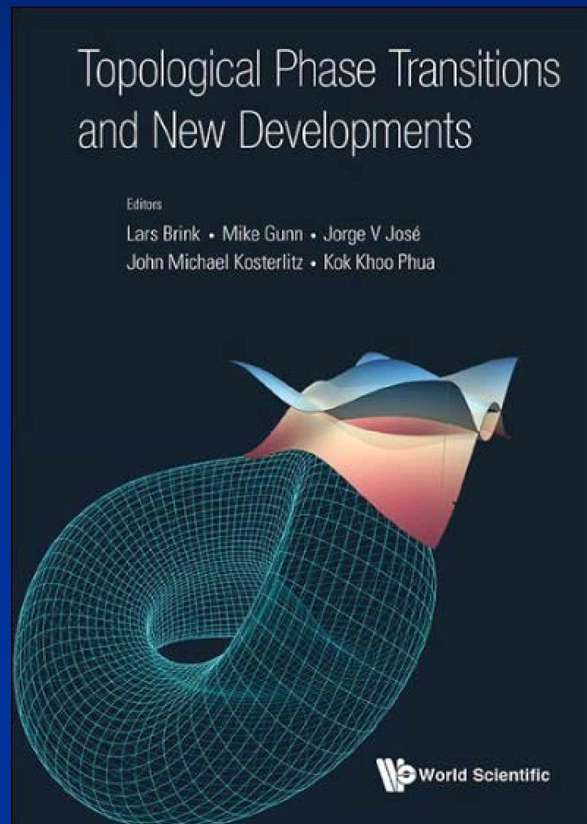
M. Endres,[1*] M. Cheneau,[1] T. Fukuhara,[1] C. Weitenberg,[1] P. Schauß,[1] C. Gross,[1] L. Mazza,[1] M. C. Bañuls,[1] L. Pollet,[2] I. Bloch,[1,3] S. Kuhr[1,4]

Science (2011)





Topological excitations is the norm in 1D



Topological Phase Transitions and New Developments, pp. 147-164 (2018)

Clean and dirty bosons in 1D lattices

Disorder (equilibrium)

Example: localization of 1D interacting bosons

$$H = \sum_{i=1}^{N} \frac{P_i^2}{2m} + \frac{1}{2} \sum_{i \neq j} V(R_i - R_j) + \sum_{i=1}^{N} D(R_i)$$

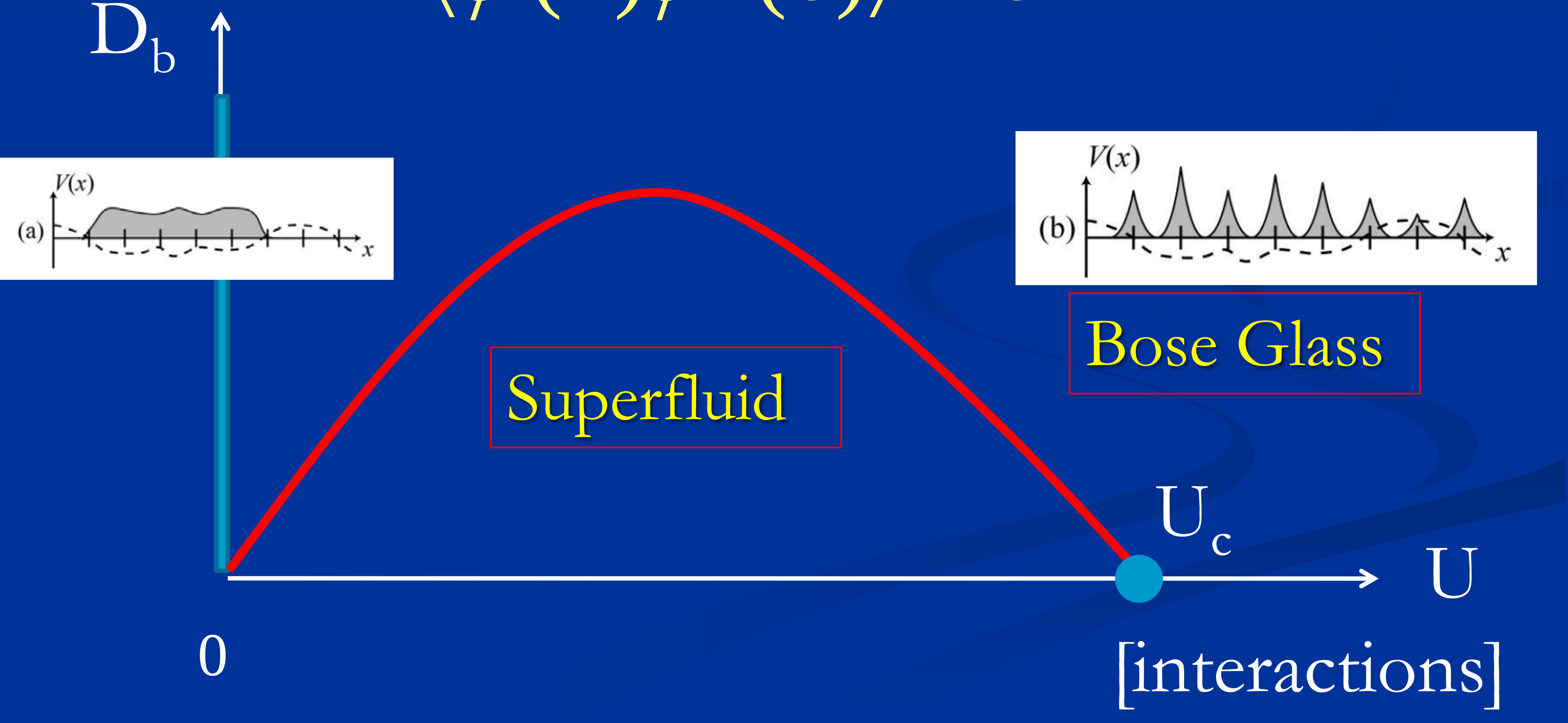
TG + H. J. Schulz EPL 3 1287 (1987); PRB 37 325 (1988)

Competition Disorder vs Interactions

Existence of a Many-Body localized phase: Bose glass

Bose glass phase

$$\langle \psi(x)\psi^\dagger(0)\rangle \sim e^{-x/\xi}$$

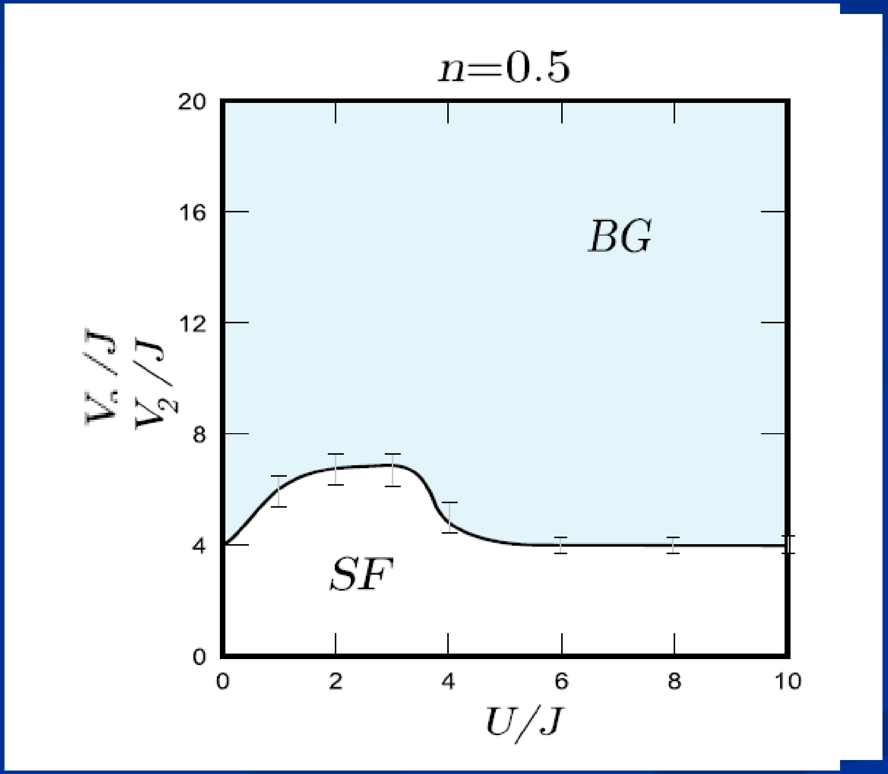


Other potentials: Biperiodics

$$V(x) = V_0 \cos(Q_0 x) + V_1 \cos(Q_1 x)$$

- U= 0
  Aubry-André model
- Localization transition

Effect of interactions?
Same as ``true" disorder ?



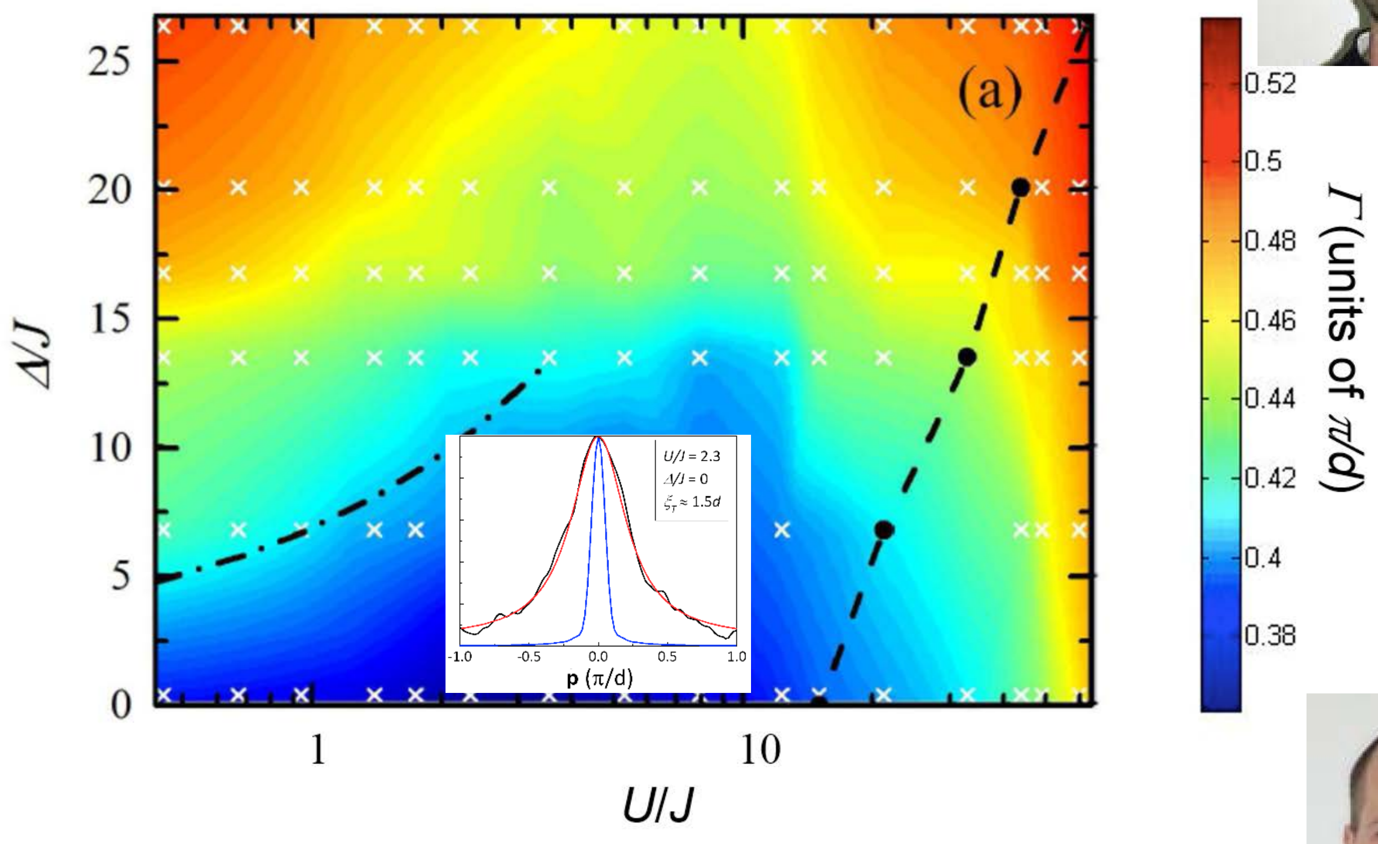
J. Vidal, D. Mouhanna, TG PRL 83 3908 (1999); PRB 65 014201 (2001)

