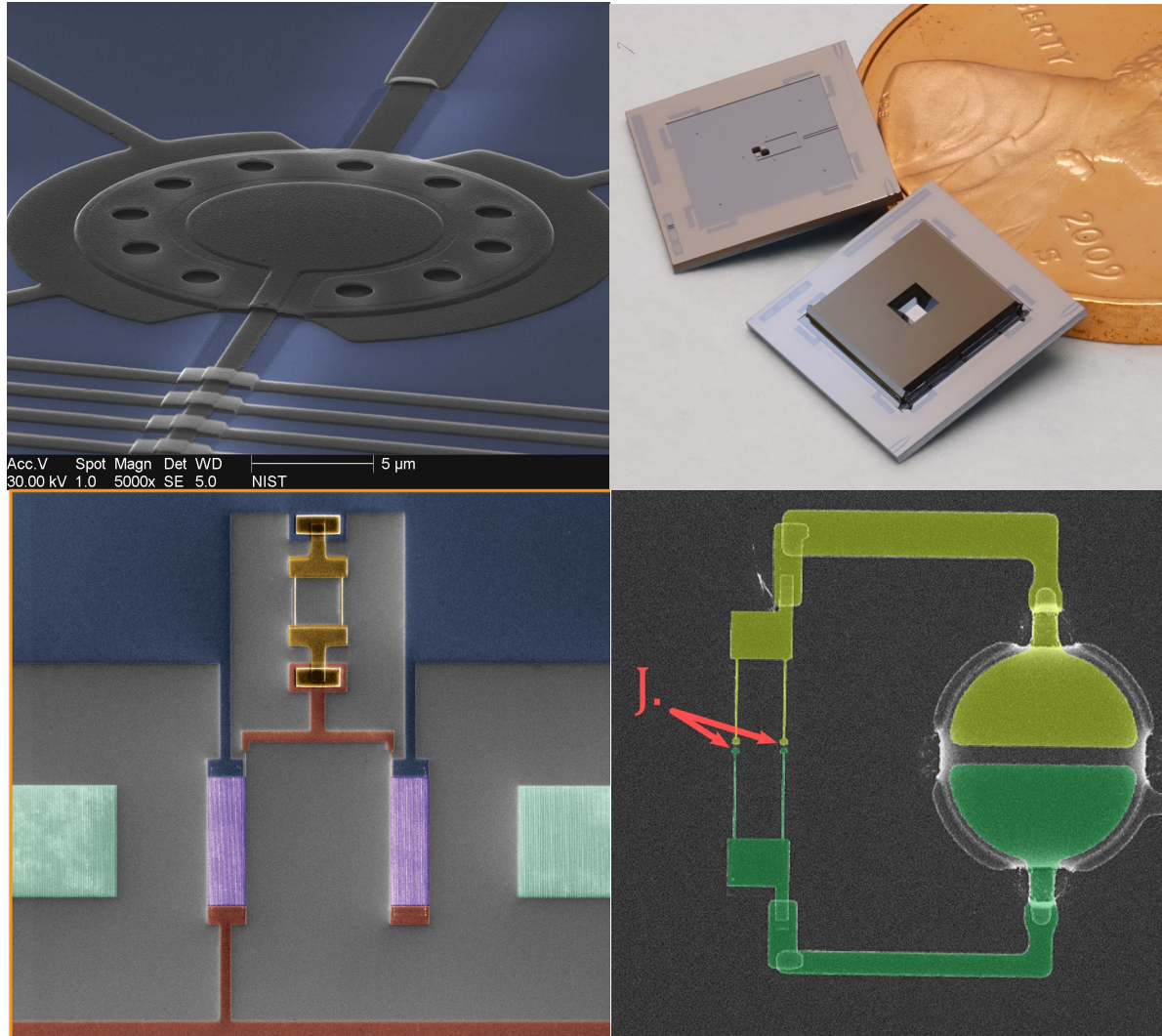
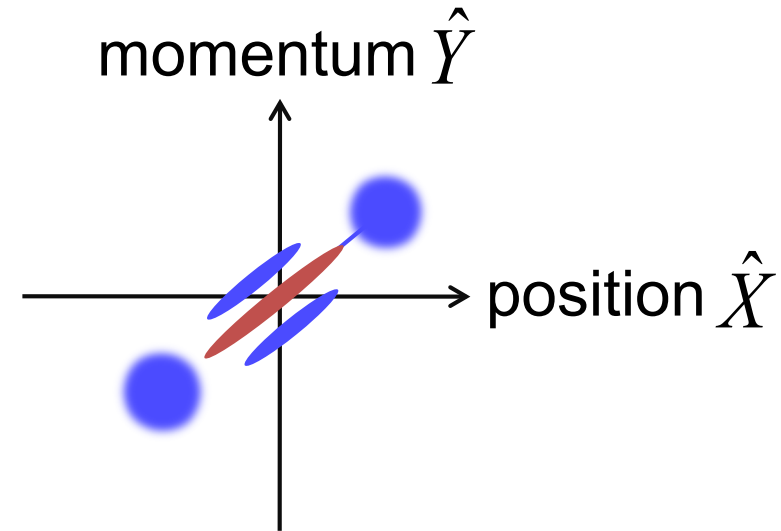
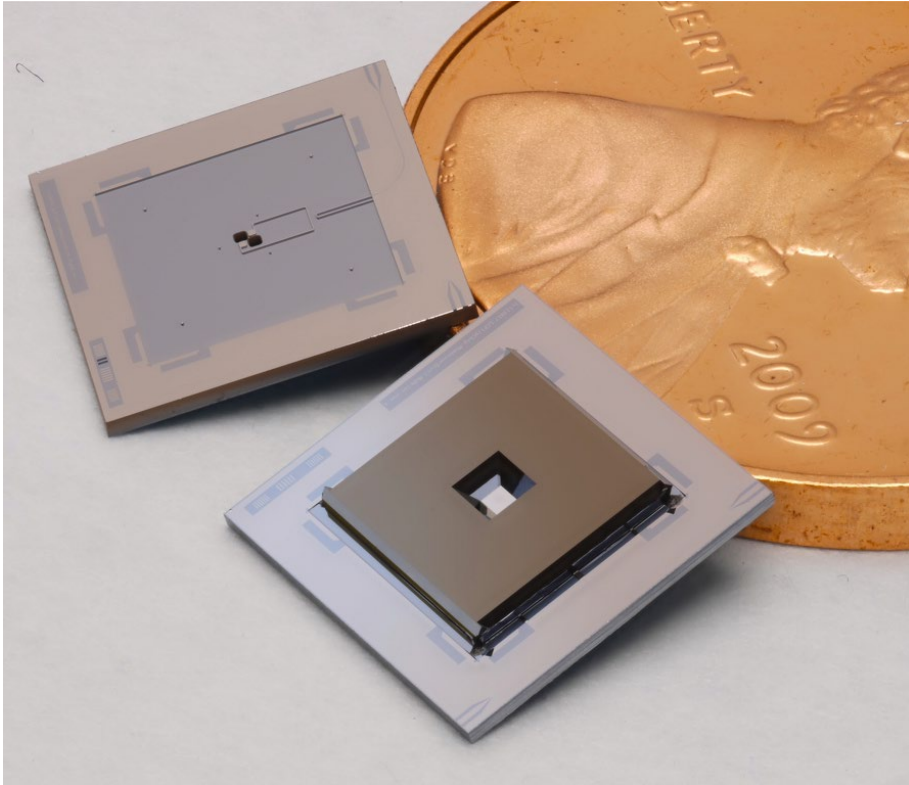


Quantum electromechanics



from linear quantum acoustics to
quantum phononics

Quantum superposition of macroscopically distinguishable states



mechanical coherent state

classical amplitude and phase

quantum uncertainty blob

mechanical “Schrödinger cat” state

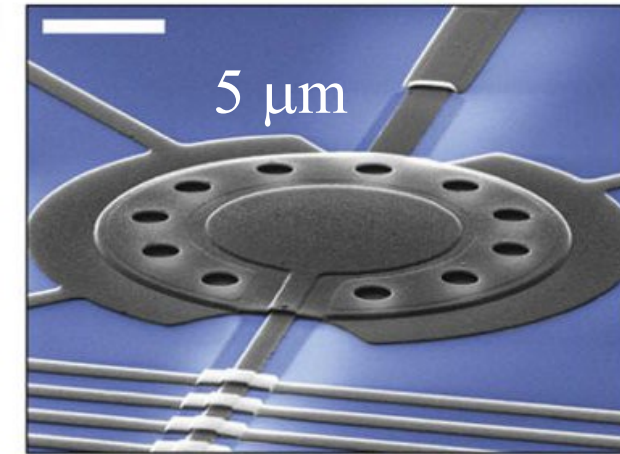
Wigner function negativity

Quantum measurement of macroscopic mechanical oscillators

standard quantum limit (SQL)

quantum non-demolition measurements (QND)

back action evasion (BAE)



On the measurement of a weak classical force coupled to a quantum-mechanical oscillator. I. Issues of principle

Carlton M. Caves, Kip S. Thorne, Ronald W. P. Drever, Vernon D. Sandberg, and Mark Zimmermann
Rev. Mod. Phys. **52**, 341 – Published 1 April 1980

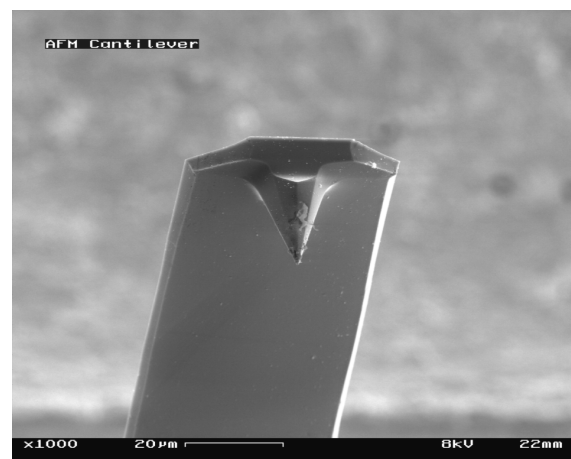
Measurement of Motion beyond the Quantum Limit by Transient Amplification

R. D. Delaney, A. P. Reed, R. W. Andrews, and K. W. Lehnert
Phys. Rev. Lett. 123, 183603 (2019)

Universal transducers/sensors in the quantum regime

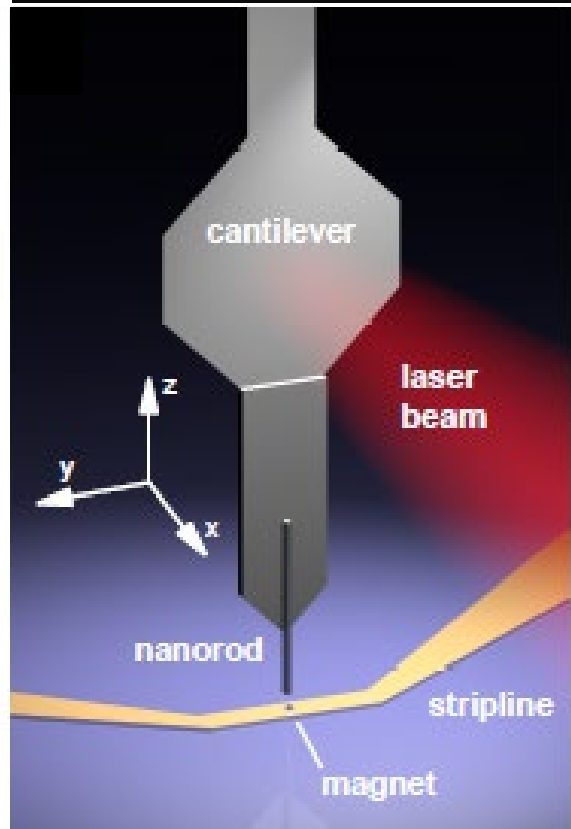
Atomic Force Microscope (AFM)

map atom scale surface forces to laser intensity



Magnetic Resonance Force Microscopy (MRFM)

image 3D nuclear spin density



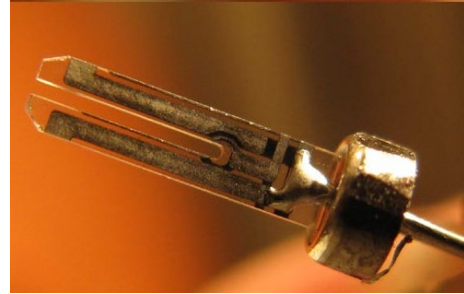
Acoustical information processing in the quantum regime?

sound waves: slow and small

$$v_s = c/100,000$$

“micro”- waves

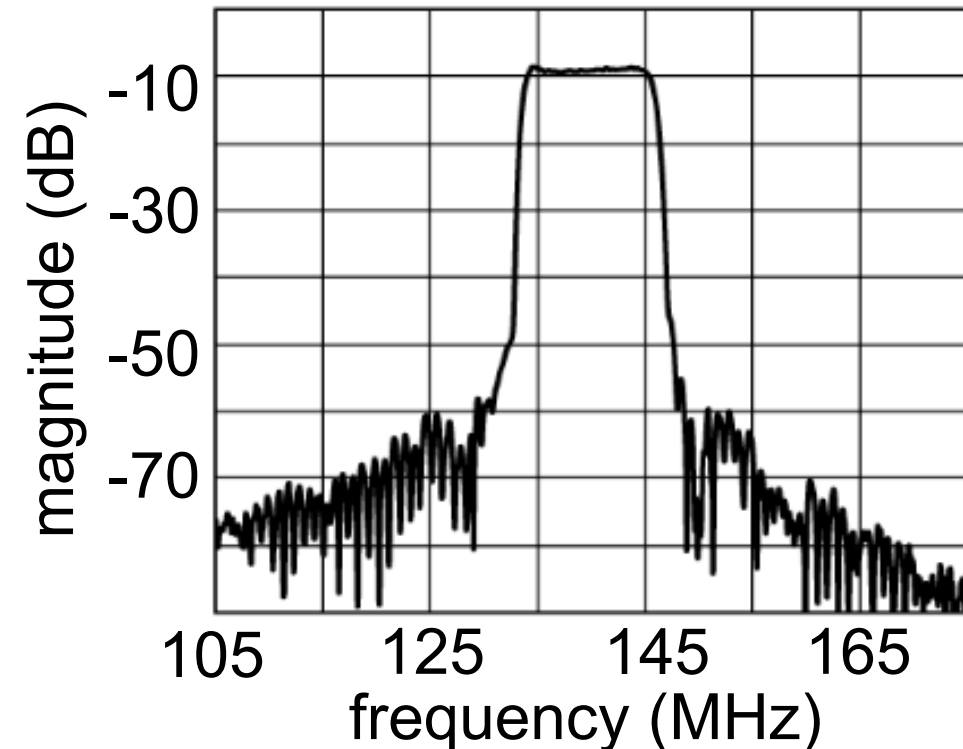
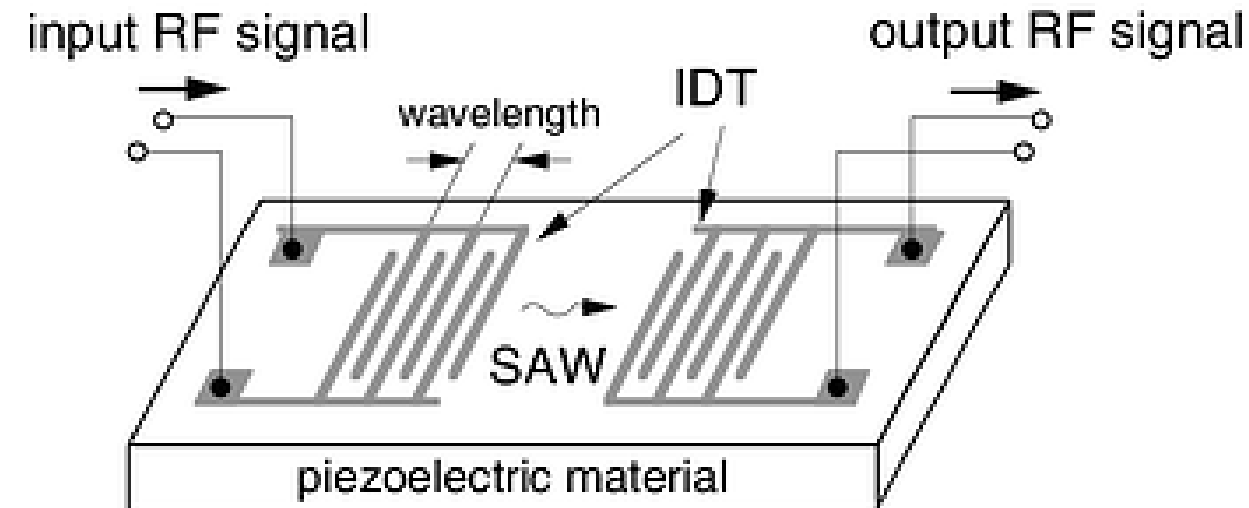
timing



delay



filtering



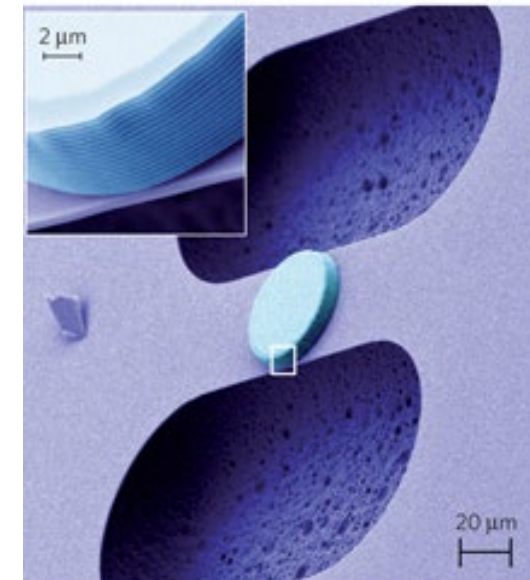
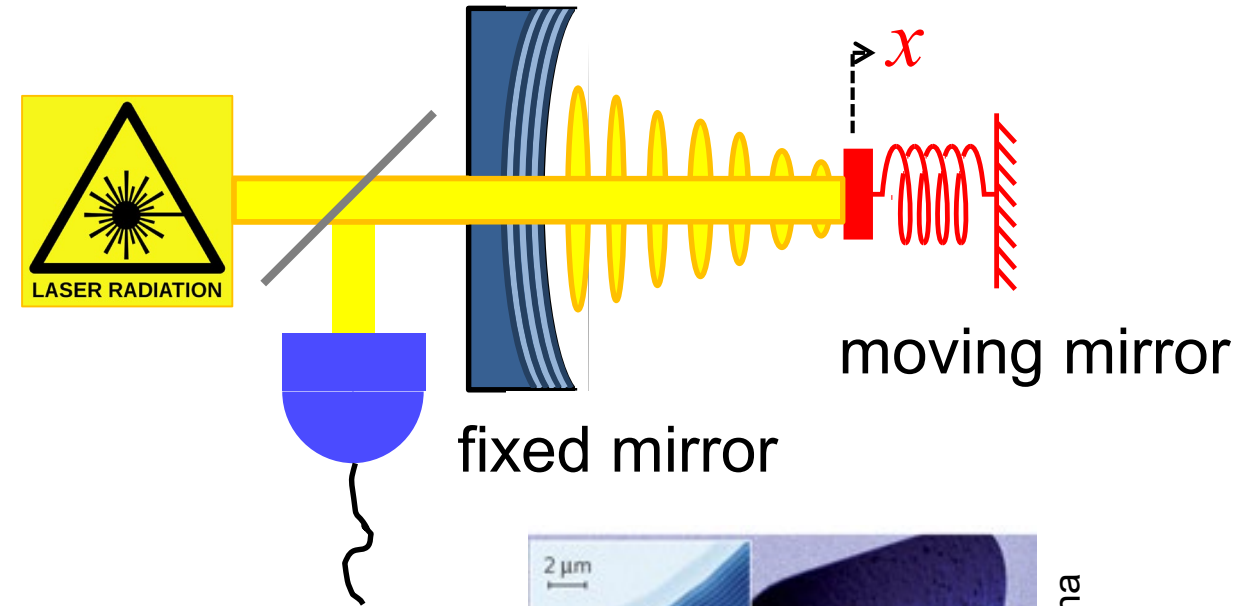
quantum electromechanics and optomechanics

Optical fields precisely control and measure macroscopic motion (optomechanics)

motion of mechanical oscillator
control with radiation force
infer through optical phase

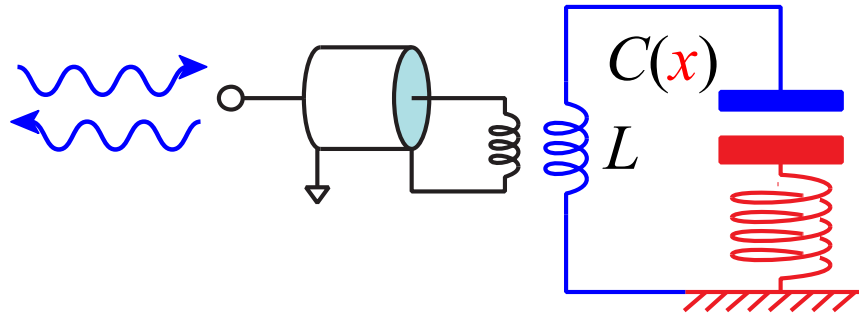


LIGO: Hanford



Aspelmeyer lab, IQOQI, Vienna

Quantum electromechanics: control and measure motion with electrical circuits



motion of compliant LC circuit
control with electrostatic force
infer through electrical phase

microwave frequency electrical circuits

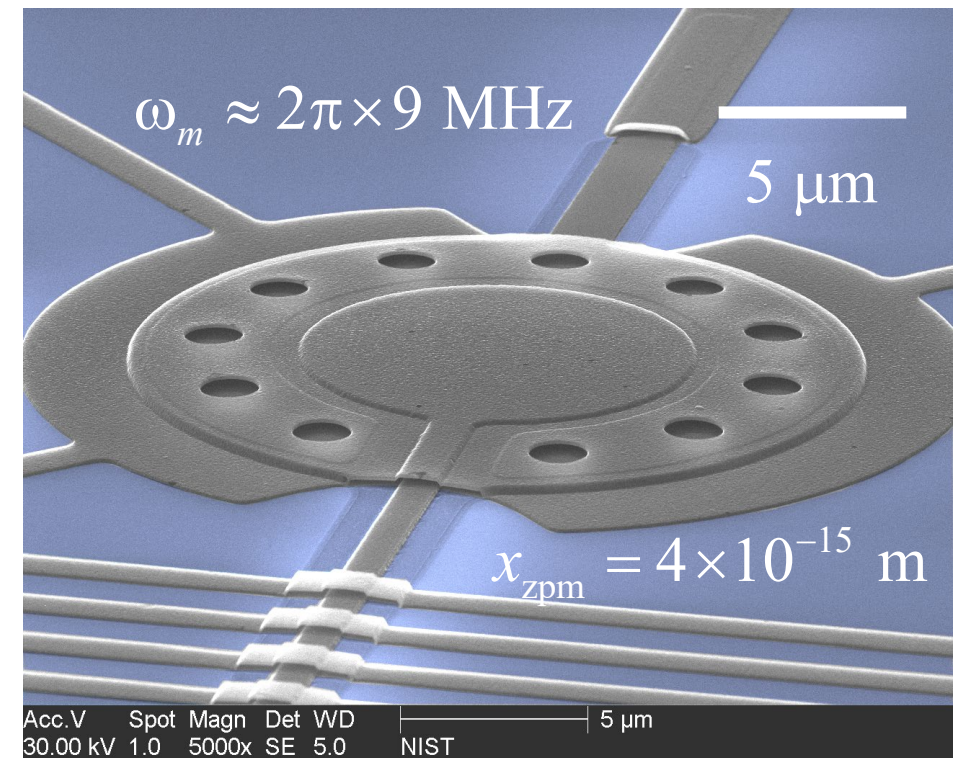
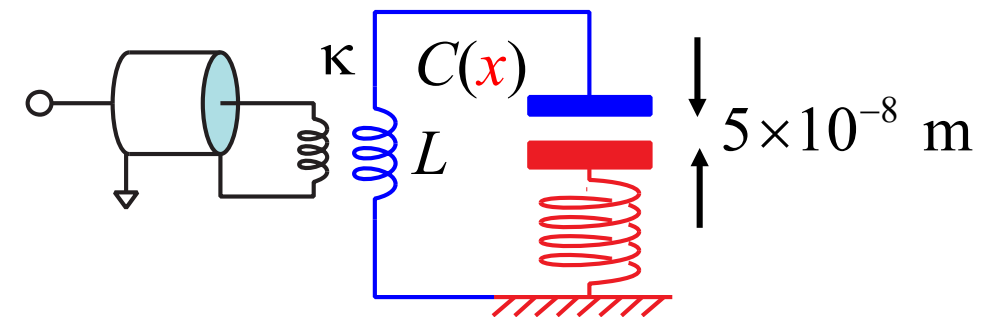
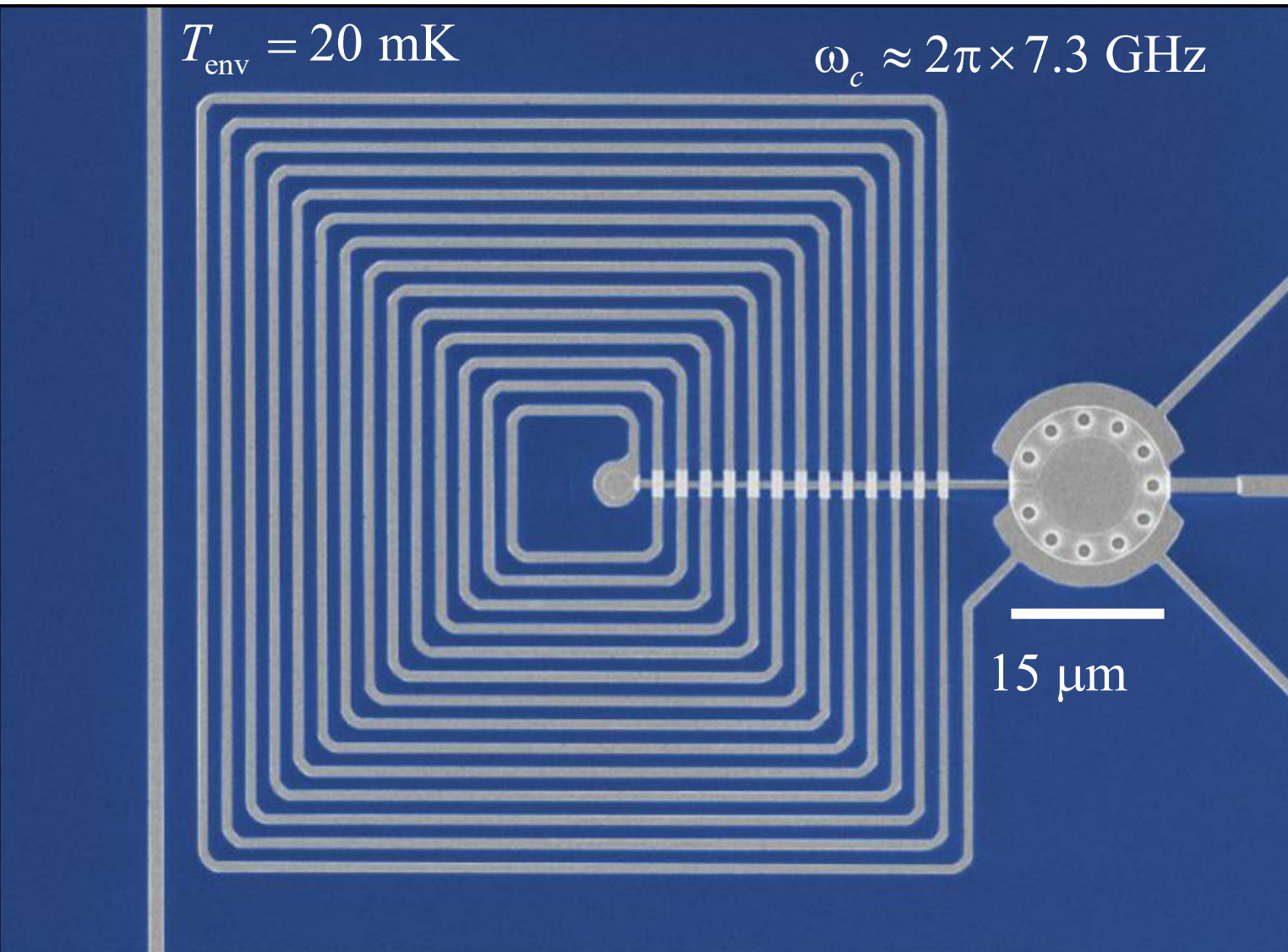
quantum operation at $T_{\text{env}} \ll 1$ K

interface with superconducting qubits



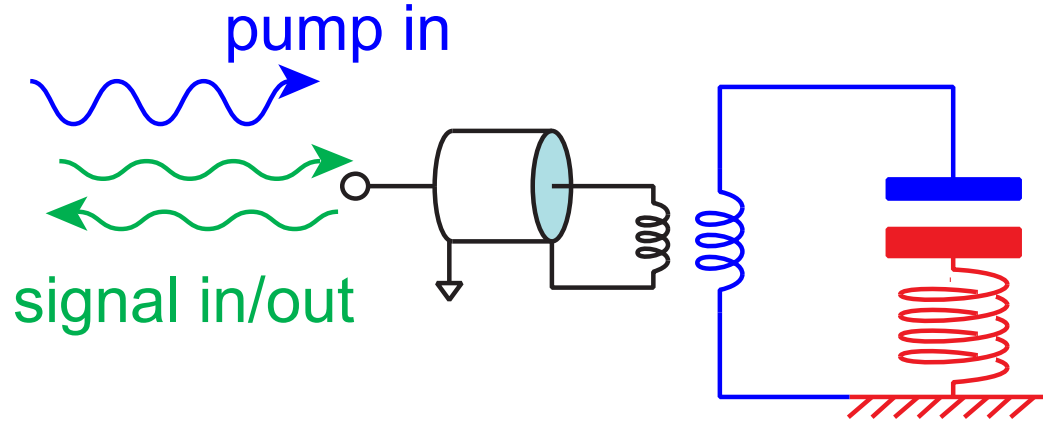
transmon qubit
in cavity

Vibrating membrane electromechanics

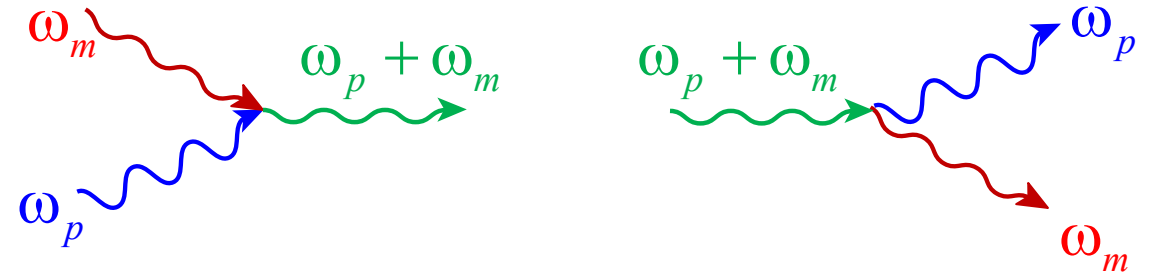


$$g_0 = 2\pi \times 200 \text{ Hz}$$

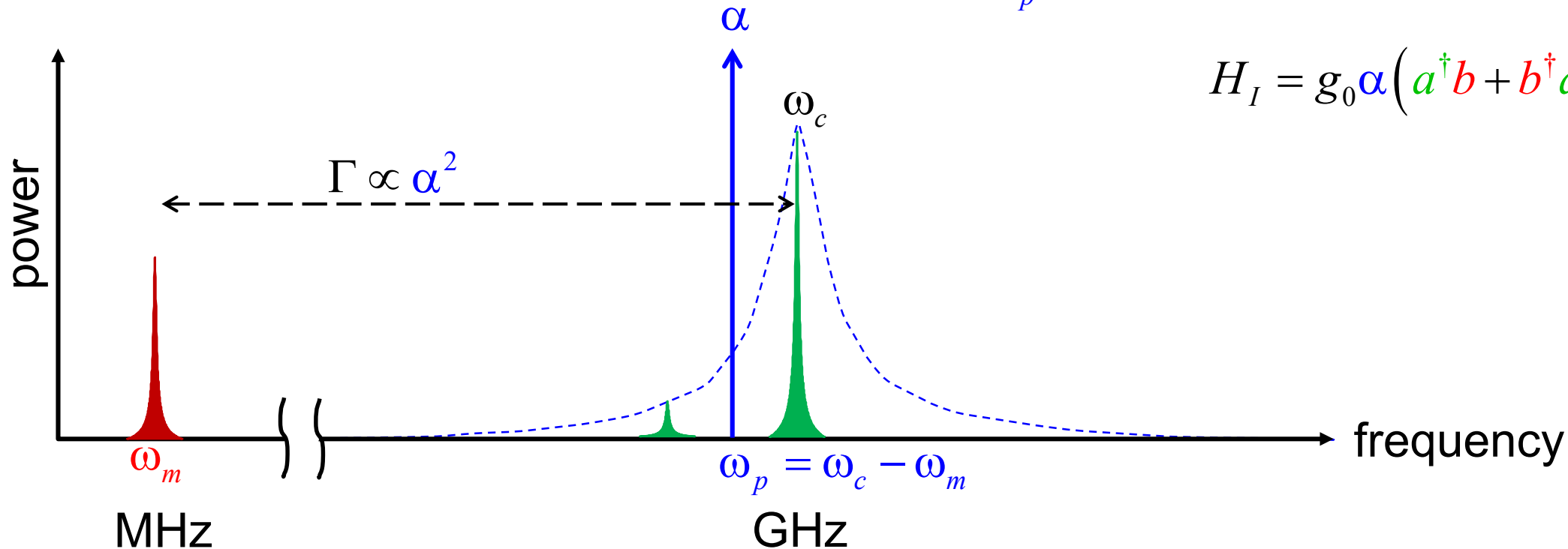
Pump creates parametric, linear coupling



