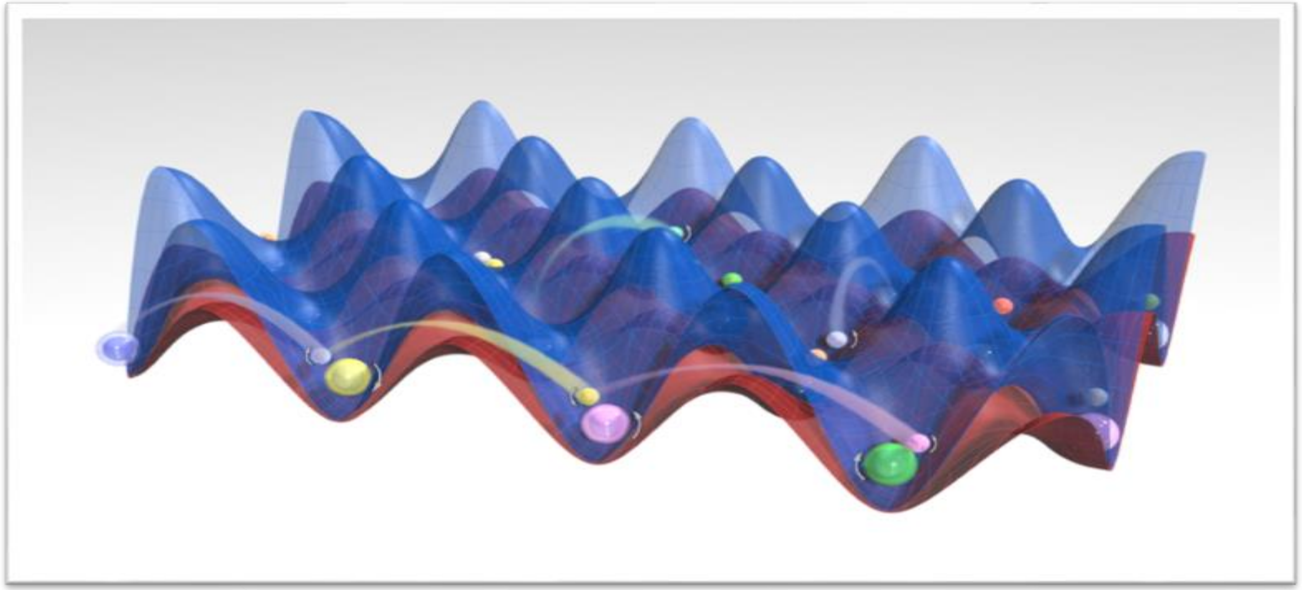
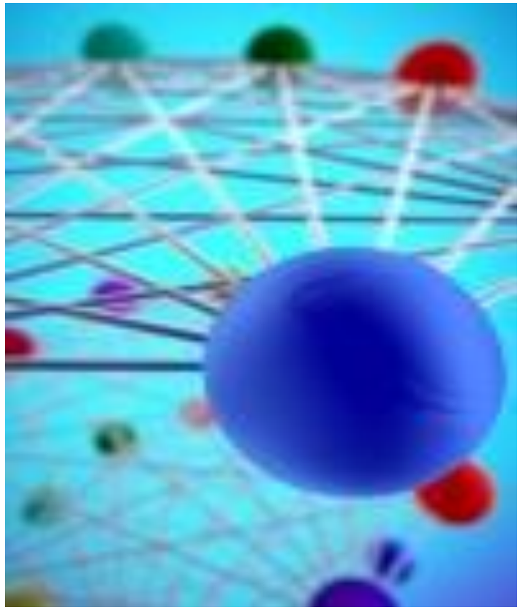
New perspectives on quantum simulation with Alkaline-earth atoms

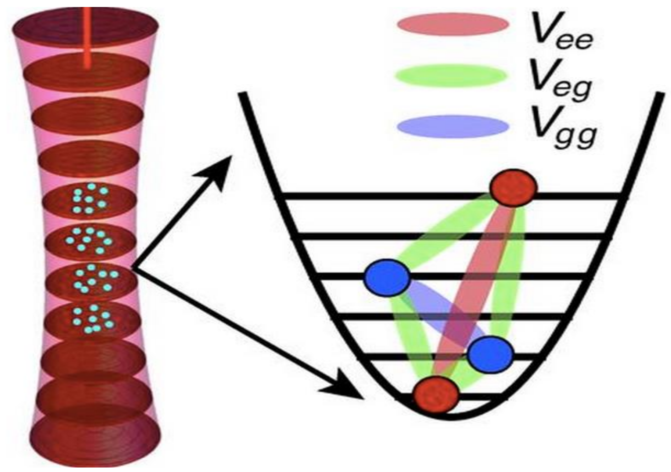
Ana Maria Rey

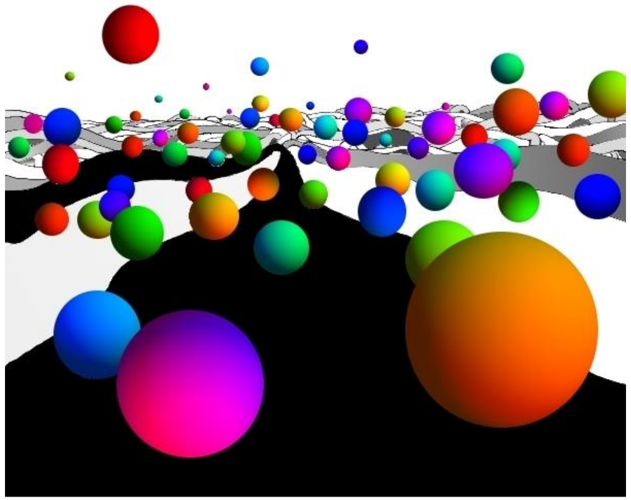


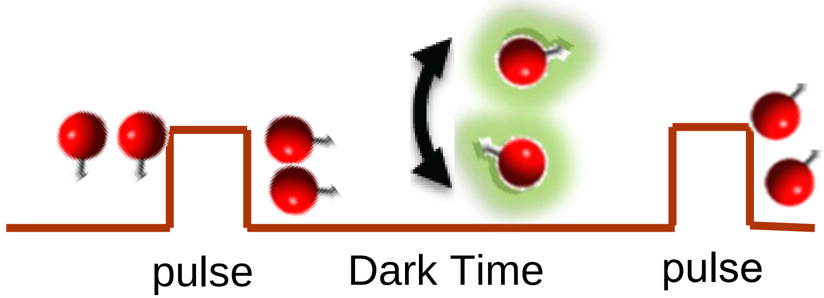


Boulder Summer School, July 19-23 (2021)