G. Roux et al. PRA 78 023628 (2008);
T. Roscilde, Phys. Rev. A 77, 063605 2008;
X. Deng et al PRA 78, 013625 (2008);

Quasi-periodics and interactions

C. D'Errico, E. Lucioni et al. PRL (2014);

L. Gori et al PRA 93 033650 (2016)

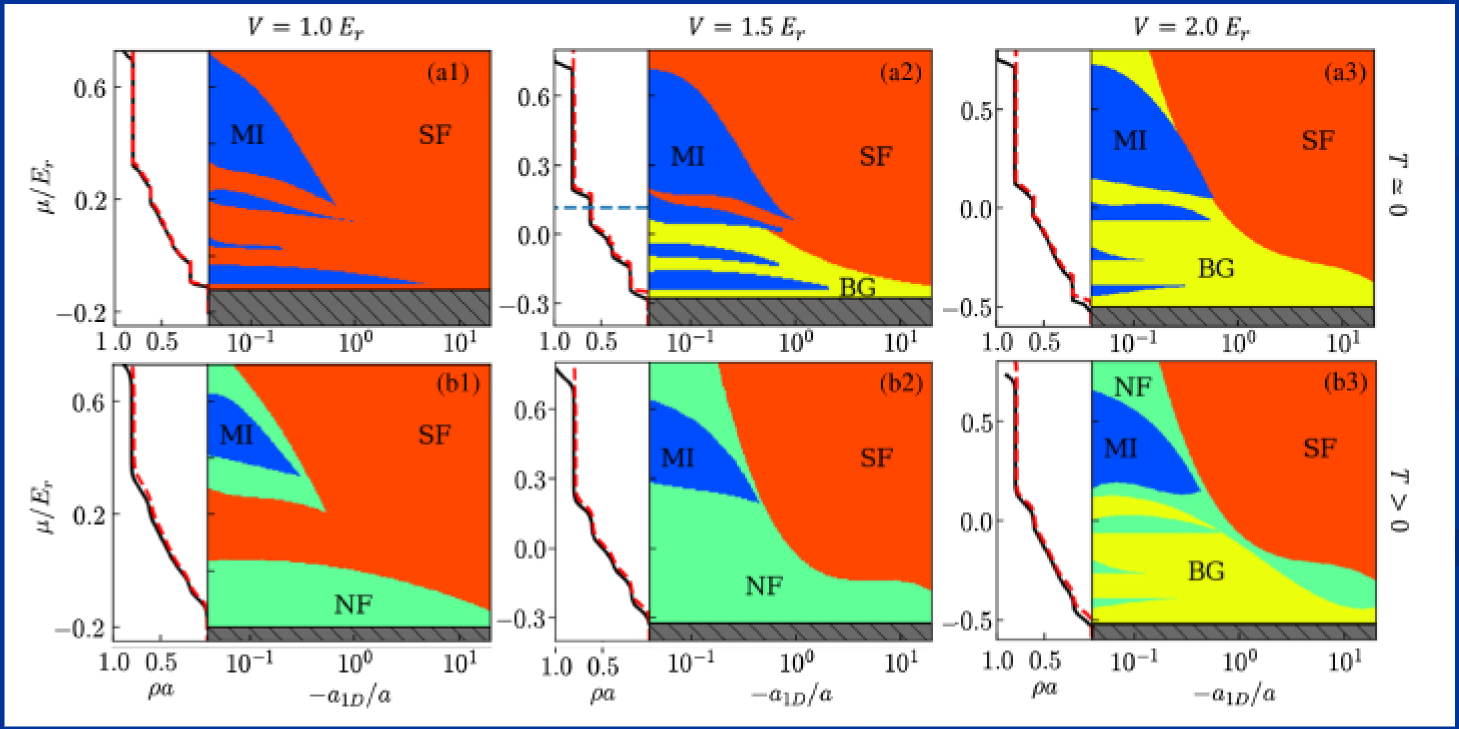


$$V(x) = V_0 \cos(Q_0 x) + V_0 \cos(Q_1 x)$$

PHYSICAL REVIEW LETTERS **125**, 060401 (2020)

Lieb-Liniger Bosons in a Shallow Quasiperiodic Potential: Bose Glass Phase and Fractal Mott Lobes

Hepeng Yao,[1] Thierry Giamarchi,[2] and Laurent Sanchez-Palencia[1]



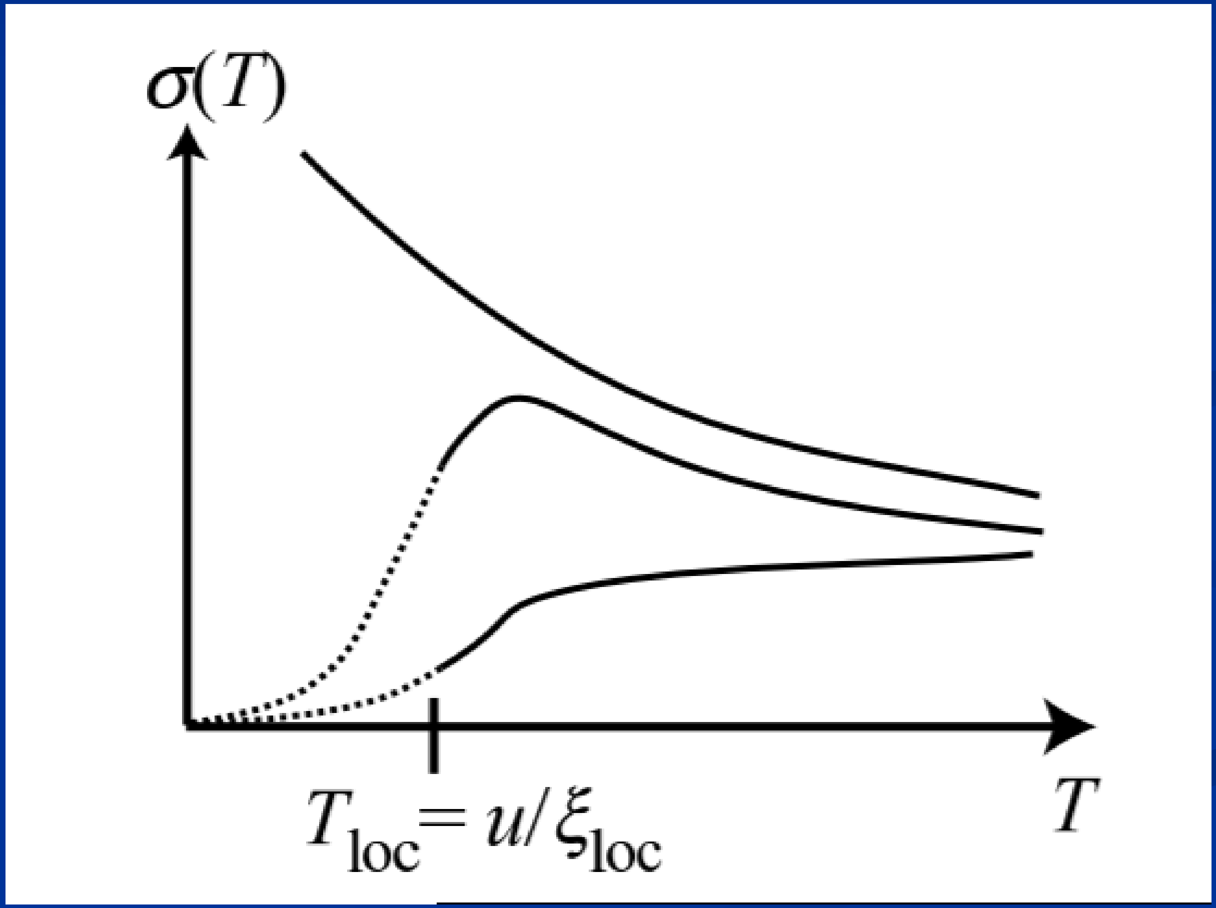
d.c. transport at finite T

- In 1D all states are exponentially localized
- Finite T: sweeps energy E with probability f(E)
- No interactions $\sigma(T)=0$ for all T !
- Needs coupling to a thermal bath (phonons, etc.)
- Or with interactions can the system be its own thermal bath ?

With a bath

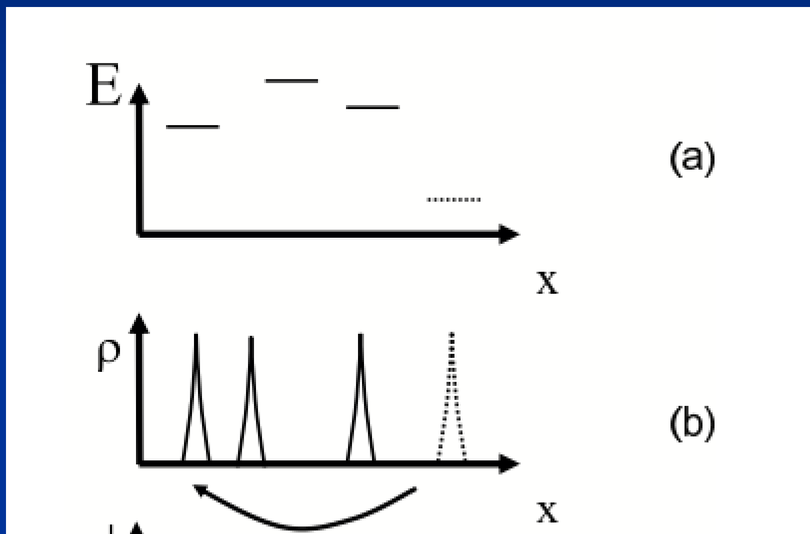
d.c. transport (high T)

TG + H. J. Schulz EPL 3 1287 (1987); PRB 37 325 (1988)



Transport with a Thermostat

- Mott variable range hopping



$$e^{-\beta E} e^{-L/\xi_{\rm loc}}$$

$$N_0 L^d E$$

$$\sigma \propto e^{-\frac{\beta}{N_0 L^d} - L/\xi_{\rm loc}}$$

$$\sigma \propto e^{-(d+1)\left(\frac{\beta}{N_0 d^d \xi_{\rm loc}^d}\right)^{\frac{1}{d+1}}}$$

- 1D with interactions

T. Nattermann, TG, P. Le doussal, PRL 91, 056603 (2003)
arXiv:cond-mat/0403487

$$\sigma(T) \propto e^{-(S^*/\hbar)} = \exp\left[-\frac{4\pi}{K^*}\sqrt{2\beta\Delta}\right].$$

Without a bath

- With interactions can other particles be a ``bath" for one particle ?

- Can the system reach the thermodynamic equilibrium ? And explore ergodically the phase space ?

Many body localization

Basko, Aleiner, Altshuler; Gornyi, Mirlin Polyakov; Huse, ……

Dmitry A. Abanin, Ehud Altman, Immanuel Bloch, and Maksym Serbyn, Rev. Mod. Phys. **91**, 021001 (2019)

No thermostat $\sigma(T) = 0$ if $T<T^*$ even with interactions

The system is not ergodic even at finite temperatures/energies

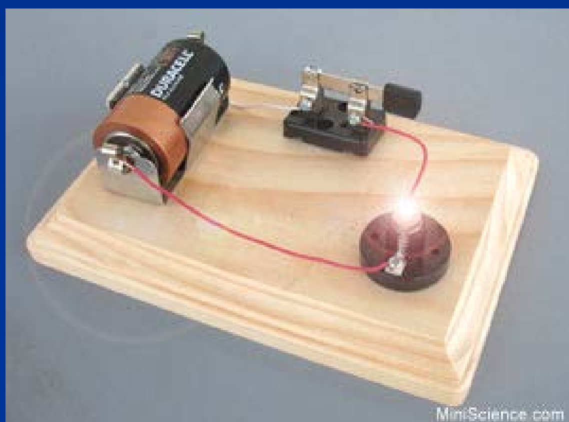
Take home message

- Remarkable interplay between localization and interactions
- Consequences and challenges both in equilibrium (LIP) and out of equilibrium (MBL).
- Experimental possibilities to explore these phenomena
- Need better tools !

Transport

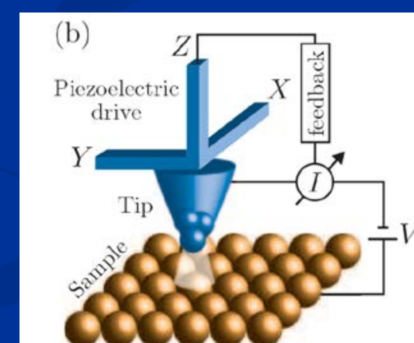
Transport

- Condensed matter: a «routine» probe for materials



- Theoretically: complicated !
  Out of equilibrium

- Typical situation



- Often (but not always !) : linear response I=G V

Questions

- I=f(V) Reflects the properties of the system (interferences – metal, insulator, etc.)
- Methods ?

  Kubo; Memory function; Landauer; Keldysh; …..
- Expectations for small V:

$$V = RI$$

$$R = R_{contact} + R_{system}$$

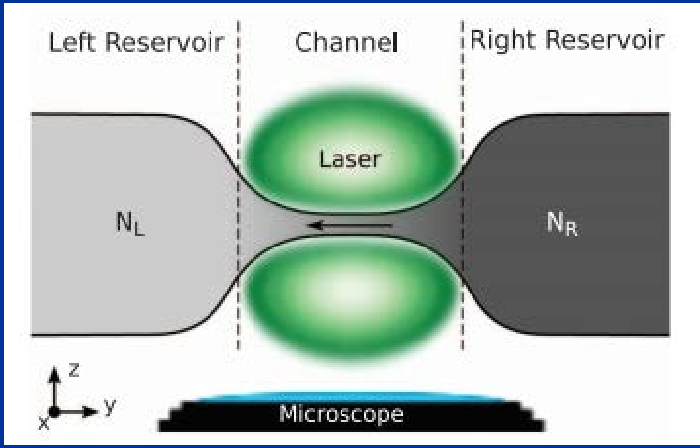
- Other transport(s): spin, temperature, etc.