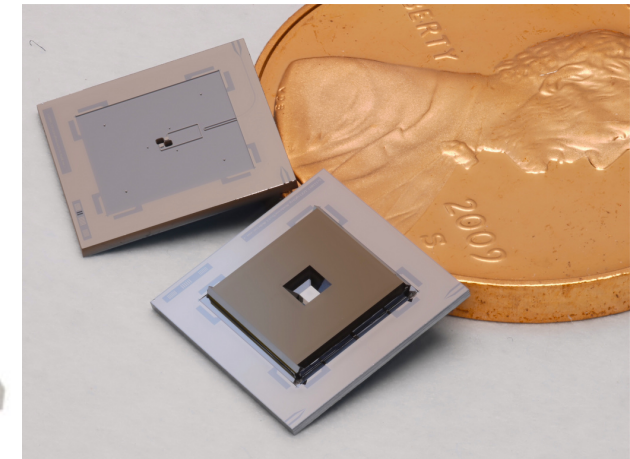
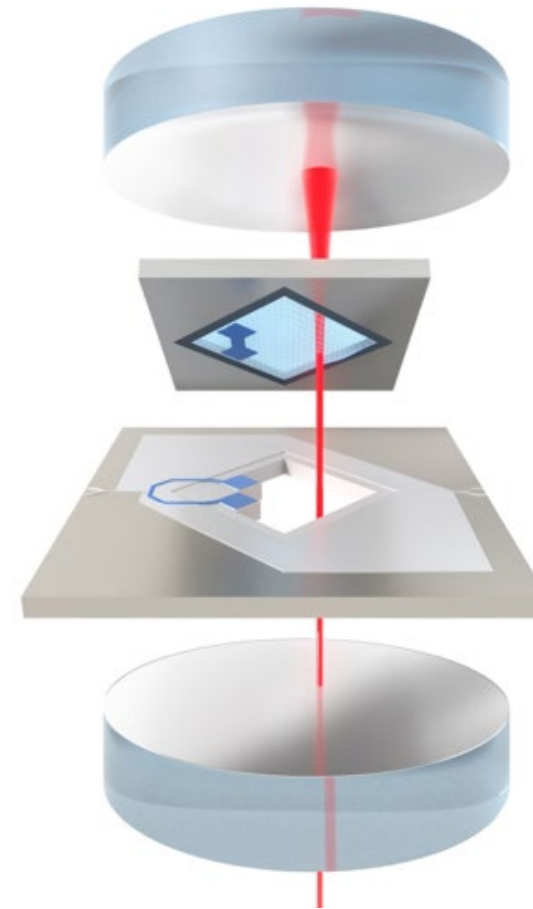
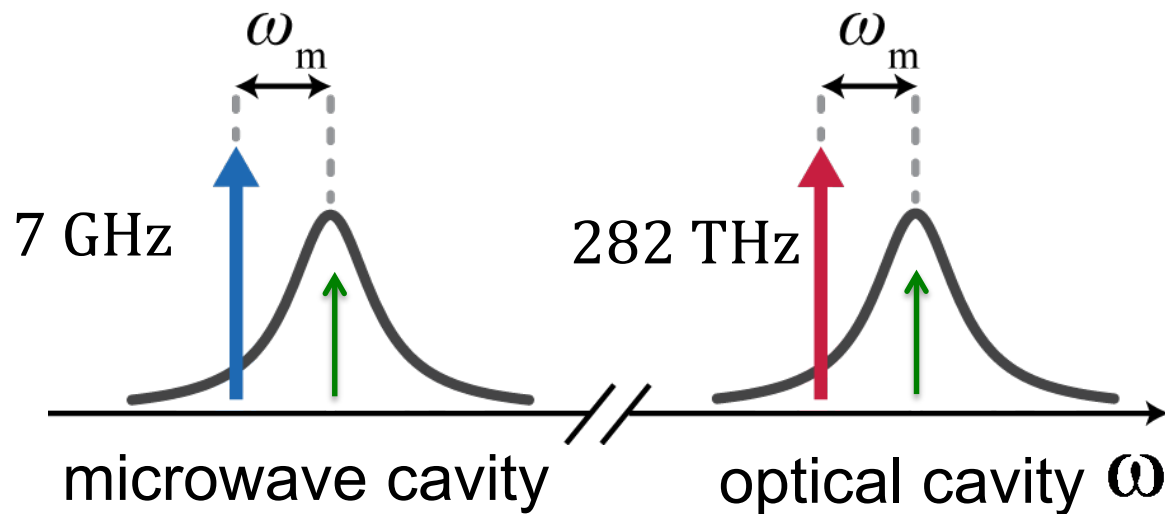
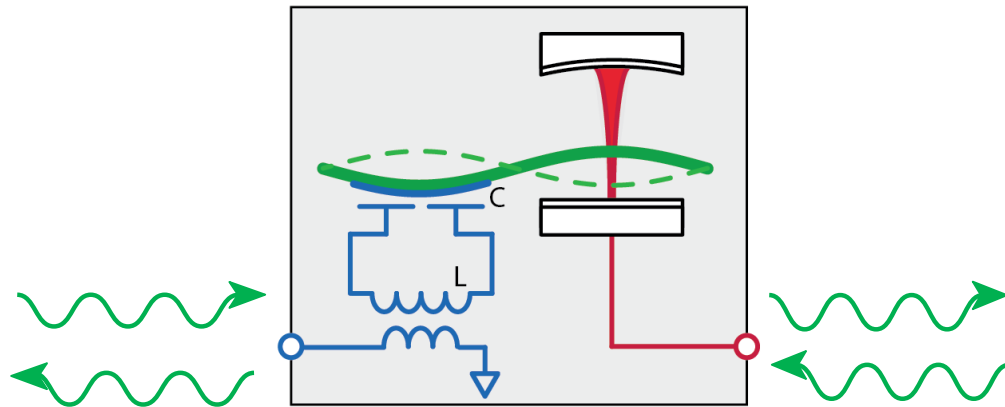
swap phonons and photons



$$H_I = g_0 \alpha (a^\dagger b + b^\dagger a)$$



Coherent conversion of microwave to optical fields

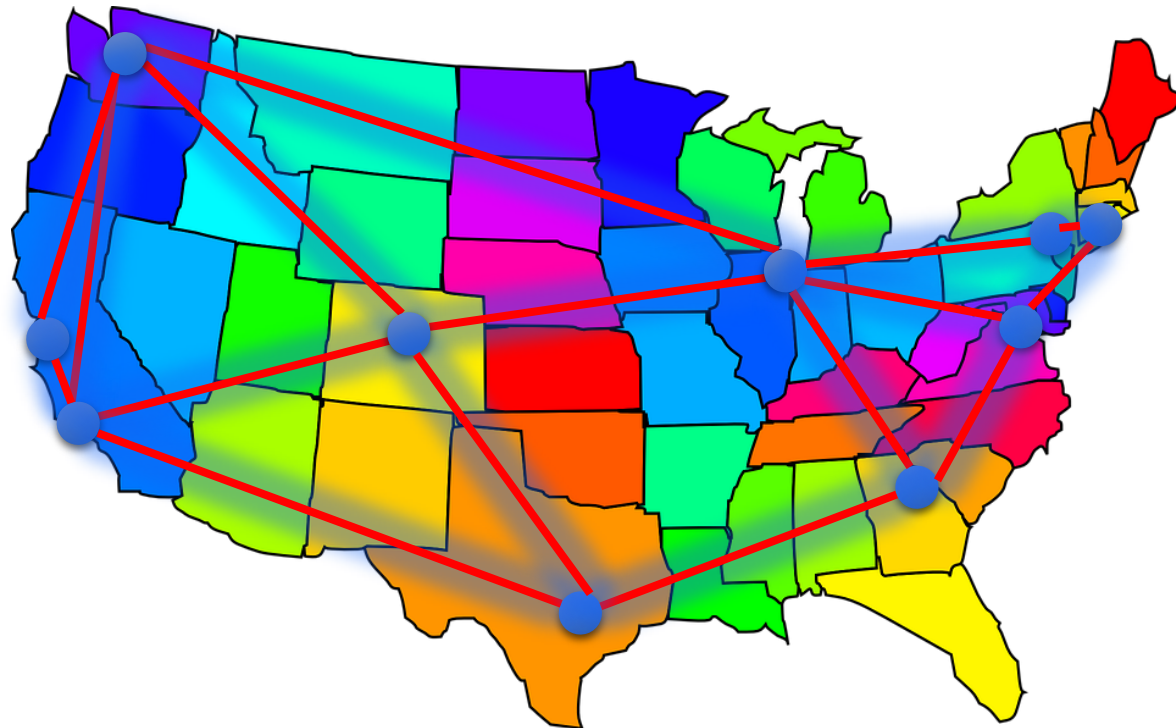


Quantum link between superconducting qubits and light enables a quantum communication network

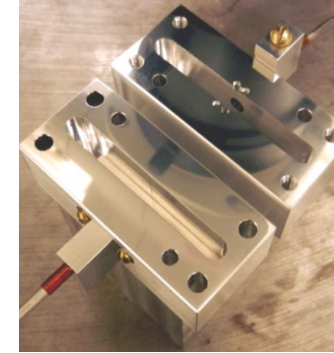
quantum network

secure communication

processing power exponential in nodes

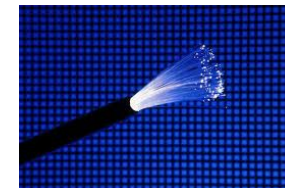


nodes



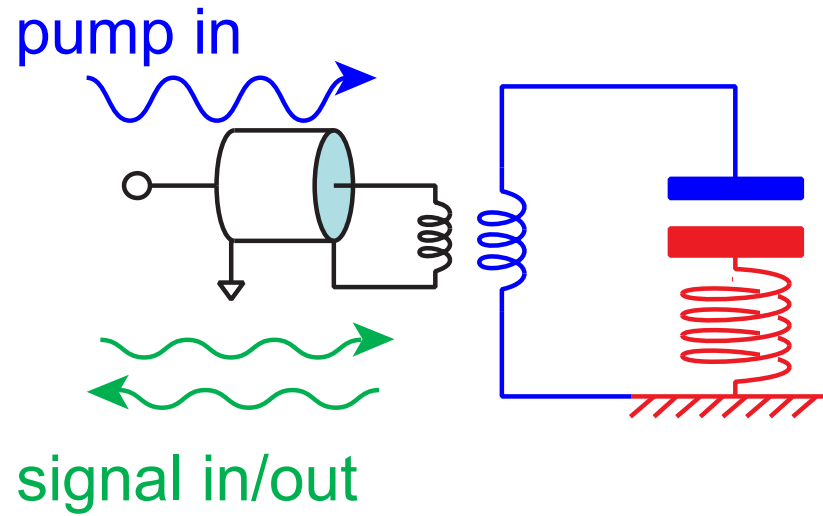
superconducting qubits
(process and store information)

links

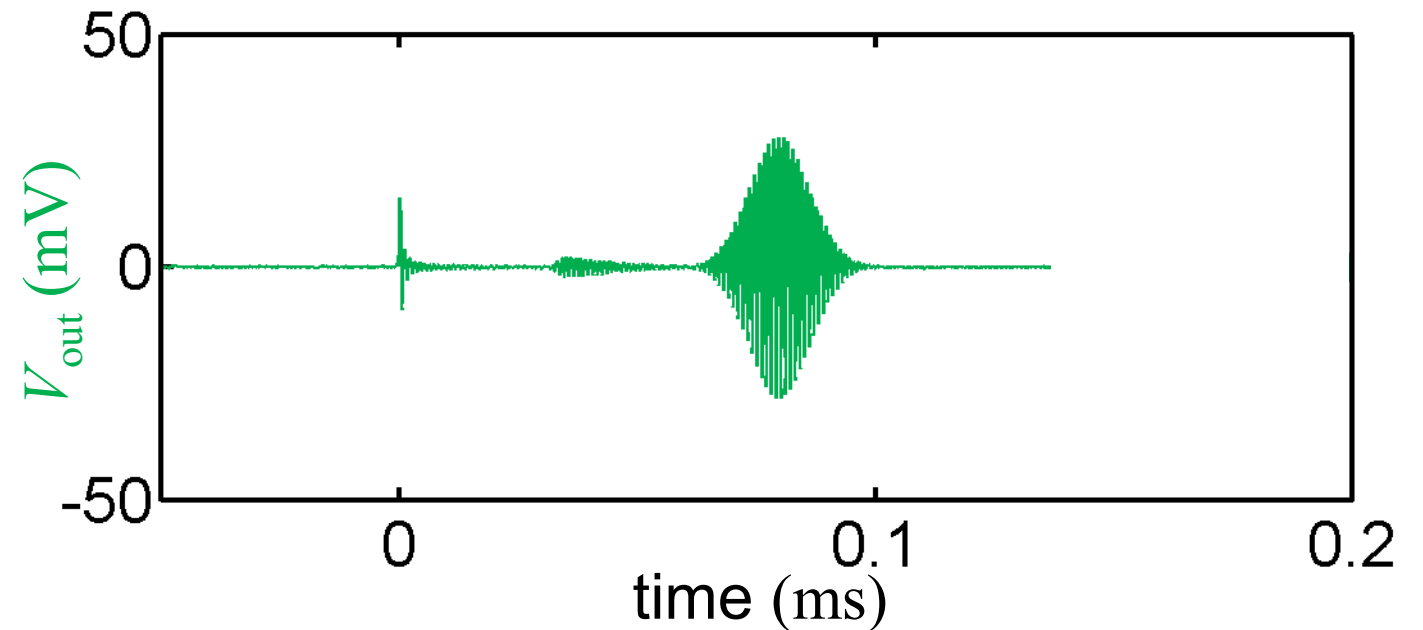
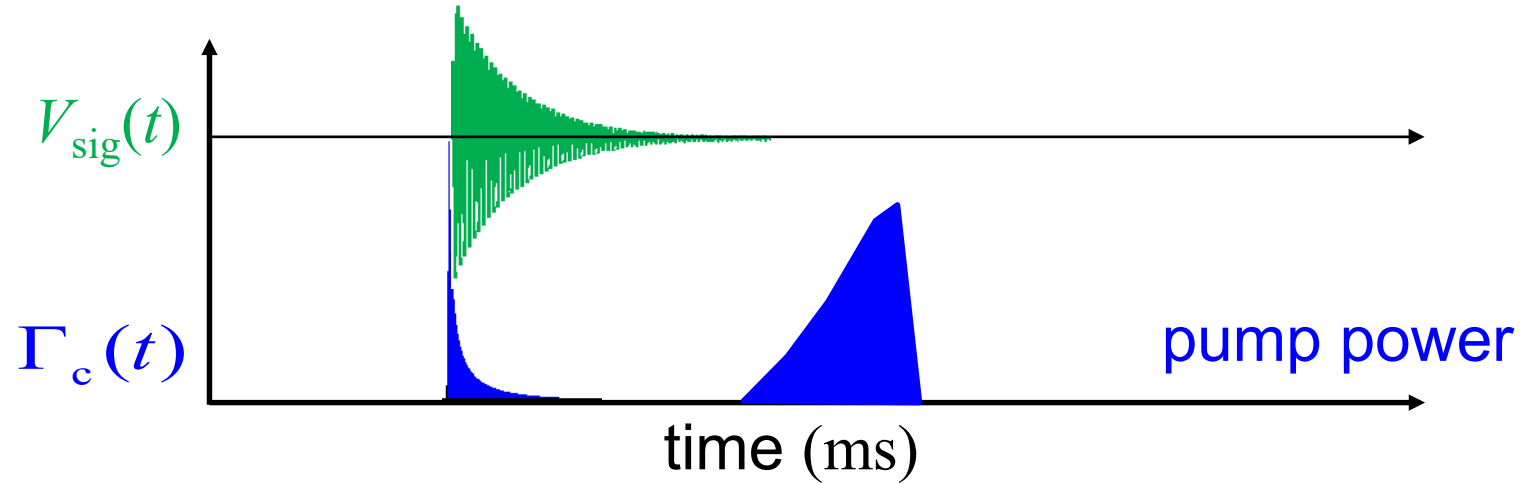


optics (transmit)

Microwave signal processing in the quantum regime



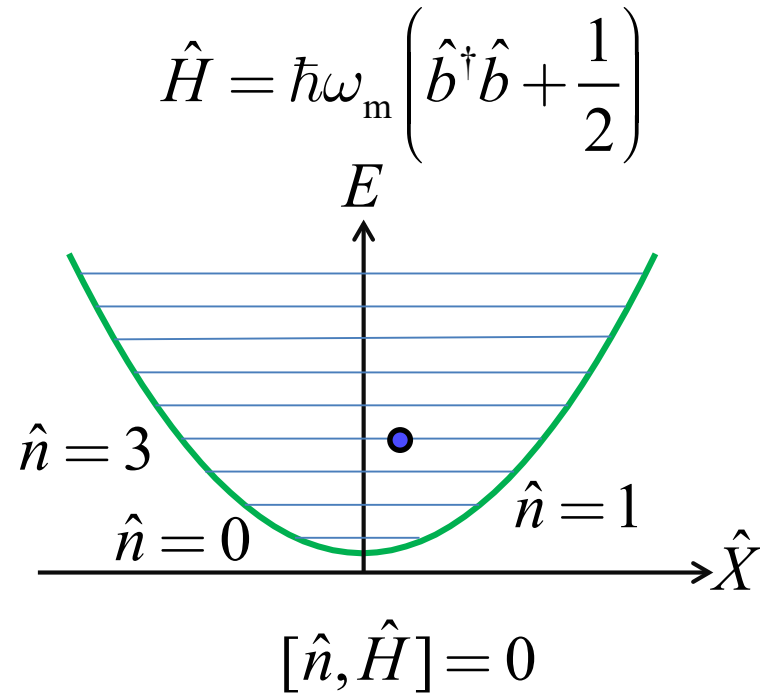
storage/delay
frequency conversion
temporal mode filtering
entanglement
amplification



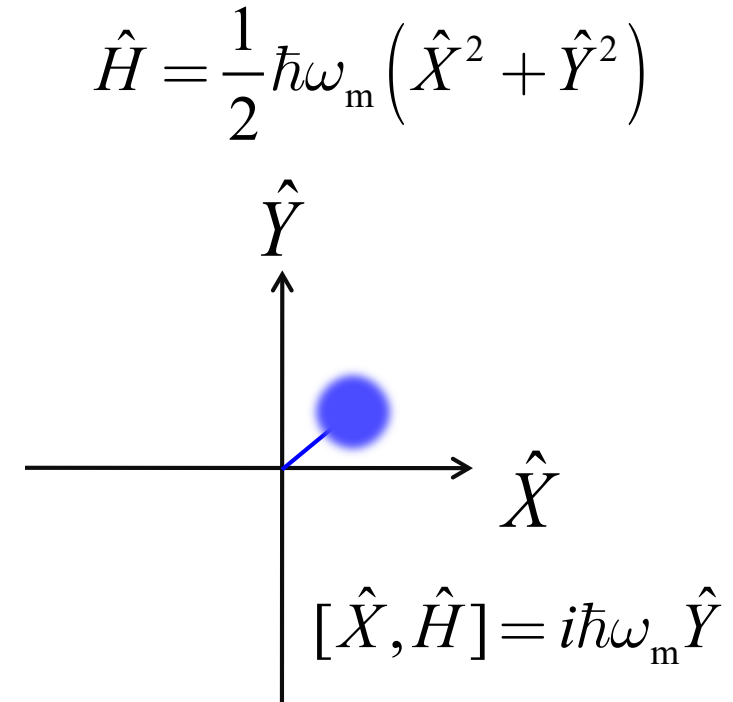
quantum phononics

Is a single phonon detectable?

the quantum harmonic oscillator



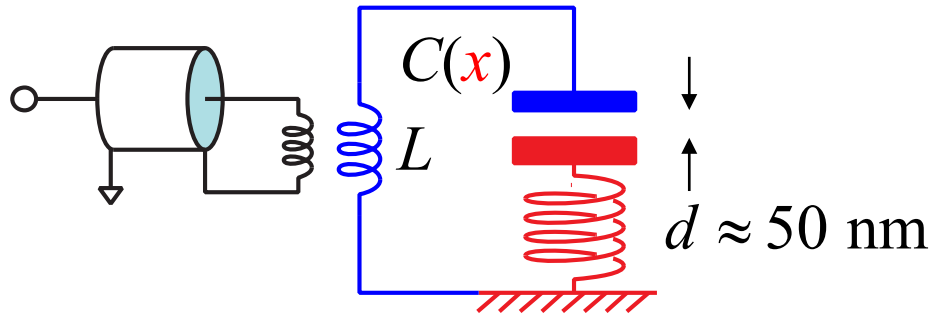
student's preference



nature's preference

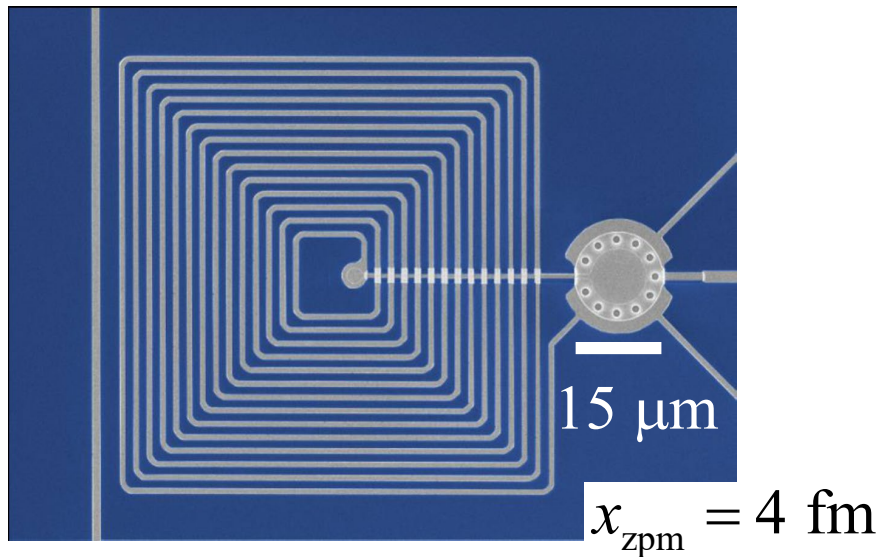
single phonons: require strong non-linearity

Vacuum electromechanical coupling is nonlinear but weak



$$H_I = F_{\text{el}} \cdot x = \frac{1}{2} V^2 \frac{\partial C}{\partial x} \cdot x$$

$$H_I = \frac{1}{4} \hbar \omega_e \cdot \frac{x_{\text{zpm}}}{d} a^\dagger a (b + b^\dagger)$$

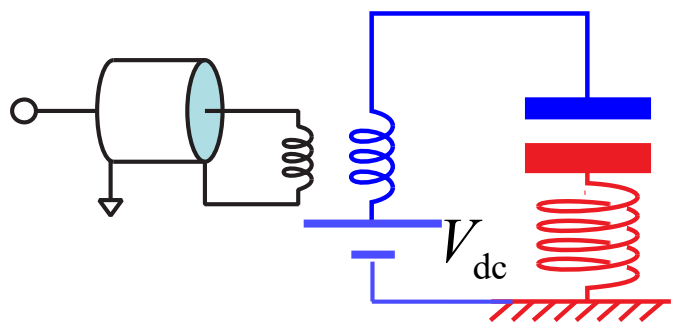


$$\omega_e = 2\pi \times 7 \text{ GHz}$$

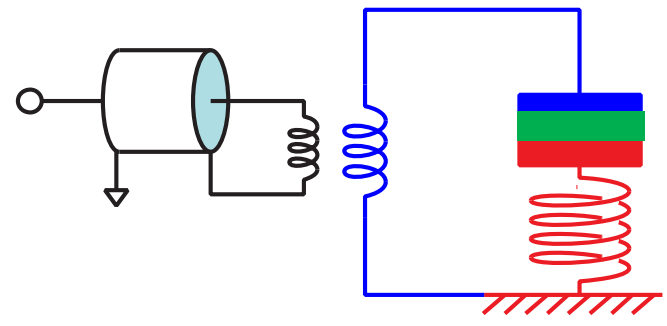
$$\hbar g_0 = \frac{1}{4} \hbar \omega_e \cdot \frac{x_{\text{zpm}}}{d} \approx 2\pi \hbar \times 200 \text{ Hz}$$

Overcoming small vacuum electromechanical coupling

apply static voltage



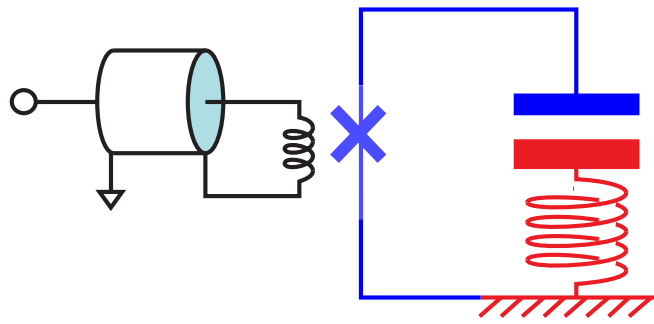
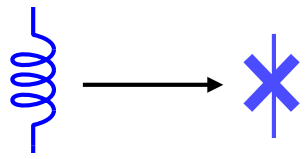
or use piezo-electric coupling



$$H_I = g_0 \left(a + a^\dagger \right) \left(b + b^\dagger \right)$$

$$g_0 \approx 2\pi \times 10 \text{ MHz}$$

add electrical non-linearity



$$H_I = g_0 \sigma_x \left(b + b^\dagger \right)$$

superconducting qubit
coupled mechanics

Outline

mechanical oscillator coupled to a charge qubit: a particle in a quantum potential

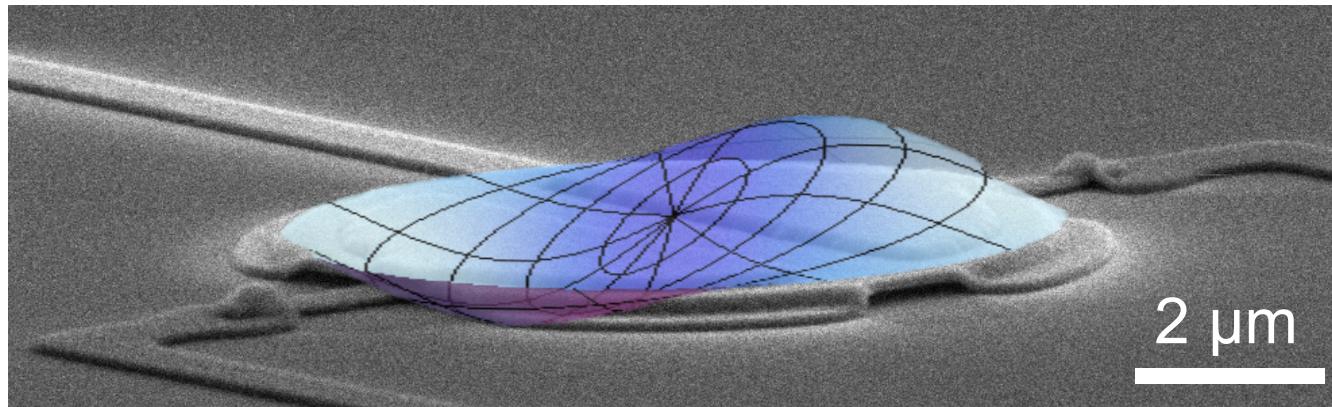
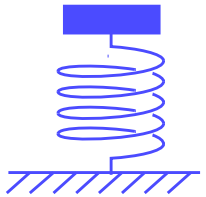
energy sensitive detector of mechanical oscillator