Agenda



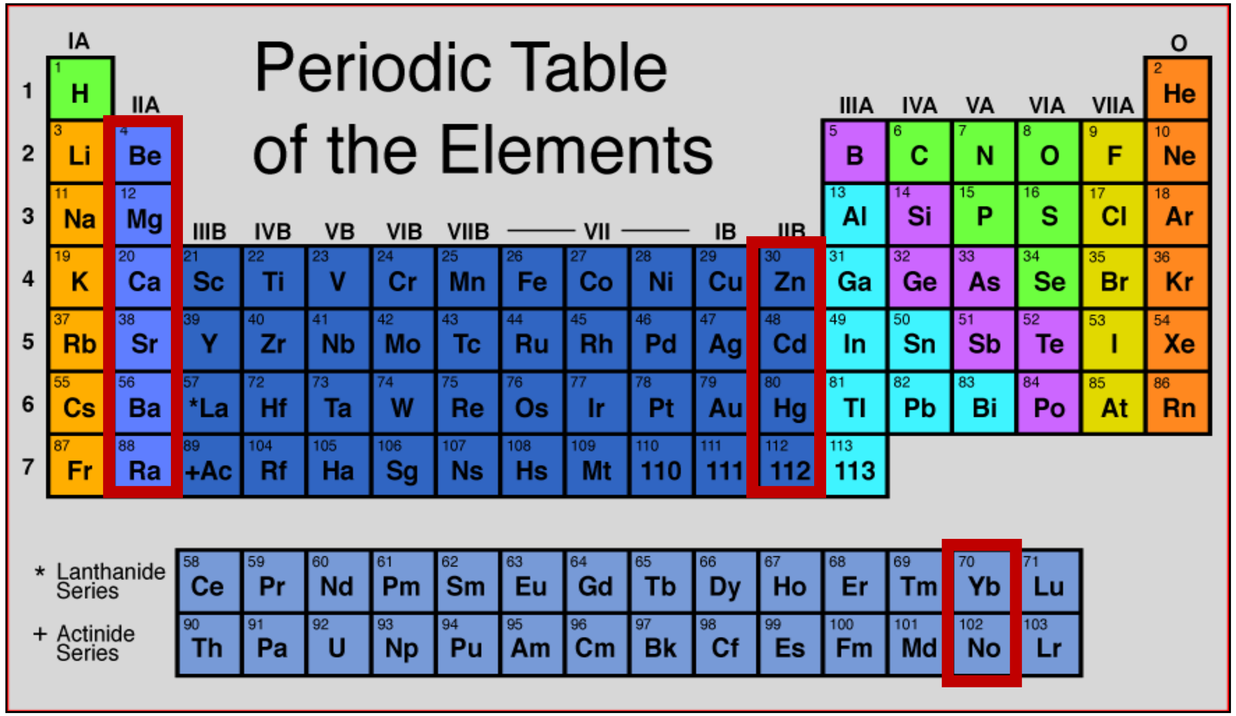


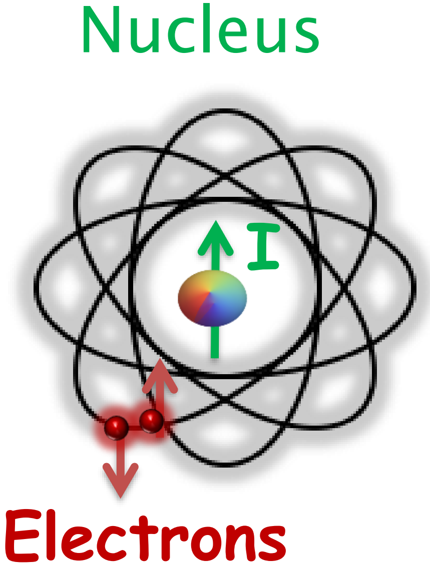


- Brief overview of alkaline earth-atoms and atomic clocks
- Collisions and SU(N) Interactions
- Density shifts in spin polarized gases
- Probing SU(N) orbital magnetism
- Spin-Orbit Coupling

Alkaline Earth (-like) Atoms: AEA

A TALE OF TWIN ELECTRONS





Atom	Nuclear spin
^{171}Yb	$\frac{1}{2}$
^{173}Yb	$\frac{5}{2}$
^{87}Sr	$\frac{9}{2}$

Fermionic isotopes have nuclear spin I > 0.

Why?: Bosonic isotopes have nuclei with even number of protons and neutrons: I=0

Fermionic Isotopes have even number of protons and odd number of neutrons

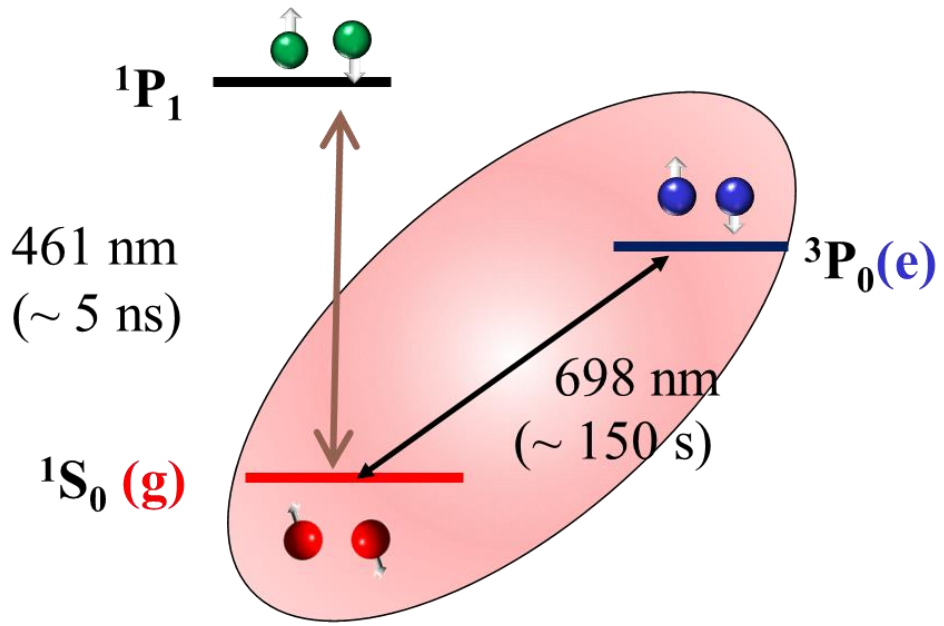
Alkaline earth(-like) atoms

Unique atomic structure

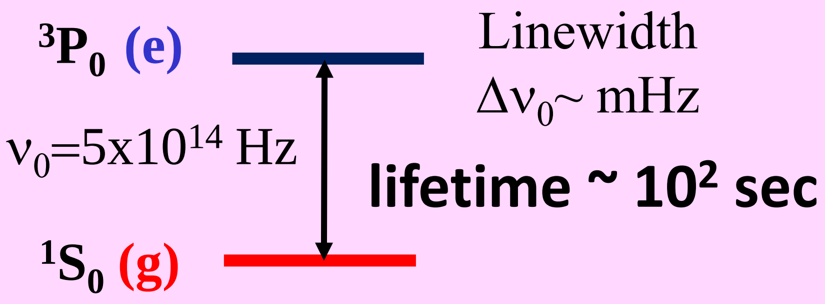
- Long lived metastable $^3P_0$ state: $^3P_0$- $^1S_0$ is a dipole and spin forbidden transition with a **linewidth ~ mHz** . ⇛ **Spectral resolution**

Dipole (J=0→ J=0) and Spin forbidden (S=0→ S=1) transition

Strontium: ^{87} Sr



Metastable states



Quality factor: $Q=\nu_0/\Delta\nu_0>10^{17}$

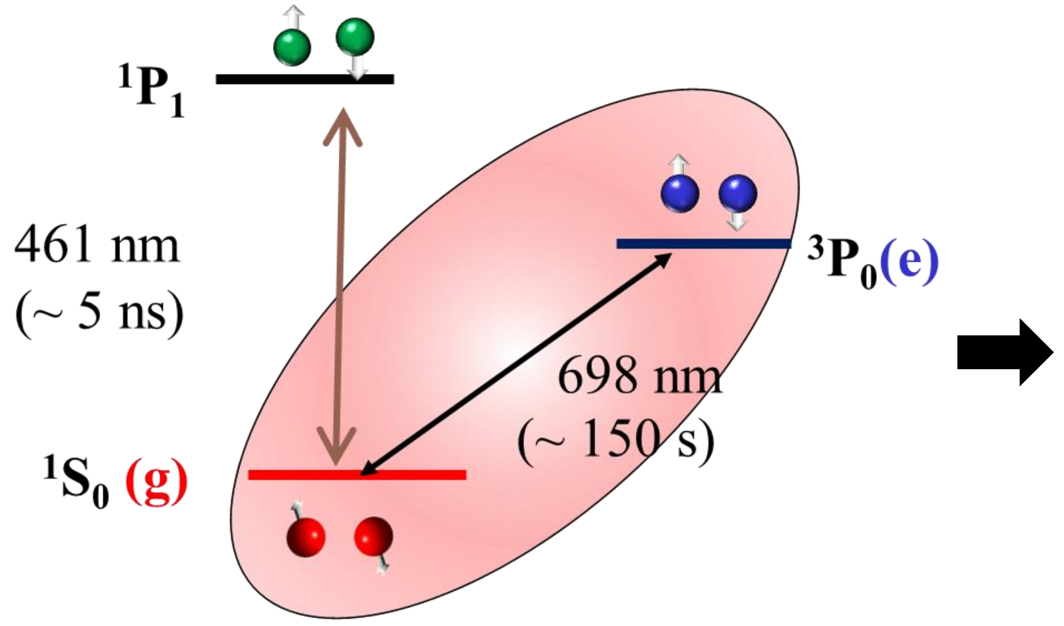
Once set, it swings during the entire age of the universe