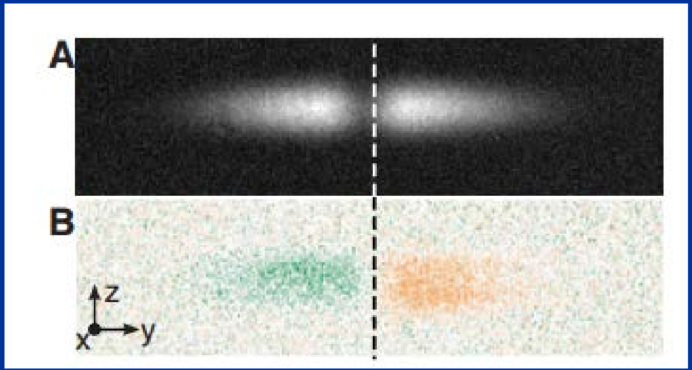
Cold atoms

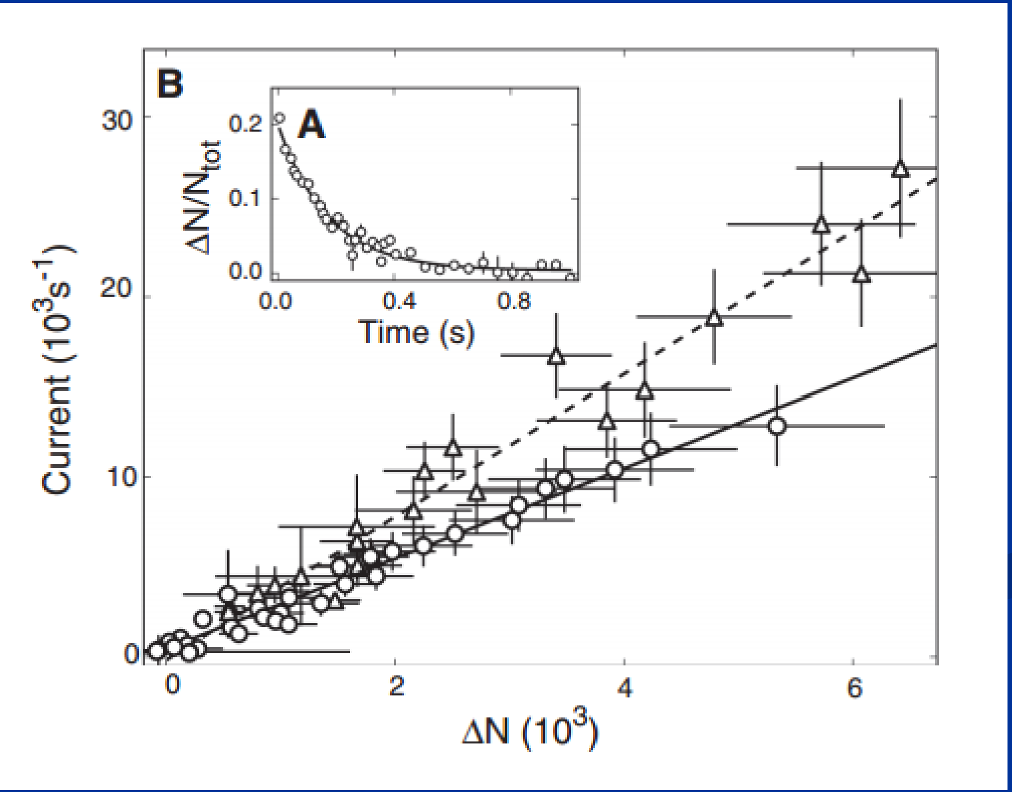
Conduction of Ultracold Fermions Through a Mesoscopic Channel

Jean-Philippe Brantut, Jakob Meineke, David Stadler, Sebastian Krinner, Tilman Esslinger*

SCIENCE VOL 337 31 AUGUST 2012







1D ballistic: quantized conductance

VOLUME 60, NUMBER 9 PHYSICAL REVIEW LETTERS 29 FEBRUARY 1988

Quantized Conductance of Point Contacts in a Two-Dimensional Electron Gas

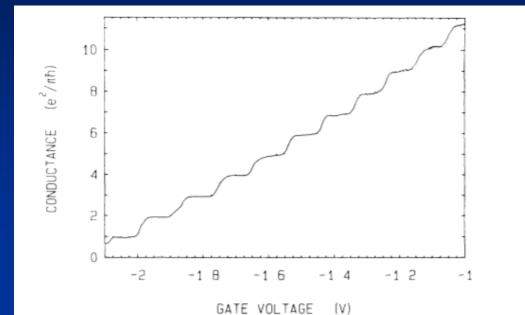
B. J. van Wees
Department of Applied Physics, Delft University of Technology, 2628 CJ Delft, The Netherlands

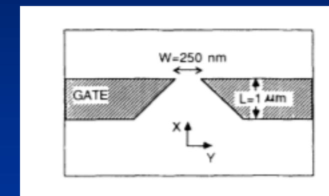
H. van Houten, C. W. J. Beenakker, and J. G. Williamson,
Philips Research Laboratories, 5600 JA Eindhoven, The Netherlands

L. P. Kouwenhoven and D. van der Marel
Department of Applied Physics, Delft University of Technology, 2628 CJ Delft, The Netherlands

and

C. T. Foxon
Philips Research Laboratories, Redhill, Surrey RH1 5HA, United Kingdom
(Received 31 December 1987)

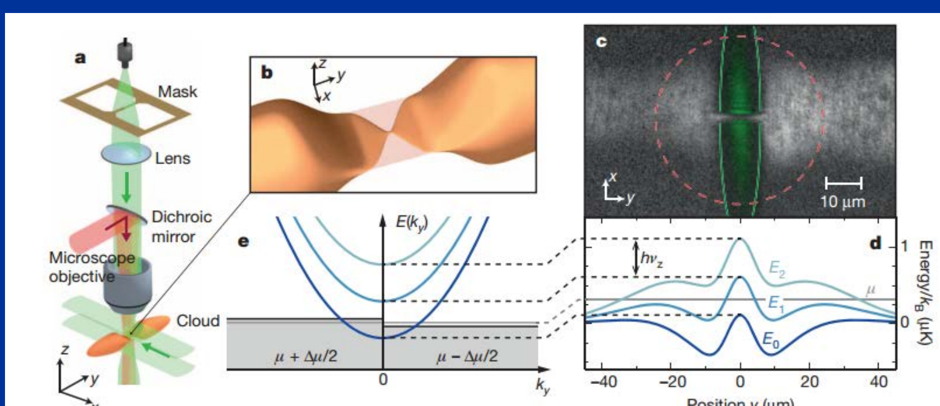


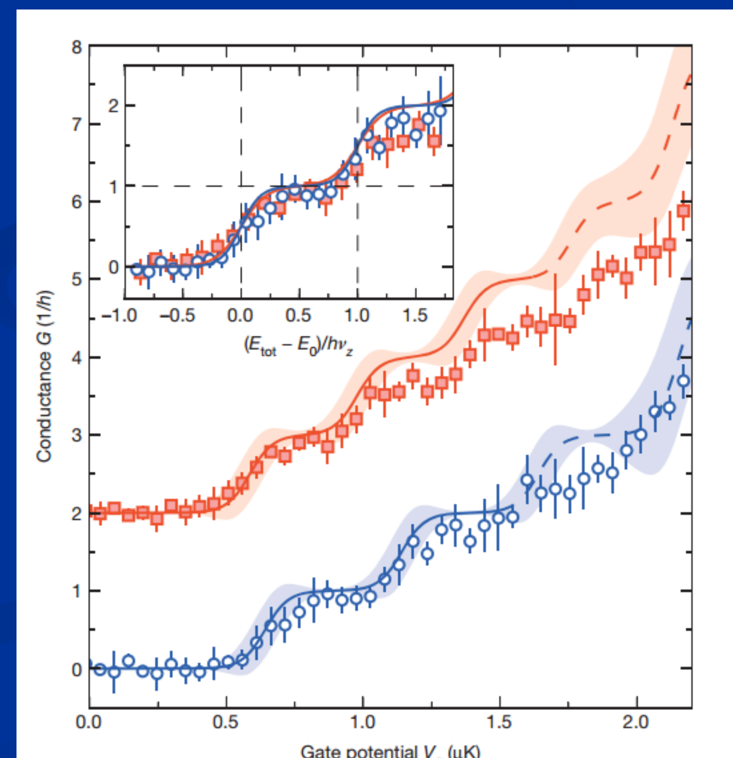


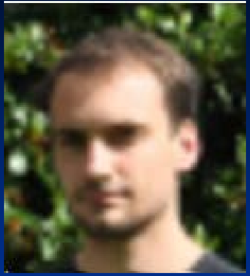
Observation of quantized conductance in neutral matter

Sebastian Krinner[1], David Stadler[1], Dominik Husmann[1], Jean-Philippe Brantut[1] & Tilman Esslinger[1]

64 | NATURE | VOL 517 | 1 JANUARY 2015

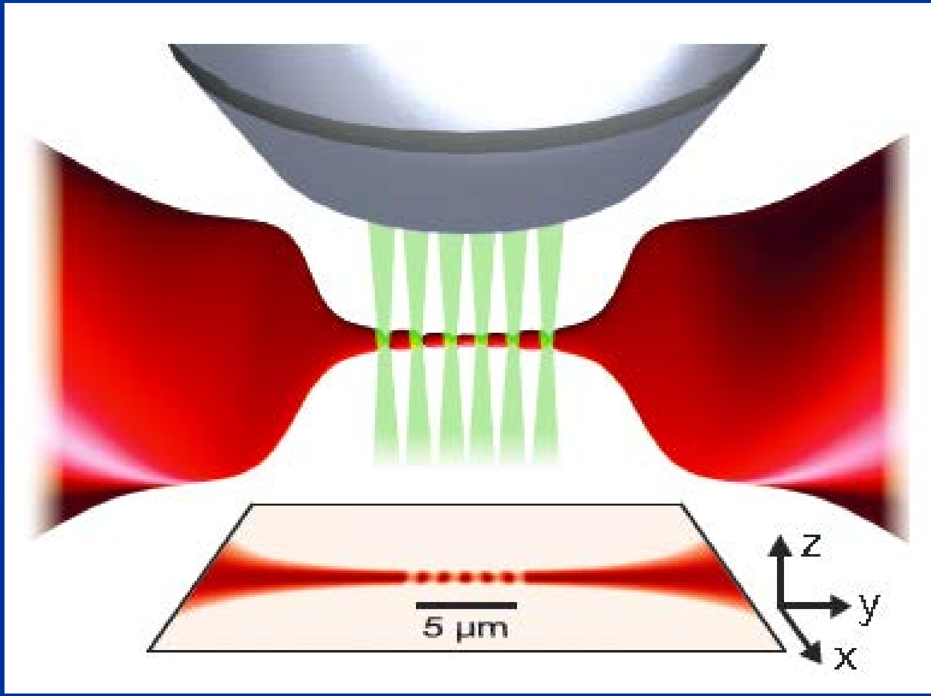


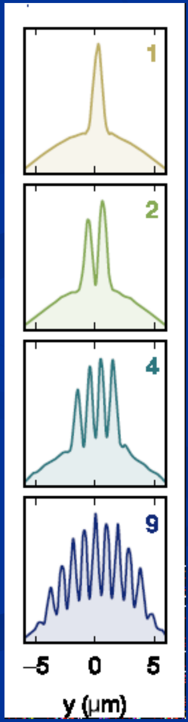




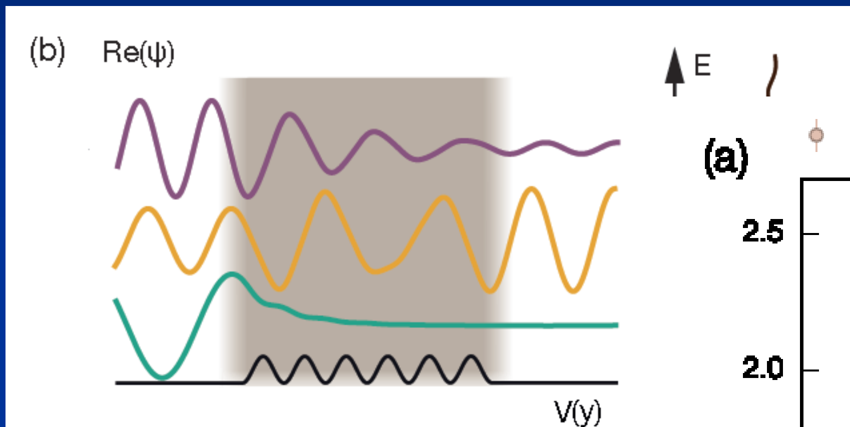
Atomtronic

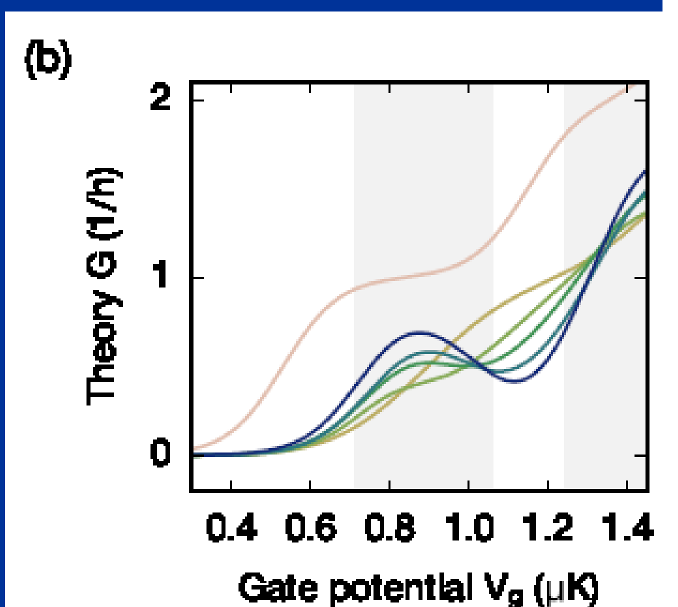
M. Lebrat, P. Grisins et al., PRX 8 011053 (18)



