Outline — Lecture 1

Introduction

1. **Many-Body Localisation in 1D & 2D**
   - Probing MBL transition CDW & Domain Wall Dynamics
2. **2D MBL with Coupling to a Finite Bath**
   - CDW Dynamics in the presence of a finite bath
3. **Probing Entanglement Dynamics in MBL**
4. **Evidence for (Avalanche) Instabilities**

Outline — Lecture 2

1. **Probing Thermalization Dynamics in QMB**
2. **Thermalization to Negative Absolute Temperature**

Non-Equilibrium Dynamics

3. **Lieb Robinson Bounds**
4. **From Hubbard to Spin Dynamics**

Outline — Lecture 3

1. **Bound Magnons**
2. **Spin-Charge Fractionalization in Fermi Hubbard Chains**
3. **Connection to Ground State Non-Local Order**
4. **Kardar-Parisi-Zhang Universality**

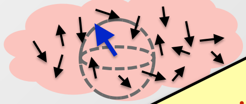
MBL — Motivation

Thermalization

Quantum correlations in local d.o.f are rapidly lost as these get entangled with the rest of the system.

Classical hydro description of remaining slow modes (conserved quantities, and order parameters).

Many-body localization



Local quantum i... persists ind...

Statistical Physics Fails!

Ne... y **quantum** description of the long time dynamics!

?

The many-body localization transition = elusive interface between quantum and classical worlds

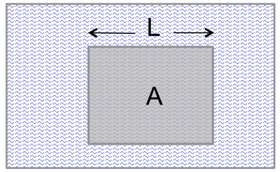
Gornyi et al. Phys. Rev. Lett. 2005
Basko et al. Ann. of Physics 2006
Nandkishore et al., Annu. Rev. Cond. Mat. 2015
Altman et al. Annu. Rev. Cond. Mat. 2015

MBL — Eigenstate Thermalisation Hypothesis

Deutsch (91), Srednicki (94,98), Rigol, Dunjko & Olshanii (2009), D'Alessio, Kafri, Polkovnikov, Rigol, Adv. Phys. **65**, 239 (2016)

$$\rho_A = \frac{1}{Z_A} e^{-\beta H_A}$$

$$S_A \equiv \mathrm{tr}\left[\rho_A \ln \rho_A\right] \propto L^d$$



Are there scenarios when this fails?

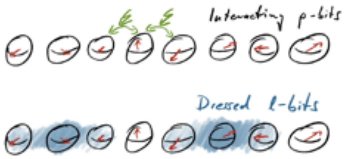
System fails to act as its own heat bath!

Nandkishore et al., Annu. Rev. Cond. Mat. 2015; Altman et al. Annu. Rev. Cond. Mat. 2015,

MBL — Generic Failure of Thermalization

Generic: Disorder + Interactions + High Energy Density

$$\hat{H} = \sum_i h_i \hat{S}_i^z + \sum_{i,j} J_{i,j} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$$



$$\hat{H}_{\text{l-bit}} = \sum_i \hat{T}_i^z + \sum_{i,j} \tilde{J}_{i,j} \hat{T}_i^z \hat{T}_j^z + \sum_{n=1}^{\infty} \sum_{i,j,\{k\}} K_{i\{k\}j}^{(n)} \hat{T}_i^z \hat{T}_{k_1}^z \ldots \hat{T}_{k_n}^z \hat{T}_j^z$$

"l-bits" are **quasi-local integrals of motion**

J.Z. Imbrie, Jour. Stat. Phys. 163:998-1048 (2016)
"l-bits": Serbyn, PRL 2013 | Huse, PRB 2014 | Nandkinshore, Annu. Rev. Cond. Mat. 2015

Approaching Many-Body Localization from Disordered Luttinger Liquids
C. Karrasch, J. E. Moore
Subjects: Strongly Correlated Electrons (cond-mat.str-el)
28. arXiv:1506.00592 [pdf, other]
Protection of topological order by symmetry and many-body
Andrew C. Potter, Ashvin Vishwanath
Comments: 17 pages, 4 figures
Subjects: Disordered Systems and Neural Networks (cond-m
29. arXiv:1505.07089 [pdf, other]
Dynamics of many-body localisation in a translation invarian
Merlijn van Horssen, Emanuele Levi, Juan P. Garrahan
Comments: 5 pages, 4 figures
Subjects: Statistical Mechanics (cond-mat.stat-mech); Quan
30. arXiv:1505.06343 [pdf, ps, other]
Many-body ground state localization and coexistence of loca
Yucheng Wang, Haiping Hu, Shu Chen
Comments: 5 pages, 6 figures
Subjects: Disordered Systems and Neural Networks (cond-m
31. arXiv:1505.05386 [pdf, other]
Revisiting Many-body Localization with Random Networks o
Benoît Descamps, Frank Verstraete
Comments: 3 figures
Subjects: Quantum Physics (quant-ph)
32. arXiv:1505.05147 [pdf, other]
Many-Body Localization of Symmetry Protected Topological
Kevin Slagle, Zhen Bi, Yi-Zhuang You, Cenke Xu
Comments: 5 pages 2 figures
Subjects: Strongly Correlated Electrons (cond-mat.str-el)
33. arXiv:1505.02028 [pdf, other]
Out-of-equilibrium states and quasi-many-body localization i
L. Barbiero, C. Menotti, A. Recati, L. Santos
Comments: 5 pages, 4 figures
Subjects: Quantum Gases (cond-mat.quant-gas)
34. arXiv:1504.06872 [pdf, other]
Total correlations of the diagonal ensemble herald the many-body localization transition
J. Goold, C. Gogolin, S. R. Clark, J. Eisert, A. Scardicchio, A. Silva
Comm...
Subj... ysics (quant-ph)
35. a...
Many... mobility edge
Xiao...
Com...
Subj... t.quant-gas); Quantum Physics (quant-ph)
36. ar...
Many body localization in the presence of a single particle mobility edge
Ranjan Modak, Subroto Mukerjee
Comments: 5 pages, 6 figures
Subjects: Disordered Systems and Neural Networks (cond-mat.dis-nn); Statistical Mechanics (cond-mat.stat-mech); Strongly Correlated Electrons (cond-mat.str-el)
37. arXiv:1503.06508 [pdf, ps, other]
Localization in a random $x-y$ model with the long-range interaction: Intermediate case between single particle and many-body problems
Alexander L. Burin
Comments: Modified version after review
Subjects: Disordered Systems and Neural Networks (cond-mat.dis-nn)
38. arXiv:1503.06147 [pdf, other]
Many-body localization characterized from a one-particle perspective

Pioneering work:
I.V. Gornyi, A. D. Mirlin, and D. G. Polyakov, PRL (2005).
D. M. Basko, I. L. Aleiner, B. L. Altschuler, Ann. Phys. (2006).

Review/intro:
D.A. Huse, R. Nandkishore, V. Oganesyan,
Annu. Rev. Cond. Mat. **6**, 15 (2015)

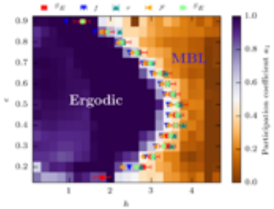
R. Vosk & E. Altman,
Annu. Rev. Cond. Mat. **6**, 383 (2015)

D. Abanin, E. Altman, I.B., M. Serbyn (2019)
Rev. Mod. Phys. **91**, 021001 (2019)

Experiments: Cold Atoms, Ions, NV Centers, Electronic Systems…

A New Type of Phase Transition

The toy model of MBL: $\hat{H} = \sum_i h_i \hat{S}_i^z + \sum_{ij} J_{ij} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$



Numerics from Luitz et al. PRB 2015

Ergodic - MBL transition not visible in thermodynamic (equilibrium) quantities

Properties of the many-body eigenstates change (area law entanglement)

Experiment: Need to probe the dynamics at high energy density

Early work: Altschuler et al, Ann. Phys. 2006
Oganesyan + Huse 2007 | Znidaric et al, PRB 2007 | Pal + Huse 2010

Interesting Questions Connected to MBL

- Nature of the phase transition (universality, diverging scales, rare regions …)
  Pal + Huse, PRB 2010 | Agarwal, PRL 2015 | Potter PRX 2015 | Vosk, PRX 2015 | Luitz, PRB 2016 …
- Entanglement dynamics in the MBL phase
  Žnidarič, PRB 2008 | Bardarson, PRL 2012 | Serbyn, PRL 2013 | Vosk, PRL 2013 | Nanduri PRB 2014 ...
- Local integrals of motion
  Serbyn, PRL 2013 | Huse, PRB 2014 | Chandran, PRB 2015 | Ros, Nucl. Phys. B 2015 …
- Stability to environmental couplings
  Nandkishore, PRB 2014 | Huse, PRB 2015 | Johri, PRL 2015 | Levi, PRL 2016 | Fischer, PRL 2016 | Luitz, PRL 2017 ...
- Coupling to small "baths"
  Nandkishore, PRB 2015 | Hyatt, PRB 2017
- Extensions of MBL to Floquet systems (time crystals, SPT phases)
  Ponte, PRL 2015 | Else, PRB 2016 | von Keyserlingk, PRB 2016 | Khemani, PRL 2016 | Yao, PRL 2017 ...

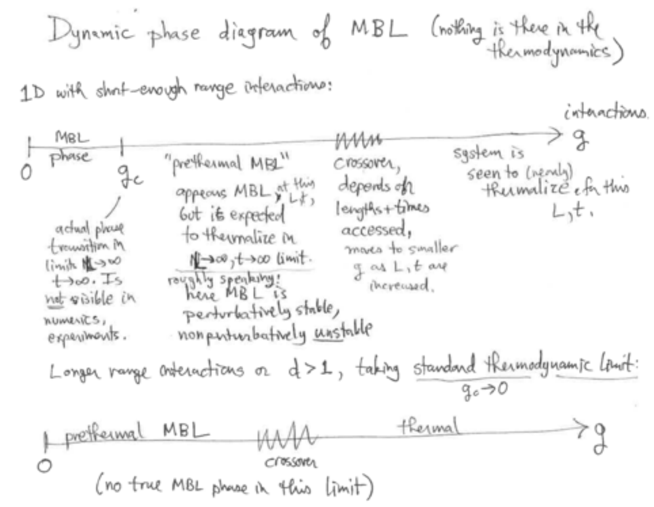
Important Points

Very little theoretically known about MBL in d>1
(**stability of MBL in d>1 unclear**)

Calls for particularly precise characterization of the experiments
(validation through a quantum simulator)

Experiments (almost) isolated from environment
but **small residual coupling** limits observation time (>1000 t)

Phase Diagram - D. Huse Lecture (?)

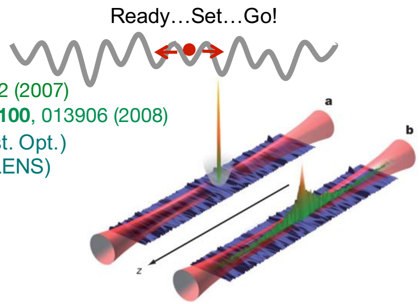


A. Morningstar et al. arXiv 2107.05642

MBL

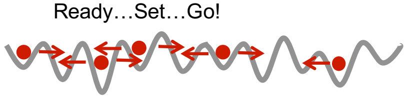
Measuring Localisation

Anderson localization:

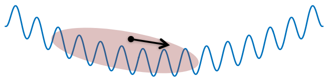


T. Schwartz et al. Nature **446**, 52 (2007)
Y. Lahini, et al. Phys. Rev. Lett. **100**, 013906 (2008)
J. Billy et. al. Nature 2008 (Inst. Opt.)
G. Roati et. al. Nature 2008 (LENS)

Many-body localization:



Fastest timescale: local probe!



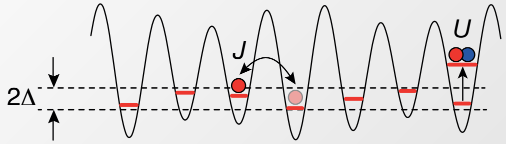
Slowest timescale: global probe

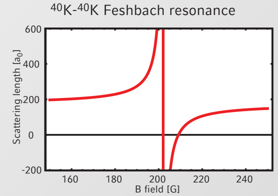
Kondov et al. (DeMarco) Phys. Rev. Lett. 114, 083002 (2015)

MBL

1D Quasi-Disordered Fermi-Hubbard

$$H = -J\sum_{i,\sigma}\left(\hat{c}^\dagger_{i,\sigma}\hat{c}_{i+1,\sigma} + H.c.\right) + \Delta\sum_{i,\sigma} sin\left(2\pi\alpha i + \phi\right)\hat{c}^\dagger_{i,\sigma}\hat{c}_{i,\sigma} + U\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$





Without interactions U=0 : Aubry-André model

- Homogenous tunneling but quasi-random onsite energies
- α is the incommensurability ratio, irrational, in the experiment ≈ 0.721

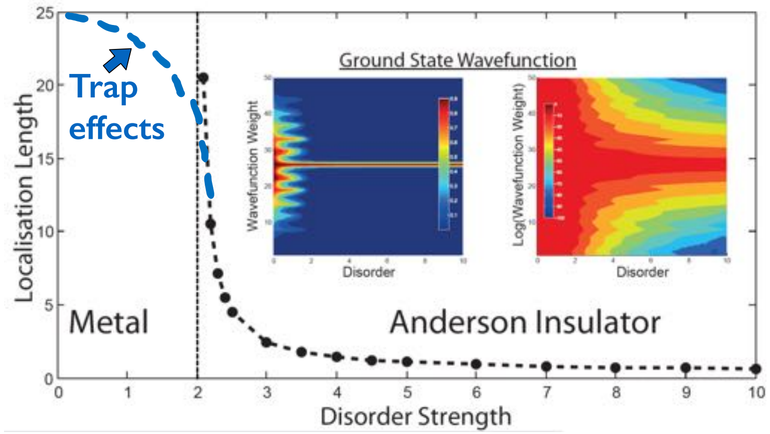
All eigenstates extended for Δ/J < 2

All eigenstates exponentially localised for Δ/J > 2

G. Roati et al. Nature (2008)

MBL

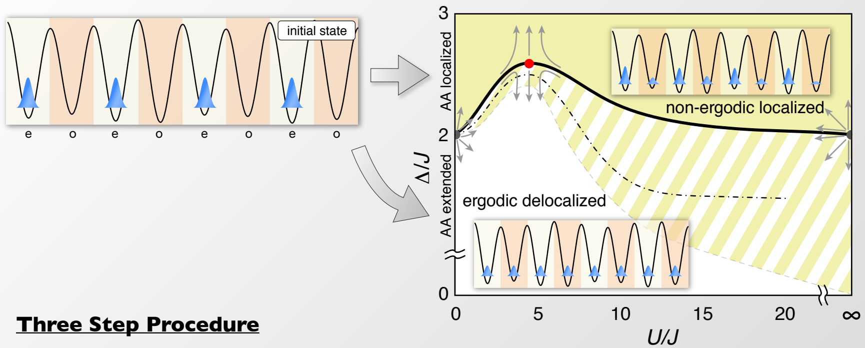
Single Particle Orbitals



$$\xi_{sp} = \ln^{-1}(\Delta/2J)$$

MBL

Probing the Interacting Aubry-André Model



Three Step Procedure

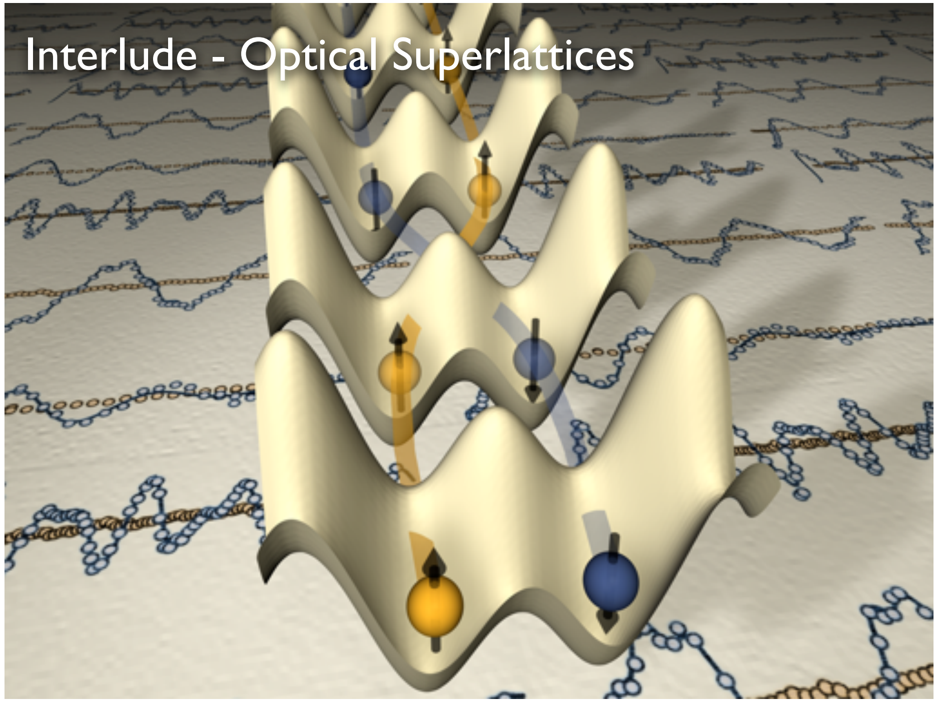
1) Prepare CDW (with different doublon densities)
2) Evolve in disorder
3) Readout CDW (disorder averaged)

Main Observable: **Imbalance** $I = \dfrac{N_e - N_o}{N_e + N_o}$

Hamming Distance

$D(t) = \frac{N}{2}\left[1 - I(t)\right]$

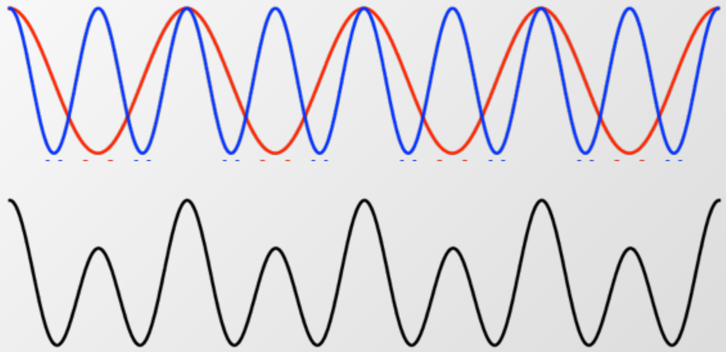
(see P. Hauke & M. Heyl, PRB 2015)



Optical Superlattices

Superimpose two standing waves **with controllable phase & amplitude.**

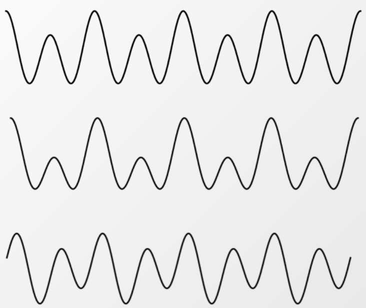
1530 nm + 765 nm



Array of double wells

Note: two non-equivalent sites in unit cell!

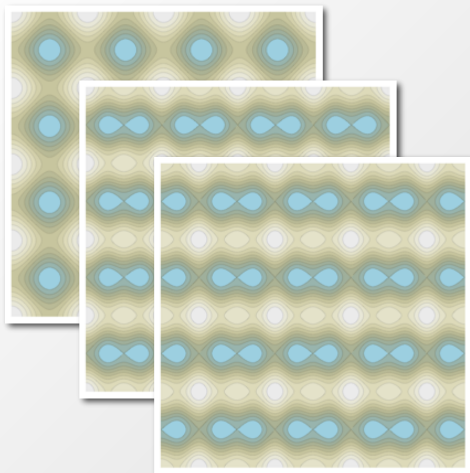
Controllable Parameters



- **Original**
- **Intra & Interwell Barrier Depth**
- **Potential Bias**

All parameters can be changed dynamically & in-situ!

2D Superlattice Geometries (1 SL)

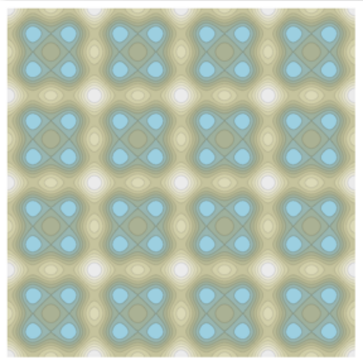


Controllable couplings and dynamics

Superlattice useful for controlling and detecting spin correlations

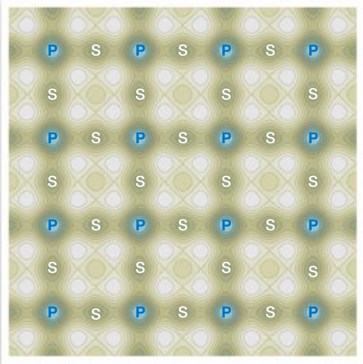
Optical Lattices

2D Superlattice Geometries (2 SL)



Coupled Plaquette Systems

see B. Paredes & I. Bloch, PRA **77**, 23603 (2008)
S. Trebst et al., PRL **96**, 250402 (2006)

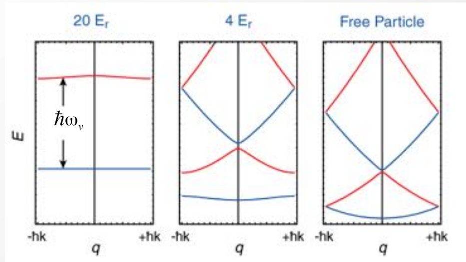


Higher Lattice Orbital Physics

see V. Liu, A. Ho, C. Wu and others work
exp: related to A. Hemmerich's exp.

