Quantum Optics and Information

Alexey V. Gorshkov

Joint Quantum Institute (JQI)
Joint Center for Quantum Information and Computer Science (QuICS)
NIST and University of Maryland

2021 Boulder Summer School
July 19, 20, 21
2021

Outline for the three lectures

- interacting photons in Rydberg media

(• dynamics of quantum systems with long-range interactions)

Interacting photons in Rydberg media

References:

Homework: check these out, especially the last two

Quantum optics:

- lecture notes for Misha Lukin's class Modern Atomic and Optical Physics II, compiled by Lily Childress: https://lukin.physics.harvard.edu/teaching
- Meystre and Sargent, "Elements of Quantum Optics"
- Loudon, "Quantum Theory of Light"

Waveguide QED:

- Roy, Wilson, Firstenberg, "Colloquium: Strongly interacting photons in one-dimensional continuum," RMP 89, 021001 (2017)

Interacting photons in Rydberg media:

- Murray, Pohl, "Quantum and Nonlinear Optics in Strongly Interacting Atomic Ensembles", Adv. At., Mol., Opt. Phys. 65, 321 (2016)

Thanks to colleagues whose slides I am borrowing:
Misha Lukin, Bill Phillips, Thomas Pohl,…

Outline

- motivation and basic idea
- E&M field quantization
- propagation of light through atomic ensembles; electromagnetically induced transparency (EIT)
- Rydberg atoms
- basic idea revisited
- photon interacting with stationary excitation
  - on resonance: single-photon switch, subtractor
  - off resonance: two-photon quantum gate
- dynamics of multiple photons
  - on resonance: source of single photons
  - off resonance: two-photon gate, bound states, many-body physics
- more applications

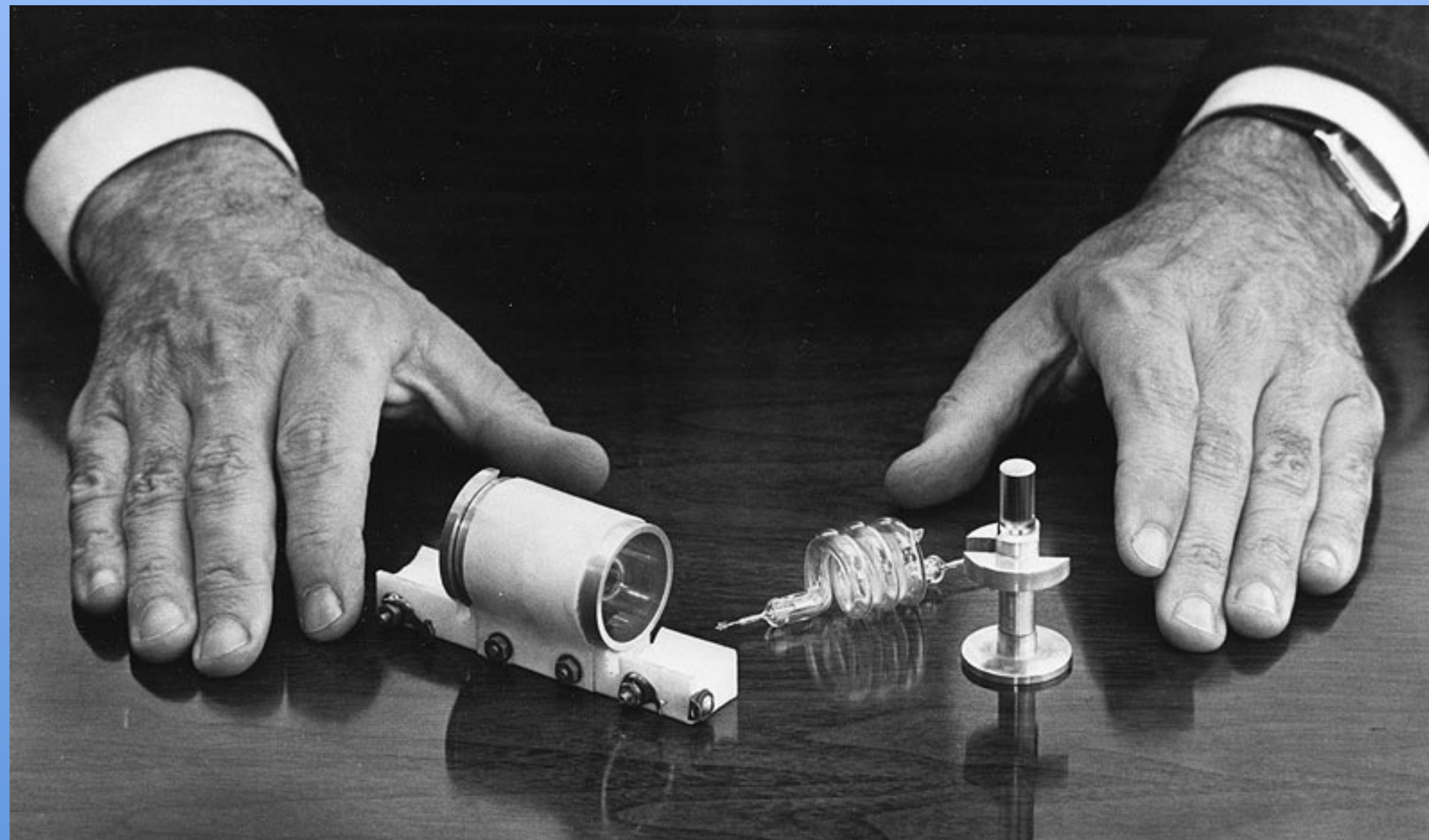
Outline

- motivation and basic idea
- E&M field quantization
- propagation of light through atomic ensembles; electromagnetically induced transparency (EIT)
- Rydberg atoms
- basic idea revisited
- photon interacting with stationary excitation
  - on resonance: single-photon switch, subtractor
  - off resonance: two-photon quantum gate
- dynamics of multiple photons
  - on resonance: source of single photons
  - off resonance: two-photon gate, bound states, many-body physics
- more applications

Photons interact only in Sci-Fi

But light CAN act on light under the right circumstances

One of the first scientific achievements following the first laser…



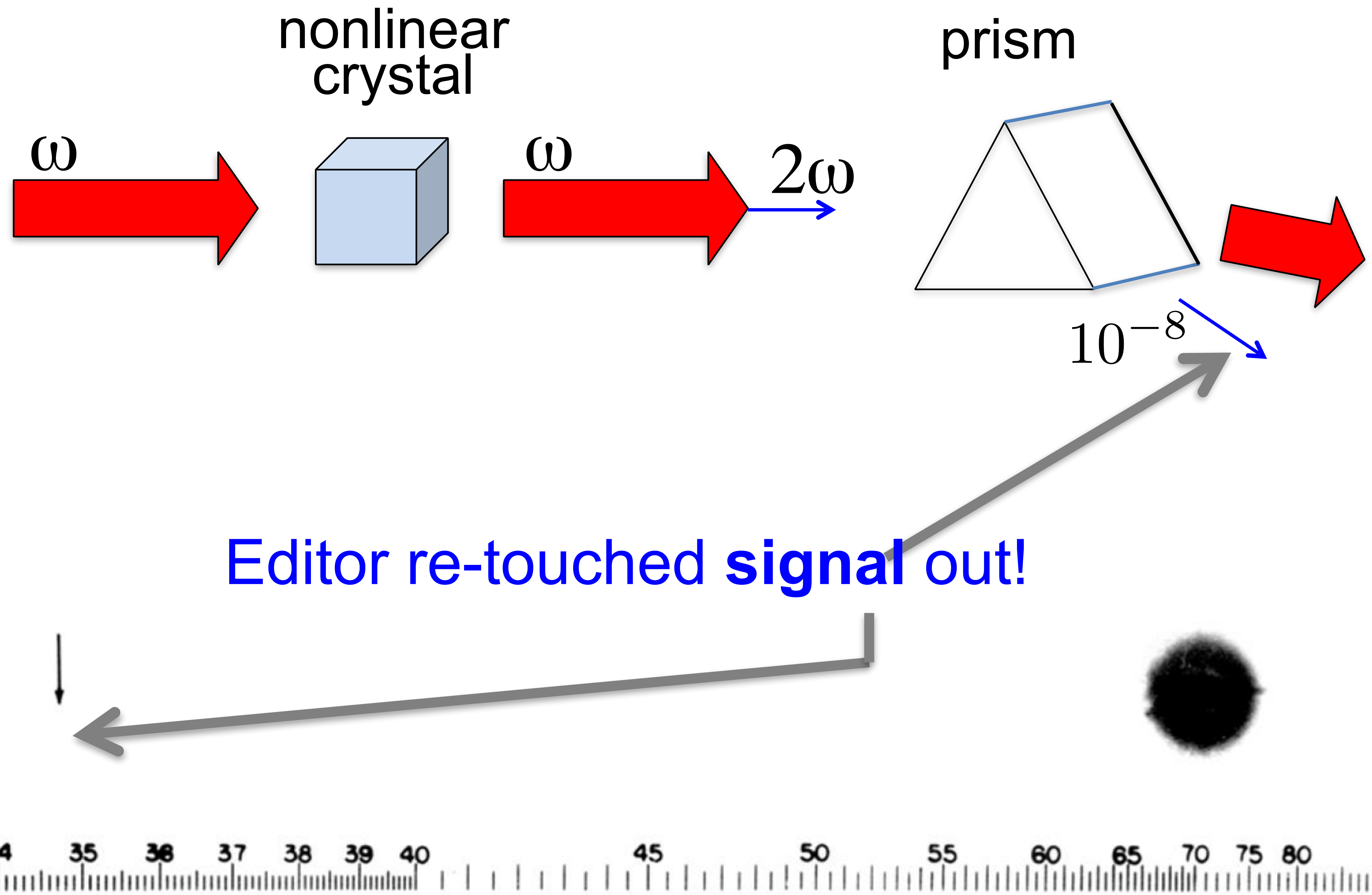
…was the first demonstration of non-linear optics

VOLUME 7, NUMBER 4 PHYSICAL REVIEW LETTERS AUGUST 15, 1961

GENERATION OF OPTICAL HARMONICS*

P. A. Franken, A. E. Hill, C. W. Peters, and G. Weinreich
The Harrison M. Randall Laboratory of Physics, The University of Michigan, Ann Arbor, Michigan
(Received July 21, 1961)

The first non-linear optics needed the first laser

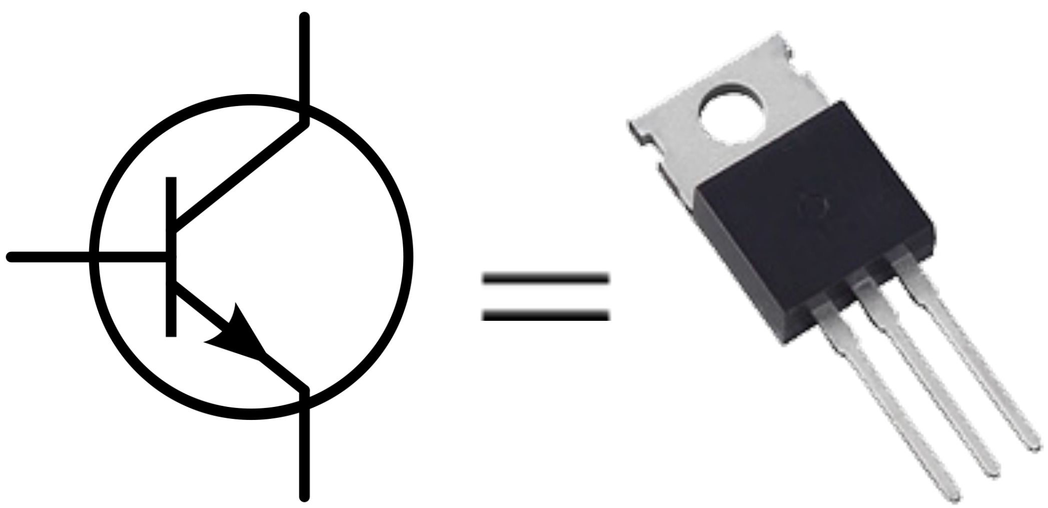


Electrons vs. Photons

$$F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$$ (large)

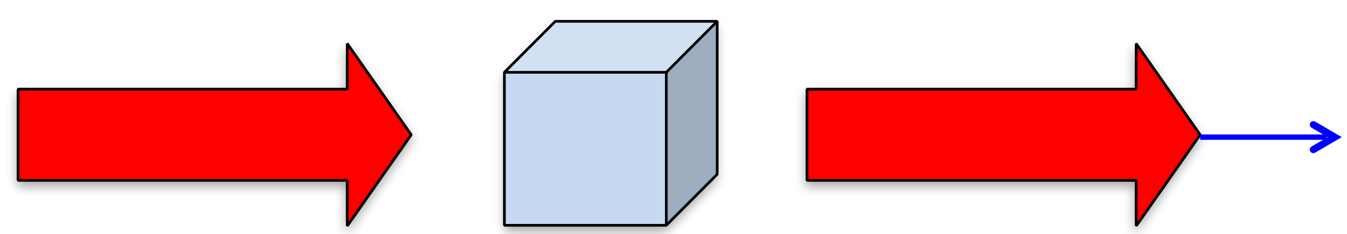
photons hardly interact

transistor



small electrical signals
control huge currents

nonlinear
crystal

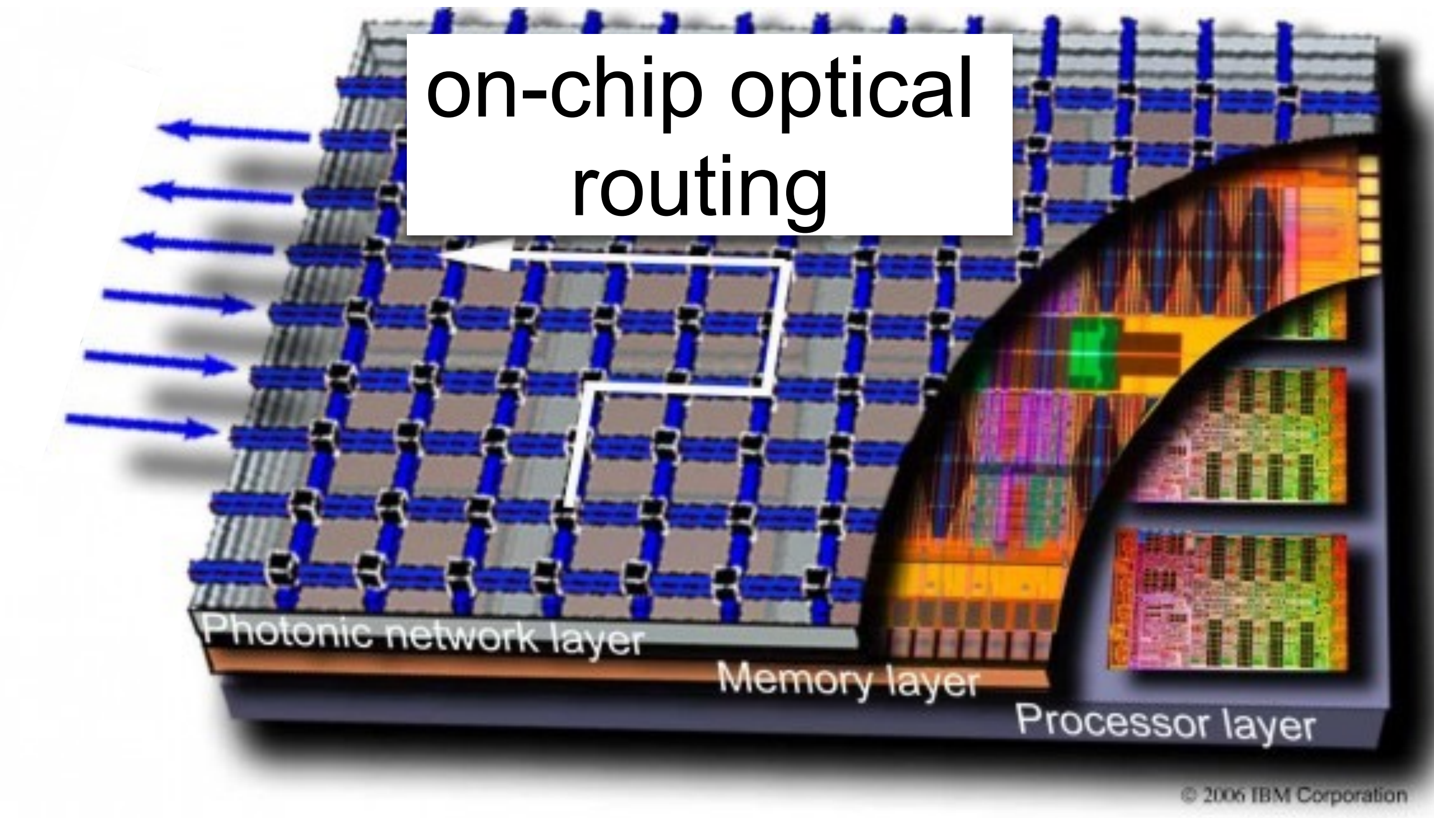


even huge optical intensities
only control tiny optical signals

Electronics vs. Photonics

electrons
process information

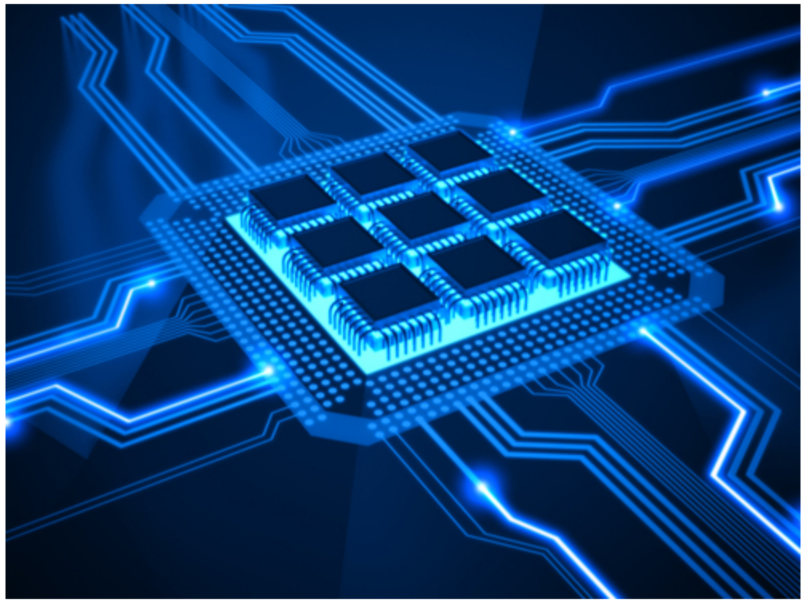
photons
transmit information



Electronics vs. Photonics

electrons
process information

photons
transmit information



converting photons to electrical signals is “expensive”

Electronics vs. Photonics

electrons
process information

photons
transmit information

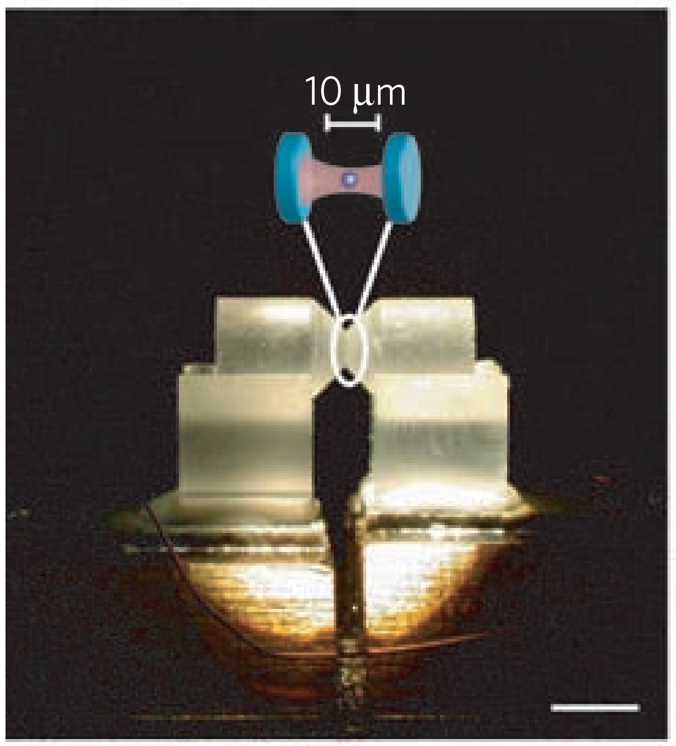
Can we make photons process
without conversion?

- photon-photon interactions too weak for processing information

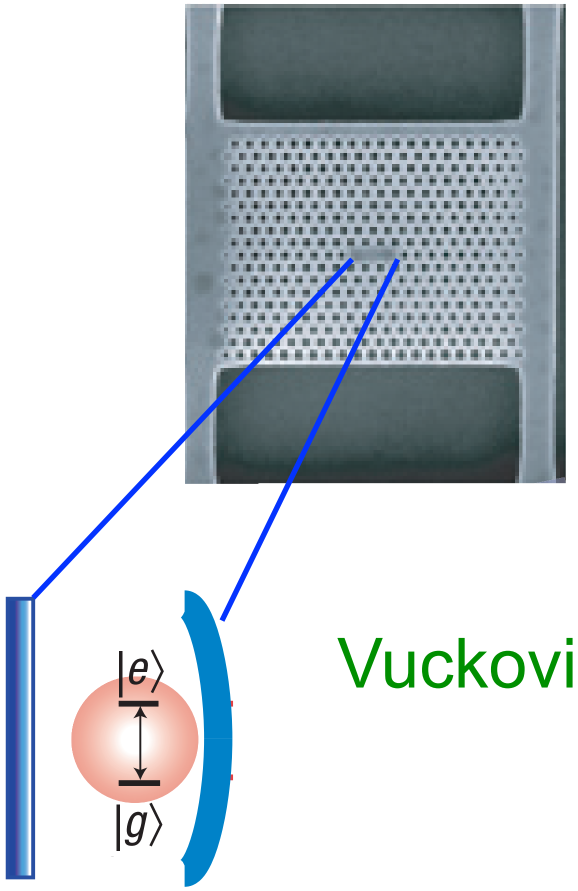
Photon-photon interactions

Typical approach to achieving interactions between optical photons:

- nonlinearity induced by individual atoms (or artificial atoms)



Kimble @ Caltech



Vuckovic @ Stanford

Hard!

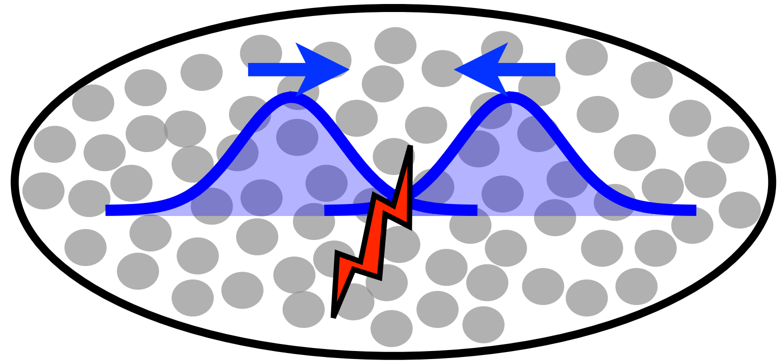
Photon-photon interactions

Typical approach to achieving interactions between optical photons:

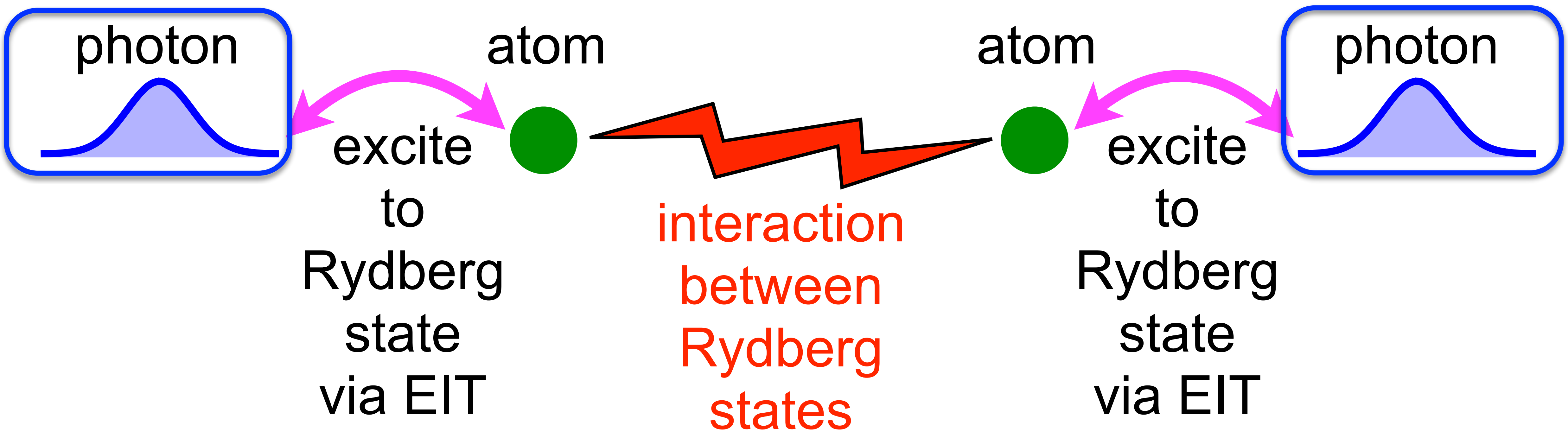
- nonlinearity induced by individual atoms (or artificial atoms)

This talk: Map strong atom-atom interactions onto strong photon-photon interactions

Medium where photons interact strongly



Map strong atom-atom interactions onto strong photon-photon interactions



EIT = electromagnetically induced transparency

Experiments: Adams, Kuzmich, Lukin & Vuletic, Pfau & Löw, Grangier, Weidemüller, Hofferberth, Dürr & Rempe, Simon, Firstenberg, Ourjoumtsev, H. de Riedmatten, etc…
Theory: Kurizki, Fleischhauer, Petrosyan, Mølmer, Pohl, Lesanovsky, Kennedy, Brion, Büchler, Sørensen, most experimental groups above, etc...