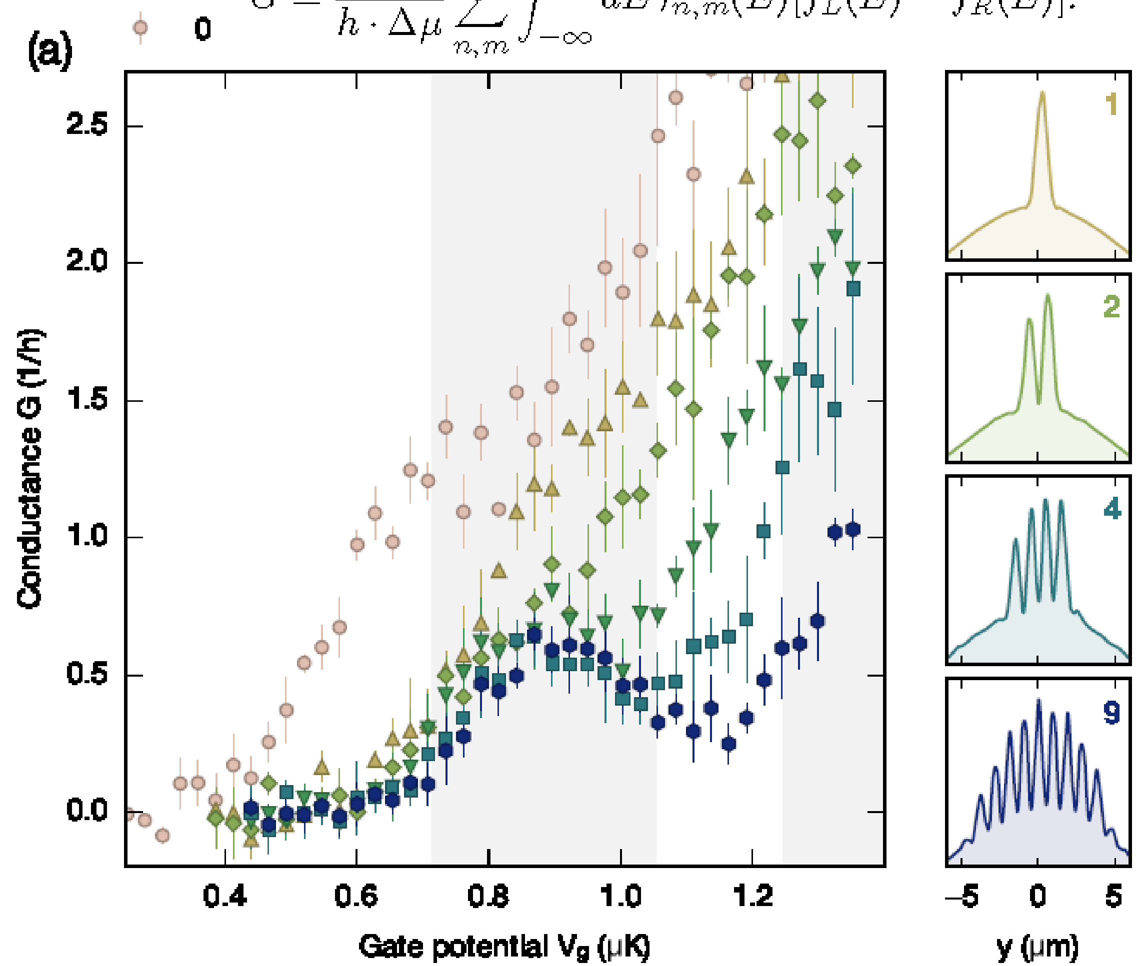


No interactions: band insulator





$$G = \frac{1}{h \cdot \Delta\mu} \sum_{n,m} \int_{-\infty}^{\infty} dE\, \mathcal{T}_{n,m}(E)\,[f_L(E) - f_R(E)].$$



What happens with interactions?

$$H = H_{\mathrm{GY}} + H_{\mathrm{lattice}},$$

$$H_{\mathrm{GY}} = -\frac{\hbar^2}{2m}\sum_i \frac{\partial^2}{\partial y_i^2} + g_1 \sum_{i<j} \delta(y_i - y_j),$$

$$H_{\mathrm{lattice}} = \int dy\, V(y)\,\rho(y),$$

Luther-Emery liquid

- Gap in the spin sector (singlet pairing)

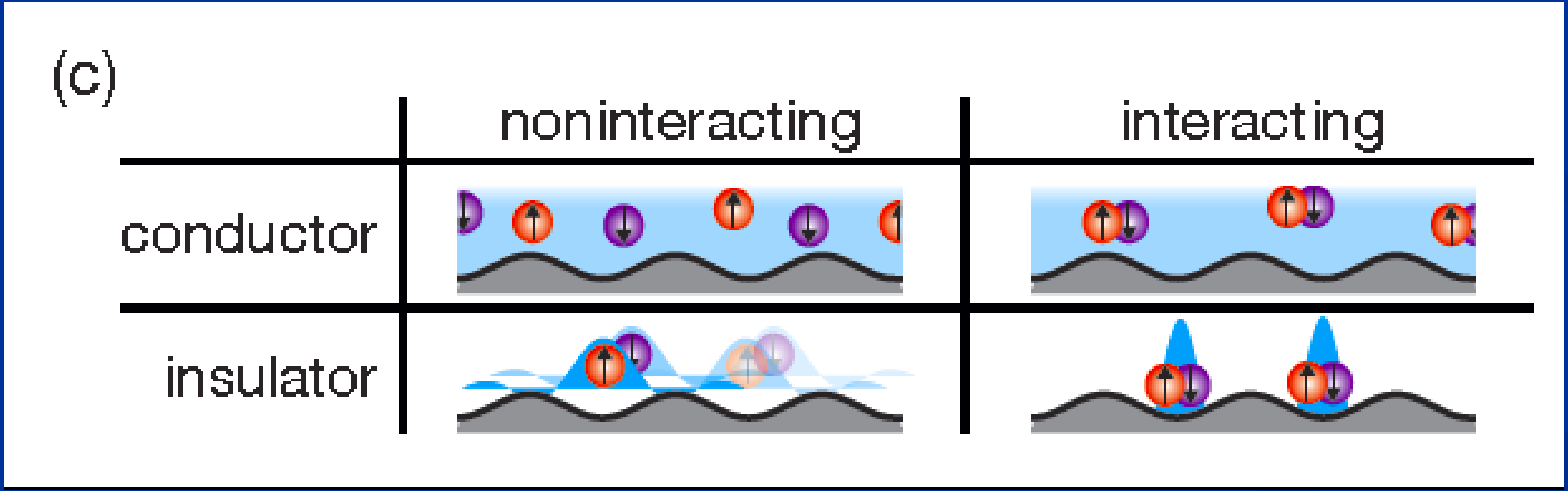
$$\rho(y) = \rho_0 - \frac{\sqrt{2}}{\pi}\nabla\phi_c(y) + 2\rho_0 f_s \cos\left(2k_F y - \sqrt{2}\phi_c(y)\right) + 2C\rho_0 \cos\left(4k_F y - 2\sqrt{2}\phi_c(y)\right),$$

- Conductance determined by the charge sector

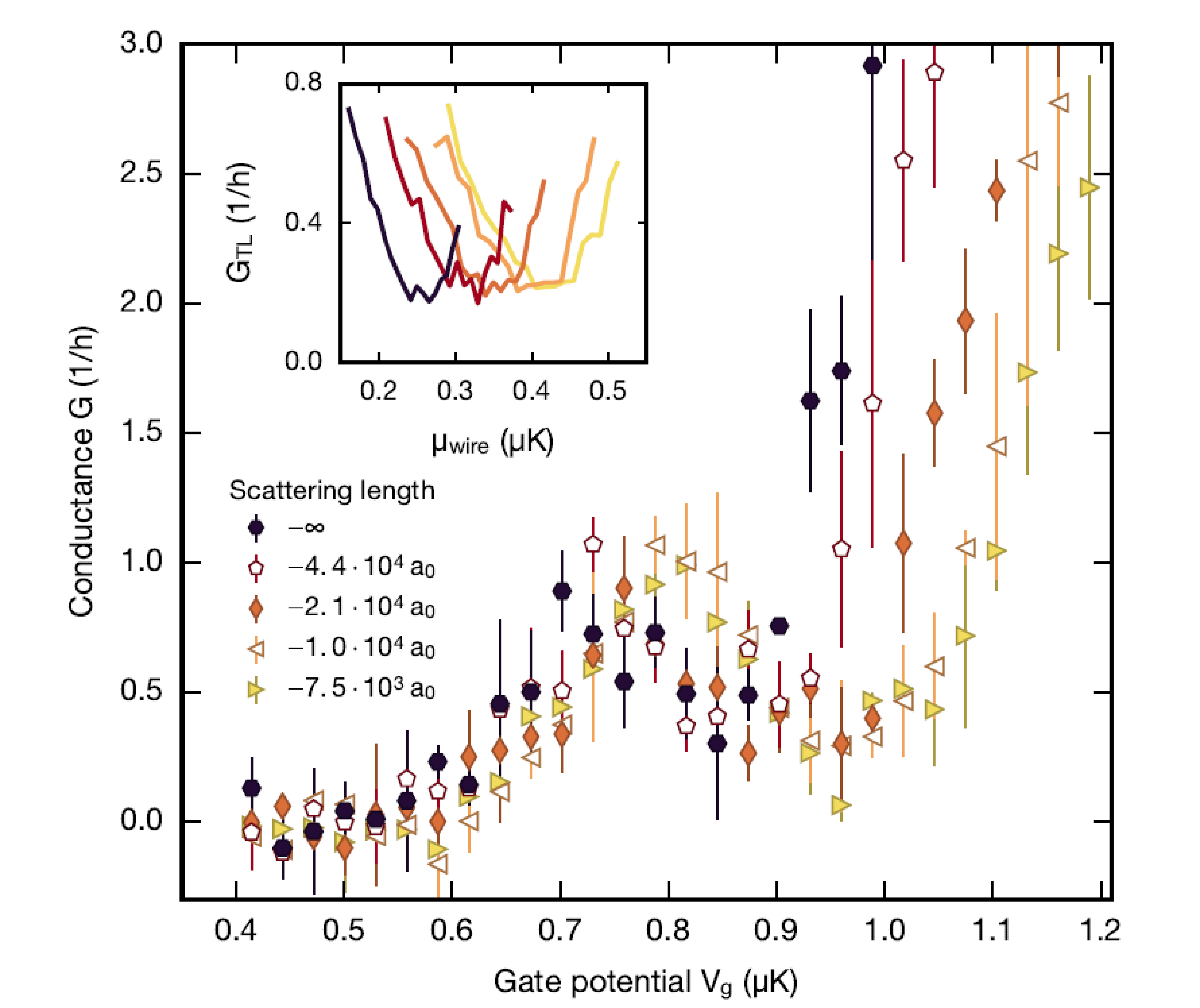
$$\mathcal{H}_\mu(y) = -\mu(y)\rho(y) = \mu(y)\frac{\sqrt{2}}{\pi}\nabla\phi_c,$$

$$I_{\uparrow\downarrow}(y) = \frac{\sqrt{2}}{\pi}\partial_t\phi_c(y,t)$$

Many-body insulator ``pinned” L.E. liquid



Experimental evidence for L.E. liquid

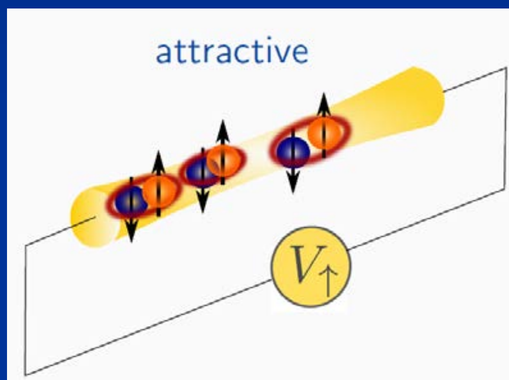


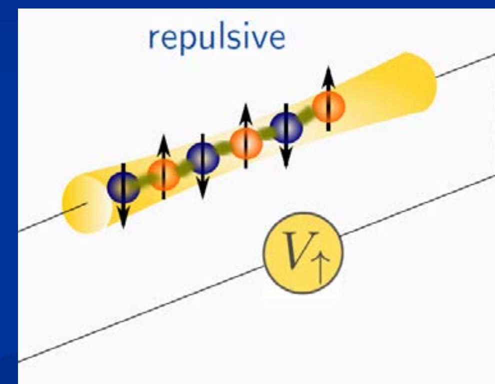
Spin transport

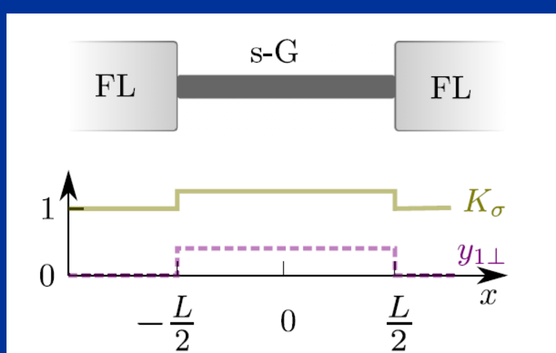
A.-M. Visuri, M. Lebrat, S. Häusler, L. Corman and TG, PRR **2**, 023062 (2020)

Spin transport and spin drag

$$J_\sigma = \langle j_\uparrow - j_\downarrow \rangle = G_\sigma(\mu_\uparrow - \mu_\downarrow)$$







$$\begin{pmatrix} I_\uparrow \\ I_\downarrow \end{pmatrix} = \begin{pmatrix} G_{\uparrow\uparrow} & \Gamma \\ \Gamma & G_{\downarrow\downarrow} \end{pmatrix} \begin{pmatrix} \Delta\mu_\uparrow \\ \Delta\mu_\downarrow \end{pmatrix}$$

Spin Drag: $\Delta\mu_\uparrow$ $I_\downarrow$ $I_\downarrow = \Gamma\Delta\mu_\uparrow$