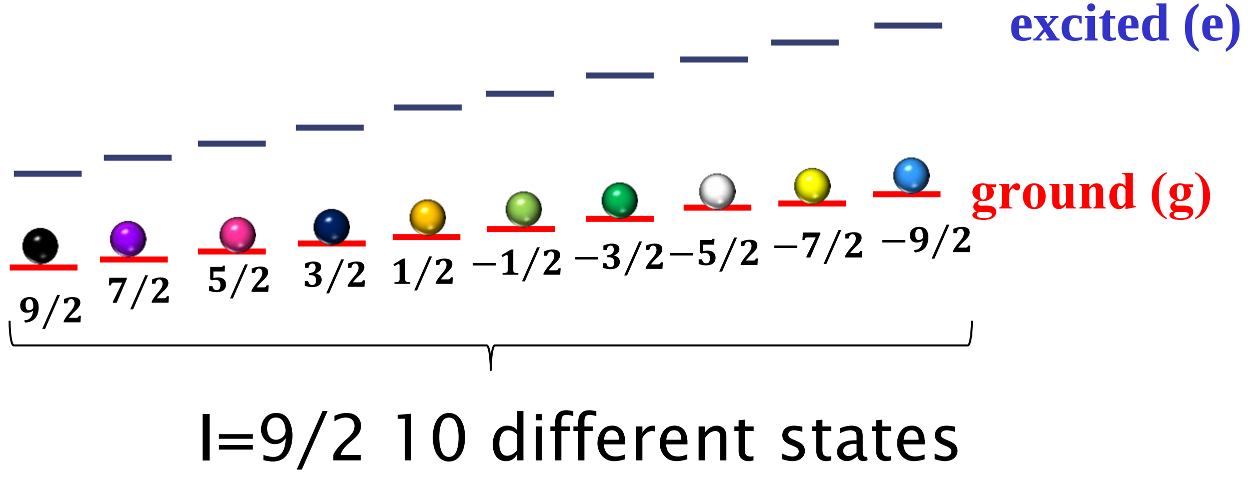
Alkaline earth(-like) atoms

Unique atomic structure

- Long lived metastable ${}^3P_0$ state: ${}^3P_0 - {}^1S_0$ is a dipole and spin forbidden transition with a **linewidth ~ mHz** . ⇛ **Spectral resolution**
- Total electronic angular momentum **J=0** → Only nuclear spin **I**

Strontium: ^{87} Sr

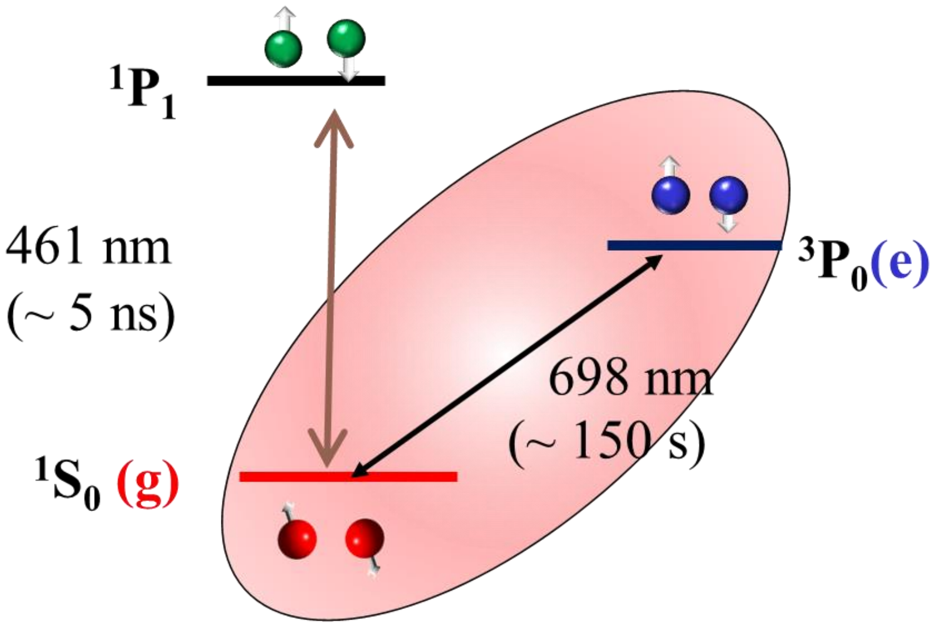


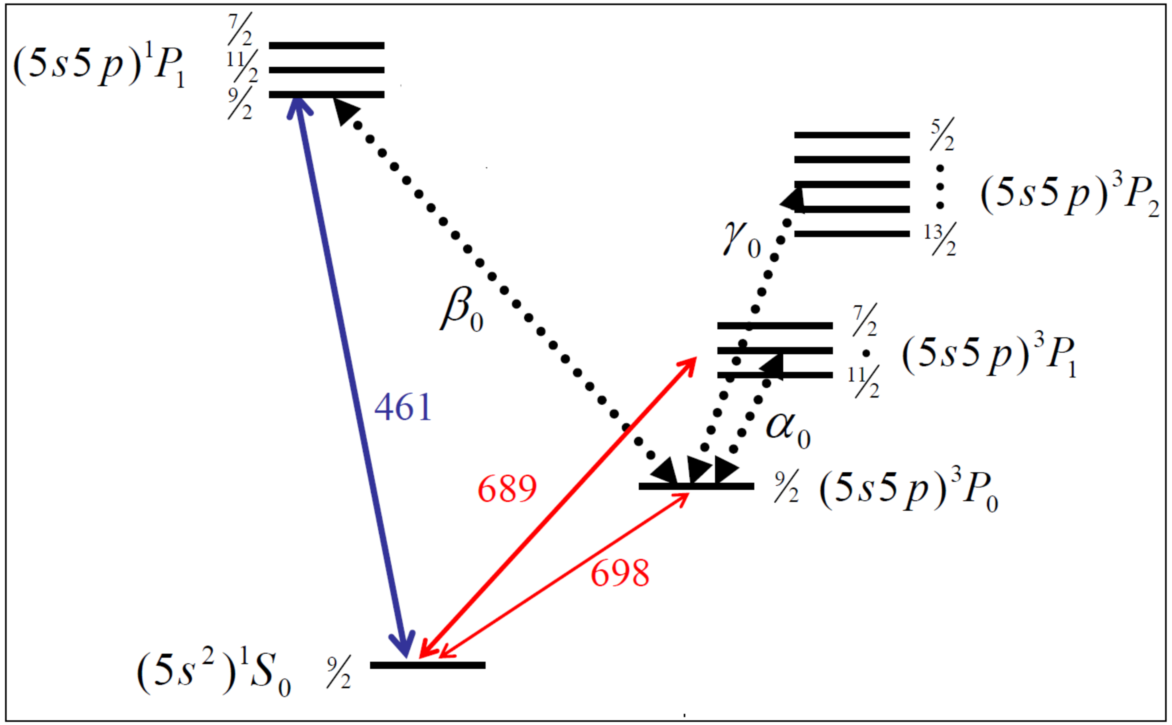


Why alkaline-earth (like) atoms?

(Jun Ye : Presentation)

Strontium: ^{87} Sr





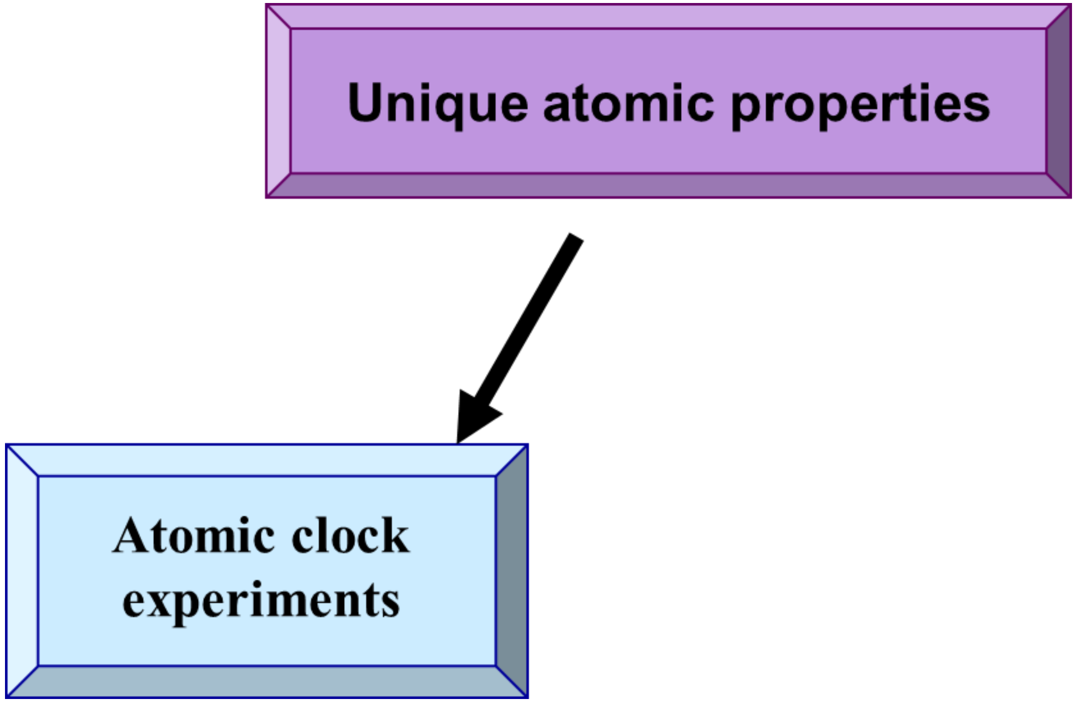
$$^3P_0 = {}^3P_0^{(0)} + a\,{}^3P_1^{(0)} + b\,{}^3P_2^{(0)} + c\,{}^1P_1^{(0)}$$