Outline

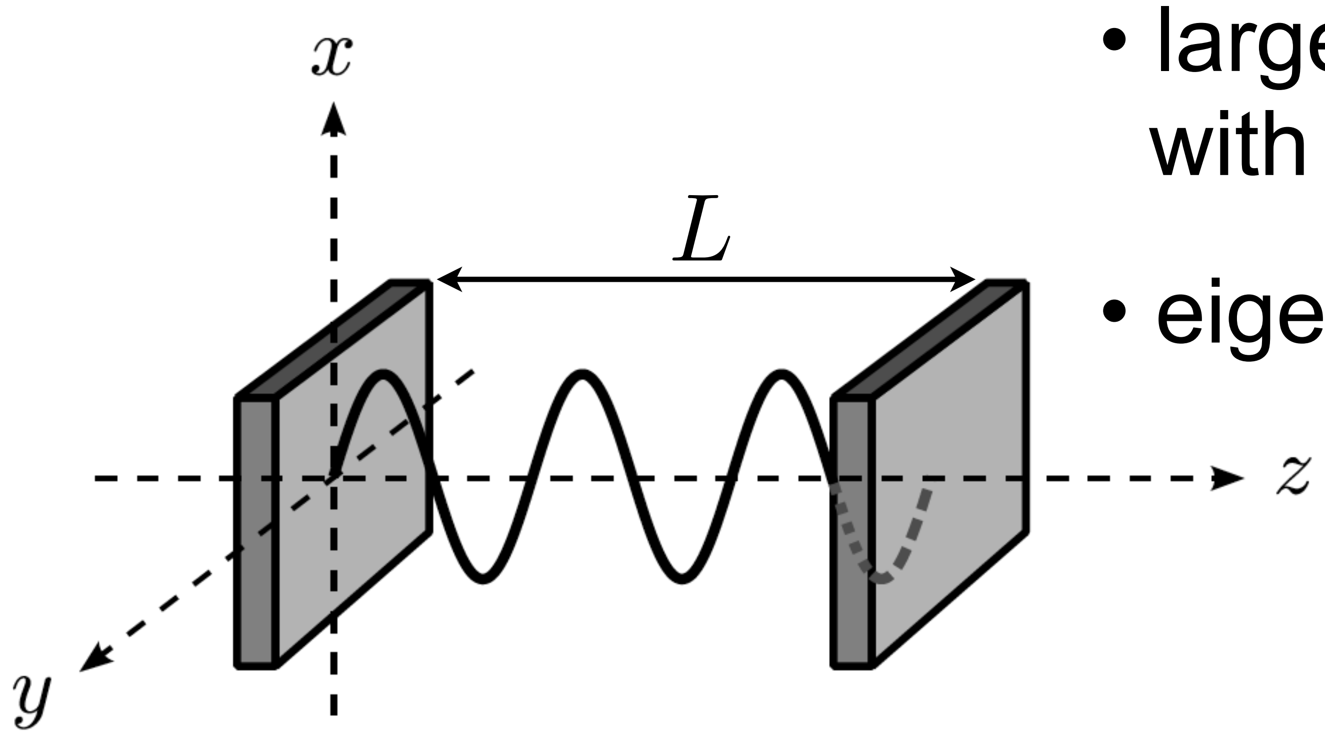
- motivation and basic idea
- E&M field quantization
- propagation of light through atomic ensembles; electromagnetically induced transparency (EIT)
- Rydberg atoms
- basic idea revisited
- photon interacting with stationary excitation
  - on resonance: single-photon switch, subtractor
  - off resonance: two-photon quantum gate
- dynamics of multiple photons
  - on resonance: source of single photons
  - off resonance: two-photon gate, bound states, many-body physics
- more applications

E&M field quantization

- Lukin/Childress lecture notes
- Meystre and Sargent, ``Elements of Quantum Optics"
- consider free field (no sources)
- Maxwell's equations $\Rightarrow$ wave equation

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

- knowing $\mathbf{E}$, find $\mathbf{B}$ via $\nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$ (SI units used)



- large cavity of length L & volume V, with $\mathbf{E} = 0$ on mirrors
- eigenmodes = standing waves

$$\sin(k_j z) \qquad k_j L = \pi j$$

$$j = 1, 2, 3, \ldots$$

(• could do periodic boundary conditions => running waves)

• consider $\hat{x}$-polarized field

$$E_x(z,t) = \sum_j \underbrace{\sqrt{\frac{2\nu_j^2}{\epsilon_0 V}}}_{A_j} q_j(t) \sin k_j z \qquad \nu_j = ck_j$$

amplitude

$$\Rightarrow \; B_y(z,t) = \frac{1}{c^2} \sum_j \frac{\dot{q}_j(t)}{k_j} A_j \cos k_j z$$

- classical energy:

$$H = \frac{1}{2}\int dV \left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)$$

$$= \sum_j \frac{\nu_j^2 q_j^2}{2} + \frac{\dot{q}_j^2}{2} \rightarrow \frac{1}{2}\sum_j \left(\nu_j^2 \hat{q}_j^2 + \hat{p}_j^2\right) = \sum_j \hbar\nu_j \left(\hat{a}_j^\dagger \hat{a}_j + \frac{1}{2}\right)$$

- independent harmonic oscillators with frequency ν_j, unit mass, position q_j

- <u>Quantization:</u> $q_j \rightarrow \hat{q}_j \quad \dot{q}_j \rightarrow \hat{p}_j \qquad [\hat{q}_j, \hat{p}_{j'}] = i\hbar\delta_{j,j'}$

$$[\hat{a}_j, \hat{a}_{j'}^\dagger] = \delta_{j,j'}$$

- creation/annihilation:

$$\hat{a}_j = \frac{1}{\sqrt{2\hbar\nu_j}}\left(\nu_j \hat{q}_j + i\hat{p}_j\right) \qquad \hat{q}_j = \sqrt{\frac{\hbar}{2\nu_j}}\left(\hat{a}_j + \hat{a}_j^\dagger\right)$$

$$\hat{a}_j^\dagger = \frac{1}{\sqrt{2\hbar\nu_j}}\left(\nu_j \hat{q}_j - i\hat{p}_j\right) \qquad \hat{p}_j = -i\sqrt{\frac{\hbar\nu_j}{2}}\left(\hat{a}_j - \hat{a}_j^\dagger\right)$$

- $\hat{x}$ component of electric field operator

$$\hat{E}_x(z) = \sum_j \underbrace{A_j \sqrt{\frac{\hbar}{2\nu_j}}}_{\sqrt{\frac{\hbar\nu_j}{\epsilon_0 V}}} (\hat{a}_j + \hat{a}_j^\dagger) \sin k_j z$$

$\sqrt{\frac{\hbar\nu_j}{\epsilon_0 V}}$ = electric field per photon

(makes sense: $\hbar\nu_j \sim \mathrm{energy} \sim \epsilon_0 E^2 V$)

- for running waves, including all polarizations & directions

$$\hat{\mathbf{E}}(\mathbf{r}) = \hat{\mathcal{E}}(\mathbf{r}) + \hat{\mathcal{E}}^\dagger(\mathbf{r})$$

$$\hat{\mathcal{E}}(\mathbf{r}) = \sum_{\mathbf{k},\alpha} \epsilon_\alpha \sqrt{\frac{\hbar\nu_j}{2\epsilon_0 V}} \hat{a}_{\mathbf{k},\alpha} e^{i\mathbf{k}\cdot\mathbf{r}}$$

transverse polarization

Atom-field interactions

- starting point: dipole Hamiltonian for a 2-level atom

$|2\rangle$

$$\hat{V}_{af} = -\hat{\mathbf{E}} \cdot \hat{\mathbf{d}}$$
$$= -(\hat{\mathcal{E}} + \hat{\mathcal{E}}^\dagger) \cdot (\langle 2|\hat{\mathbf{d}}|1\rangle |2\rangle\langle 1| + \langle 1|\hat{\mathbf{d}}|2\rangle |1\rangle\langle 2|)$$

$|1\rangle$

- 4 types of terms:

$$\hat{a}|2\rangle\langle 1| \qquad \cancel{\hat{a}|1\rangle\langle 2|} \qquad \cancel{\hat{a}^\dagger|2\rangle\langle 1|} \qquad \hat{a}^\dagger|1\rangle\langle 2|$$

(Heisenberg evolution under $\hat{H} = \hbar\nu_j \hat{a}_j^\dagger \hat{a}_j$: $\quad \hat{a}_j(t) = \hat{a}_j(0)e^{-i\nu_j t}$)

- RWA ≃ energy conservation
- with RWA: $\quad \hat{V}_{af} = -\sum_{\mathbf{k},\alpha} \hbar g_{\mathbf{k},\alpha} |2\rangle\langle 1| \hat{a}_{j,\alpha} + \hbar g^*_{\mathbf{k},\alpha} |1\rangle\langle 2| \hat{a}^\dagger_{\mathbf{k},\alpha}$
- single-photon Rabi frequency: $\quad g_{\mathbf{k},\alpha} = \frac{\mu_\alpha}{\hbar}\sqrt{\frac{\hbar\nu_j}{2\epsilon_0 V}}e^{i\mathbf{k}\cdot\mathbf{r}}$

(• if standing wave mode, $|g| \propto \sin(kz)$) $\qquad \mu_\alpha = \langle 2|d_\alpha|1\rangle$

Remarks

- no sources ($\nabla \cdot \mathbf{E} = 0$), wave equation

  $\Rightarrow$ didn't need $\mathbf{A}$; used $\hat{V}_{af} = -\hat{\mathbf{E}} \cdot \hat{\mathbf{d}}$

  $\Rightarrow$ didn't need to choose gauge

- with sources ($\nabla \cdot \mathbf{E} \neq 0$), no wave equation

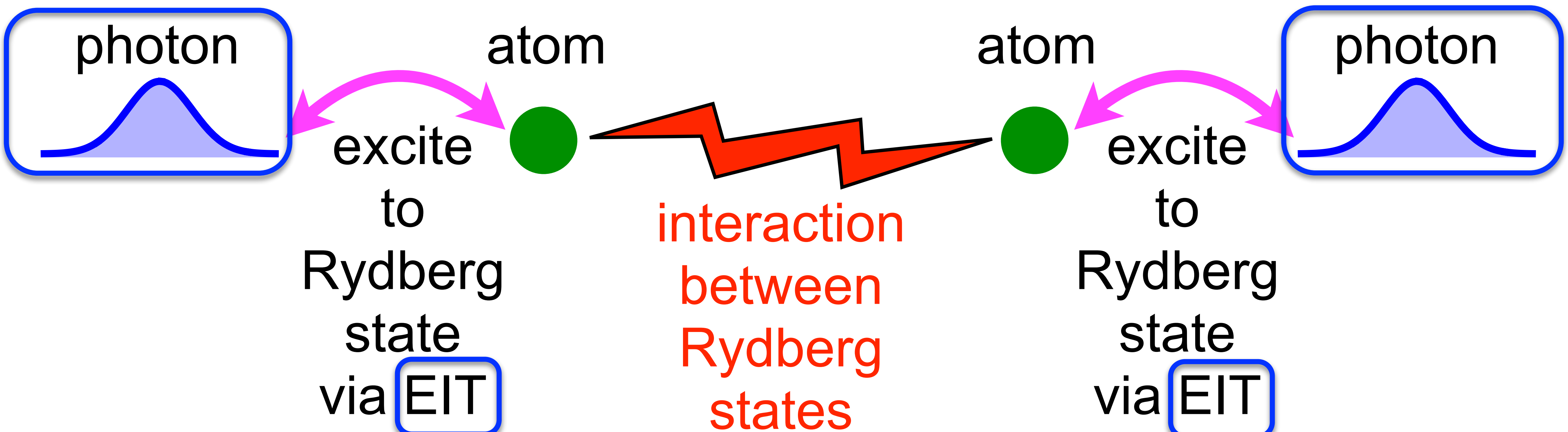
  $\Rightarrow$ need $\mathbf{A}$

  $\Rightarrow$ choose Coulomb gauge $\nabla \cdot \mathbf{A} = 0$

  $\Rightarrow$ quantized similarly to this lecture

  [see Cohen-Tannoudji et al., "Photons and Atoms"]

Medium where photons interact strongly

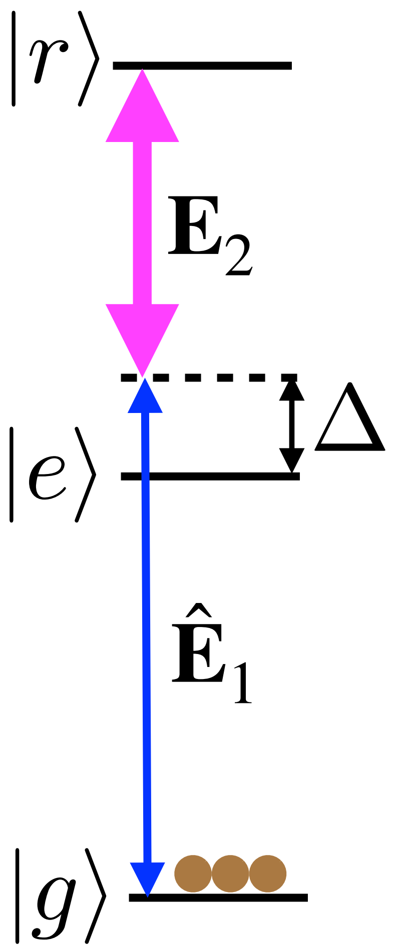


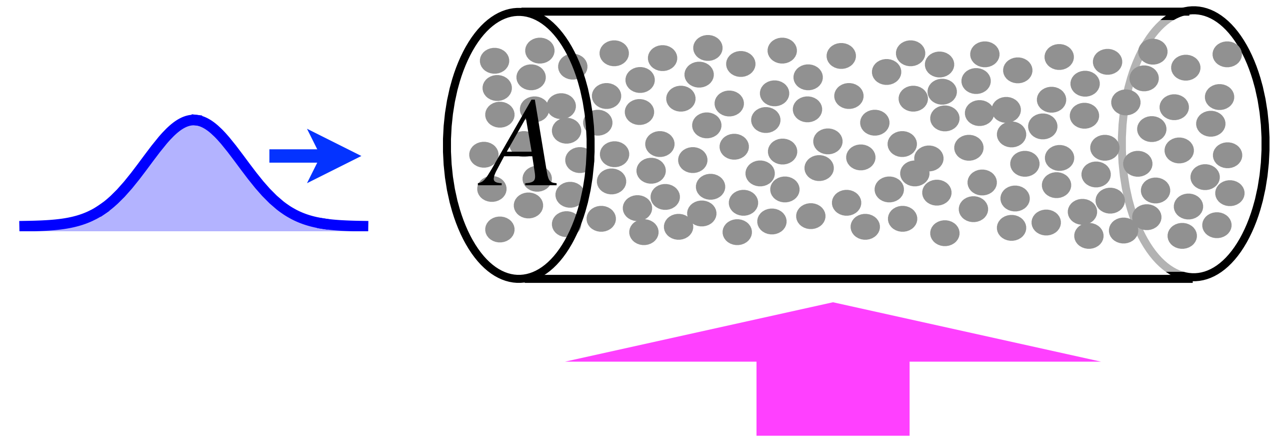
EIT = electromagnetically induced transparency

Outline

- motivation and basic idea
- E&M field quantization
- propagation of light through atomic ensembles; electromagnetically induced transparency (EIT)
- Rydberg atoms
- basic idea revisited
- photon interacting with stationary excitation
  - on resonance: single-photon switch, subtractor
  - off resonance: two-photon quantum gate
- dynamics of multiple photons
  - on resonance: source of single photons
  - off resonance: two-photon gate, bound states, many-body physics
- more applications

Three-level medium





A = cross section of beam and of ensemble

$\omega_1 = \omega_{eg} + \Delta$ $\qquad$ $\omega_2 = \omega_{re} - \Delta$

$$\hat{\mathbf{E}}_1(z) = \epsilon_1 \left(\frac{\hbar\omega_1}{4\pi c \epsilon_0 A} \right)^{1/2} \int d\omega \hat{a}_\omega e^{i\omega z/c} + h.c.$$

Loudon, "Quantum Theory of Light"

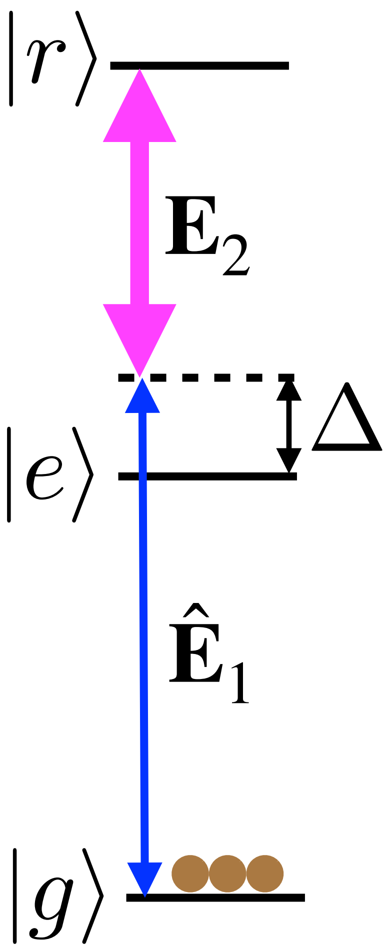
$$\left[\hat{a}_\omega, \hat{a}^\dagger_{\omega'} \right] = \delta(\omega - \omega')$$

$$\mathbf{E}_2(t) = \epsilon_2 \mathscr{E}_2(t) \cos(\omega_2 t)$$

Fleischhauer, Lukin, PRA 65, 022314 (2002)

AVG, Adre, Lukin, Sorensen, PRA 76, 033805 (2007)

Three-level medium



$$\hat{H} = \hat{H}_0 + \hat{V}$$

$$\hat{H}_0 = \int d\omega \hbar\omega \hat{a}^\dagger_\omega \hat{a}_\omega + \sum_{i=1}^{N} \left(\hbar\omega_{rg}\hat{\sigma}^i_{rr} + \hbar\omega_{eg}\hat{\sigma}^i_{ee} \right)$$

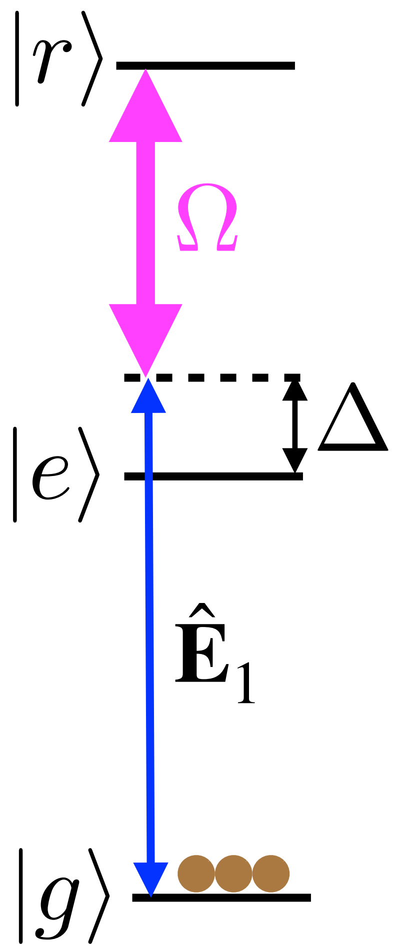
$$\hat{V} = -\sum_{i=1}^{N} \hat{\mathbf{d}}_i \cdot [\mathbf{E}_2(t) + \hat{\mathbf{E}}_1(z_i)]$$

$$= -\hbar \sum_{i=1}^{N} \left(\Omega(t)\hat{\sigma}^i_{re} e^{-i\omega_2 t} + g\sqrt{\frac{1}{2\pi c}} \int d\omega \hat{a}_\omega e^{i\omega z_i/c} \hat{\sigma}^i_{eg} + h.c. \right)$$

$$\hat{\sigma}^i_{\mu\nu} = |\mu\rangle_{ii}\langle\nu|$$

$\Omega(t) = \langle r|(\hat{\mathbf{d}} \cdot \epsilon_2)|e\rangle \mathscr{E}_2(t)/(2\hbar)$ = Rabi frequency

Three-level medium



$$\hat{H} = \hat{H}_0 + \hat{V}$$

$$\hat{H}_0 = \int d\omega \hbar\omega \hat{a}^\dagger_\omega \hat{a}_\omega + \sum_{i=1}^{N} \left(\hbar\omega_{rg}\hat{\sigma}^i_{rr} + \hbar\omega_{eg}\hat{\sigma}^i_{ee} \right)$$

$$\hat{V} = -\sum_{i=1}^{N} \hat{\mathbf{d}}_i \cdot [\mathbf{E}_2(t) + \hat{\mathbf{E}}_1(z_i)]$$

$$= -\hbar \sum_{i=1}^{N} \left(\Omega(t)\hat{\sigma}^i_{re} e^{-i\omega_2 t} + g\sqrt{\frac{1}{2\pi c}} \int d\omega \hat{a}_\omega e^{i\omega z_i/c} \hat{\sigma}^i_{eg} + h.c. \right)$$

$$\hat{\sigma}^i_{\mu\nu} = |\mu\rangle_{ii}\langle\nu|$$

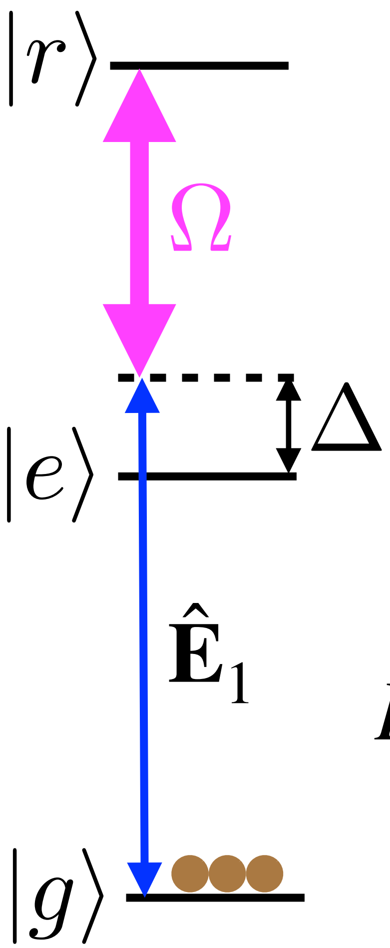
$\Omega(t) = \langle r|(\hat{\mathbf{d}} \cdot \epsilon_2)|e\rangle \mathscr{E}_2(t)/(2\hbar)$ = Rabi frequency

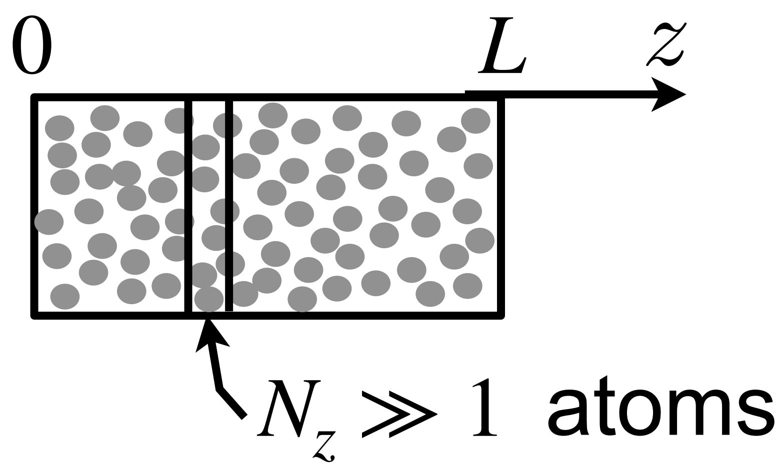
$$g = \langle e|(\hat{\mathbf{d}} \cdot \epsilon_1)|g\rangle \sqrt{\frac{\omega_1}{2\hbar\epsilon_0 A}}$$

- π pulse takes time $\pi/(2\Omega)$
- I will often set $\hbar = 1$

Three-level medium

define slowly varying operators





- thin enough that fields continuous
- assume almost all atoms in ground state at all times
- n = atom density

homework exercise

$$\hat{P}^\dagger(z,t) = \sqrt{n}\frac{1}{N_z}\sum_{i=1}^{N_z}\hat{\sigma}^i_{eg}(t)e^{-i\omega_1(t-z_i/c)} \qquad \left[\hat{P}(z,t),\hat{P}^\dagger(z',t)\right] = \delta(z-z')$$

creates $|e\rangle$ excitation at z

$$\hat{S}^\dagger(z,t) = \sqrt{n}\frac{1}{N_z}\sum_{i=1}^{N_z}\hat{\sigma}^i_{rg}(t)e^{-i\omega_1(t-z_i/c)-i\omega_2 t} \qquad \left[\hat{S}(z,t),\hat{S}^\dagger(z',t)\right] = \delta(z-z')$$

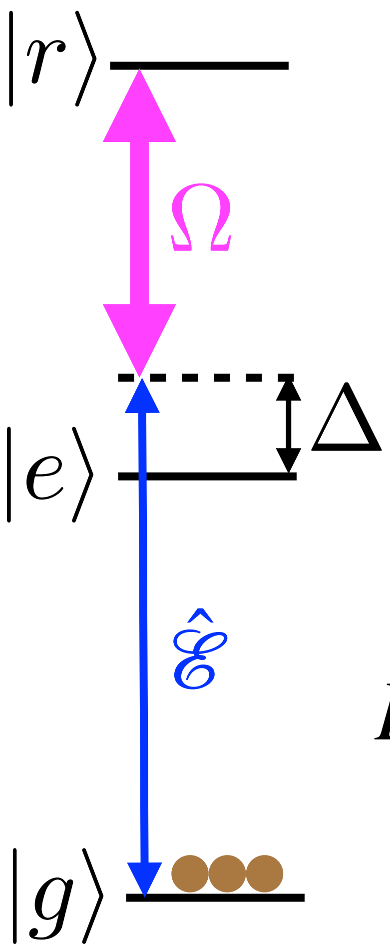
creates $|r\rangle$ excitation at z

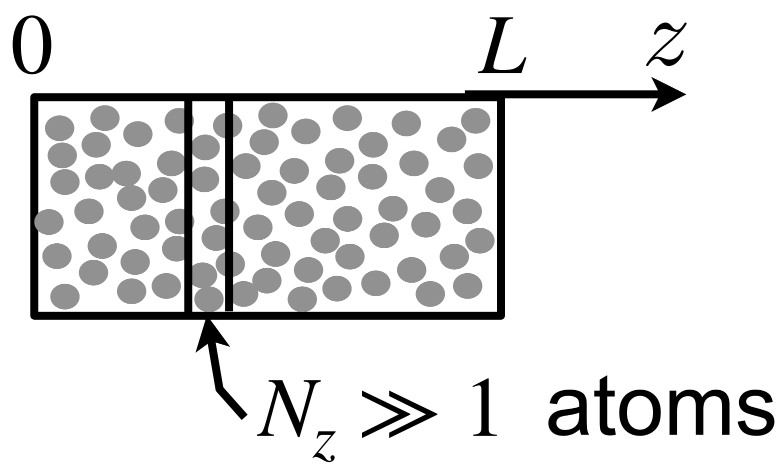
$$\hat{\mathscr{E}}^\dagger(z,t) = \sqrt{\frac{1}{2\pi c}}e^{-i\omega_1(t-z/c)}\int d\omega\,\hat{a}^\dagger_\omega(t)e^{-i\omega z/c}$$

creates photon at z

Three-level medium

define slowly varying operators





- thin enough that fields continuous
- assume almost all atoms in ground state at all times
- n = atom density

homework exercise

$$\hat{P}^\dagger(z,t) = \sqrt{n}\frac{1}{N_z}\sum_{i=1}^{N_z}\hat{\sigma}^i_{eg}(t)e^{-i\omega_1(t-z_i/c)} \qquad \left[\hat{P}(z,t),\hat{P}^\dagger(z',t)\right] = \delta(z-z')$$

creates $|e\rangle$ excitation at z

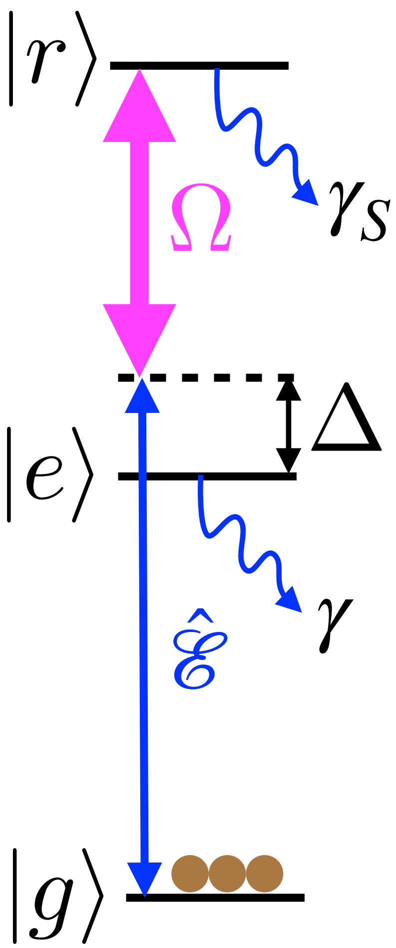
$$\hat{S}^\dagger(z,t) = \sqrt{n}\frac{1}{N_z}\sum_{i=1}^{N_z}\hat{\sigma}^i_{rg}(t)e^{-i\omega_1(t-z_i/c)-i\omega_2 t} \qquad \left[\hat{S}(z,t),\hat{S}^\dagger(z',t)\right] = \delta(z-z')$$

creates $|r\rangle$ excitation at z

$$\hat{\mathscr{E}}^\dagger(z,t) = \sqrt{\frac{1}{2\pi c}}e^{-i\omega_1(t-z/c)}\int d\omega\,\hat{a}^\dagger_\omega(t)e^{-i\omega z/c} \qquad \left[\hat{\mathscr{E}}(z,t),\hat{\mathscr{E}}^\dagger(z',t)\right] = \delta(z-z')$$

creates photon at z

Three-level medium



collective enhancement

$$\frac{\hat H}{\hbar} = -ic\int dz\, \hat{\mathcal{E}}^\dagger(z)\frac{\partial}{\partial z}\hat{\mathcal{E}}(z)$$
$$+\int dz\left[-\Delta \hat P^\dagger(z)\hat P(z) - \left(\Omega(t)\hat S^\dagger(z)\hat P(z) + g\sqrt{n}\,\hat P^\dagger(z)\hat{\mathcal{E}}(z) + h.c.\right)\right]$$

Heisenberg evolution:

$$(\partial_t + c\partial_z)\hat{\mathcal{E}} = ig\sqrt{n}\hat P$$

$$\partial_t \hat P = i\Delta\hat P + ig\sqrt{n}\hat{\mathcal{E}} + i\Omega^*\hat S - \gamma\hat P + \sqrt{2\gamma}\hat F_P$$

$$\partial_t \hat S = i\Omega\hat P - \gamma_s\hat S + \sqrt{2\gamma_s}\hat F_S$$

Langevin noise

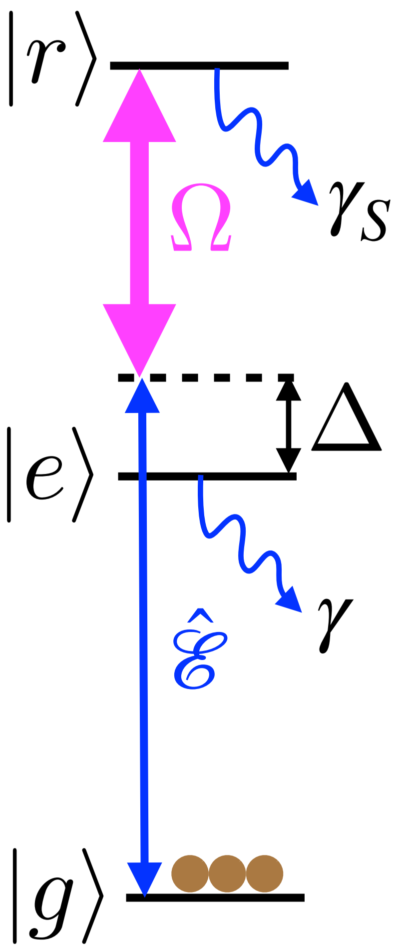
only nonzero noise correlations are:

$$\langle \hat F_P(z,t)\hat F_P^\dagger(z',t')\rangle = \delta(z-z')\delta(t-t')$$

$$\langle \hat F\rangle = \langle \hat F\hat F\rangle = \langle \hat F^\dagger \hat F\rangle = 0$$

$$\langle \hat F_S(z,t)\hat F_S^\dagger(z',t')\rangle = \delta(z-z')\delta(t-t')$$