stabilizing a non-classical state of motion

electromechanical device as an artificial molecule

mechanical oscillator coupled to a charge qubit

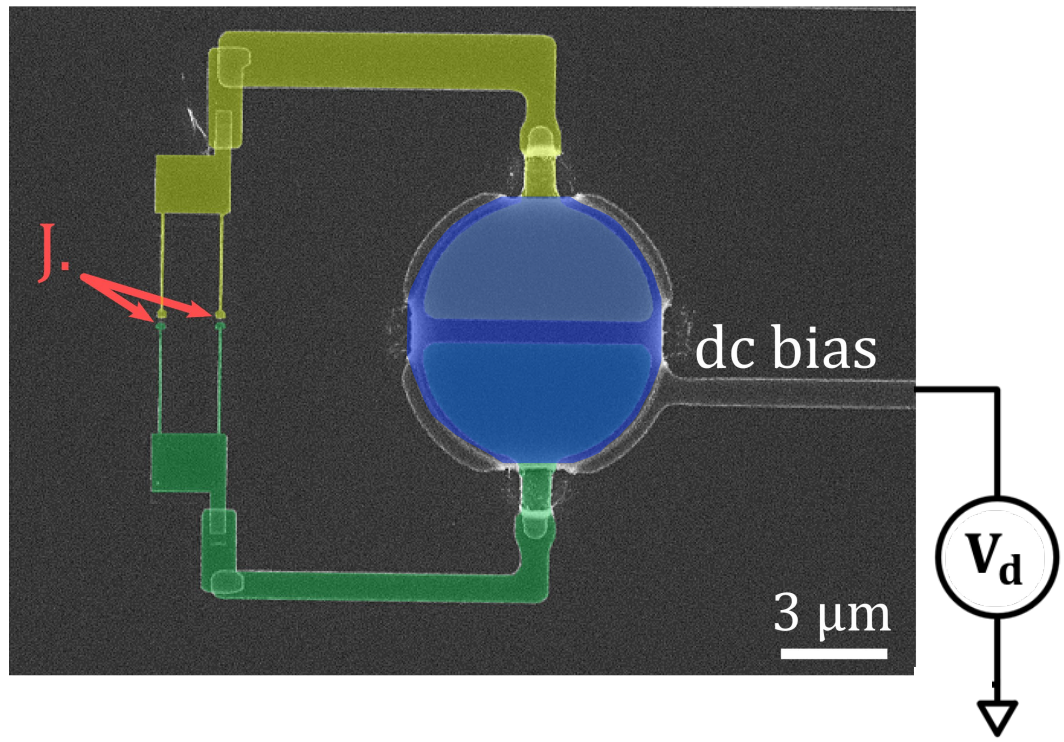
Mechanics: vibrating aluminum membrane



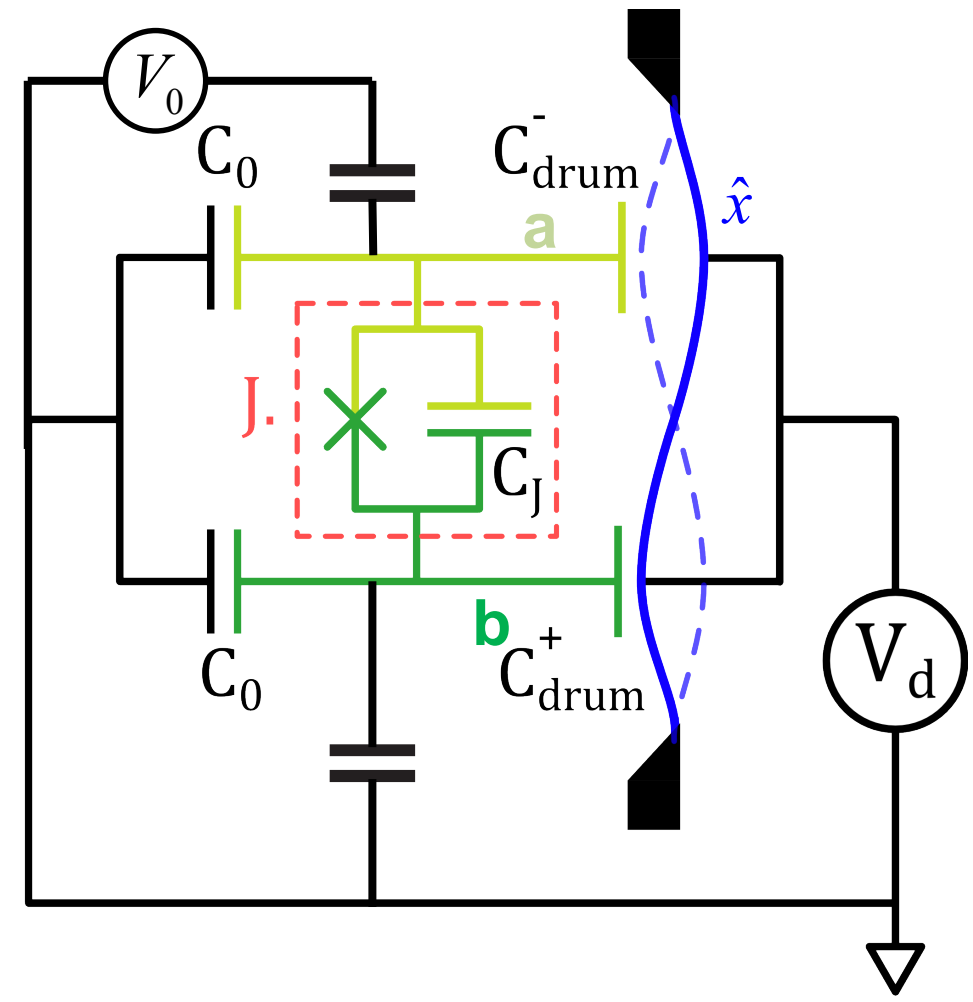
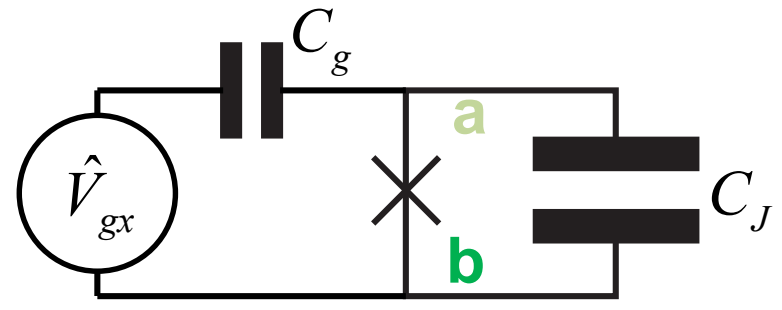
$$\omega_m = 2\pi \times 25 \text{ MHz}$$

antisymmetric 2:1 mode

Coupling an Al drum oscillator to a Cooper pair box qubit



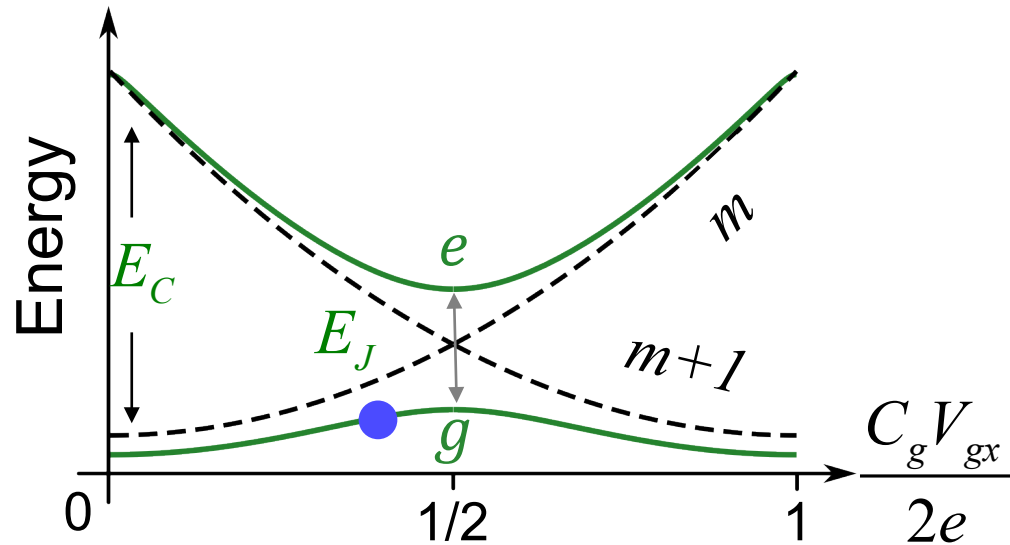
Thevenin equivalent



$$\hat{V}_{gx} = \left(\frac{\partial V}{\partial x} + \frac{\partial V_g}{\partial x} \right) \hat{x}$$

gate voltage is oscillator coordinate

Mechanical oscillator moving in a qubit potential



oscillator particle in qubit potential

$$\omega_m \ll \omega_q$$

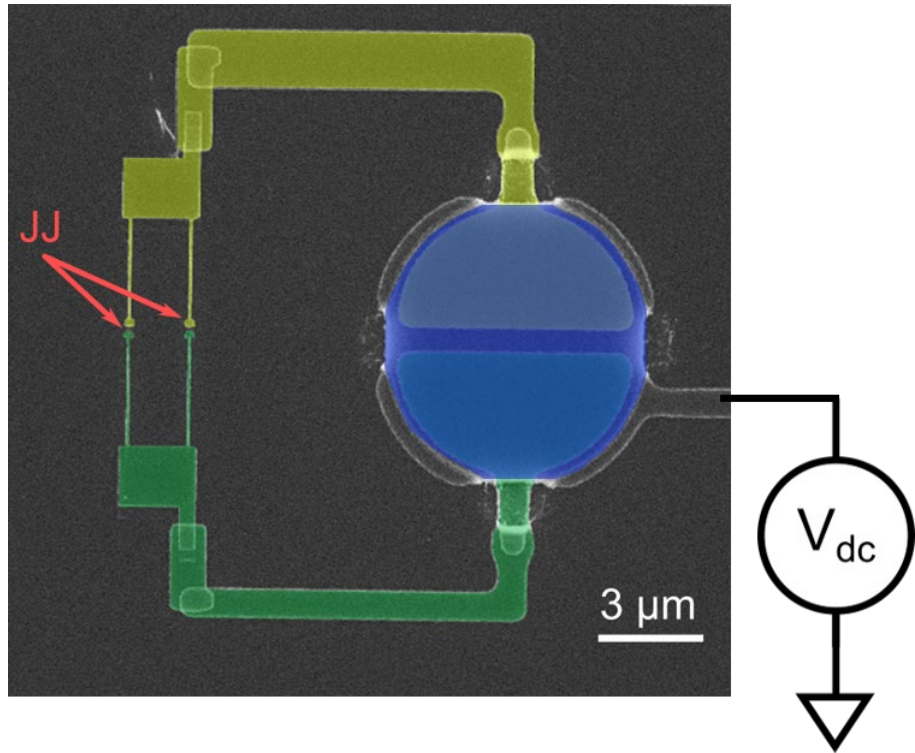
$$\frac{C_g V_0}{2e} \neq \frac{1}{2} \quad \text{qubit dependent force}$$

$$\hat{H}_I \propto \hat{\sigma}_z \hat{x} = \hat{\sigma}_z (b^\dagger + b)$$

$$\frac{C_g V_0}{2e} = \frac{1}{2} \quad \text{qubit dependent spring constant}$$

$$\hat{H} = \frac{1}{2} \left(\hbar \omega_q \hat{\sigma}_z + \hat{p}^2 / m + k \hat{x}^2 \right) + \frac{1}{2} k_q \hat{x}^2 \hat{\sigma}_z$$

Ultrastrong coupling, dispersive limit of Rabi Hamiltonian



at charge degeneracy

$$H_{\text{Rabi}} / \hbar = \frac{1}{2} \omega_q \sigma_z + \omega_m b^\dagger b + \frac{1}{2} g_0 \sigma_x (b + b^\dagger)$$

$$g_0 \propto V_{\text{dc}}$$

$$V_{\text{dc}} = 6 \text{ V} \Rightarrow g_0 \approx 2\pi \times 22 \text{ MHz}$$

$$g_0 \sim \omega_m$$

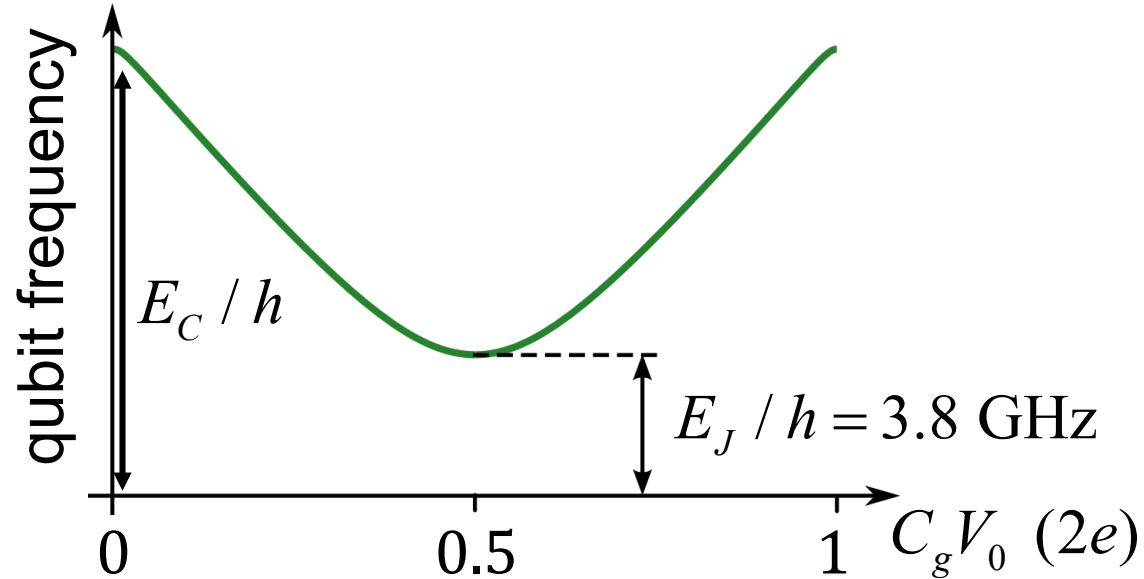
$$\omega_q + \omega_m \approx \omega_q - \omega_m$$

$$\chi_m = \frac{2g_0^2}{\omega_m} \approx 2\pi \times 260 \text{ kHz}$$

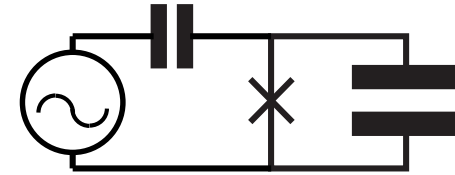
$$\hat{H}_I = \chi_m \hat{\sigma}_z \hat{x}^2 = \underbrace{\chi_m \hat{\sigma}_z b^\dagger b}_{\text{Stark shift}} + \underbrace{\frac{1}{2} \chi_m \hat{\sigma}_z (b^{\dagger 2} + b^2)}_{\text{+ sidebands}}$$

energy sensitive detector of mechanical
oscillator

Motional dispersion of qubit emulated with classical drive



$$V_{\text{dc}} = 0; \partial_x V_g = 0$$

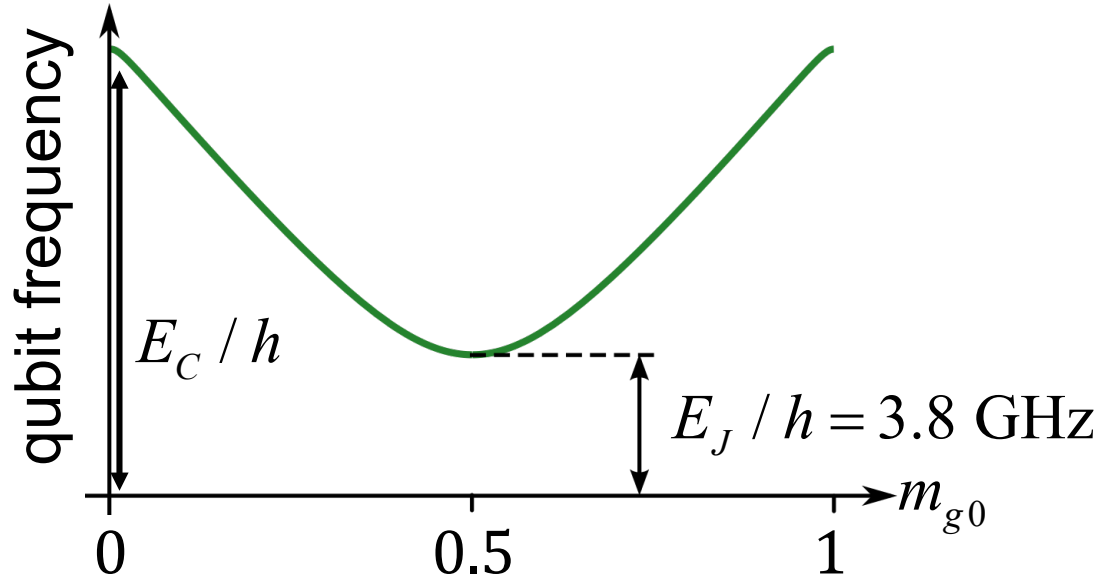


$$\frac{C_g V_0}{2e} = \frac{1}{2} + \lambda \cos(\omega_m t)$$

$$\hbar \omega_q(t) = \sqrt{E_J^2 + [E_C \lambda \cos(\omega_m t)]^2}$$

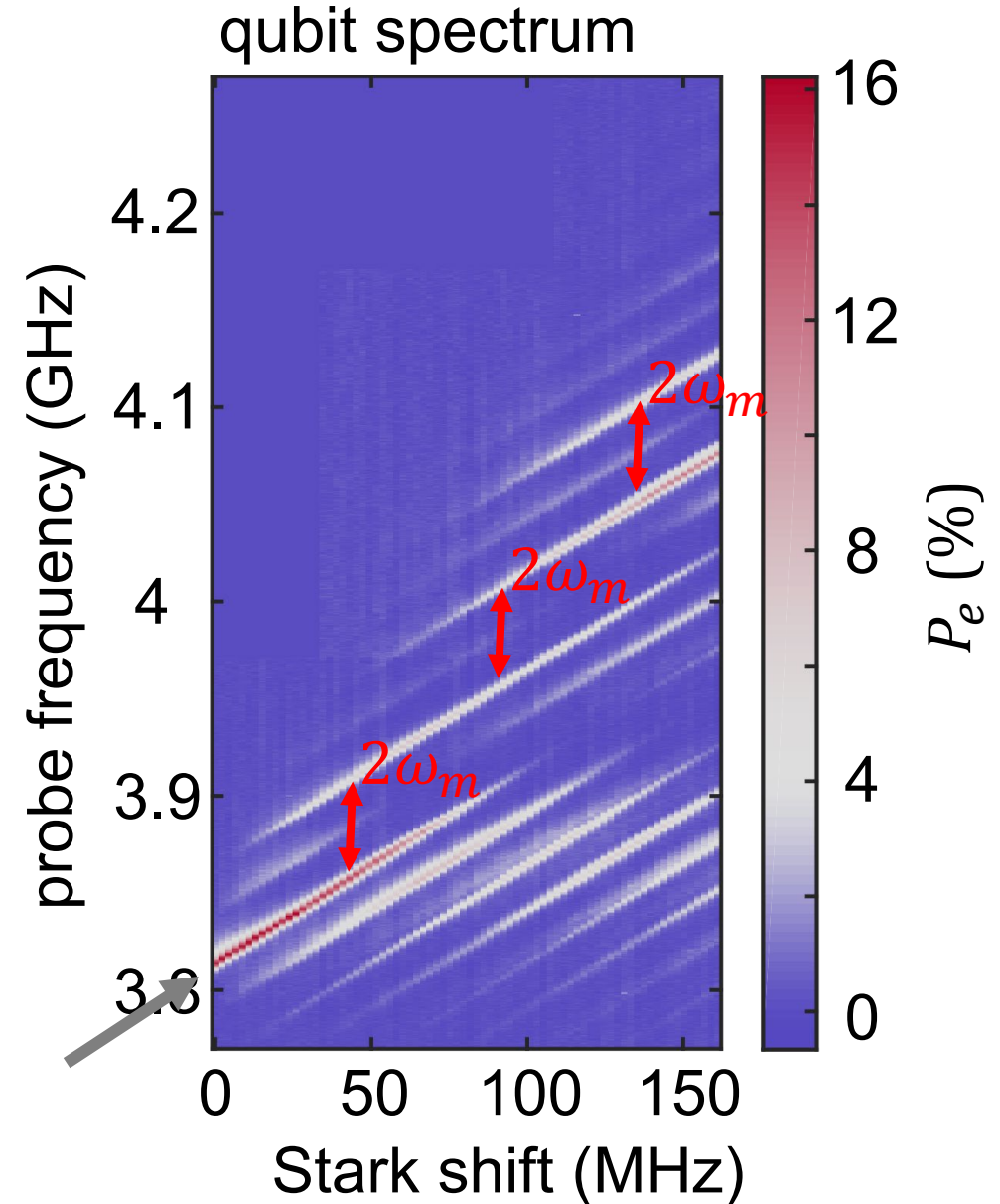
$$\hbar \omega_q(t) = E_J + \underbrace{\left[\frac{E_C^2}{E_J} \lambda^2 \right]}_{\text{Stark shift}} + \underbrace{\frac{E_C^2}{E_J} \lambda^2 \cos(2\omega_m t)}_{\text{sidebands}} + O(\lambda^4)$$

Motional modulation of qubit emulated with classical drive

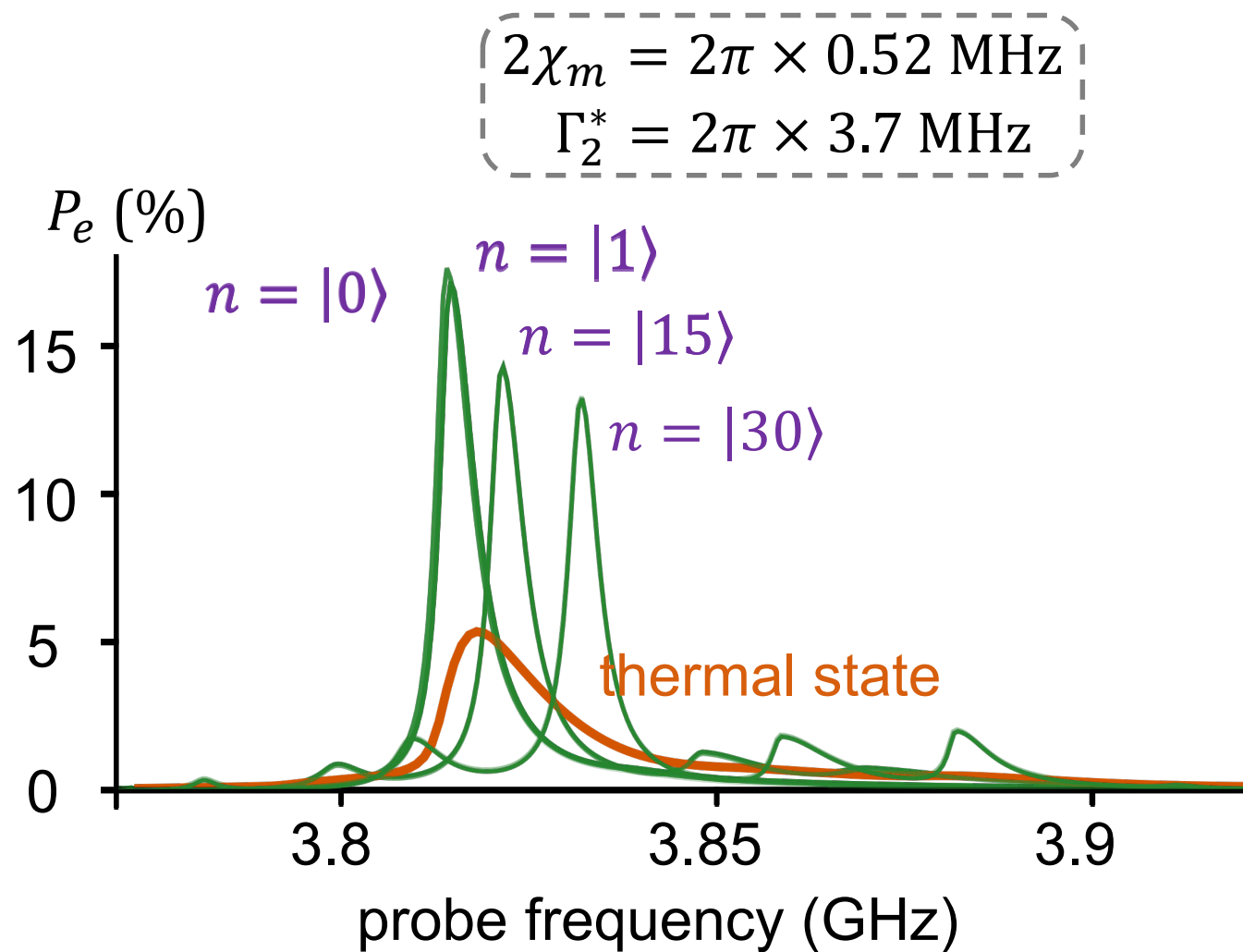
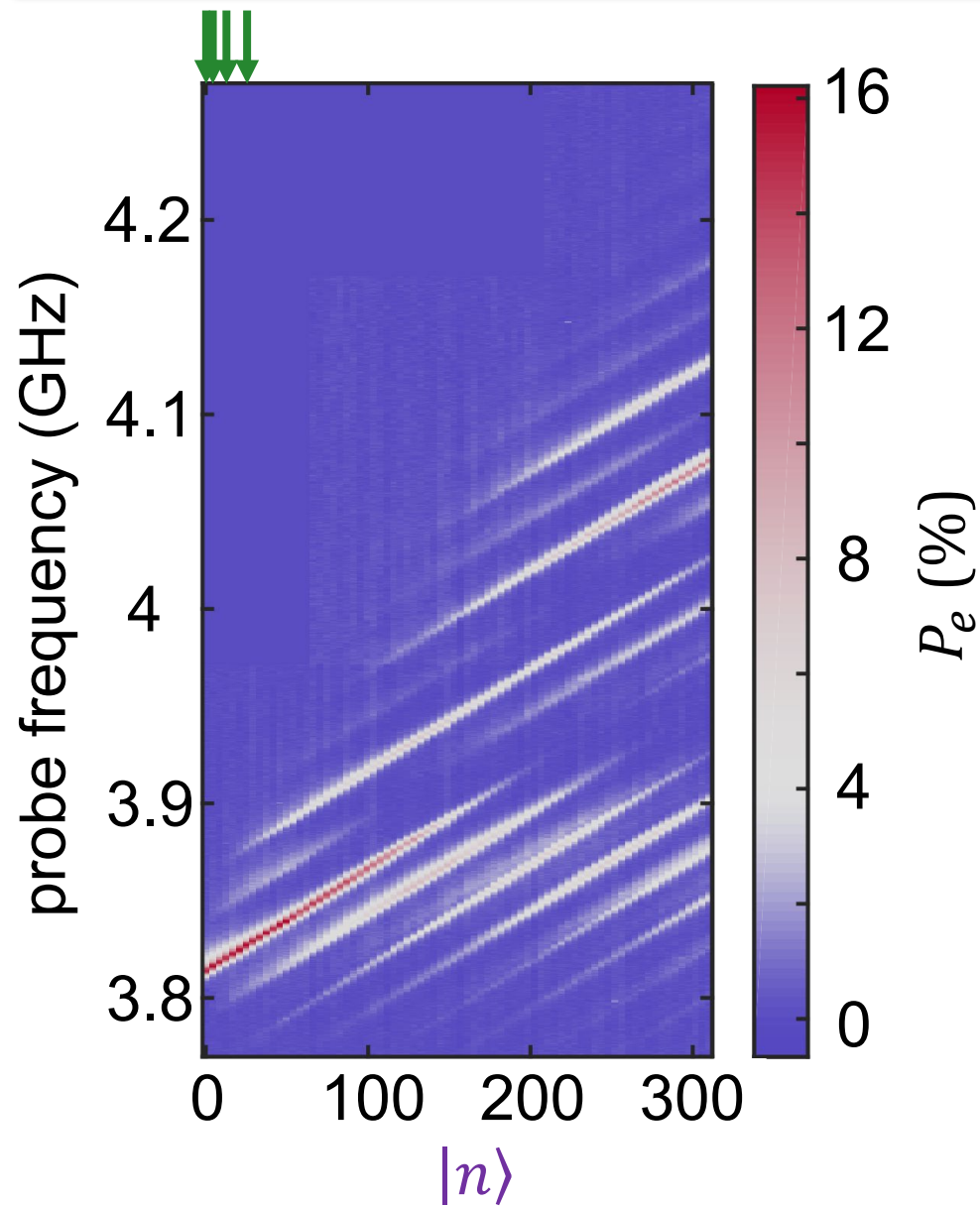


$$\hbar\omega_q(t) = \sqrt{E_J^2 + [E_C \lambda \cos(\omega_m t)]^2}$$

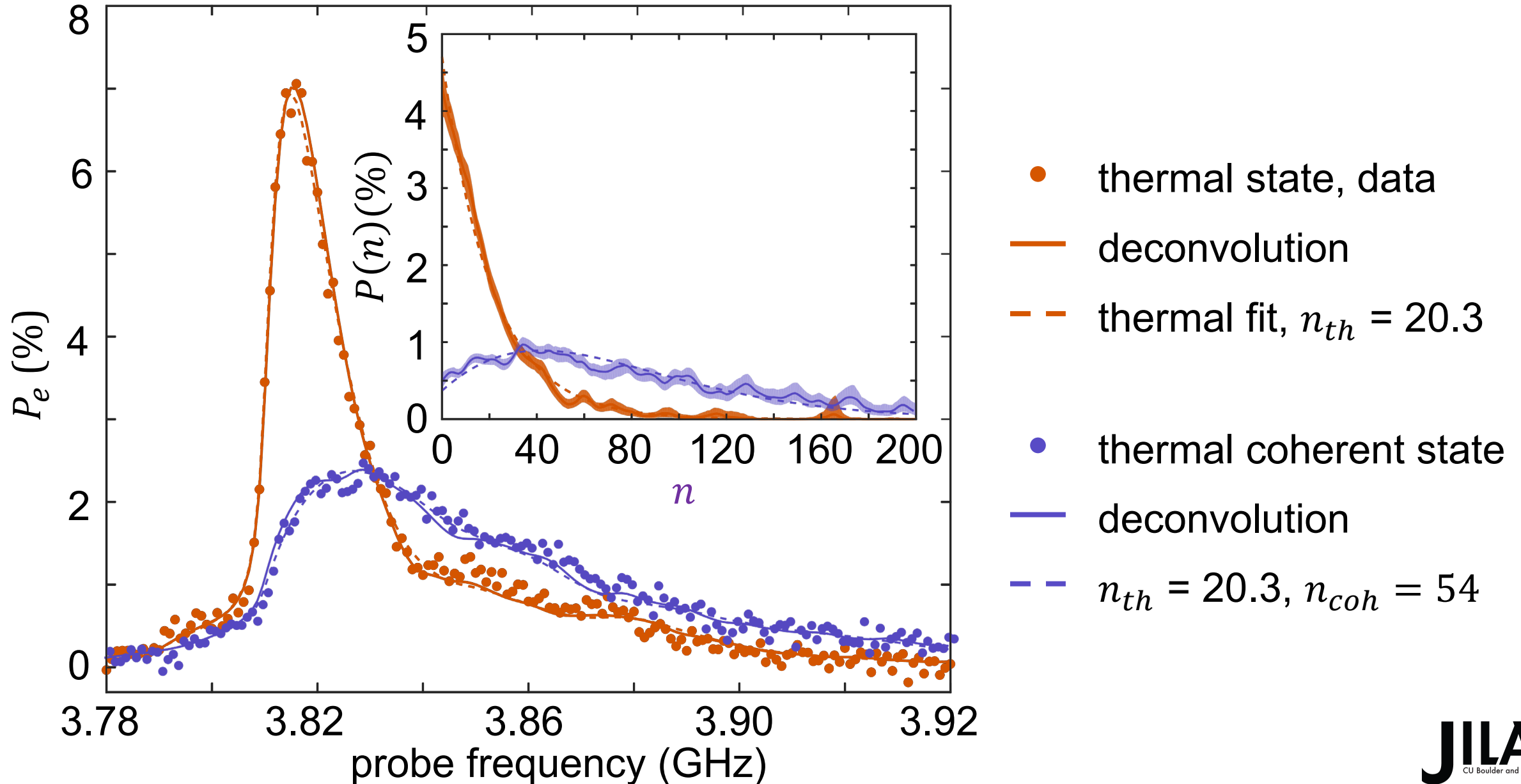
$$\hbar\omega_q(t) = E_J + \underbrace{\left[\frac{E_C^2}{E_J} \lambda^2 \right]}_{\text{Stark shift}} + \underbrace{\frac{E_C^2}{E_J} \lambda^2 \cos(2\omega_m t)}_{\text{sidebands}} + O(\lambda^4)$$