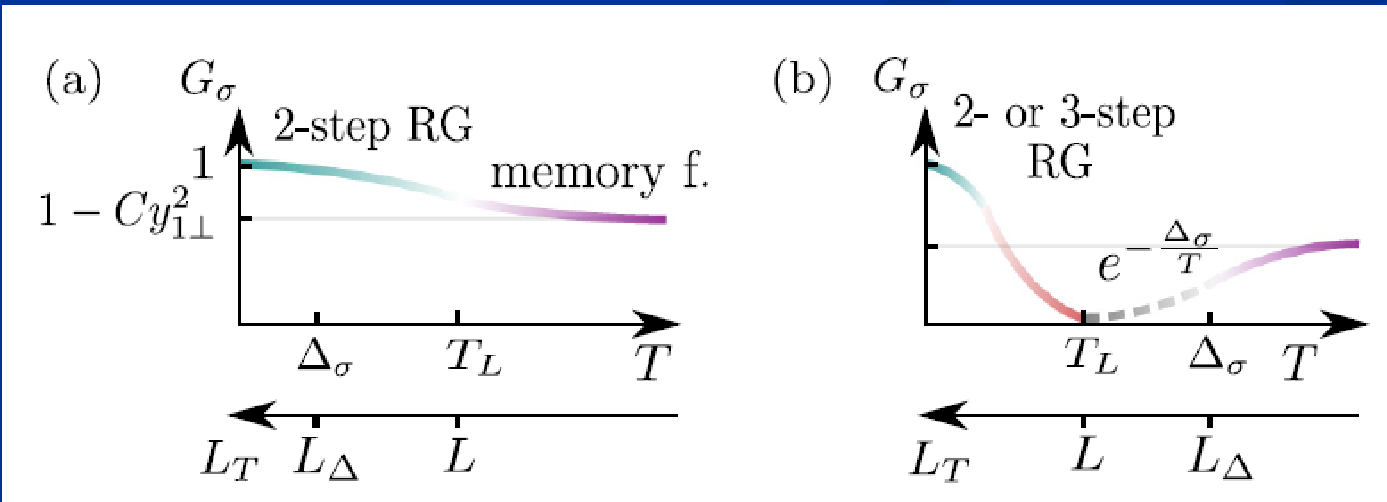
Solution (using Sine-Gordon)

$$H_\nu^0 = \frac{1}{2\pi}\int dx\left\{v_\nu K_\nu[\partial_x\theta_\nu(x,t)]^2 + \frac{v_\nu}{K_\nu}[\partial_x\phi_\nu(x,t)]^2\right\}.$$

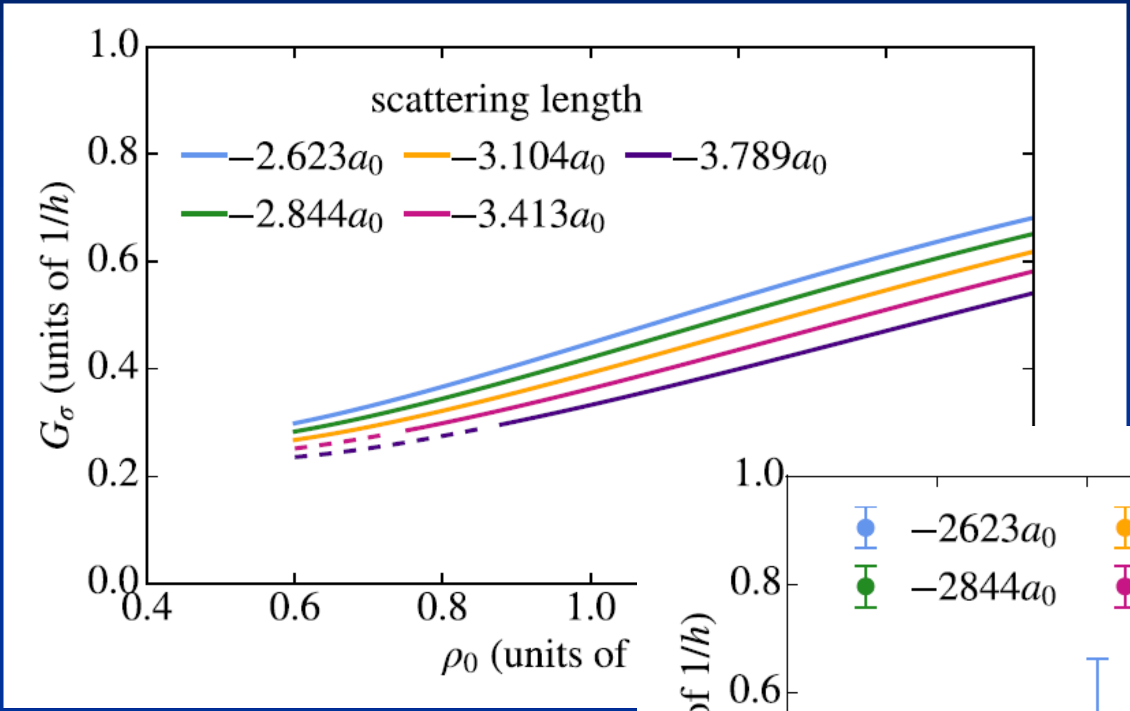
$$\frac{2g_{1\perp}}{(2\pi\alpha)^2}\int_{-\frac{L}{2}}^{\frac{L}{2}} dx\cos[2\sqrt{2}\phi_\sigma(x,t)],$$

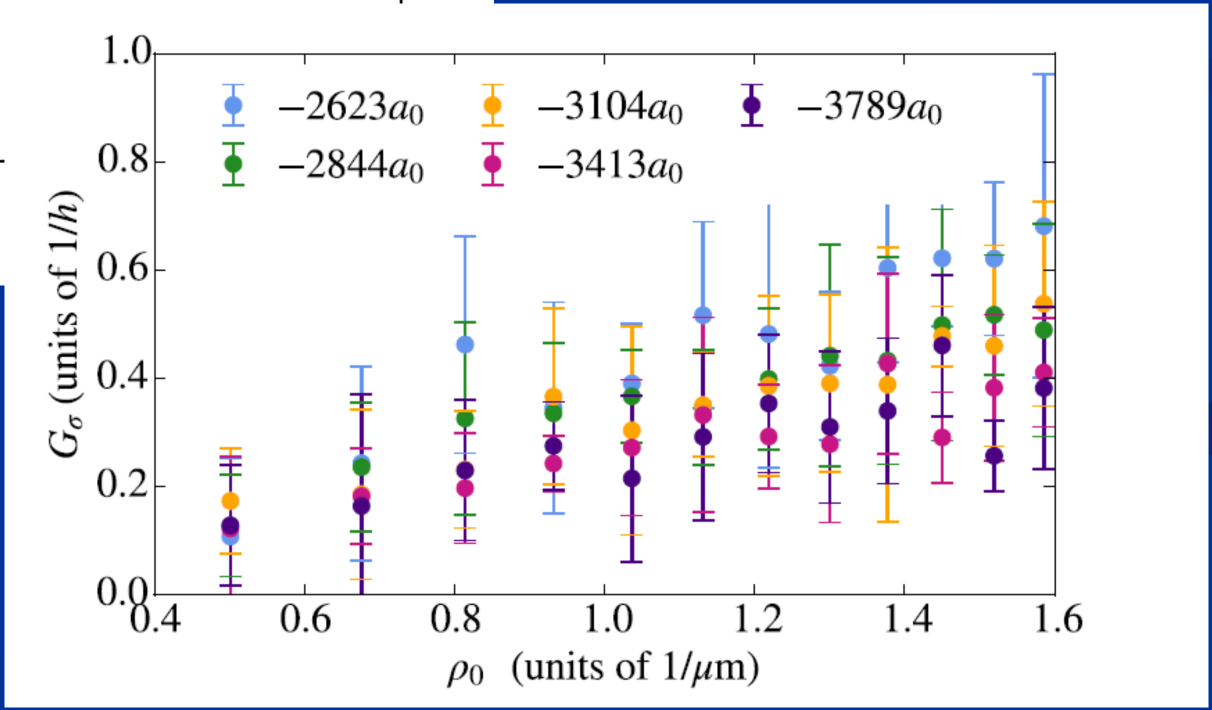
$J \propto \partial_t \phi_\sigma$

	weak coupling $L \ll L_\Delta$ $\Delta_\sigma \ll T_L$	strong coupling $L_\Delta \ll L$ $T_L \ll \Delta_\sigma$
high temperature $\Delta_\sigma, T_L \ll T$	memory function $G_\sigma - 1 \propto -y_{1\perp}^2 L(T/\Lambda)^{4K_\sigma-3}$	
intermediate temperature $T_L \ll T \ll \Delta_\sigma$ $\Delta_\sigma \ll T \ll T_L$	2-step RG $G_\sigma - 1 \propto$ $-y_{1\perp}^2(T/\Lambda)^2$	$e^{-\frac{\Delta_\sigma}{T}}$
low temperature $T_f \ll T \ll T_L, \Delta_\sigma$		2-step RG $G_\sigma \propto f^2(T/\Lambda)^2$
low temperature $T \ll T_f$		3-step RG $G_\sigma - 1 \propto$ $-y_{1\perp}^2(T/\Lambda)^2$



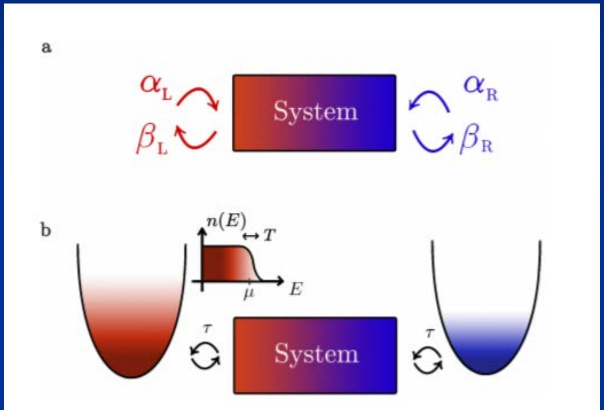
Comparison with experiments



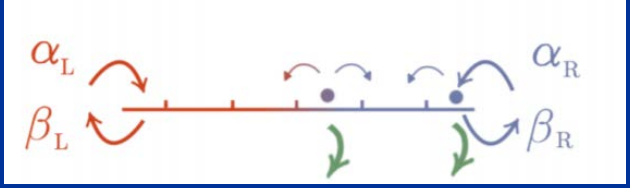


Lindblad reservoirs and losses

T. Jin, M. Filippone, TG PRB 102 205131 (2021)



$$\alpha = \frac{1}{2}\Delta\left[1+\tanh\left(\frac{\mu}{2T}\right)\right],$$
$$\beta = \frac{1}{2}\Delta\left[1-\tanh\left(\frac{\mu}{2T}\right)\right].$$

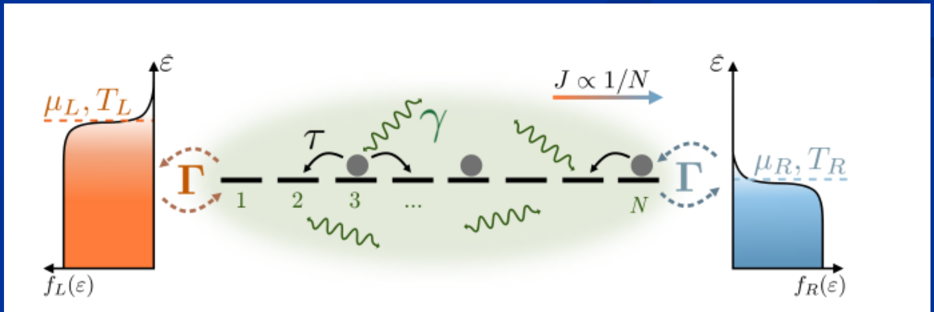


$$J_{\mathcal{L}} = \frac{1}{2}\left\{(\alpha_L - \beta_L - \alpha_R + \beta_R) + \frac{i}{2}\int \frac{d\epsilon}{2\pi}\mathrm{Tr}\Big[\Big((\alpha_L+\beta_L)\boldsymbol{\gamma_L} - (\alpha_R+\beta_R)\boldsymbol{\gamma_R}\Big)\boldsymbol{G^{\mathcal{K}}}\Big]\right\},$$
$$J_{D,\mathcal{L}} = \frac{i}{2}\int \frac{d\epsilon}{2\pi}\mathrm{Tr}\Big[\Big((\alpha_L+\beta_L)\boldsymbol{\gamma_L} + (\alpha_R+\beta_R)\boldsymbol{\gamma_R}\Big)\boldsymbol{G^{\mathcal{K}}}\Big].$$

T. Jin, J. Ferreira, M. Filippone, TG arxiv/2106.14765

$$\int dxdt\,\xi(x,t)\rho(x,t)$$

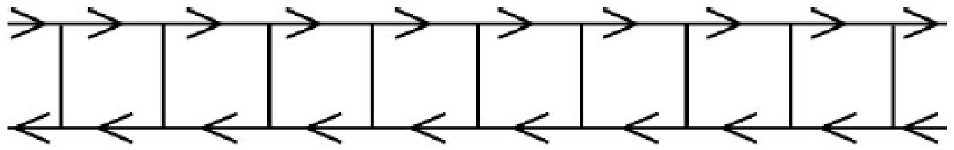
Ballistic ?
Diffusive ?