Band Mapping

Mapping the Population of the Energy Bands onto the Brillouin Zones

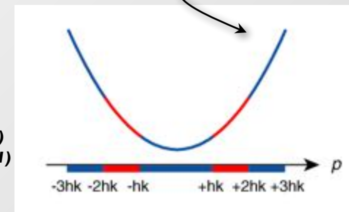


Crystal momentum

Crystal momentum is conserved while lowering the lattice depth adiabatically !

Population of n^{th} band is mapped onto n^{th} Brillouin zone !

A. Kastberg et al. PRL 74, 1542 (1995)
M. Greiner et al. PRL 87, 160405 (2001)

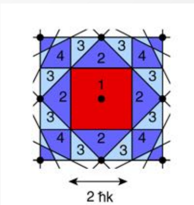


Free particle momentum

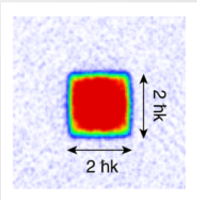
Band Mapping

Experimental Results

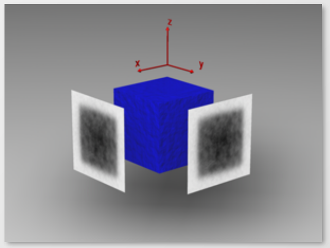
Brillouin Zones in 2D



Momentum distribution of a dephased condensate after turning off the lattice potential adiabtically



2D

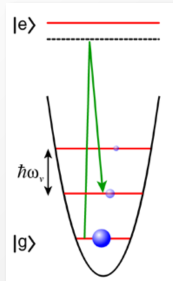


3D

Optical Lattices

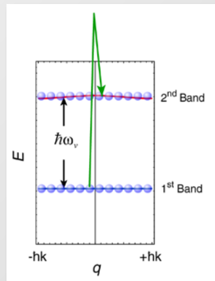
Populating Higher Energy Bands

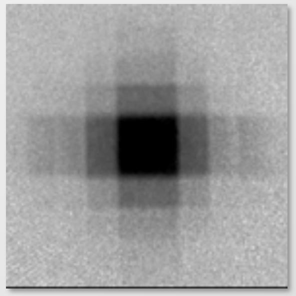
Single lattice site



Stimulated Raman transitions between vibrational levels are used to populate higher energy bands.

Energy bands



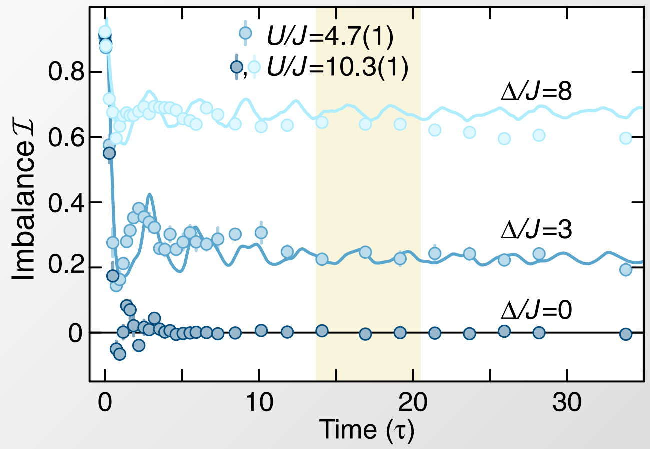


Measured Momentum Distribution !

End Interlude

MBL

Time Evolution



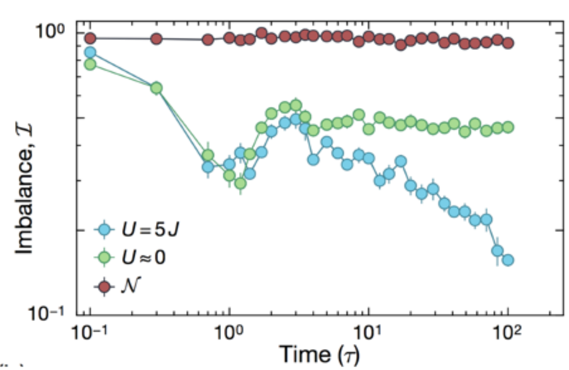
Non-ergodic, non-thermalizing quantum evolution !

M. Schreiber et al. Science **349**, 842 (2015)
P. Bordia et al. Phys. Rev. Lett. **116**, 140401 (2016)

Relaxation

2D Quasiperiodic

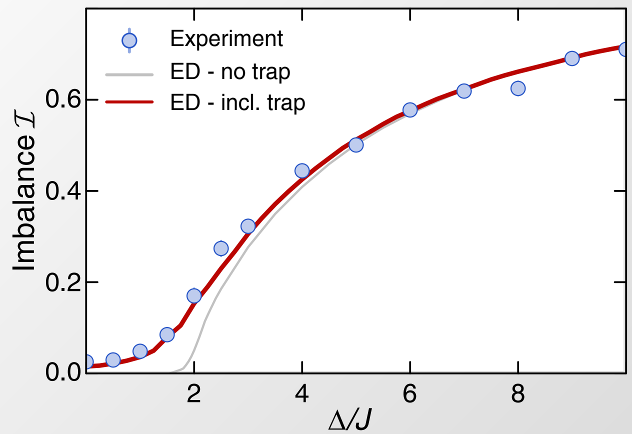
$\Delta = 5J$



Noninteracting localised - Interacting non-localised

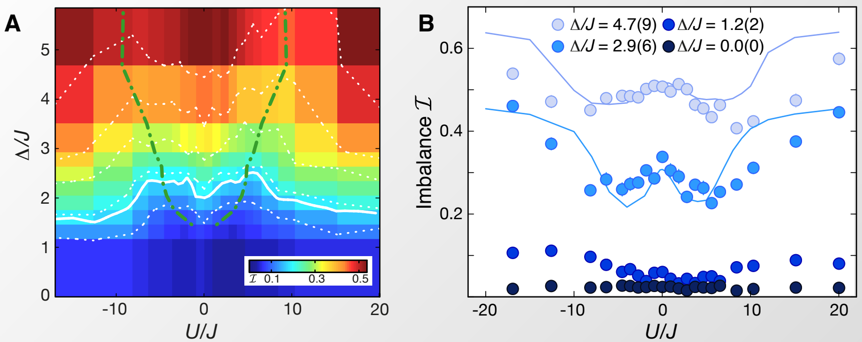
MBL

U=0 - Anderson Localization



MBL

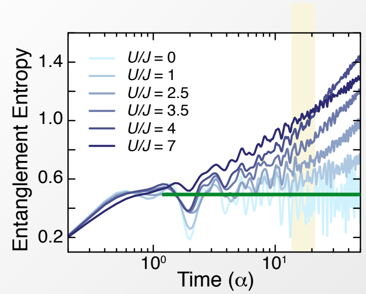
Imbalance vs U/J



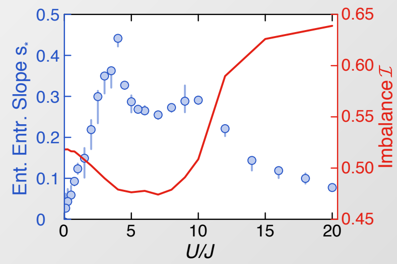
1) Localisation for all Interactions
2) Characteristic W-shape
3) Dynamical U vs -U symmetry

MBL

Numerics - Entanglement Entropy



Characteristic log(t) entanglement entropy growth



Maximum in entanglement entropy slope connected to minimum of imbalance

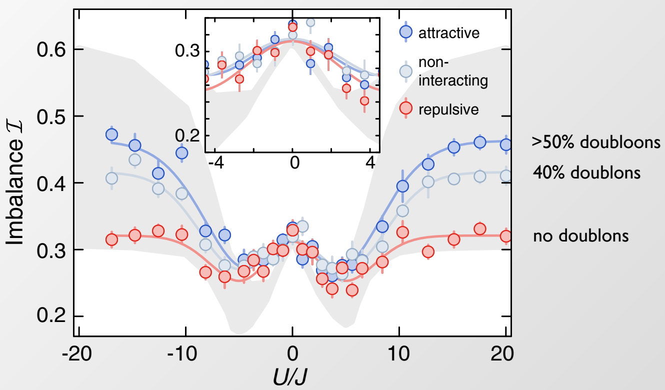
DMRG Simulations $\Delta/J = 5$

$$S = s_0 + s^* \log(t/\tau)$$

M Žnidarič, T Prosen, P Prelovšek, Phys. Rev. B (2008)
JH Bardarson, F Pollmann, JE Moore PRL (2012)
M Serbyn, Z Papic, DA Abanin PRL (2013)

MBL

Influence of Initial Doublon Fraction



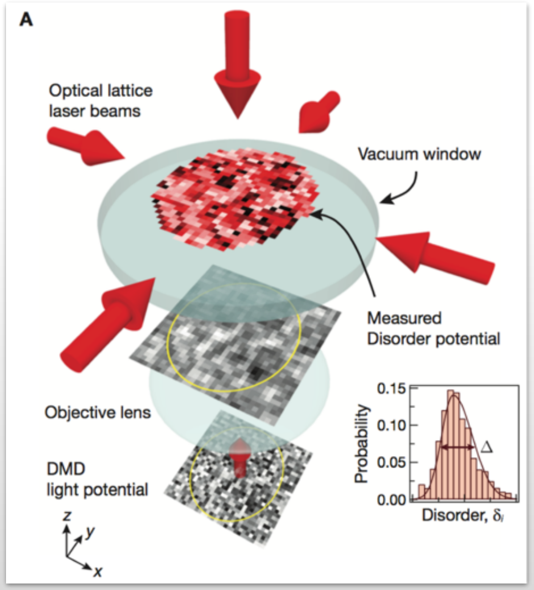
>50% doubloons
40% doublons
no doublons

Kinetic energy of doublons for large U $J_{\rm dbl} = J^2/U$

Doublons see effectively larger disorder $\frac{J_{\rm dbl}}{\Delta} \ll \frac{J}{\Delta}$

Probing Many-Body Localisation in 2D


System Summary



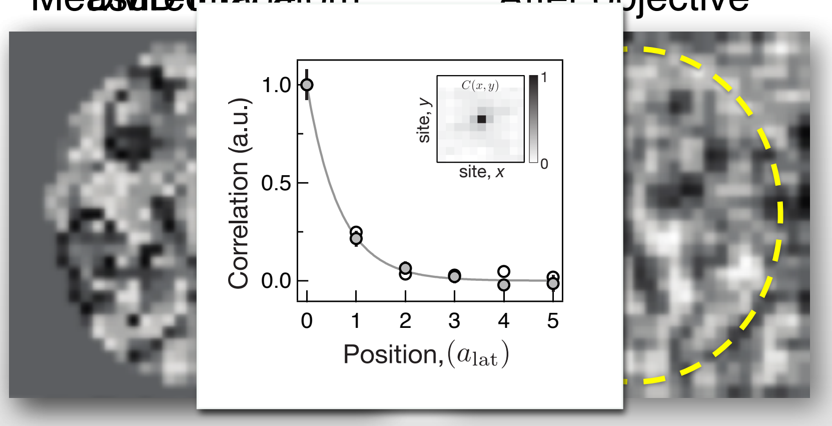
1. Prepare Domain Wall
   (no tunneling dynamics)
2. Turn on disorder potential
3. Lower the lattice depth
   (near critical point)
4. Measure atomic distribution

* Tunneling time is 6.4 ms.
* Disorder is changed for each image.
* Take 100 picture for averaging.

$$U = 24J$$

$$\Delta = 0 - 20J$$

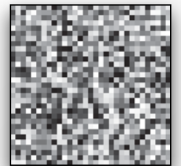
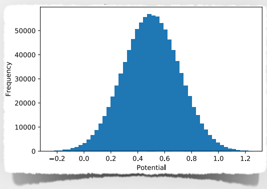
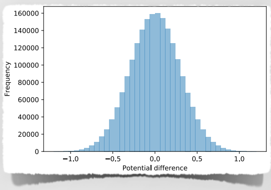
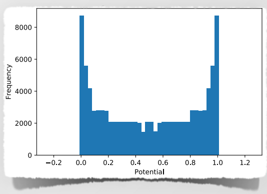
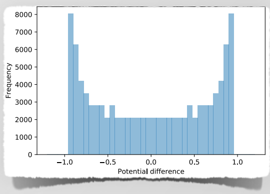
Disorder Potential



Excellent characterization of disorder !!

The setup

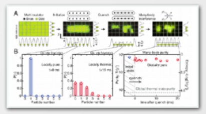
Other potentials

Types of potential	Potentials hist.	NN difference hist.
 True random		
 1D Quasiperiodic		

MBL

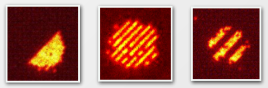
Probing Non-Thermalization in AMO System

Very hard to probe whether the system has thermalised!
(but possible for small system sizes)



A. Kaufman *et al.* Science 2016

1) Start with recognisable (density) pattern



2) Evolve until steady state is reached (*always exp. limited)

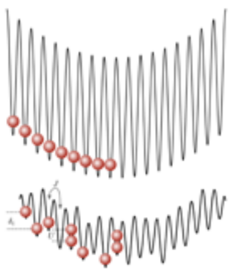
3) Analyse if any remnant pattern detectable

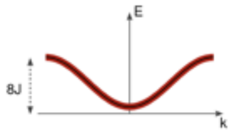
Probing Thermalization far from Equilibrium

Evolution under 2d disordered Bose-Hubbard:

$$\hat{H} = -J\sum_{<i,j>} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i (\delta_i + V_i)\hat{n}_i$$

System preparation far from equilibrium



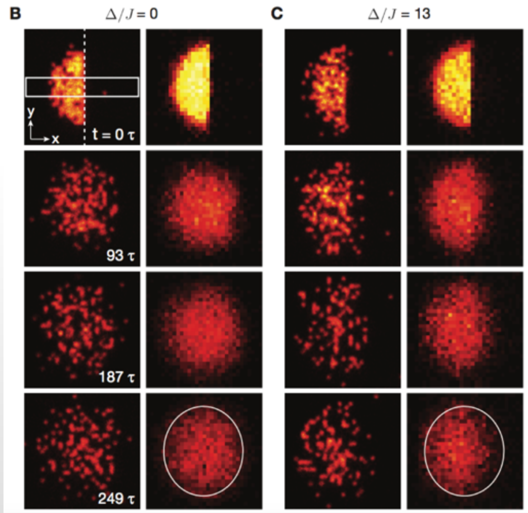


Infinite temperature wrt.

kinetic energy and disorder

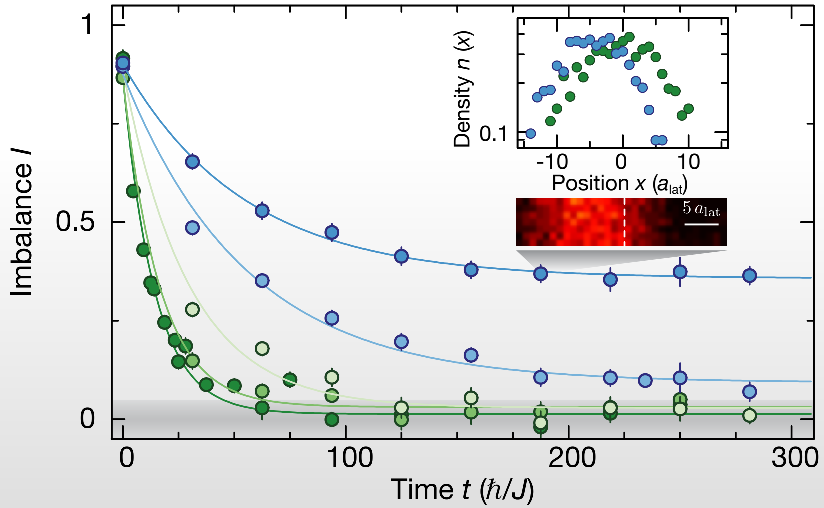
See also experiments on ground state Bose-Glass:
Kondov et al. Phys. Rev. Lett. (2015)

Domain Wall Dynamics



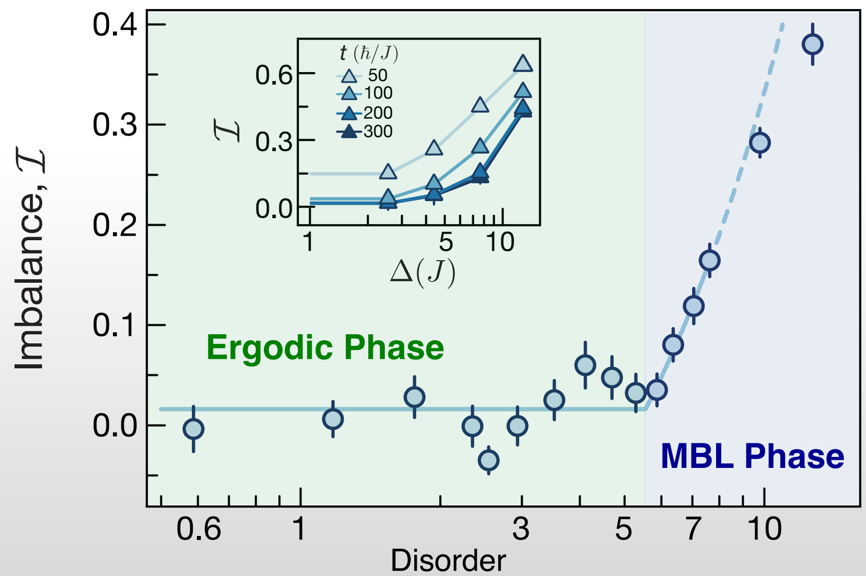
Without disorder **With disorder**

Domain Wall Imbalance

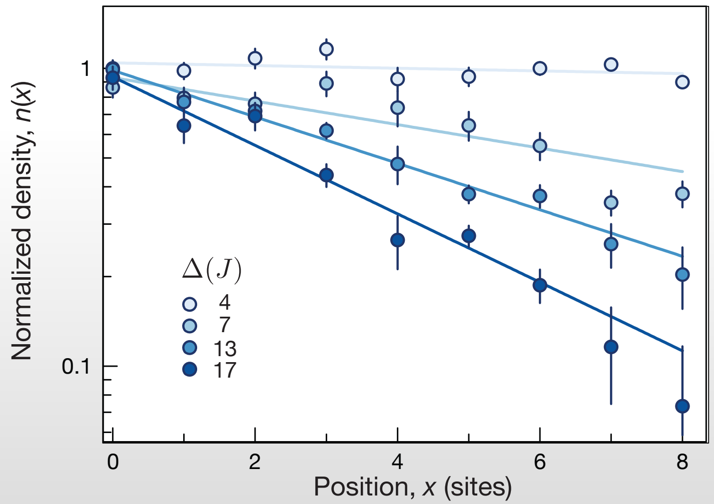


Numerics: **1d+ladder** tDMRG: Hausschild *et al.* PRB (2016); **2d** T. Wahl *et al.* arXiv:1711.02678, Yan et al PRL (2017)