Qubit spectrum encodes phonon number distribution

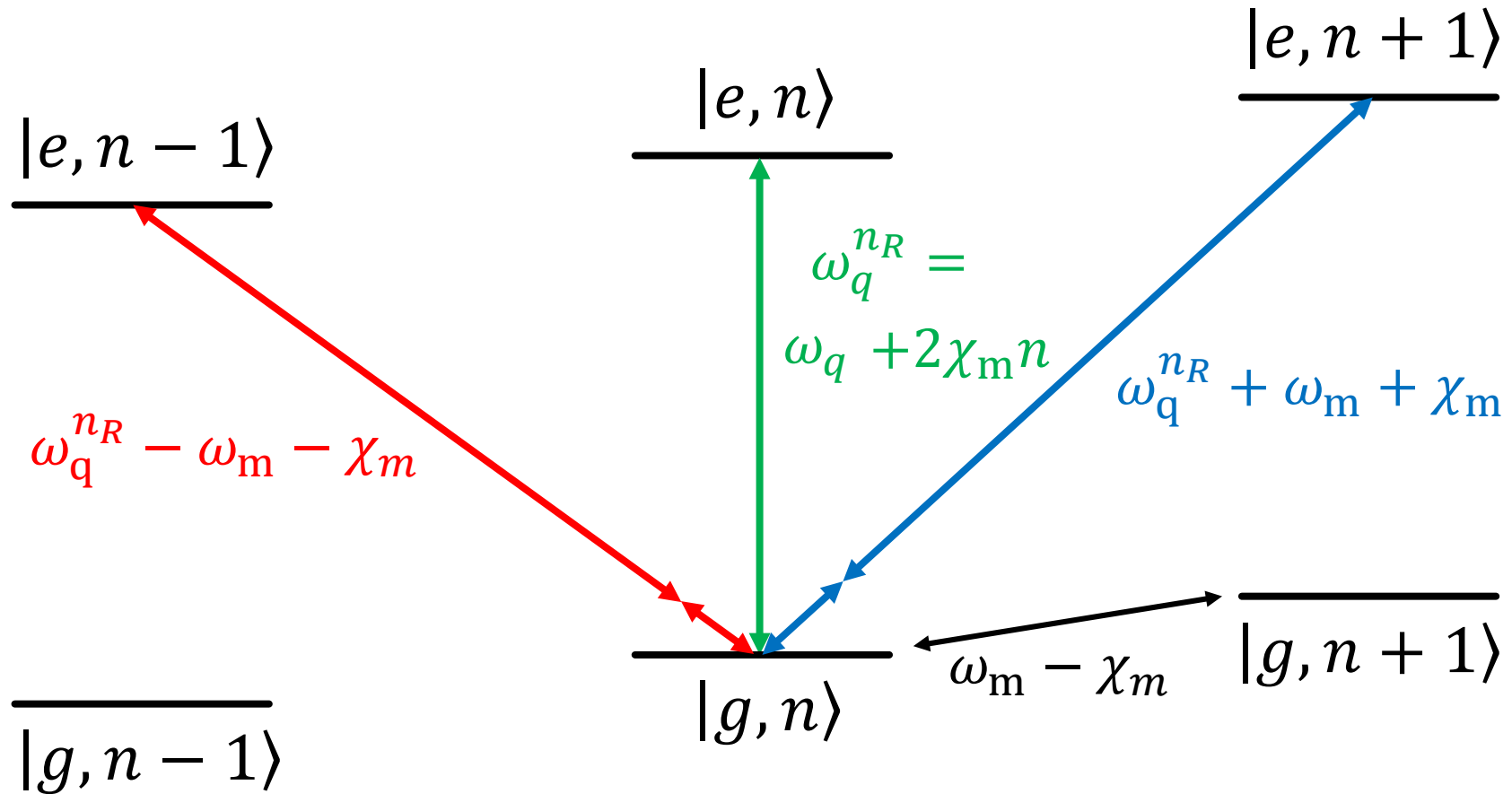


De-convolution reveals phonon statistics

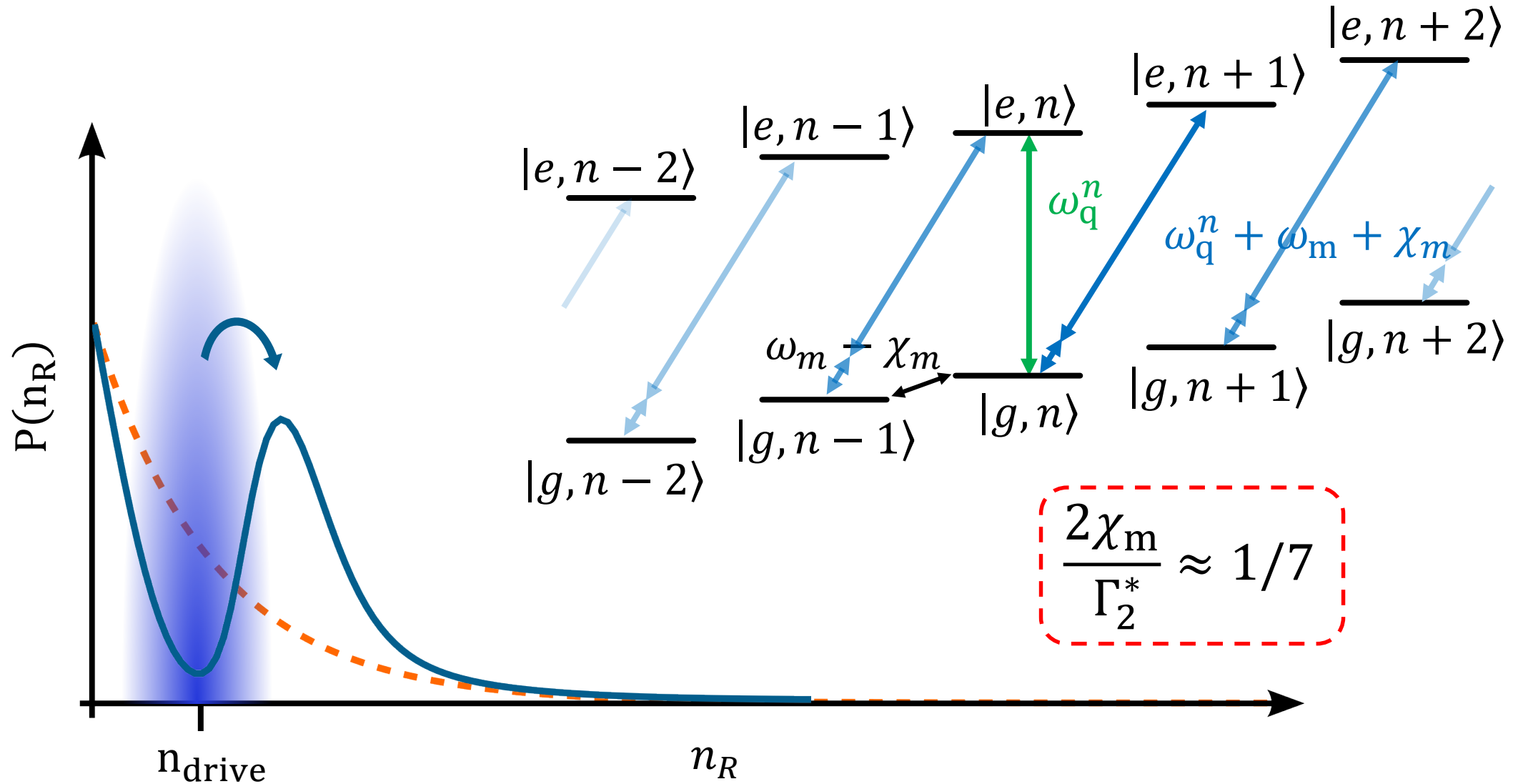


stabilizing a phonon number squeezed state

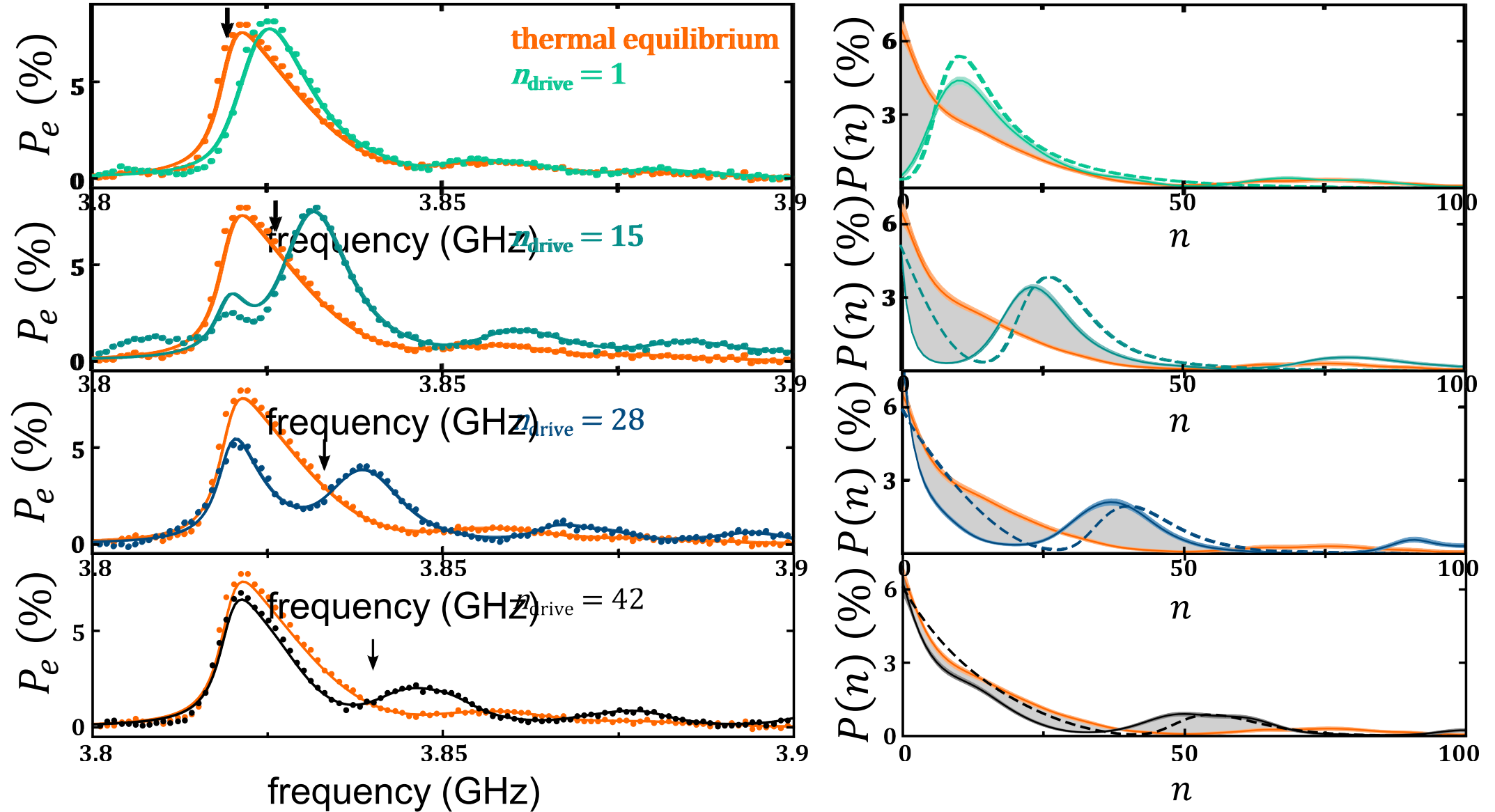
Number sensitive sideband transitions



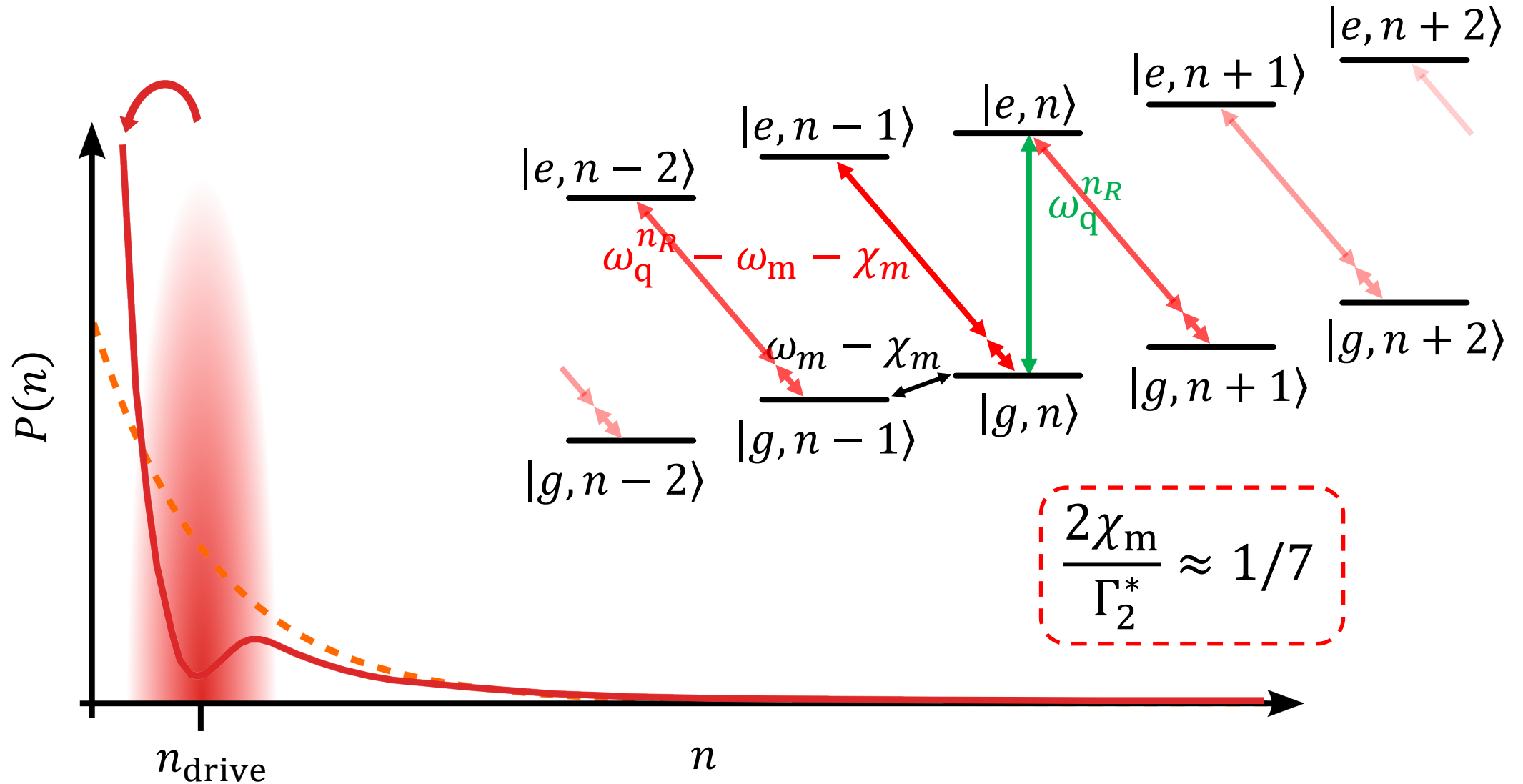
Sideband drive moves population selectively



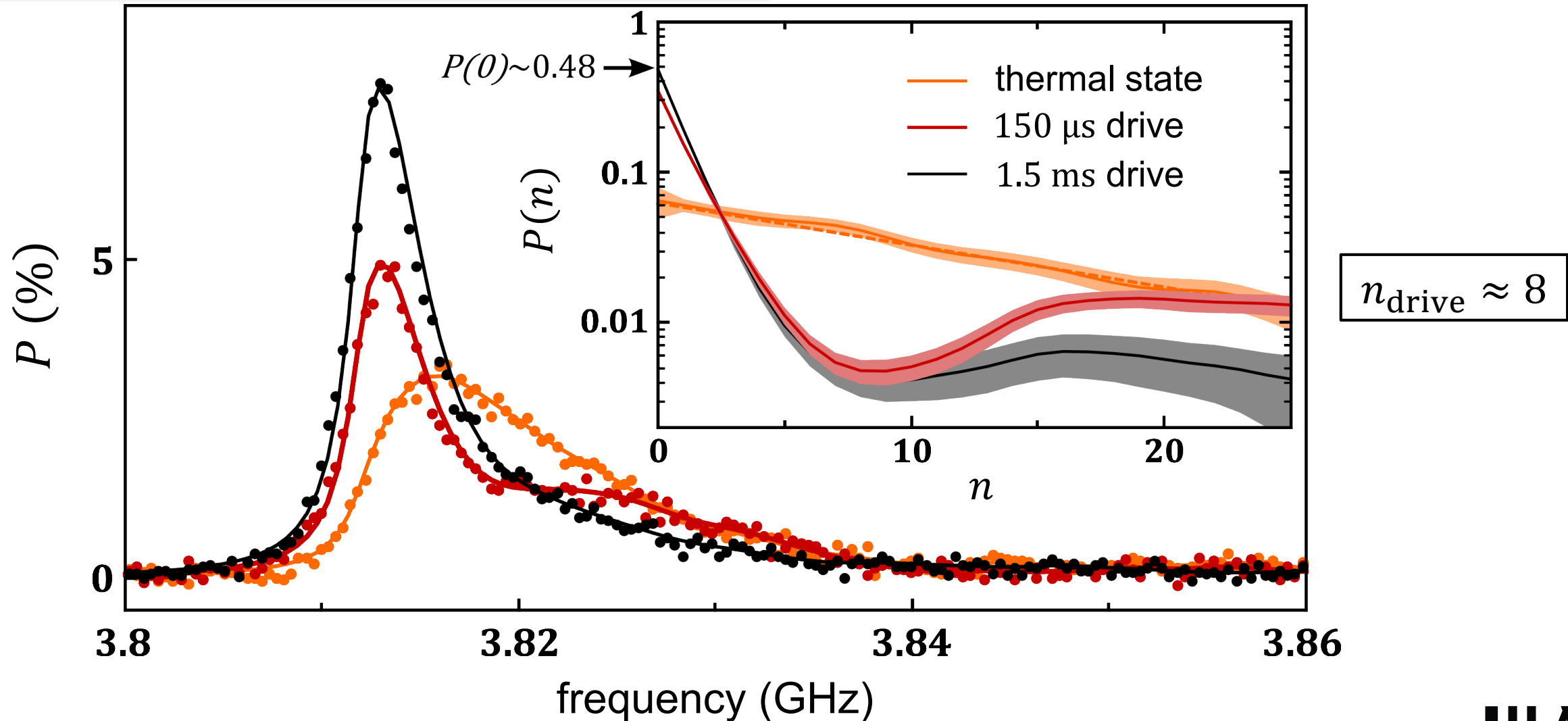
Number sensitive blue sideband



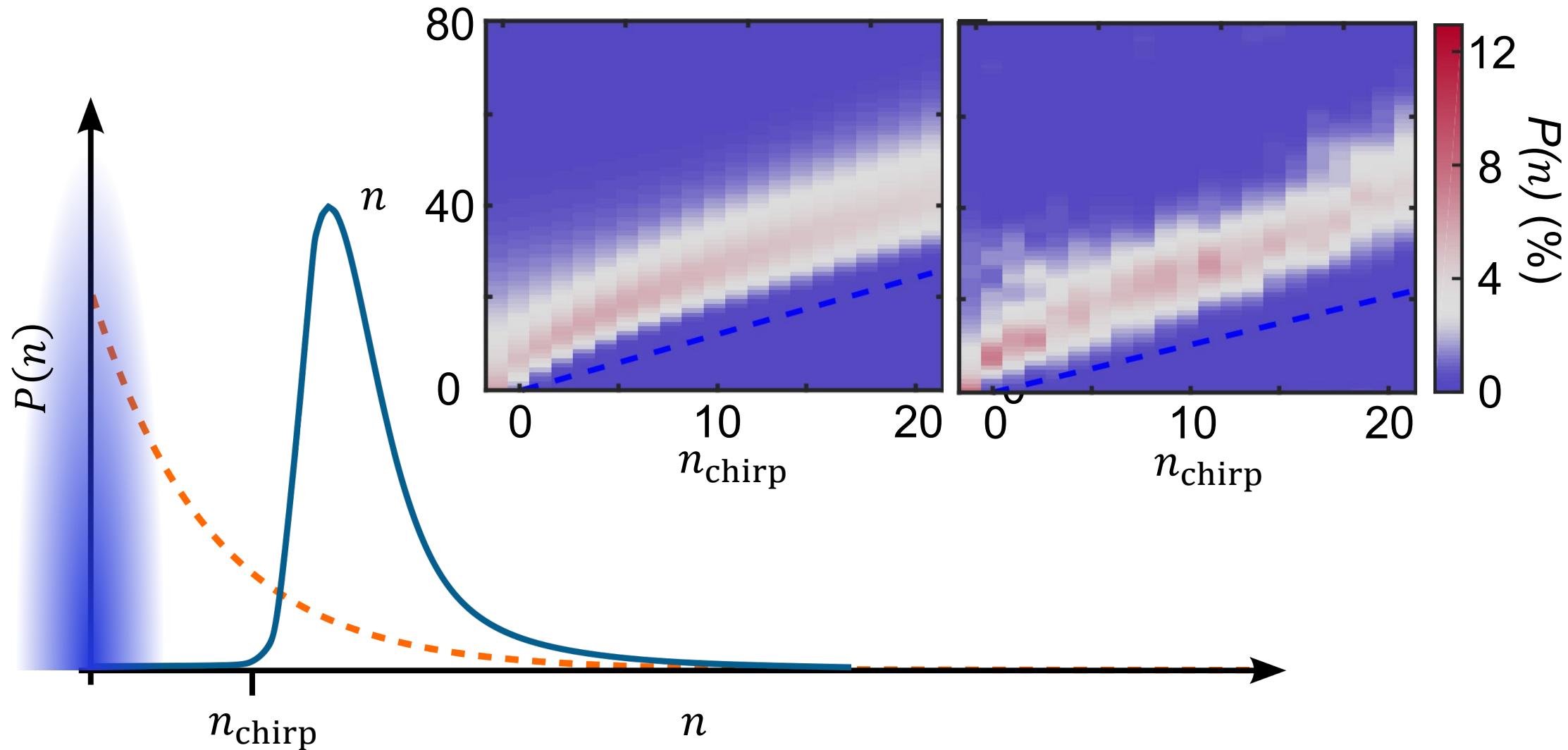
Sideband drive moves population selectively



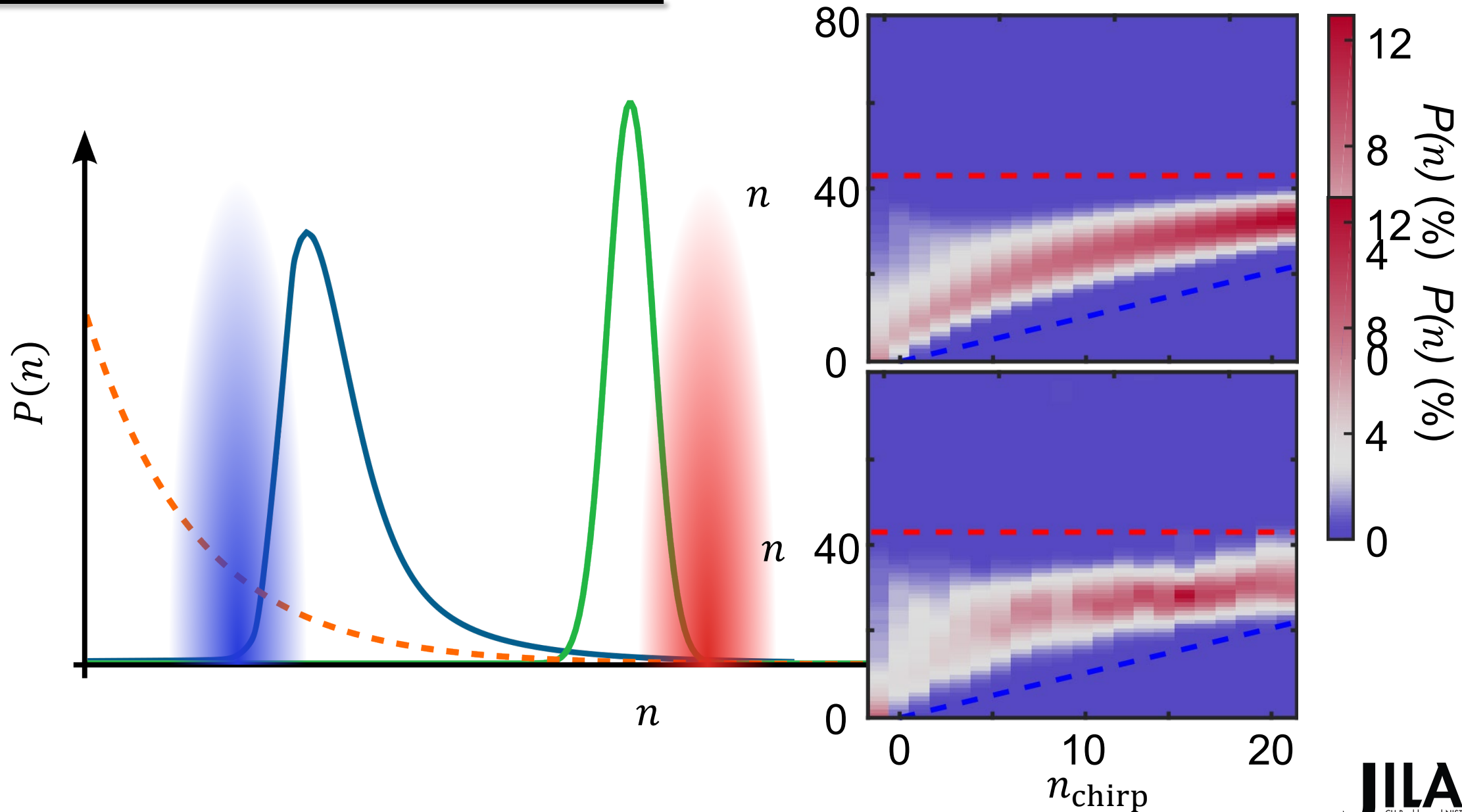
Number sensitive red sideband : cooling a mechanical oscillator with a two level system



Chirped blue sideband displace thermal distribution

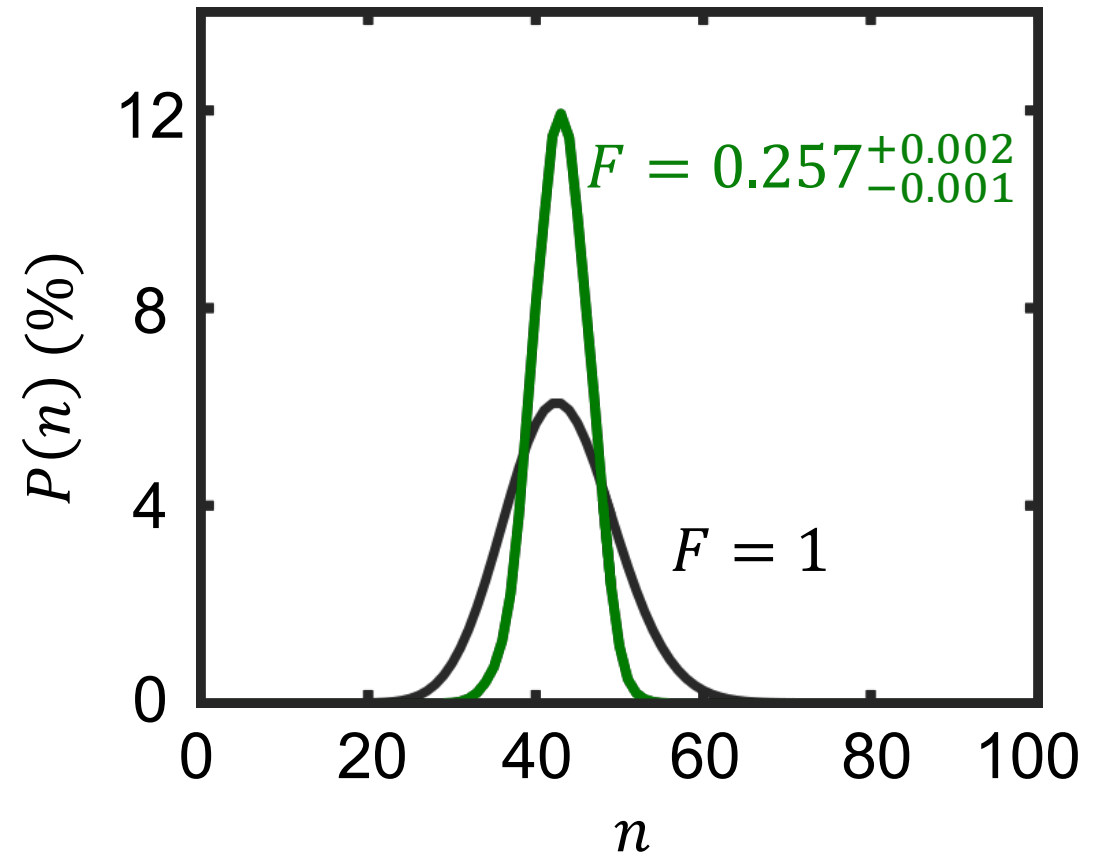
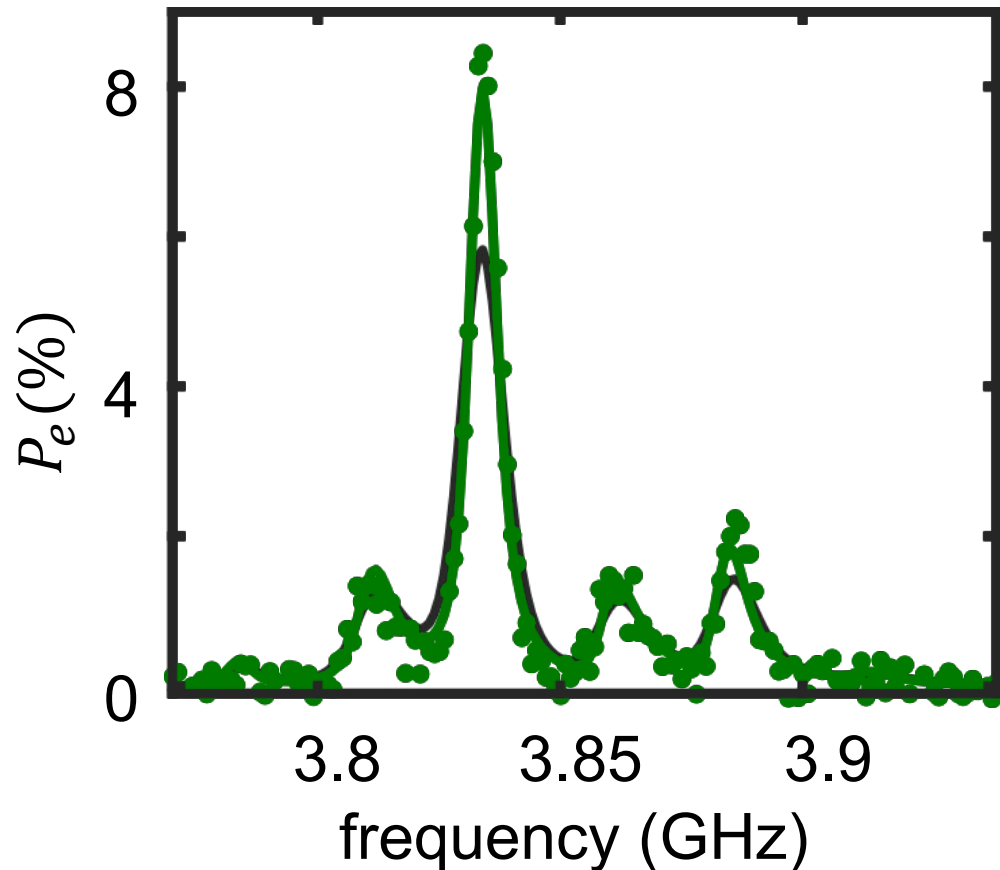


Squeezing the number distribution with sideband drives



Number squeezed state, dissipatively stabilized

Fano factor $F = \frac{\text{var}(n)}{\langle n \rangle}$



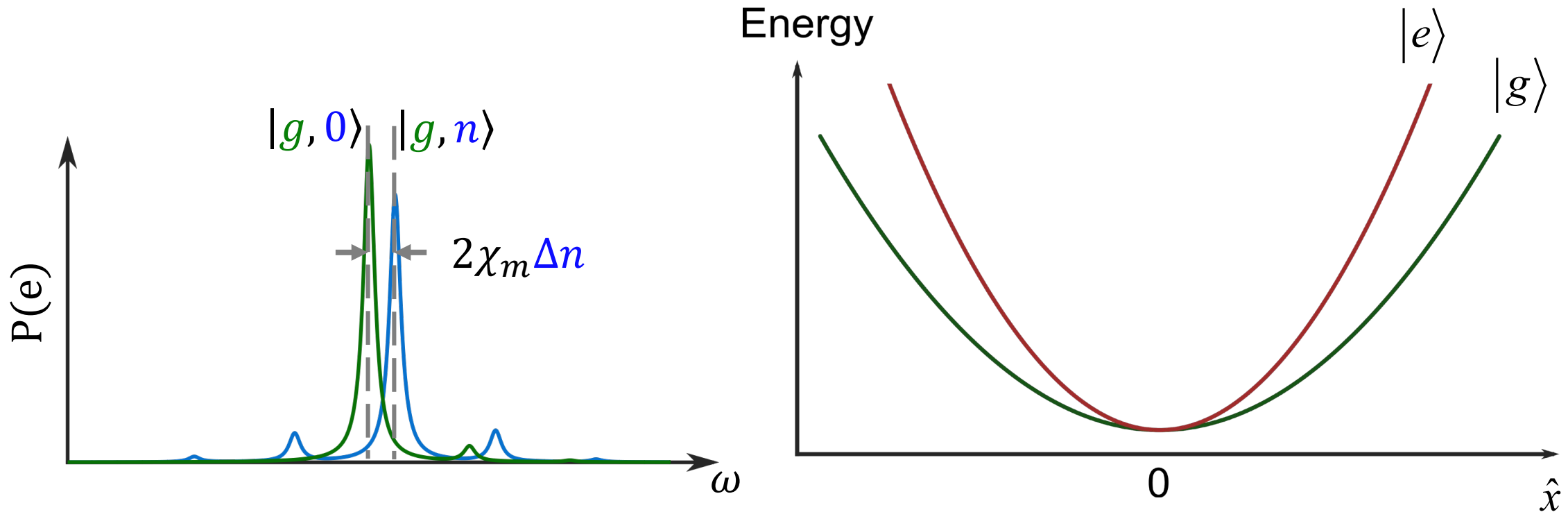
vibronic transitions in electromechanics

