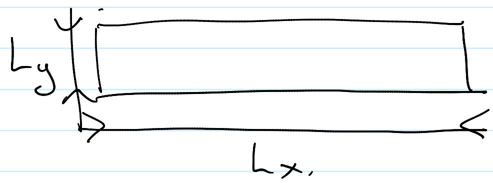
Monday, 26 July 2021 17:14

I.] What is a 1D System:

Confinement.

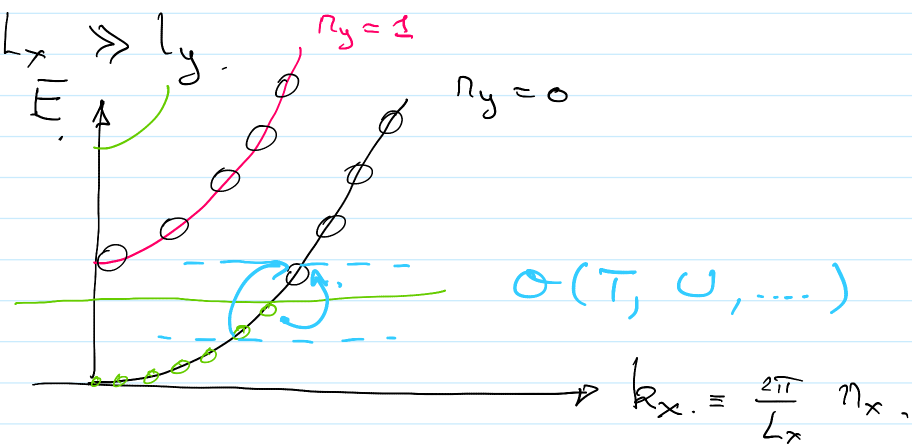


$$k_x = \frac{2\pi}{L_x} n_x \qquad k_y = \frac{2\pi}{L_y} n_y .$$

$$E = \frac{k_x^2}{2m} + \frac{k_y^2}{2m}$$

$$= \frac{(2\pi)^2}{L_x^2} \frac{1}{2m} n_x^2 + \frac{(2\pi)^2}{L_y^2} \frac{1}{2m} n_y^2 .$$

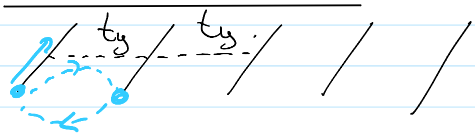
$L_x \gg l_y$.



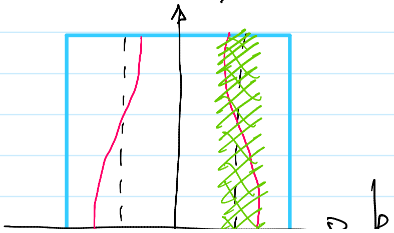
Scattering, thermal etc n_y is constant.

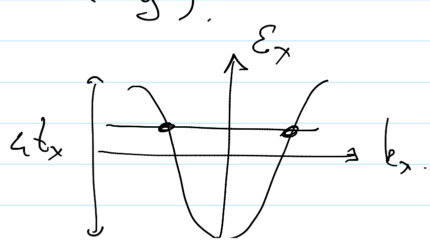
$$\psi(x,y) = \psi(x) \underbrace{\boxed{\phi(y)}}_{n_y = 0}$$

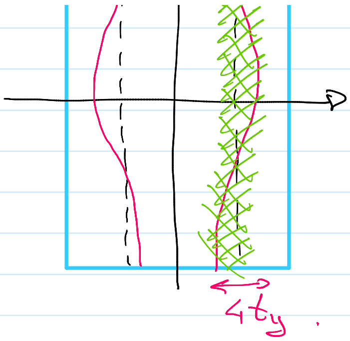
Incoherence:

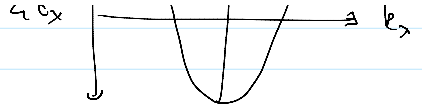


$$E = \varepsilon_x - 2t_y \cos(k_y) . \qquad E_F .$$





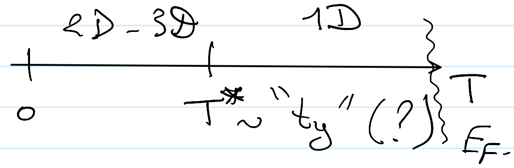




$T \gg 4t_y$.