Three-level medium



$$(\partial_t + c\partial_z)\hat{\mathscr{E}} = ig\sqrt{n}\hat{P}$$

$$\partial_t \hat{P} = -(\gamma - i\Delta)\hat{P} + ig\sqrt{n}\hat{\mathscr{E}} + i\Omega^* \hat{S} + \sqrt{2\gamma}\hat{F}_P$$

$$\partial_t \hat{S} = -\gamma_s \hat{S} + i\Omega\hat{P} + \sqrt{2\gamma_s}\hat{F}_S$$

- assume all atoms initially in ground state, i.e. no P or S excitations
- assume 1 incoming photon

$$|\psi(t)\rangle = \int dz E(z,t)\hat{\mathscr{E}}^\dagger(z)\,|\,0\rangle + \int dz P(z,t)\hat{P}^\dagger(z)\,|\,0\rangle + \int dz S(z,t)\hat{S}^\dagger(z)\,|\,0\rangle$$

$$(\partial_t + c\partial_z)E = ig\sqrt{n}P \qquad E(z,t=0) = P(z,t=0) = S(z,t=0) = 0$$

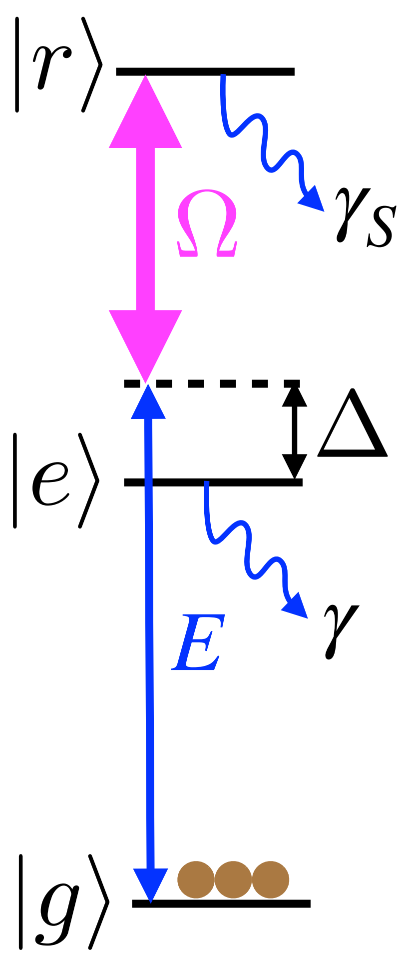
$$\partial_t P = -(\gamma - i\Delta)P + ig\sqrt{n}E + i\Omega^* S$$

$$E(z=0,t) = E_{in}(t)$$

$$\partial_t S = -\gamma_s S + i\Omega P$$

- same as equations for coherent input

Three-level medium



$$(\partial_t + c\partial_z)\hat{\mathscr{E}} = ig\sqrt{n}\hat{P}$$

$$\partial_t \hat{P} = -(\gamma - i\Delta)\hat{P} + ig\sqrt{n}\hat{\mathscr{E}} + i\Omega^* \hat{S} + \sqrt{2\gamma}\hat{F}_P$$

$$\partial_t \hat{S} = -\gamma_s \hat{S} + i\Omega\hat{P} + \sqrt{2\gamma_s}\hat{F}_S$$

- assume all atoms initially in ground state, i.e. no P or S excitations
- assume 1 incoming photon

$$|\psi(t)\rangle = \int dz E(z,t)\hat{\mathscr{E}}^\dagger(z)|0\rangle + \int dz P(z,t)\hat{P}^\dagger(z)|0\rangle + \int dz S(z,t)\hat{S}^\dagger(z)|0\rangle$$

$$(\partial_t + c\partial_z)E = ig\sqrt{n}P$$

$$E(z,t=0) = P(z,t=0) = S(z,t=0) = 0$$

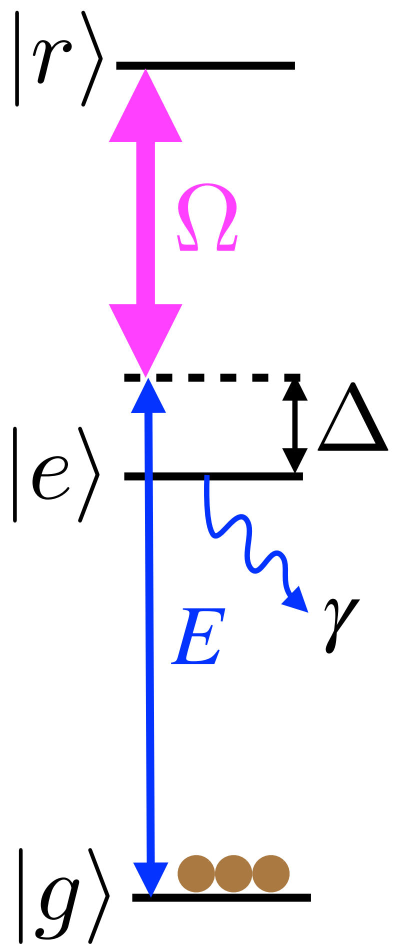
$$\partial_t P = -(\gamma - i\Delta)P + ig\sqrt{n}E + i\Omega^* S$$

$$\partial_t S = -\cancel{\gamma_s S} + i\Omega P$$

$$E(z=0,t) = E_{in}(t)$$

- same as equations for coherent input

No atoms



$$(\partial_t + c\partial_z)E = ig\sqrt{n}P$$

$$\partial_t P = -(\gamma - i\Delta)P + ig\sqrt{n}E + i\Omega^* S$$

$$\partial_t S = i\Omega P$$

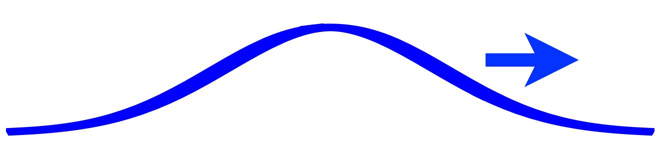
sanity check: no atoms

$$g\sqrt{n} = 0$$

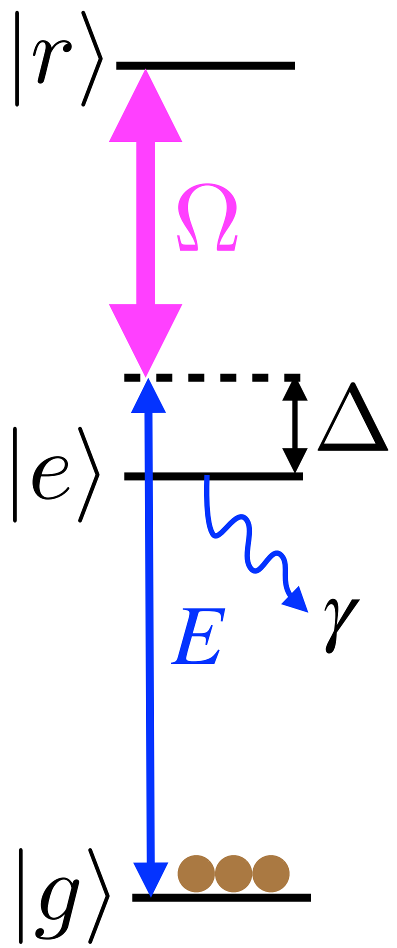
$$(\partial_t + c\partial_z)E = 0$$

$$E(z,t) = E(0,t - z/c)$$

undistorted propagation at c



Two-level medium



$$(\partial_t + c\partial_z)E = ig\sqrt{n}P$$

$$\partial_t P = -(\gamma - i\Delta)P + ig\sqrt{n}E + i\Omega^* S$$

$$\partial_t S = i\Omega P$$

assume: resonant incoming photon, no control

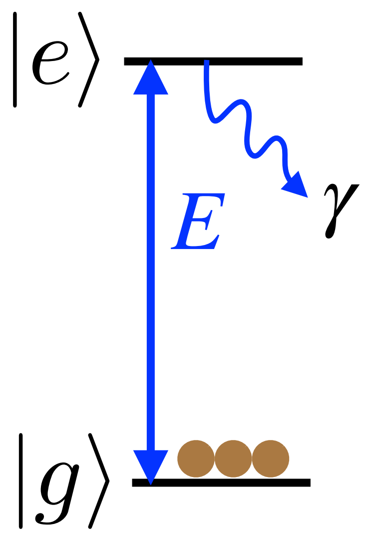
$\Delta = 0$ $\Omega = 0$

Two-level medium

$$(\partial_t + c\partial_z)E = ig\sqrt{n}P$$

$$\partial_t P = -(\gamma - i\Delta)P + ig\sqrt{n}E + i\Omega^* S$$

$$\partial_t S = i\Omega P$$



assume: resonant incoming photon, no control

$$\Delta = 0 \qquad \Omega = 0$$

$$(\partial_t + c\partial_z)E = ig\sqrt{n}P$$

$$\partial_t P = -\gamma P + ig\sqrt{n}E$$

$$E(z,t) = \int d\omega \tilde{E}(z,\omega)e^{-i\omega t}$$

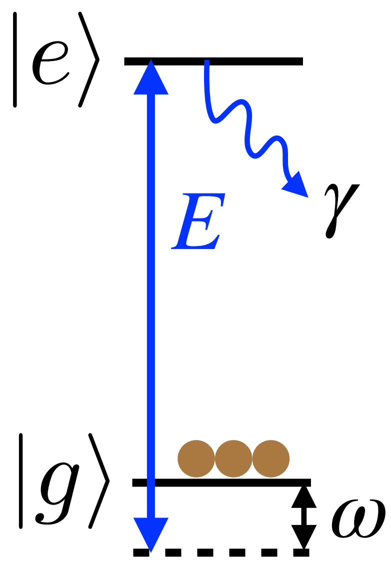
$$P(z,t) = \int d\omega \tilde{P}(z,\omega)e^{-i\omega t}$$

Two-level medium

$$(\partial_t + c\partial_z)E = ig\sqrt{n}P$$

$$\partial_t P = -(\gamma - i\Delta)P + ig\sqrt{n}E + i\Omega^* S$$

$$\partial_t S = i\Omega P$$



assume: resonant incoming photon, no control

$$\Delta = 0 \qquad\qquad \Omega = 0$$

$$(\partial_t + c\partial_z)E = ig\sqrt{n}P$$

$$\partial_t P = -\gamma P + ig\sqrt{n}E$$

$$E(z,t) = \int d\omega \tilde{E}(z,\omega)e^{-i\omega t}$$

$$P(z,t) = \int d\omega \tilde{P}(z,\omega)e^{-i\omega t}$$

$$(-i\omega + c\partial_z)\tilde{E} = ig\sqrt{n}\tilde{P}$$

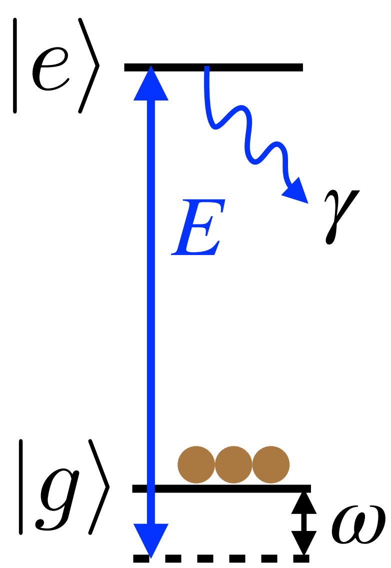
$$-i\omega\tilde{P} = -\gamma\tilde{P} + ig\sqrt{n}\tilde{E}$$

Two-level medium

$$(-i\omega + c\partial_z)\tilde{E} = ig\sqrt{n}\tilde{P}$$

$$-i\omega\tilde{P} = -\gamma\tilde{P} + ig\sqrt{n}\tilde{E}$$

$$\tilde{P} = \frac{ig\sqrt{n}}{\gamma - i\omega}\tilde{E}$$



$$c\partial_z\tilde{E} = \left(i\omega - \frac{g^2 n}{\gamma - i\omega}\right)\tilde{E}$$

$$\tilde{E}(L,\omega) = \tilde{E}(0,\omega)\ \exp\left[i\omega\frac{L}{c} - \frac{g^2 nL/c}{\gamma - i\omega}\right]$$

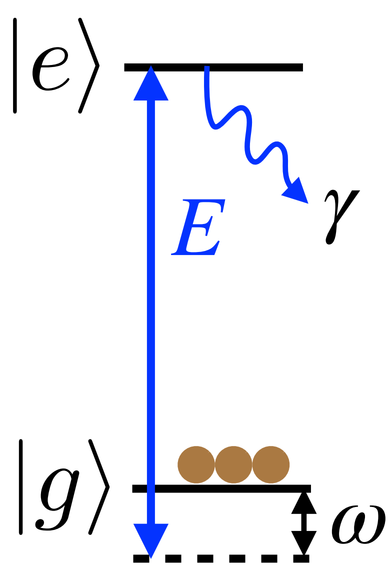
$$\tilde{E}(L,\omega) = \tilde{E}(0,\omega)\ \exp\left[i\omega\frac{L}{c} - \frac{d\gamma}{\gamma - i\omega}\right]$$

$$d = \frac{g^2 nL}{\gamma c}$$

$$|\tilde{E}(L,\omega)|^2 = |\tilde{E}(0,\omega)|^2\ \exp\left[-\frac{2d}{1 + (\omega/\gamma)^2}\right]$$

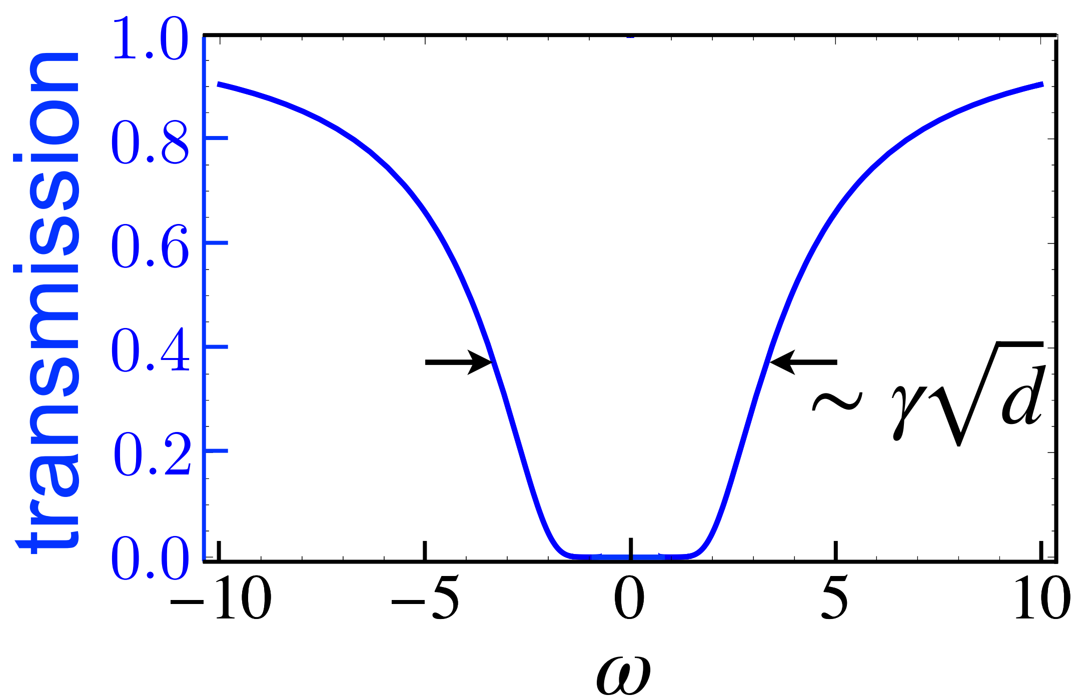
Two-level medium

$$|\tilde{E}(L,\omega)|^2 = |\tilde{E}(0,\omega)|^2 \exp\left[-\frac{2d}{1+(\omega/\gamma)^2}\right]$$

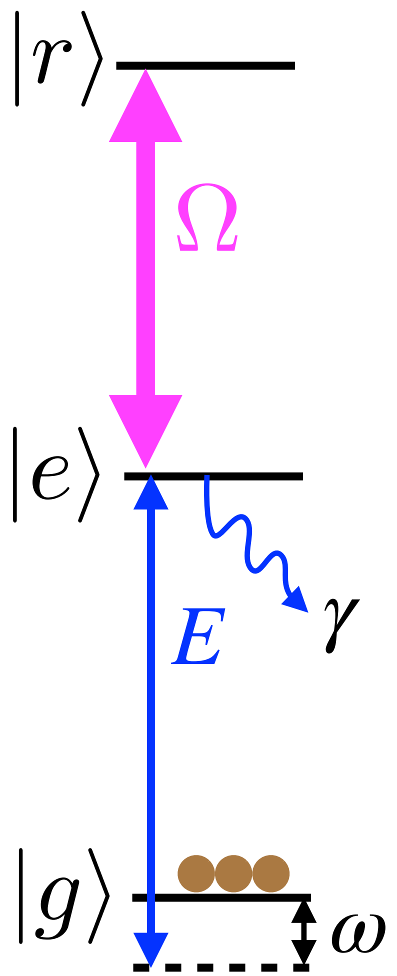


- on resonance: $I_{out} = I_{in} e^{-2d}$
- $2d =$ optical depth
- assume $d \gg 1$

absorption line



Electromagnetically induced transparency (EIT)



$$(\partial_t + c\partial_z)E = ig\sqrt{n}P$$

$$\partial_t P = -\gamma P + ig\sqrt{n}E + i\Omega^* S$$

$$\partial_t S = i\Omega P$$

- assume Ω real

$$(-i\omega + c\partial_z)\tilde{E} = ig\sqrt{n}\tilde{P}$$

$$-i\omega\tilde{P} = -\gamma\tilde{P} + ig\sqrt{n}\tilde{E} + i\Omega\tilde{S}$$

$$-i\omega\tilde{S} = i\Omega\tilde{P}$$

$$E(z,t) = \int d\omega \tilde{E}(z,\omega)e^{-i\omega t}$$

$$P(z,t) = \int d\omega \tilde{P}(z,\omega)e^{-i\omega t}$$

$$S(z,t) = \int d\omega \tilde{S}(z,\omega)e^{-i\omega t}$$

- at $\omega = 0$: $\tilde{P} = 0$

$$\tilde{S} = -\frac{g\sqrt{n}}{\Omega}\tilde{E}$$

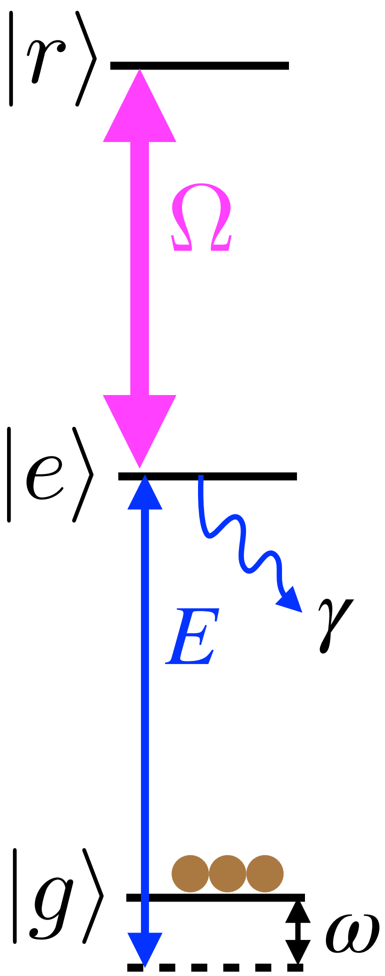
$\partial_z\tilde{E} = 0$ perfect transmission, i.e. no scattering

Dark-state polariton: coupled atom-photon excitation

[Fleishhauer & Lukin, 2000, 2002]

- destructive interference

Electromagnetically induced transparency (EIT)



$$(-i\omega + c\partial_z)\tilde{E} = ig\sqrt{n}\tilde{P}$$

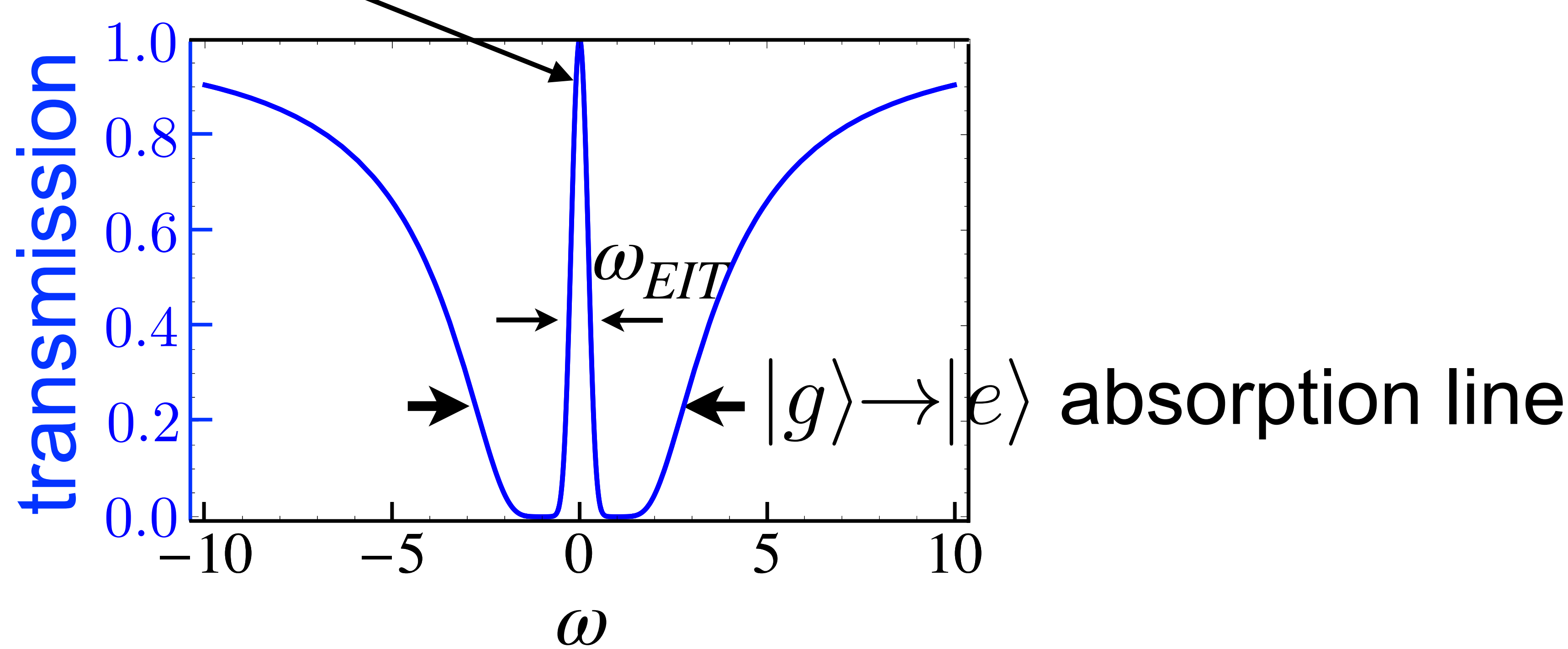
$$-i\omega\tilde{P} = -\gamma\tilde{P} + ig\sqrt{n}\tilde{E} + i\Omega\tilde{S}$$

$$-i\omega\tilde{S} = i\Omega\tilde{P}$$

- near $\omega = 0$

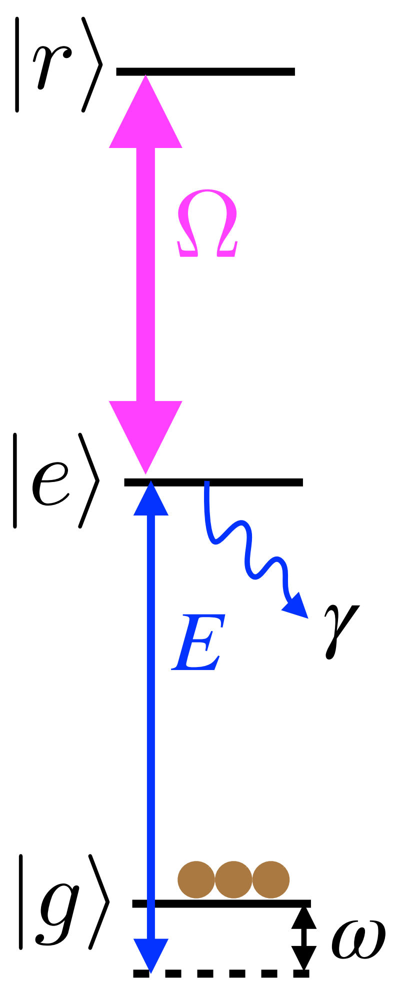
$$|\tilde{E}(L,\omega)|^2 \approx |\tilde{E}(0,\omega)|^2 \exp\left[-\frac{2d\gamma^2\omega^2}{\Omega^4}\right]$$

homework exercise



- EIT transparency window of bandwidth $\omega_{EIT} \sim \dfrac{\Omega^2}{\gamma\sqrt{d}}$

Electromagnetically induced transparency (EIT)



$$(-i\omega + c\partial_z)\tilde{E} = ig\sqrt{n}\tilde{P}$$

$$-i\omega\tilde{P} = -\gamma\tilde{P} + ig\sqrt{n}\tilde{E} + i\Omega\tilde{S}$$

$$-i\omega\tilde{S} = i\Omega\tilde{P}$$

- near $\omega = 0$

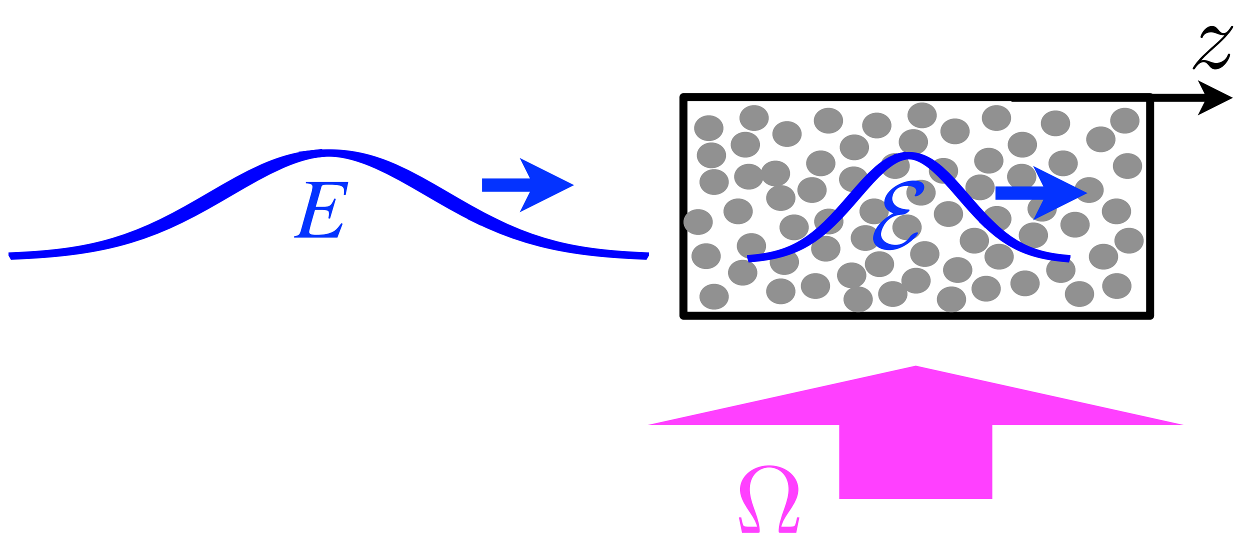
homework exercise

$$\partial_z\tilde{E} \approx i\frac{\omega}{v_g}\tilde{E} \qquad v_g \approx \frac{\Omega^2}{g^2 n}c \ll c$$

$$(\partial_t + v_g\partial_z)E = 0$$

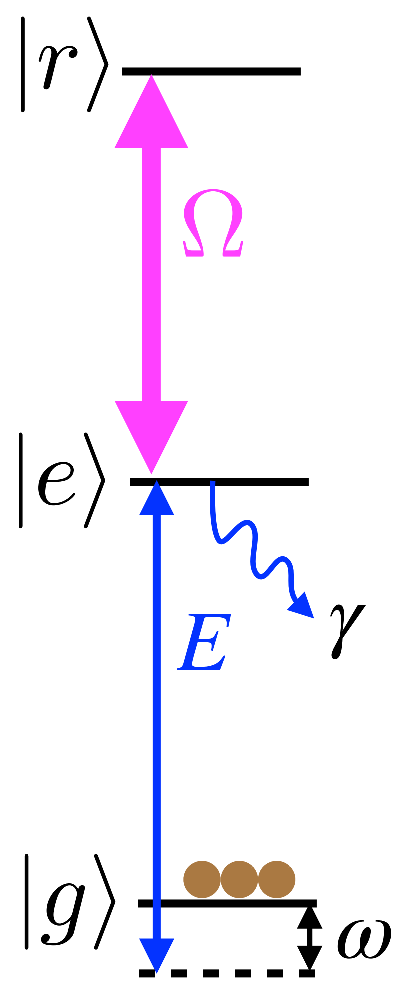
reduced group velocity

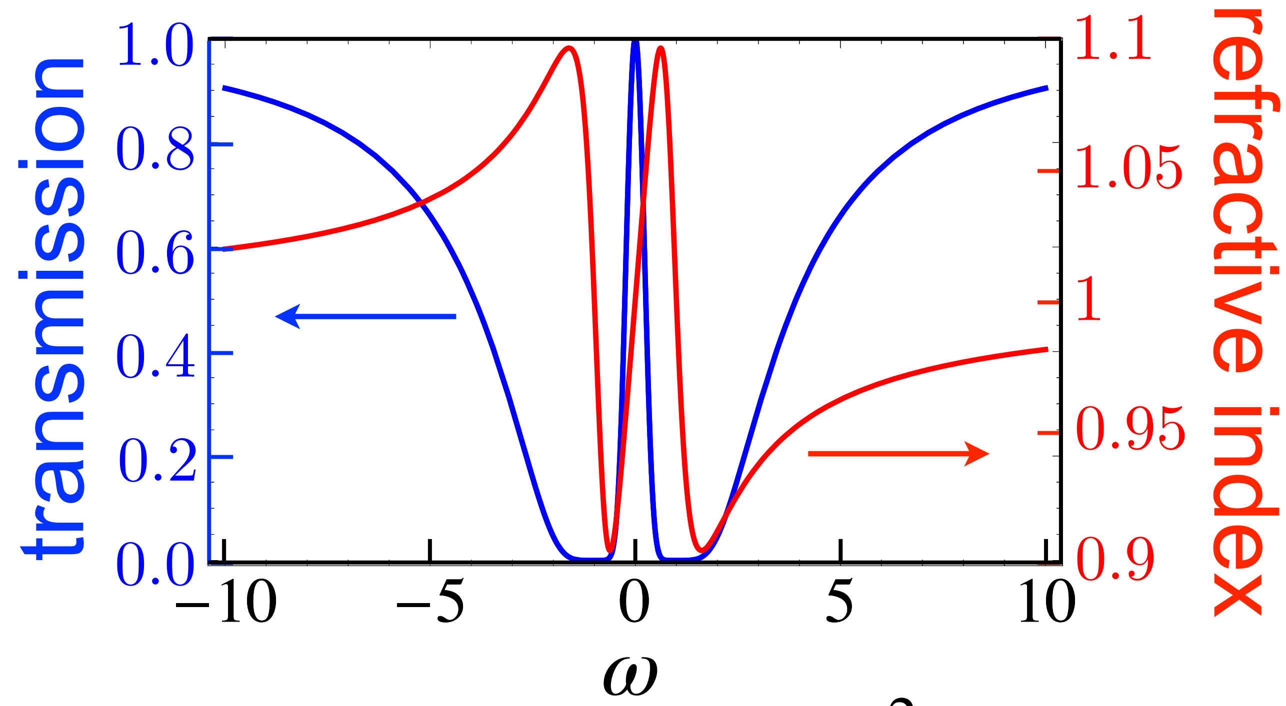
"slow light"