Qubit response to quantized motion

$$H_{\text{eff}} = \frac{1}{2} \omega_q \sigma_z + \frac{\hat{p}^2}{2m} + \frac{1}{2} k \hat{x}^2 + \frac{1}{2} k \left(\frac{2\chi_m}{\omega_m} \right) \sigma_z \hat{x}^2$$

$$\chi_m = 2 \frac{g^2}{\omega_q} \approx 2\pi \times 260 \text{ kHz}$$

$$\omega_m(\sigma_z) = \sqrt{k(\sigma_z)/m} \approx \omega_m^0 + \sigma_z \chi_m$$



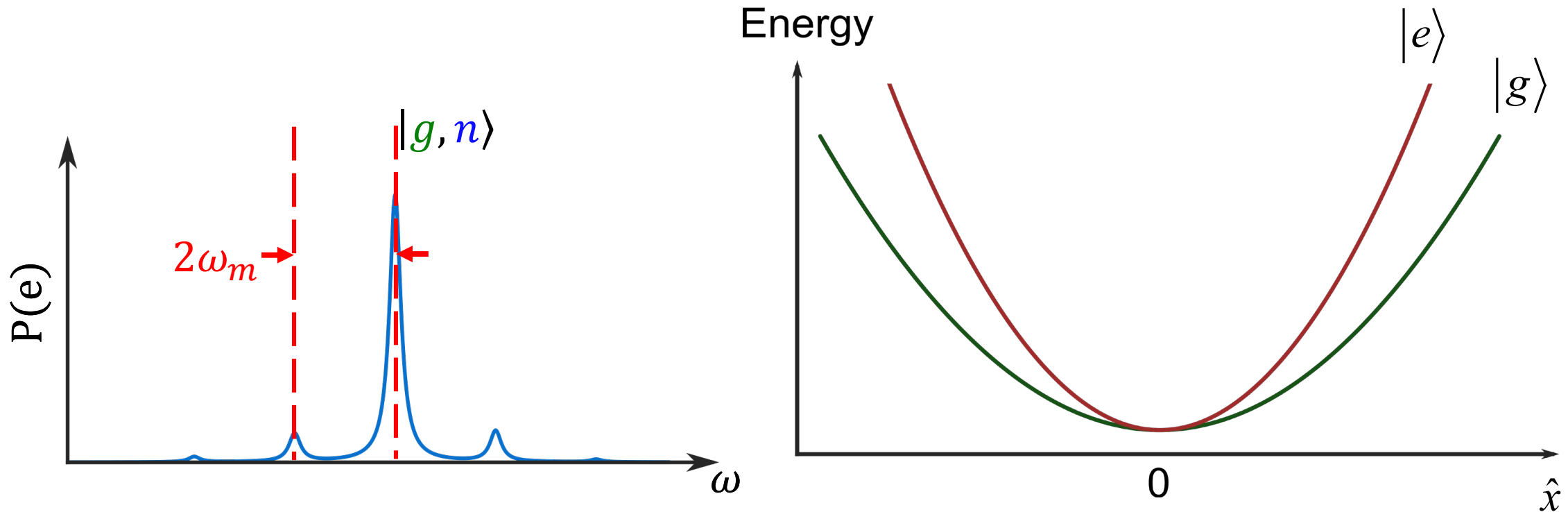
Qubit response to quantized motion

$$H_{\text{eff}} = \frac{1}{2} \omega_q \sigma_z + \frac{\hat{p}^2}{2m} + \frac{1}{2} k \hat{X}^2 + \frac{1}{2} k \left(\frac{2\chi_m}{\omega_m} \right) \sigma_z \hat{X}^2$$

$$\chi_m = 2 \frac{g^2}{\omega_q} \approx 2\pi \times 260 \text{ kHz}$$

$$\omega_m(\sigma_z) = \sqrt{k(\sigma_z)/m} \approx \omega_m^0 + \sigma_z \chi_m$$

$$Z_m(\sigma_z) = \sqrt{k(\sigma_z) \times m} = p_{\text{zpf}}/x_{\text{zpf}}$$



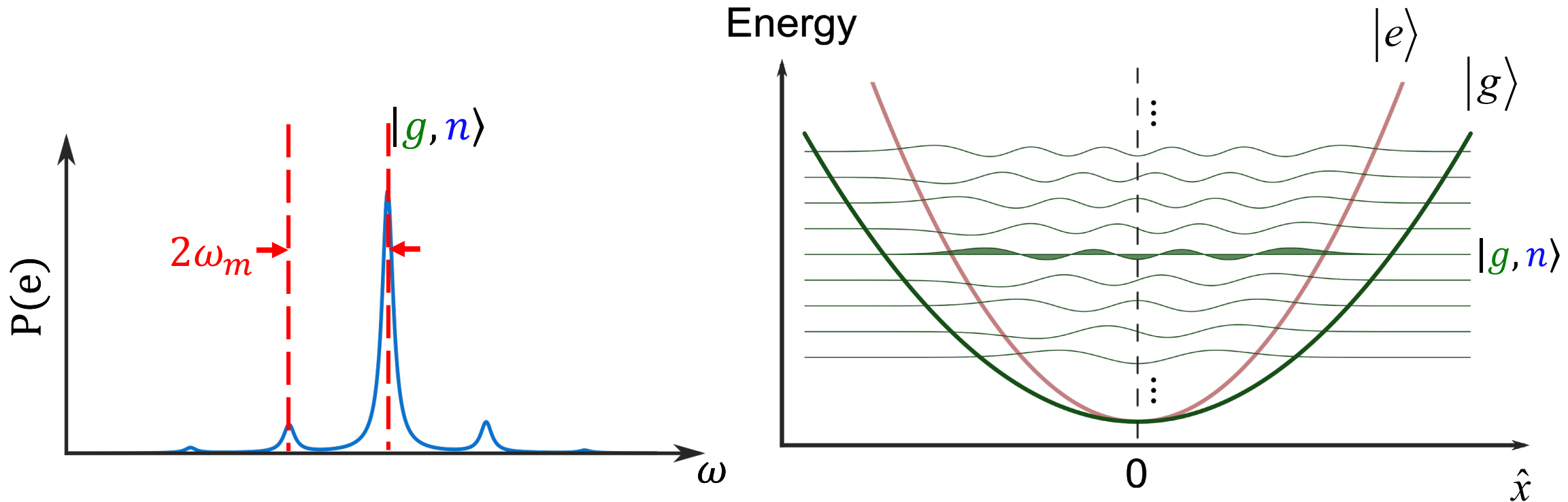
Qubit response to quantized motion

$$H_{\text{eff}} = \frac{1}{2} \omega_q \sigma_z + \frac{\hat{p}^2}{2m} + \frac{1}{2} k \hat{x}^2 + \frac{1}{2} k \left(\frac{2\chi_m}{\omega_m} \right) \sigma_z \hat{x}^2$$

$$\chi_m = 2 \frac{g^2}{\omega_q} \approx 2\pi \times 260 \text{ kHz}$$

$$\omega_m(\sigma_z) = \sqrt{k(\sigma_z)/m} \approx \omega_m^0 + \sigma_z \chi_m$$

$$Z_m(\sigma_z) = \sqrt{k(\sigma_z) \times m} = p_{\text{zpf}}/x_{\text{zpf}}$$



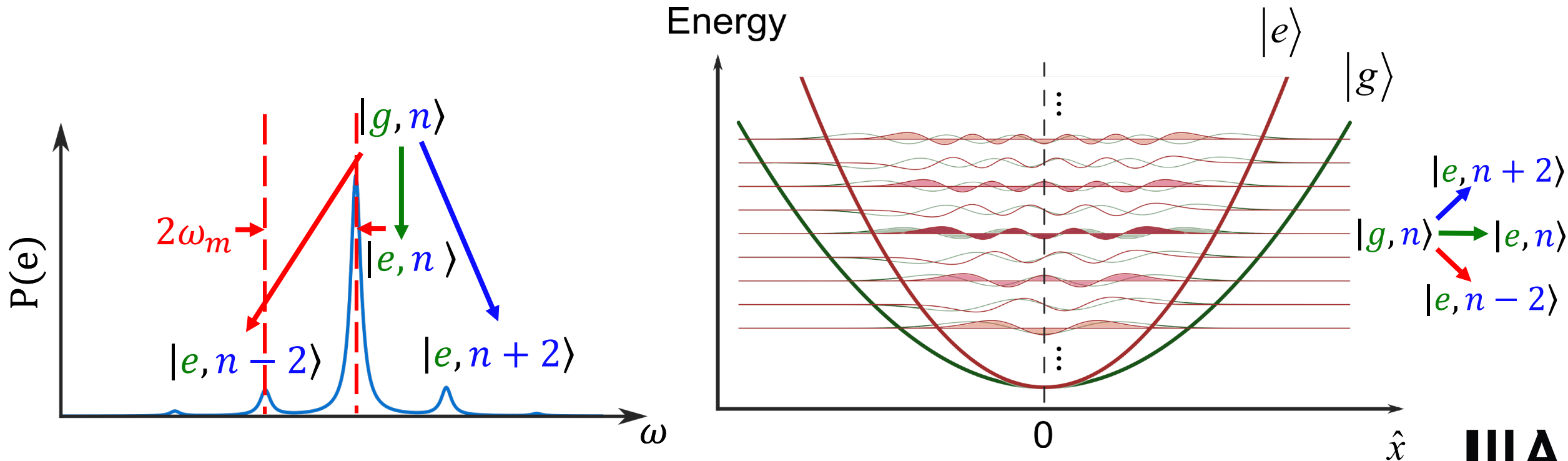
Qubit response to quantized motion

$$H_{\text{eff}} = \frac{1}{2} \omega_q \sigma_z + \frac{\hat{p}^2}{2m} + \frac{1}{2} k \hat{x}^2 + \frac{1}{2} k \left(\frac{2\chi_m}{\omega_m} \right) \sigma_z \hat{x}^2$$

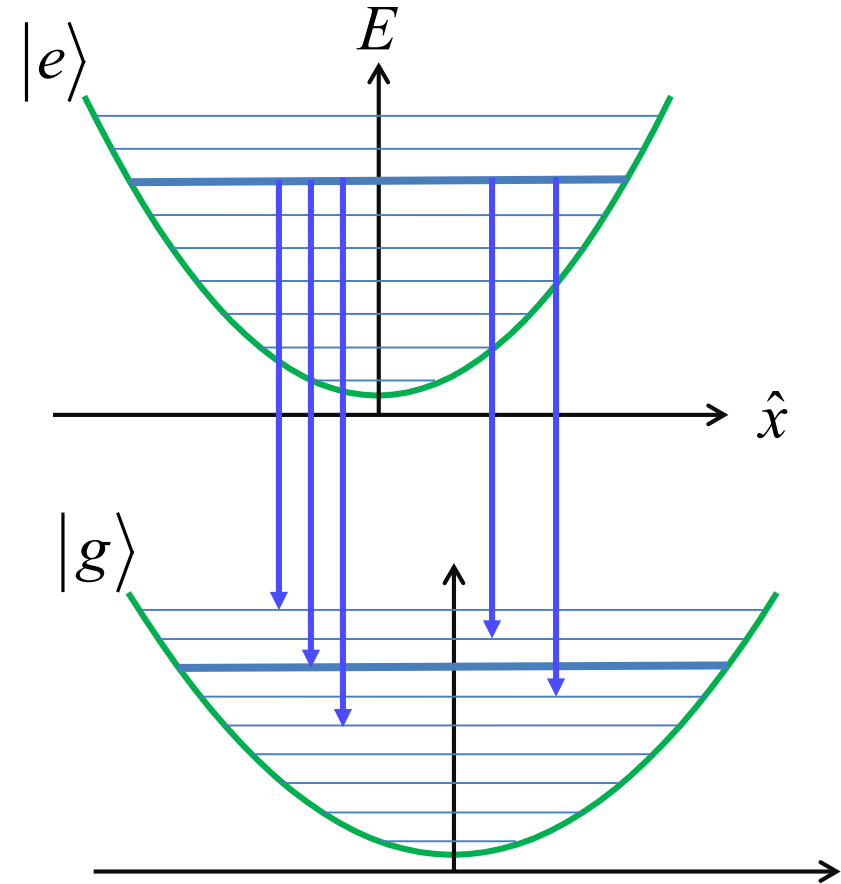
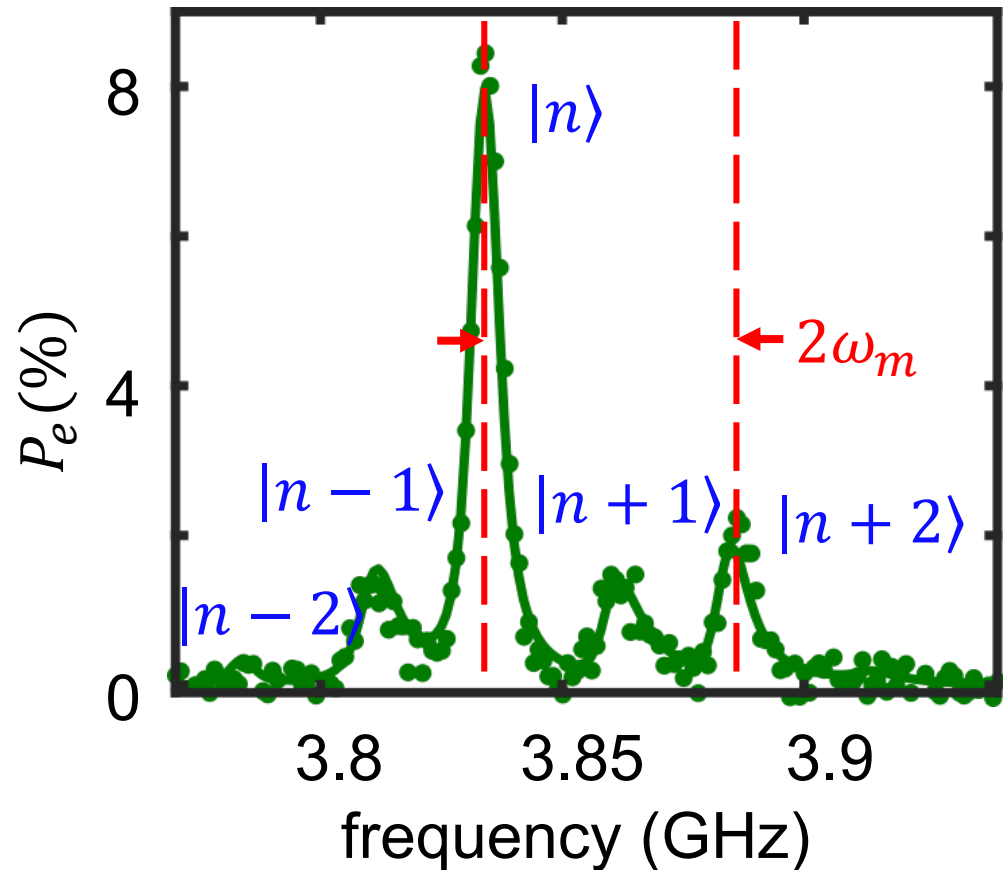
$$\chi_m = 2 \frac{g^2}{\omega_q} \approx 2\pi \times 260 \text{ kHz}$$

$$\omega_m(\sigma_z) = \sqrt{k(\sigma_z)/m} \approx \omega_m^0 + \sigma_z \chi_m$$

$$Z_m(\sigma_z) = \sqrt{k(\sigma_z) \times m} = p_{\text{zpf}}/x_{\text{zpf}}$$



Qubit spectrum acquires vibronic sidebands

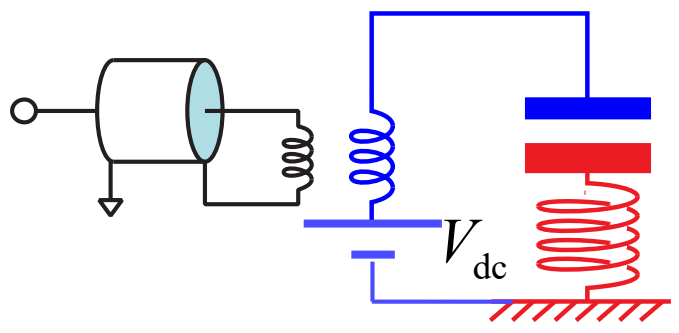


resolving the mechanical recoil of a qubit transition: Franck – Condon physics

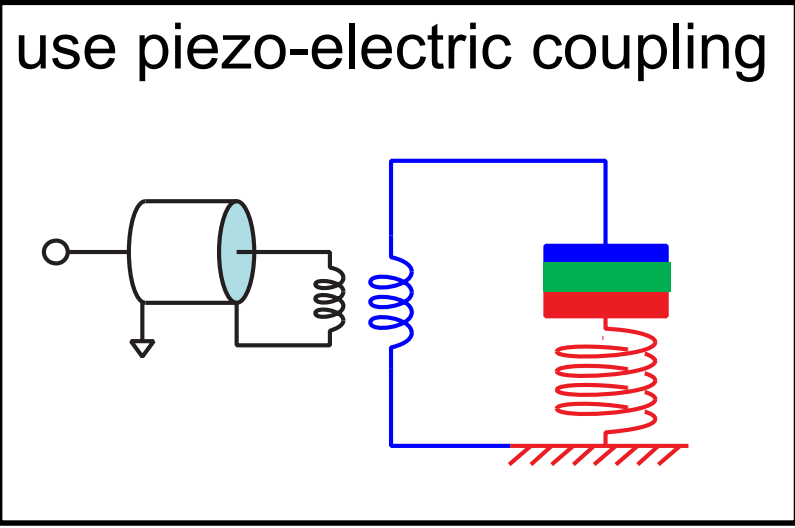
from artificial atoms to artificial molecules

Overcoming small vacuum electromechanical coupling

apply static voltage



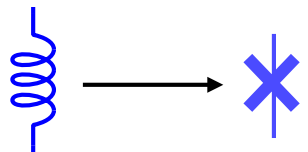
or



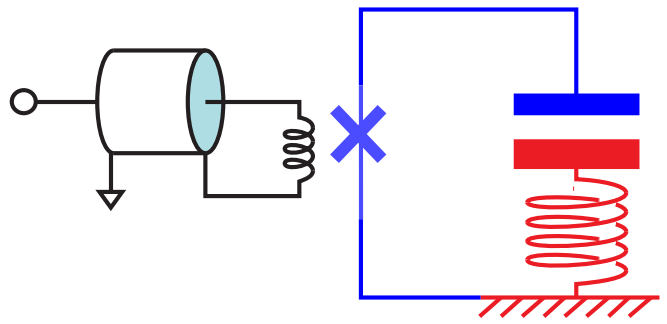
$$H_I = g_0 \left(a + a^\dagger \right) \left(b + b^\dagger \right)$$

$$g_0 \approx 2\pi \times 10 \text{ MHz}$$

add electrical non-linearity



superconducting qubit coupled mechanics



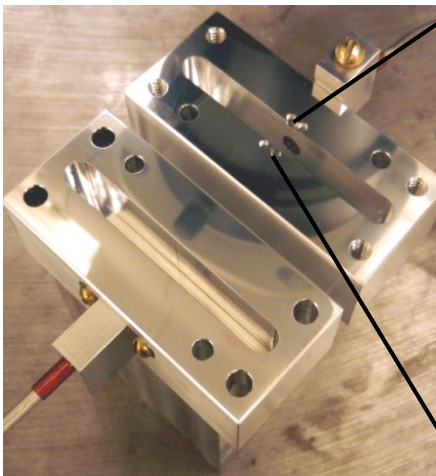
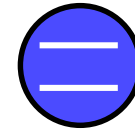
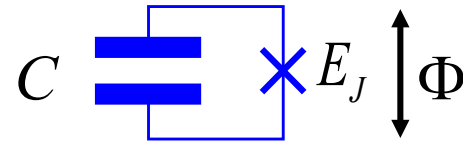
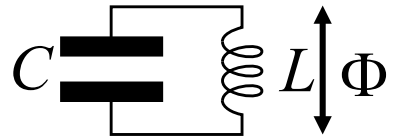
$$H_I = g_0 \sigma_x \left(b + b^\dagger \right)$$

accessing single phonons with piezomechanics

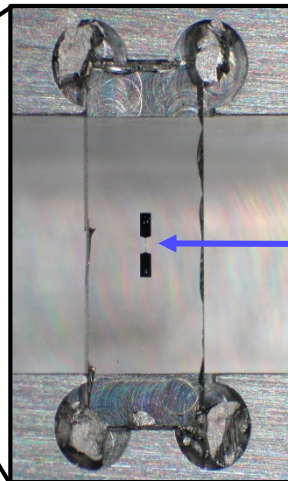
Transmon qubit: a strongly nonlinear LC circuit

$$H = \frac{\Phi^2}{2L} + \frac{Q^2}{2C}$$

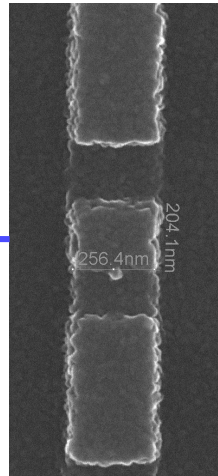
$$H = \frac{E_J}{2} \left(1 - \cos \left(2\pi \frac{\Phi}{\Phi_0} \right) \right) + \frac{Q^2}{2C}$$



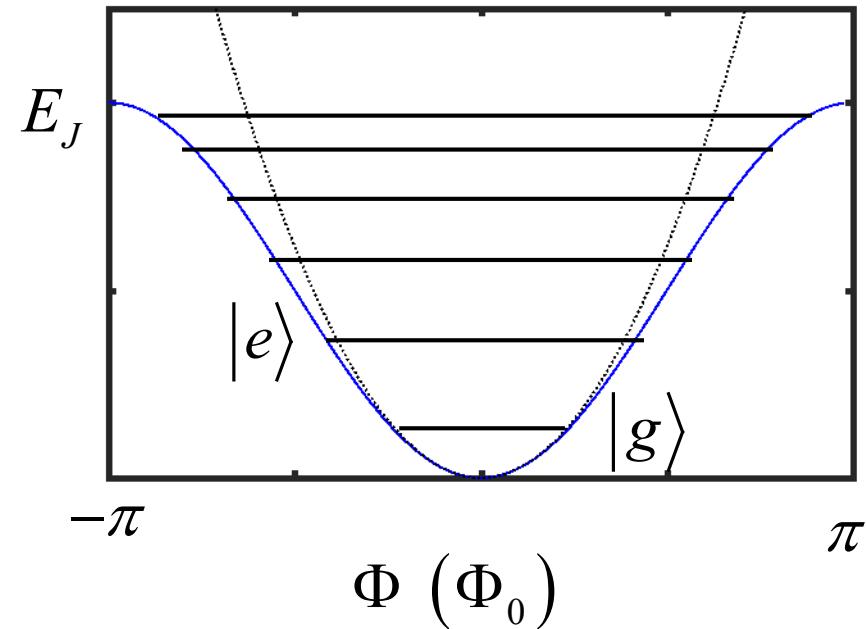
Al cavity



Al antenna
on sapphire



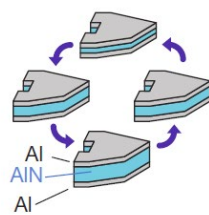
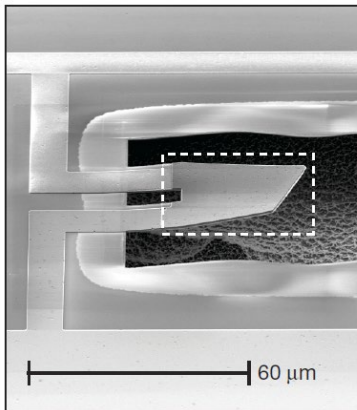
Al/AlOx/Al
Josephson junction



Circuit quantum acoustodynamics (CQAD): Superconducting qubits and piezo-electric materials

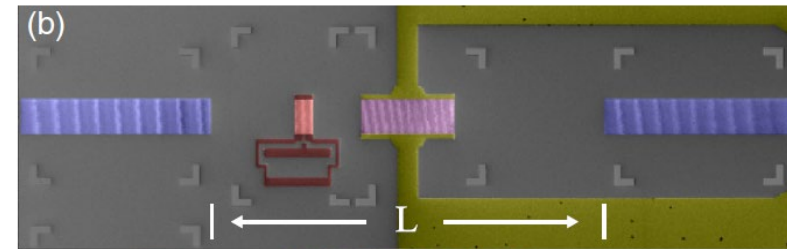
strong resonant coupling (qutons and phonbits) $g_0 > \max[\kappa, \gamma]$

bulk waves



A. O'Connell, A. Cleland
UCSB, 2011

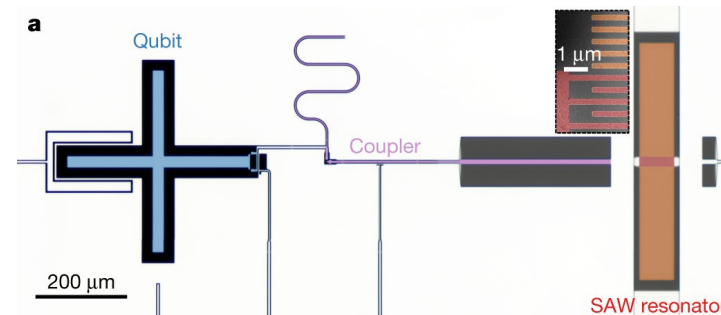
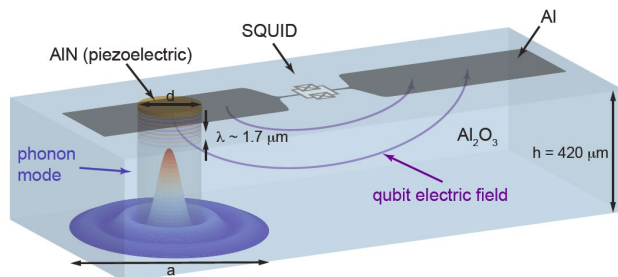
surface waves



B. Moores, KWL,
JILA, 2018

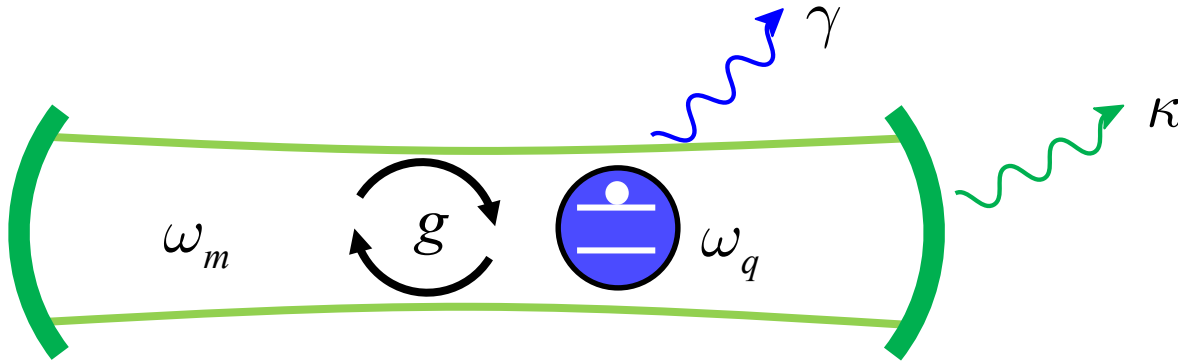


Y. Chu, R. Schoelkopf
Yale, 2017



K. Satzinger, A. Cleland
Chicago, 2018

Count phonons in the dispersive limit of CQAD



$$\hat{H} / \hbar \approx \frac{1}{2} \omega_q \hat{\sigma}_z + \omega_m \hat{b}^\dagger \hat{b} + g_0 \hat{\sigma}_x (\hat{b}^\dagger + \hat{b})$$

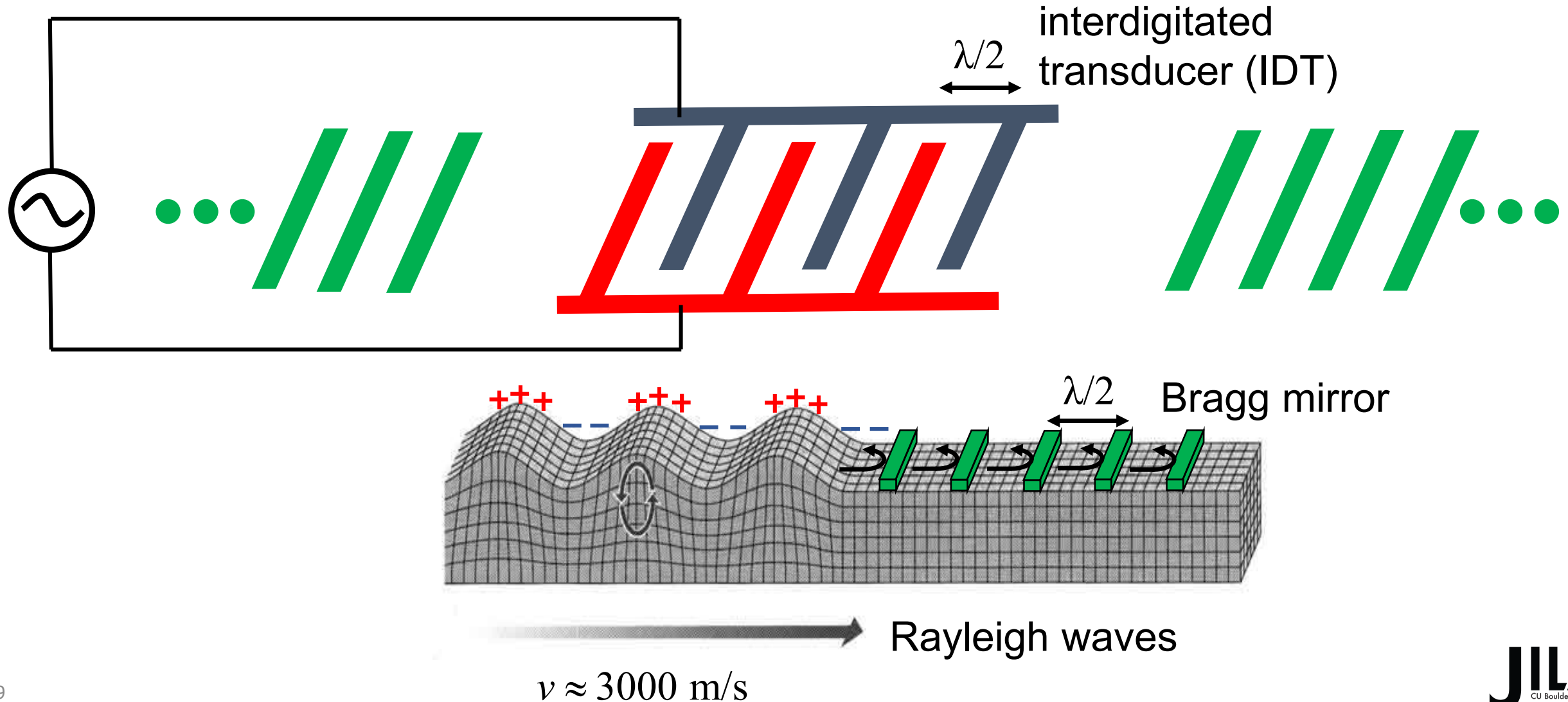
dispersive limit $\omega_q - \omega_m = \Delta \gg g$

$$\hat{H} / \hbar \approx \frac{1}{2} \omega_q \hat{\sigma}_z + \omega_m \hat{b}^\dagger \hat{b} + \chi \hat{\sigma}_z \hat{b}^\dagger \hat{b} \quad \chi = \frac{g^2}{\Delta}$$