Artificial gauge fields

Meissner effect in bosonic ladders

E. Orignac, TG, PRB 64 144515 (2001)



(a)

$$H = -t_{\parallel} \sum_{i,p=1,2} \left(b^{\dagger}_{i+1,p} e^{ie^* a A_{\parallel,p}(i)} b_{i,p} + b^{\dagger}_{i,p} e^{-ie^* a A_{\parallel,p}(i)} b_{i+1,p} \right)$$

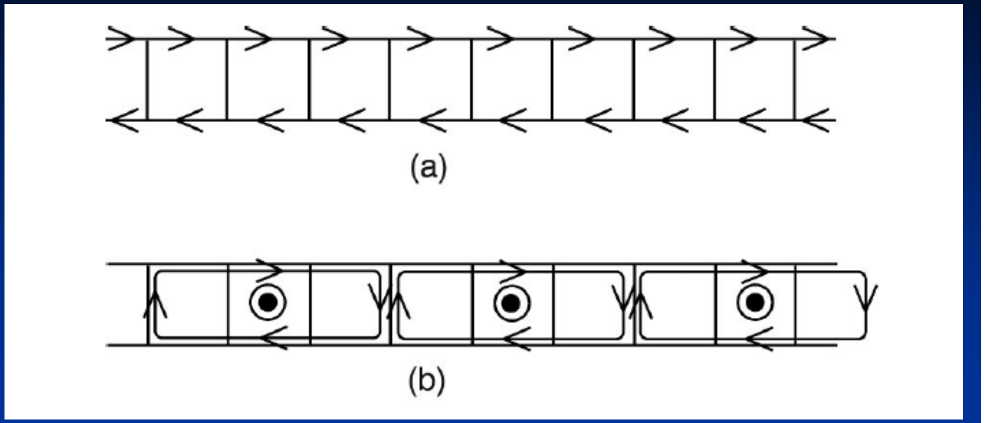
$$-t_{\perp} \sum_{i} \left(b^{\dagger}_{i,2} e^{ie^* A_{\perp}(i)} b_{i,1} + b^{\dagger}_{i,1} e^{-ie^* A_{\perp}(i)} b_{i,2} \right)$$

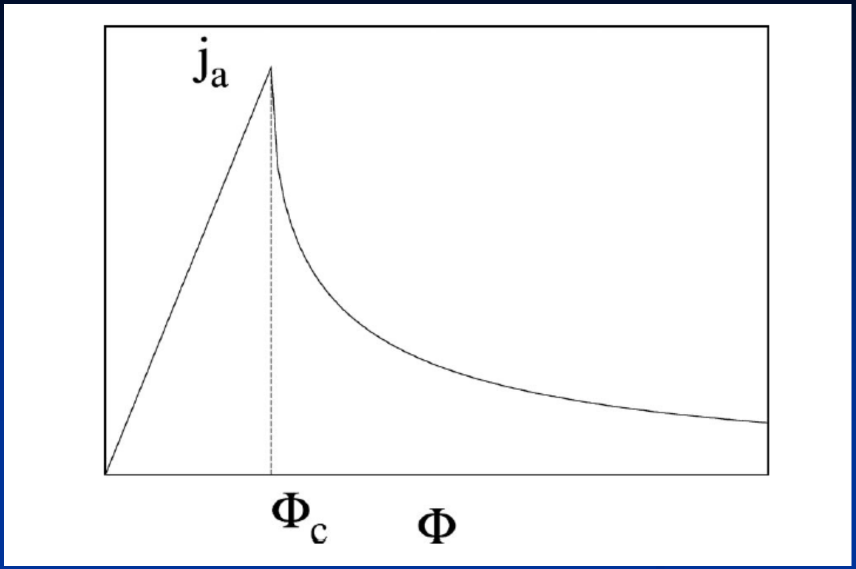
$$+U \sum_{i,p} n_{i,p}(n_{i,p} - 1) + V n_{i,1} n_{i,2}, \qquad (1)$$

$$\int \vec{A} \cdot \overrightarrow{dl} = \Phi$$

$$H = H^0_s + H^0_a - \frac{t_{\perp}}{\pi a} \int dx \cos[\sqrt{2}\,\theta_a + e^* A_{\perp}(x)]$$

$$+ \frac{2Va}{(2\pi a)^2} \int dx \cos\sqrt{8}\,\phi_a ,$$



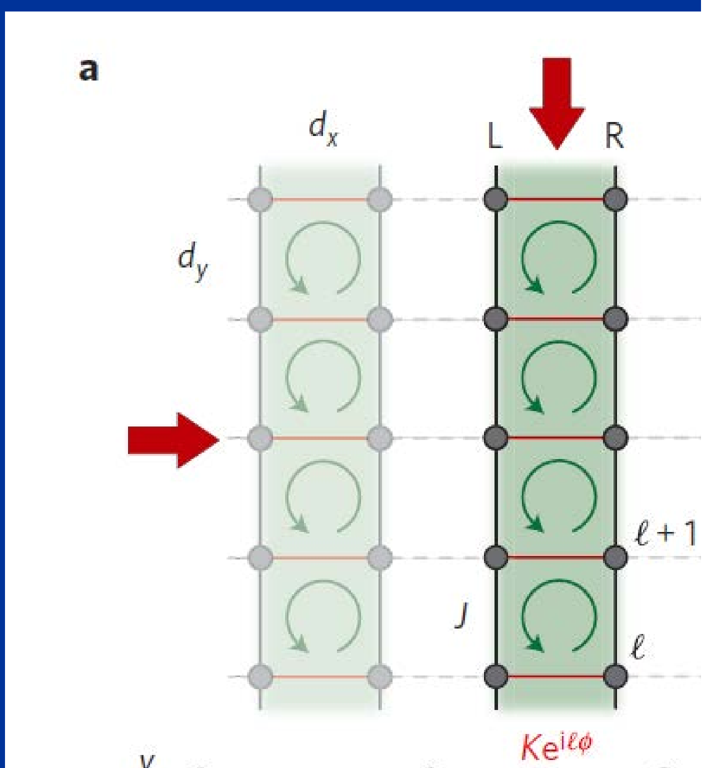


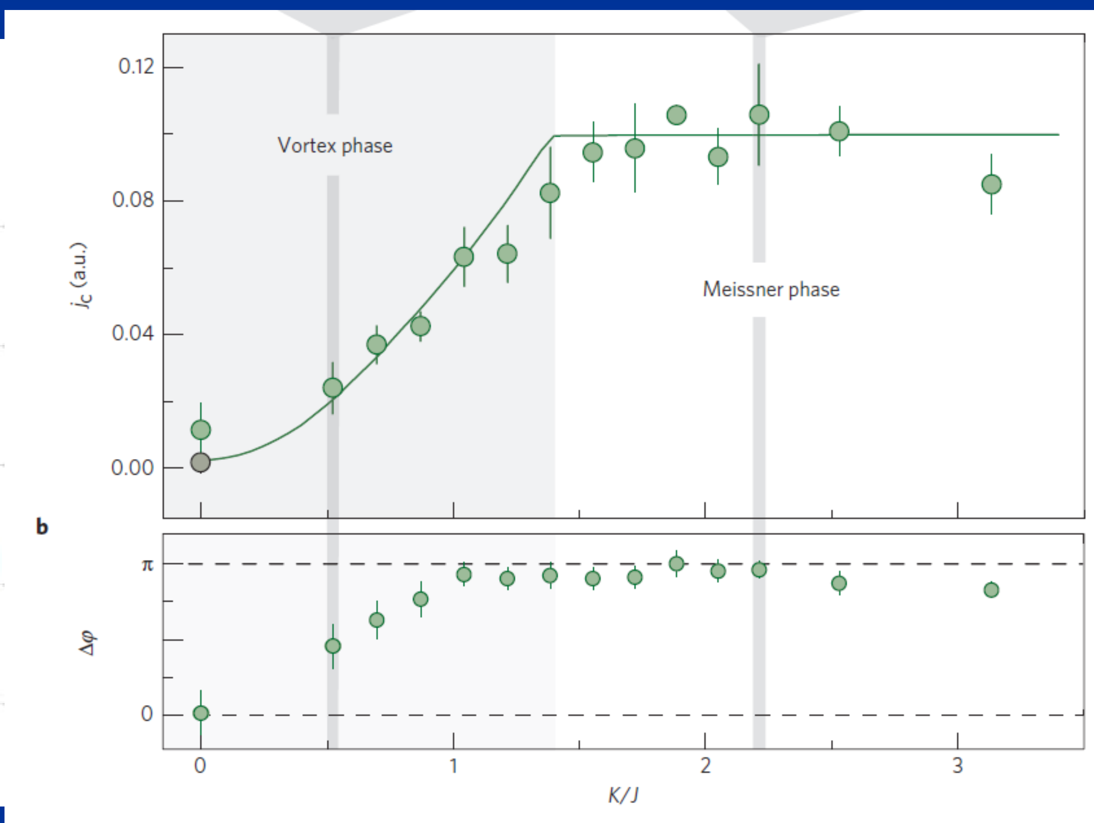
Orbital currents (``Meissner” effect)

Field ``Hc1”: appearance of vortices

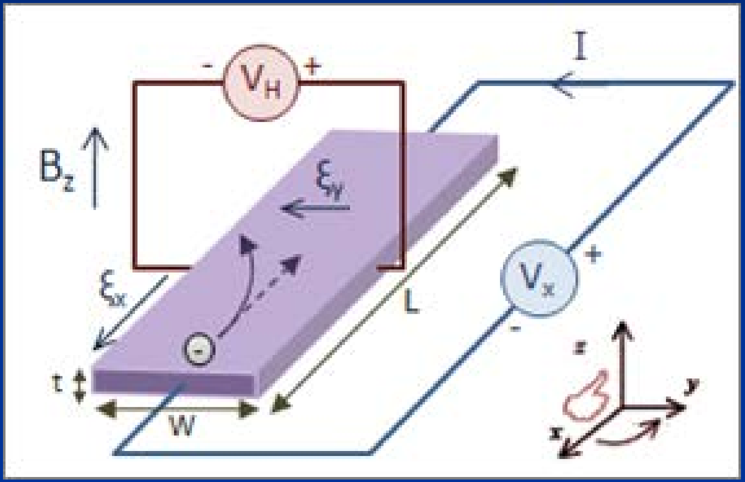
Artificial gauge field (cold atoms)

M Atala et al. Nat Phys, 10 588 (2014)





Hall effect



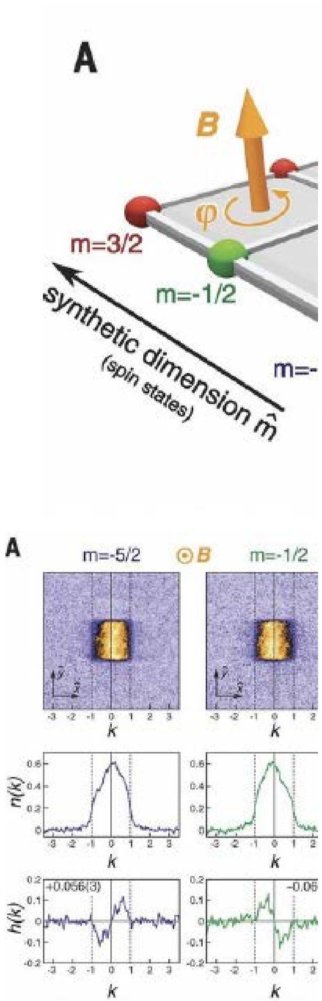
Non interacting ``simple"

$$R_h = \frac{V_\perp}{I_\parallel B} \propto \frac{1}{n}$$

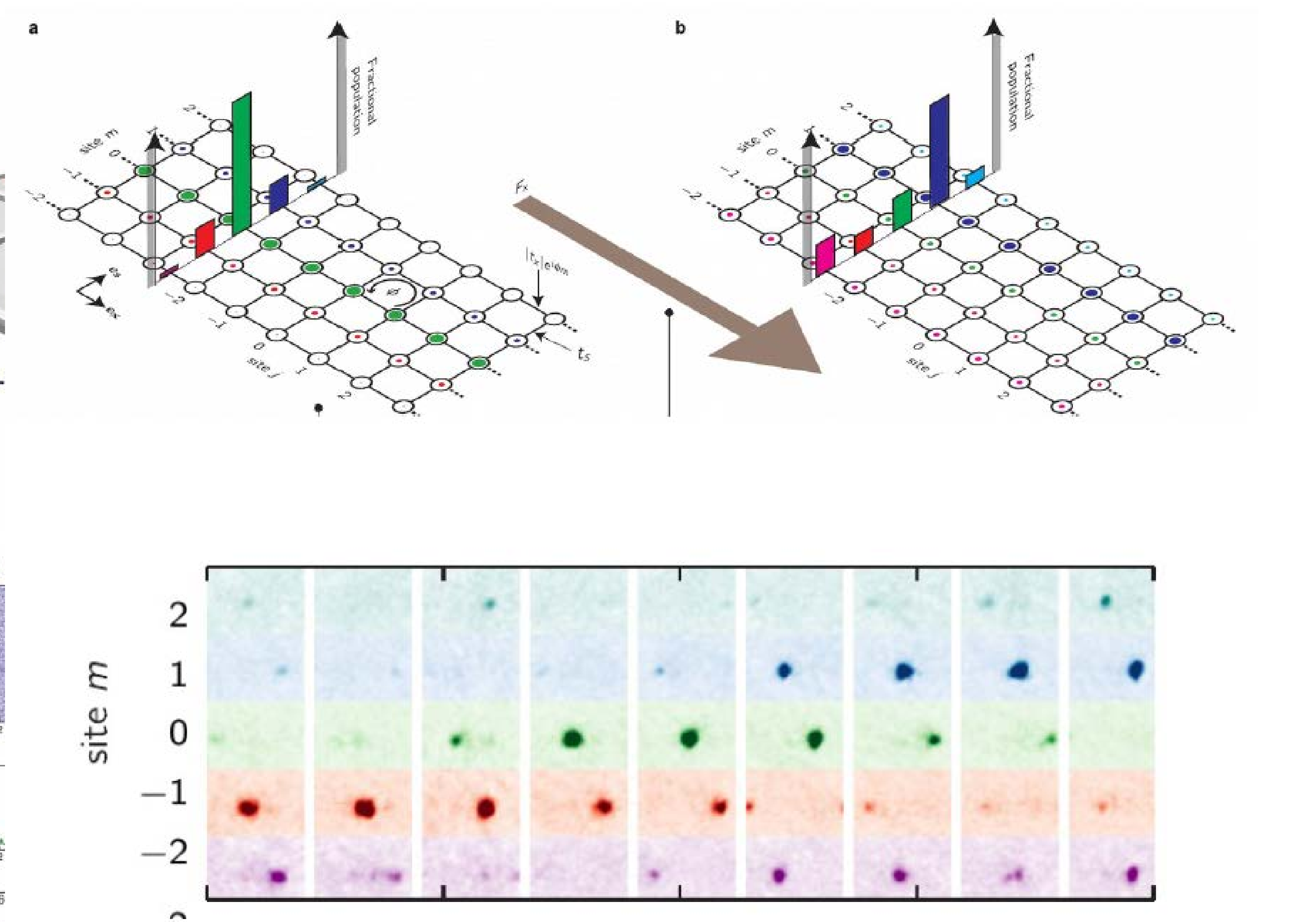
No interactions: curvature of fermi surface - topological formula (Thouless-Kohmoto)