- pulse compression

Electromagnetically induced transparency (EIT)



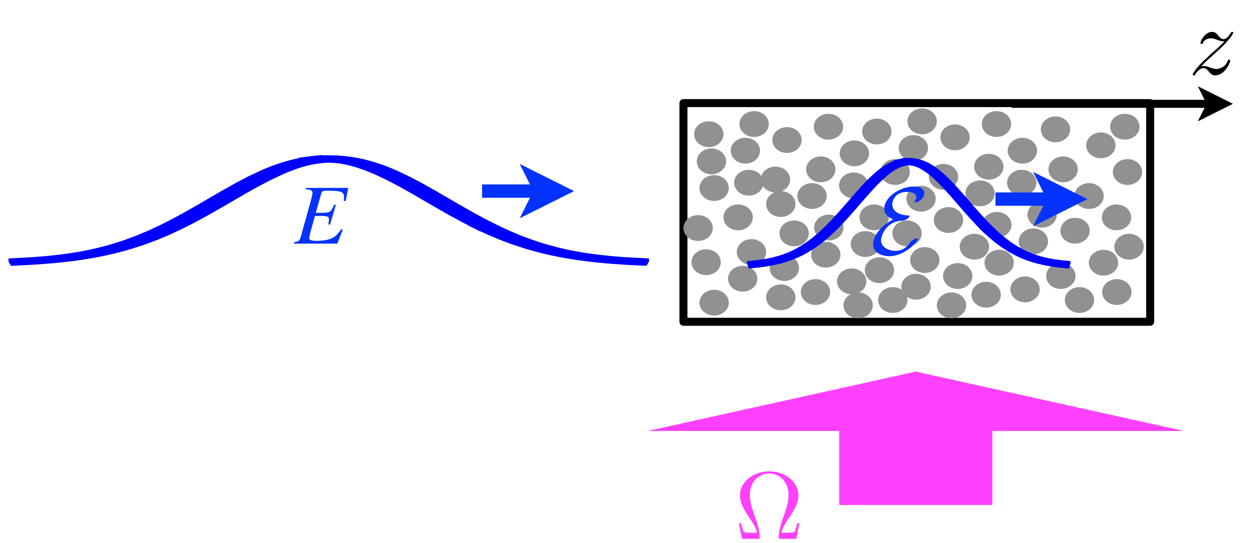


$$\partial_z \tilde{E} \approx i \frac{\omega}{v_g} \tilde{E} \qquad v_g \approx \frac{\Omega^2}{g^2 n} c \ll c$$

$$(\partial_t + v_g \partial_z) E = 0$$

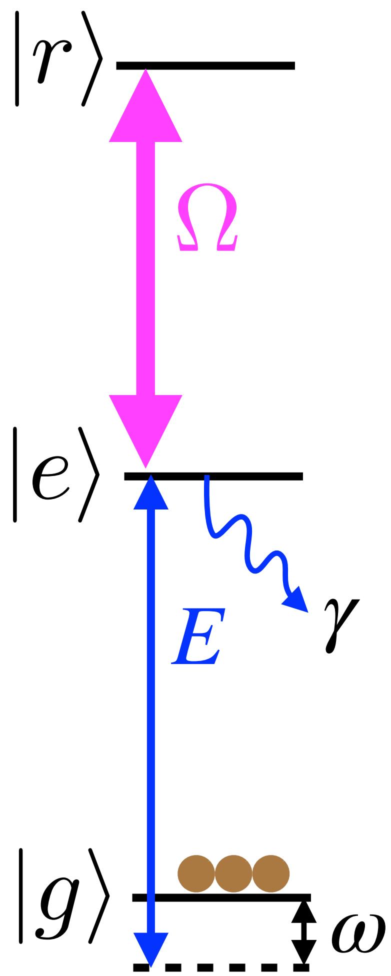
reduced group velocity

"slow light"



- pulse compression

Photon storage and retrieval



- dark state polariton

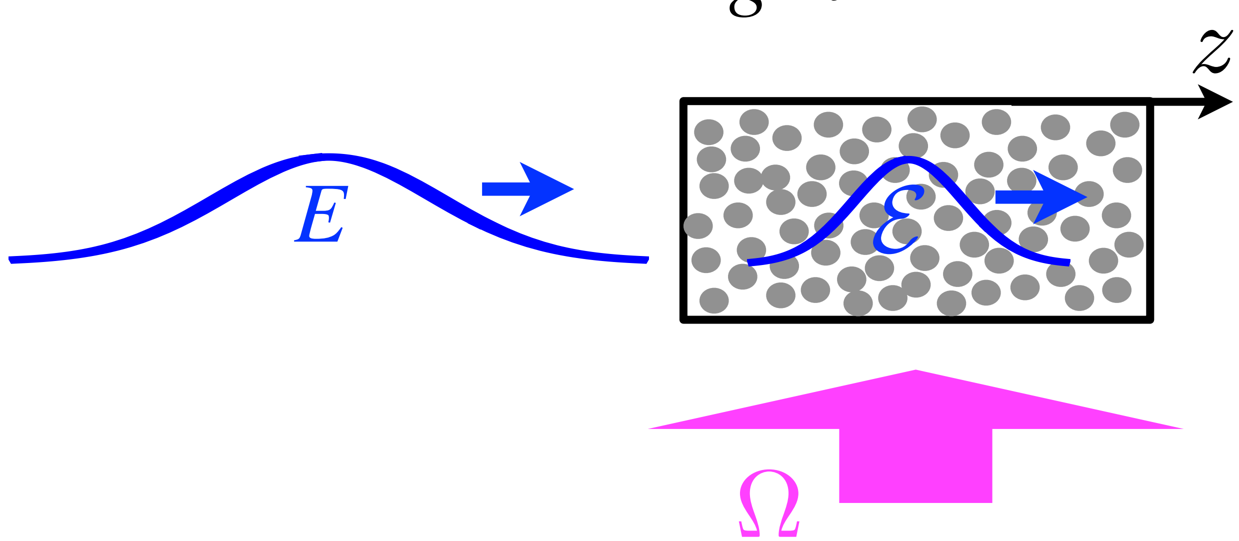
$$|\psi\rangle \sim \int dz f(z - v_g t)\left(\Omega \hat{\mathscr{E}}^\dagger(z) - g\sqrt{n}\hat{S}^\dagger(z)\right)|0\rangle$$

- while pulse is inside medium, turn Ω off

$$|\psi\rangle \sim \int dz f(z) S^\dagger(z)|0\rangle$$ photon stored in “spinwave”

- when turn Ω back on, photon is retrieved

$$v_g \approx \frac{\Omega^2}{g^2 n} c$$ reduced group velocity



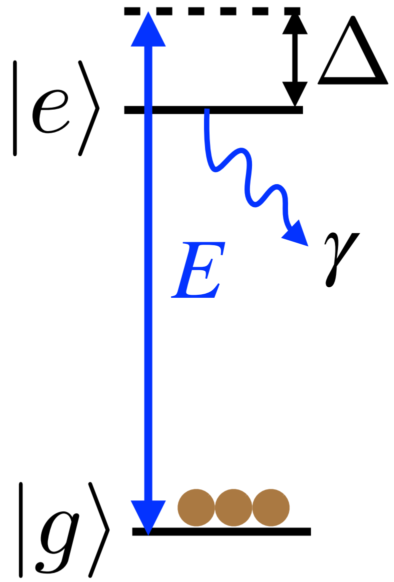
- pulse compression

Off-resonant two-level medium

$$(\partial_t + c\partial_z)E = ig\sqrt{n}P$$

$$\partial_t P = -(\gamma - i\Delta)P + ig\sqrt{n}E$$

large Δ



- Fourier transform in time & drop γ:

$$(-i\omega + c\partial_z)\tilde{E} = ig\sqrt{n}\tilde{P}$$

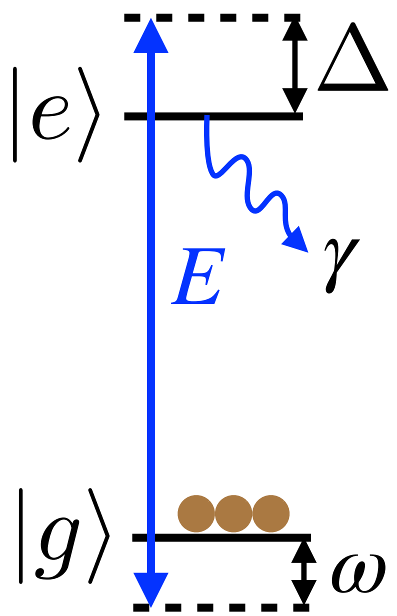
$$-i\omega\tilde{P} \approx i\Delta\tilde{P} + ig\sqrt{n}\tilde{E}$$

Off-resonant two-level medium

$$(\partial_t + c\partial_z)E = ig\sqrt{n}P$$

$$\partial_t P = -(\gamma - i\Delta)P + ig\sqrt{n}E$$

large Δ



- Fourier transform in time & drop γ :

$$(-i\omega + c\partial_z)\tilde{E} = ig\sqrt{n}\tilde{P}$$

$$-i\omega\tilde{P} \approx i\Delta\tilde{P} + ig\sqrt{n}\tilde{E}$$

- near $\omega = 0$:

$$c\partial_z\tilde{E} \approx ig\sqrt{n}\tilde{P}$$

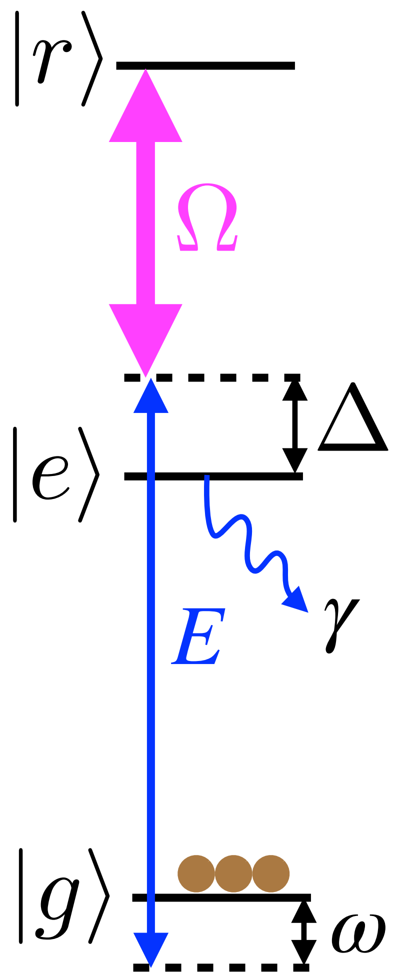
$$0 \approx i\Delta\tilde{P} + ig\sqrt{n}\tilde{E} \qquad \tilde{P} \approx -\frac{g\sqrt{n}}{\Delta}\tilde{E}$$

$$\partial_z\tilde{E} \approx -i\frac{g^2 n}{c\Delta}\tilde{E}$$

$$\tilde{E}(z = L, \omega \approx 0) \approx \tilde{E}(z = 0, \omega \approx 0)\ \exp\left[-i\frac{d\gamma}{\Delta}\right]$$

- atoms imprint a phase on photon

EIT with large single-photon detuning



$$(-i\omega + c\partial_z)\tilde{E} = ig\sqrt{n}\tilde{P}$$

$$-i\omega\tilde{P} = -(\gamma - i\Delta)\tilde{P} + ig\sqrt{n}\tilde{E} + i\Omega\tilde{S}$$

$$-i\omega\tilde{S} = i\Omega\tilde{P}$$

large Δ

- at $\omega = 0$:

same as for $\Delta = 0$

$$\tilde{P} = 0 \qquad \tilde{S} = -\frac{g\sqrt{n}}{\Omega}\tilde{E}$$

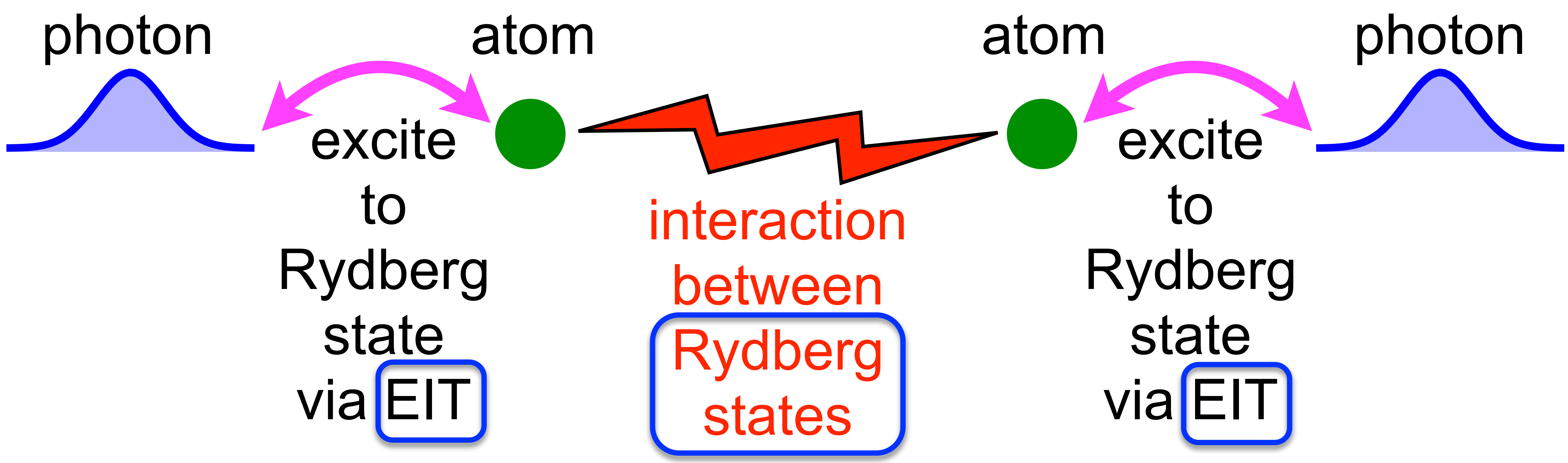
$\partial_z\tilde{E} = 0$ perfect transmission due to EIT

Dark-state polariton: coupled atom-photon excitation
[Fleishhauer & Lukin, 2000, 2002]

$(\partial_t + v_g\partial_z)E = 0$ reduced group velocity

"slow light"

Medium where photons interact strongly



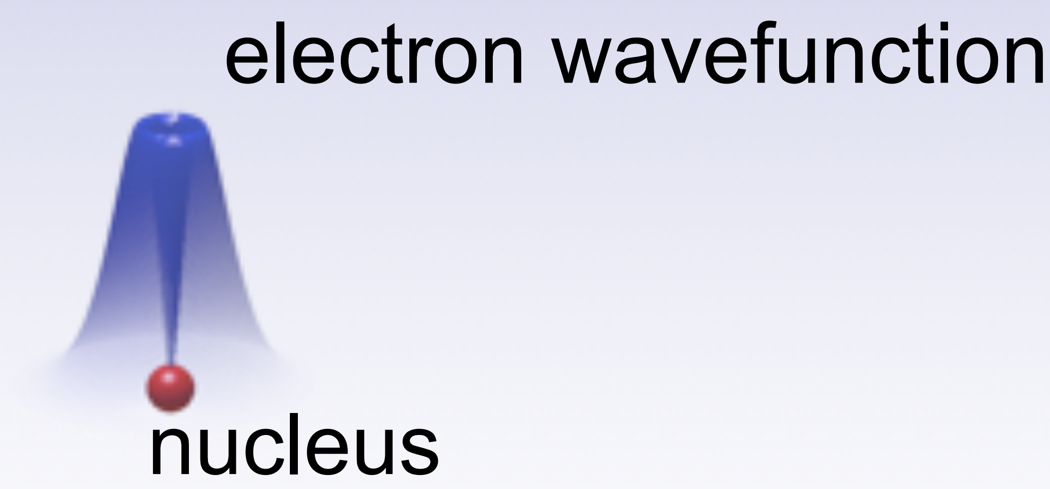
EIT = electromagnetically induced transparency

Summer school lectures by Browaeys, Hazzard,
and possibly Kaufman, Bakr, etc…

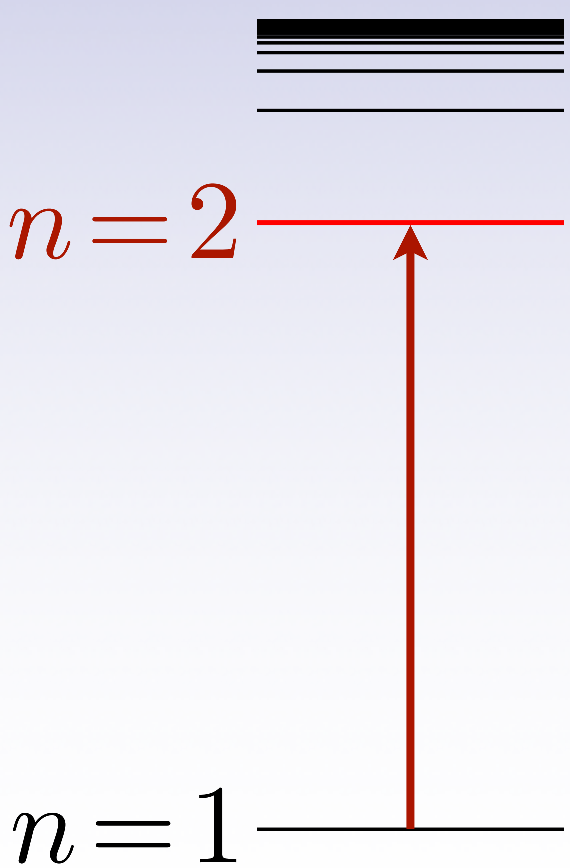
Outline

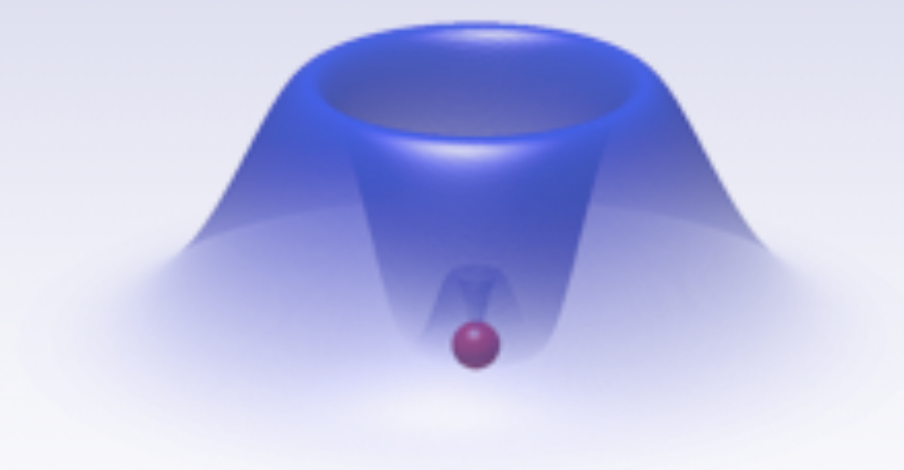
- motivation and basic idea
- E&M field quantization
- propagation of light through atomic ensembles; electromagnetically induced transparency (EIT)
- Rydberg atoms
- basic idea revisited
- photon interacting with stationary excitation
  - on resonance: single-photon switch, subtractor
  - off resonance: two-photon quantum gate
- dynamics of multiple photons
  - on resonance: source of single photons
  - off resonance: two-photon gate, bound states, many-body physics
- more applications

electronic levels in atom:

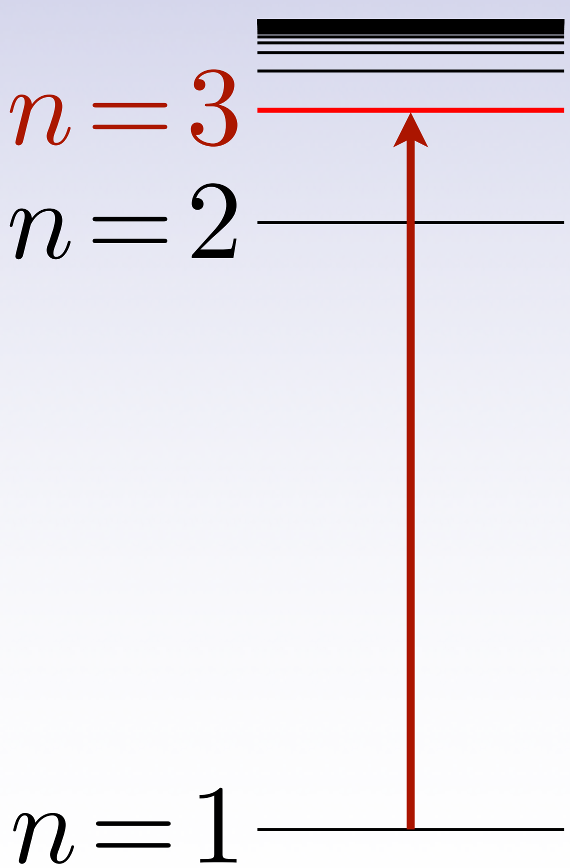


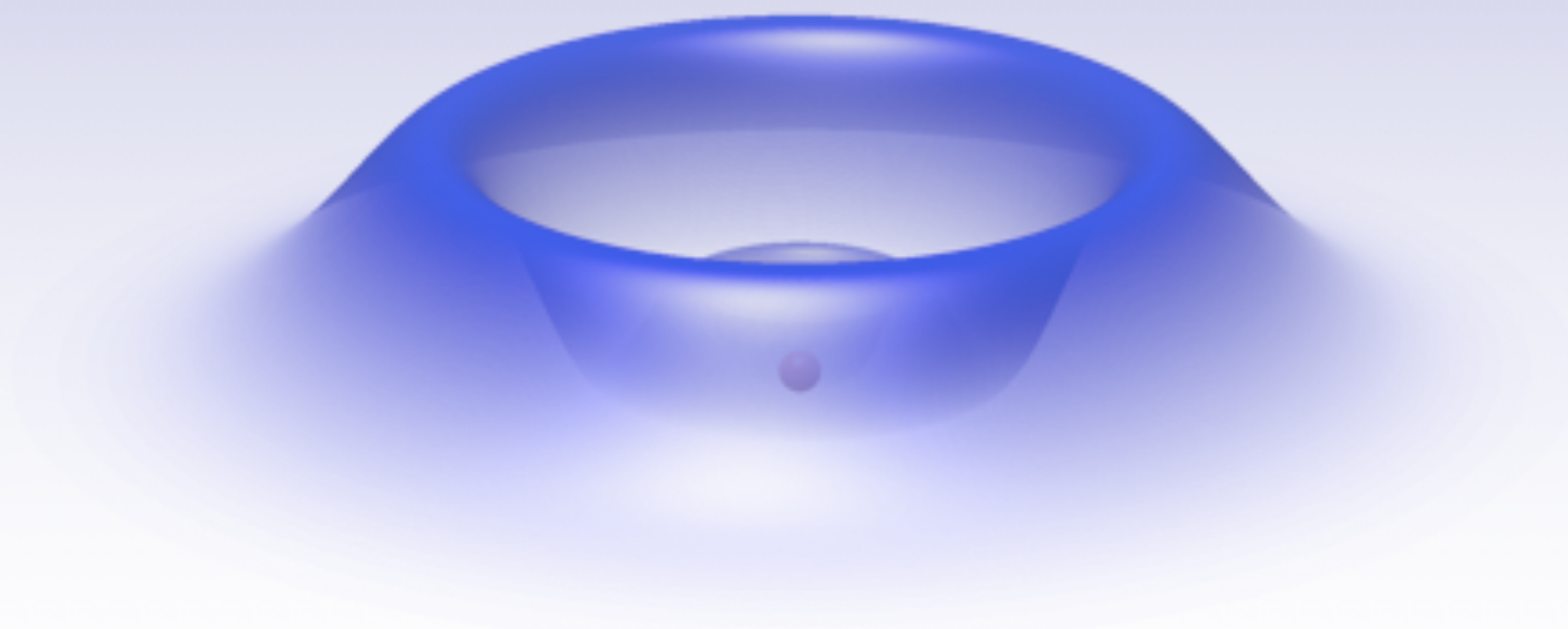
electronic levels in atom:



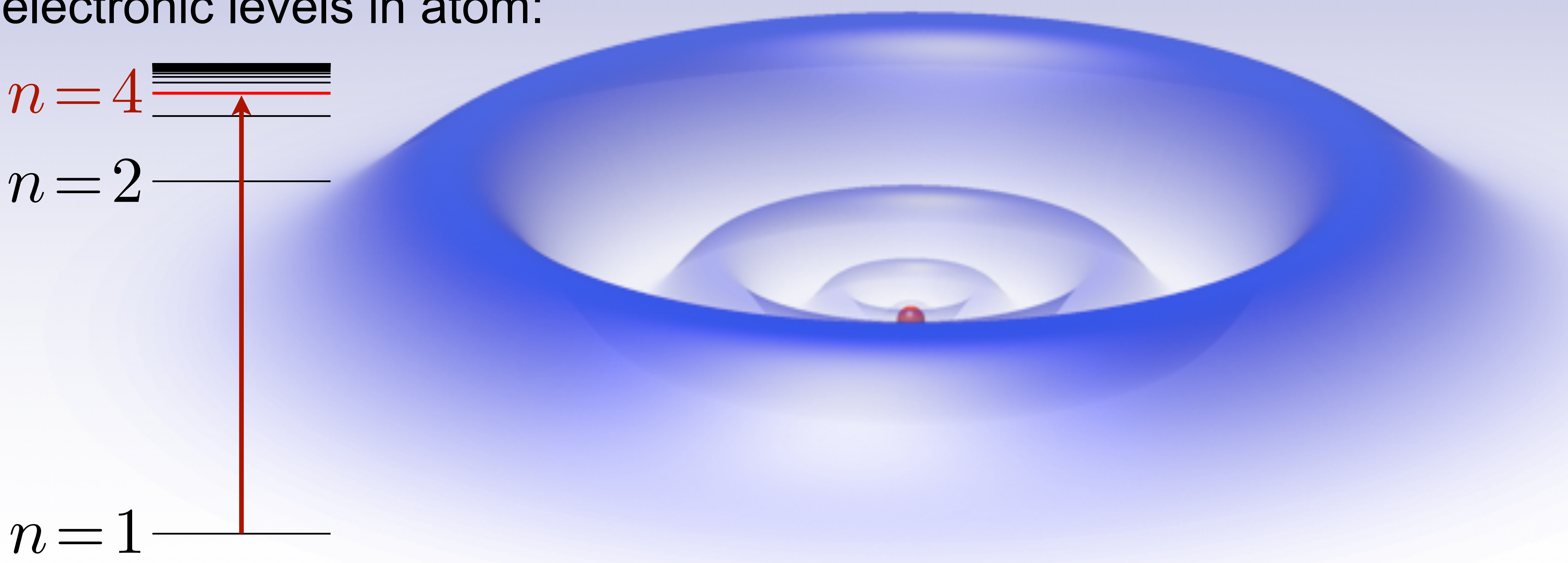


electronic levels in atom:

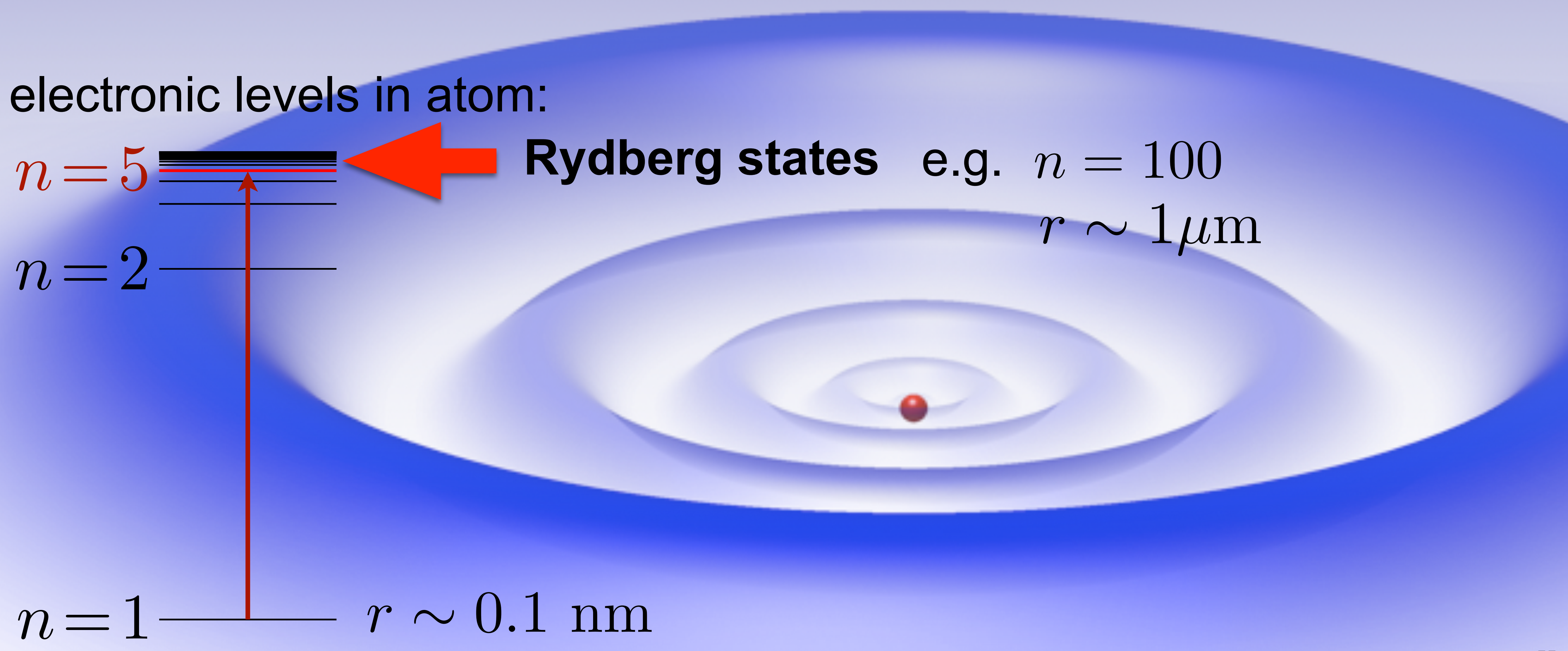


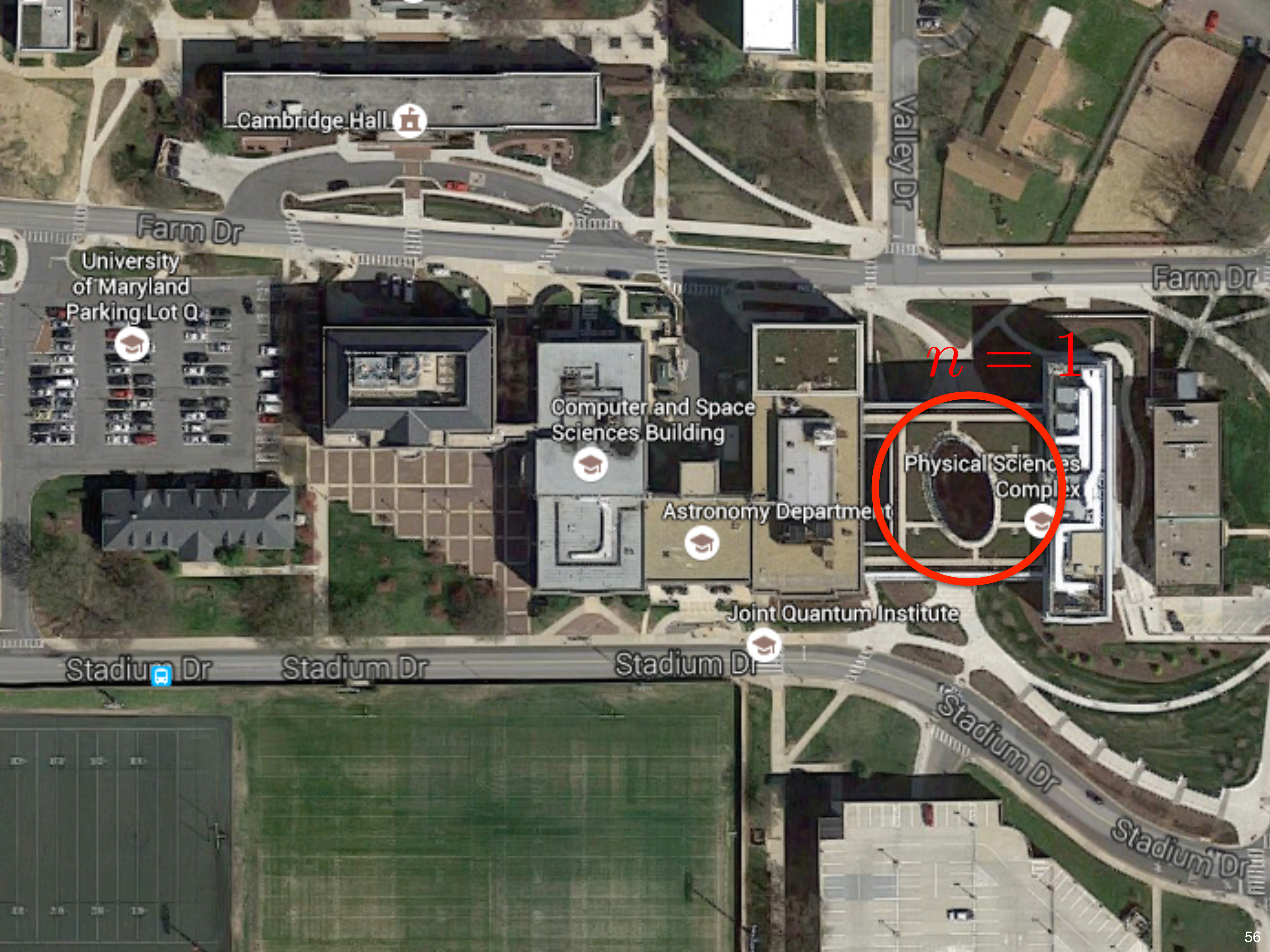


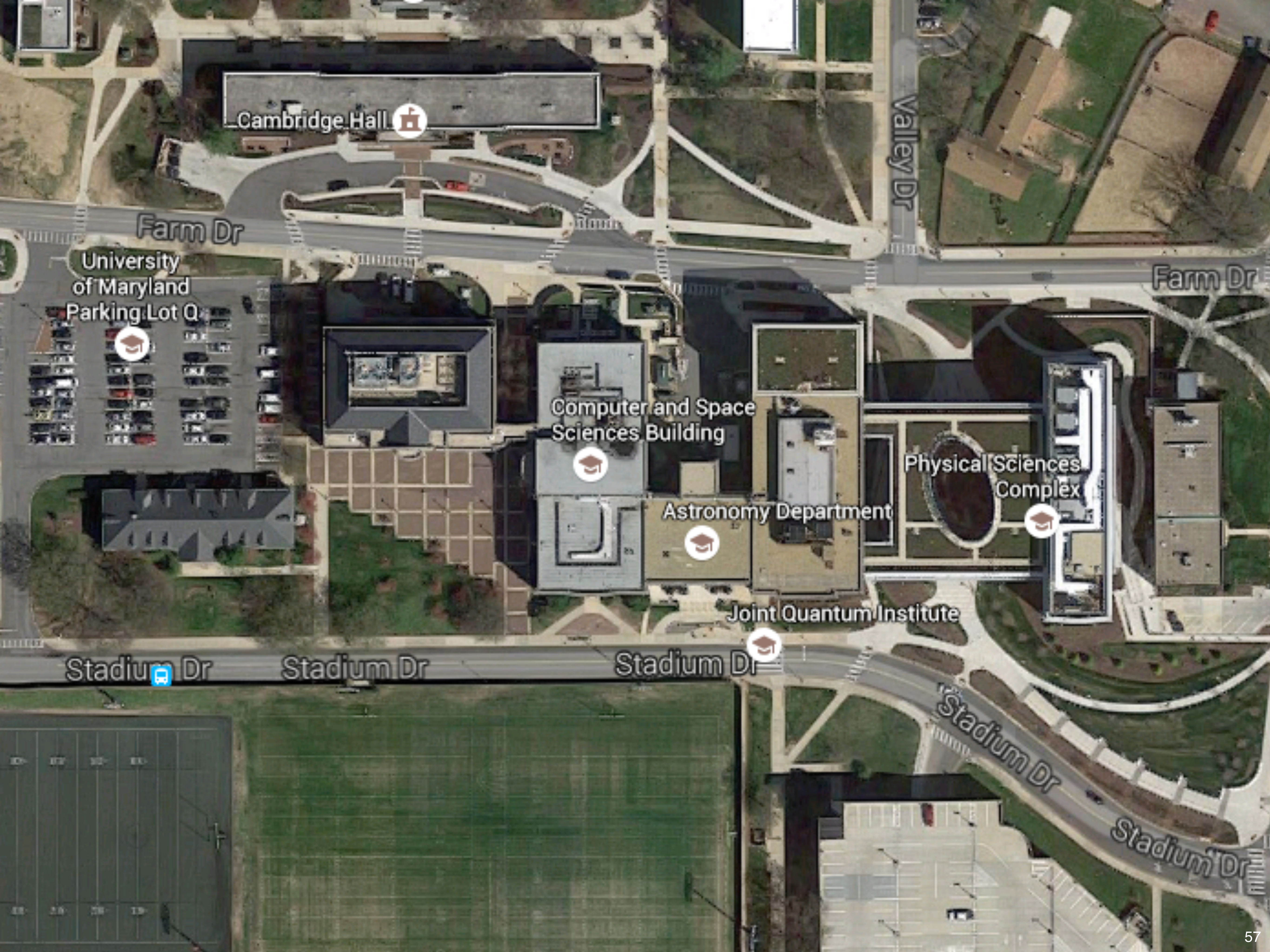
electronic levels in atom:

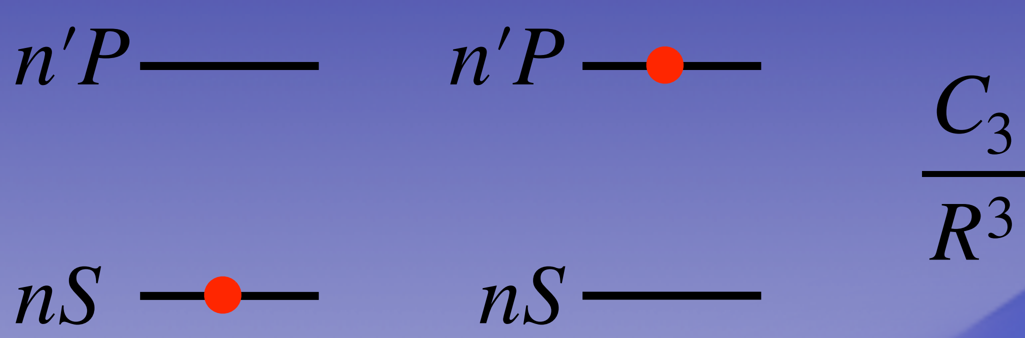


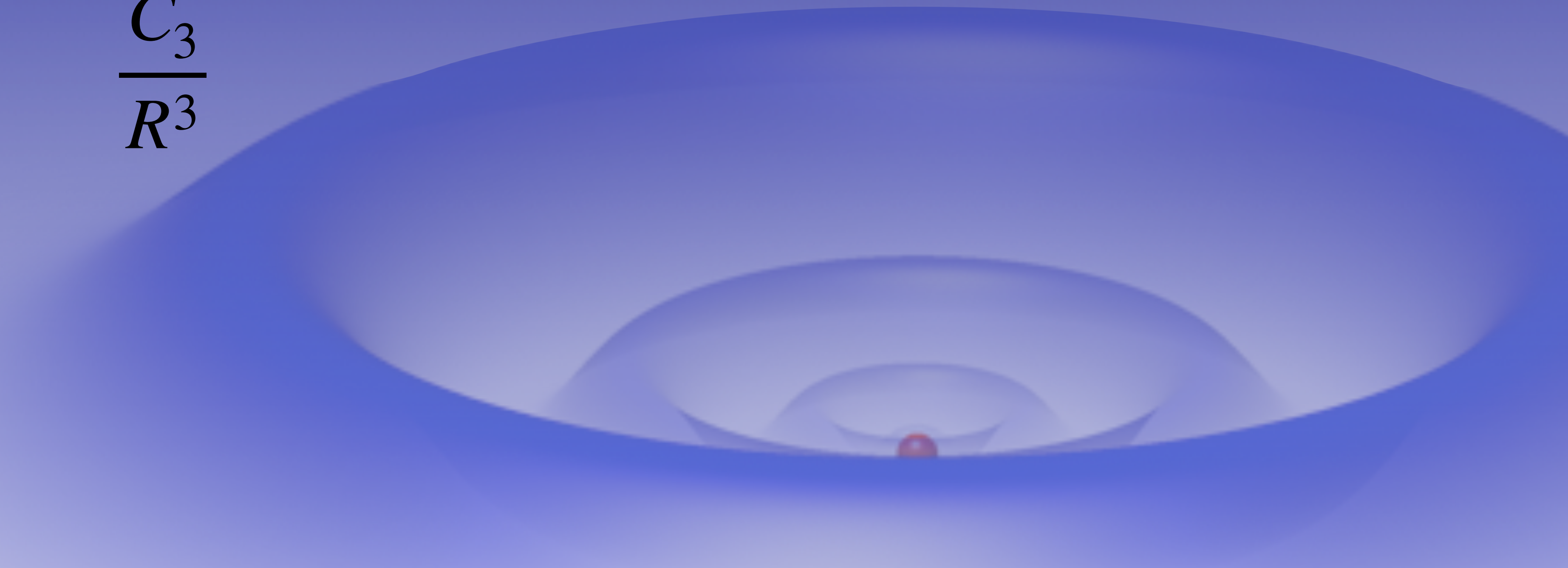
• large size: $r \sim n^2$

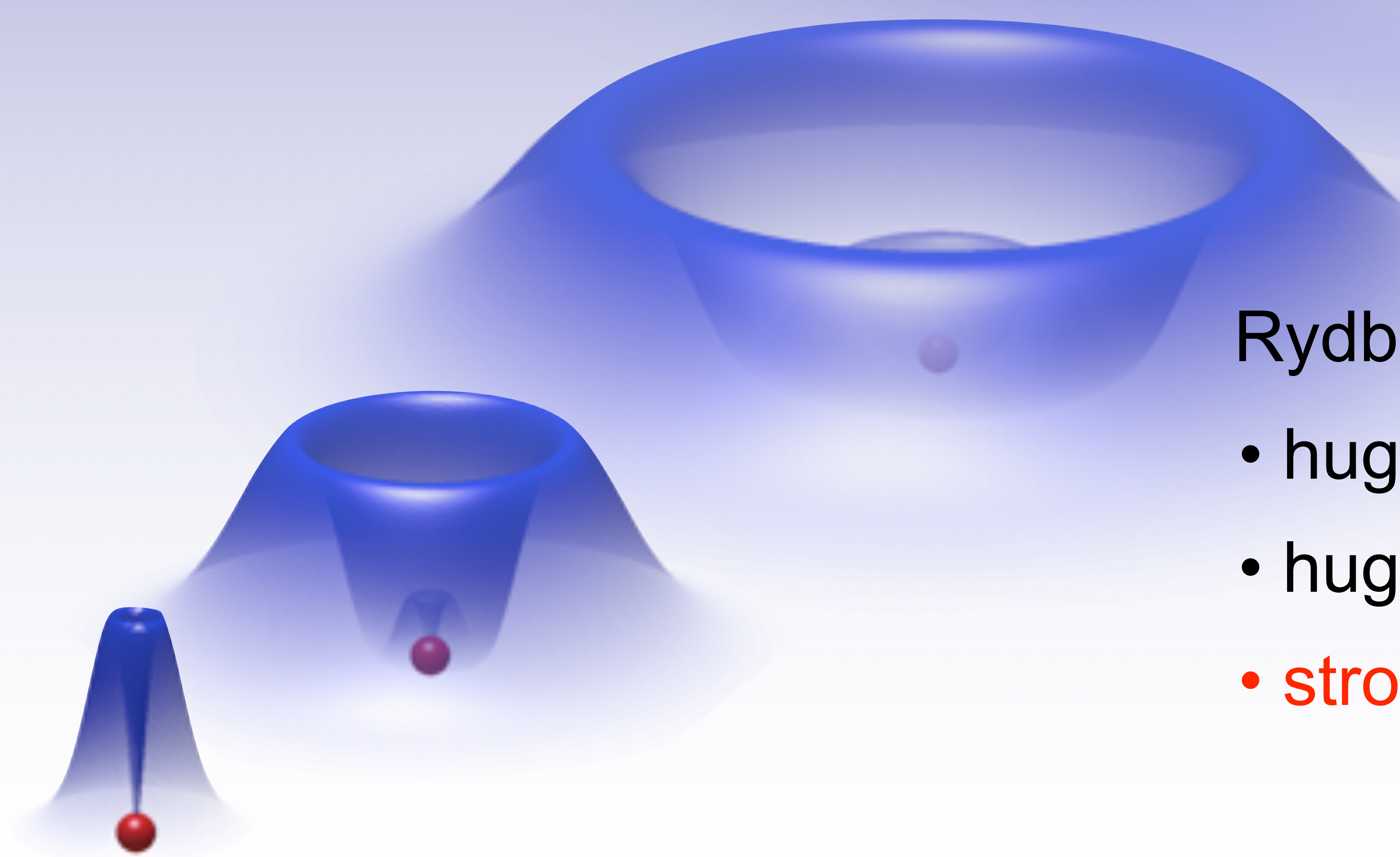



Cambridge Hall
Farm Dr
Valley Dr
Farm Dr
University of Maryland Parking Lot Q
$n = 1$
Computer and Space Sciences Building
Physical Sciences Complex
Astronomy Department
Joint Quantum Institute
Stadium Dr
Stadium Dr
Stadium Dr
Stadium Dr
Stadium Dr


Cambridge Hall
Farm Dr
Valley Dr
Farm Dr
University of Maryland Parking Lot Q
Computer and Space Sciences Building
Physical Sciences Complex
Astronomy Department
Joint Quantum Institute
Stadium Dr
Stadium Dr
Stadium Dr
Stadium Dr
Stadium Dr

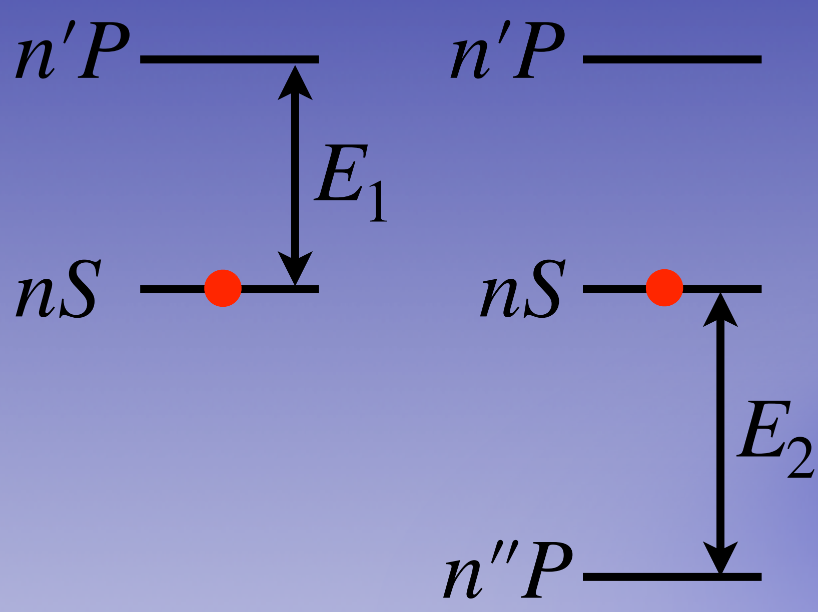




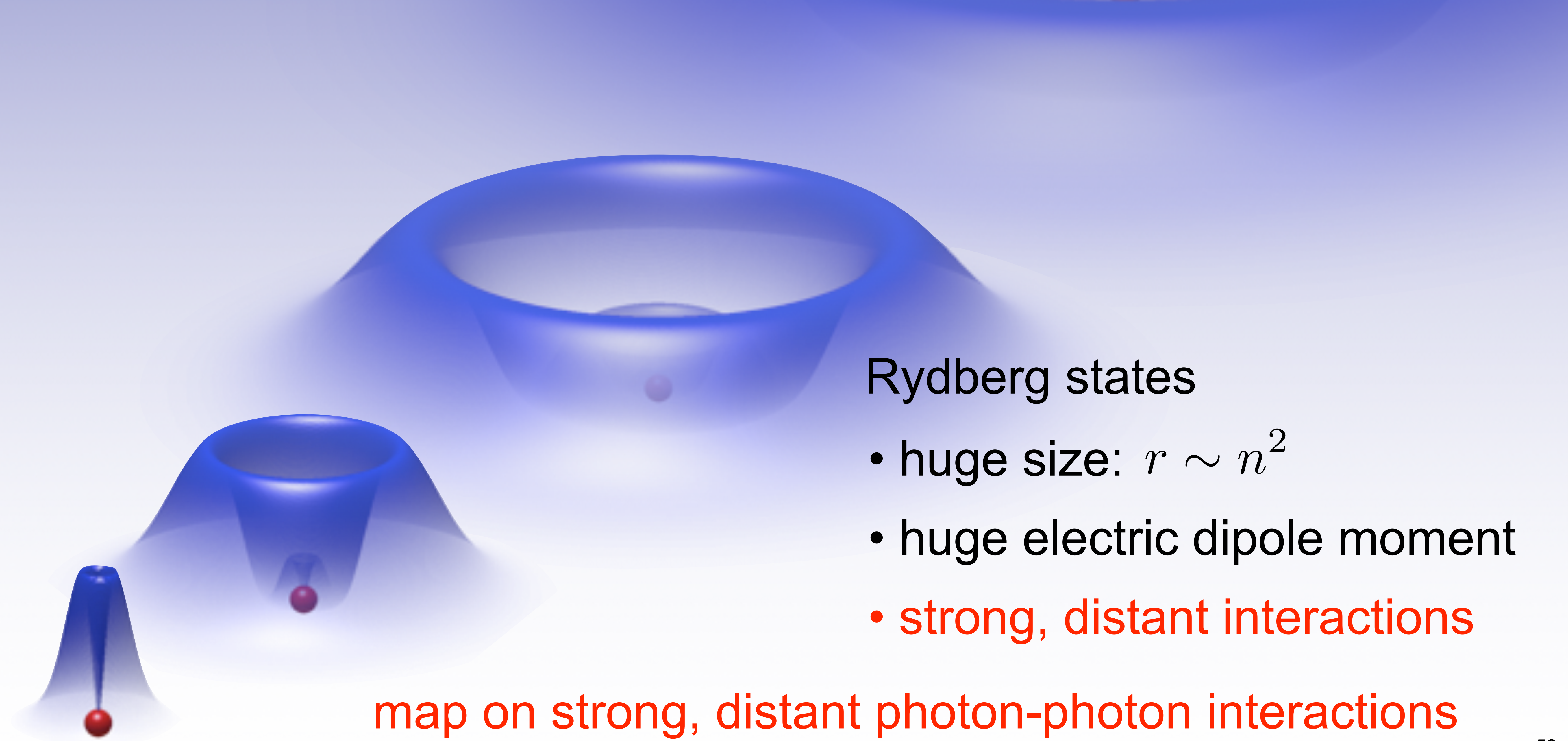


Rydberg states

- huge size: $r \sim n^2$
- huge electric dipole moment
- strong, distant interactions



$$\sim \frac{(C_3/R^3)^2}{E_2 - E_1} \sim \frac{C_6}{R^6}$$



Rydberg states

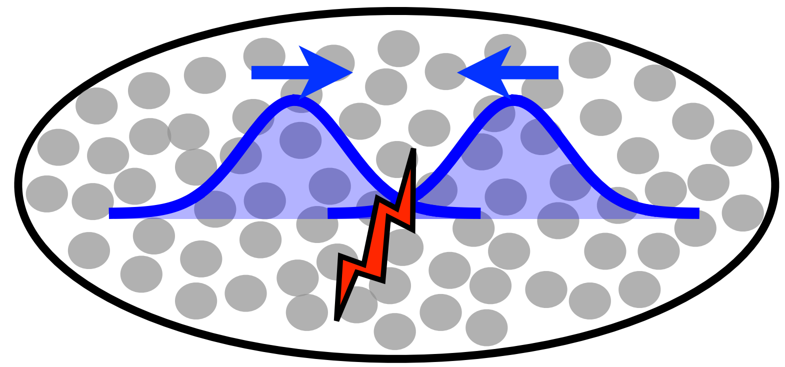
- huge size: $r \sim n^2$
- huge electric dipole moment
- strong, distant interactions

map on strong, distant photon-photon interactions

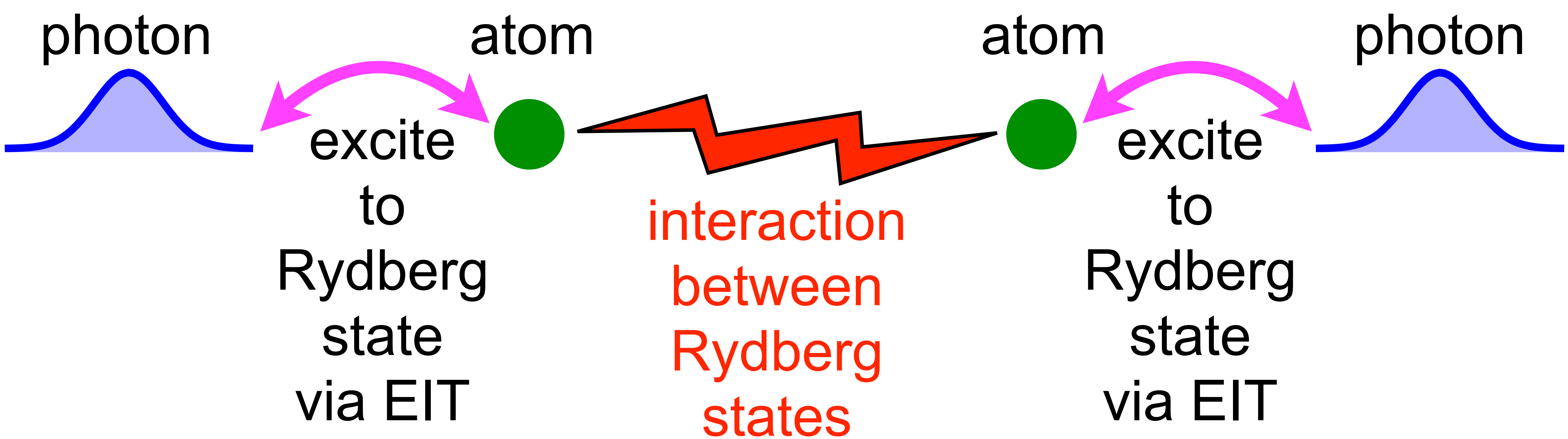
Outline

- motivation and basic idea
- E&M field quantization
- propagation of light through atomic ensembles; electromagnetically induced transparency (EIT)
- Rydberg atoms
- basic idea revisited
- photon interacting with stationary excitation
  - on resonance: single-photon switch, subtractor
  - off resonance: two-photon quantum gate
- dynamics of multiple photons
  - on resonance: source of single photons
  - off resonance: two-photon gate, bound states, many-body physics
- more applications