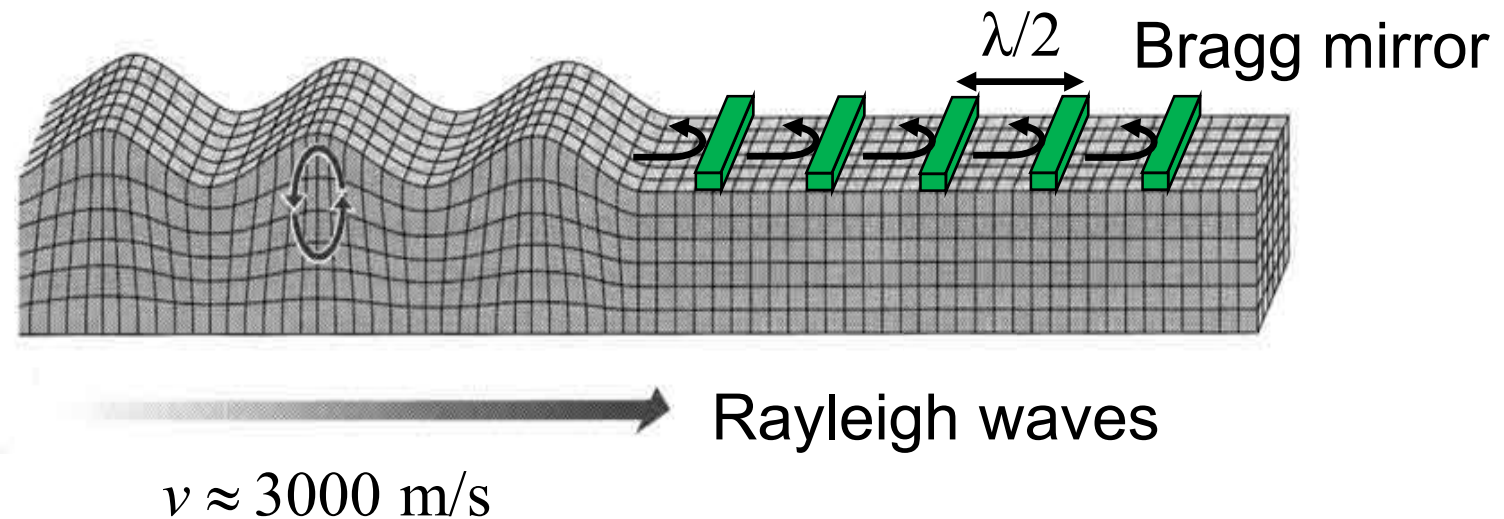
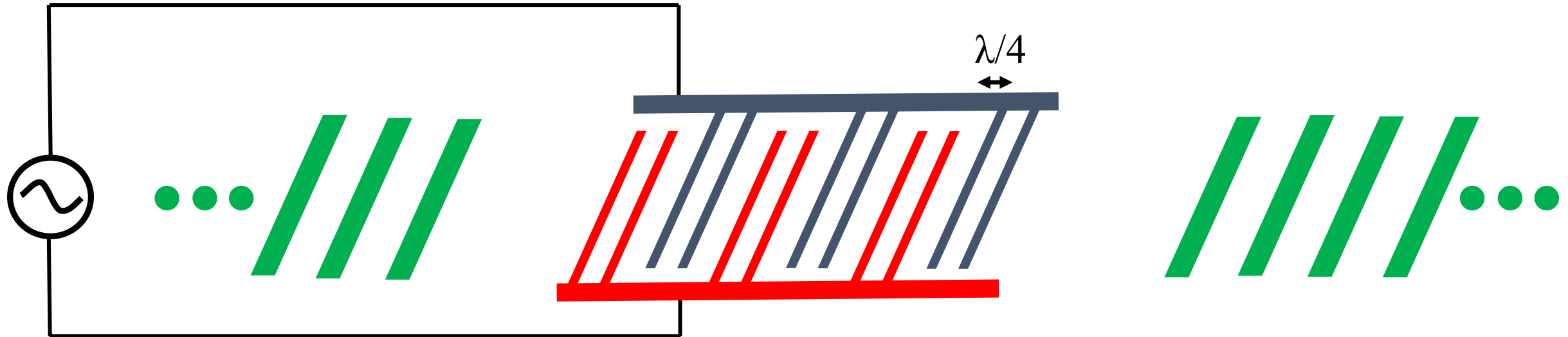
resolve single phonon acoustical Stark shift

$$\chi > \max[\kappa, \gamma]$$

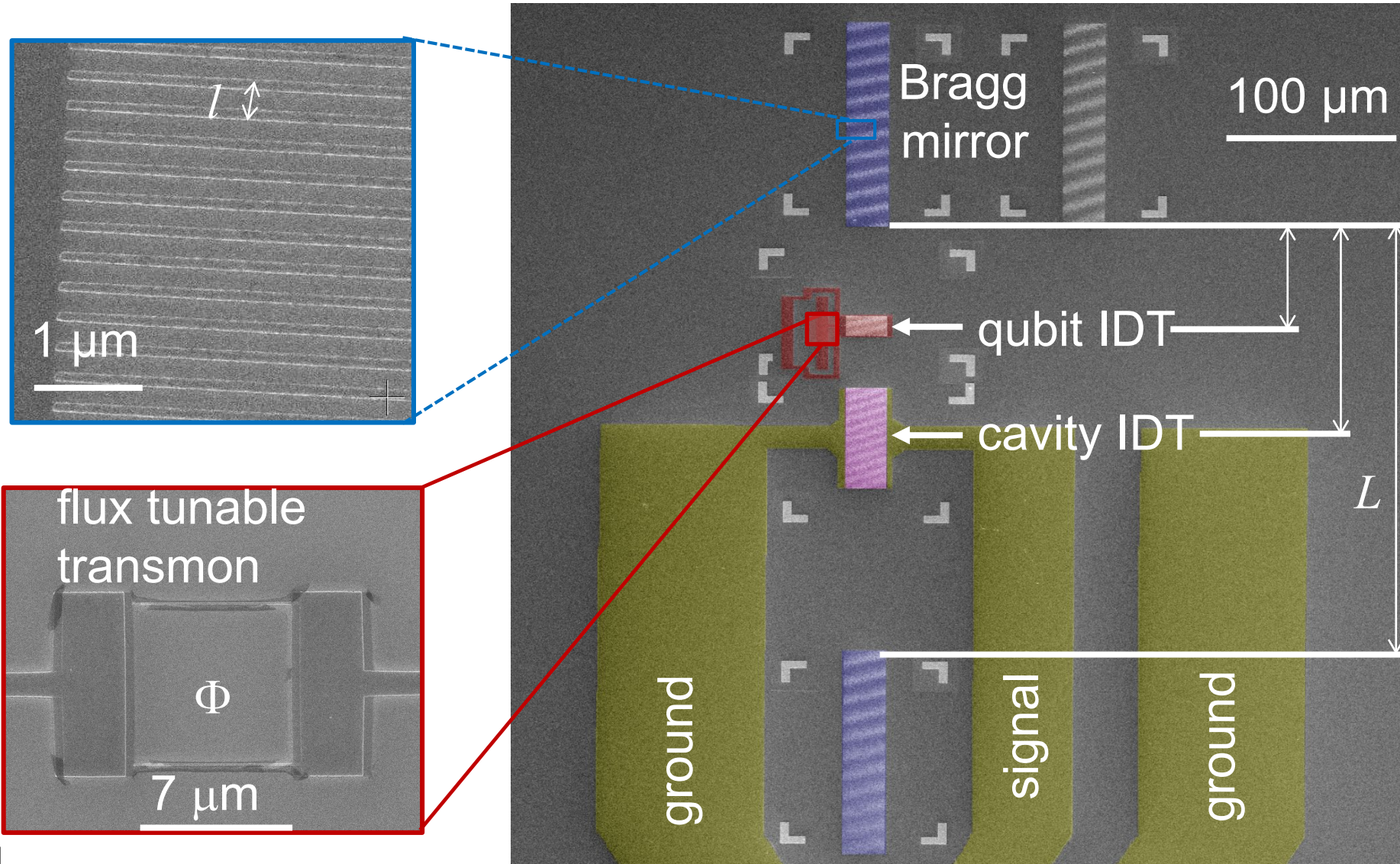
SAW waves confined between mirrors form multimode cavities



Split finger transducers launch SAWs without reflecting them



Transmon qubit coupled piezoelectrically to multimode SAW cavity in GaAs



$$L = 400 \mu\text{m}$$

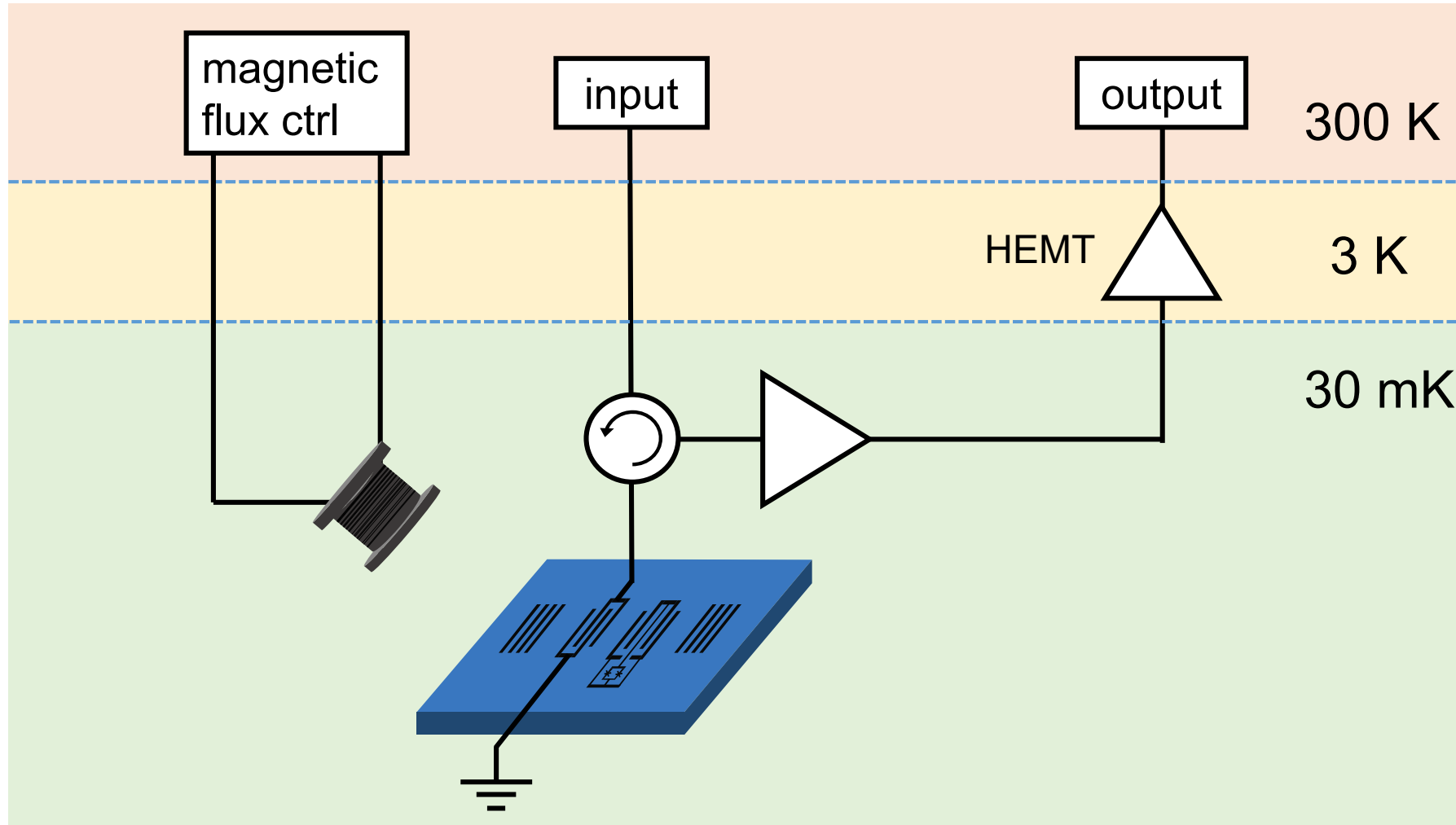
$$\text{FSR: } 4.8 \text{ MHz}$$

$$l = 380 \text{ nm}$$

$$\omega_{\text{cav}} \approx 2\pi \times 4.25 \text{ GHz}$$

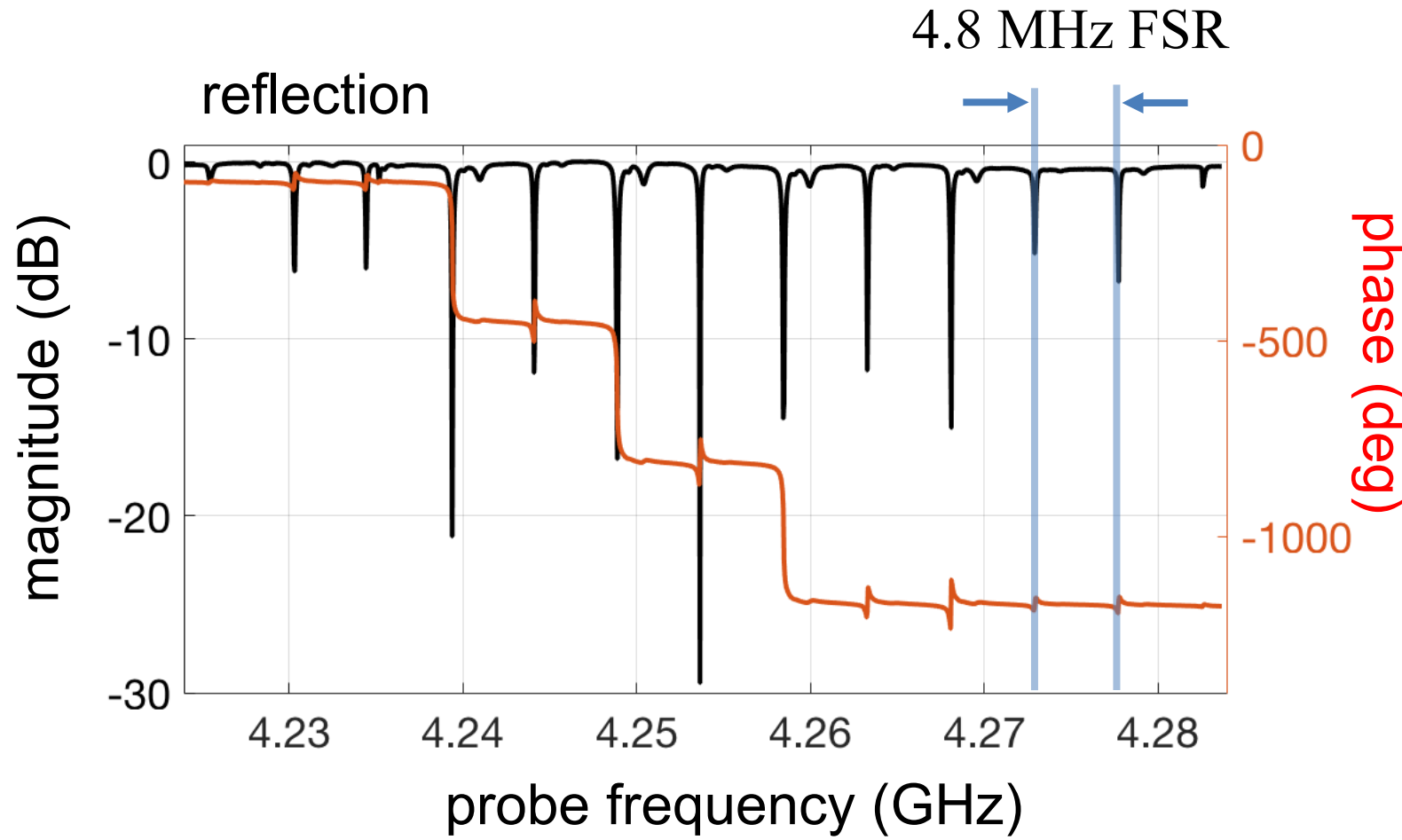
mirror bandwidth
50 MHz

All acoustical measurement and control of qubit



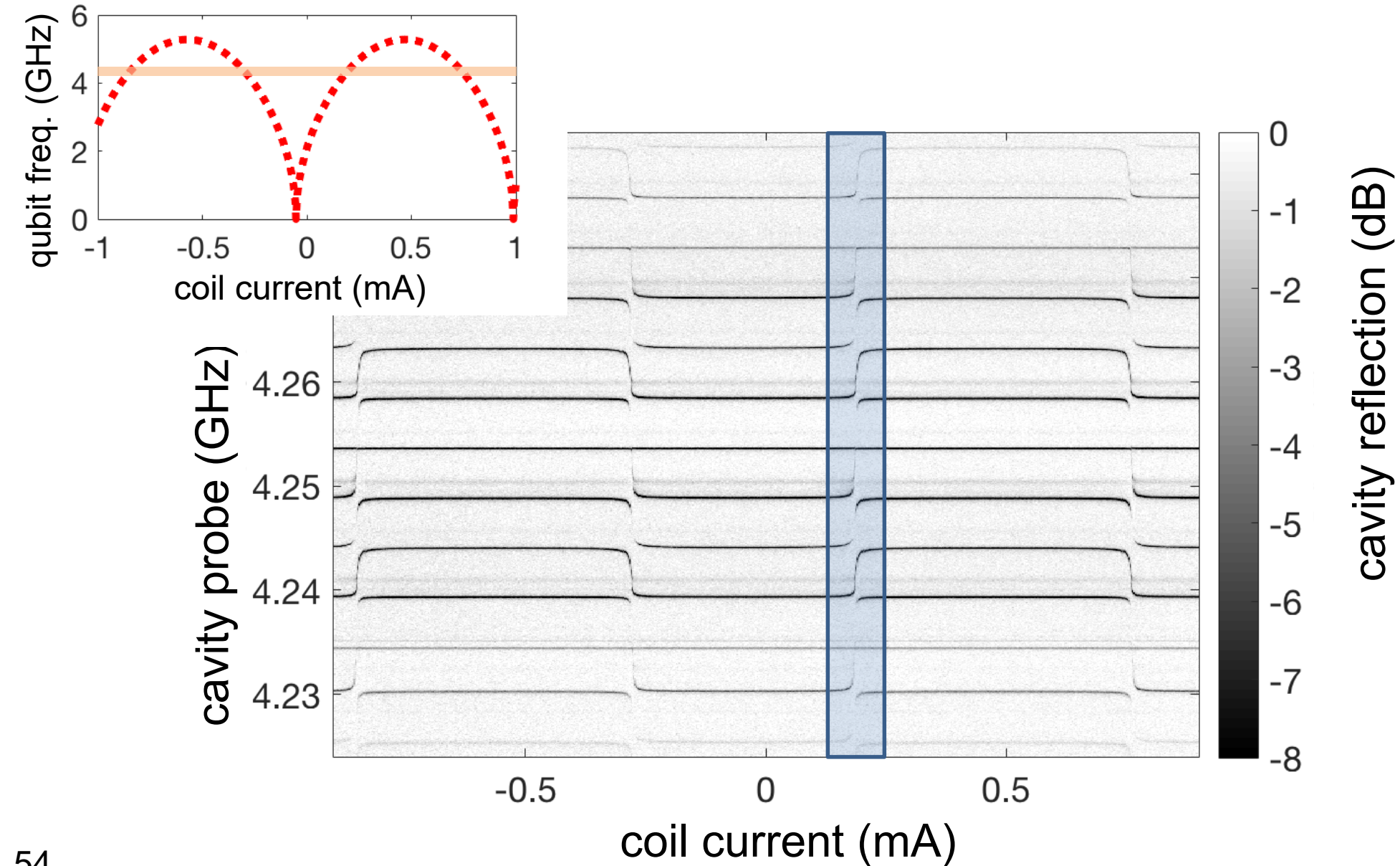
measure SAW cavity reflection

Cavity reflection reveals spectrum of high-Q longitudinal SAW mode

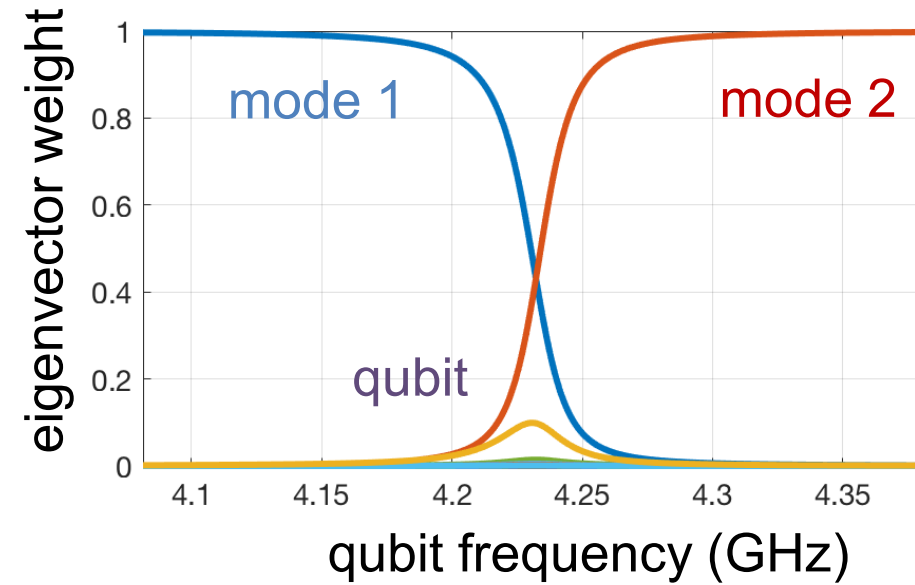
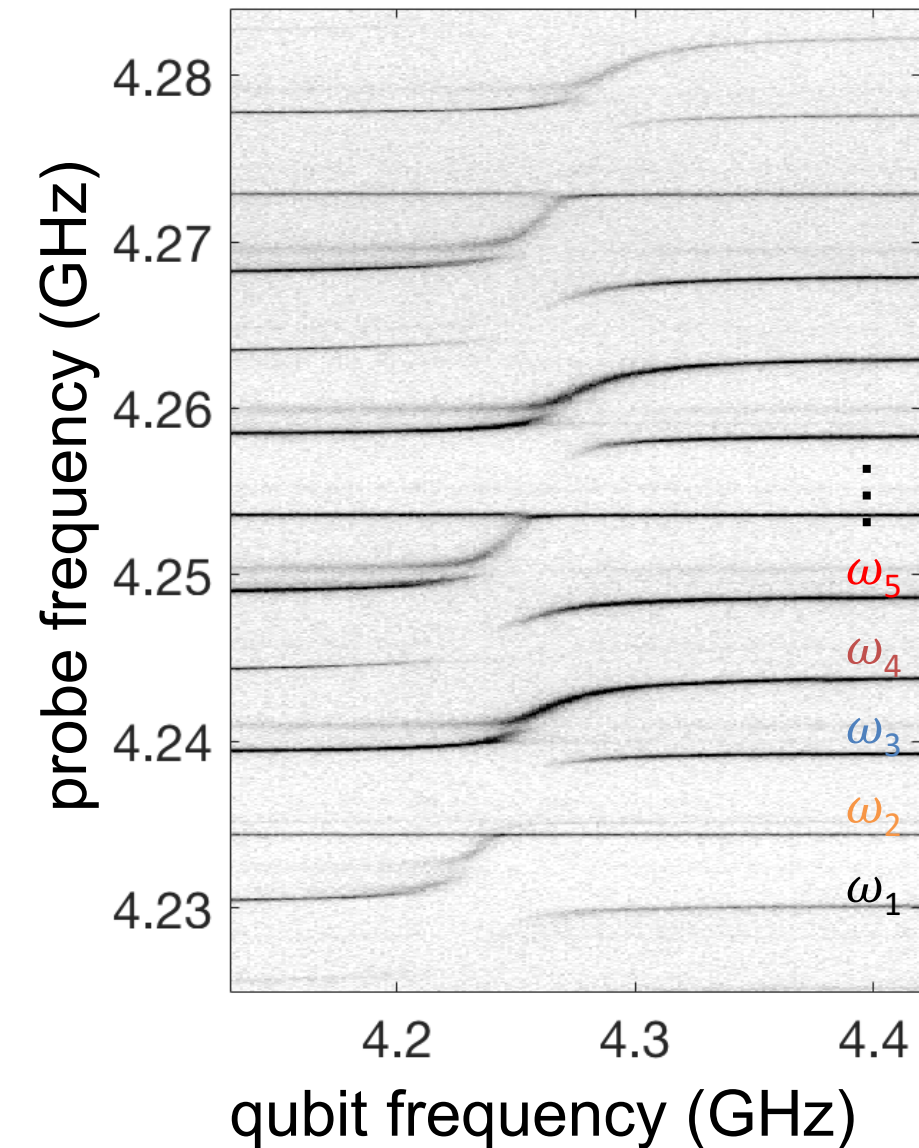


$$\kappa \approx 2\pi \times 100 \text{ kHz}$$

CQAD system in the strong multimode regime



Qubit-cavity avoided crossing show coherent coupling



strong coupling limit: $g_0 \approx 2\pi \times 6.5 \text{ MHz}$

$$g_0 > \max(\kappa, \gamma)$$

multimode limit: intrinsic qubit linewidth:

$$g_0 > \omega_{\text{fsr}} \quad \frac{\gamma}{2\pi} = 1.1 \text{ MHz}$$

Dense acoustical modes: too much of good thing?

strong dispersive limit

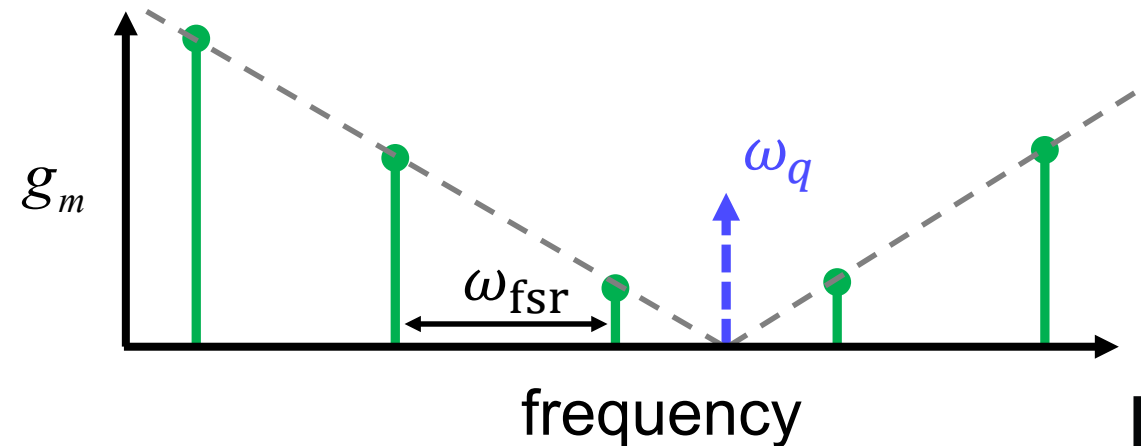
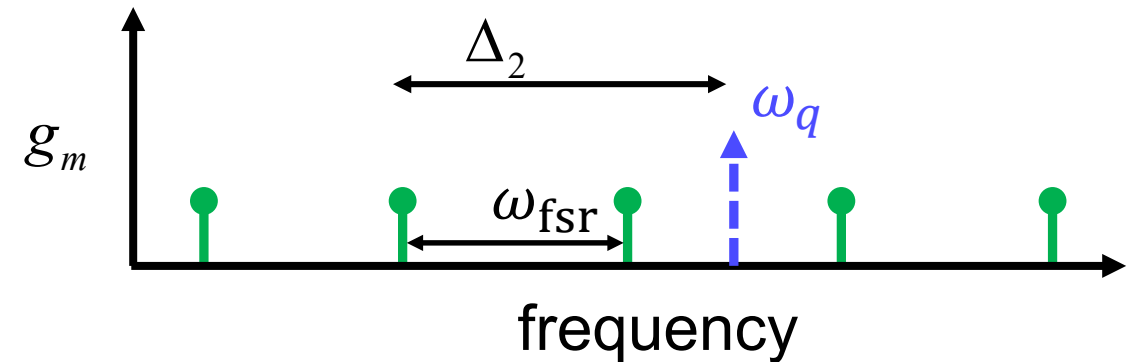
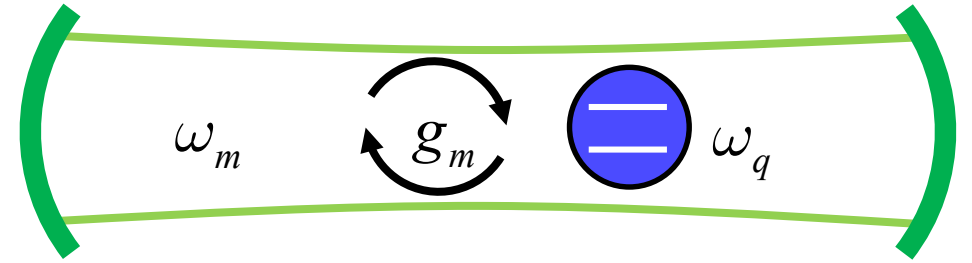
$$g_m \ll \Delta_m \quad \chi_m = \frac{g_m^2}{\Delta_m} \gg \kappa_m$$

uniform coupling

$$g_m \ll \omega_{\text{fsr}}$$

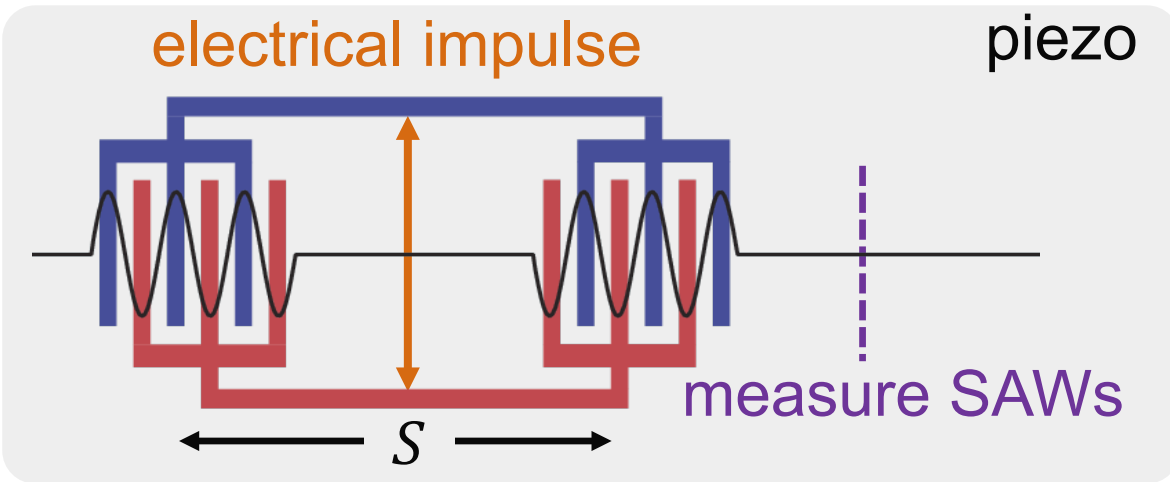
frequency-sensitive coupling

$$g_m = \frac{\Delta_m}{10}$$



Acoustical Ramsey interferometric coupling

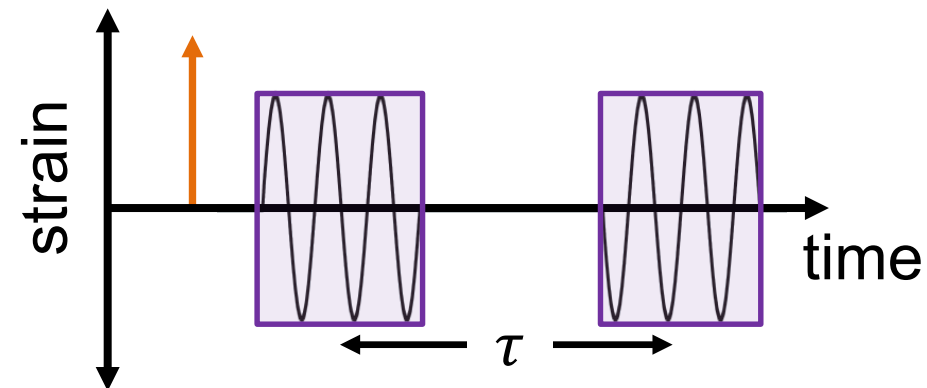
space



split IDT:
double-slit diffraction

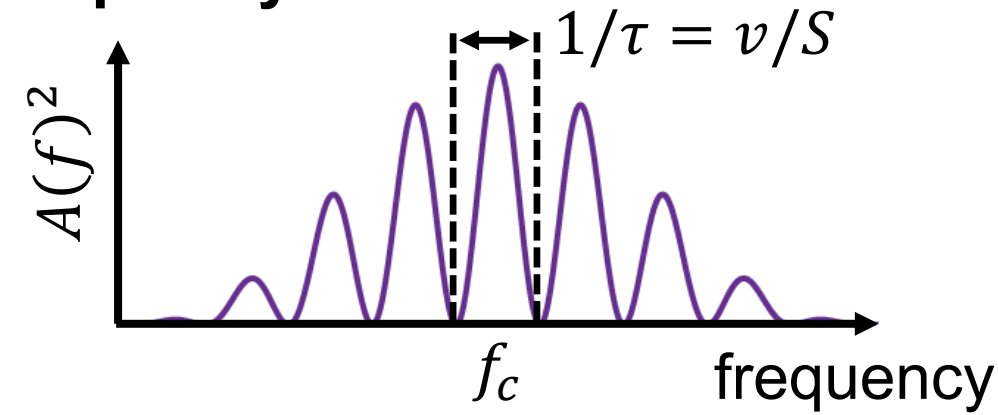
frequency domain control
via IDT geometry

time

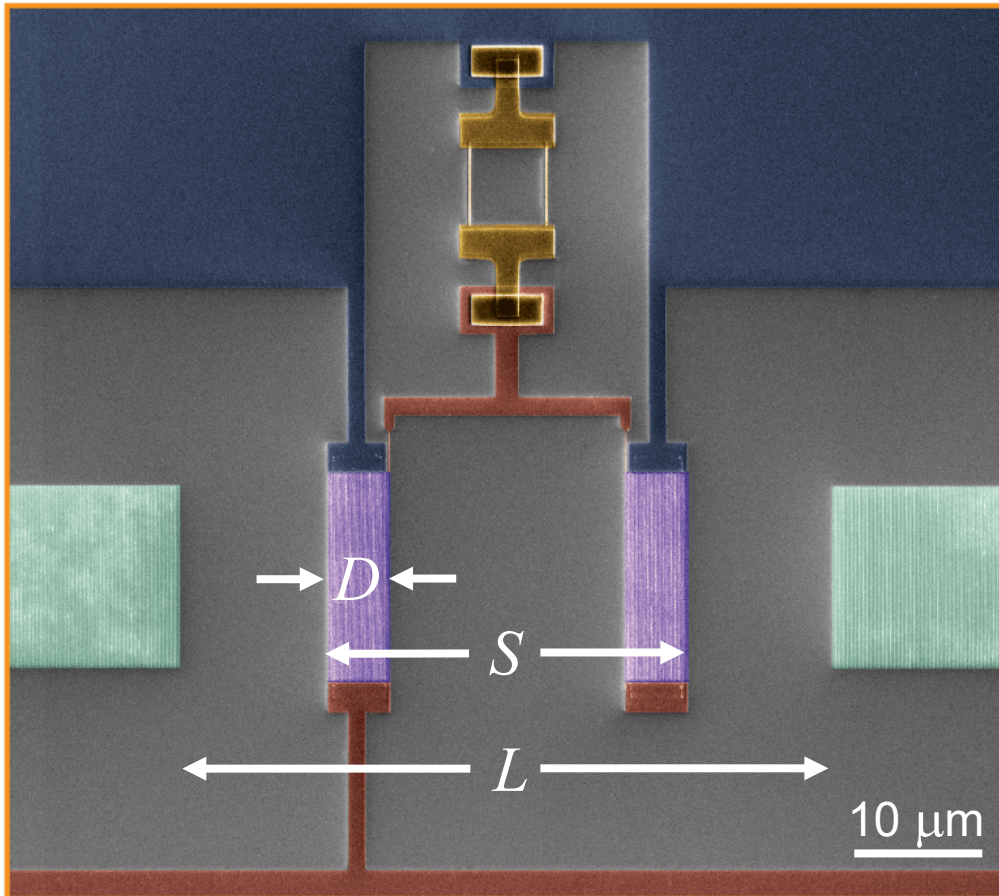
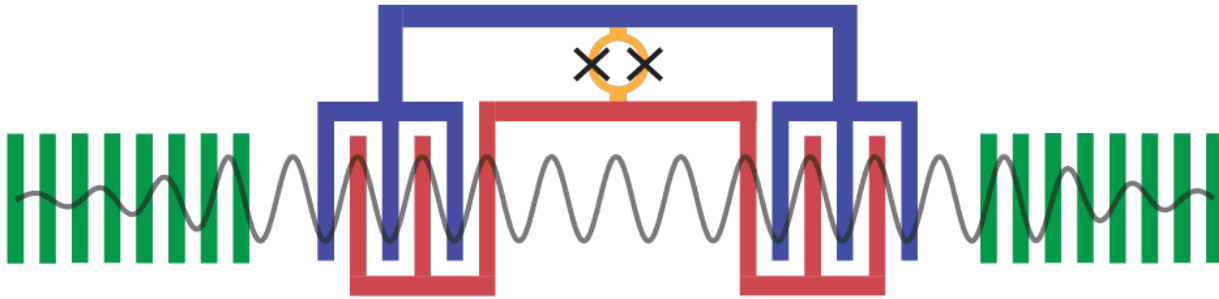