With interactions: open question

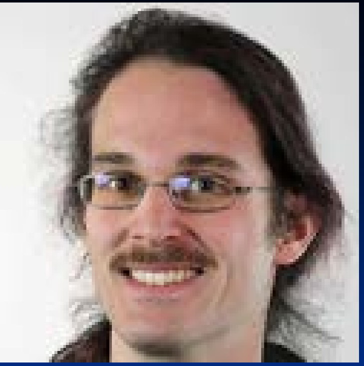
Hall effect: measurements



Fallani's group (Florence) **Scien**

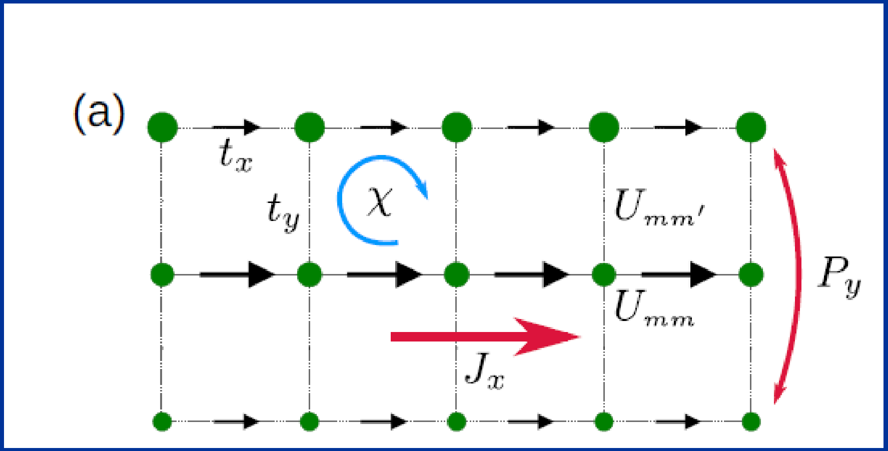


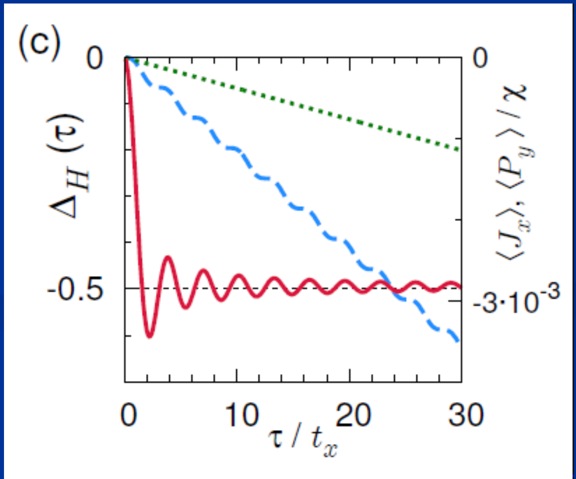
Spielman's group (Maryland) **arXiv:1804.06345**

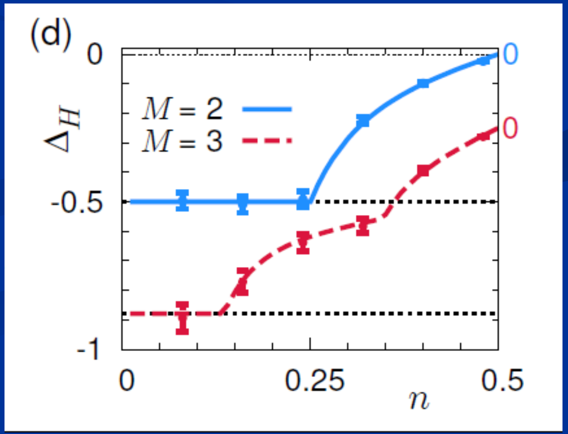


Hall effect on ladders

S. Greshner, M. Filippone, TG,
PRL 122, 083402 (19)





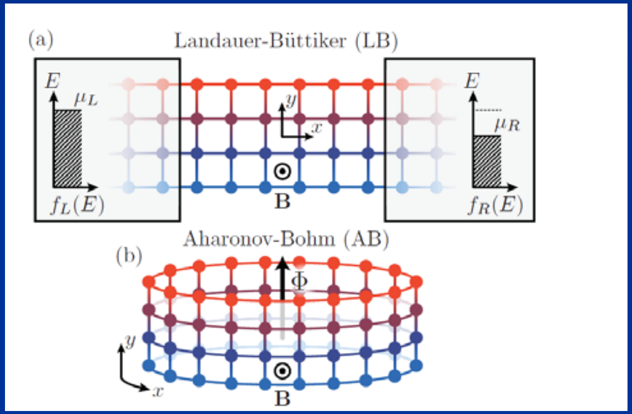


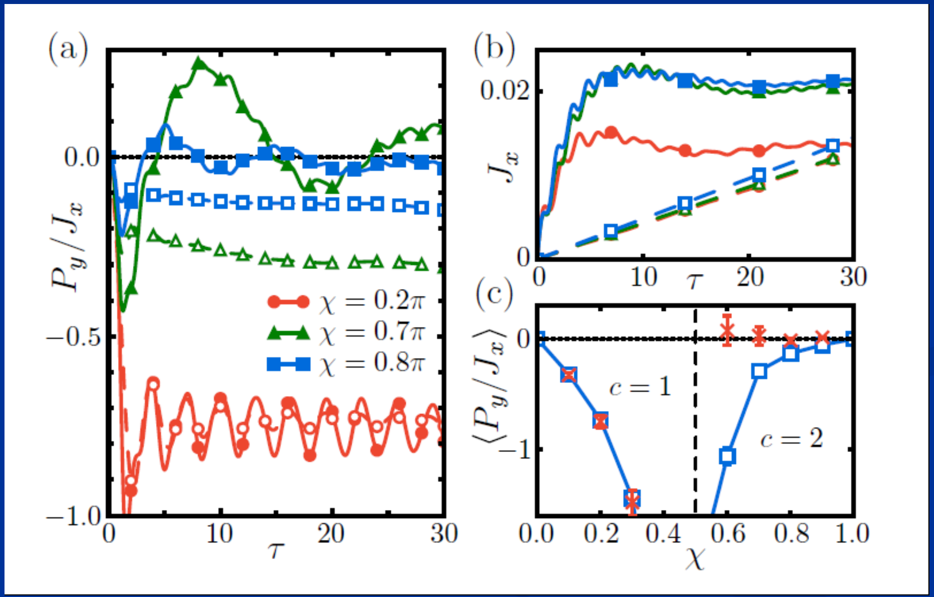
Analytic: calculation on a ring

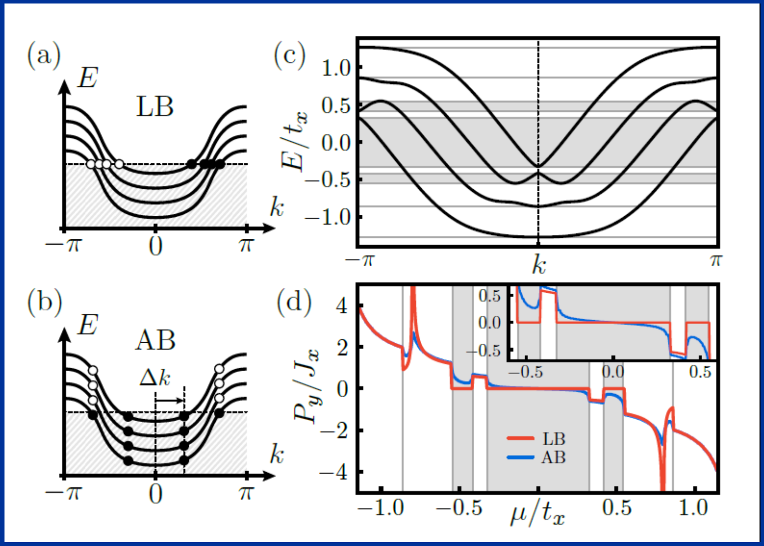
Experiments: out of equilibrium

Vanishing Hall response

M. Filippone, C.E. Bardyn, S. Greshner, TG, PRL 123, 086803 (19)



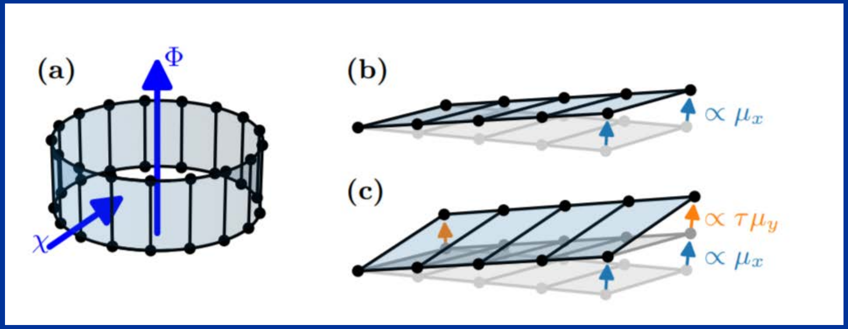




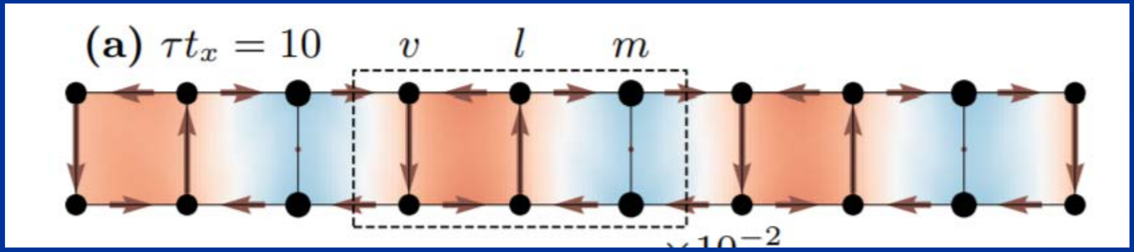
Exact zero of the Hall effect for LB geometry

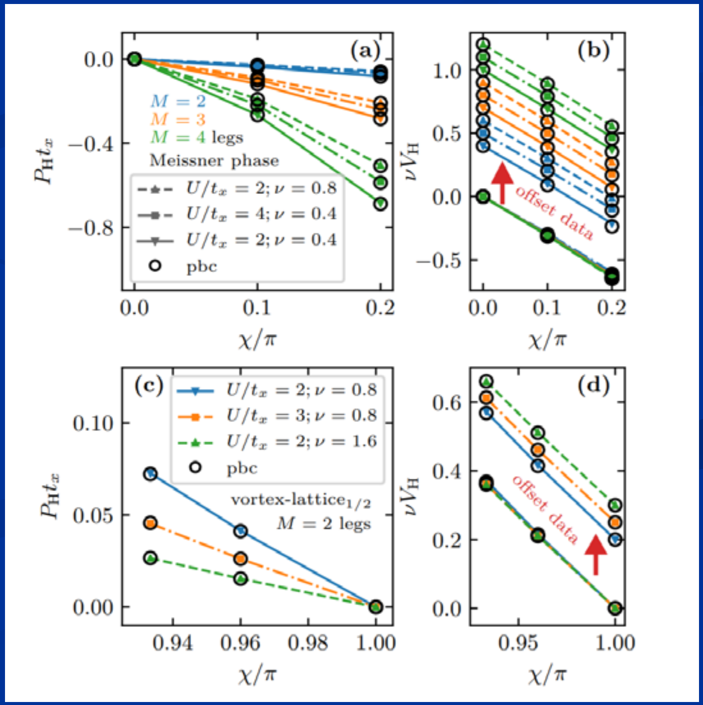
Hall votage vs polarization

M. Buser, S. Greshner, U. Schollwöck,
TG, PRL 126, 030501 (21)



Practical protocol to compute Hall **voltage** Vh from Hall **polarization** Ph

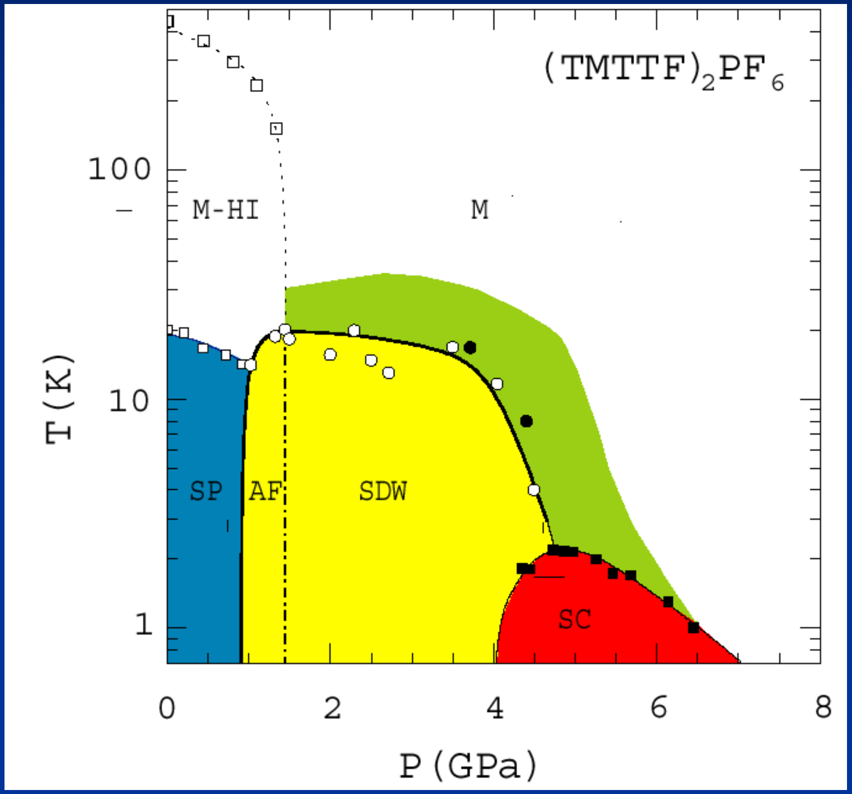


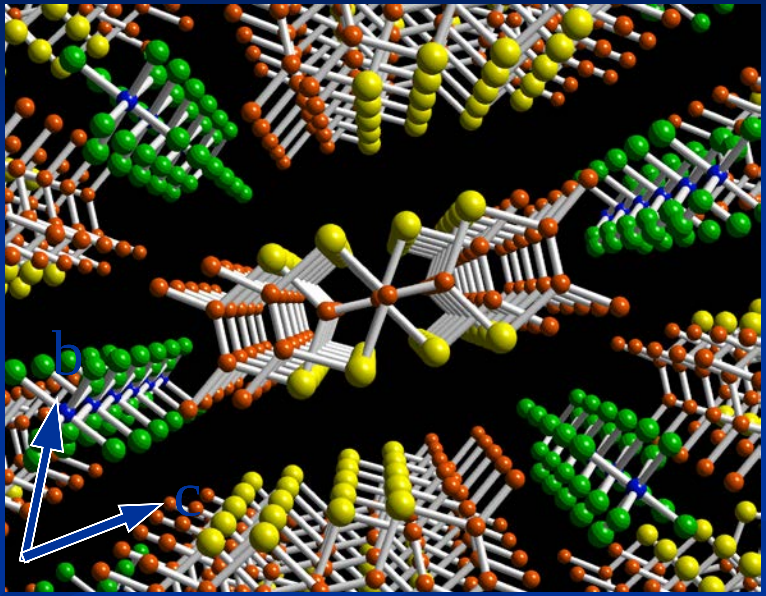


Dimensional crossover

- Largely open physics
- Strong difficulties to treat (analytics, numeric)
- Allows to incorporate SOC and magnetic field effects
- Many studies limited to non-interacting case
- Relevant for many experimental systems