Medium where photons interact strongly

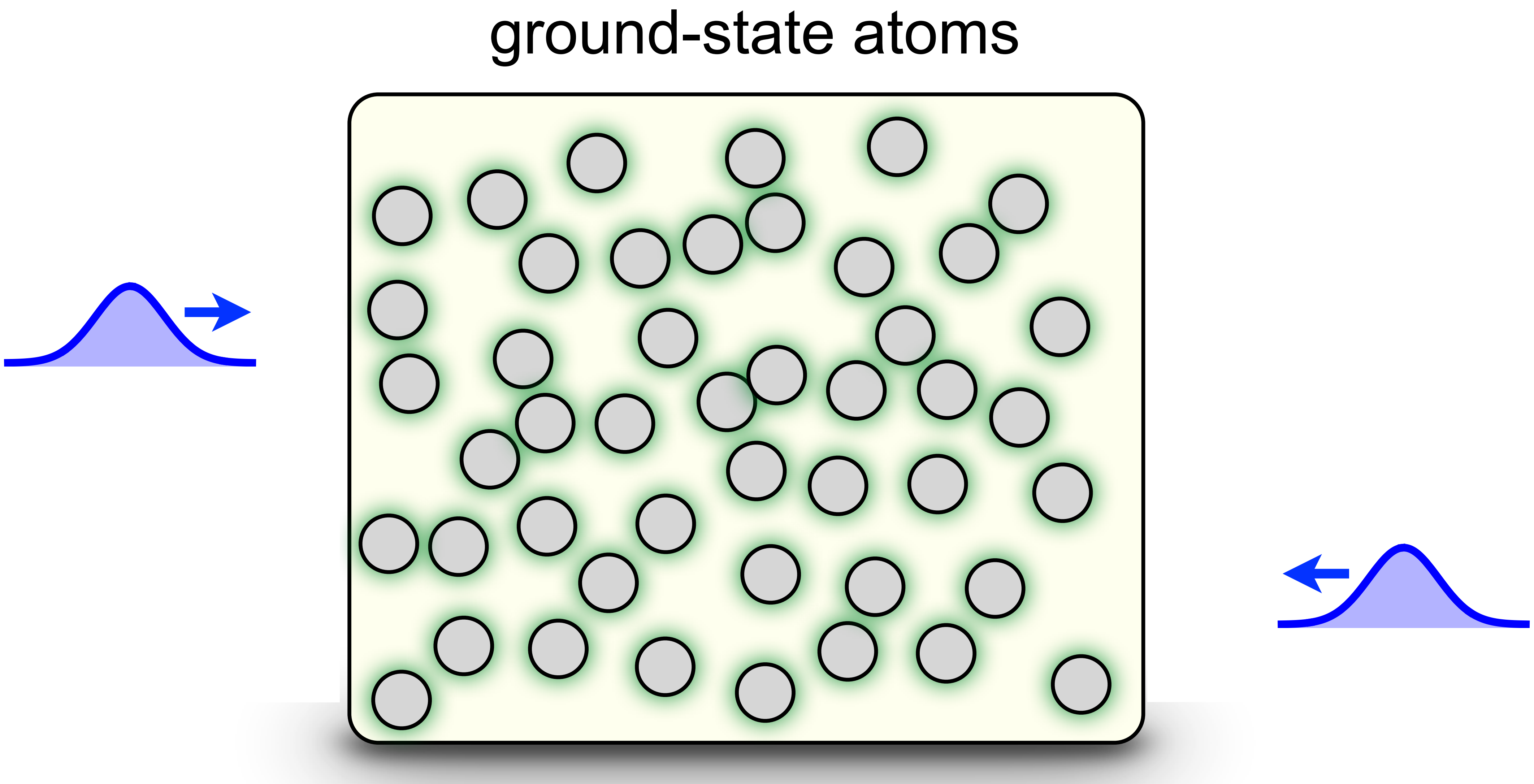


Map strong atom-atom interactions onto strong photon-photon interactions



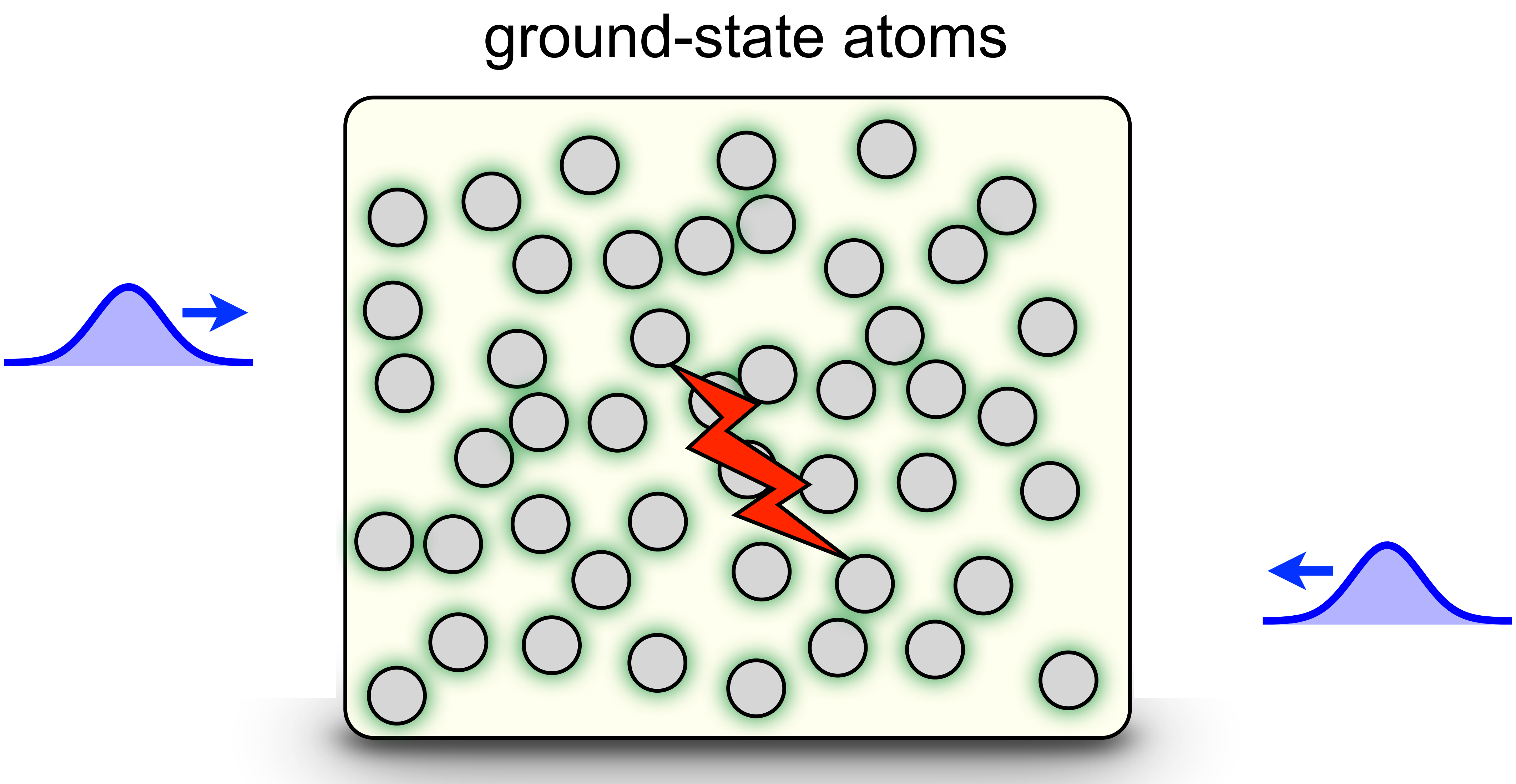
EIT = electromagnetically induced transparency

Basic idea



- one photon (polariton) drags along a Rydberg excitation

Basic idea



• one photon (polariton) drags along a Rydberg excitation

• another photon drags along a Rydberg excitation

• Rydberg excitations feel strong, distant interactions

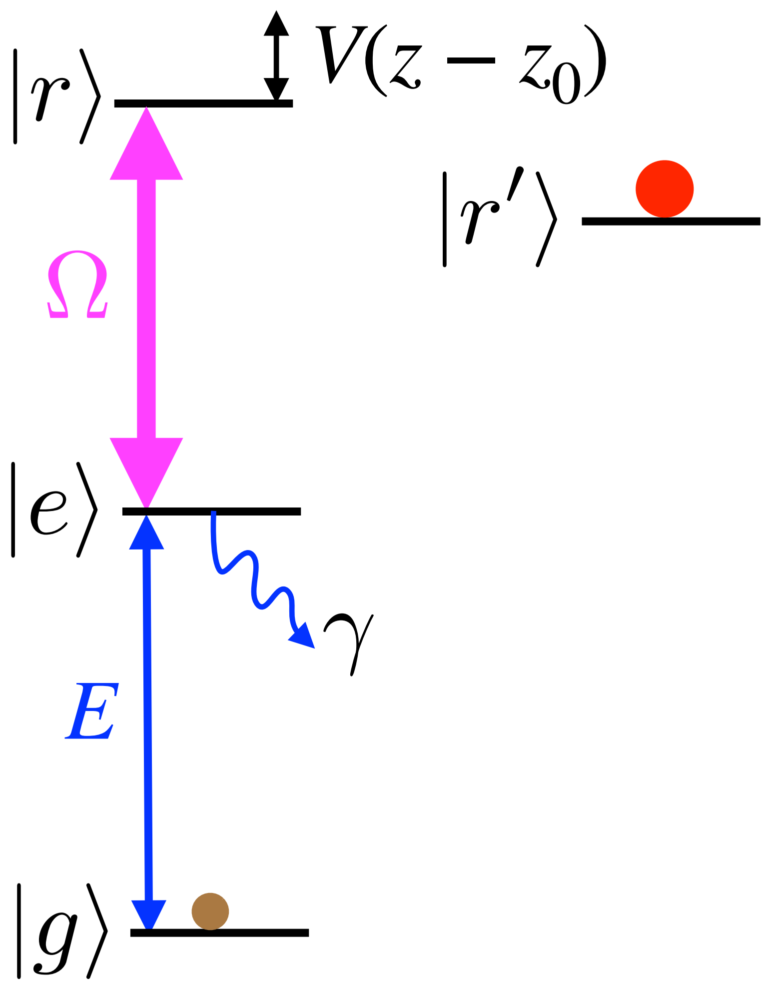
⇒ strong, distant photon-photon interactions

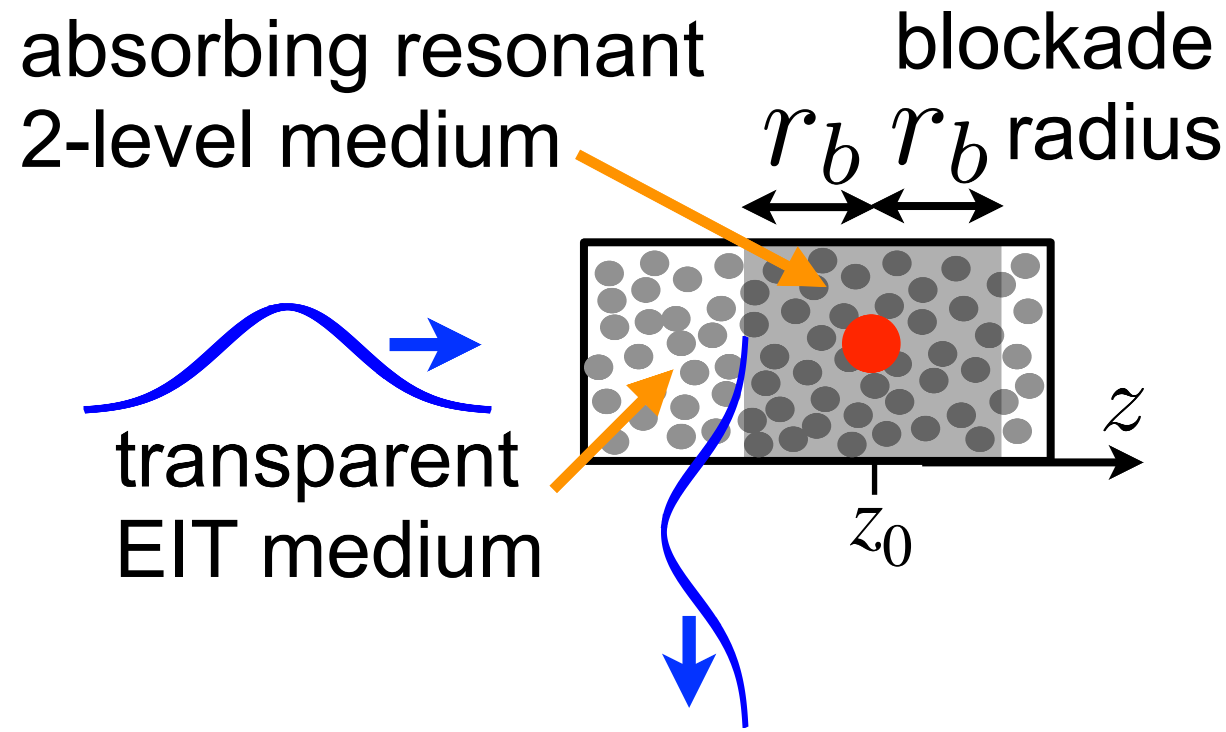
Outline

- motivation and basic idea
- E&M field quantization
- propagation of light through atomic ensembles; electromagnetically induced transparency (EIT)
- Rydberg atoms
- basic idea revisited
- photon interacting with stationary excitation
  - on resonance: single-photon switch, subtractor
  - off resonance: two-photon quantum gate
- dynamics of multiple photons
  - on resonance: source of single photons
  - off resonance: two-photon gate, bound states, many-body physics
- more applications

Photon interacting with stationary excitation

$|r\rangle, |r'\rangle$ = Rydberg states



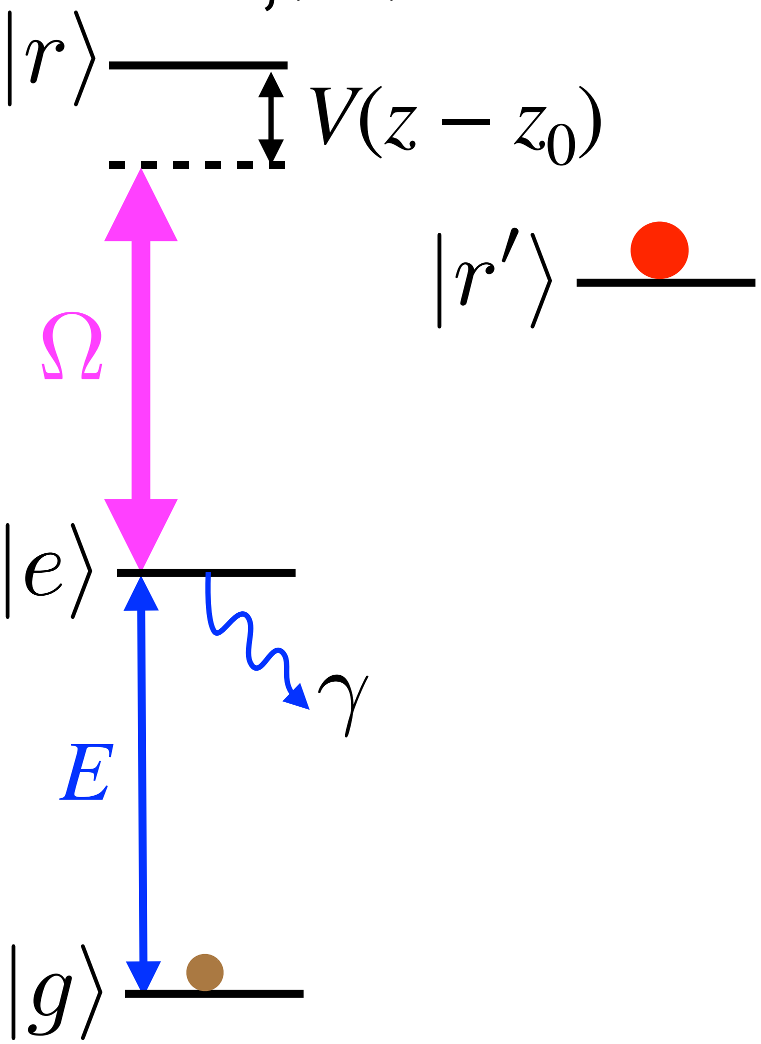


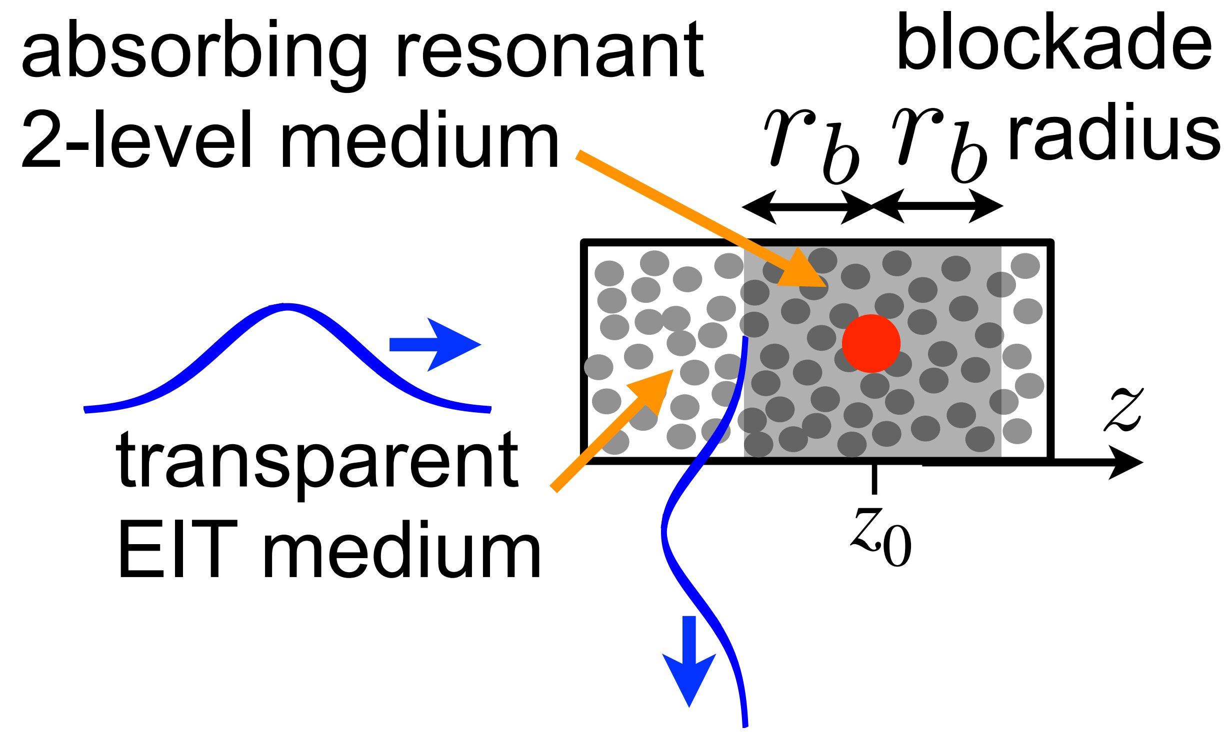
- control atom at z_0 prepared in $|r'\rangle$
- atoms in $|r\rangle$ experience van der Waals potential $V(z-z_0) = \dfrac{C_6}{(z-z_0)^6}$

AVG et al, PRL 107, 133602 (2011)

Photon interacting with stationary excitation

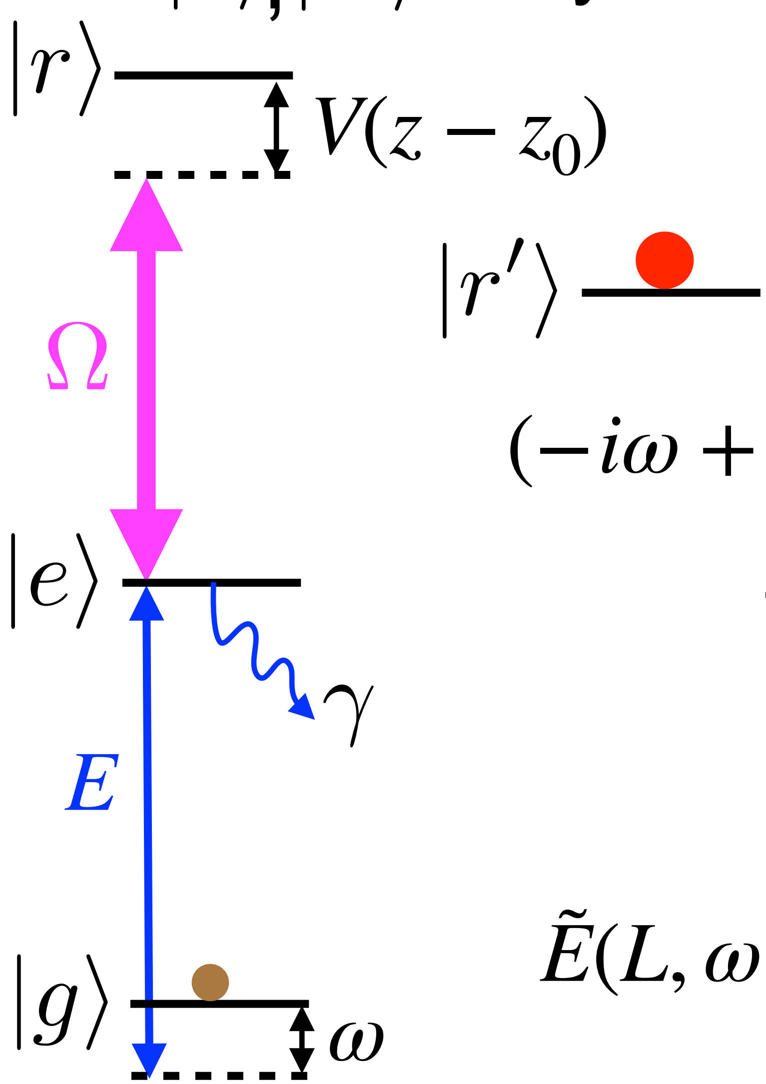
$|r\rangle, |r'\rangle$ = Rydberg states

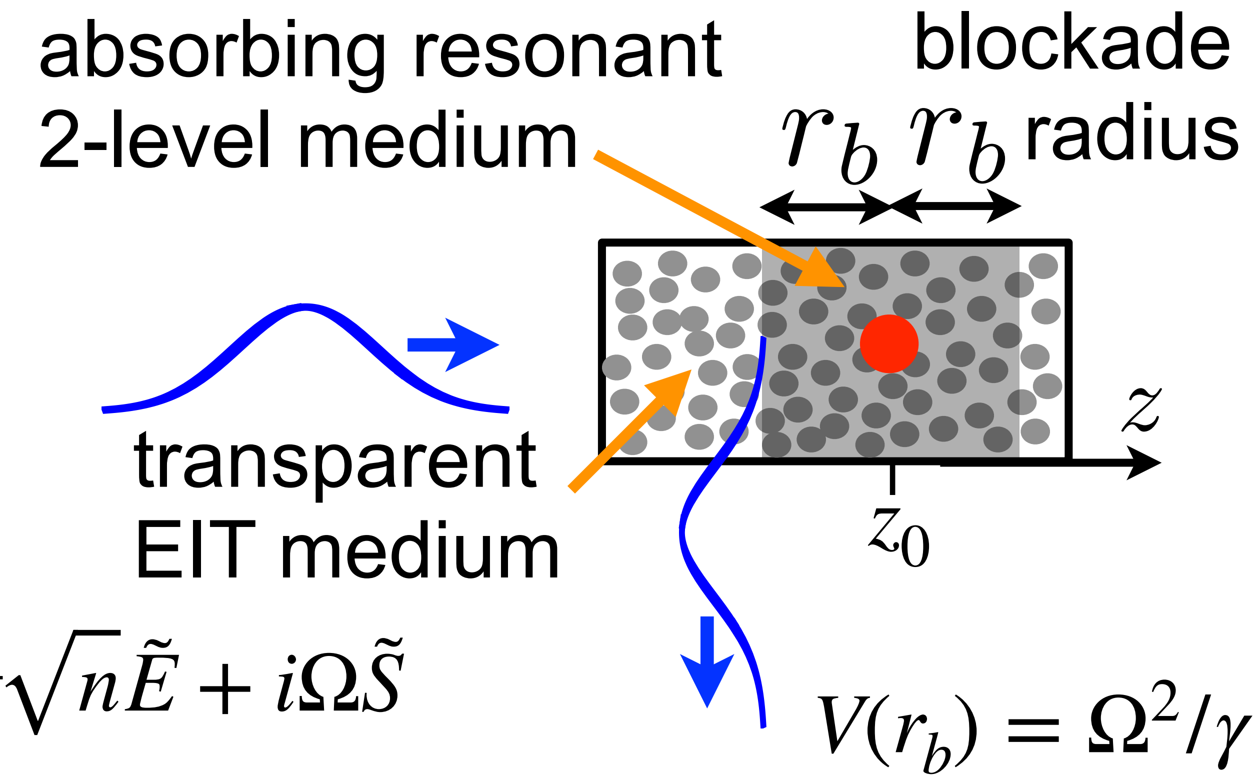




Photon interacting with stationary excitation

$|r\rangle, |r'\rangle$ = Rydberg states





$$(-i\omega + c\partial_z)\tilde{E} = ig\sqrt{n}\tilde{P}$$

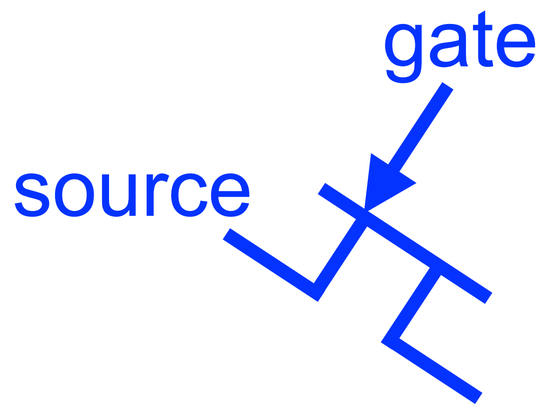
$$-i\omega\tilde{P} = -\gamma\tilde{P} + ig\sqrt{n}\tilde{E} + i\Omega\tilde{S}$$

$$-i\omega\tilde{S} = -iV(z-z_0)\tilde{S} + i\Omega\tilde{P}$$

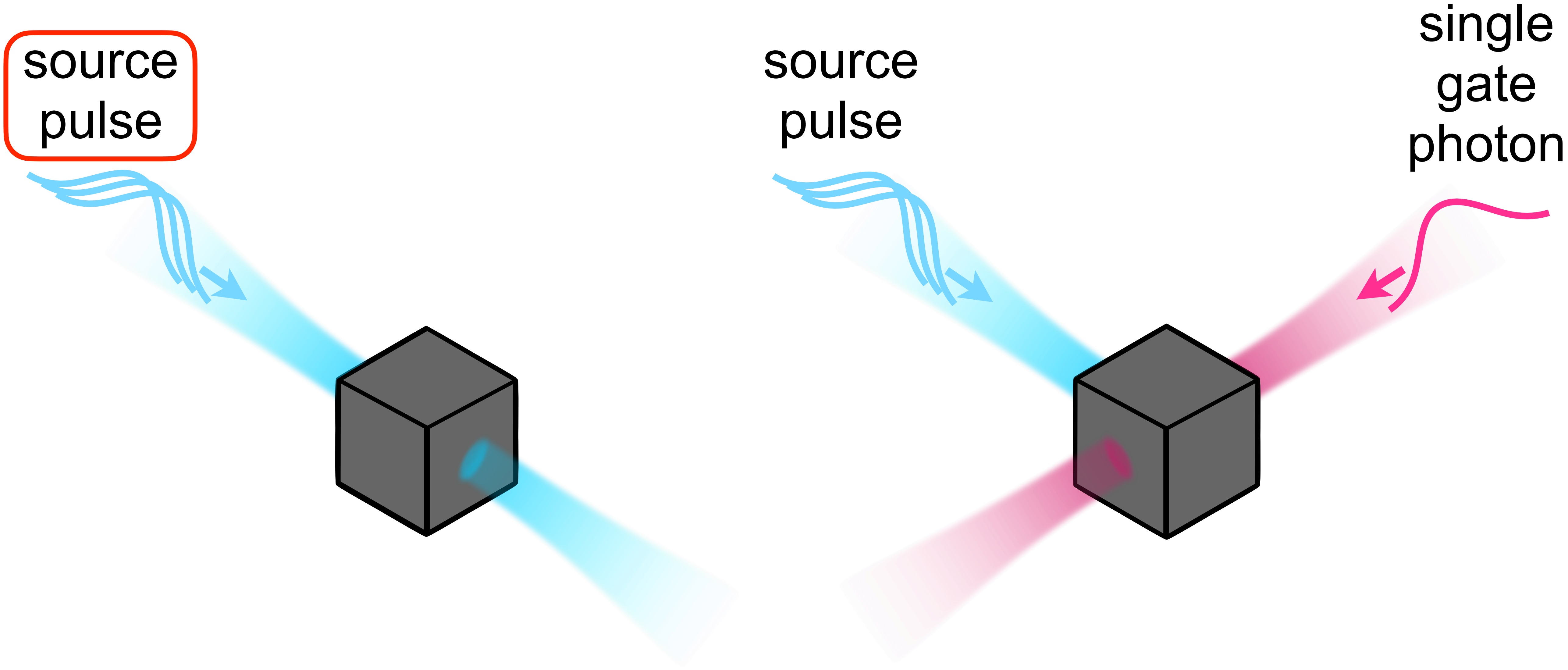
$$\tilde{E}(L,\omega=0) = \tilde{E}(0,\omega=0)\ \exp\left[-i\frac{1}{L}\int_0^L dz\frac{d\gamma V(z-z_0)}{\Omega^2 + i\gamma V(z-z_0)}\right]$$

- $C_6 = 0$: perfect transmission (EIT)
- $C_6 \to \infty$: $\tilde{E}(L,\omega=0) = \tilde{E}(0,\omega=0)\, e^{-d}$ resonant 2-level medium of length L and optical depth d
- general C_6:
  - $V(z-z_0) < \Omega^2/\gamma \Rightarrow$ EIT
  - $V(z-z_0) > \Omega^2/\gamma \Rightarrow$ resonant 2-level medium