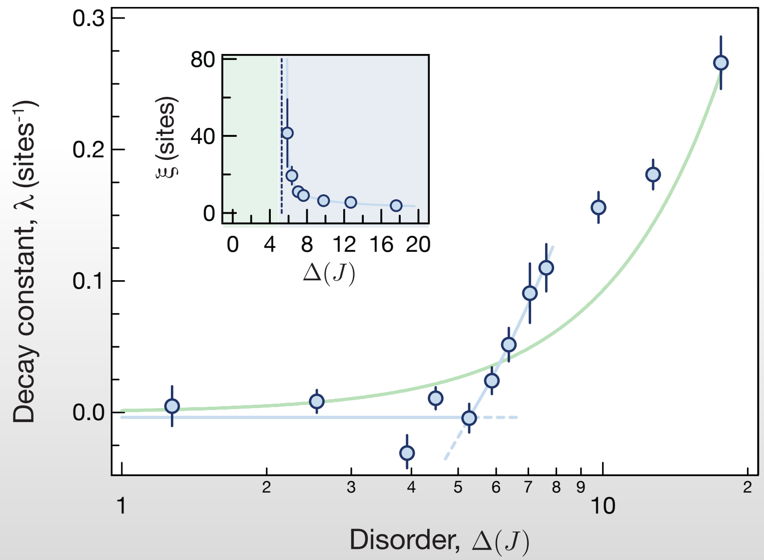
Delocalization-to-Localization



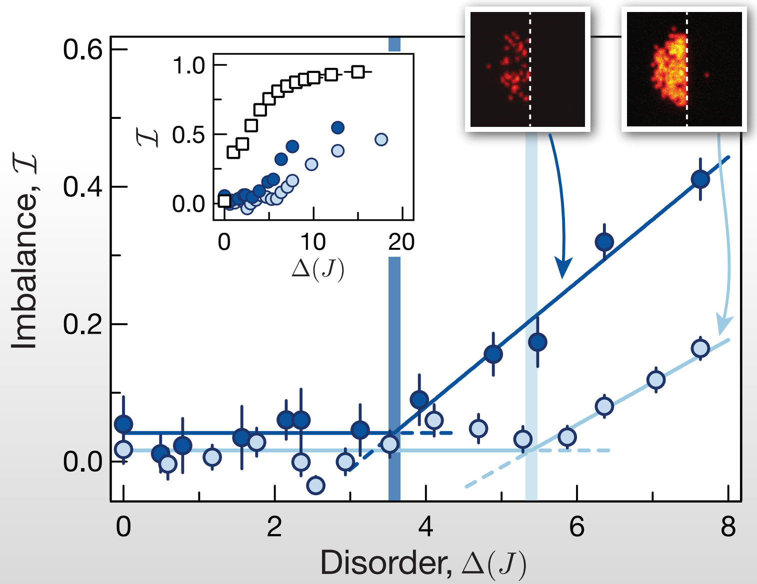
Disorder Effect in Real Space II



Diverging Length Scale



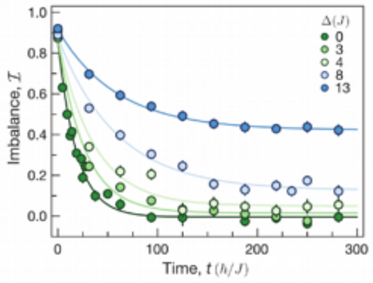
Role of Interaction

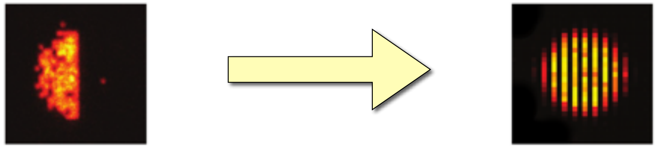


Probing MBL on Different Length and Timescales

MBL

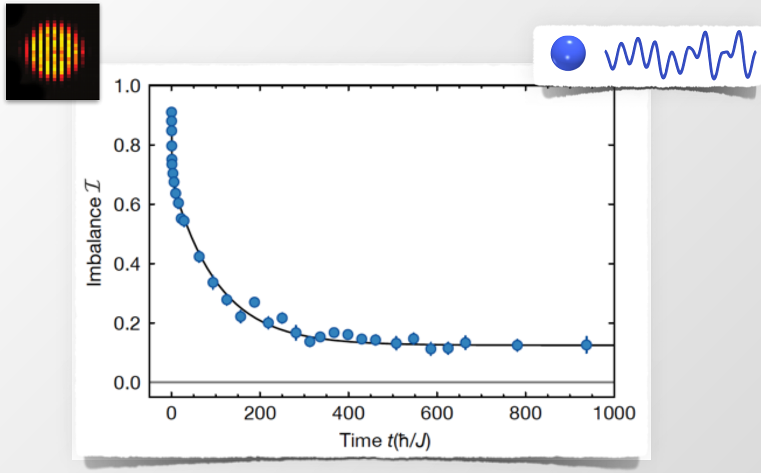
Better Separation of Timescales





MBL with a small bath

Dynamics without mixture

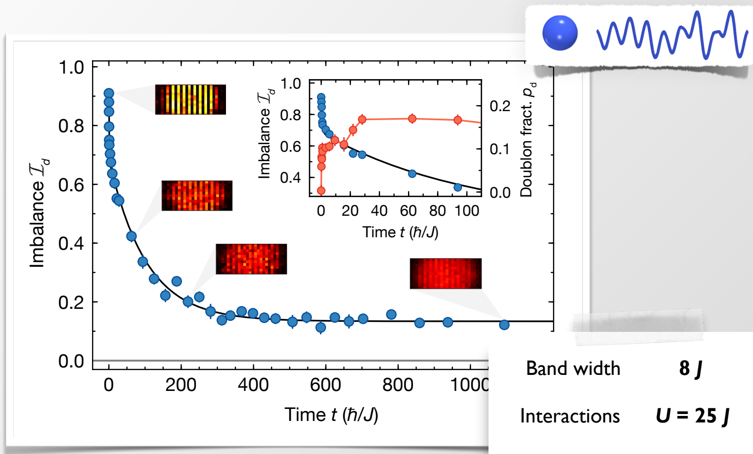


τ_1~0.6 ħ/J τ_2~105 ħ/J τ_3 > 2500 ħ/J

A. Rubio-Abadal, J-y. Choi *et al.*, PRX 2019

MBL with a small bath

Dynamics without mixture



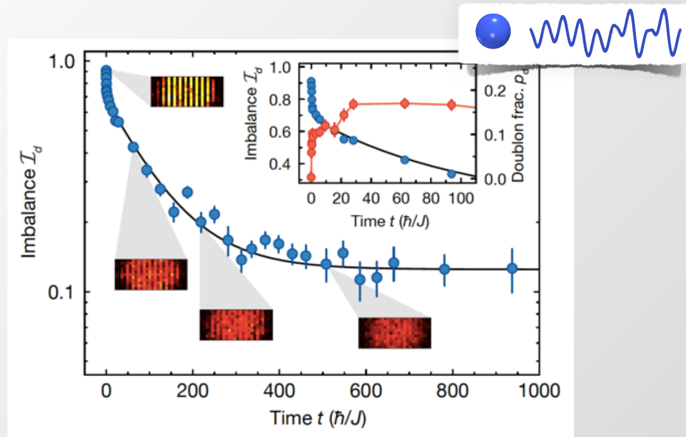
Band width	8 J
Interactions	U = 25 J
Disorder	Δ = 28 J

τ_1~0.6 ħ/J τ_2~105 ħ/J τ_3

A. Rubio-Abadal, J-y. Choi *et al.*, PRX 2019; arXiv:1805.00056

MBL with a small bath

Dynamics without mixture

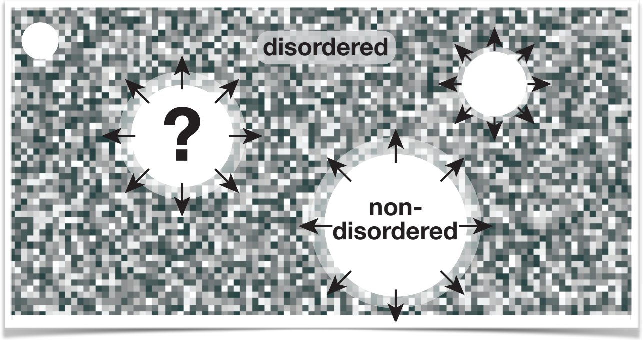


τ_1~0.6 ħ/J τ_2~105 ħ/J τ_3 > 2500 ħ/J

A. Rubio-Abadal, J-y. Choi *et al.*, arXiv:1805.00056

MBL

Probing MBL Instabilities



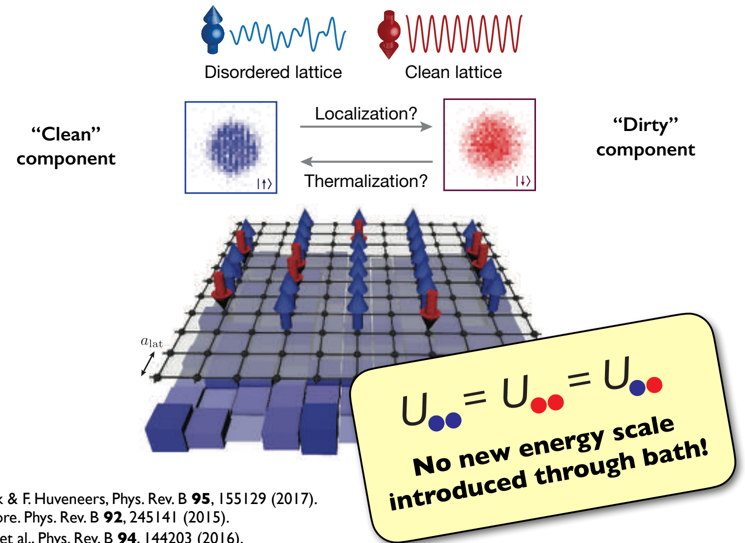
Engineered disorder with **controlled non-disordered (ergodic) grains!**

Avalanches?
Stability?
Range?
Timescales of Instability?
⋮

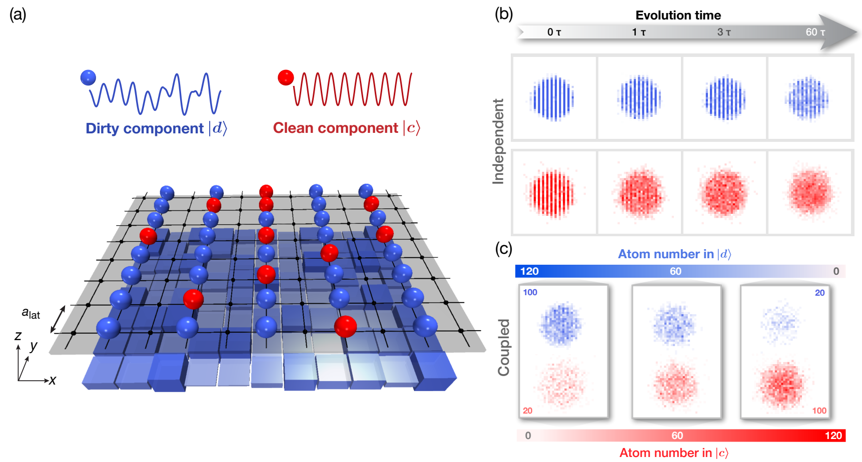
Stability vs Coupling to an Atomic Bath and "MBL Proximity Effect"

Motivation

MBL Coupled to a Finite Bath



W. De Roeck & F. Huveneers, Phys. Rev. B **95**, 155129 (2017).
R. Nandkishore. Phys. Rev. B **92**, 245141 (2015).
A. Chandran et al., Phys. Rev. B **94**, 144203 (2016).
K. Hyatt et al., Phys. Rev. B **95**, 035132 (2017).
K. Agarwal *et al.*, AdP, 1600326 (2017).



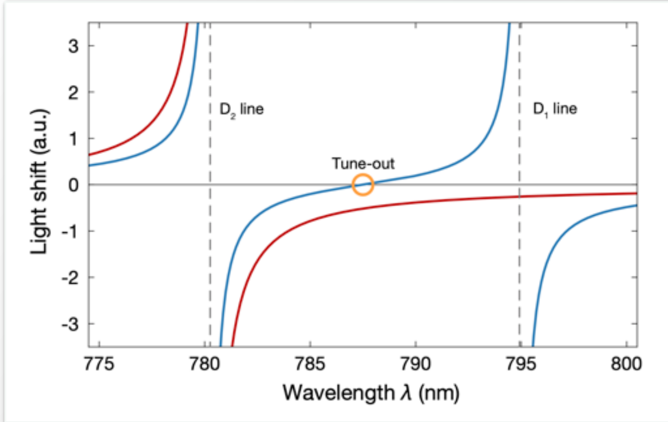
1) Deloc + Loc = Deloc

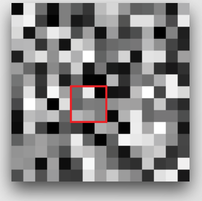
2) Deloc + Loc = Loc

3) Deloc + Loc = Deloc + Loc

Introduction

Spin dependent disorder potential



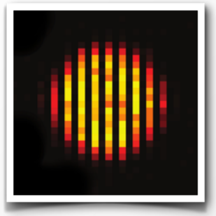


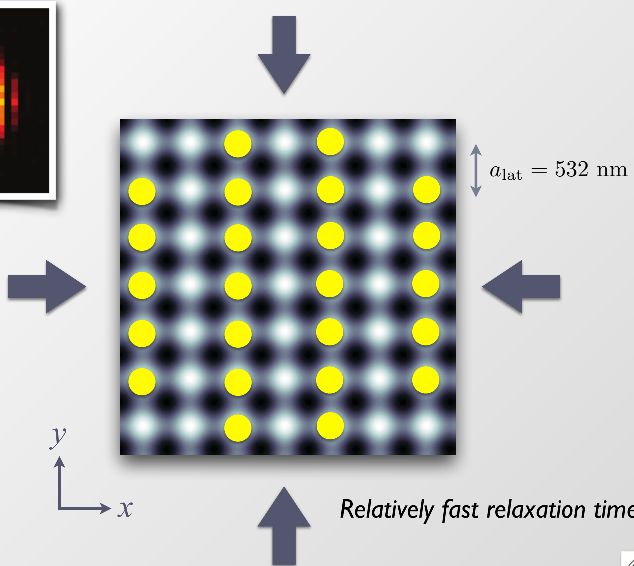
Down spin can be considered as a heat bath

$$U = U_{\downarrow\downarrow} \approx U_{\uparrow\uparrow} \approx U_{\uparrow\downarrow}$$

Introduction

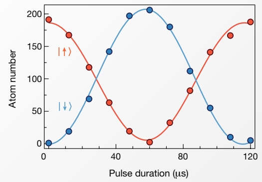
Charge Density Wave

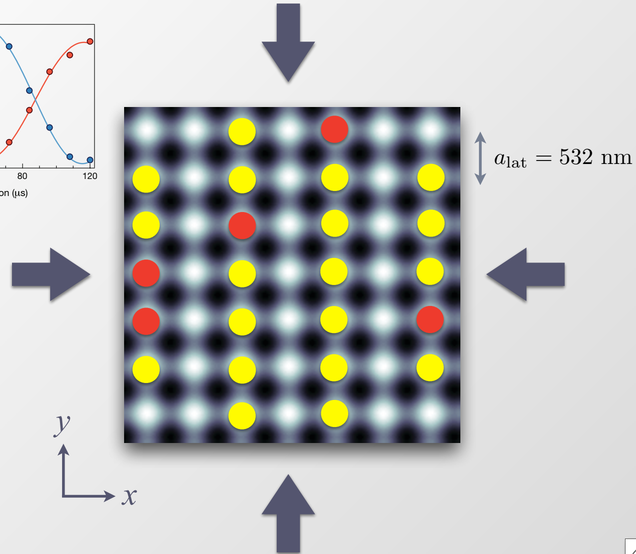




Relatively fast relaxation time

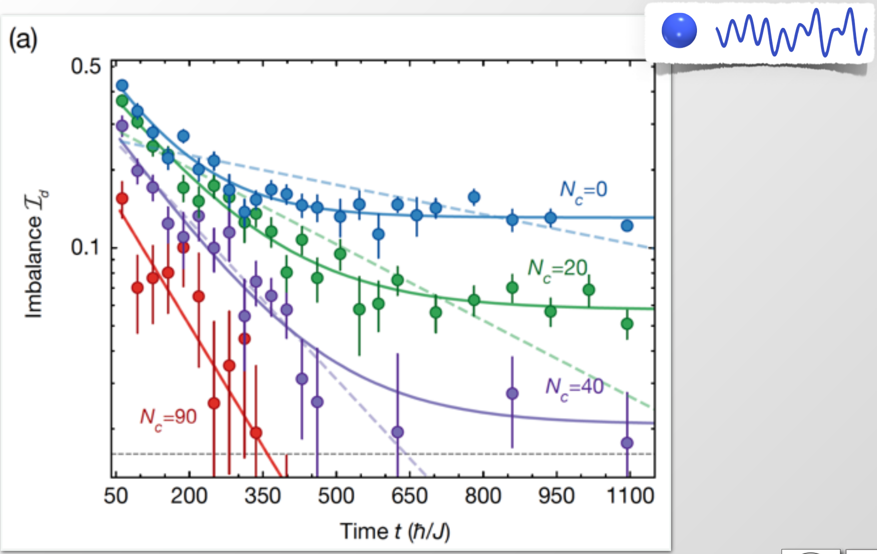
Ergodic spin grains





MBL with a small bath

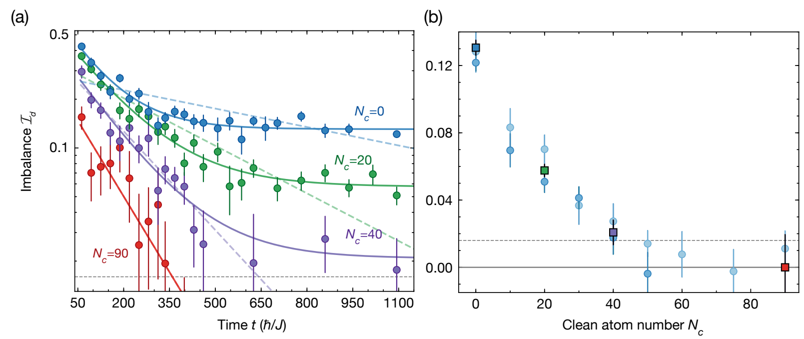
Dynamics of the dirty component



A. Rubio-Abadal, J-y. Choi *et al.*, PRX 2019

MBL Bath

Stability of MBL vs Size of Bath



Imbalance stable for small admixtures

Larger bath size kills the imbalance ⇨ **Localization non-trivial !**

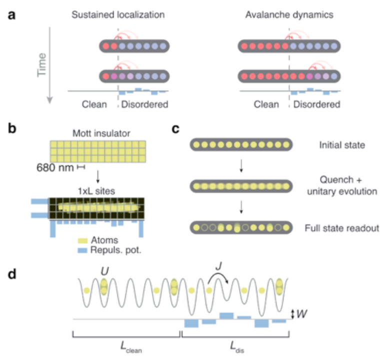
1) Deloc + Loc = Deloc Larger bath

2) Deloc + Loc = Loc Small bath

3) Deloc + Loc = Deloc + Loc Small bath

MBL

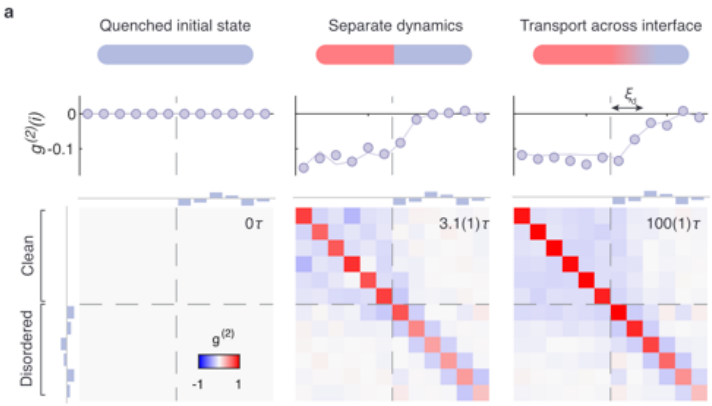
MBL Instabilities



$$\hat{\mathcal{H}} = -J\sum_i \left(\hat{a}_i^\dagger \hat{a}_{i+1} + h.c.\right) + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) + W\sum_{i\in L_{\text{dis}}} h_i \hat{n}_i,$$

MBL

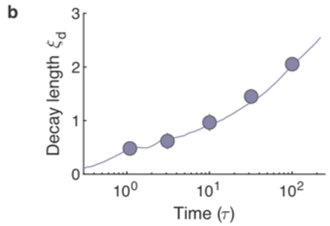
MBL Coupled to Bath



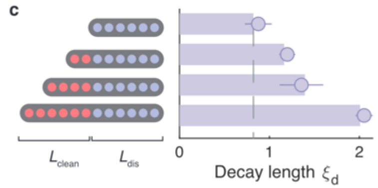
$$g^{(2)}(i,j) = \langle \hat{n}_i \hat{n}_j \rangle - \langle \hat{n}_i \rangle \langle \hat{n}_j \rangle$$

MBL

MBL Coupled to Bath



Decay length as a function of **evolution time**

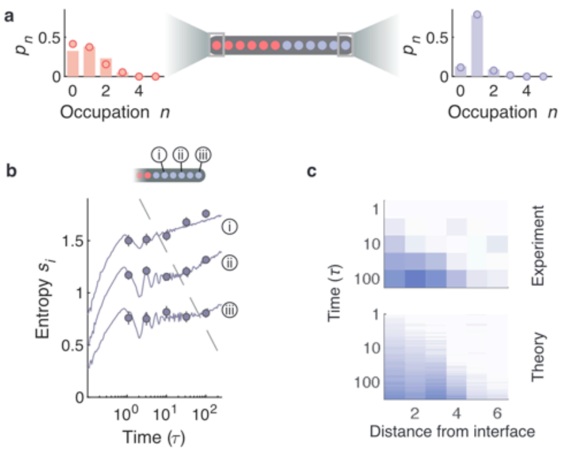


Decay length as a function of **bath size**

MBL

MBL Coupled to Bath

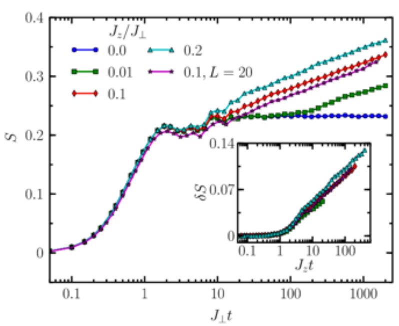
Local Particle Number Fluctuations



$$s_i = -\sum_n p_n \log p_n / \langle \hat{n}_i \rangle$$

Probing Dynamical Entanglement Features

Entanglement Entropy Growth



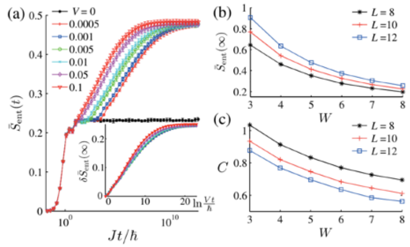
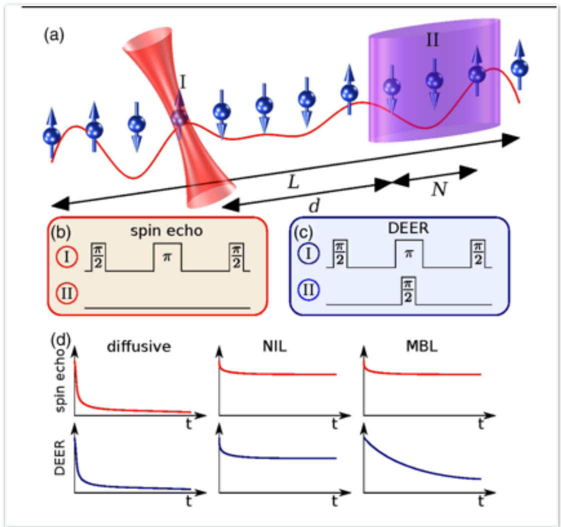


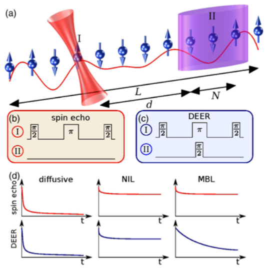
FIG. 2 (color online). (a) Averaged entanglement entropy of initial product states, in which all fermions are localized at some sites, shows a characteristic logarithmic growth on long time scales (system size is $L=12$, $W=5$). Growth rate is found to be proportional to $\ln(Vt/\hbar)$ (inset). Saturated entanglement (b) and the ratio $C=\bar{S}_{\mathrm{ent}}(\infty)/\bar{S}_{\mathrm{diag}}$ (c) decrease with W (for fixed $V=0.01$).