Cannot jump coherently from one chain to the next if

$T \gg t_y \Rightarrow 1D$

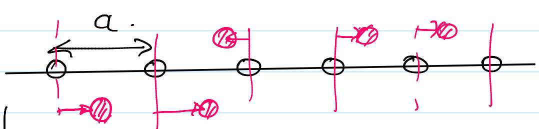


II.] How to describe:

bosons + contact.

$$H = \int dx \, \frac{\nabla\psi^\dagger \nabla\psi}{2m} + g \cdot \int dx \, (\underbrace{\rho(x)}_{\psi^\dagger(x)\psi(x)})^2$$

▶ Collective coordinates

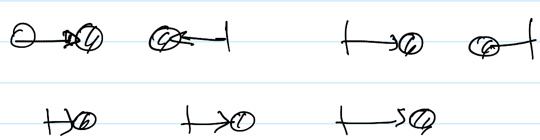


u_i $\quad R_i^0 \quad x = R_i^0 + u_i$

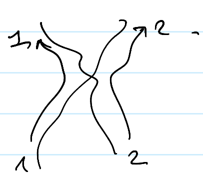
↓ Long wavelength theory for the displacements.

u_i

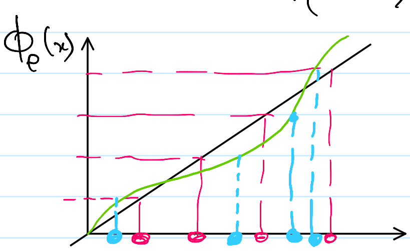
$u(x)$



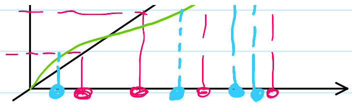
\# Number particles from left to right.



Field. $\phi_e(x) = 2\pi n_i$ at the position of particle i



$$\phi_e(x) = 2\pi\rho_0 x - 2\phi(x)$$



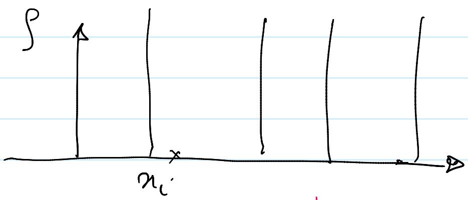
$$\rho(x) = \sum_i \delta(x - x_i) . \quad \text{operator giving the position of particle } i$$

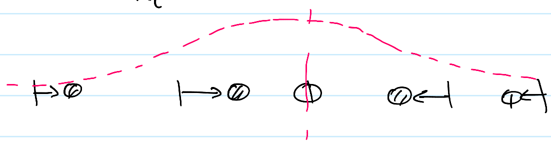
$$= \sum_n |\nabla \phi_e(x)| \, \delta(\phi_e(x) - 2\pi n)$$

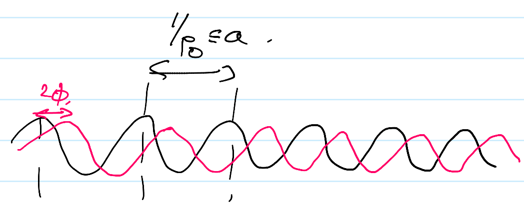
$$\delta(f(x)) \equiv \frac{1}{|f'(x_i)|} \delta(x - x_i).$$

x_i zeros. $f(x_i) = 0$

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi} \nabla \phi(x)\right] \sum_p e^{i 2p(\pi \rho_0 x - 2\phi(x))} .$$







$\cos(2(\pi\rho_0 x - 2\phi(x)))$

ϕ is ordered density is ordered.

$\psi(x) = [\rho(x)]^{1/2} e^{i\Theta(x)}$. : Θ operator giving the phase.

Hamiltonian:

$$\int dx \, \frac{\nabla\psi^\dagger \nabla\psi}{2m} \equiv \frac{\rho_0}{2m} \int dx \, (\nabla\Theta)^2 .$$

$$g \int dx \, (\rho(x) - \rho_0)^2 = g \int dx \, \frac{1}{\pi^2} (\nabla\phi)^2 + \text{oscillating} .$$

$$\begin{cases} H = \left[\underbrace{\frac{\rho_0}{2m} \int dx \, (\nabla\Theta)^2}_{uK.} + \underbrace{\frac{g}{\pi^2}}_{u/K.} (\nabla\phi)^2\right] \\ \left[\frac{1}{\pi} \nabla\phi_x , \Theta_{x'}\right] = -i \delta(x - x') \end{cases}$$

Fully solved the Lieb-Lininger model !

u : dimension of a velocity.

K : dimensionless.

$$\sum_k \frac{1}{K}\left[\frac{1}{u}\omega^2 + u k^2\right]\phi_k^*\phi_k \equiv S$$

u : Sound velocity.

$$\begin{cases} uK \equiv \dfrac{\rho_0}{2m}. & K \sim 1/\sqrt{g}. \\ \dfrac{u}{K} \sim g. & u \sim \sqrt{g}. \end{cases}$$

g small.

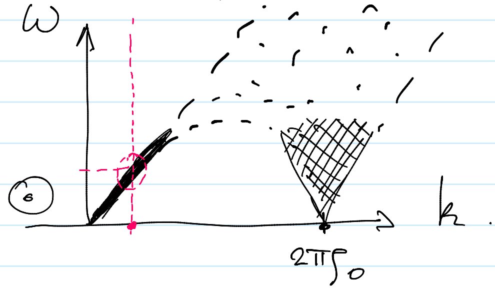
- Linear Spectrum $\omega \sim uk$.