- $d_b = 2dr_b/L$ blockaded optical depth
- $d_b \gg 1$ incoming photon scatters

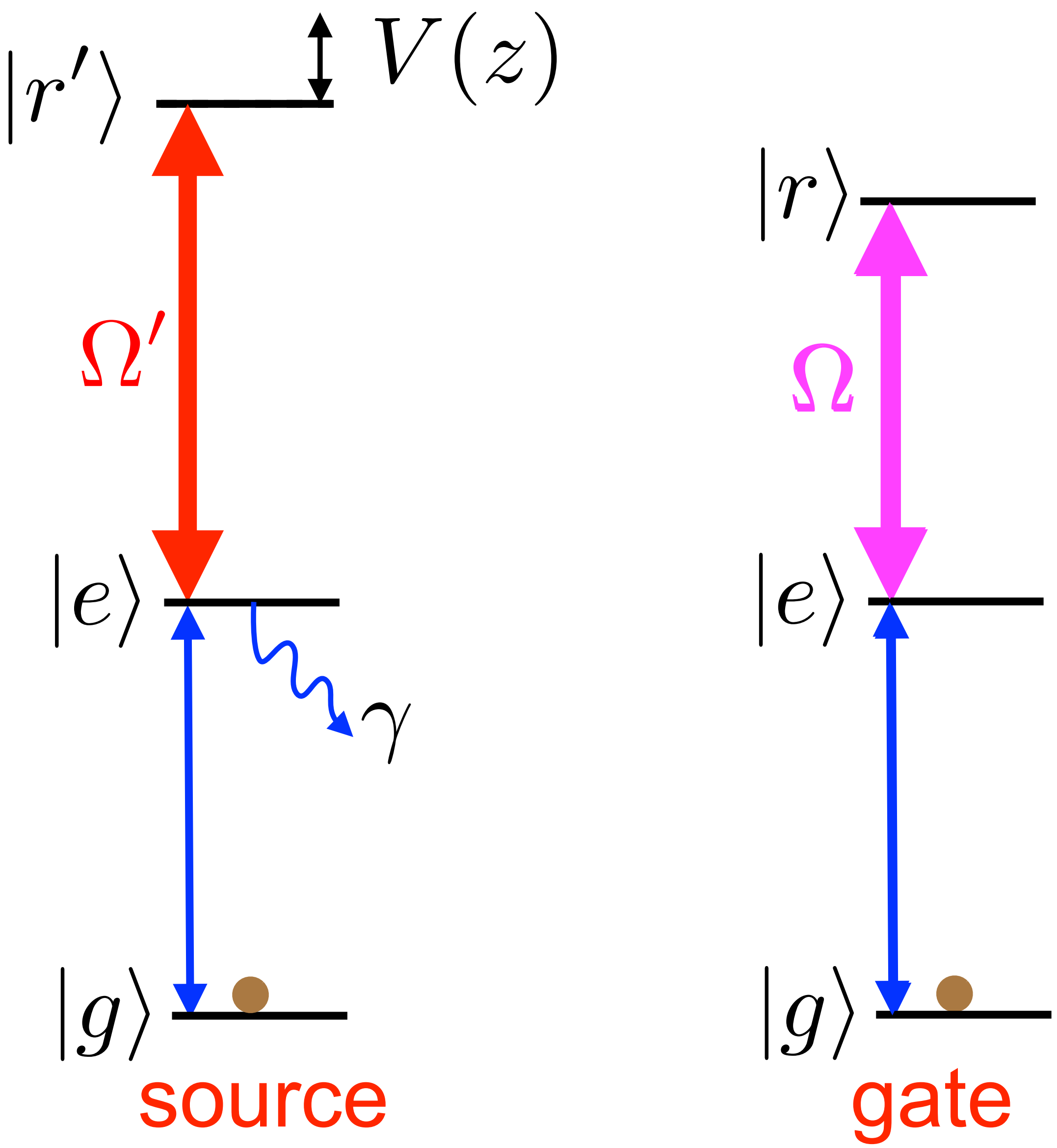


Single-photon switch



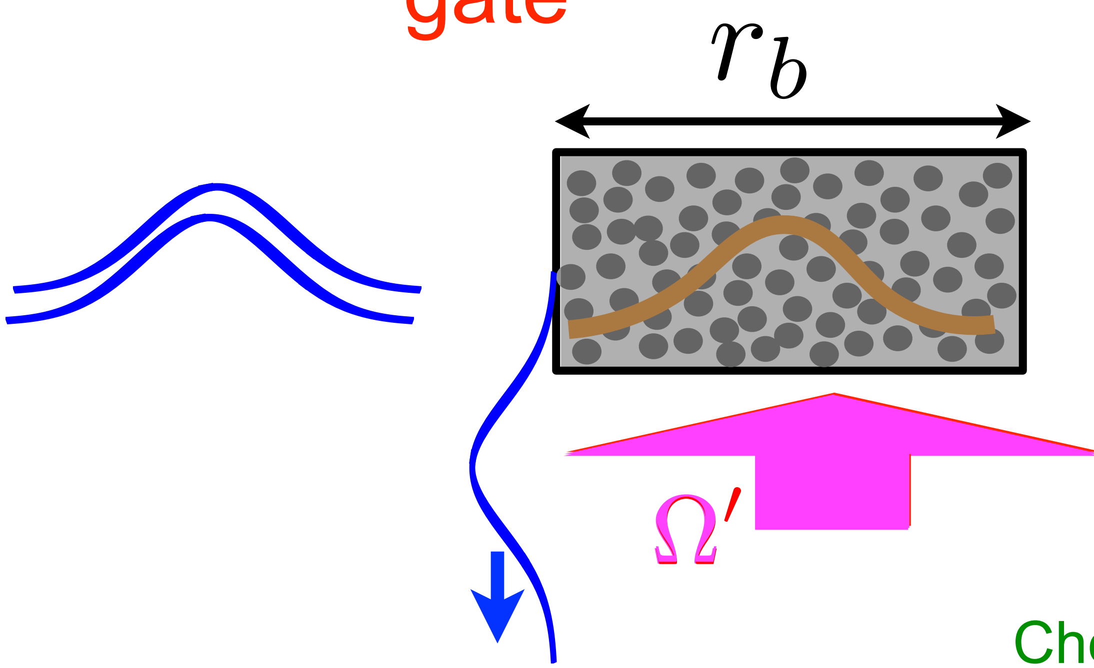
Applications: classical optical information processing, ideal non-demolition photon detector, …

Single-photon switch



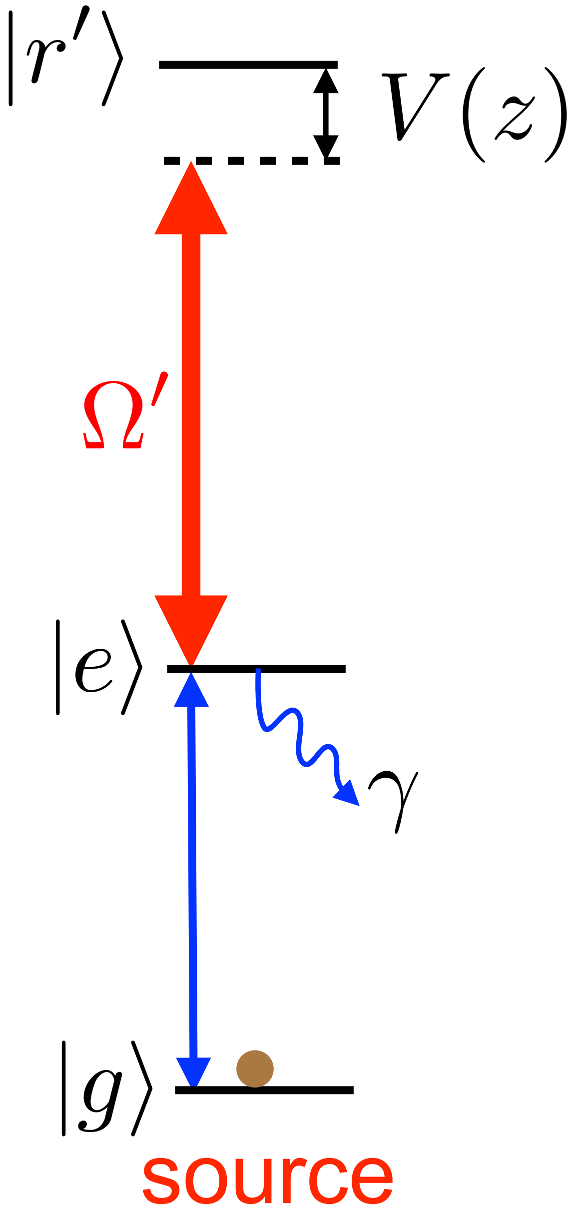
- spin wave $\sum_i C(z_i)|gg\ldots r_i \ldots g\rangle$

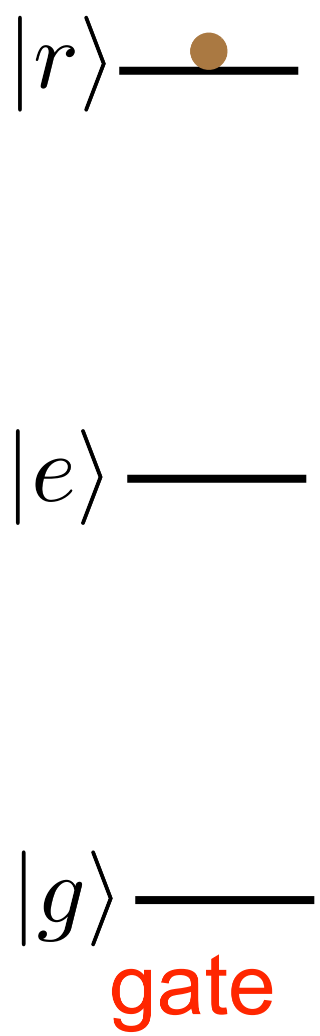
theory: Murray, AVG, Pohl, NJP (2016)
Li, Lesanovsky, PRA (2015)
experiment: Gorniaczyk et al (Hofferberth), Nat Commun (2016) [also Rempe]



Another switch expt:
Chen et al (Vuletic), Science 341, 768 (2013)

Single-photon switch

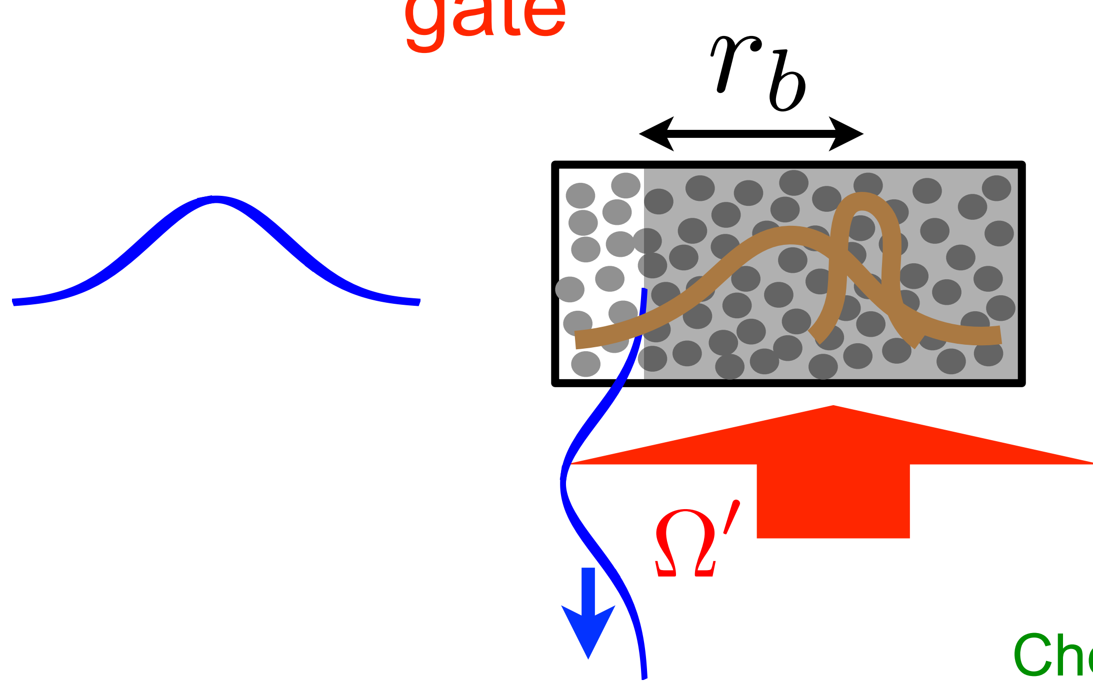




- spin wave $\sum_i C(z_i)|gg\ldots r_i \ldots g\rangle$
- localized excitation

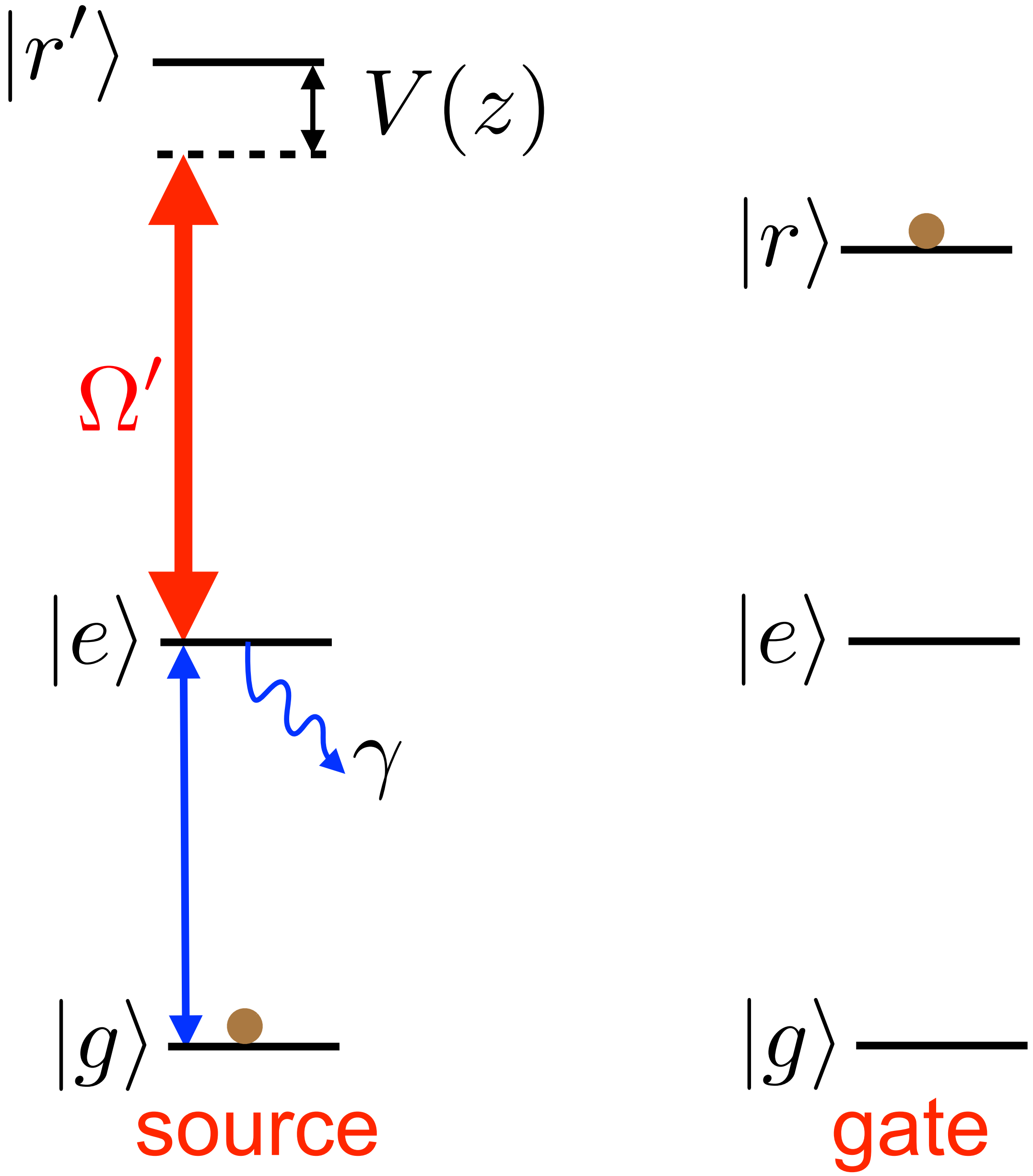
theory: Murray, AVG, Pohl, NJP (2016)
Li, Lesanovsky, PRA (2015)
experiment: Gorniaczyk et al (Hofferberth), Nat Commun (2016) [also Rempe]

- indeed can't retrieve spin wave



Another switch expt:
Chen et al (Vuletic), Science 341, 768 (2013)

Single-photon switch



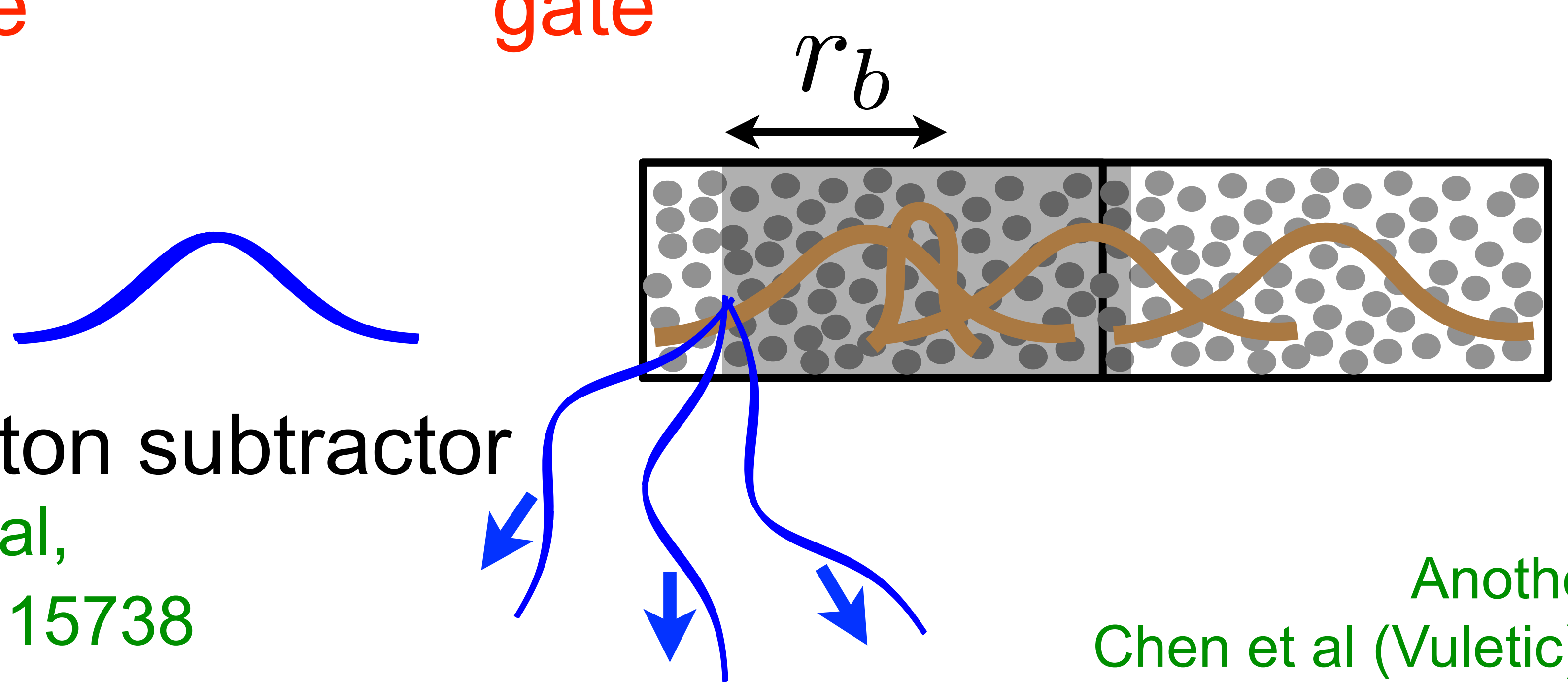
- spin wave $\sum_i C(z_i)|gg\ldots r_i \ldots g\rangle$
- localized excitation

theory: Murray, AVG, Pohl, NJP (2016)
Li, Lesanovsky, PRA (2015)
experiment: Gorniaczyk et al (Hofferberth), Nat Commun (2016) [also Rempe]

\- indeed can't retrieve spin wave

- $\boxed{\text{single-photon subtractor}}$ (theory&expt)

[Murray et al, PRL 120, 113601 (2018)]



- multi-photon subtractor

Stiesdal et al, arXiv:2103.15738

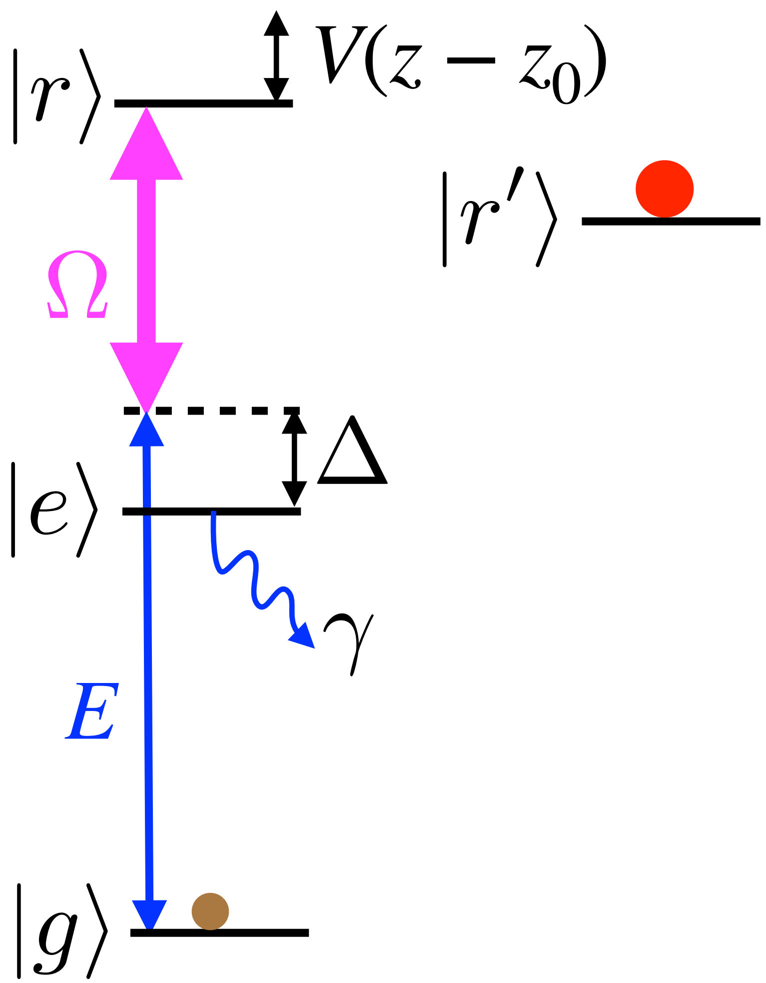
Another switch expt:
Chen et al (Vuletic), Science 341, 768 (2013)

Outline

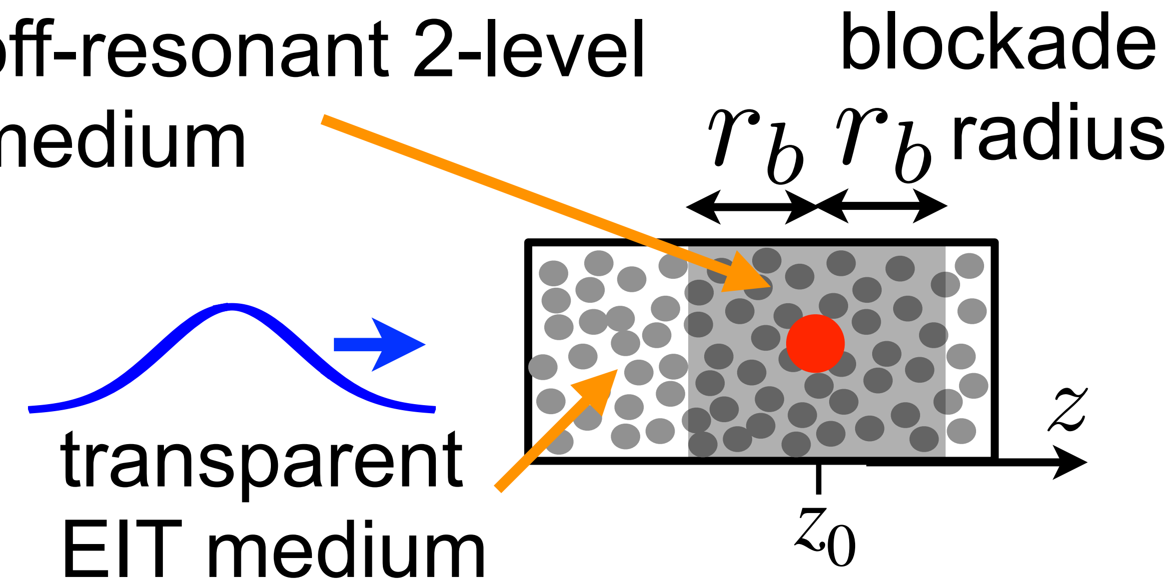
- motivation and basic idea
- E&M field quantization
- propagation of light through atomic ensembles; electromagnetically induced transparency (EIT)
- Rydberg atoms
- basic idea revisited
- photon interacting with stationary excitation
  - on resonance: single-photon switch, subtractor
  - off resonance: two-photon quantum gate
- dynamics of multiple photons
  - on resonance: source of single photons
  - off resonance: two-photon gate, bound states, many-body physics
- more applications

Photon interacting with stationary excitation

$|r\rangle, |r'\rangle$ = Rydberg states



$\Delta \gg \gamma$

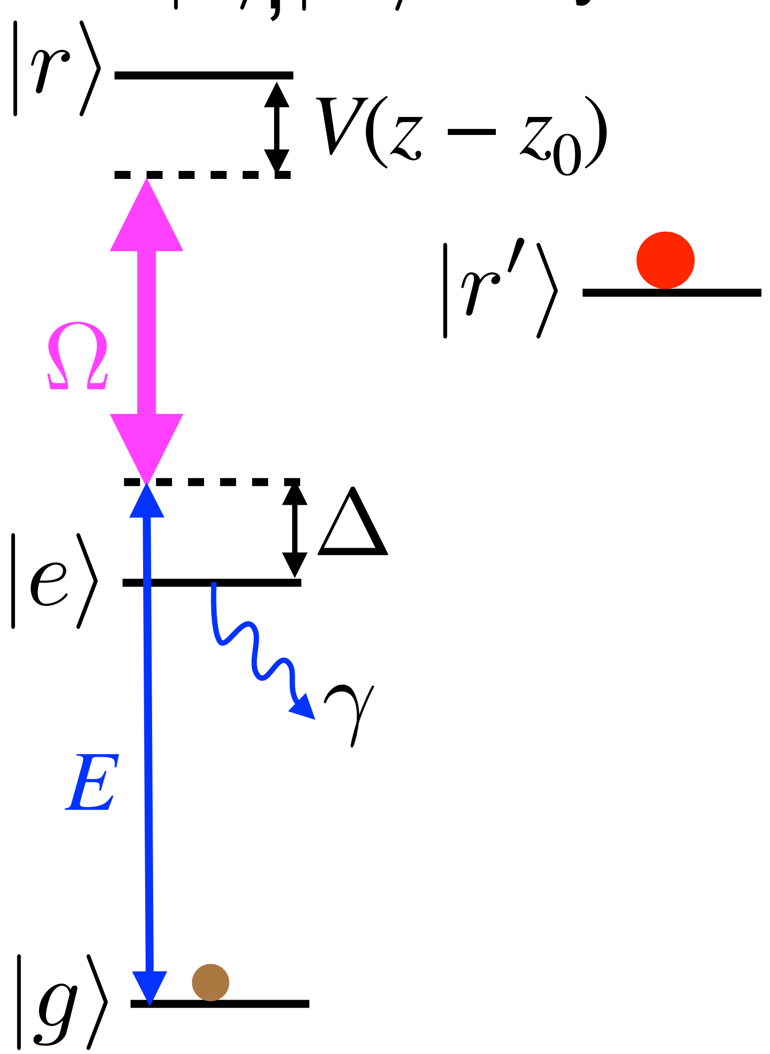


- control atom at z_0 prepared in $|r'\rangle$
- atoms in $|r\rangle$ experience van der Waals potential $V(z-z_0) = \dfrac{C_6}{(z-z_0)^6}$