Interaction induced entanglement of different spatial regions of system
log(t) direct consequence of exponentially decaying interactions strength of LIOMs

M Žnidarič, T Prosen, P Prelovšek, Phys. Rev. B (2008)
JH Bardarson, F Pollmann, JE Moore PRL (2012)
M Serbyn, Z Papic, DA Abanin PRL (2013)



M. Serbyn PRL (2014)



Spin-Echo and DEER Pulse Sequence

Simple Model : 2-l-bit system

Initial State	①	②	Step
	$\Uparrow$	$\frac{1}{\sqrt{2}}(\Uparrow+\Downarrow)$	1
$\frac{\pi}{2}$-Pulse ①	$\frac{1}{\sqrt{2}}(\Uparrow+\Downarrow)$	$\frac{1}{\sqrt{2}}(\Uparrow+\Downarrow)$	2
Evolve for $t_{/2}$ with H_{MBL}	$\frac{1}{2}e^{iJ_{12}t_{/2}}(\Uparrow\Uparrow+\Downarrow\Downarrow)+\frac{1}{2}e^{-iJ_{12}t_{/2}}(\Uparrow\Downarrow+\Downarrow\Uparrow)$		3
π-Pulse ①	$\frac{1}{2}e^{iJ_{12}t_{/2}}(\Downarrow\Uparrow+\Uparrow\Downarrow)+\frac{1}{2}e^{-iJ_{12}t_{/2}}(\Downarrow\Downarrow+\Uparrow\Uparrow)$		4
Evolve for $t_{/2}$ with H_{MBL}	$\frac{1}{2}(\Downarrow\Uparrow+\Uparrow\Downarrow)+\frac{1}{2}(\Downarrow\Downarrow+\Uparrow\Uparrow)$		5
$\frac{\pi}{2}$-Pulse ①	$\Downarrow$	$\frac{1}{\sqrt{2}}(\Uparrow+\Downarrow)$	6

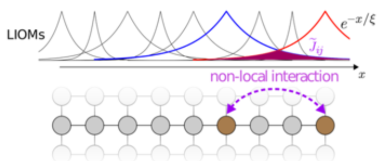
DEER $\frac{\pi}{2}$-Pulse on ②

Entanglement Dynamics SC Processor

Direct measurement of non-local interactions in the many-body localized phase

arXiv:1910.06024v2 [cond-mat.dis-nn] 8 Jul 2020

The interplay of interactions and strong disorder can lead to an exotic quantum many-body localized (MBL) phase. Beyond the absence of transport, the MBL phase has distinctive signatures, such as slow dephasing and logarithmic entanglement growth; they commonly result in slow and subtle modification of the dynamics, making their measurement challenging. Here, we experimentally characterize these properties of the MBL phase in a system of coupled superconducting qubits. By implementing phase sensitive techniques, we map out the structure of local integrals of motion in the MBL phase. Tomographic reconstruction of single and two qubit density matrices allowed us to determine the spatial and temporal entanglement growth between the localized sites. In addition, we study the preservation of entanglement in the MBL phase. The interferometric protocols implemented here measure affirmative correlations and allow us to exclude artifacts due to the imperfect isolation of the system. By measuring elusive MBL quantities, our work highlights the advantages of phase sensitive measurements in studying novel phases of matter.

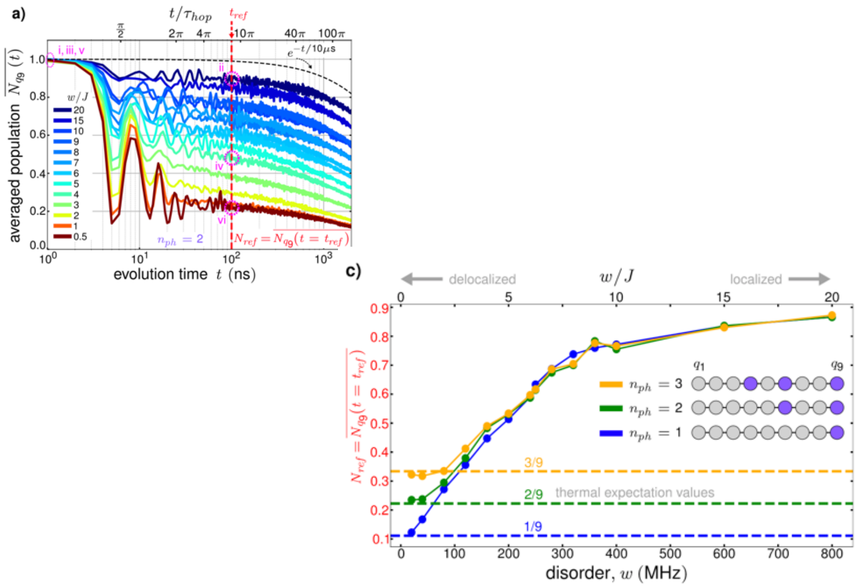


9-site 1D chain

$$H_{\mathrm{BH}}=\underbrace{\sum_i^{n_Q} h_i a_i^\dagger a_i}_{\text{on-site detuning}}+\underbrace{\frac{U}{2}\sum_i^{n_Q} a_i^\dagger a_i(a_i^\dagger a_i-1)}_{\text{Hubbard interaction}}+\underbrace{J\sum_{\langle i,j\rangle}\left(a_i^\dagger a_j+\text{h.c.}\right)}_{\text{NN coupling / hopping}},$$

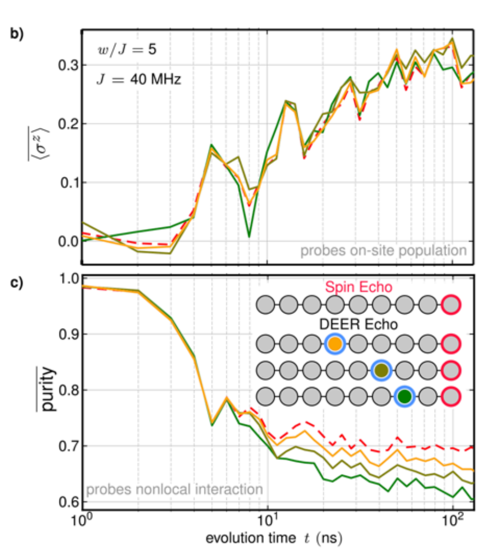
MBL

Population Measurement



MBL

Spin-Echo & DEER



Population

Coherence/
Single Qubit Purity

Big Open Questions

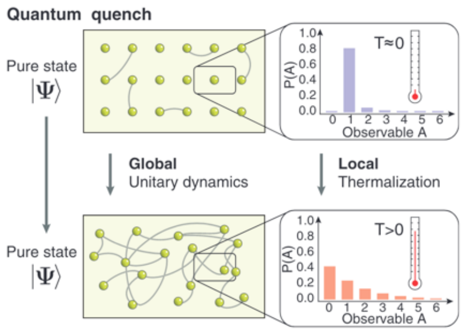
- Stability of MBL
- Nature of Transport in Ergodic Phase
- Definition of Localization Length
- Finite Coupling to Bath
- Phase Diagram

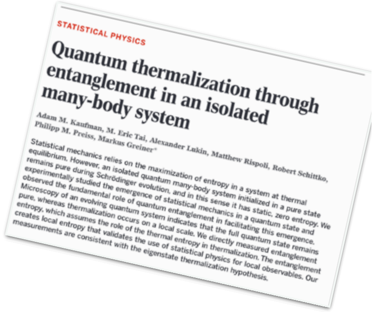
Experimental Advances

- Longer timescales
- Larger systems
- Structured disorder
- Improved isolation from environment

Probing Thermalization in a QMB System

Thermalization in an Isolated QMB System




STATISTICAL PHYSICS

Quantum thermalization through entanglement in an isolated many-body system

A. Kaufmann *et al.* Science 2016

ETH assumes that locally: $\rho_A = \frac{1}{Z_A} e^{-\beta E_n}$

$$S_{VN} = -\,tr\left(\rho_A \log \rho_A\right) = S_{th}$$

Remaining System acts as "Thermal Reservoir" for smaller subsystem.

Entanglement Entropy **is** Thermal Entropy in a QMB System

To Show:

1) Global State Remains Pure

2) Locally, system looks thermal

3) Probe local vN Entanglement Entropy

(or related quantity)

A. Kaufmann *et al.* Science 2016

Probing Renyi Entropy

Probing State Purity via Many-Body Quantum Interference (here for bosons)

two identical copies

t or $\hat{H}$

BS

Beam Splitter Operation between copies

Purity measurement

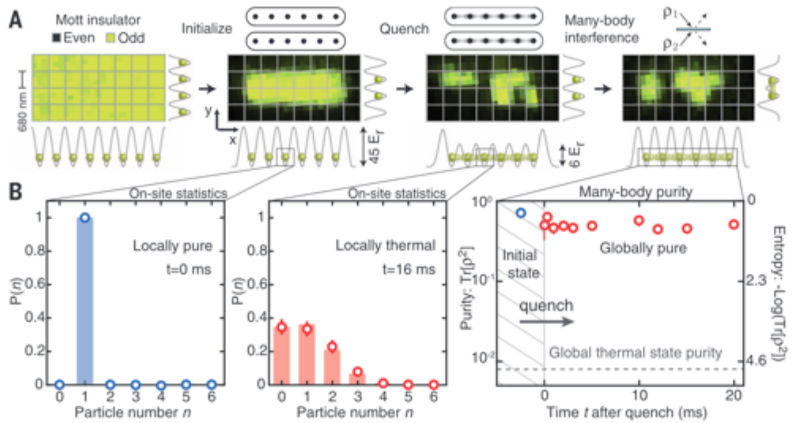
Purity	$tr\left(\rho^2\right)$
Renyi-2 Entropy	$S_2(\rho) = -\log\left(tr\left(\rho^2\right)\right)$

Bounds von Neumann Entropy

$$S_{vN} \geq S_2(\rho)$$

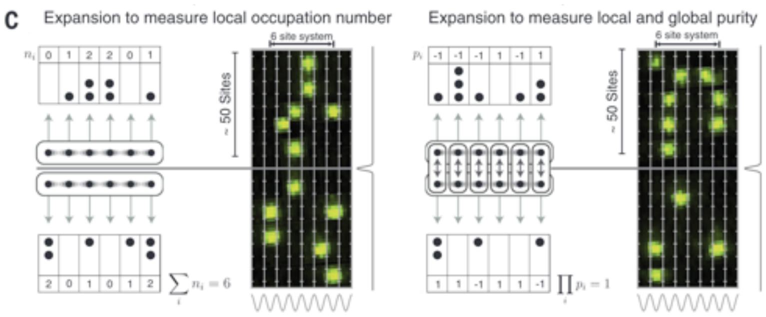
see C.M. Alves & D. Jaksch PRL 2004
A. Daley et al. PRL 2012

Thermalization in an Isolated QMB System

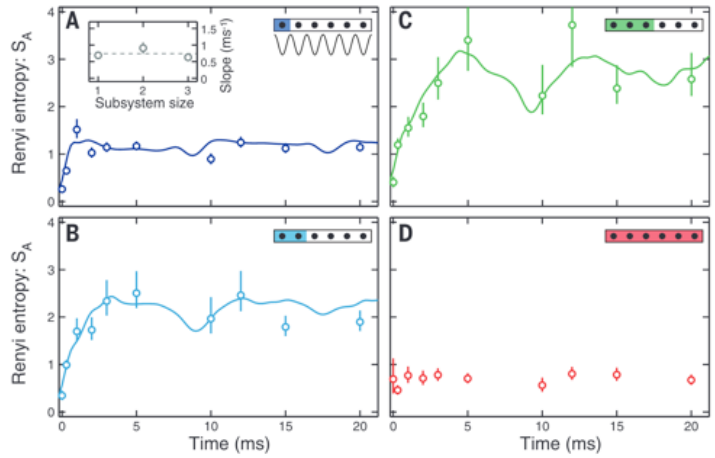


Measure Renyi-2 Entropy

$$tr(\rho^2) = \langle \hat{P} \rangle = \langle \Pi_i p_i^{(k)} \rangle$$

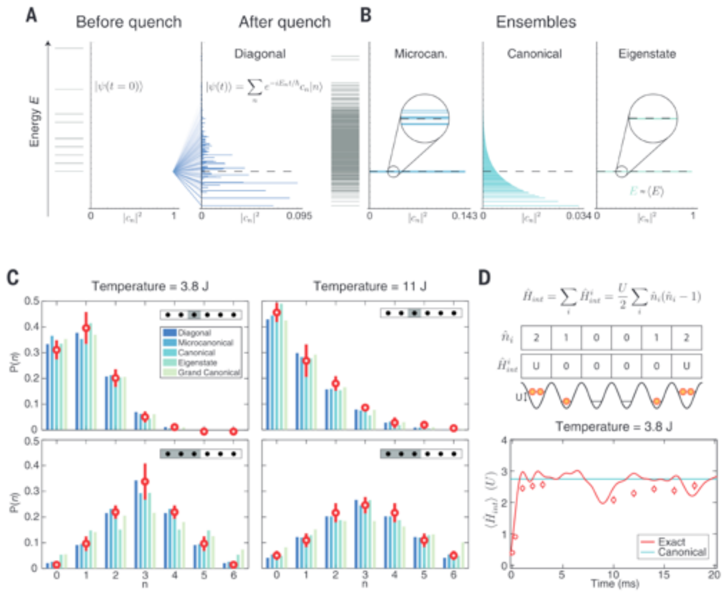


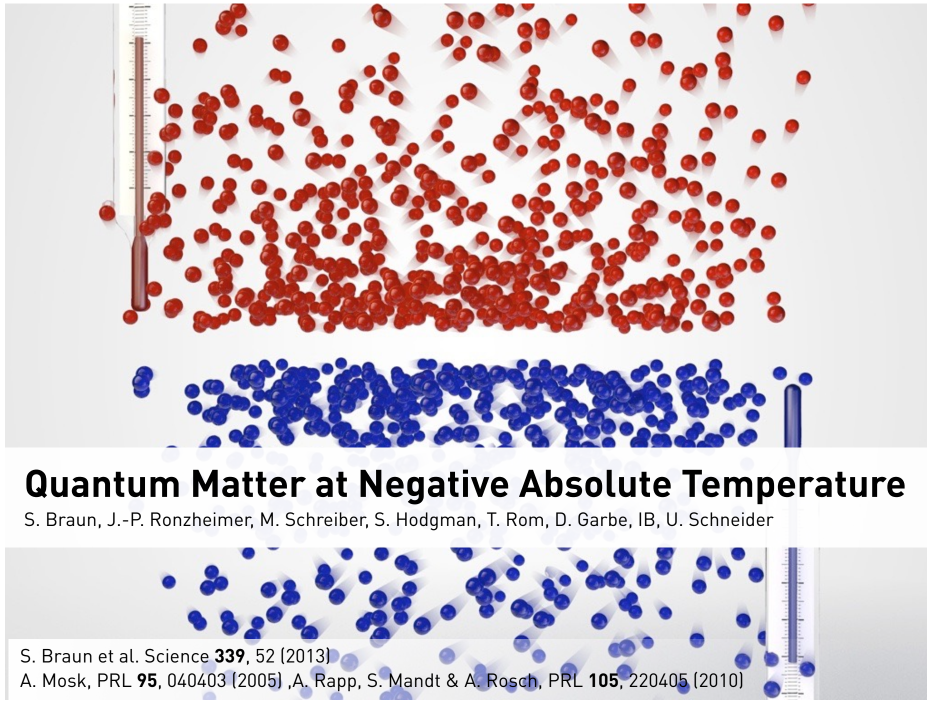
Thermalization in an Isolated QMB System



Thermalization

Thermalization in an Isolated QMB System





Quantum Matter at Negative Absolute Temperature

S. Braun, J.-P. Ronzheimer, M. Schreiber, S. Hodgman, T. Rom, D. Garbe, IB, U. Schneider

S. Braun et al. Science **339**, 52 (2013)

A. Mosk, PRL **95**, 040403 (2005) ,A. Rapp, S. Mandt & A. Rosch, PRL **105**, 220405 (2010)

Negative Temperature

Thermodynamic Definition of Temperature

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right.$$

mperature

ith Energy

erature

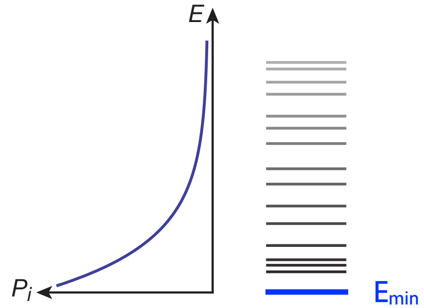
s with Energy

Warning: Temperature does not measure energy content!!!

Thermodynamic theorems apply in negative as well as positive temperature regime!

Negative Temperatures

Requirements

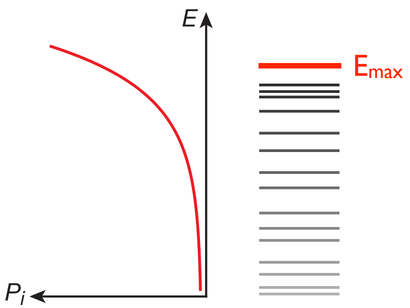


$$P_i \propto e^{-\frac{E_i}{k_B T}}$$

For positive temperatures, we require lower energy bound E_{min}!

Negative Temperatures

Requirements

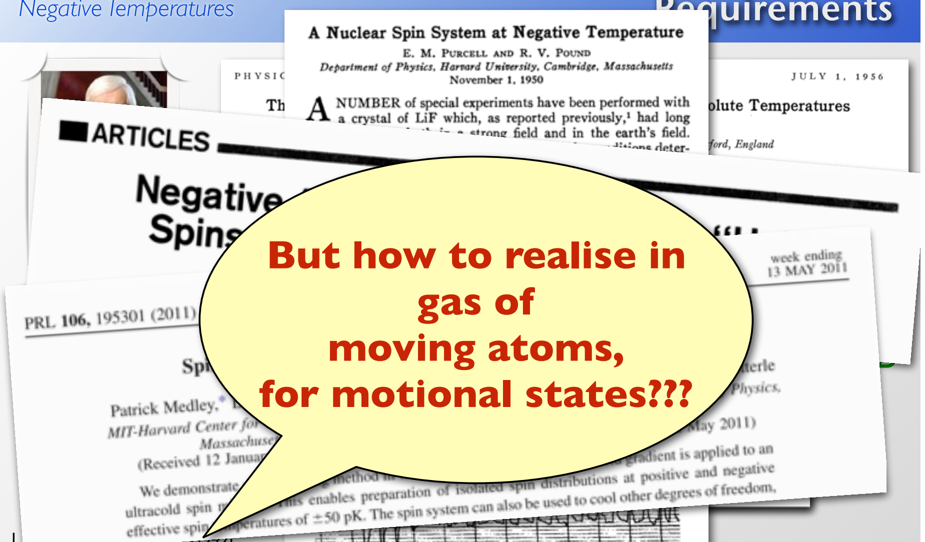


$$P_i \propto e^{-\frac{E_i}{k_B(-T)}}$$

For negative temperatures, we require upper energy bound E_{max}!