Correlations :

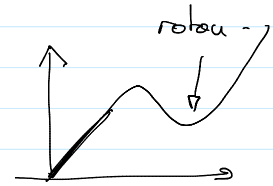
$$G = \langle \delta\rho(x,t)\,\delta\rho(0,0)\rangle \sim \text{"}\frac{1}{(x-vt)^2}\text{"} + \cos(2\pi\rho_0 x)\left(\frac{1}{r}\right)^{2K}.$$

$(\rho - \rho_0)$

$G(\omega, k)$.



$\omega \simeq uk$.



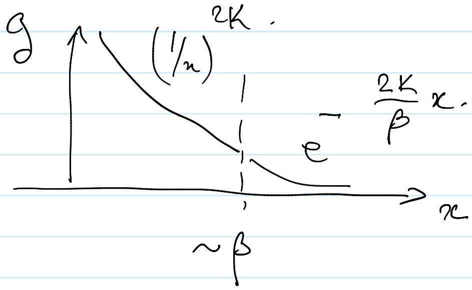
$\gamma = \infty$ Tonks limit. hard core bosons.

- finite temperature.

$$\langle e^{i2\phi(x)}\, e^{-i2\phi(0)}\rangle \simeq e^{-2K\,\mathrm{Log}[x]}$$

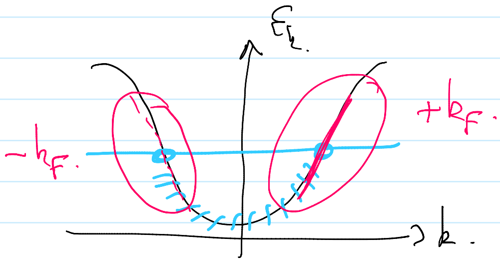
$\delta\rho(x)\quad \delta\rho(0)$

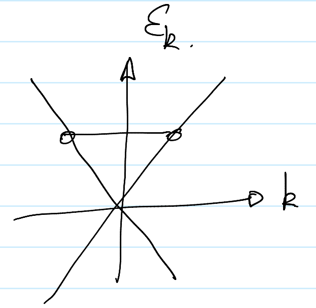
$$e^{-2K\,\mathrm{Log}[\beta \sinh(x/\beta)]}$$



$\sim \beta$

Fermions;





III.] Tomonaga-Luttinger liquids.

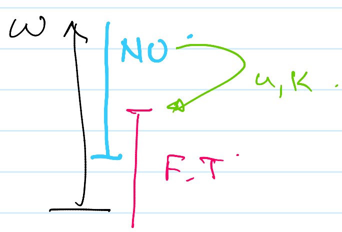
form of H and the operators is exact.

u and K must be determined from the microscopic model.
[Amplitudes]

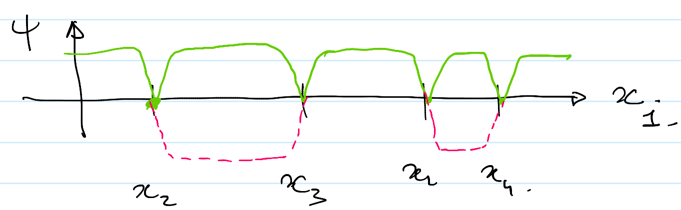
$\cancel{K \sim 1/\sqrt{g}}$.

- Short range / Short time $\Rightarrow$ use numerics. (or BA, ...)
- Use the exact solutions [BA, numerics, etc] to compute the exact values of u an K [Amplitudes]

↳ exact field theory.



Tonks limit: hard core bosons.



$\rightarrow$ Free fermion problem.

$\psi_B(x_1 \ldots x_N)$. $\psi_F(x_1 \ldots x_N)$

$|\psi|^2$. $\rho(x)$ $j(x)$ .... Same fermions and bosons.

$$\langle \psi_B(x)\, \psi_B^\dagger(0) \rangle \neq \langle \psi_F(x)\, \psi_F^\dagger(0) \rangle$$

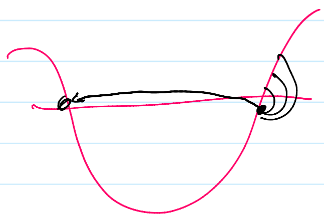
$(1/x)^{\frac{1}{2K}}$.

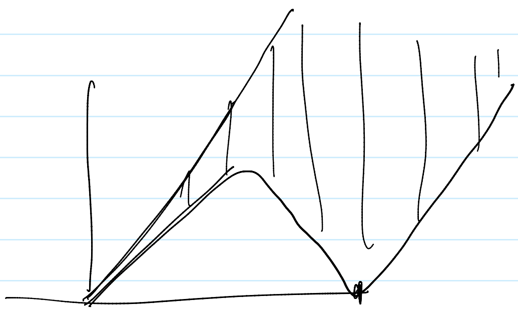
$(\;)^{2}$. $(\;)^{2K}$.

$\left(1/x\right)^{2K}$.

$\langle \delta\rho_F(x)\,\delta\rho_F(0)\rangle \sim \cos(2k_F x)\left(\frac{1}{x}\right)^{2}$. $\left(\frac{1}{x}\right)^{2K}$.

$K[g=\infty]=1$.





$2k_F$.