Fourier transform

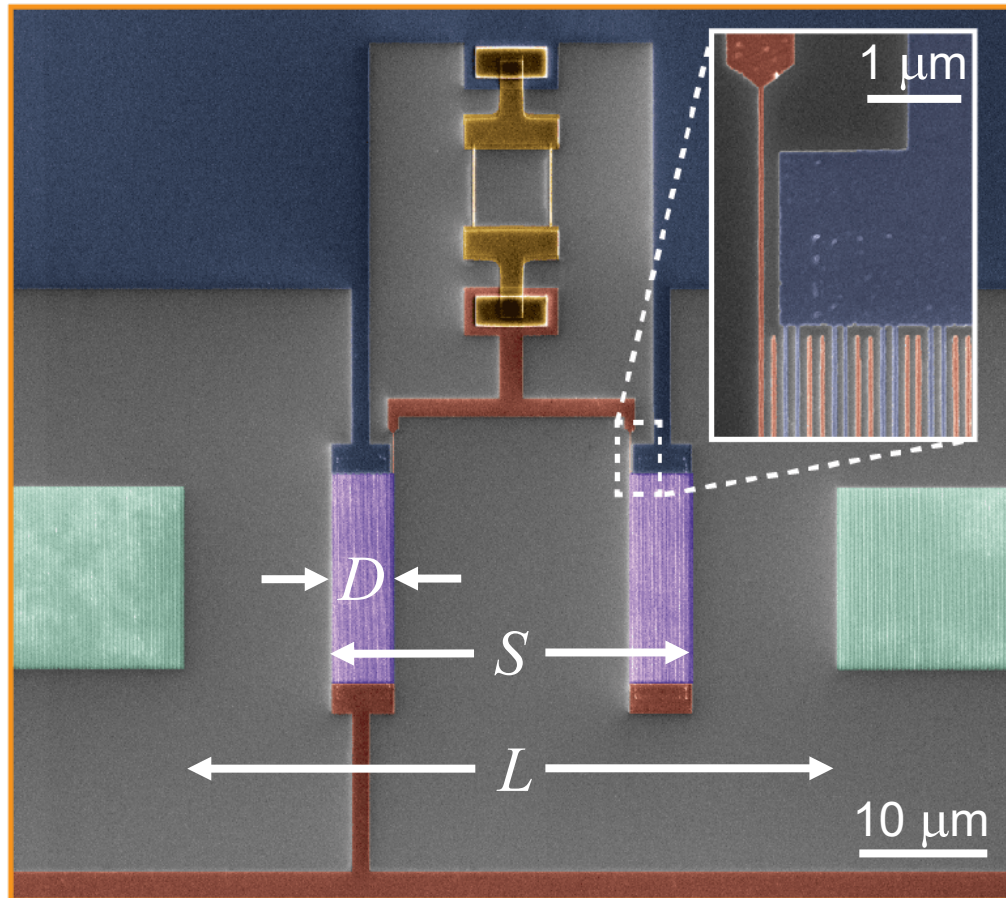
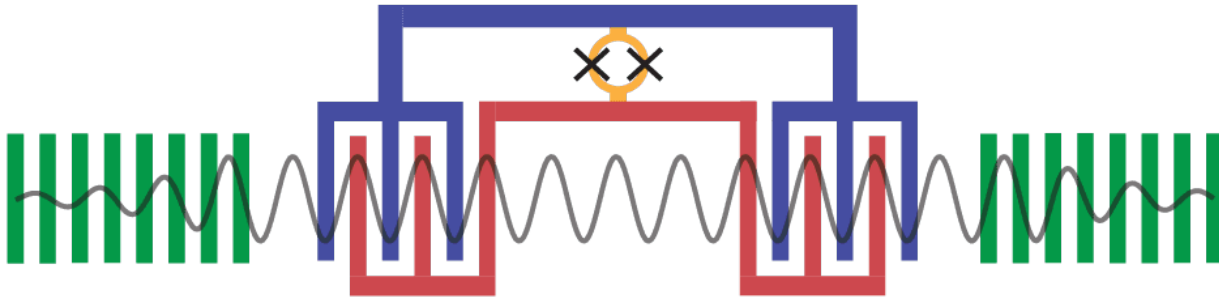
frequency



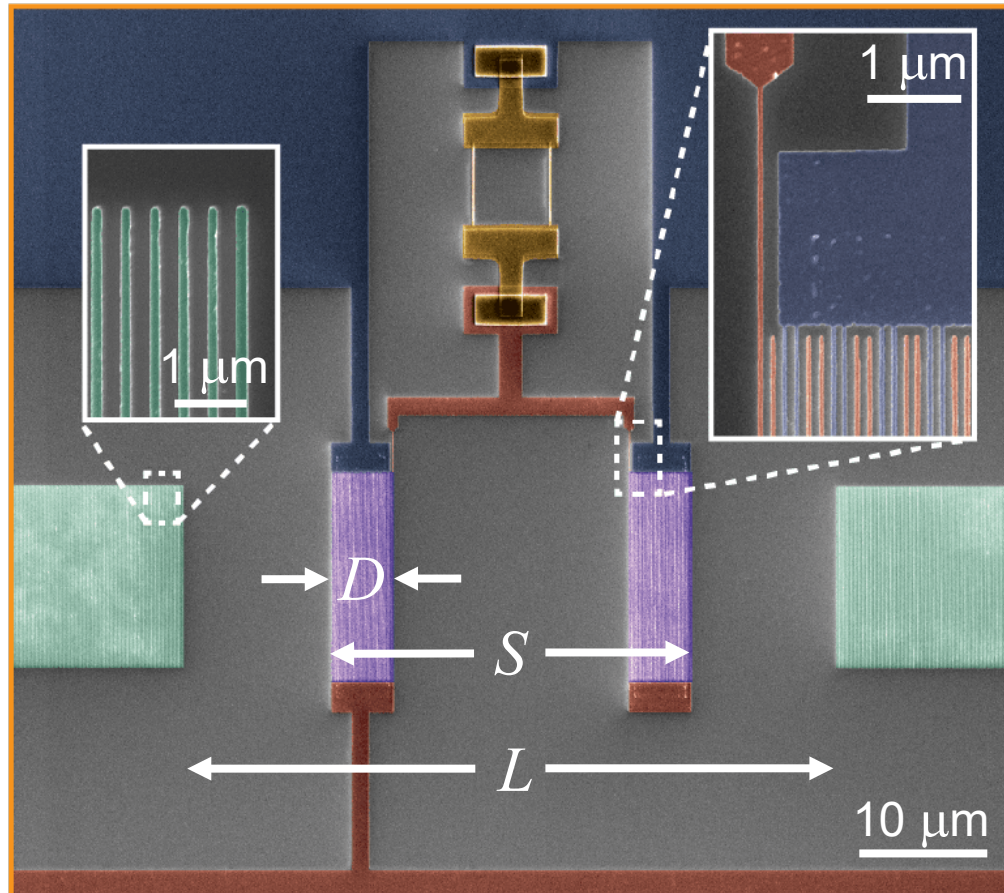
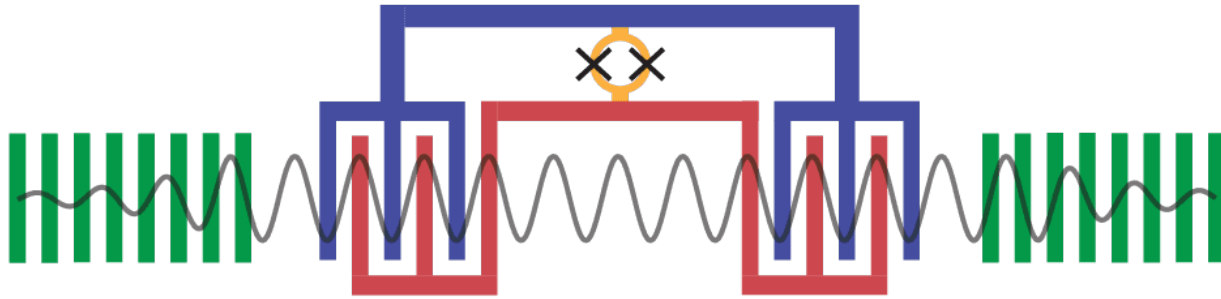
Ramsey qubit: realizing frequency sensitive couplings



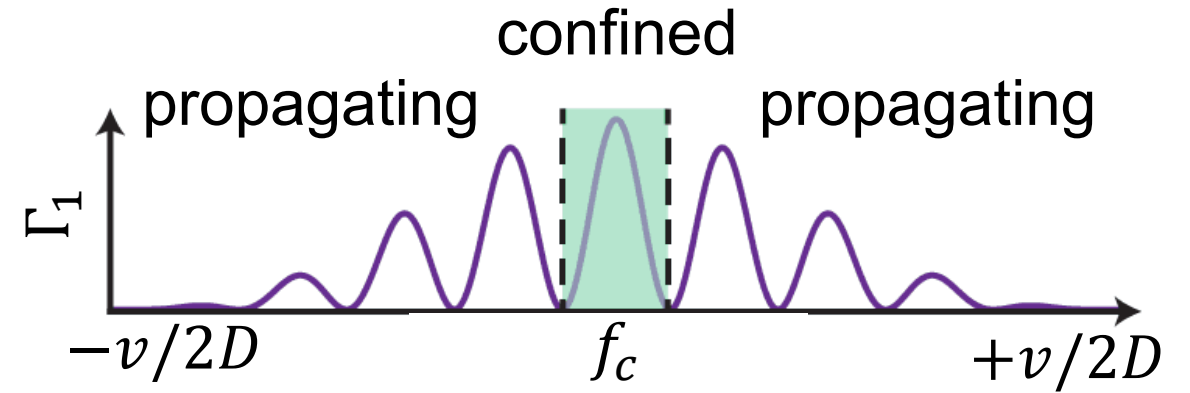
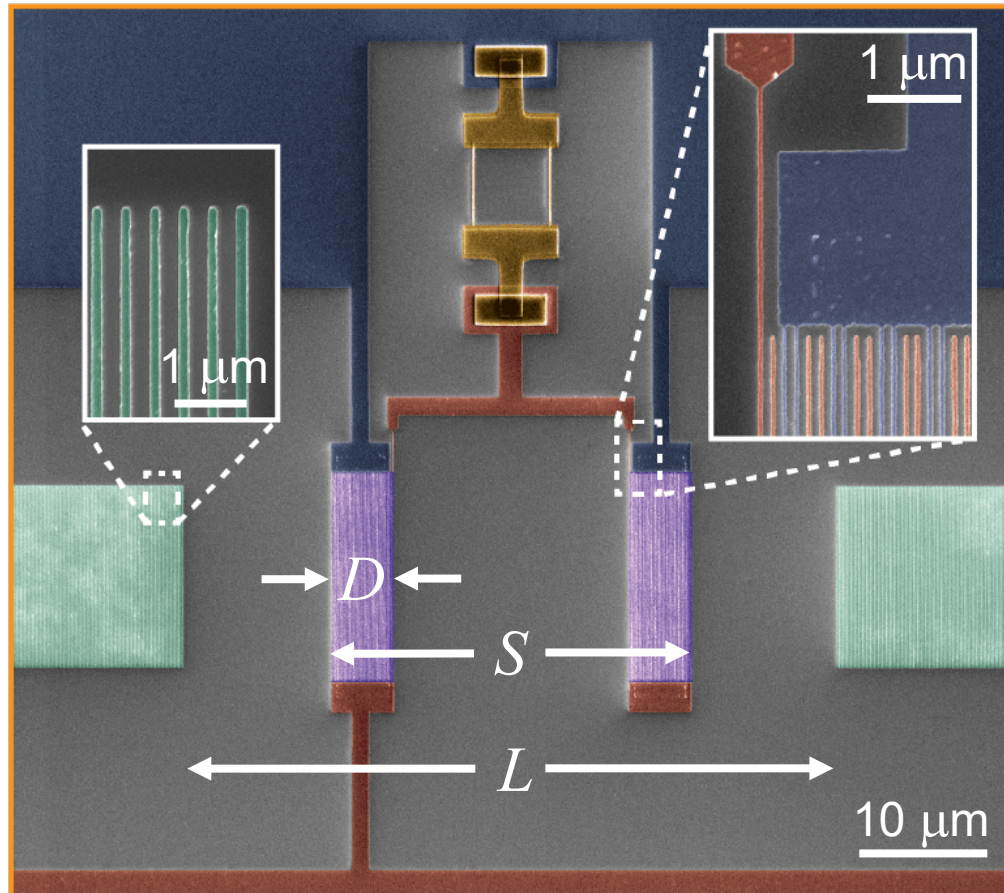
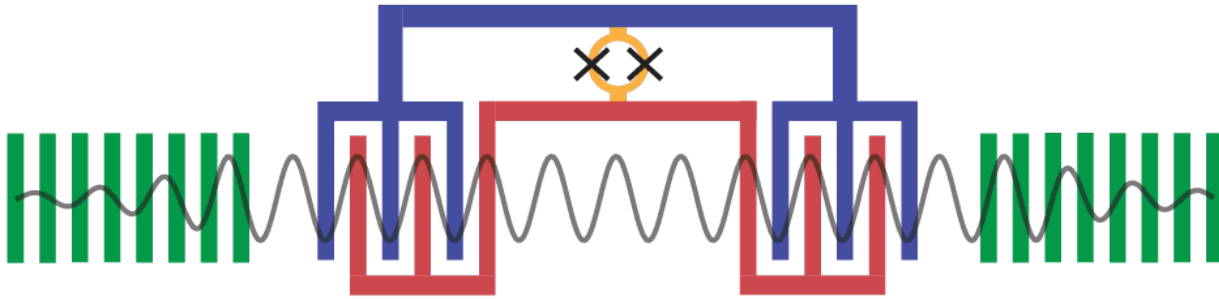
Ramsey qubit: realizing frequency sensitive couplings



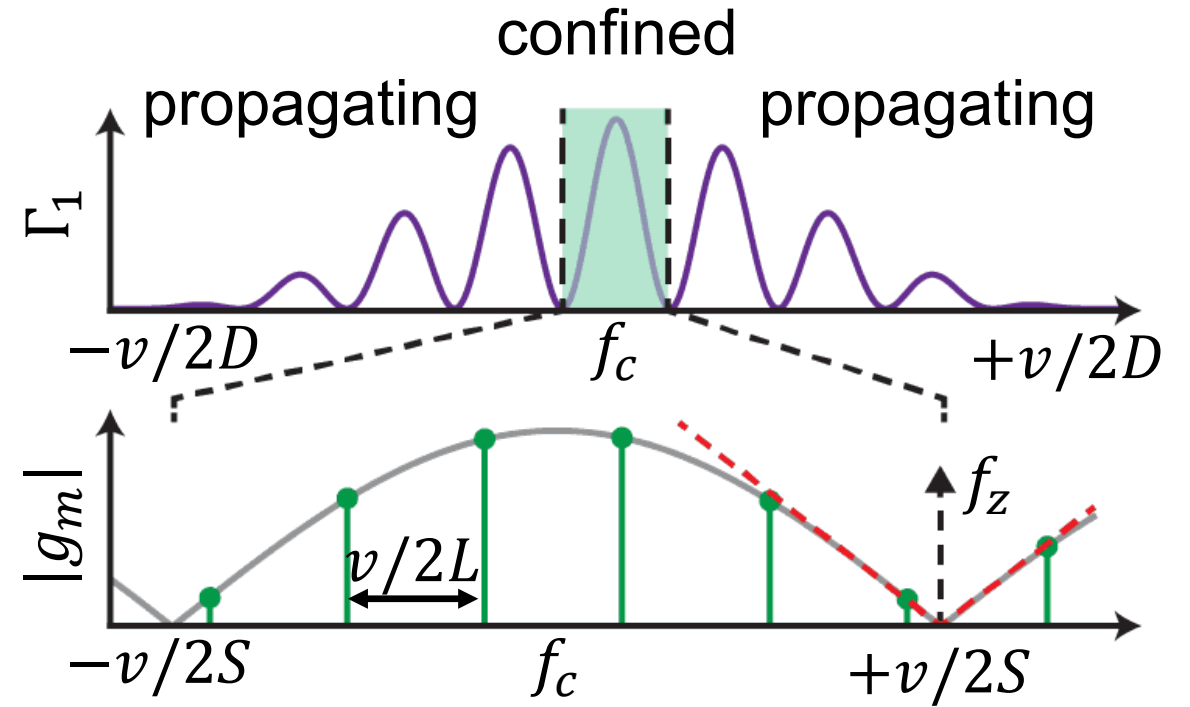
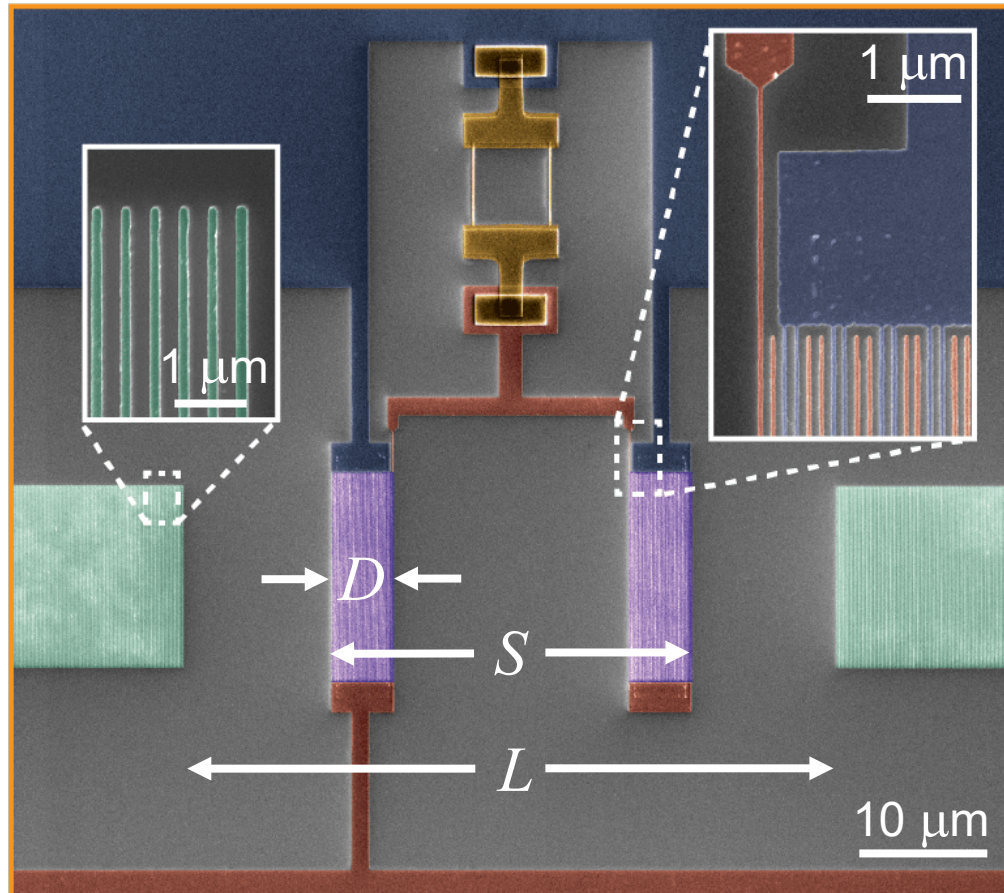
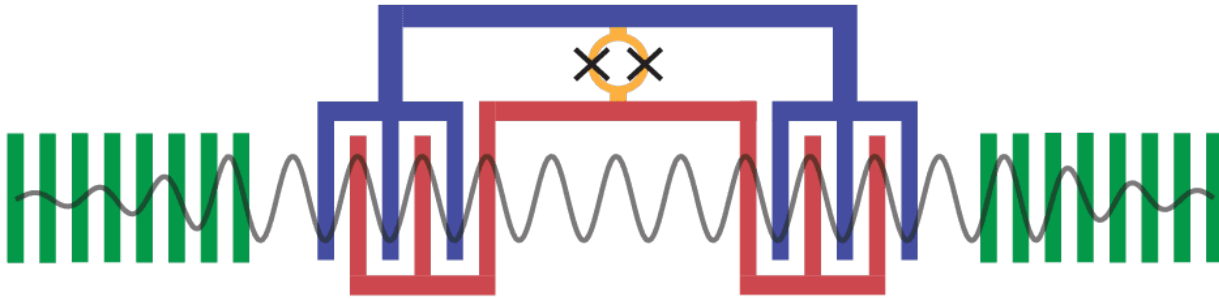
Ramsey qubit: realizing frequency sensitive couplings



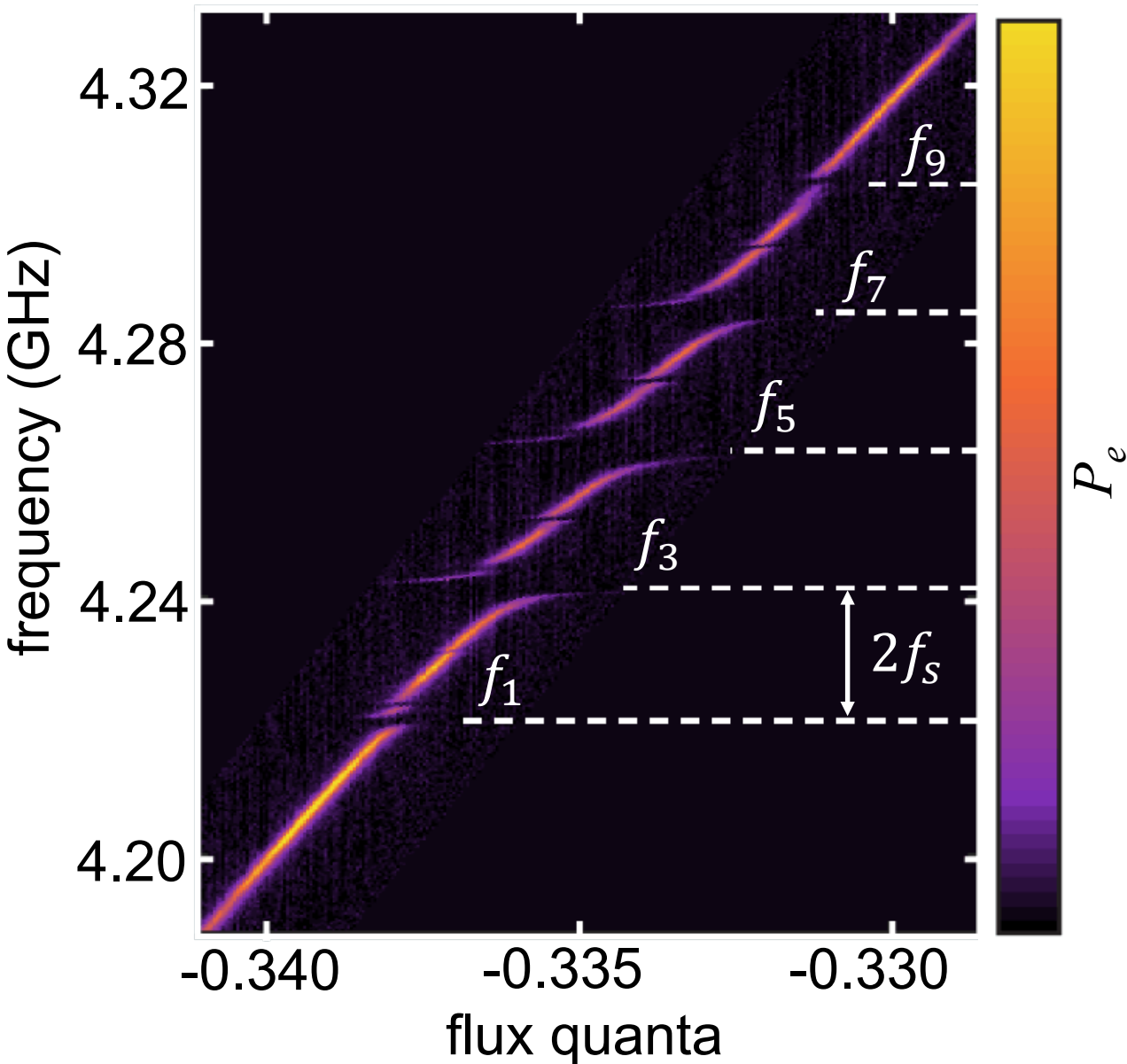
Ramsey qubit: realizing frequency sensitive couplings



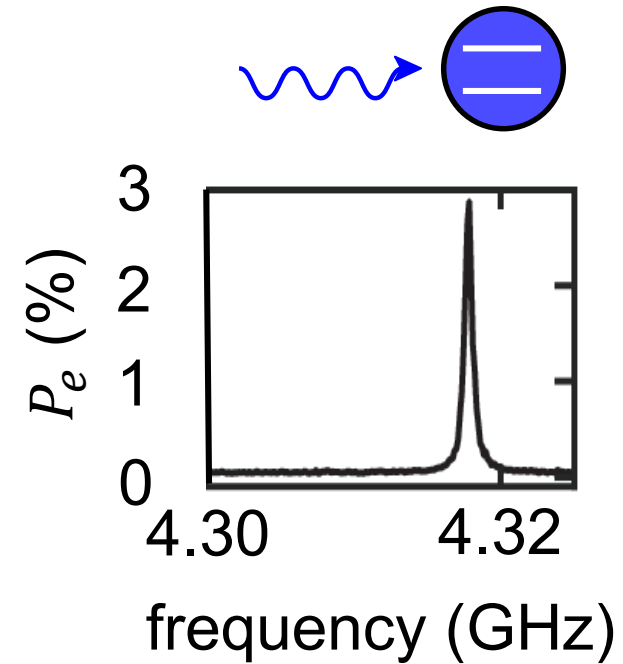
Ramsey qubit: realizing frequency sensitive couplings



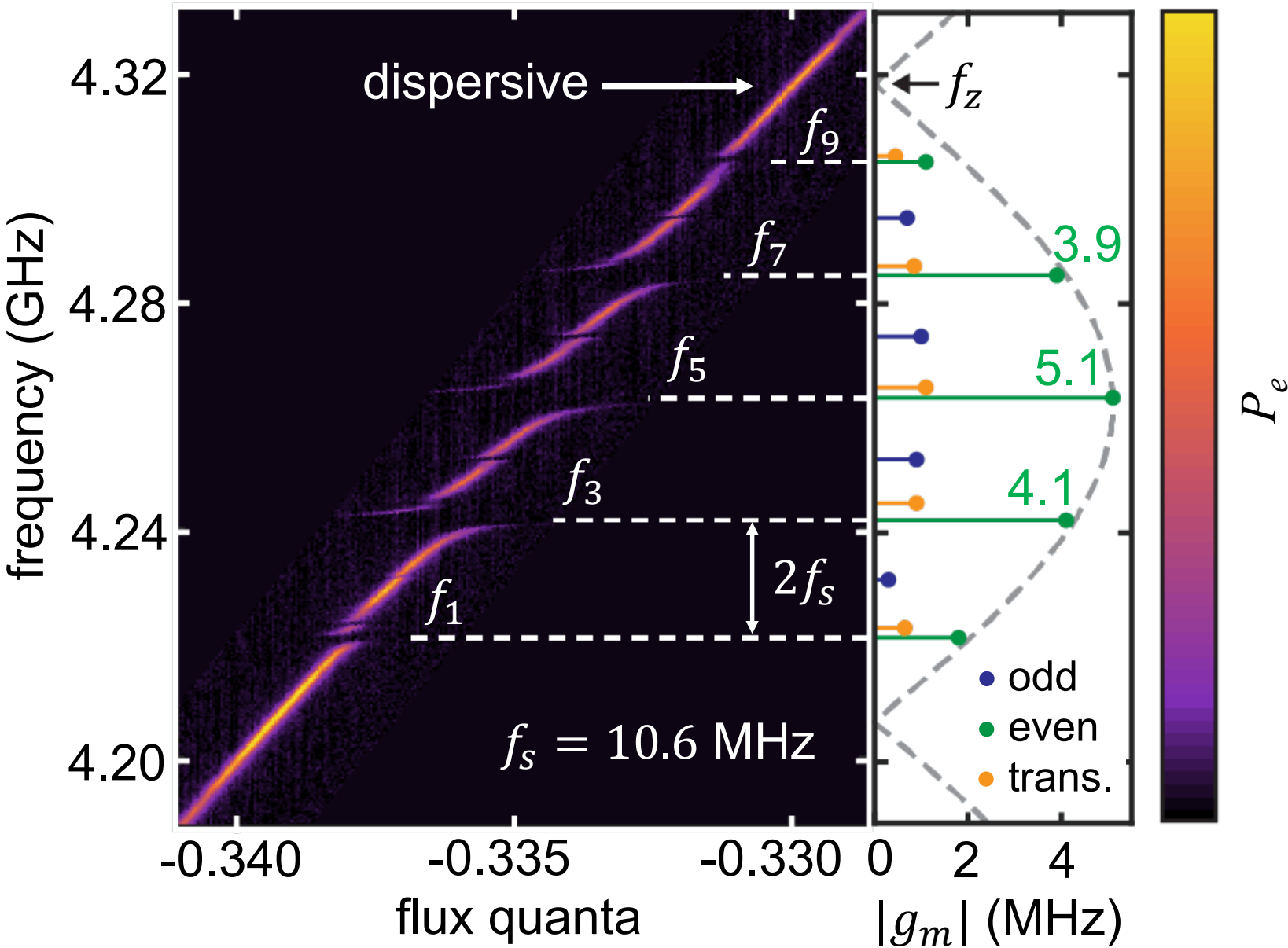
Tune qubit through acoustical cavity resonances



qubit spectroscopy



Measured coupling strengths depend on frequency



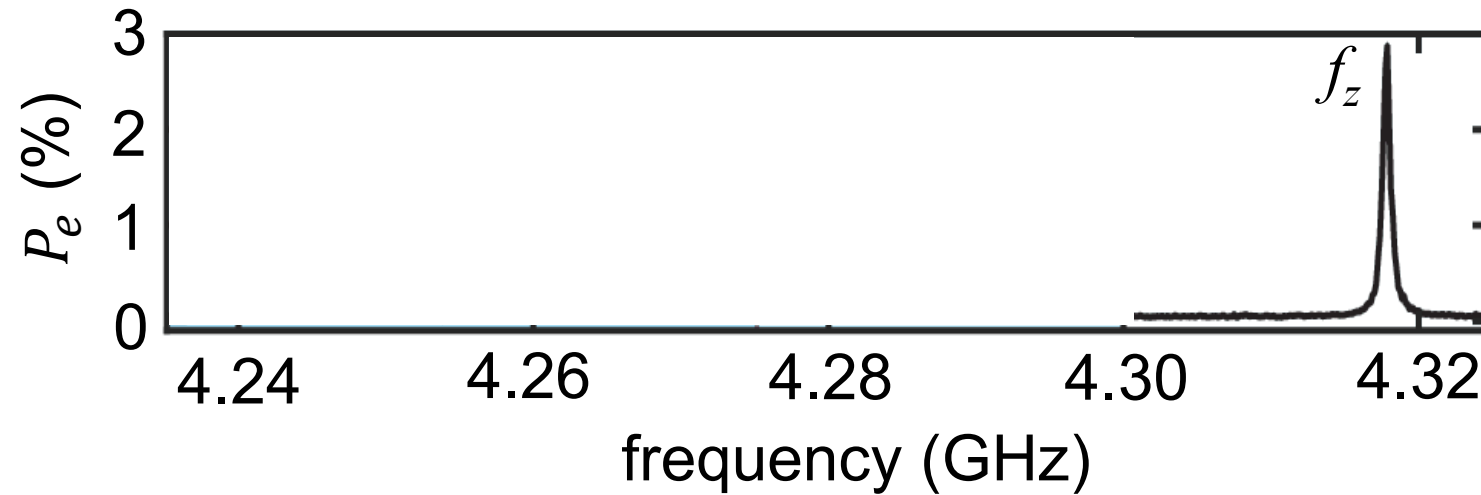
qubit tuned across mirror bandwidth (100 MHz)

three strongly coupled modes

mode-dependent coupling
dispersive “windows”

Characterizing the system with the qubit tuned to f_z

Qubit tuned to zero in coupling at f_z .