Why? Because Hyperfine Interactions $H = AI \cdot J$

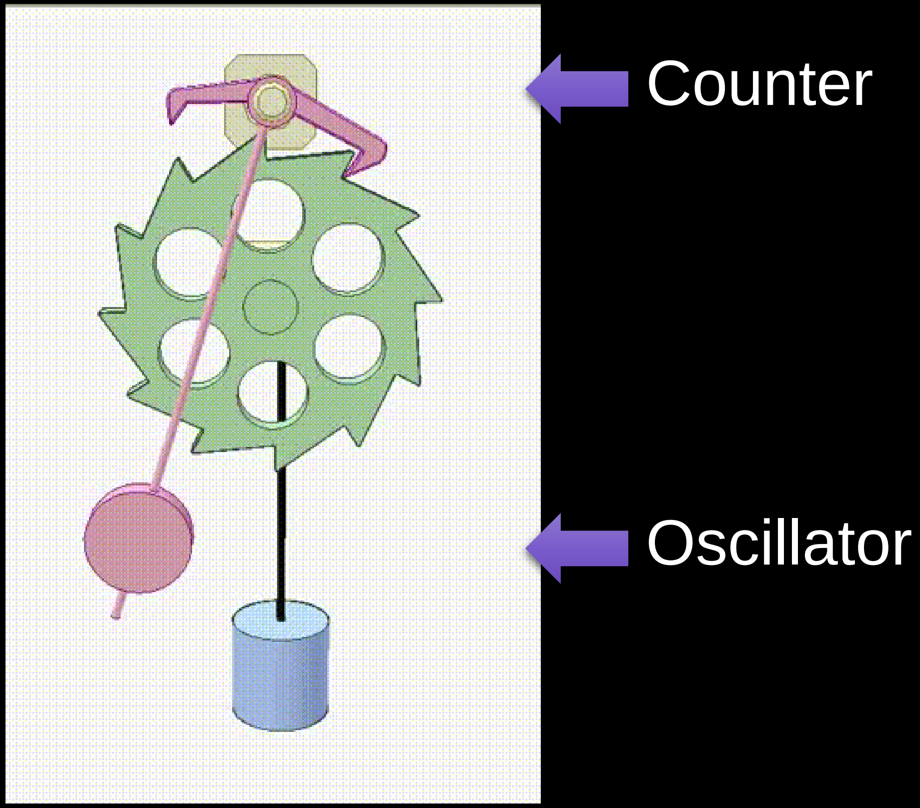
How about Strontium: ^{88} Sr?

Why alkaline-earth (like) atoms?

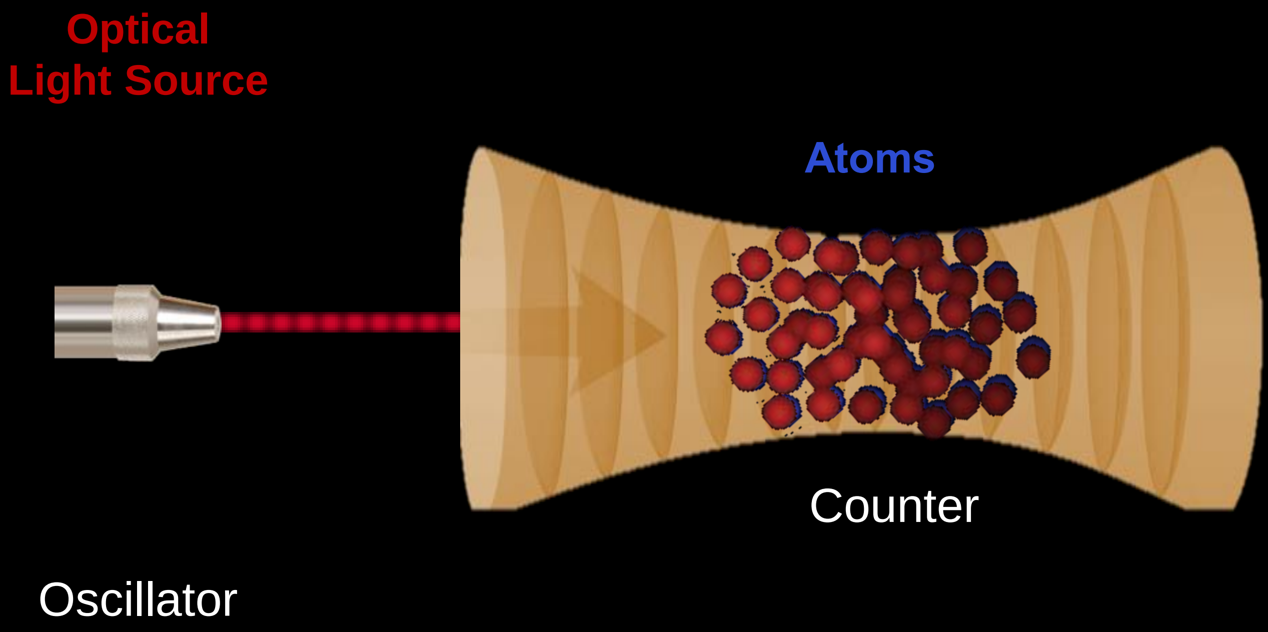
(Jun Ye : Presentation)



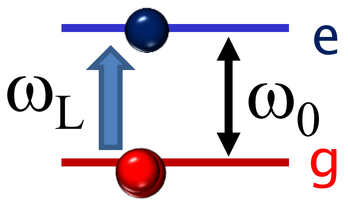
All Clocks



Atomic Clocks



Ramsey Spectroscopy



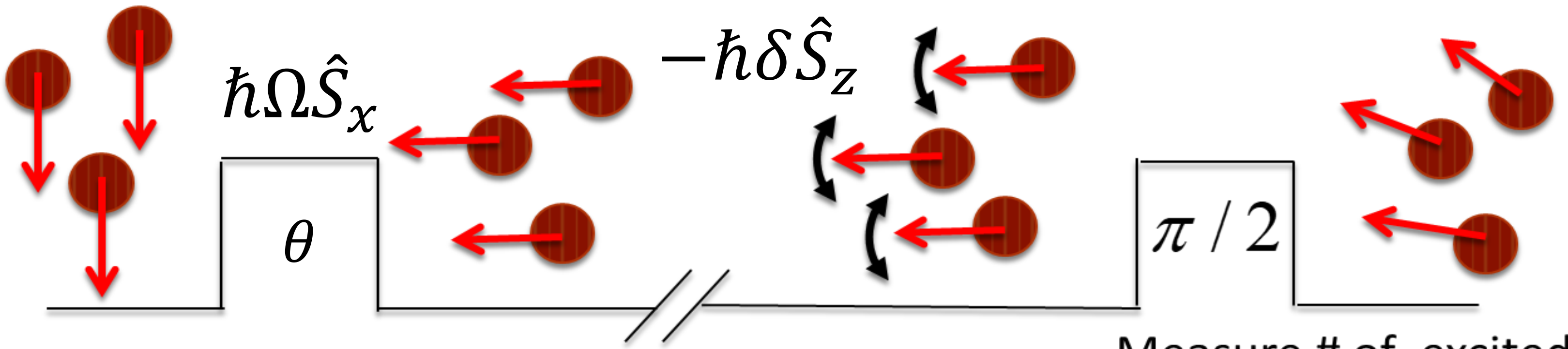
$\delta = (\omega_L - \omega_0)$

Detuning

N. Ramsey. Nobel prize 1989

See handwritten notes

Ramsey Spectroscopy



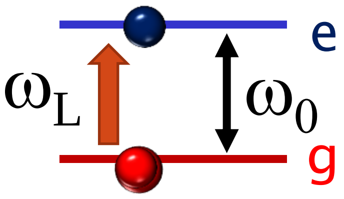
Pulse area $\theta = \Omega t$ Dark time Measure # of excited atoms