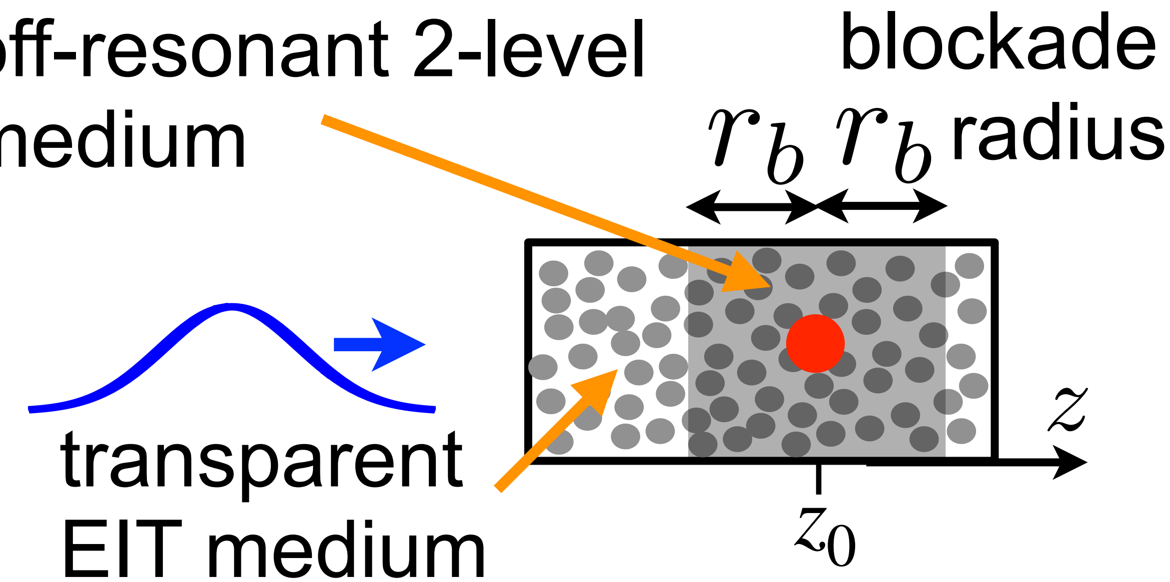
AVG et al, PRL 107, 133602 (2011)

Photon interacting with stationary excitation

$|r\rangle, |r'\rangle$ = Rydberg states

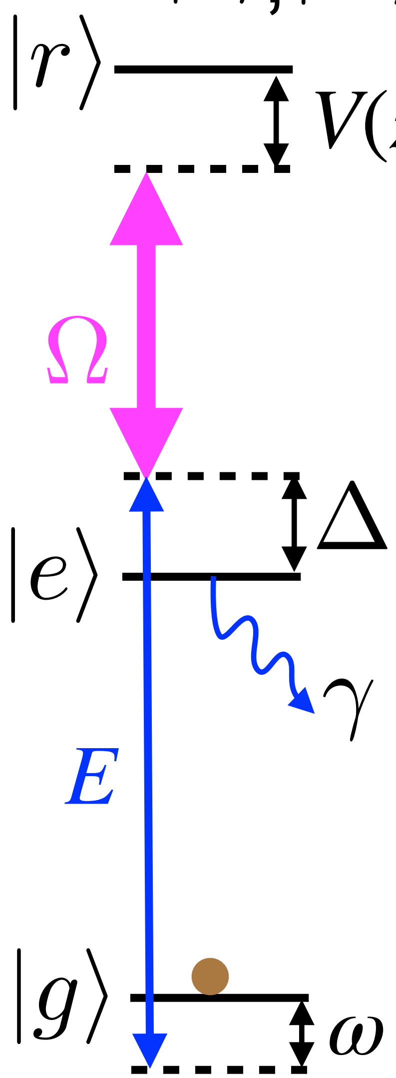


$\Delta \gg \gamma$

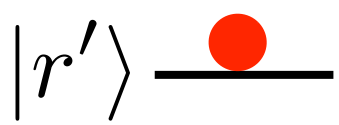


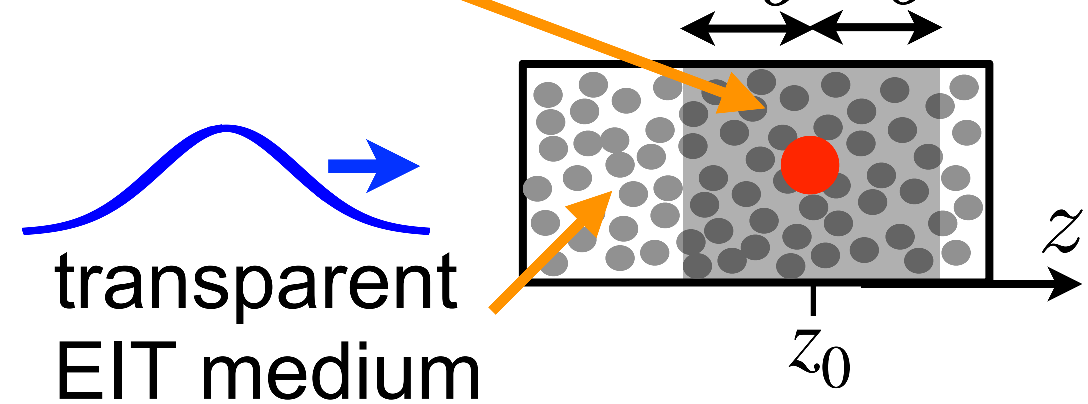
Photon interacting with stationary excitation

$|r\rangle, |r'\rangle$ = Rydberg states



$\Delta \gg \gamma$





$$(-i\omega + c\partial_z)\tilde{E} = ig\sqrt{n}\tilde{P}$$

$$-i\omega\tilde{P} = i\Delta\tilde{P} + ig\sqrt{n}\tilde{E} + i\Omega\tilde{S}$$

$$-i\omega\tilde{S} = -iV(z-z_0)\tilde{S} + i\Omega\tilde{P}$$

$$V(r_b) = \Omega^2/\Delta$$

$$\tilde{E}(L,\omega=0) = \tilde{E}(0,\omega=0)\ \exp\left[-i\frac{1}{L}\int_0^L dz\frac{d\gamma V(z-z_0)}{\Omega^2+\Delta V(z-z_0)}\right]$$

- $C_6 = 0$: no phase picked up (EIT)
- $C_6 \to \infty$: $\tilde{E}(L,\omega=0) = \tilde{E}(0,\omega=0)\,e^{-id\gamma/\Delta}$

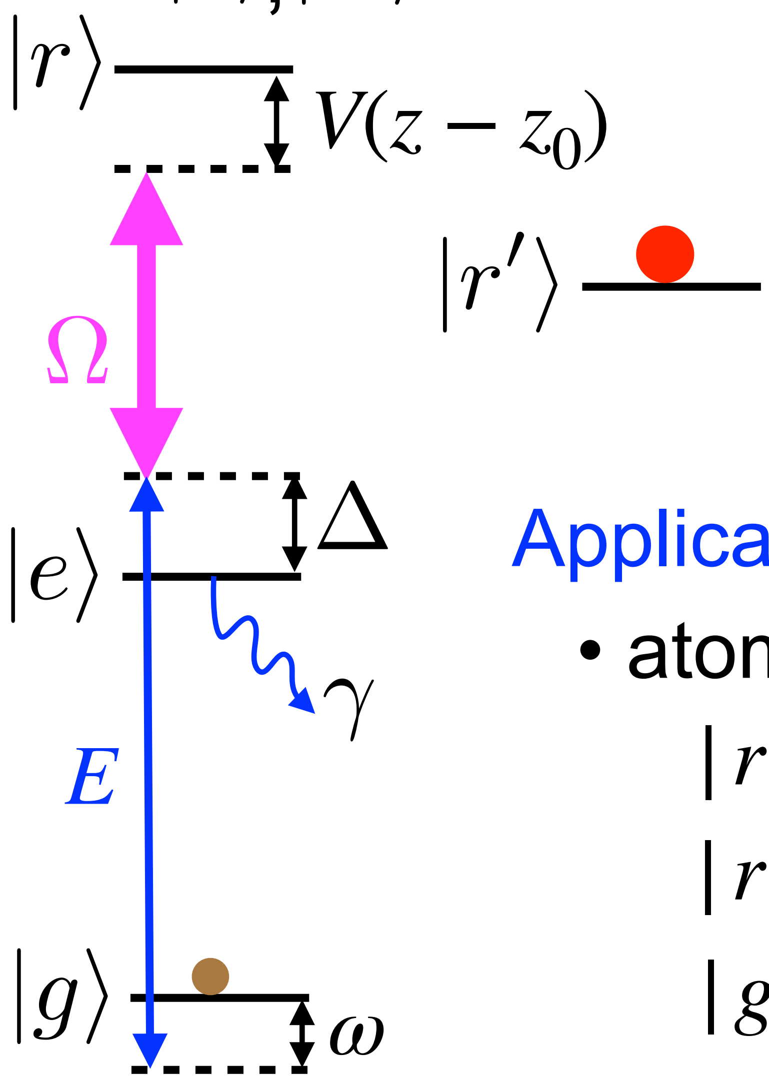
off-resonant 2-level medium of length L and optical depth d

- general C_6:
  - $V(z-z_0) < \Omega^2/\Delta \Rightarrow$ EIT
  - $V(z-z_0) > \Omega^2/\Delta \Rightarrow$ off-resonant 2-level medium

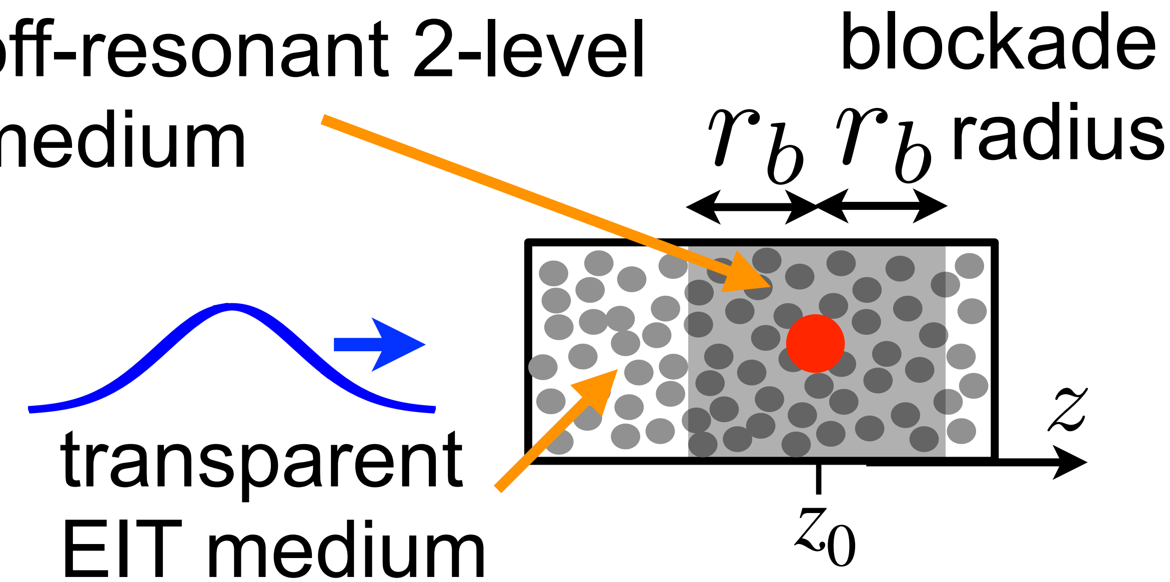
- $d_b = 2dr_b/L$ blockaded optical depth
- $|r'\rangle$ imprints phase $d_b\gamma/\Delta$ on photon

Photon interacting with stationary excitation

$|r\rangle, |r'\rangle$ = Rydberg states



$\Delta \gg \gamma$



$V(r_b) = \Omega^2/\Delta$

Applications:

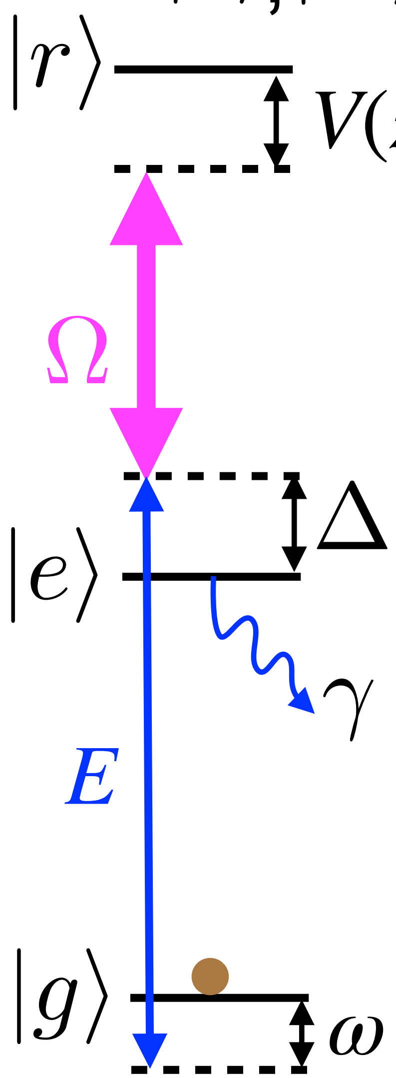
- atom-photon gate

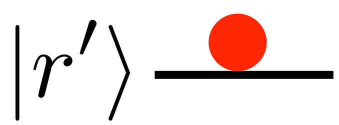
$$|r'\rangle|1\rangle \to -|r'\rangle|1\rangle$$
$$|r'\rangle|0\rangle \to |r'\rangle|0\rangle$$
$$|g\rangle|1\rangle \to |g\rangle|1\rangle$$
$$|g\rangle|0\rangle \to |g\rangle|0\rangle$$

- $d_b = 2dr_b/L$ blockaded optical depth
- $|r'\rangle$ imprints phase $d_b\gamma/\Delta$ on photon

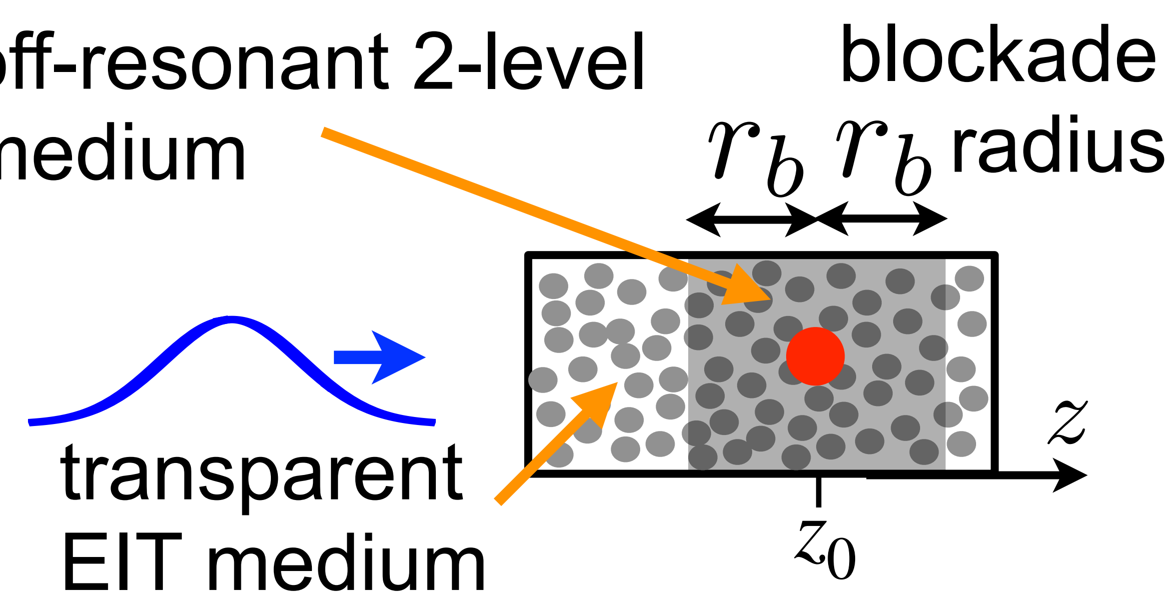
Photon interacting with stationary excitation

$|r\rangle, |r'\rangle$ = Rydberg states





$\Delta \gg \gamma$



Applications:

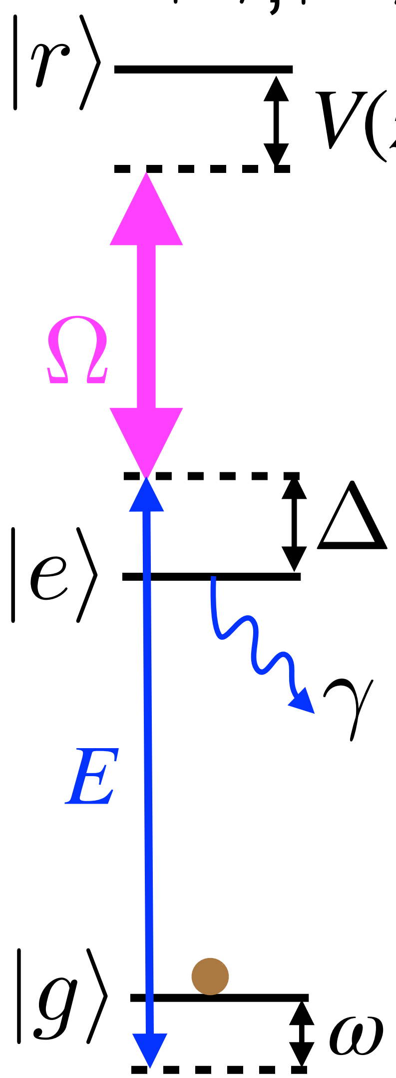
- atom-photon gate
- photon-photon gate using Duan, Kimble, PRL (2004)

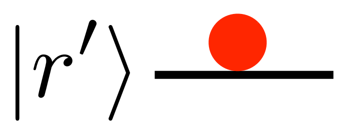
$V(r_b) = \Omega^2/\Delta$

- $d_b = 2dr_b/L$ blockaded optical depth
- $|r'\rangle$ imprints phase $d_b\gamma/\Delta$ on photon

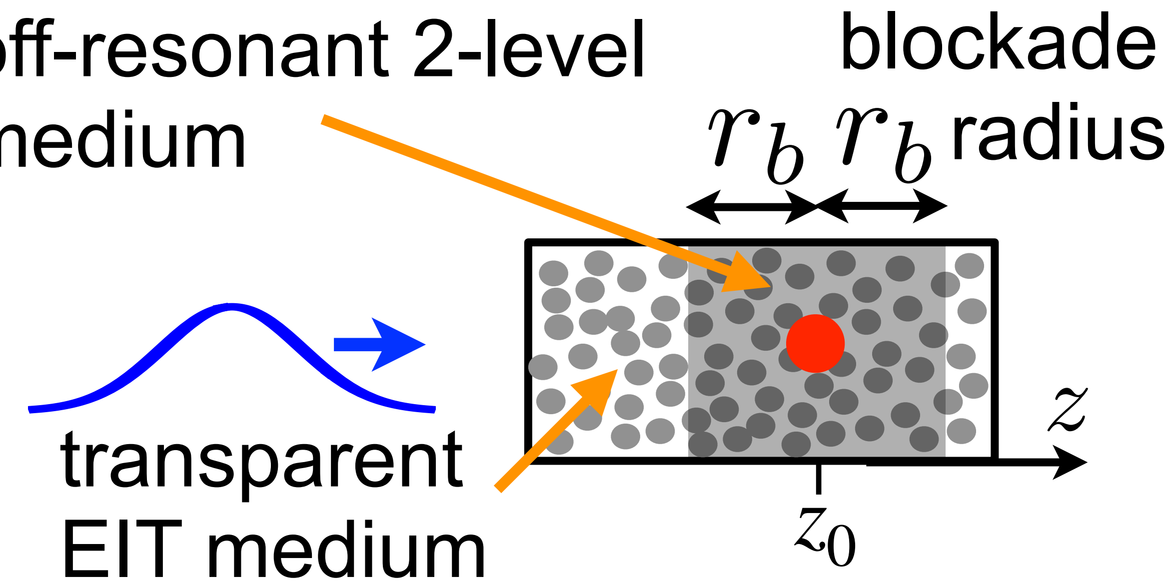
Photon interacting with stationary excitation

$|r\rangle, |r'\rangle$ = Rydberg states





$\Delta \gg \gamma$



$V(r_b) = \Omega^2/\Delta$

Applications:

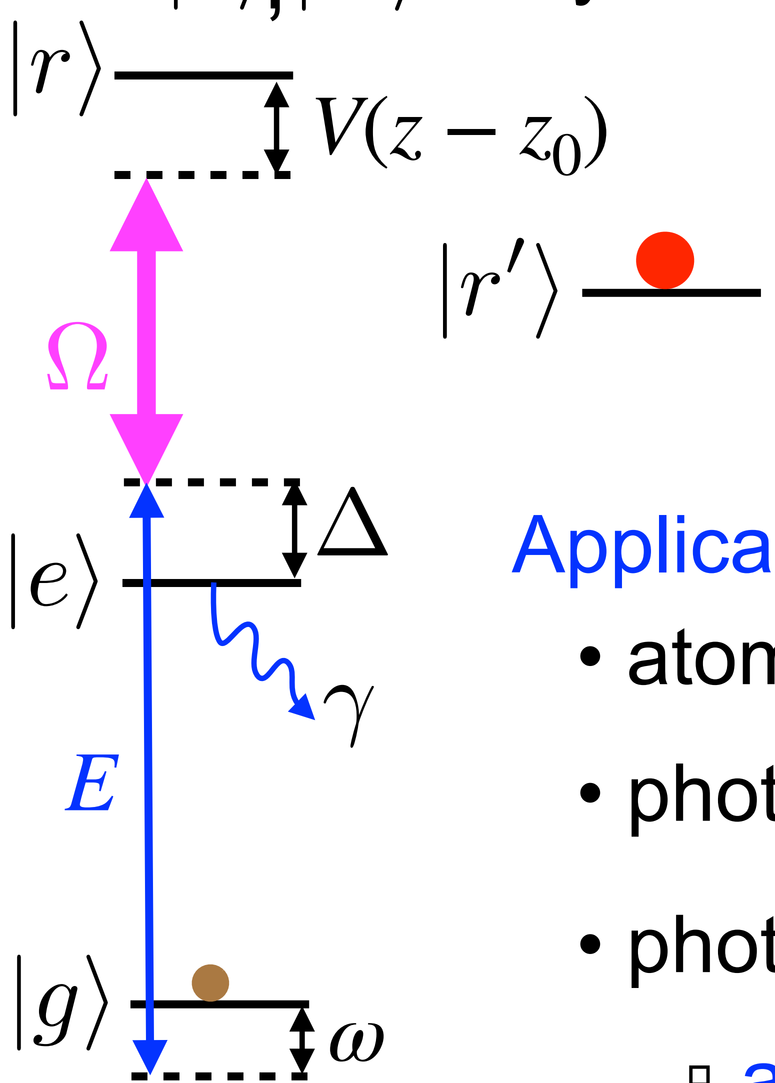
- atom-photon gate
- photon-photon gate using Duan, Kimble, PRL (2004)
- photon-photon gate using storage and retrieval:
  - store control photon as $|r'\rangle$ spinwave
  - run target photon through as above
  - retrieve control photon

- $d_b = 2dr_b/L$ blockaded optical depth
- $|r'\rangle$ imprints phase $d_b\gamma/\Delta$ on photon

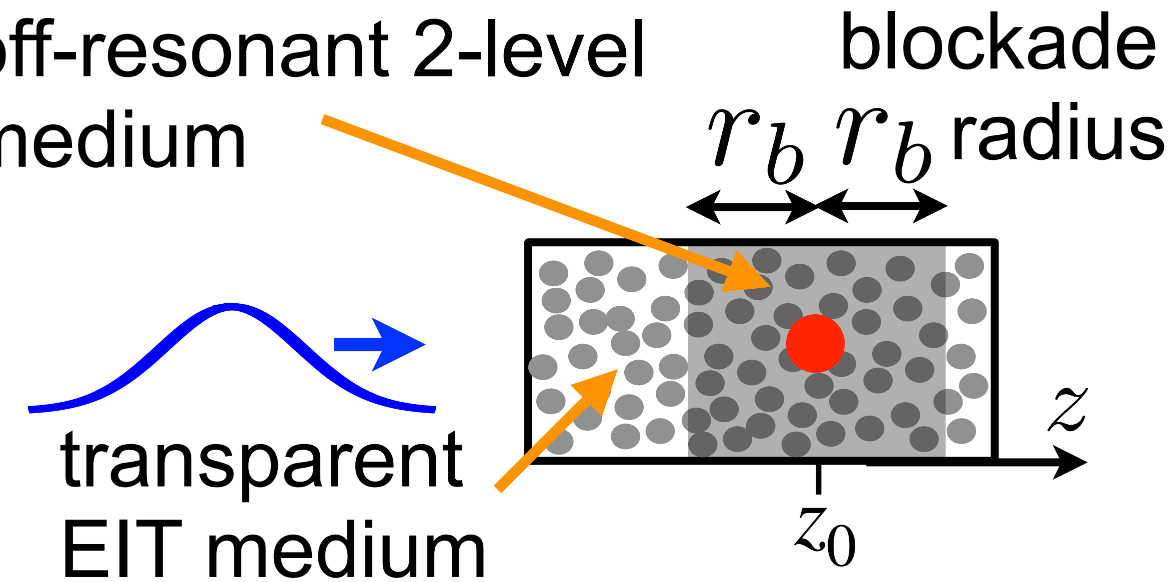
Experimental demonstration:
Tiarks et al., Sci. Adv. 2, e1600036 (2016)
Tiarks et al., Nat Phys 15, 124 (2019)
Similar gate: Thompson et al, Nature 542, 206 (2017)
See also: Busche et al, Nature Phys 13, 655 (2017)

Photon interacting with stationary excitation

$|r\rangle, |r'\rangle$ = Rydberg states



$\Delta \gg \gamma$



$V(r_b) = \Omega^2/\Delta$

Applications:

- atom-photon gate
- photon-photon gate using Duan, Kimble, PRL (2004)
- photon-photon gate using storage and retrieval:
  - applications: quantum computing, networking, …

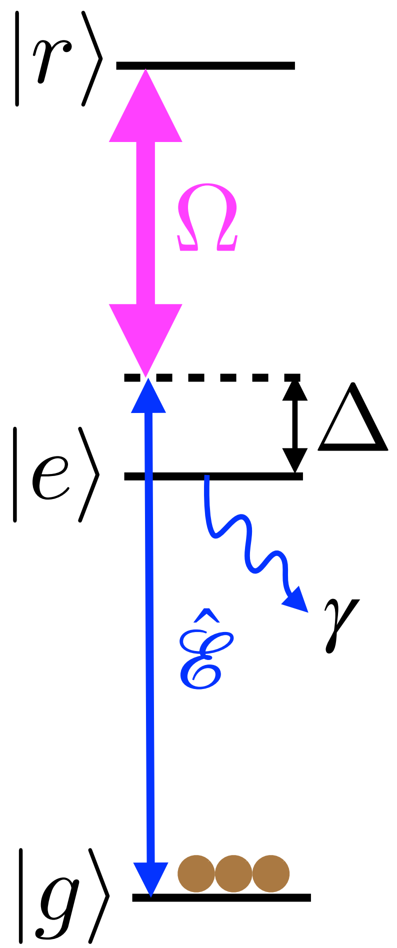
Experimental demonstration:
Tiarks et al., Sci. Adv. 2, e1600036 (2016)
Tiarks et al., Nat Phys 15, 124 (2019)
Similar gate: Thompson et al, Nature 542, 206 (2017)
See also: Busche et al, Nature Phys 13, 655 (2017)

- $d_b = 2dr_b/L$ blockaded optical depth
- $|r'\rangle$ imprints phase $d_b\gamma/\Delta$ on photon

Outline

- motivation and basic idea
- E&M field quantization
- propagation of light through atomic ensembles; electromagnetically induced transparency (EIT)
- Rydberg atoms
- basic idea revisited
- photon interacting with stationary excitation
  - on resonance: single-photon switch, subtractor
  - off resonance: two-photon quantum gate
- dynamics of multiple photons
  - on resonance: source of single photons
  - off resonance: two-photon gate, bound states, many-body physics
- more applications

Dynamics of multiple polaritons



$$\frac{\hat{H}}{\hbar} = -ic\int dz\hat{\mathscr{E}}^\dagger(z)\frac{\partial}{\partial z}\hat{\mathscr{E}}(z)$$

$$+\int dz\left[-\Delta\hat{P}^\dagger(z)\hat{P}(z) - \left(\Omega(t)\hat{S}^\dagger(z)\hat{P}(z) + g\sqrt{n}\hat{P}^\dagger(z)\hat{\mathscr{E}}(z) + h.c.\right)\right]$$

$$+\frac{1}{2}\int dzdz'\hat{S}^\dagger(z)\hat{S}^\dagger(z')V(z-z')\hat{S}(z')\hat{S}(z)$$

- e.g. 2 incoming photons:

$$|\psi(t)\rangle = \frac{1}{2}\int dzdz'EE(z,z',t)\hat{\mathscr{E}}^\dagger(z)\hat{\mathscr{E}}^\dagger(z')|0\rangle + \int dzdz'EP(z,z',t)\hat{\mathscr{E}}^\dagger(z)\hat{P}^\dagger(z')|0\rangle$$

$$+\frac{1}{2}\int dzdz'PP(z,z',t)\hat{P}^\dagger(z)\hat{P}^\dagger(z')|0\rangle + \int dzdz'ES(z,z',t)\hat{\mathscr{E}}^\dagger(z)\hat{S}^\dagger(z')|0\rangle$$

$$+\frac{1}{2}\int dzdz'SS(z,z',t)\hat{S}^\dagger(z)\hat{S}^\dagger(z')|0\rangle + \int dzdz'PS(z,z',t)\hat{P}^\dagger(z)\hat{S}^\dagger(z')|0\rangle$$

Outline

- motivation and basic idea
- E&M field quantization
- propagation of light through atomic ensembles; electromagnetically induced transparency (EIT)
- Rydberg atoms
- basic idea revisited
- photon interacting with stationary excitation
  - on resonance: single-photon switch, subtractor
  - off resonance: two-photon quantum gate
- dynamics of multiple photons
  - on resonance: source of single photons
  - off resonance: two-photon gate, bound states, many-body physics
- more applications