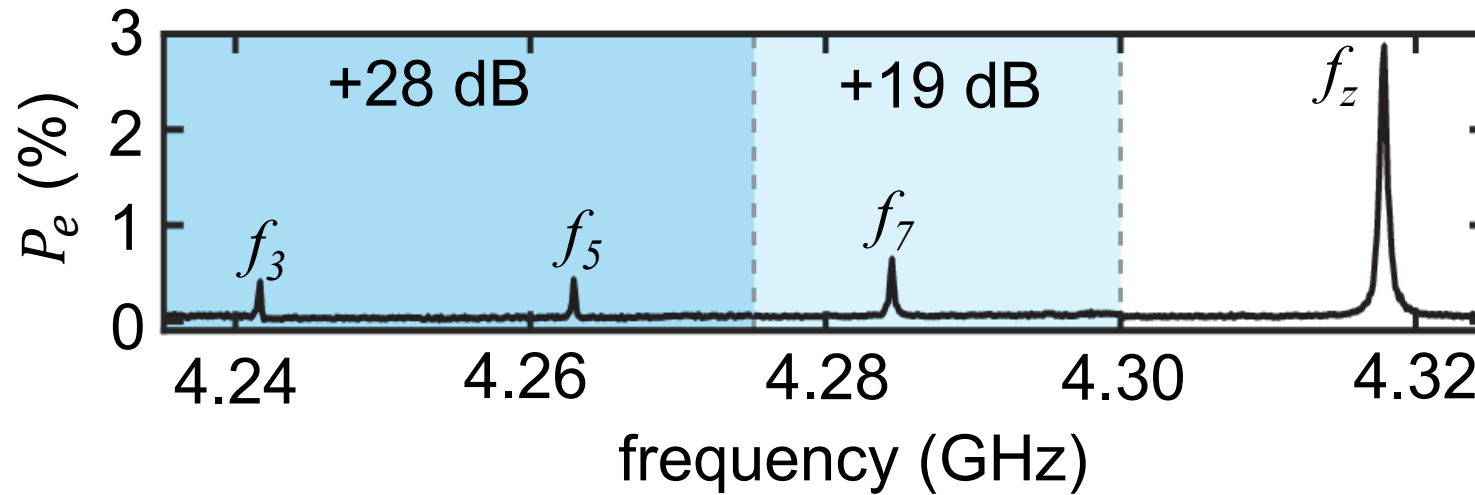


qubit linewidth

$$\gamma = 550 \text{ kHz}$$

Characterizing the system with the qubit tuned to f_z

Qubit tuned to zero in coupling at f_z .



qubit linewidth

$$\gamma = 550 \text{ kHz}$$

acoustic linewidths

$$\kappa_{3,5,7} \approx 250 \text{ kHz}$$

dispersive with all acoustic modes

$$\Delta_3/g_3 = 18$$

$$\Delta_5/g_5 = 11$$

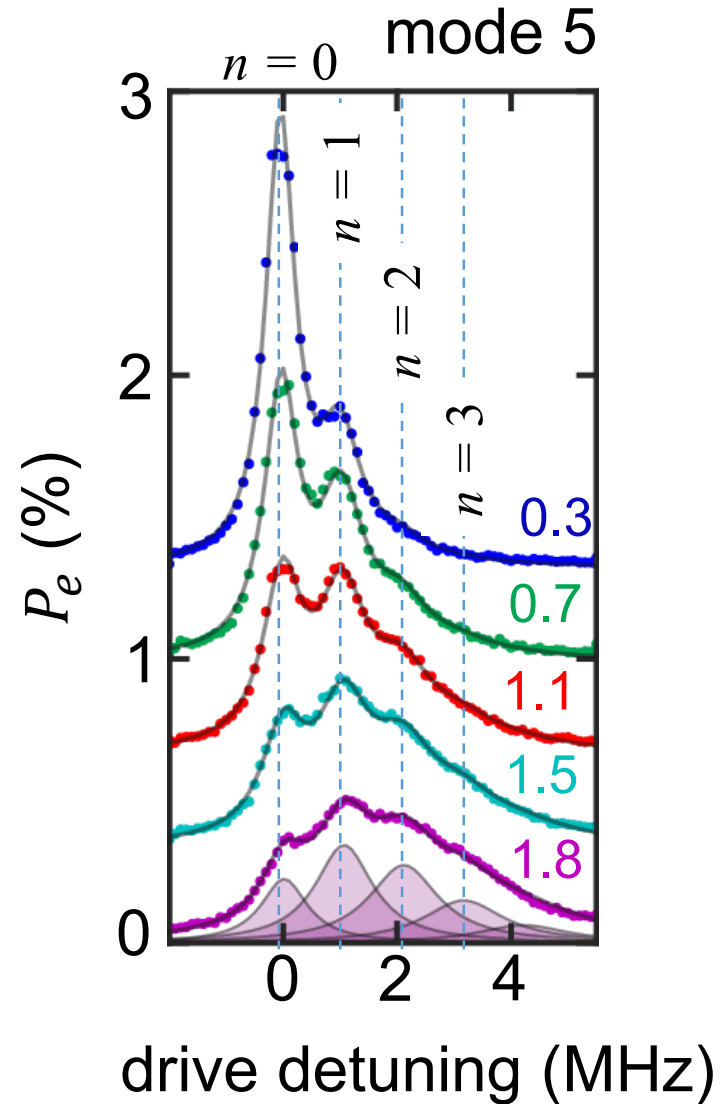
$$\Delta_7/g_7 = 8.5$$

$$\Delta_9/g_9 = 12$$

excite acoustic modes through qubit

Resolving the acoustical Stark shift of a single phonon!

excite acoustical mode, measure qubit spectrum

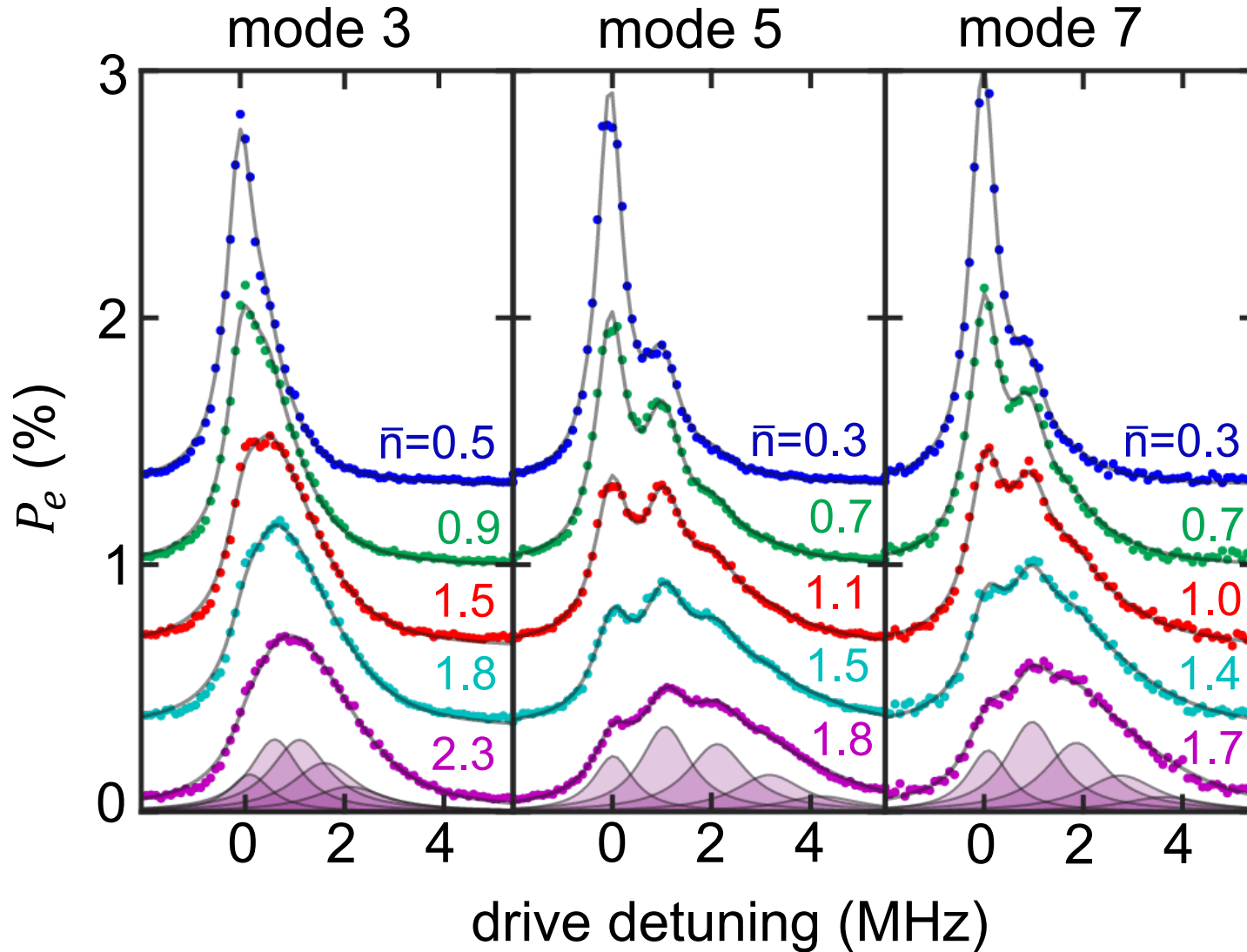


strong dispersive regime

$$\chi > \max[\kappa, \gamma]$$

Strong dispersive regime for two cavity modes

excite acoustical mode, measure qubit spectrum



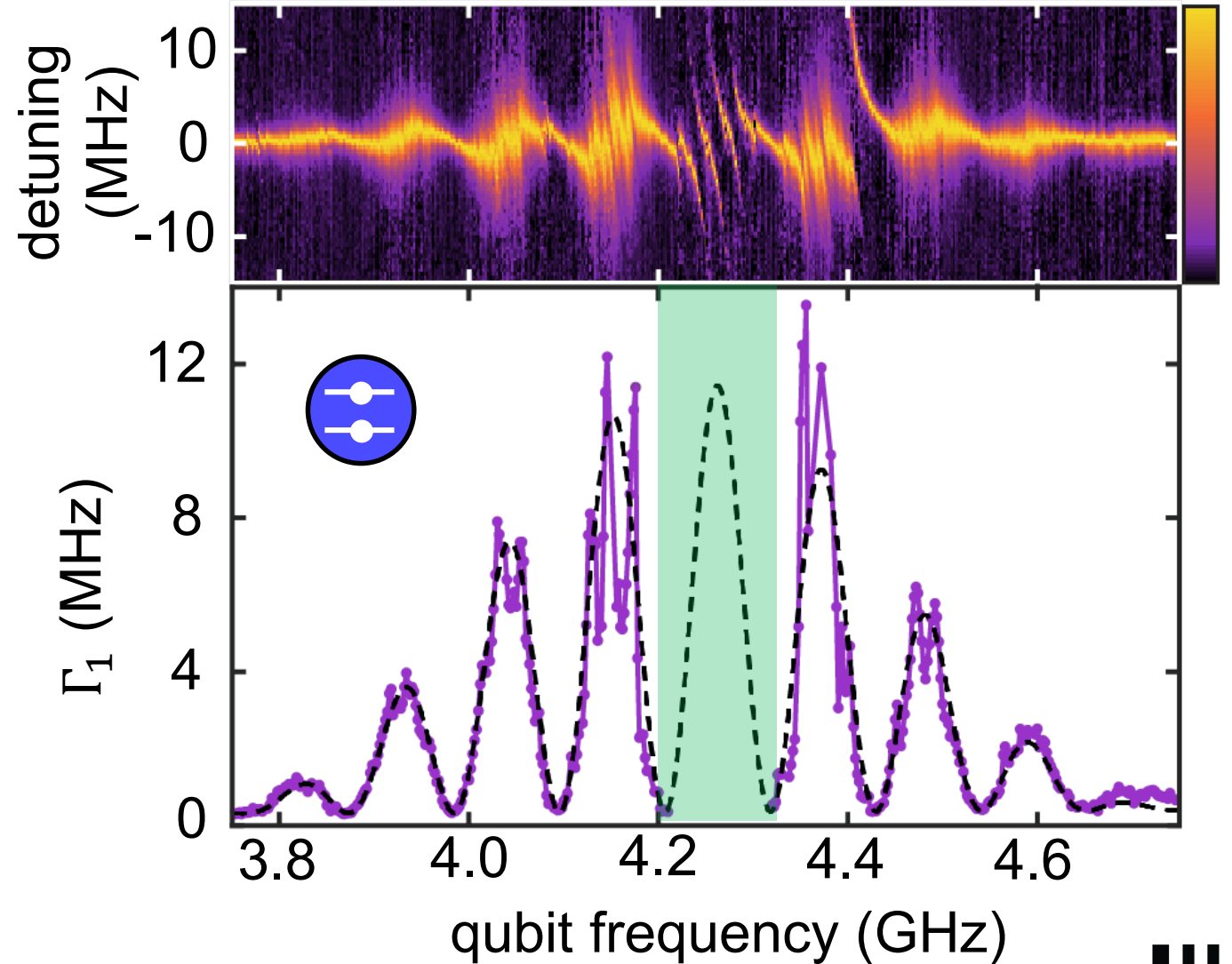
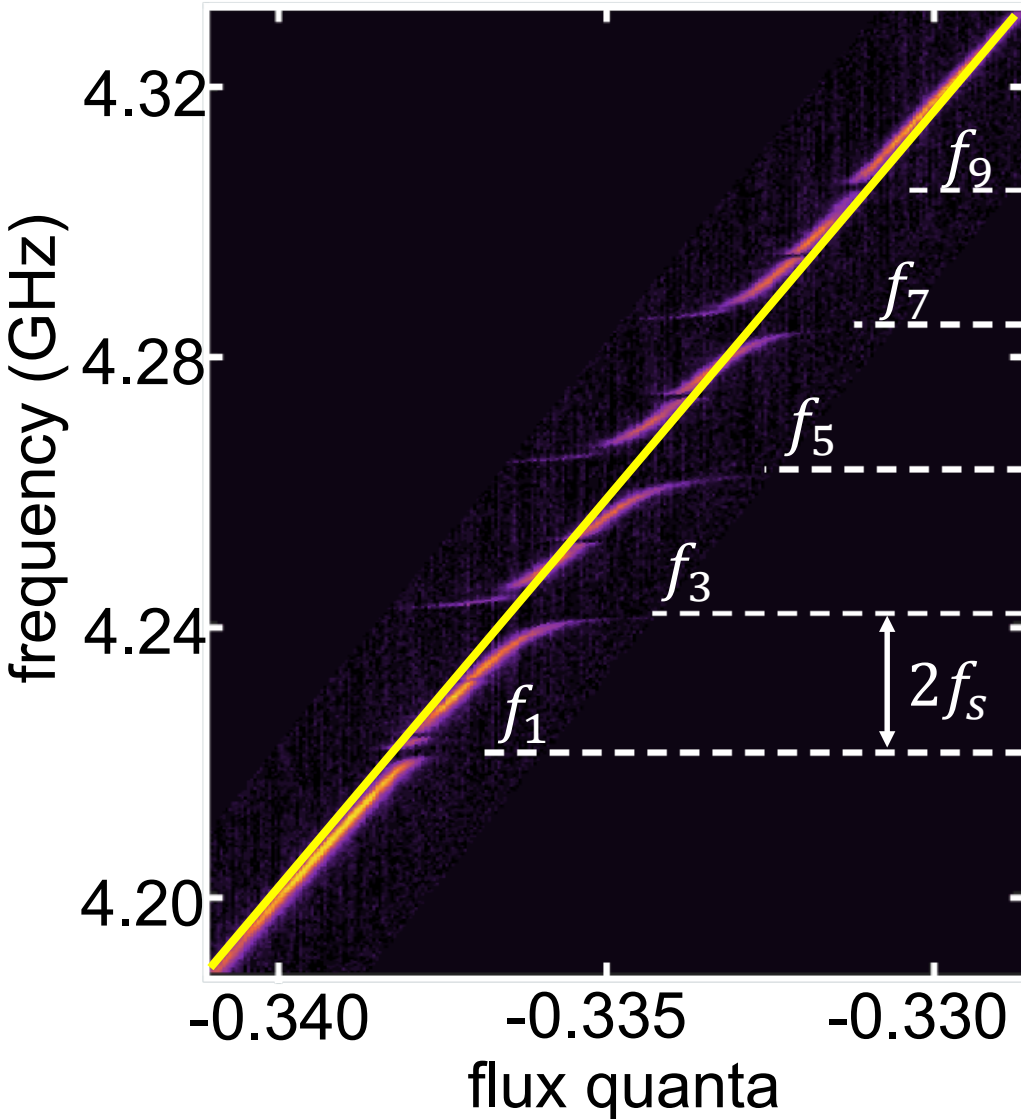
resolved peaks for modes 5, 7

qubit linewidth $\gamma = 550$ kHz

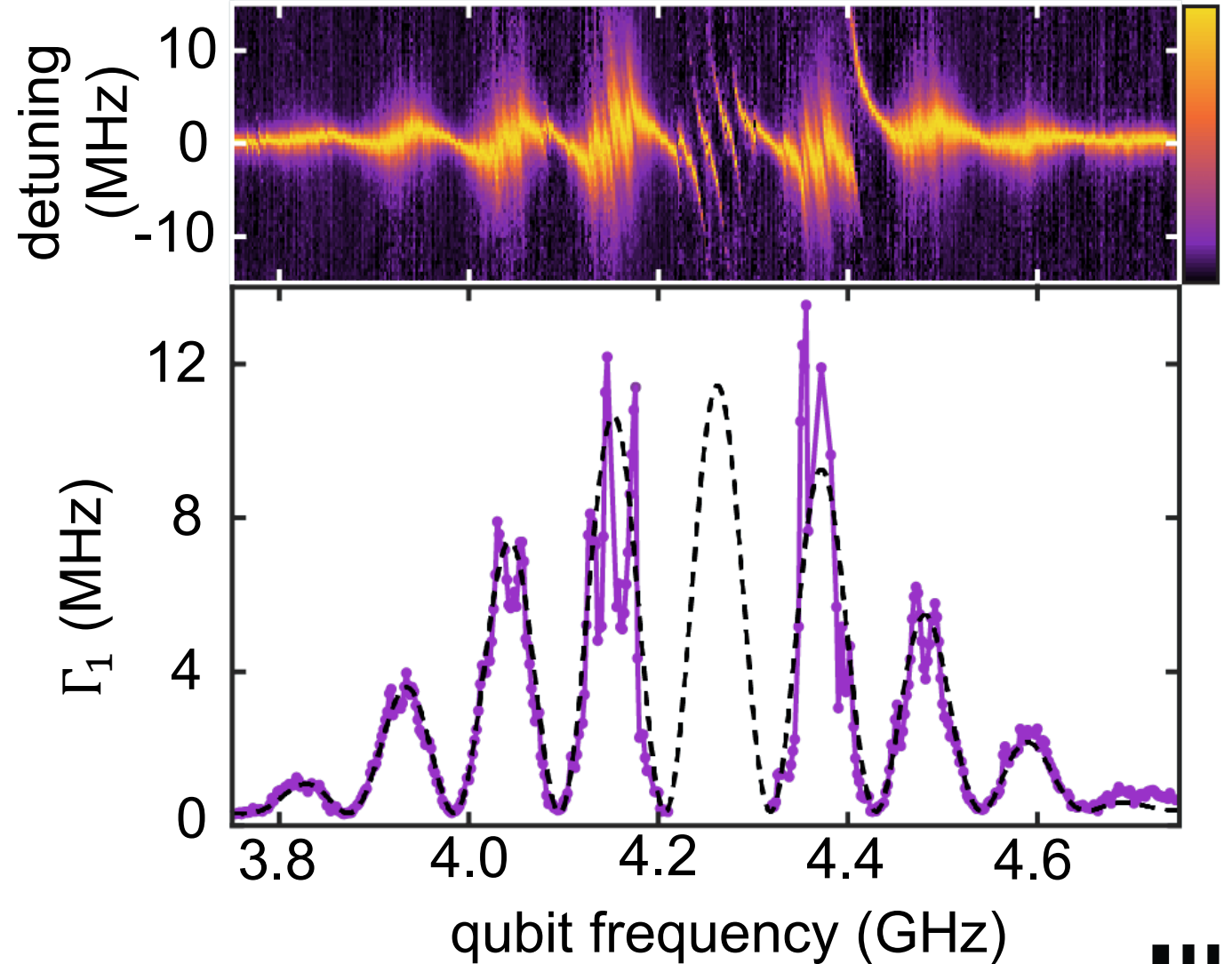
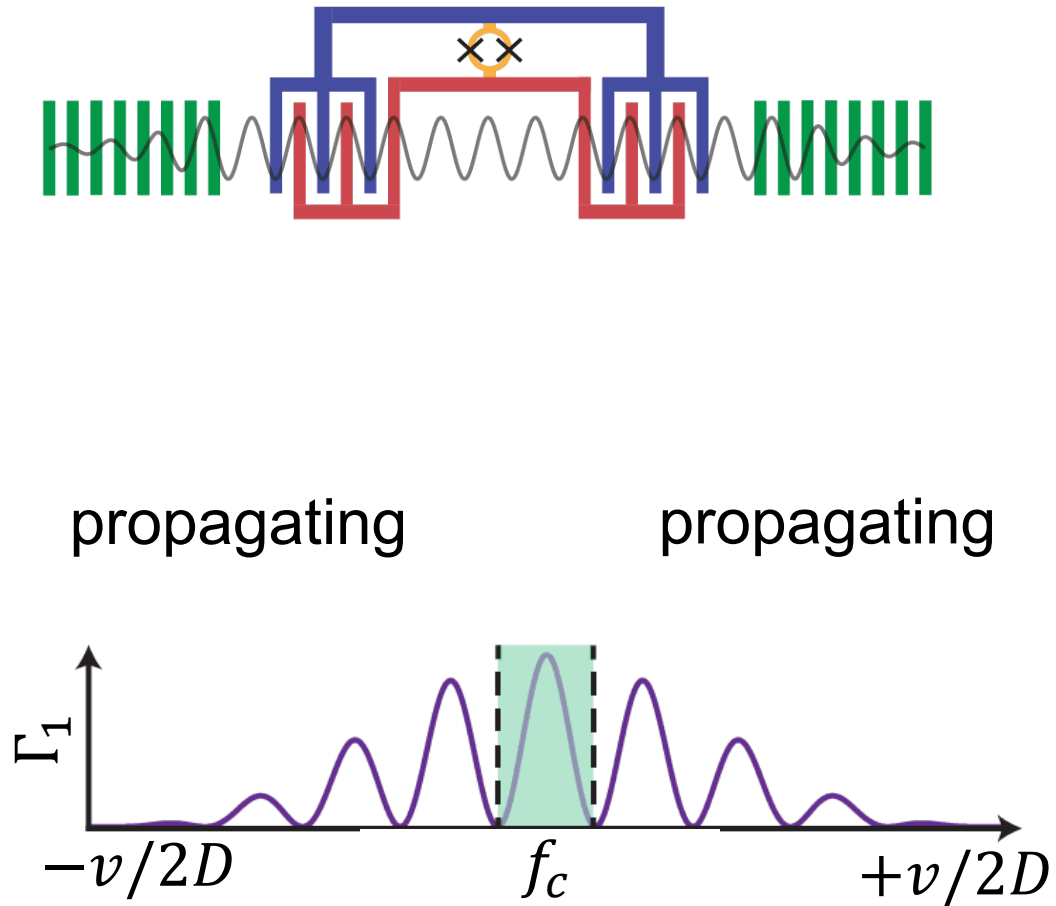
dispersive shifts

$$2\chi_{3,5,7} = 500, 1050, 890 \text{ kHz}$$

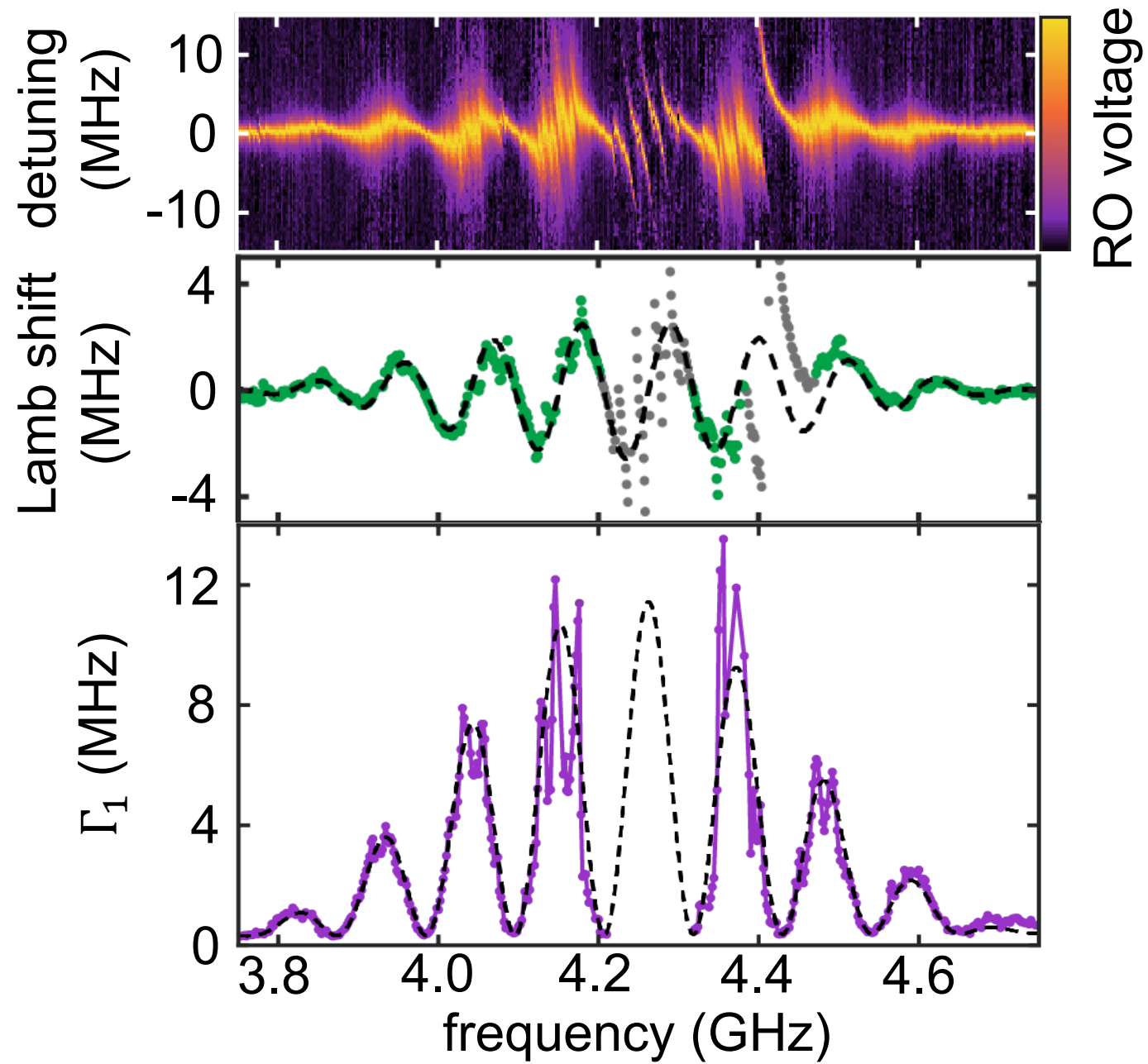
Single phonon self-interference!



Single phonon self-interference!



Acoustical Lamb shift



dissipation dispersion related by
Kramers-Kronig

frequency shift

frequency dependent dissipation

Conclusion

linear quantum electromechanics

non-linear quantum electromechanics

resolving single acoustical phonons

acoustical qubits have designer couplings

observation of acoustic Purcell effect and Lamb shift



Collaborators

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John Teufel

Kevin Silverman

Graduate students

[Lucas Sletten](#)

XiZheng Ma

Peter Burns

Max Urmey

Eric Rosenthal

Dan Palken

Kelly Wurtz

Sarang Mittal

Alec Emser

Rob Delaney

Ziyi Zhou

Post-docs

[Brad Moores](#)

[Brendon Rose](#)

[Jeremie Viennot](#)

Ben Brubaker

Jon Kindem

Undergraduates

Justin White

Adam Farmer