Deconfinement

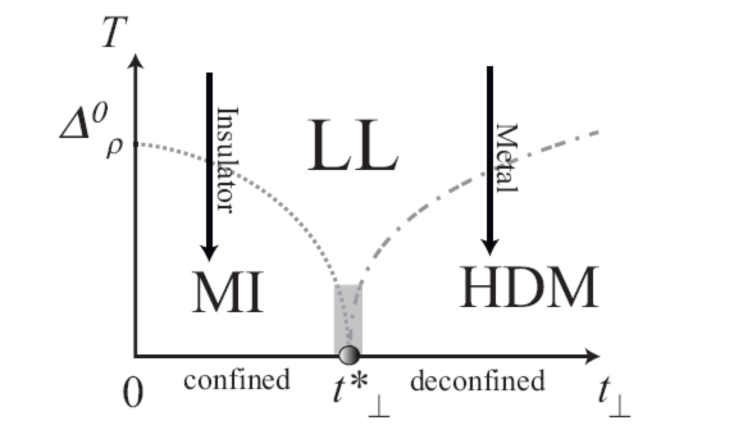


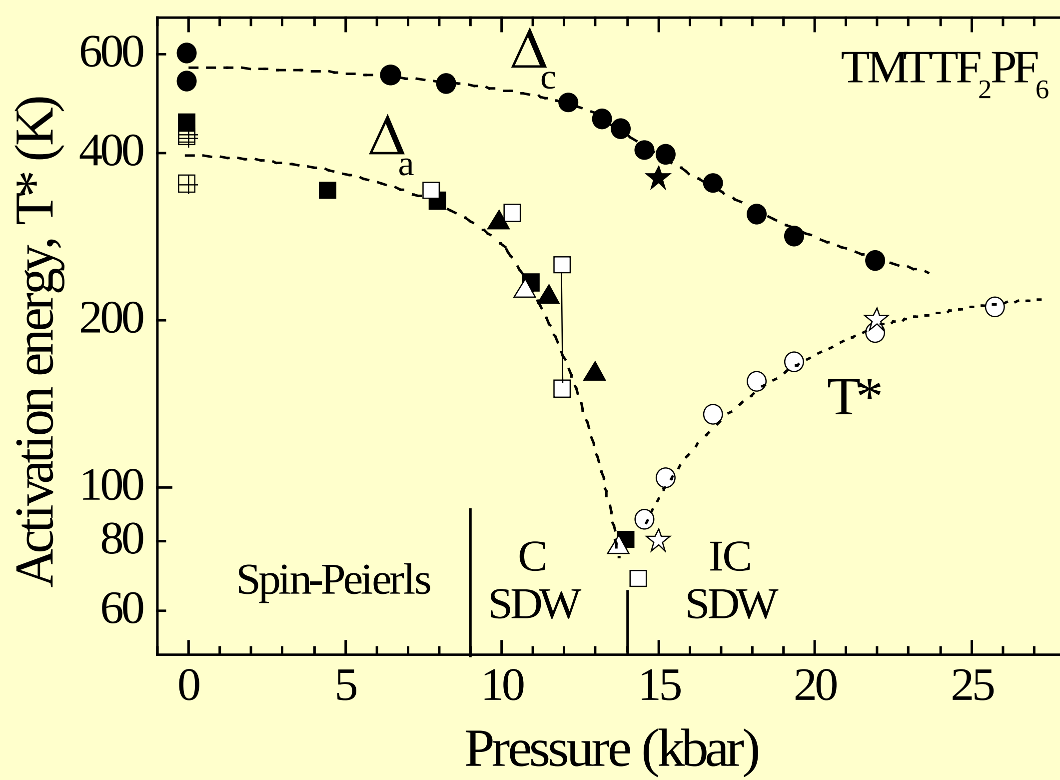


TG Chemical Review 104 5037 (2004)

D. Jaccard et al., J. Phys. C, 13 L89 (2001)

Deconfinement



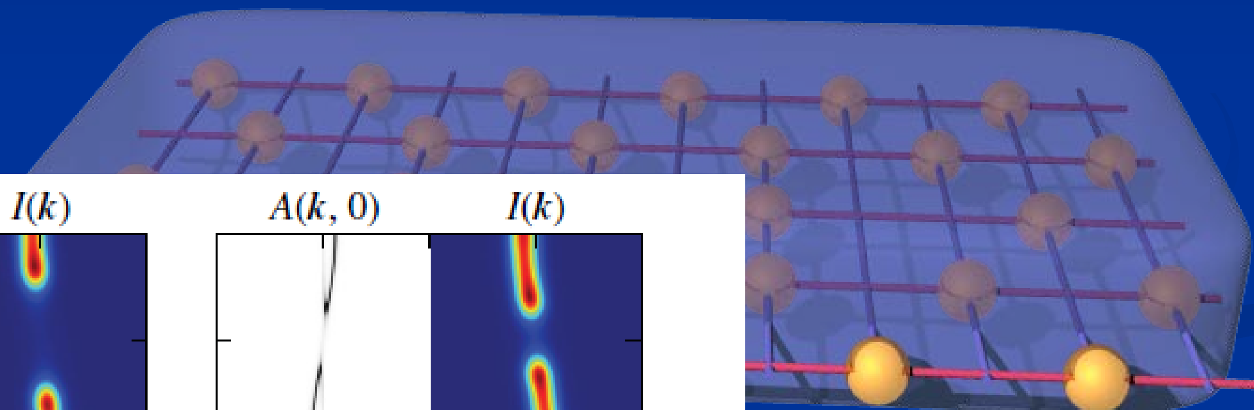


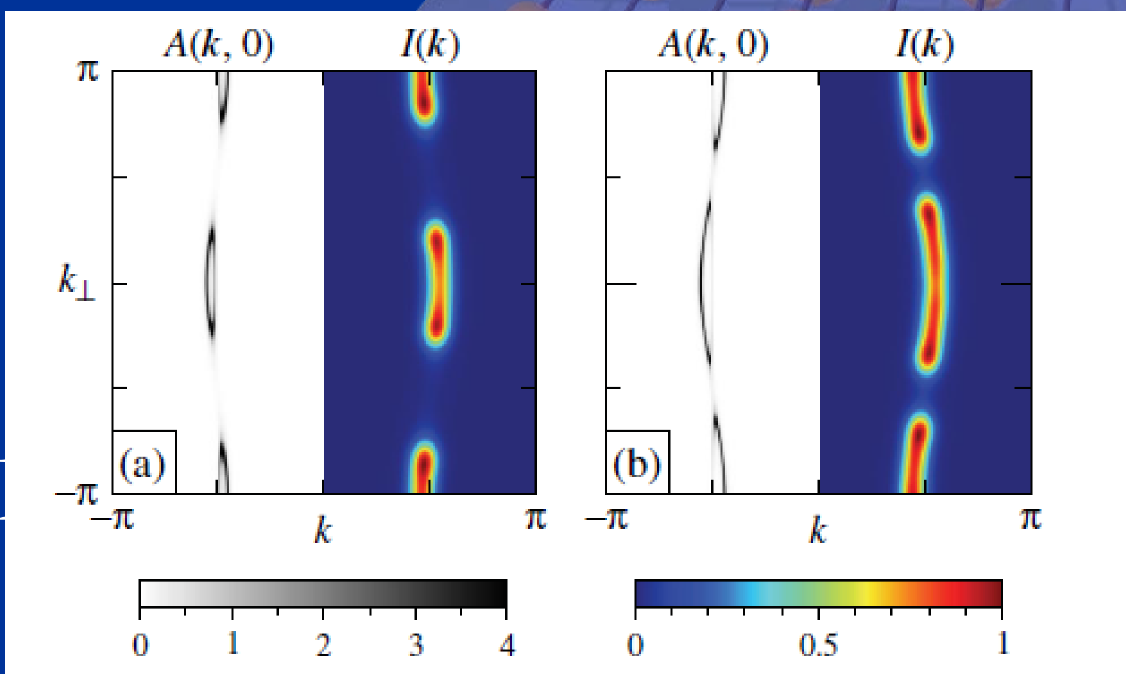
TG Chemical Review 104 5037 (2004)

P. Auban-Senzier, D. Jérome, C. Carcel and J.M. Fabre J de Physique IV, (2004)
A. Pashkin, M. Dressel, M. Hanfland, C. A. Kuntscher, PRB 81 125109 (2010)

Chain (Cluster) - DMFT

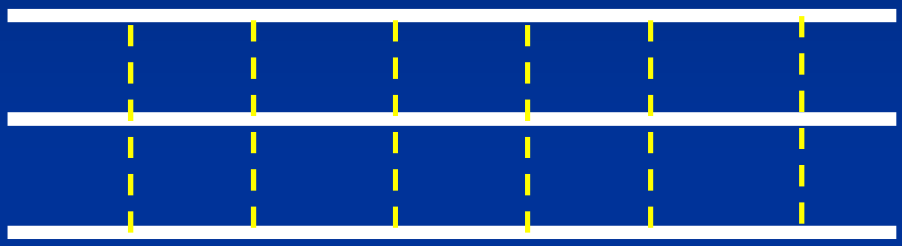
S. Biermann, A. Georges, A. Lichtenstein, TG, PRL 87 276405 (2001)
C. Berthod et al. PRL 97, 136401 (2006)





determined

Transverse transport



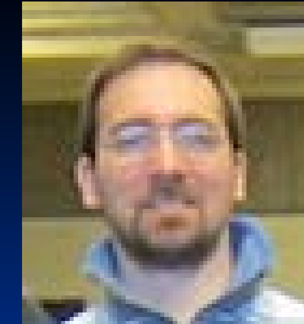
Perpendicular hopping t'

T > t': tunneling, not usual transport

$$\sigma(\omega,T) \propto (\omega,T)^{2\alpha-1}$$

$$\alpha = \tfrac{1}{4}(K + K^{-1}) - \tfrac{1}{2}$$

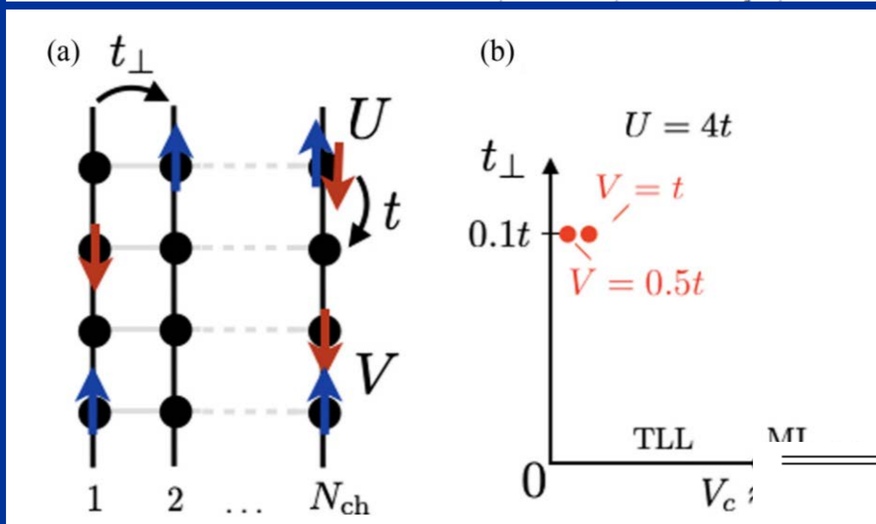
DMRG



PHYSICAL REVIEW B **100**, 075138 (2019)

Understanding repulsively mediated superconductivity of correlated electrons via massively parallel density matrix renormalization group

A. Kantian,[1] M. Dolfi,[2,3,*] M. Troyer,[2] and T. Giamarchi[4]



	$\overline{\Delta}_s[N_{ch}]$ $[t \times 10^{-3}]$			
	Straight extrapolation		Extr. + estimates	
N_{ch}	$V/t = 0.5$	$V/t = 1$	$V/t = 0.5$	$V/t = 1$
2	5.59	7.59	5.36	7.31
4	9.68	10.2	10.77(10)[a]	10.268(11)
6	9.37	11.4	4.3(1.4)[b]	11.20(97)
8	9.45		7.6(1.3)[c]	

Conclusions

- Tour of one dimensional physics
- Luttinger liquid theory provides a framework to study this physics, and to go beyond
- Beautiful open problems: out of equilibrium, disorder, coupled chains, impurities, etc. ……
- Requires interplay of analytical and numerical techniques (and new ideas!) to make progress
- Remarkable and complementary realization in condensed matter and cold atomic gases