N. Ramsey. Nobel prize 1989

$$\langle\hat{\mathbf{S}}\rangle = \tfrac{1}{2}\{0, \sin\theta, -\cos\theta\}$$

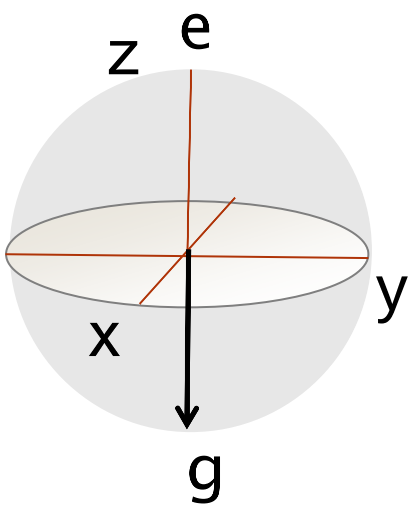
$$\langle \hat{S}^-(\tau)\rangle = -\tfrac{i}{2}\sin\theta\, e^{-i\delta\tau}$$

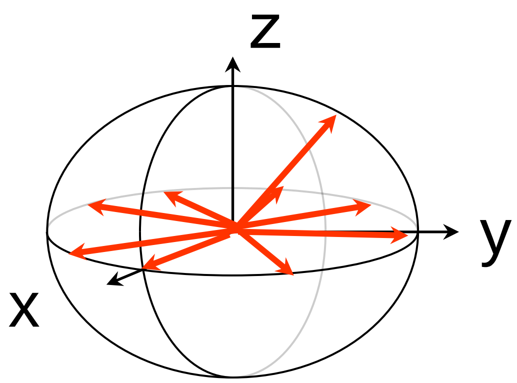
$$\langle \hat{S}^z(\tau)\rangle = \tfrac{1}{2}\sin\theta\cos(\delta\tau)$$

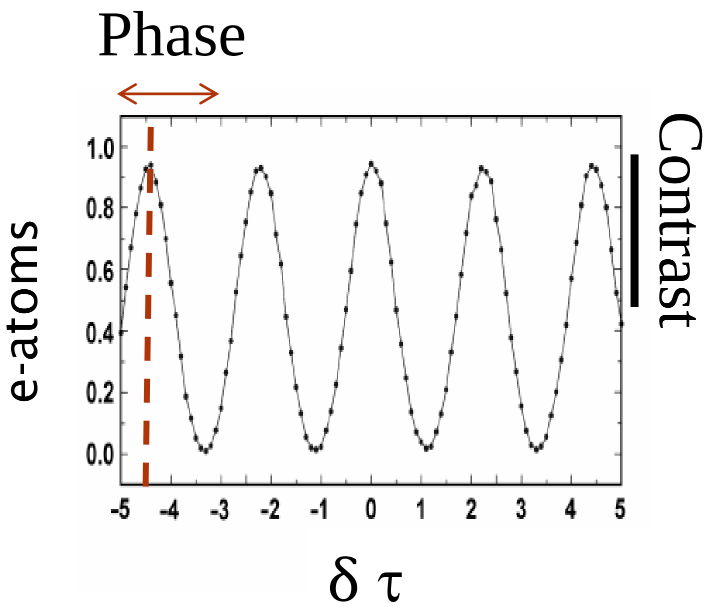


$\delta = (\omega_L - \omega_0)$

Detuning

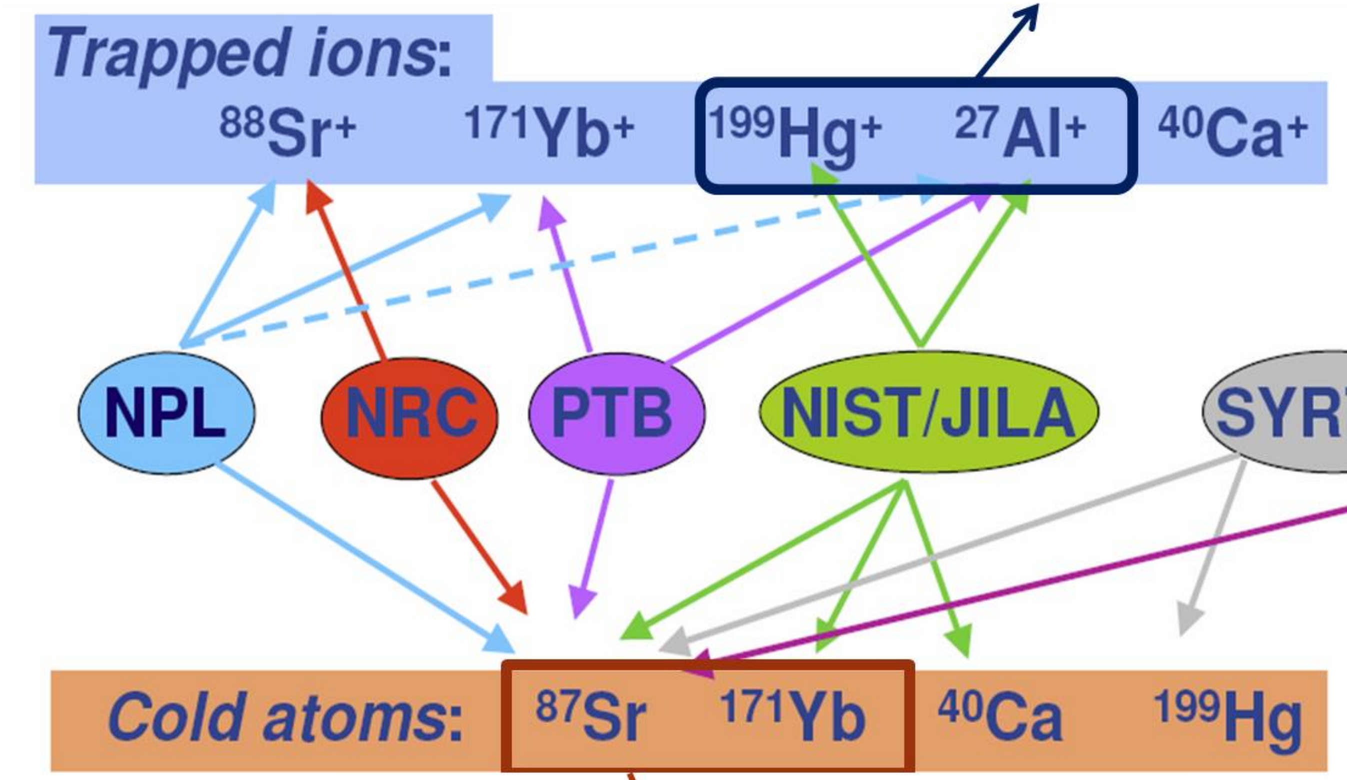






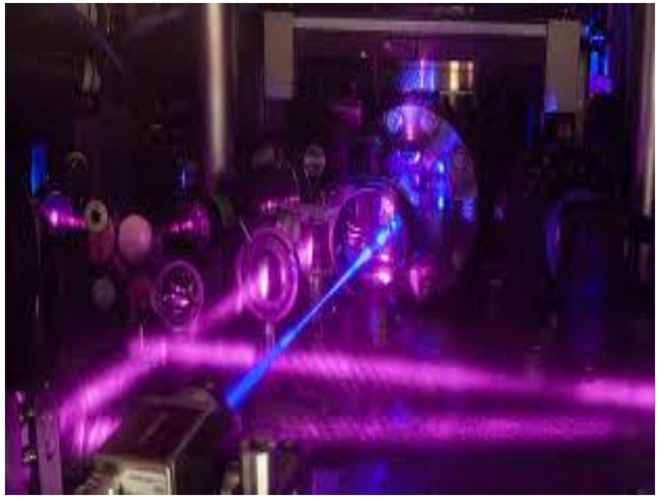
Optical clocks

- Accuracy approaching 10^{-18}
- Low stability: only single ion



List of labs not exhaustive

Neither gain nor lose one second in some 15 billion years—roughly the age of the universe.