Negative Temperatures

Requirements

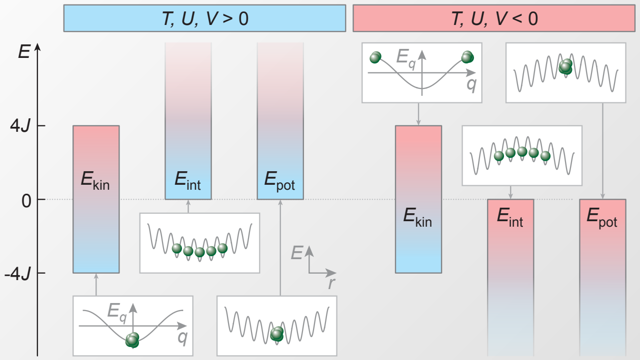


But how to realise in gas of moving atoms, for motional states???

E.M. Purcell & R.V. Pound, Phys. Rev. **81**, 2…
N. Ramsey, Phys. Rev. **103**, 20 (1956)
M.J. Klein, Phys. Rev. **104**, 589 (1956)
P. Hakonen & O. Lounasmaa, Science **265**…
P. Medley et al, Phys. Rev. Lett **106**, 195301 (2011)

Negative Temperature

Energy Bounds of the BH Model

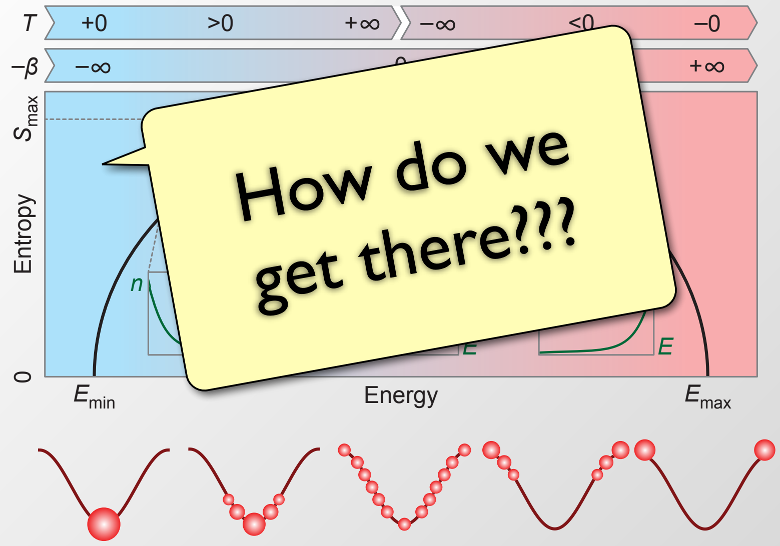


$$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + V \sum_i \mathbf{R}_i^2 \hat{n}_i$$

U,V < 0 required for upper energy bound!

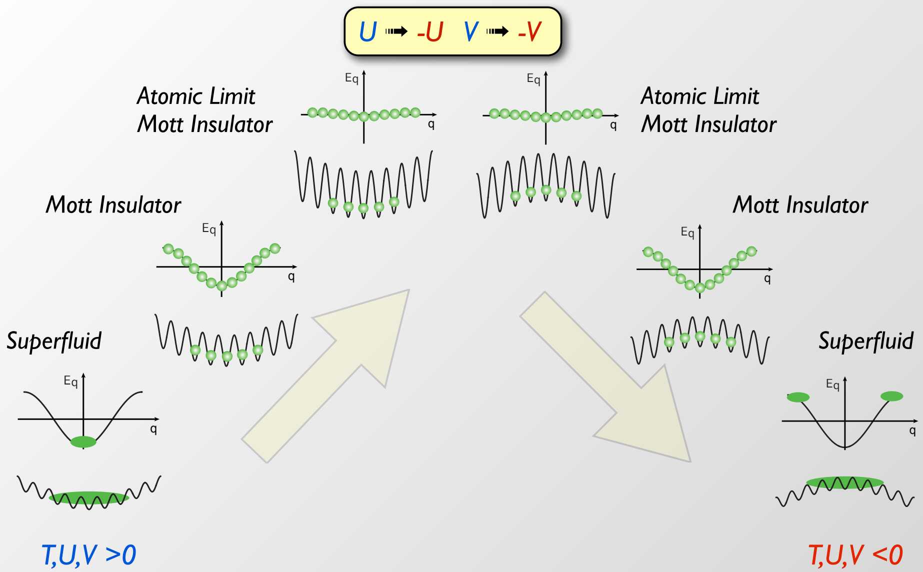
Negative Temperature

Entropy vs Energy



Negative Temperature

Experimental Sequence

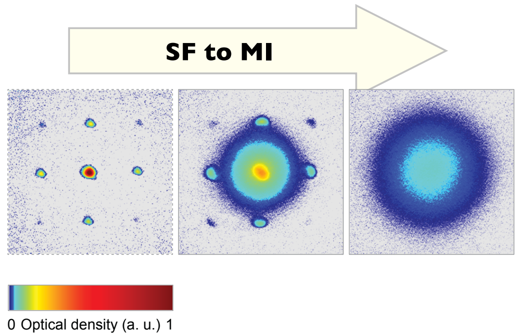


Sequence: A. Rapp, S. Mandt & A. Rosch, PRL (2010)

Negative Temperatures

Experimental Results

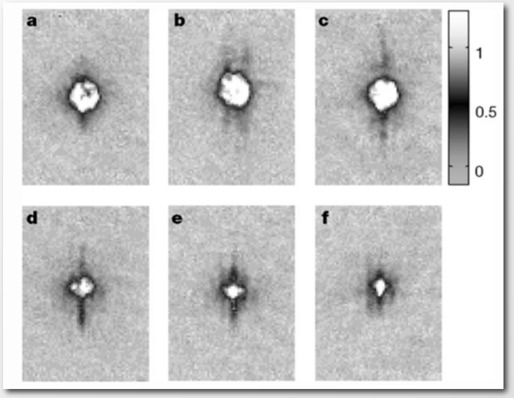
Positive Temperature w/o switching



Negative Temperature w switching

Collapse of Condensate

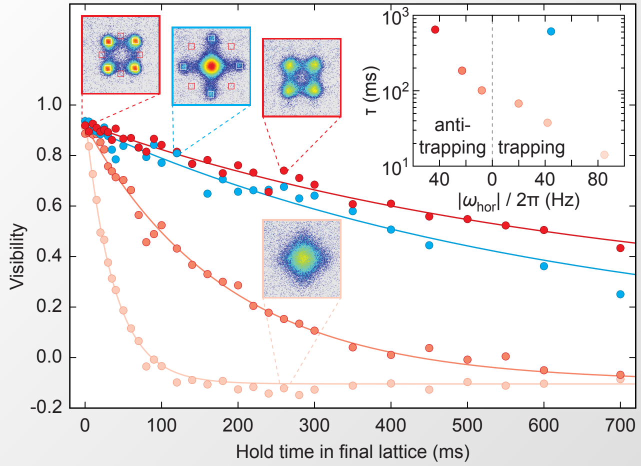
For attractive interactions (a<0), condensate collapses!



E.A. Donley et al. *Nature* **412,** 295-299 (2001)
J. M. Gerton et al. *Nature* 408, 692 (2000)

Negative Temperatures

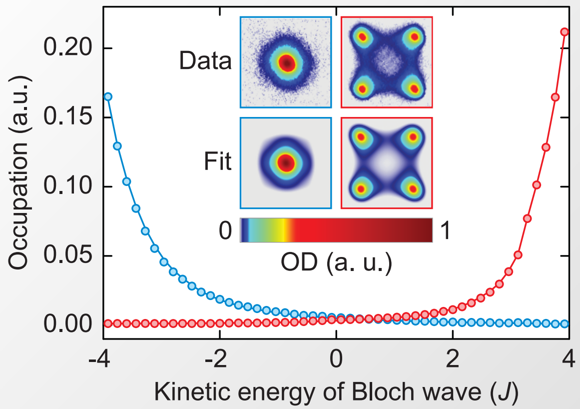
Stability



Negative Temperature State as Stable as Positive Temperature State!

Negative Temperatures

Occupation of Energy States



T=-2.2J/k_B

Kinetic energy well fitted by Bose-Einstein distribution

$$n(q_x, q_y) = \frac{1}{e^{(E_{kin}(q_x,q_y)-\mu)/k_B T} - 1}$$

$$E_{kin}(q_x, q_y) = -2J\left[\cos(q_x d) + \cos(q_y d)\right]$$

Negative Temperatures

Implications

Gases with **negative temperature** possess **negative pressure**!

$$\left.\frac{\partial S}{\partial V}\right|_E \geq 0 \qquad \text{and} \qquad dE = TdS - PdV$$

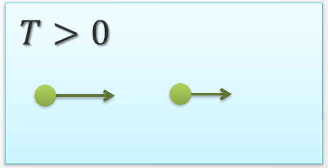
$$\Rightarrow \quad \left.\frac{\partial S}{\partial V}\right|_E = \frac{P}{T} \geq 0$$

Carnot engines **above unit efficiency!** **(but no perpetuum mobile!)**

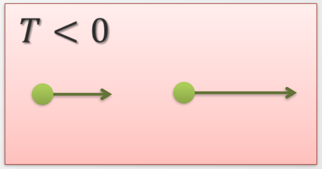
$$\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$$

Some statements for the second law of thermodynamics become invalid!

Anti-Friction at Negative Temperature



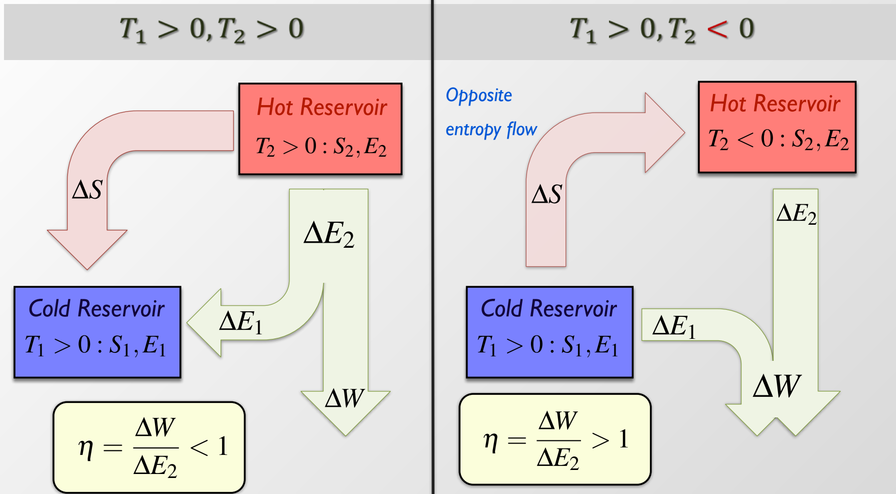
Friction:
- entropy increases
  → Medium heats up
- Particle slows down



particle spectrum is
assumed to be unbounded

Anti-Friction:
- entropy increases
  → Medium cools down
- Particle accelerates
  (but direction is randomized
  in long-term limit)

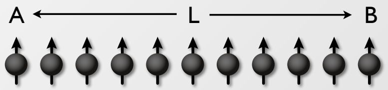
Carnot limits



- *Energy and Entropy are globally conserved!*
- *No violation of thermodynamic laws → No solution to energy problem!*

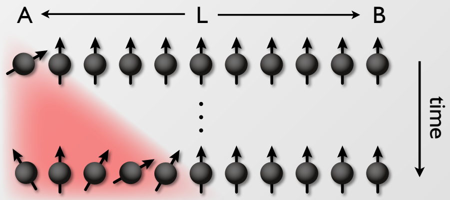
Lieb-Robinson bounds

Spin chain
short-range interactions



Lieb-Robinson bounds

Spin chain
short-range interactions

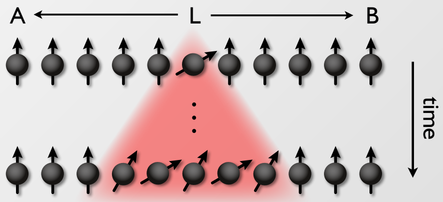


Lieb and Robinson (1972)

$$\left|[A, B(t)]\right| \leq \lambda \exp\left(\frac{vt - L}{\zeta}\right)$$

Lieb-Robinson bounds

Spin chain
short-range interactions



Bravyi, Hastings and Verstraete (2006)
Calabrese and Cardy (2006)
Eisert and Osborne (2006)
Nachtergaele, Ogata and Sims (2006)
... and many others since then

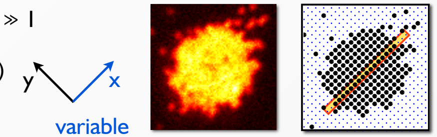
$$\left|\langle A(t)B(t)\rangle - \langle A(t)\rangle\langle B(t)\rangle\right| \le \lambda' \exp\left(\frac{vt - L/2}{\zeta'}\right)$$

the propagation of correlations is bounded by an effective light cone

1D Mott insulator out of equilibrium

1. Prepare 1D Mott insulator with U/J ≫ 1

deep lattice (20 E_r)
no tunnelling

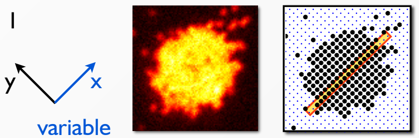


variable
lattice depth

1D Mott insulator out of equilibrium

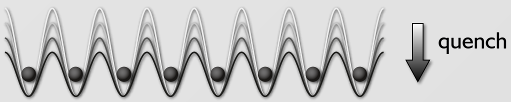
1. Prepare 1D Mott insulator with U/J ≫ 1

deep lattice (20 E_r)
no tunnelling



variable
lattice depth

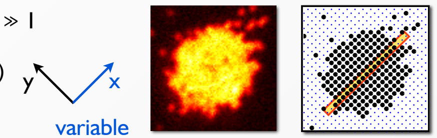
2. Lower U/J abruptly



1D Mott insulator out of equilibrium

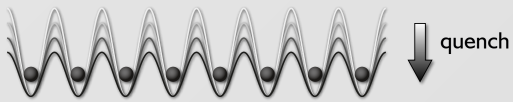
1. Prepare 1D Mott insulator with U/J ≫ 1

deep lattice (20 E_r)
no tunnelling



variable
lattice depth

2. Lower U/J abruptly



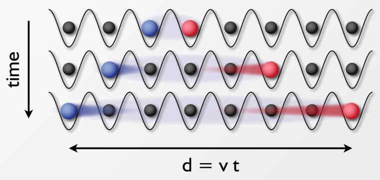
3. Record the dynamics

The initial state is highly excited. Calabrese and Cardy (2006)

Quasiparticles are emitted and propagate ballistically, carrying correlations across the system.

Light-cone like spreading of correlations

- Quasiparticle dynamics



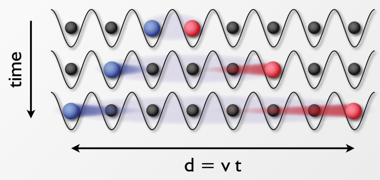
- Two-point parity correlation function

$$C_d(t) = \langle s_j(t)s_{j+d}(t)\rangle - \langle s_j(t)\rangle\langle s_{j+d}(t)\rangle \longrightarrow \begin{array}{l} \simeq 0 \text{ in the initial state} \\ >0 \text{ when } t \simeq d/v \end{array}$$

$$s_j(t) = e^{i\pi[n_j(t)-\bar{n}]} \begin{cases} +1 & \text{if} \\ -1 & \text{if} \quad \text{or} \end{cases}$$

Light-cone like spreading of correlations

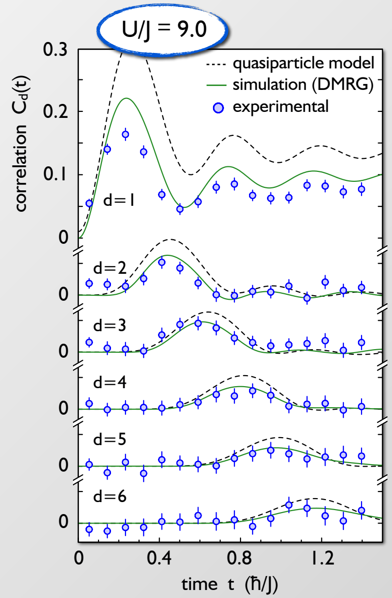
- Quasiparticle dynamics



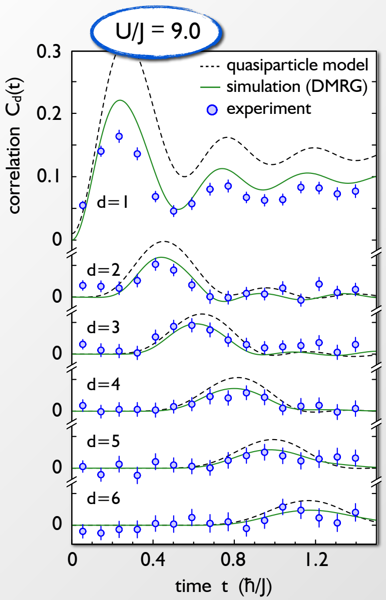
- Two-point parity correlation function

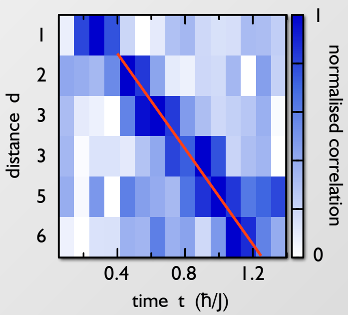
$$C_d(t) = \langle s_j(t)s_{j+d}(t)\rangle - \langle s_j(t)\rangle\langle s_{j+d}(t)\rangle$$

$$s_j(t) = e^{i\pi[n_j(t)-\bar{n}]} \begin{cases} +1 & \text{if} \\ -1 & \text{if} \quad \text{or} \end{cases}$$



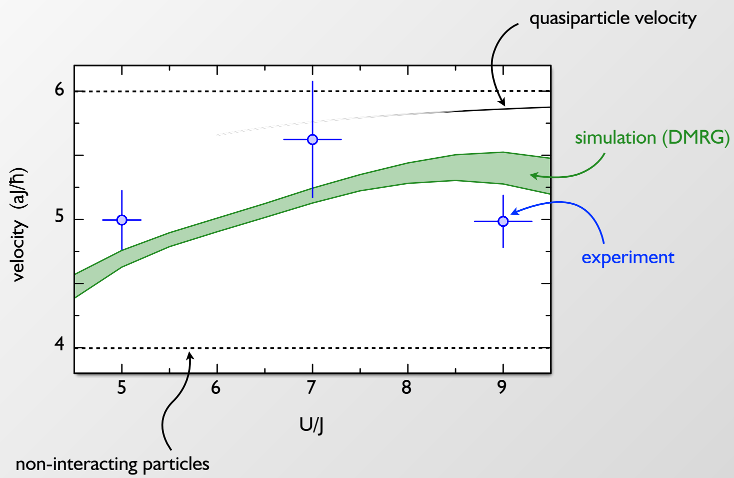
Light-cone like spreading of correlations





effective light-cone!