- Accuracy at 10^{-18}
- High stability: 10^{3-4} Many atoms

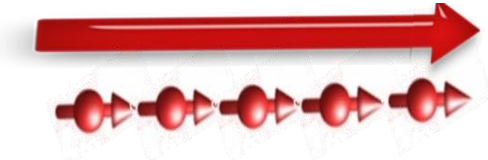
About 1000 times better than current cesium standard

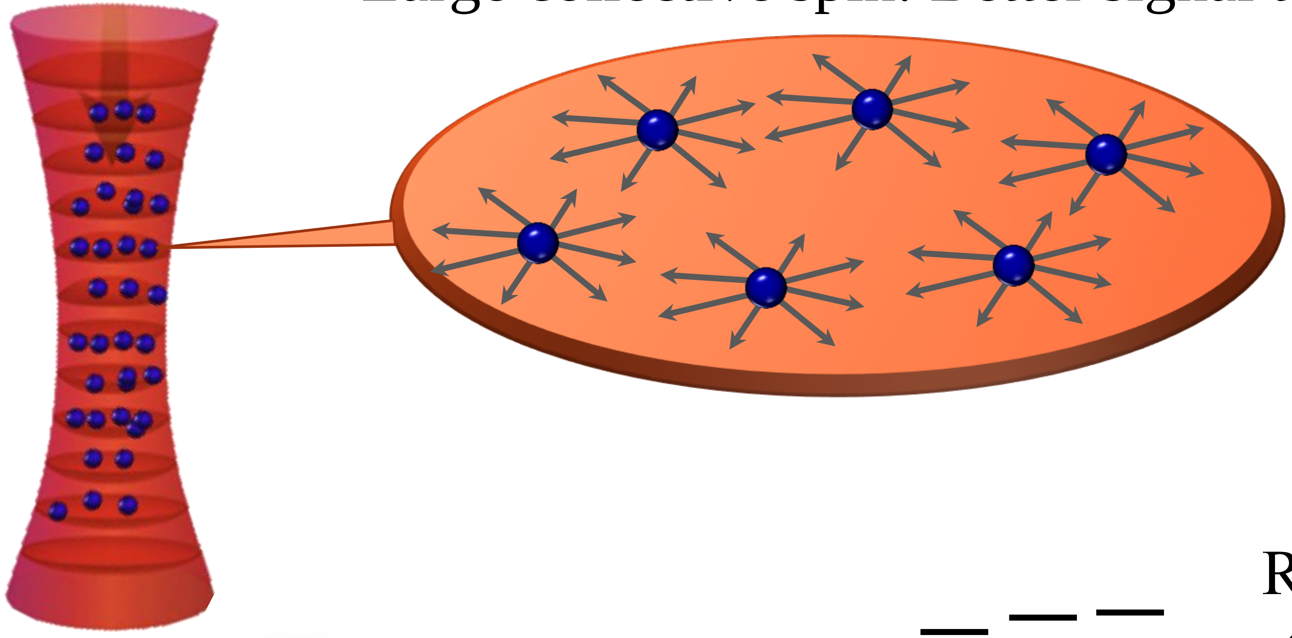
1D lattice clock: T~μK

What happens in the real experiment with many atoms?

No interaction: All the spins precess collectively

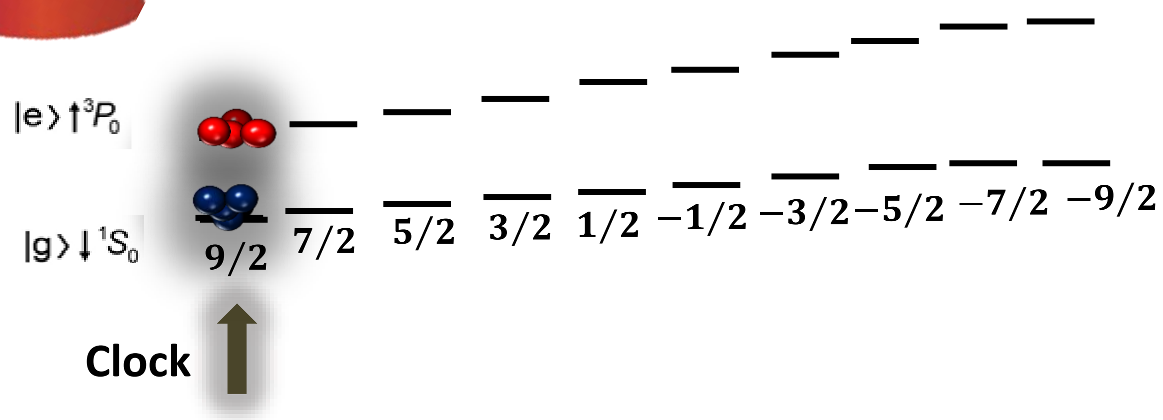
Large collective spin: Better signal to noise





$$\hat{S}^{x,y,z} = \frac{1}{2}\sum_i \hat{\sigma}_i^{x,y,z} \qquad S = N/2$$

$$\langle \hat{S}^z(\tau) \rangle = \frac{N}{2}\sin\theta\cos(\delta\tau)$$



Requirement:

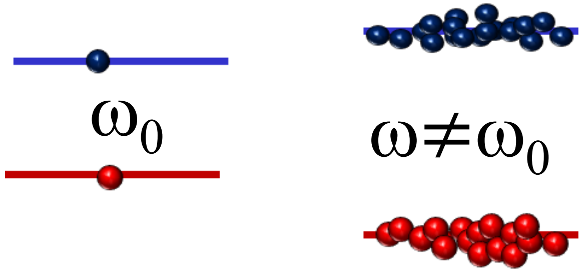
- ✓ All experience same Rabi frequency (Resource for spin-orbit coupling)
- ✓ All have same detunings: No Doppler& Stark Shifts: Deep Magic Lattice
- ❑ Atomic Collisions?

DILEMMA

More atoms better signal to noise

Atomic collisions change the frequency. Packing more atoms makes the error worse

Interactions:



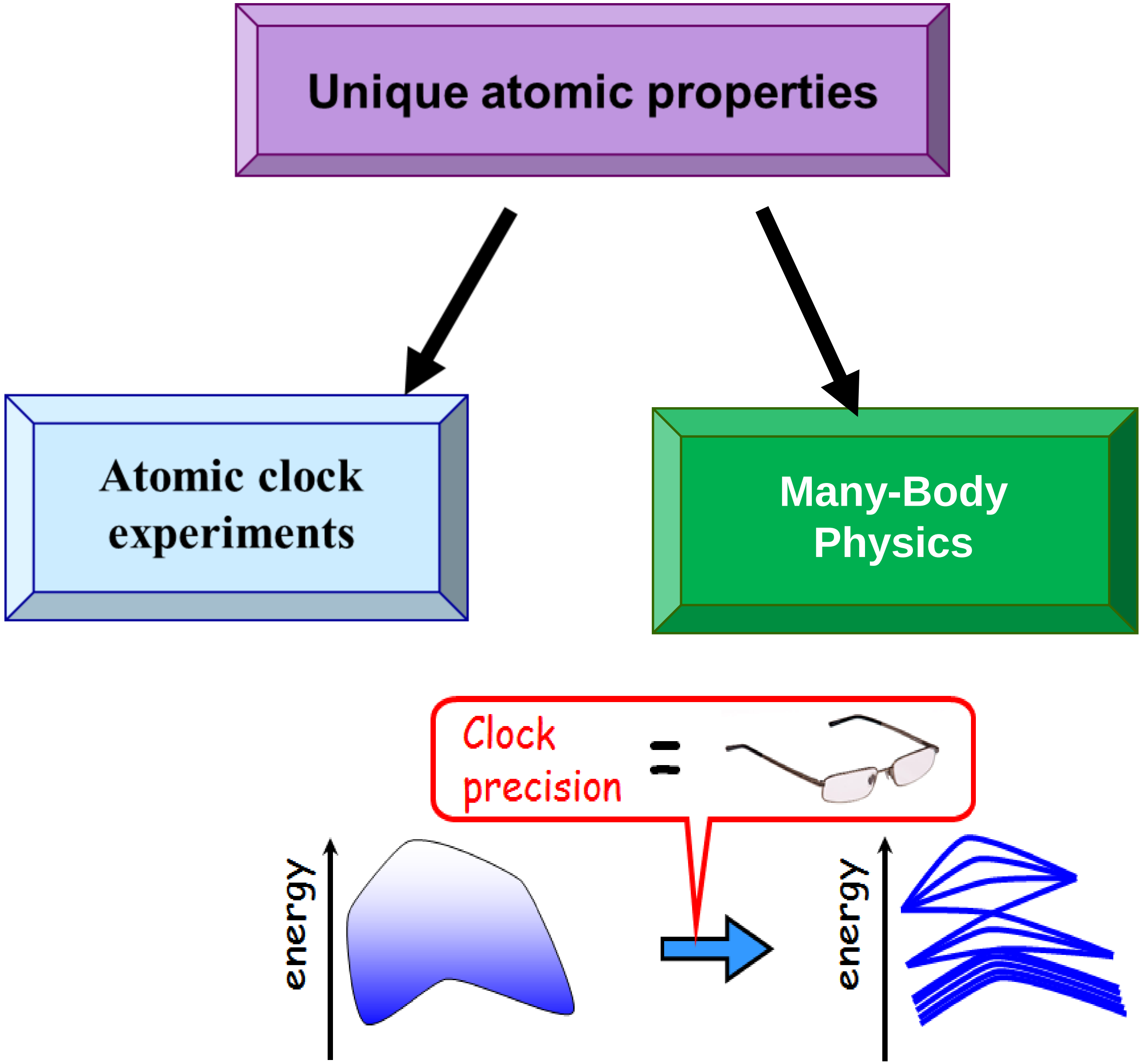
JILA:G. Campbell *et al* Science 324, 360 (09)

NIST: N. Lemke *et al* PRL 103,063001 (09)

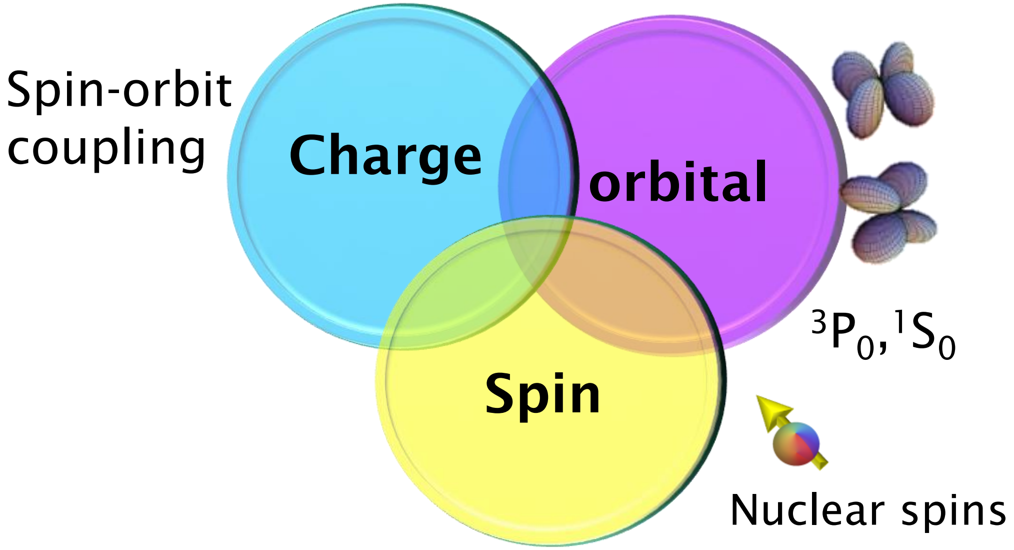
Need to understand interactions

- Degraded signal: Even in identical fermionic atoms. In 2008 gave rise to the second largest uncertainty to the 10^{-16} error budget

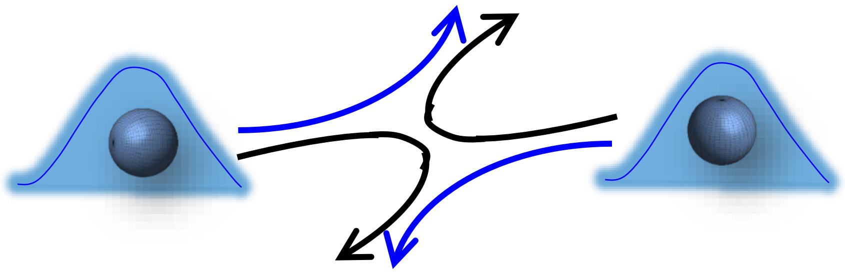
Why alkaline-earth (like) atoms?

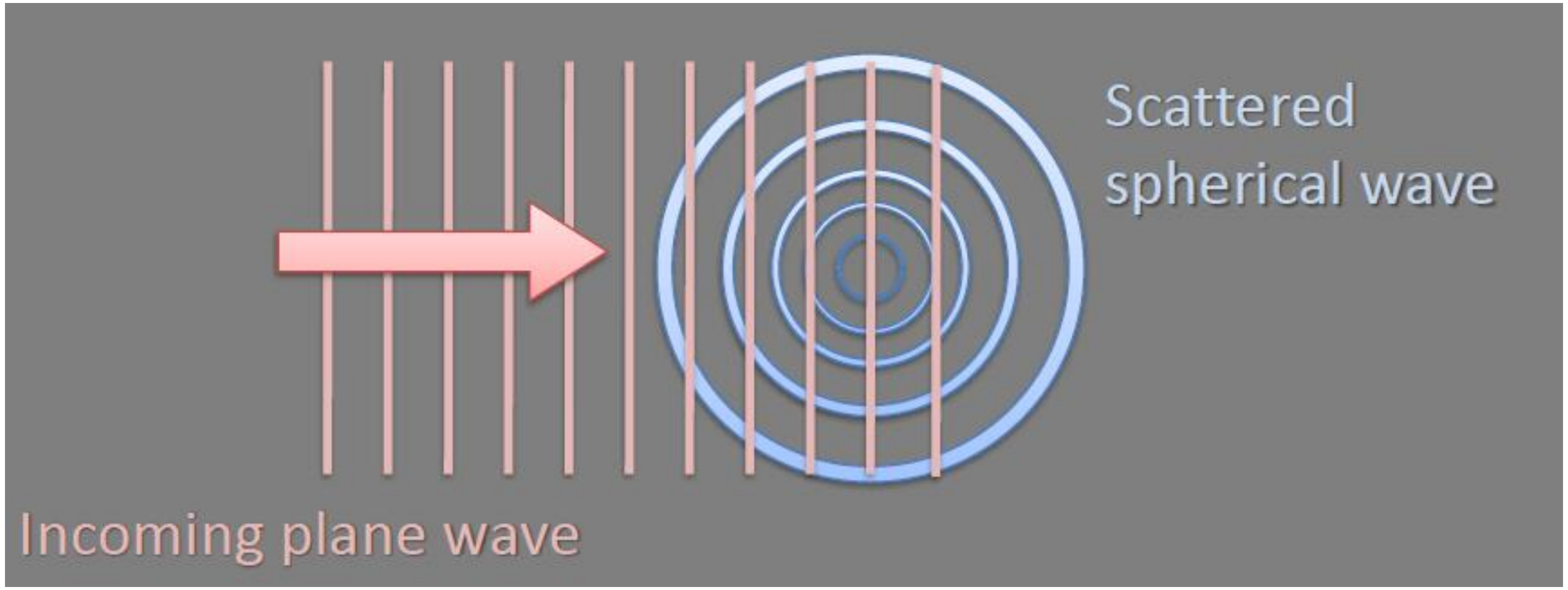


Strongly correlated materials



Basic Scattering in quantum mechanics





Goal: find scattered wave $\psi(\mathbf{r}) \propto e^{ikz} + f(\theta)\frac{e^{ikr}}{r}$ f: scattering amplitude $\frac{d\sigma}{d\Omega} = |f(\Omega)|^2$

cross section

Solve Schrödinger

$$-\frac{\hbar^2}{2M}\nabla_{\boldsymbol{R}}^2\psi - \frac{\hbar^2}{2\mu}\nabla_{\boldsymbol{r}}^2\psi + V(r)\psi = \mathrm{E}\psi \qquad \psi(\boldsymbol{R},\boldsymbol{r}) = \Psi_{CM}(\mathbf{R})\Psi(\mathbf{r})$$

$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ relative coordinate, $\mu=m/2$: reduced mass

$\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2$ Center of Mass coordinate, $M = 2m$, Total mass

Partial Wave Expansion

Angular momentum is quantized: $\ell = 0, 1, 2 \ldots$ *s*-, *p*-, waves, ...