Spreading velocity

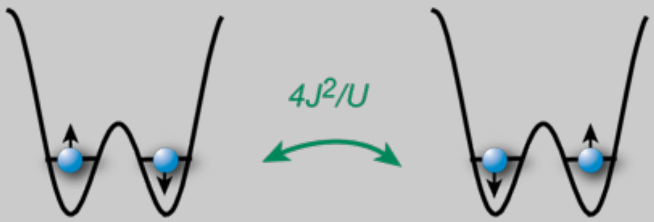


Outline Lecture 3

1. Bound Magnons
2. Spin-Charge Fractionalization in Fermi Hubbard Chains
3. Connection to Ground State Non-Local Order
4. Kardar-Parisi-Zhang Universality in Heisenberg Quantum Magnets

Superexchange Interactions

Superexchange induced flopping



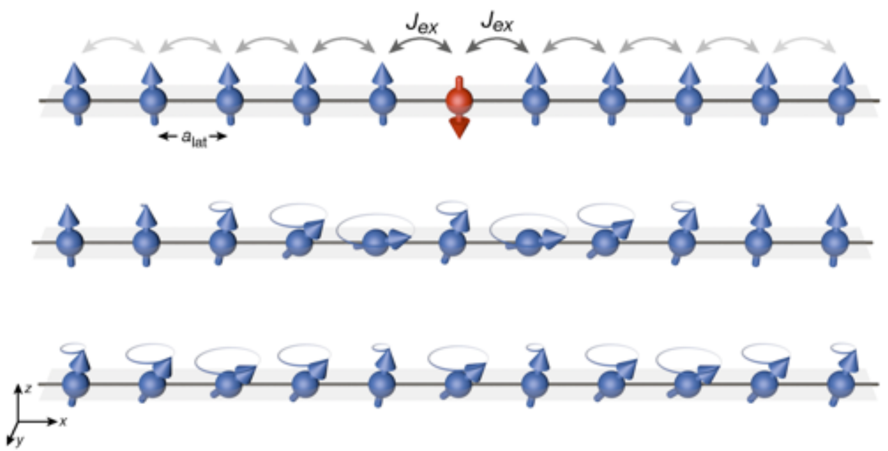
$$H_{eff} = -J_{ex}\vec{S}_i \cdot \vec{S}_j = -J_{ex}\left(\hat{S}_i^x \cdot \hat{S}_j^x + \hat{S}_i^y \cdot \hat{S}_j^y\right) - J_{ex}\hat{S}_i^z \cdot \hat{S}_j^z$$
$$= -\frac{J_{ex}}{2}\left(\hat{S}_i^+\hat{S}_j^- + \hat{S}_i^-\hat{S}_j^+\right) - J_{ex}\hat{S}_i^z\hat{S}_j^z$$

Quantum Dynamic of Mobile Single Spin Impurity

T. Fukuhara, M. Endres, M. Cheneau P. Schauss, Ch. Gross, I. Bloch, S. Kuhr, U. Schollwöck, A. Kantian, Th. Giamarchi

Sherson et al. Nature 467, 68 (2010),
see also Bakr et al. Nature (2009) & Bakr et al. Science (2010)

Spin impurity dynamics

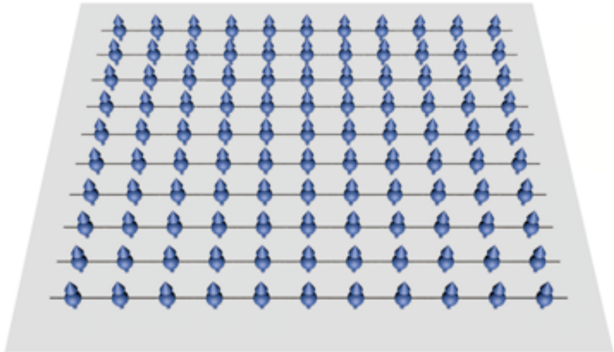


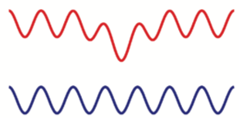
$$-J_{ex}\sum_{\langle i,j\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

Ferromagnetic Heisenberg Interaction

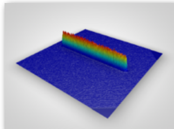
Quantum Magnetism

Spin impurity dynamics





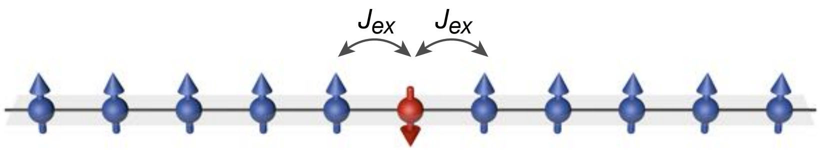
|2> = |F=2, m_F=-2>
|1> = |F=1, m_F=-1>



Line-shaped light field created with DMD SLM

Quantum Magnetism

Spin impurity dynamics



Heisenberg Hamiltonian

$$H = -J_{ex}\sum \mathbf{S}_i \cdot \mathbf{S}_j = -J_{ex}\sum\left(S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z\right)$$

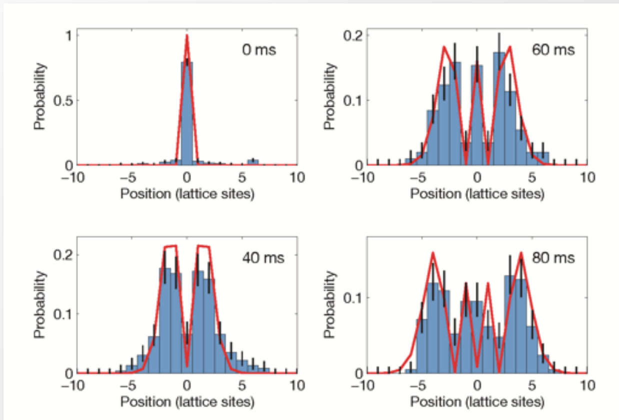
$$= -\frac{J_{ex}}{2}\sum\left(S_i^+ S_j^- + S_i^- S_j^+\right) \cancel{- J_{ex}\sum S_i^z S_j^z}$$

$$J_{ex} = 4\frac{J^2}{U}$$

$$H = -J\sum\left(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_i \hat{a}_j^\dagger\right)$$ single particle tunneling

Quantum Magnetism

Coherent quantum dynamics of single spin at zero temperature



V = 10 *Er*
U/J = 19

$$H = -\frac{J_{ex}}{2}\sum_{\langle i,j\rangle}\left(S_i^+ S_j^- + S_i^- S_j^+\right) \qquad P_j(t) = \left[\mathscr{J}_j\left(\frac{J_{ex}t}{\hbar}\right)\right]^2$$

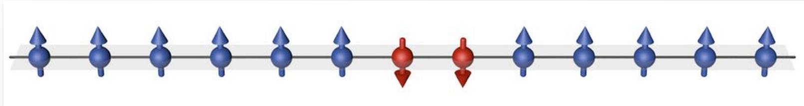
Bessel function of the first kind

Direct Observation of Magnon Bound States

T. Fukuhara, P. Schauss, S. Hild, J.Zeiher, M. Cheneau, M. Endres, I. Bloch, Ch. Gross

T. Fukuhara et al., Nature **502**, 76 (2013)
for photons: O. Firstenberg et al., Nature **502**, 71 (2013)

Magnon Bound States



Hans Bethe
(1906-2005)

There can be bound states in a Heisenberg spin chain!
Development of Bethe Ansatz.

$$H = -J_{ex}\sum_i \left(\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y\right) - \Delta \sum_i \hat{S}_i^z \hat{S}_{i+1}^z$$

$$H = -\frac{J_{ex}}{2}\sum_i \left(\hat{S}_i^+ \hat{S}_{i+1}^- + \hat{S}_i^- \hat{S}_{i+1}^+\right) - \Delta \sum_i \hat{S}_i^z \hat{S}_{i+1}^z$$

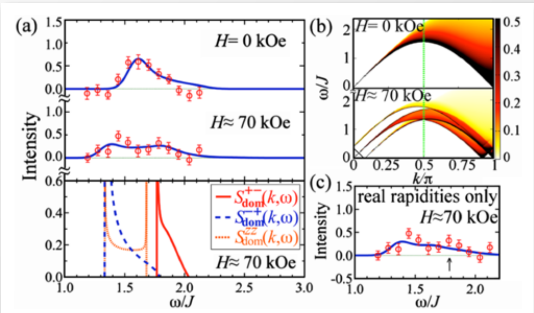
H. Bethe, Z. Phys. (1931)
M. Wortis, Phys Rev. (1963)
M. Takahashi & M. Suzuki Prog. Th. Phys. (1972)
M. Karbach, G. Müller (1997)

see also: repulsively bound pairs & interacting atoms
K.Winkler et al. Nature (2006); S. Fölling et al. Nature (2007);Y Lahini et al. PRA (2012)

A Challenge for CM Physics

Very difficult to observe in spectroscopic data in real materials!

theory **with** bound states

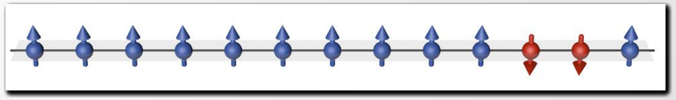


theory **without** bound states

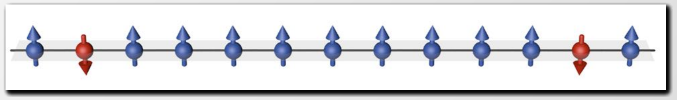
M. Kohno, Phys. Rev. Lett. **102**, 037203 (2009)

Dynamical Evolution

Bound Magnon Motion



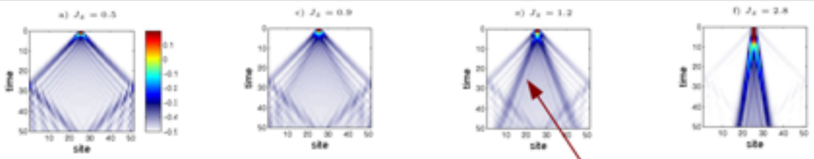
Breakup and Single Spin Motion



M. Ganahl et al., Phys. Rev. Lett. (2012)

Two-spin excitation in FM 

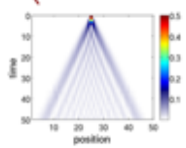
S_z



Jz =0.5 Jz =0.9 Jz =1.2 Jz =2.8

- Two distinct propagation branches beyond Jz =0.7
- New lower branch is *bound state*
- It dominates at large J_z with decreasing velocity
- Low entanglement entropy (see below)

P(↑↑)

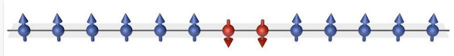


From: H.G. Evertz

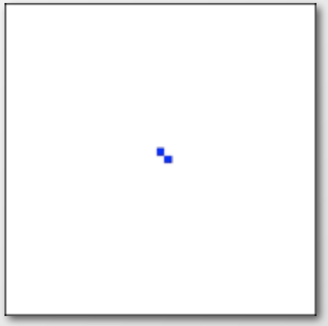
M. Ganahl et al., Phys. Rev. Lett. (2012)

Magnon Bound State

Dynamical Evolution

Initial State: 

Pair distribution evolution $P(x_1, x_2)$



$\Delta = 0$

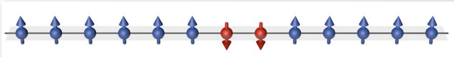
Non-Interacting

see also: two interacting atoms

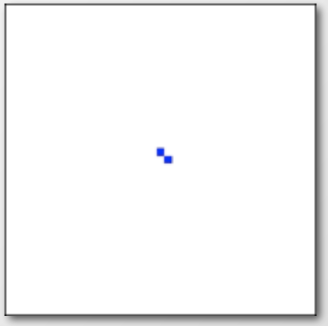
Y. Lahini et al., PRA 86, 011603 (2012)

Magnon Bound State

Dynamical Evolution

Initial State: 

Pair distribution evolution $P(x_1, x_2)$



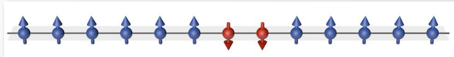
$\Delta = 1$

Interacting

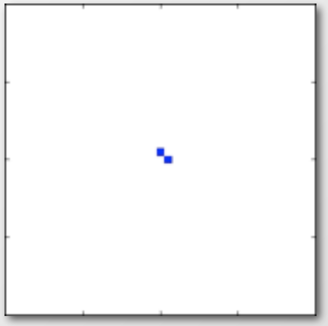
Isotropic Heisenberg

Magnon Bound State

Dynamical Evolution

Initial State: 

Pair distribution evolution $P(x_1, x_2)$



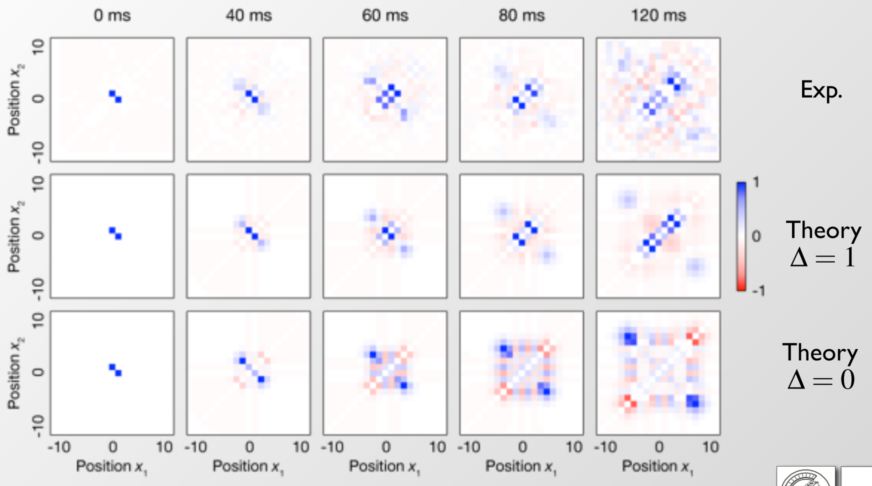
$\Delta = 1.6$

Interacting

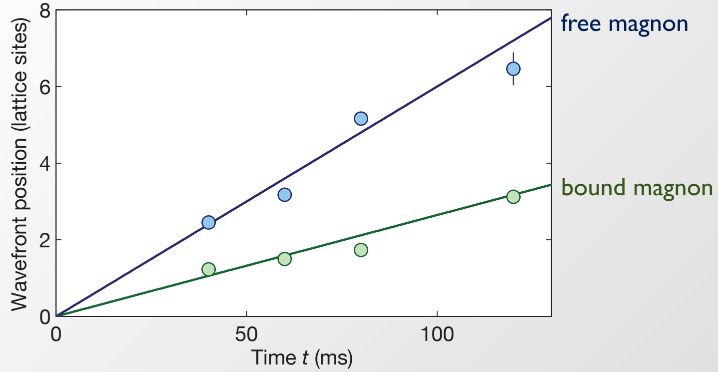
Heisenberg

Experimental Results

$$C(x_1,x_2) = P(x_1,x_2) - P(x_1)P(x_2)$$



Propagation Velocity



$$v_f = \frac{J_{ex}a}{\hbar}$$

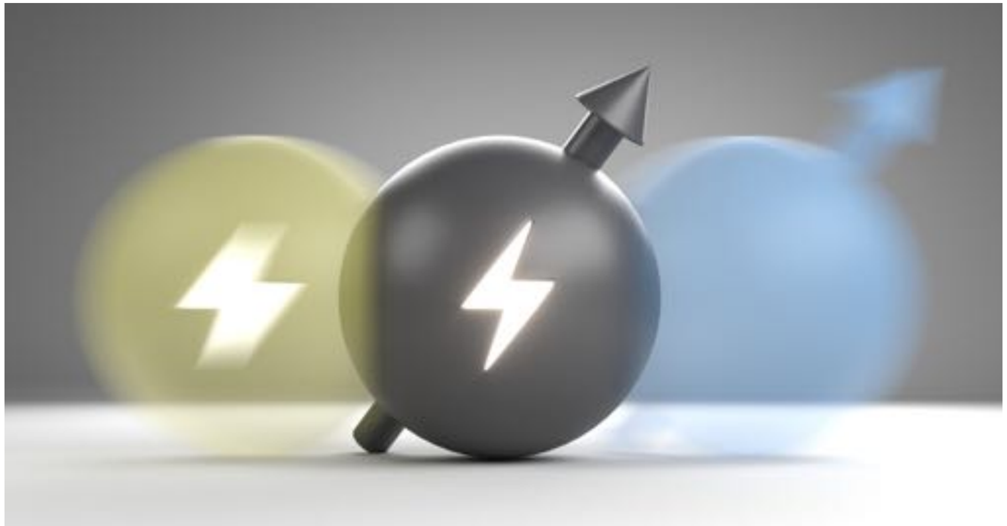
$$v_b = \frac{J_{ex}a}{2\hbar\Delta}$$

$$\Rightarrow \boxed{\frac{v_f}{v_b} = 2\Delta}$$

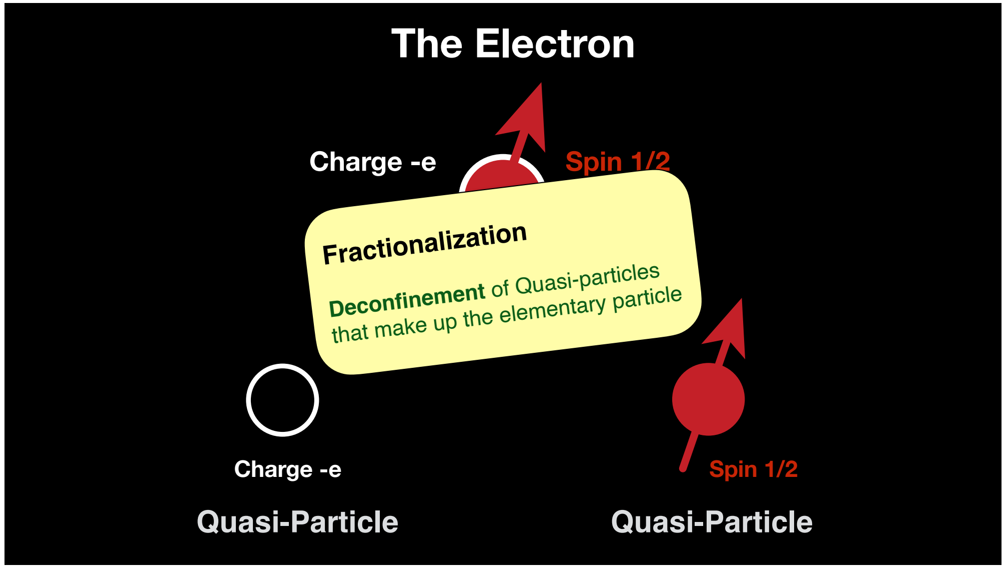
We find: $\frac{v_f}{v_b} = 2.3(3)$

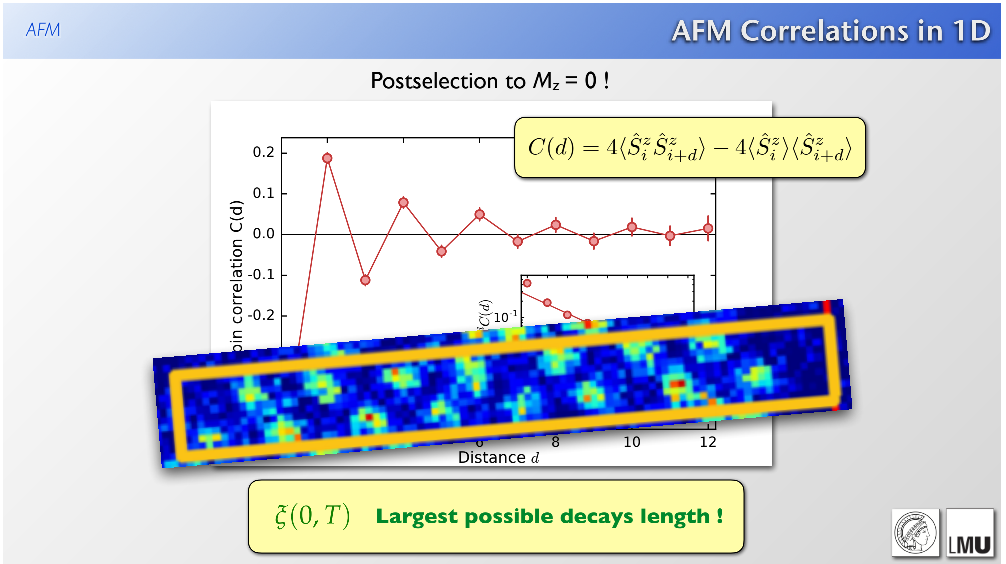
Outline

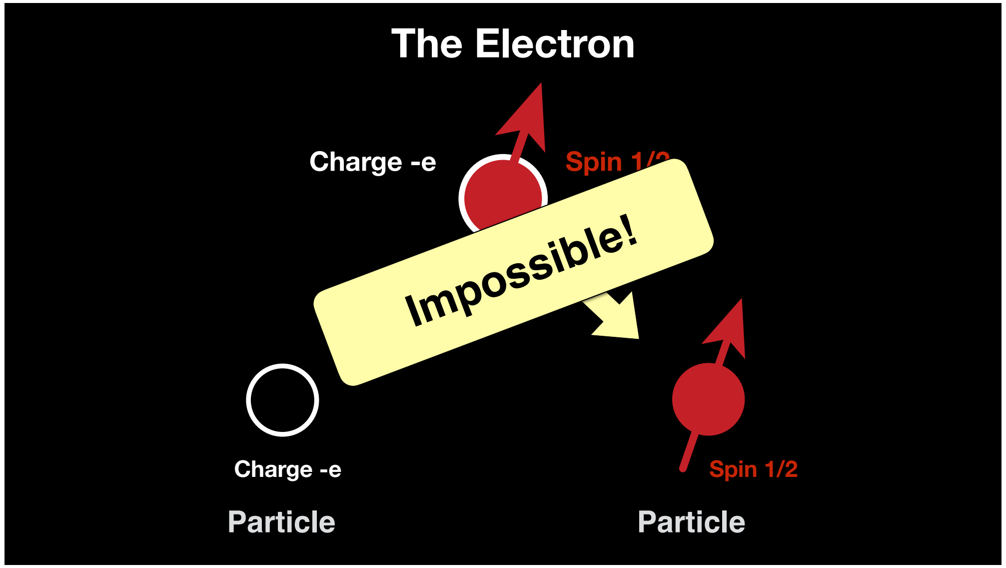
1. Bound Magnons
2. Spin-Charge Fractionalization in Fermi Hubbard Chains
3. Connection to Ground State Non-Local Order
4. Kardar-Parisi-Zhang Universality



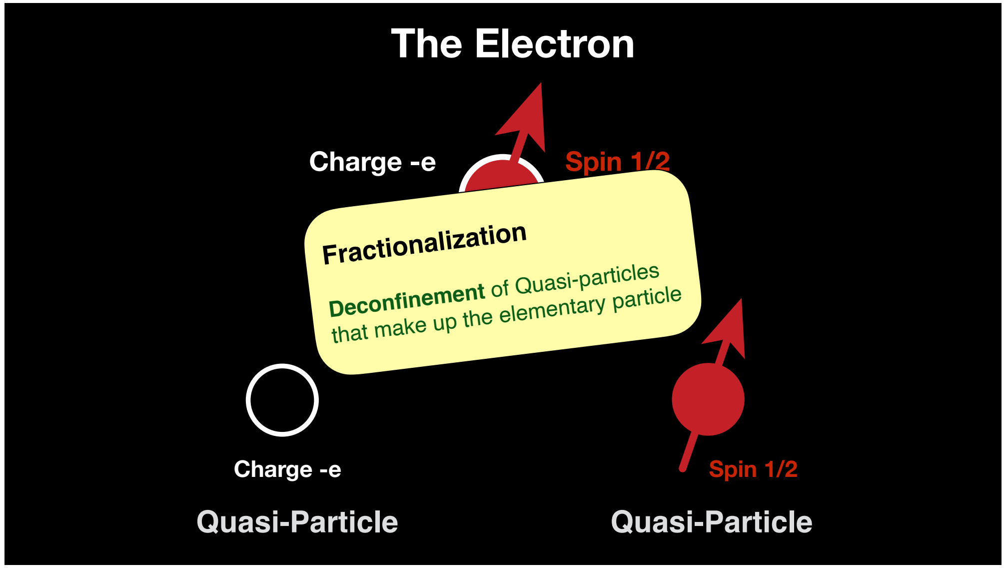
J. Vijayan *et al.* Science **367**, 186 (2020)



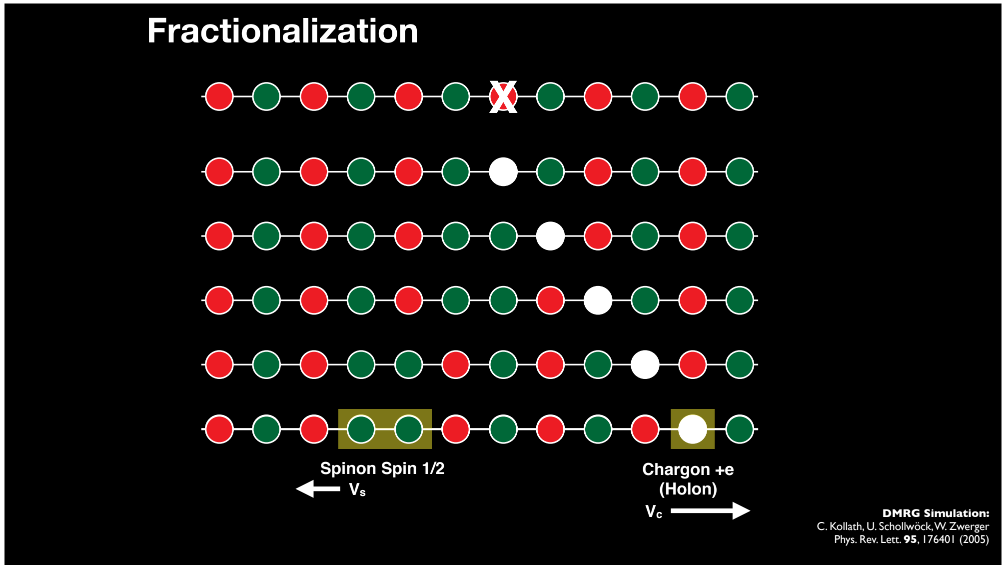




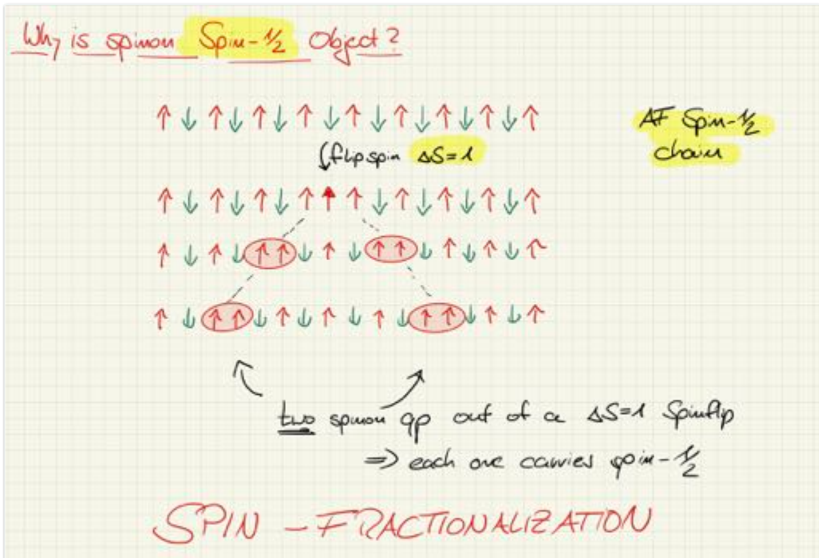
The Electron
Charge -e
Spin 1/2
Impossible!
Charge -e
Spin 1/2
Particle
Particle



The Electron
Charge -e
Spin 1/2
Fractionalization
Deconfinement of Quasi-particles that make up the elementary particle
Charge -e
Spin 1/2
Quasi-Particle
Quasi-Particle



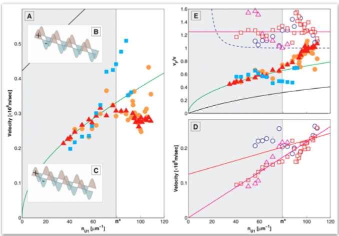
Fractionalization
Spinon Spin 1/2
V_s
Chargon +e
(Holon)
V_c
DMRG Simulation:
C. Kollath, U. Schollwöck, W. Zwerger
Phys. Rev. Lett. **95**, 176401 (2005)



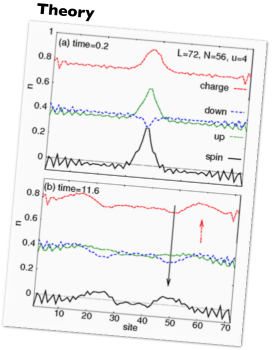
Why is spinon Spin-½ Object?
AF Spin-½ chain
flip spin $\Delta S=1$
two spinon qp out of a $\Delta S=1$ Spinflip
⇒ each one carries spin-½
SPIN - FRACTIONALIZATION

Spin-Charge Separation

Experiment



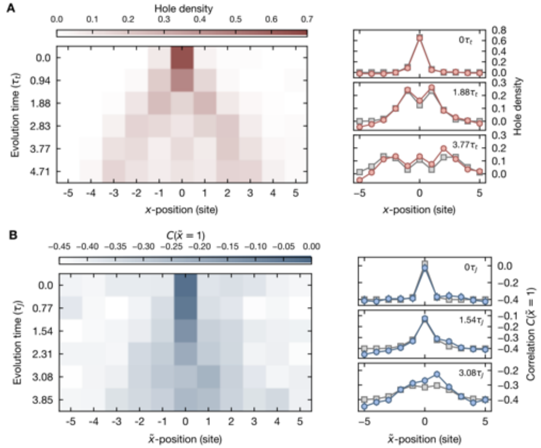
Theory



Spectroscopic determination:
C. Kim, et al. Phys. Rev. Lett. **77**, 4054 (1996)
O.M. Auslaender et al. Science **308**, 88 (2005)

DMRG Simulation:
C. Kollath, U. Schollwöck, W. Zwerger
Phys. Rev. Lett. **95**, 176401 (2005)

Dynamical Spin Charge Separation



Hole Dynamics

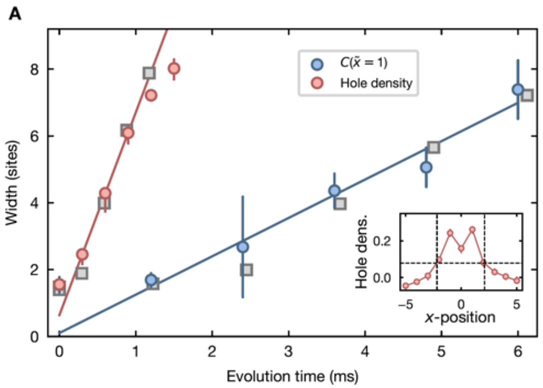
$\langle \hat{h}_i \rangle$

Spin Dynamics

$C(1) = \langle \hat{S}^z_i \hat{S}^z_{i+1} \rangle$

(squeezed space)

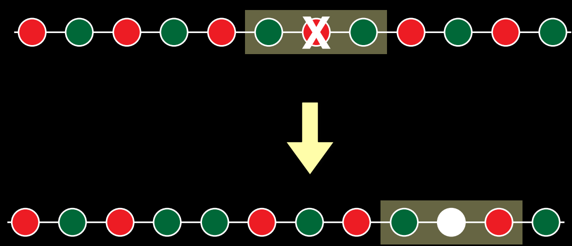
Spin & Charge Velocities



Fractionalization - Hole Shedding Spinon

Spin attached to hole

$\langle \hat{S}^z_{i-1}\, \hat{h}_i\, \hat{S}^z_{i+1} \rangle > 0$

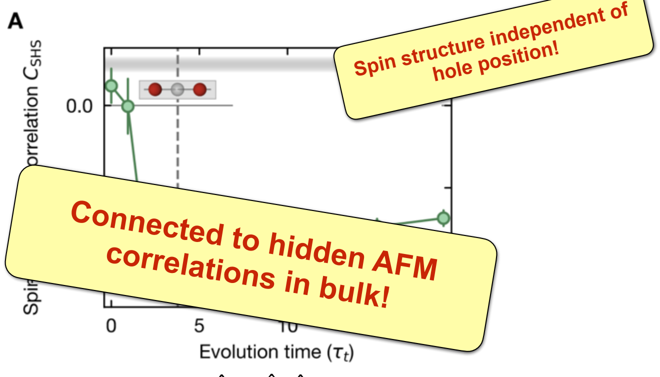


$\langle \hat{S}^z_{i-1}\, \hat{h}_i\, \hat{S}^z_{i+1} \rangle < 0$

Hole got rid of spin

SC Separation

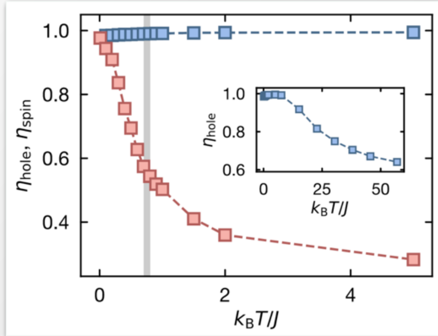
Spin-Hole-Spin Correlations



$$\langle \hat{S}^z_{i-1}\, \hat{h}_i\, \hat{S}^z_{i+1} \rangle$$

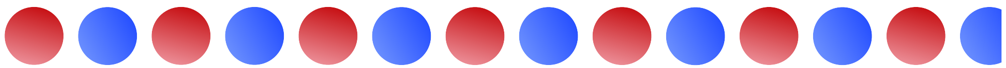
SC Separation

Fractionalization at Finite Temperatrues



Holon created with **unit efficiency**
Spinon created with **50-60% efficiency**

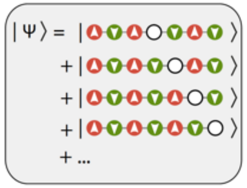
Hidden Order in the Ground State



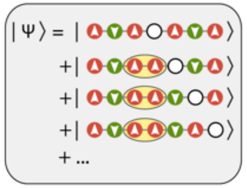
Charge & Spin Order around Hole

Minimize Energy ⇨ Two Conditions

- Holes want to delocalise
- Spins want to align antiferromagnetically

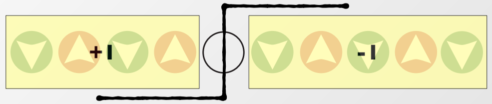


Ground State



Excited State

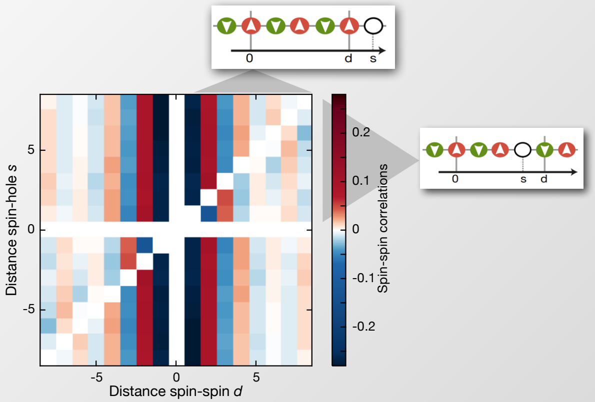
Microscopic Origin of SC —Separation



Hole introduce domain wall
“parity” kinks in AFM background!

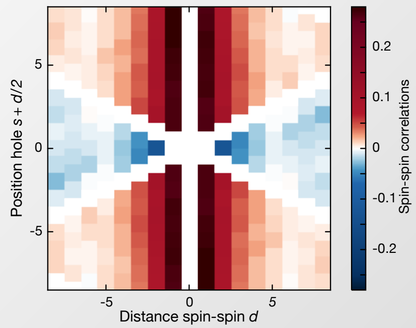
Hole = Non local topological excitations!

AFM around Holes



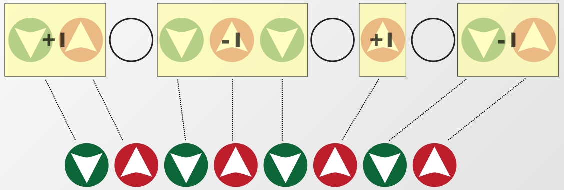
$$C_{s,h}(s,d) = \langle \hat{S}_z(i)\hat{h}_{i+s}\hat{S}_z(i+d)\rangle$$

AFM around Holes



$$C_{s,h}(s,d) = (-1)^d \langle \hat{S}_z(i)\hat{h}_{i+s}\hat{S}_z(i+d)\rangle$$

Hidden (Topological) Order in the 1D Hubbard



Heisenberg AFM in "Squeezed Space"

$$\Psi(x_1,\ldots,x_N) = \Psi_{SF}(x_1,\ldots,x_N)\Psi_{\text{Heis}}(y_1,\ldots,y_M)$$

F. Woynarovich J. Phys. C (1982)
M. Ogata & H. Shiba Phys. Rev. B (1990)

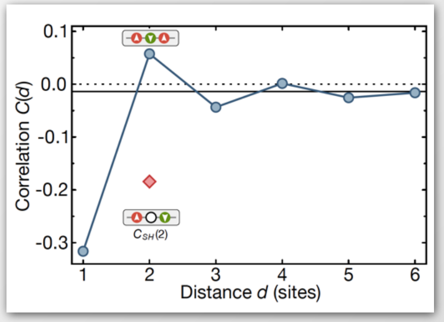
String Correlator

$$O_{top} = \langle \hat{S}^z(0)(-1)^{\sum_{j=1}^{d-1} 1-\hat{n}_j}\hat{S}_z(d)\rangle$$

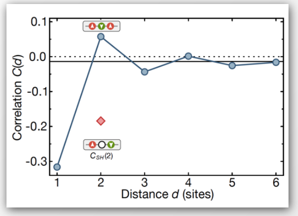
H.V. Kruis, I.P. McCulloch, Z. Nussinov & J. Zaanen EPL (2004)
H.V. Kruis, I.P. McCulloch, Z. Nussinov & J. Zaanen Phys. Rev. B (1990)

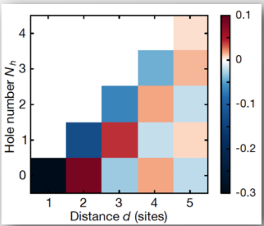
AFM order around a Single Hole

$$C_{s,h}(d=2)\langle \hat{S}_z(i)\hat{S}_z(i+2)|\hat{h}_{i+1}\rangle$$



AFM order around more holes





Spins around holes behave much as if they were direct neighbors!

Order Parameters

Typical Order Parameter in Landau Paradigm of Phase Transition

$$\lim_{|\mathbf{x}-\mathbf{y}|\to\infty}\langle \hat{A}(\mathbf{x})\hat{A}(\mathbf{y})\rangle = c$$

Order Parameter:

Examples: ...netism, AFM,...)

...unction)

General classification scheme for all phases of matter ???

Order Parameter Charac... ...ound State Correlations
Local ordering!

String Order in 1D Systems

E.g. in 1D gapped systems where $\langle \hat{A}(\mathbf{x})\hat{A}(\mathbf{y})\rangle$ decays exponentially with distance

However, they can show hidden non-local order:

$$\lim_{|\mathbf{x}-\mathbf{y}|\to\infty} \langle \hat{A}(\mathbf{x}) \left(\prod_{\mathbf{z}\in S(\mathbf{x},\mathbf{y})} \hat{B}(\mathbf{z}) \right) \hat{A}(\mathbf{y})\rangle = c$$

We say the order is hidden, because a "global view" of the underlying state is required. (**Topological Order:** X.-G. Wen)

Allows us to characterize state only via its ground state correlations!

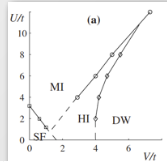
M. den Nijs, K. Rommelse, Phys. Rev. B 40, 4709 (1989).
E. Kim, G. Fa´th, J. So´lyom, D. Scalapino, Phys. Rev. B 62, 14965 (2000)
E. G. Dalla Torre, E. Berg, E. Altman, Phys. Rev. Lett. 97, 260401 (2006)
F. Anfuso, A. Rosch, Phys. Rev. B 75, 144420 (2007)
E. Berg, I E. Dalla Torre, T. Giamarchi, E. Altman, Phys. Rev. B 77, 245119 (2008)

An Example: Haldane Insulator in 1D

E. G. Dalla Torre, E. Berg, E. Altman, Phys. Rev. Lett. 97, 260401 (2006)
E. Berg, I E. Dalla Torre, T. Giamarchi, E. Altman, Phys. Rev. B 77, 245119 (2008)

$$H = -J\sum_i (\hat{a}_i^\dagger \hat{a}_{i+1} + \text{h.c.}) + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1) + V\sum \hat{n}_i\hat{n}_{i+1}$$

Bose-Hubbard with next-neighour interaction



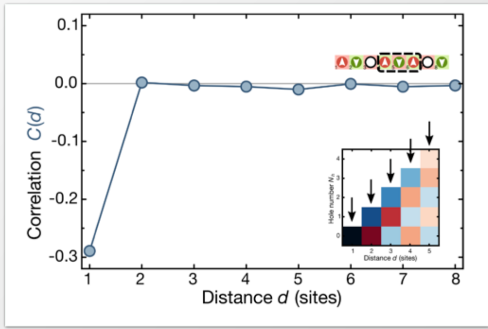
0 -1 0 0 +1 0 0 -1 0 +1 0 0 0 -1

A Hidden Antiferromagnet!

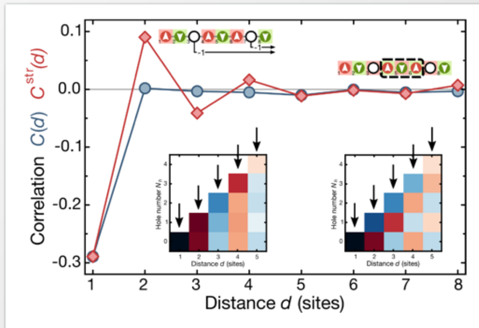
Hidden Non-local Order Captured by String Correlator

$$\mathcal{O}_S^2 = -\lim_{|i-j|\to\infty} \left\langle \delta\hat{n}_i \left(\prod_{i<k<j} e^{i\pi\delta\hat{n}_k} \right) \delta\hat{n}_j \right\rangle$$

Two Point Correlator - Doped Chains



Two Point Correlator - Doped Chains



Quantum Gas Microscopy of Kardar-Parisi-Zhang Superdiffusion

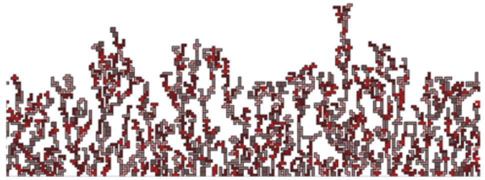
Experiment: D. Wei, A. Rubtio-Abadal, K. Srakaew, C. Gross, J. Zeiher, I.B.
Theory: B. Ye , F. Machado, J. Kemp. N. Yao, S. Gopalakrishnan **arXiv:2107.00038**

Kardar-Parisi-Zhang Equation

$$\frac{\partial h(\vec{x},t)}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta(\vec{x},t)$$

Martin Hairer
Fields Medal
(2014)

Non-linear stochastic differential equation describing **temporal change of height field**

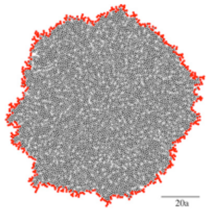


Growth of interfaces/surface growth

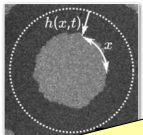
I. Corwin, Notices of the AMS **63**, 230 (2016)

M. Kardar, G. Parisi & Y.-C. Zhang PRL **56,** 889 (1986)
C.A. Tracy & H. Widom Comm. Math. Phys. **159,** 151 (1994)
C.A. Tracy & H. Widom Comm. Math. Phys. **177,** 727 (1994)
M. Prähofer & H. Spohn PRL **84**, 4882 (2000)

Growth of Interfaces

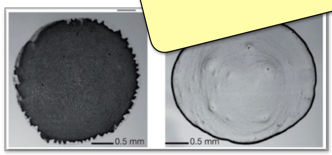


Bacterial/Tumor Growth
(Eden Growth Process)

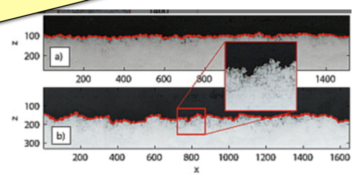


Fluff Fire, Forest Fires

Universality

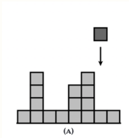


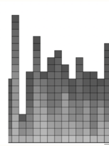
Coffee Stains



Snow Surfaces

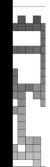
Growth of Interfaces





Random [redacted]
(non-stic[redacted]

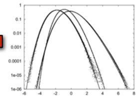
Random D[redacted]
(non-stick[redacted]

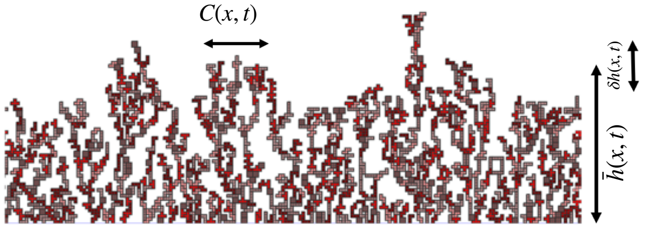


(sticky Tetris)

KPZ Universality Class (1D)

$$h(x,t) \simeq v_\infty t + A t^{1/3} \chi(X,t)$$





KPZ Universality Class

- $t^{1/3}$ height fluctuation with **GOE/GUE Tracy-Widom** Limit
- $t^{2/3}$ transv. spatial correlations

$$\delta h \propto t^{\beta} \qquad \beta = 1/3$$
$$\xi \propto t^{1/z} \qquad z = 3/2$$

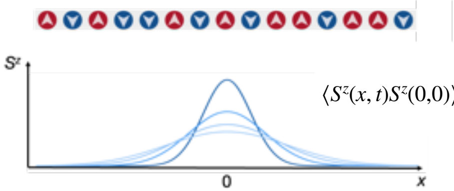
I. Corwin, Notices of the AMS **63**, 230 (2016)
K. Takeuchi Physica A **504,** 77 (2018)

A Quantum Surprise:
Infinite Temperature Spin Transport in Heisenberg Quantum Magnets is in KPZ Universality Class

Numerical Evidence

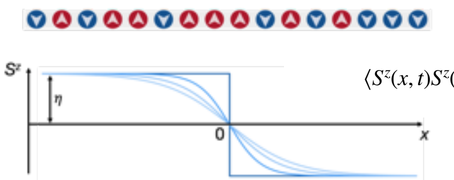
M. Ljubotina et al., Nature Comm. (2017)
M. Ljubotina et al., Phys. Rev. Lett. (2019)
Review: see V. Bulchadini, S. Gopalakrishnan, E. Ilievski arXiv:2103.0197

Numerical Evidence - Infinite T Heisenberg Dynamics



$$\langle S^z(x,t) S^z(0,0)\rangle \sim \partial^2 C(x,t) \sim f_{KPZ}\left(x/t^{1/z}\right)$$

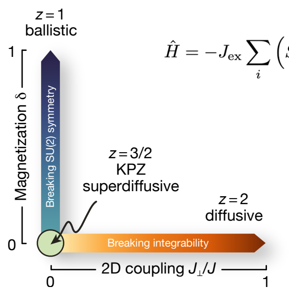
Consider **spin domain wall** at $T \to \infty$



$$\langle S^z(x,t) S^z(0,0)\rangle \overset{\eta \to 0}{\sim} \partial_x \langle S^z(x,t)\rangle_{DW}$$

M. Ljubotina et al., Nature Comm. (2017)
M. Ljubotina et al., Phys. Rev. Lett. (2019)
Review: see V. Bulchadini, S. Gopalakrishnan, E. Ilievski arXiv:2103.0197

Heisenberg Model Revisited



$$\hat{H} = -J_{\mathrm{ex}} \sum_i \left(\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y + \Delta \hat{S}_i^z \hat{S}_{i+1}^z\right)$$

Here: $\Delta = 1$

Space time scaling

$$x \sim t^{1/z}$$

- z=1 ballistic
- 1<z<2 superdiffusive
- z=2 diffusive

Numerical Evidence:
M. Ljubotina *et al.,* Nature Comm. (2017)
M. Ljubotina *et al.,* Phys. Rev. Lett. (2019)

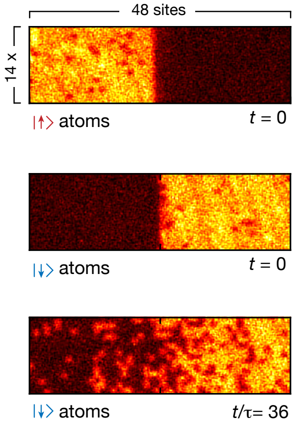
Understanding (generalied GHD, SU(2) & Integrability,…)
S. Gopalakrishnan and R. Vasseur, Phys. Rev. Lett. (2019)
J. De Nardis, Phys. Rev. Lett. (2019)
S. Gopalakrishnan, R. Vasseur, and B. Ware, PNAS (2019)
V. B. Bulchandani, Phys. Rev. B (2020)
Reviews:
B. Bertini at el. Rev. Mod. Phys. (2020)
V. B. Bulchandani, S. Gopalakrishnan, and E. Ilievski, arXiv:2103.01976

Subtle interplay of **integrability** (stable quasiparticles) & **Non-abelian SU(2) symmetry** in Heisenberg model.

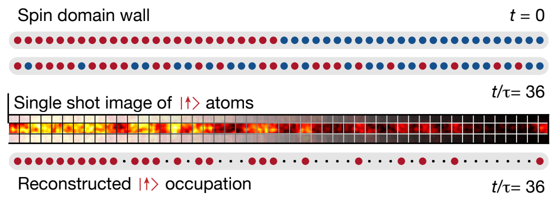
Exp.: A. Scheie *et al.* Nature Physics (2021)
Related: Transport via Spin-Spirals
S. Hild *et al.* Phys. Rev. Lett. (2014)
P.N. Jepsen *et al.* Nature (2020)

Experimental Measurement

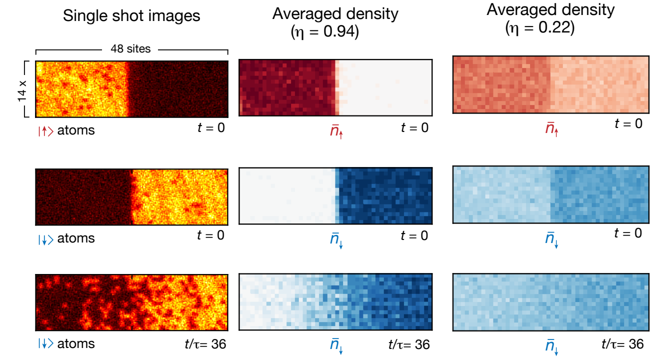
Single shot images



Box potential!
No harmonic confinement.

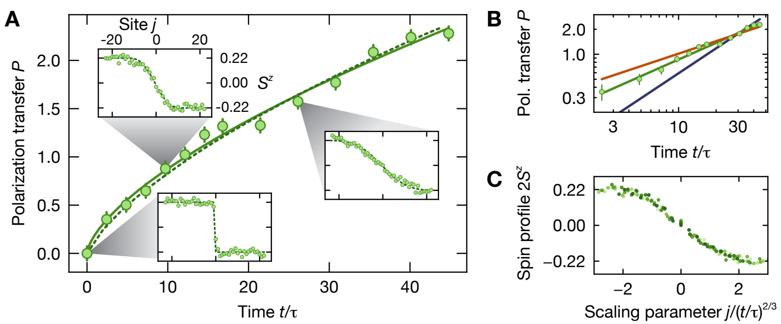


Experimental Measurement



$$\rho \sim (1+\eta\sigma_z)^{N/2} \otimes (1-\eta\sigma_z)^{N/2}$$

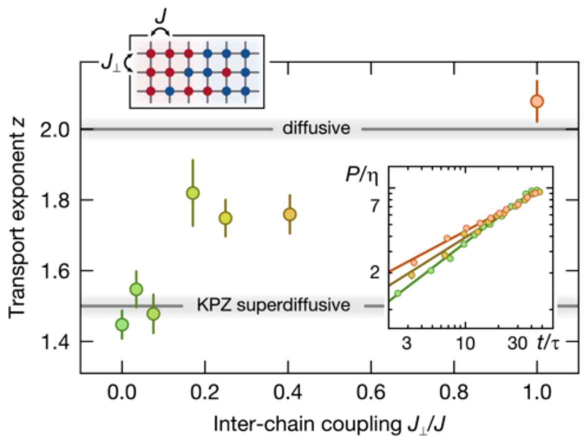
Polarisation Transfer



$$P(t) \sim t^{1/z} \qquad z_{fit} = 1.54(7)$$

M. Ljubotina *et al., Nature Comm. (2017).*
M. Ljubotina *et al., Phys. Rev. Lett.* (2019)
Exp. Spin Chains: A. Scheie *et al.*
Nat. Phys. (2021)

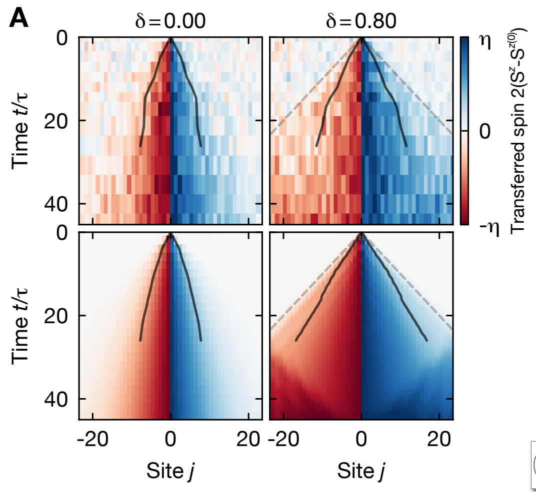
Breaking Superdiffusion - Breaking Integrability



Breaking Superdiffusion - Breaking Non-Abelian Symmetry

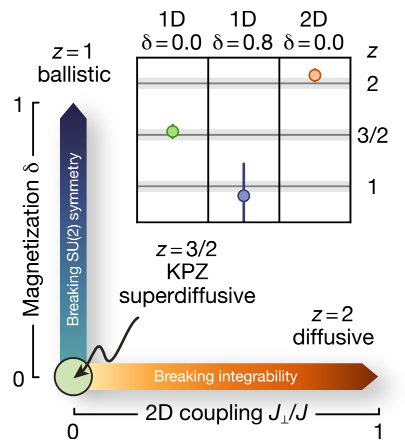
$\delta \neq 0$

Magnetizing System



Anomalous Transport

Ballistic-Superdiffusion-Diffusion



KPZ

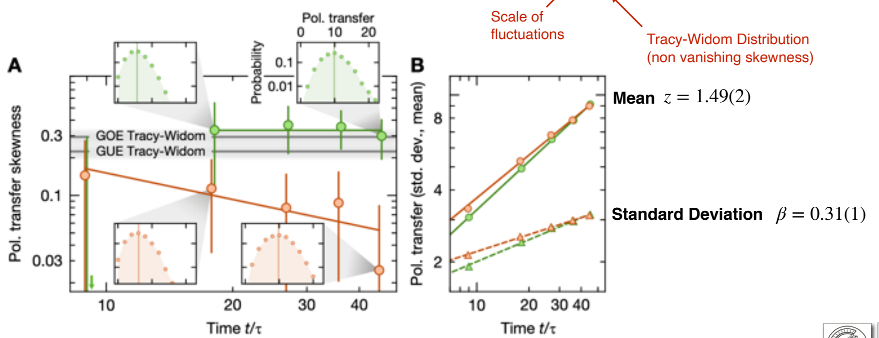
Tracy-Widom Distribution Functions

Polarization transfer ⟺ **height field (at origin) of KPZ equation**

$$\delta h = A t^{1/3} \chi(0,t)$$

Scale of fluctuations

Tracy-Widom Distribution (non vanishing skewness)



A.K. Hartmann et al. EPL **121**, 67004 (2018)

Experimental Evidence for KPZ

1. Superdiffusive transport (z=3/2) can arise from linear model!
2. Polarisation fluctuation scale (β=1/3)
3. Skewness of Polarisation fluctuation distribution constant (and compatible with Tracy-Widom GOE/GUE)
4. Breaking SU(2) symmetry OR breaking integrability destroys KPZ behaviour

arXiv:2107.00038

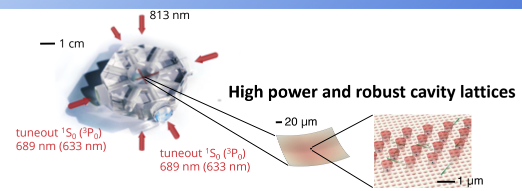
Where Next?

Scaling Up

Next 1-2 years

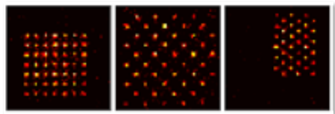
Ultracold Atoms

~10000 atoms



Tweezer Arrays

~100 atoms



Ion Traps

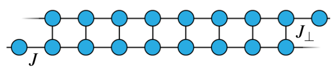
~50 atoms

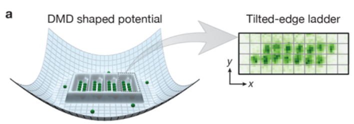
Big advantage: all atoms are the same! (built by nature)

Controllability

Flexible Geomtries and Larges Sizes

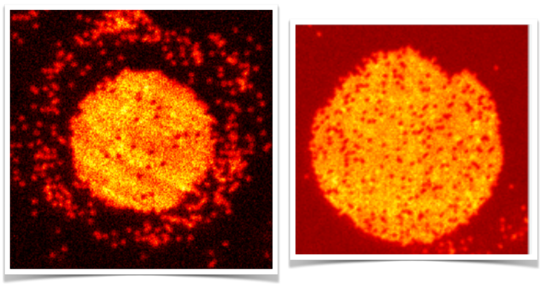
Quantum Ladders with flexible edge geometries (SPT Phases)
Haldane Spin Liquid





Fully tuneable coupling strengths

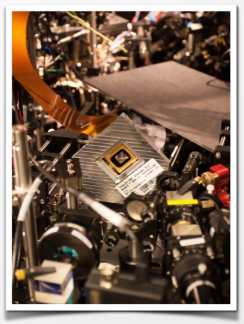
Large 2D Systems
(2500 atoms, filling >95%)



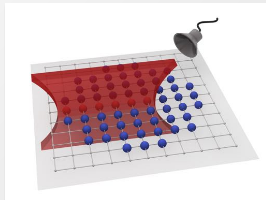
Tweezer SPT: Léséluc et al. Science 365, 6455 (2019)

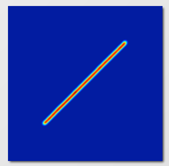
see also: Chiu et al. PRL 120, 243201 (2018)

Arbitrary Light Patterns

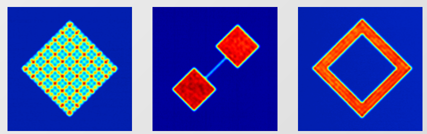


Digital Mirror Device (DMD)





Measured Light Pattern



Exotic Lattices | Quantum Wires | Box Potentials

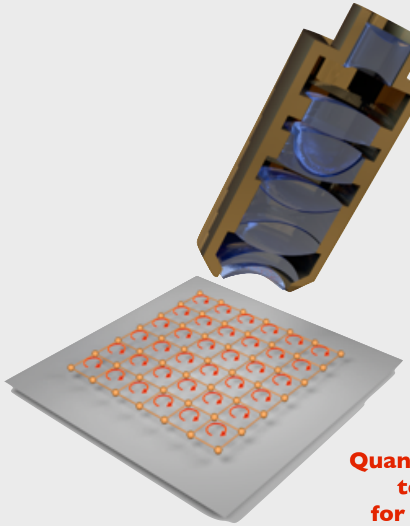
Almost Arbitrary Light Patterns Possible!

Single Spin Impurity Dynamics, Domain Walls, Quantum Wires, Novel Exotic Lattice Geometries, ...

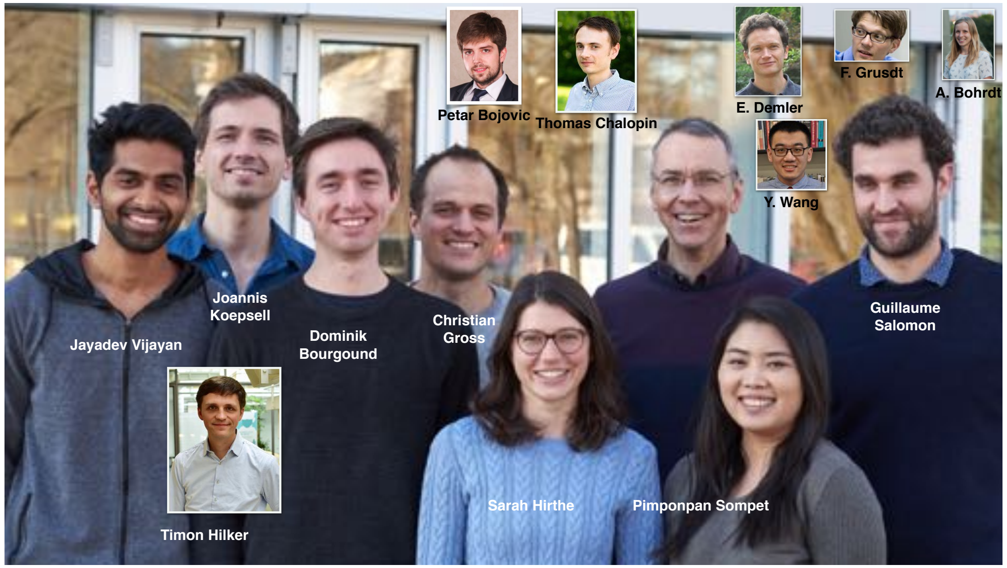
Outlook

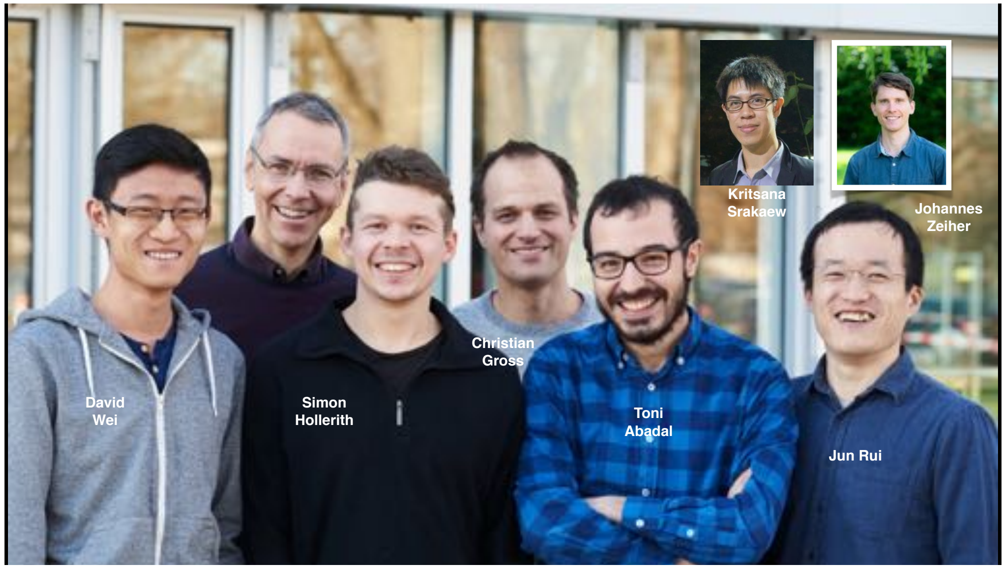
- Search for New Phases of Matter
- Extremely Strong Magnetic Field Physics
- Novel Quantum Magnets
- Controlled Quasiparticle Manipulations
- Non-Equilibrium Dynamics
  (New Types of Universality?)
- Thermalization in Isolated Quantum Systems
- Entanglement Measures in Dynamics
- Supersolids
- Cosmology - Black Hole Models?
- High Energy Physics/String Theory
- New clocks/Navigation
- Novel Quantum Light Matter Interfaces

⋮



Quantitative testbeds for theory!





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