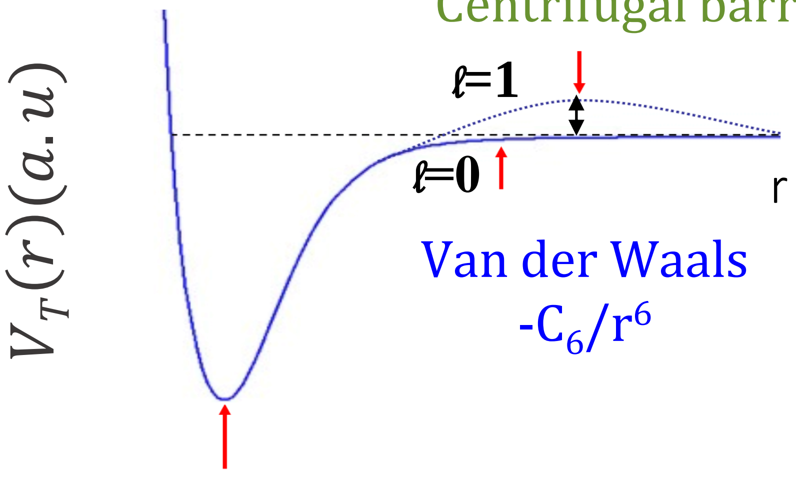
$$\Psi(r) = \sum_{\ell,m} R_{\ell,m}(r,E) Y_{\ell,m}(\theta,\varphi) \qquad \frac{\hbar^2 k^2}{2\mu} = E$$

$$V_T(r) = V(r) + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2}$$

Centrifugal barrier

$$\frac{\hbar^2}{2\mu}\left(\frac{1}{r^2}\left(r^2\frac{d}{dr}\right) + k^2 - \frac{\ell(\ell+1)}{r^2}\right)R_{\ell,m} = V(r)R_{\ell,m}$$

$$R_{\ell,m}(r \to \infty) \to \frac{C_{\ell,m}}{kr}\sin\left[kr - \ell\frac{\pi}{2} - \delta_\ell\right]$$



There is only a phase shift at long range!!

Solve Schrödinger equation for each ℓ to get δ_ℓ, $C_{\ell,m}$

$$f(\theta) = \sum_\ell (2\ell+1)P_\ell(\cos\theta)\, f_\ell \qquad f_\ell = \frac{(e^{2i\delta_\ell} - 1)}{2ik}$$

Low Energy Collisions

(1) At ultra-cold temperatures $k \to 0$ ℓ =0 collisions dominate!!

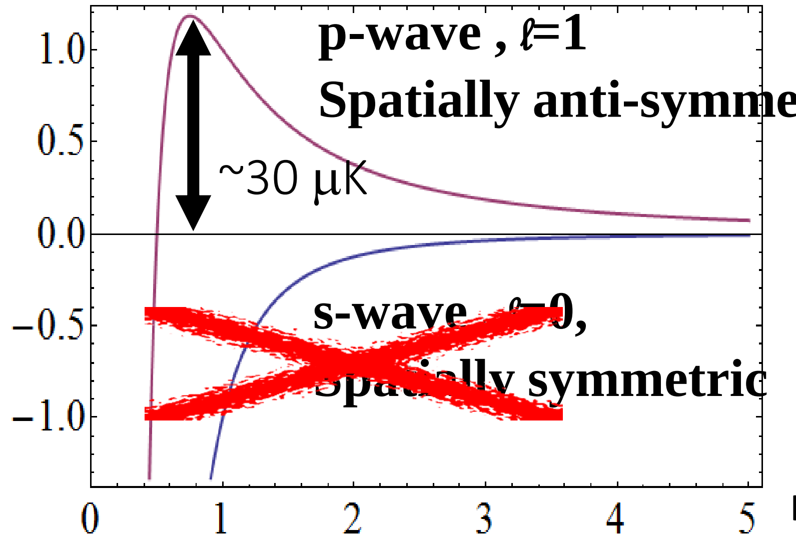
$$\frac{\delta_\ell}{k} \to A_\ell (A_\ell k)^{2\ell}$$

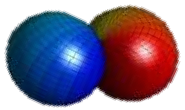
$A_0 = a$ "scattering length" Characterize *s*-wave collisions

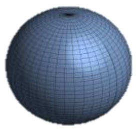
$A_1 = b^3$ "scattering volume" Characterize *p*-wave collisions

(2) Quantum statistics matter: Pauli Exclusion principle

Identical bosons: even ℓ
Identical fermions: odd ℓ







But Identical fermions

⇨ **No low energy (ℓ =0)collisions:**

⇨ **Only (*ℓ =1, p-wave*): cost energy**

Pseudo-Potential

Once there was a farmer who was not happy with his milk farm and wanted to increase the milk production. He invited 3 people to check out what is going on!

The first one was a psychologist — who observed the farm and told the farmer to paint the walls green. So that cows will be happy and produce more milk.

The farmer thought — Huh if life was that easy!

So he invited another person — the Engineer — who observed the farm and said, the milking machine is not very effective. So I will design a new one for you.

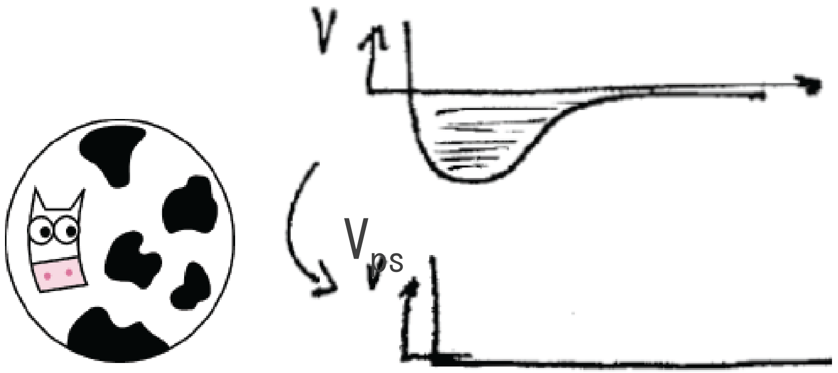
The farmer thought — Can I get a better perspective!

Well, now he invited a physicist — who looked around the place and drew a spherical cow on the board saying — Let me consider a spherical cow in a vacuum, emanating milk uniformly in all directions!

The farmer now was totally confused!

Pseudo-Potential

- Two interaction potentials V and V_{ps} are equivalent if they have the same scattering length
- So: after measuring a, b for the real system, we can model with a very simple potential.



$$\langle \mathbf{k} | V_{ps} | \mathbf{k}' \rangle = \frac{1}{L^3} \int \mathrm{d}^3\mathbf{r}\, \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{r}} V_{ps}(\mathbf{r}) \mathrm{e}^{i\mathbf{k}'\cdot\mathbf{r}},$$

$$\langle \boldsymbol{k} | V_{ps} | \boldsymbol{k}' \rangle = \frac{4\pi\hbar^2}{L^3 m} \sum_{\ell} (2\ell+1) A_\ell^{2\ell+1} k^\ell k'^\ell P_\ell(\hat{k} \cdot \hat{k}')$$

For *s*-wave collisions $\langle \boldsymbol{k} | \boldsymbol{V}_{\boldsymbol{ps}}^{\ell=0} | \boldsymbol{k}' \rangle = \dfrac{4\pi\hbar^2}{L^3 m} a$

Be careful with regularization to avoid divergencies

For *p*-wave collisions $\langle \boldsymbol{k} | \boldsymbol{V}_{\boldsymbol{ps}}^{\ell=1} | \boldsymbol{k}' \rangle = \dfrac{12\pi\hbar^2}{L^3 m} b^3 (\boldsymbol{k} \cdot \boldsymbol{k}')$

Note, it is suppressed at low energies

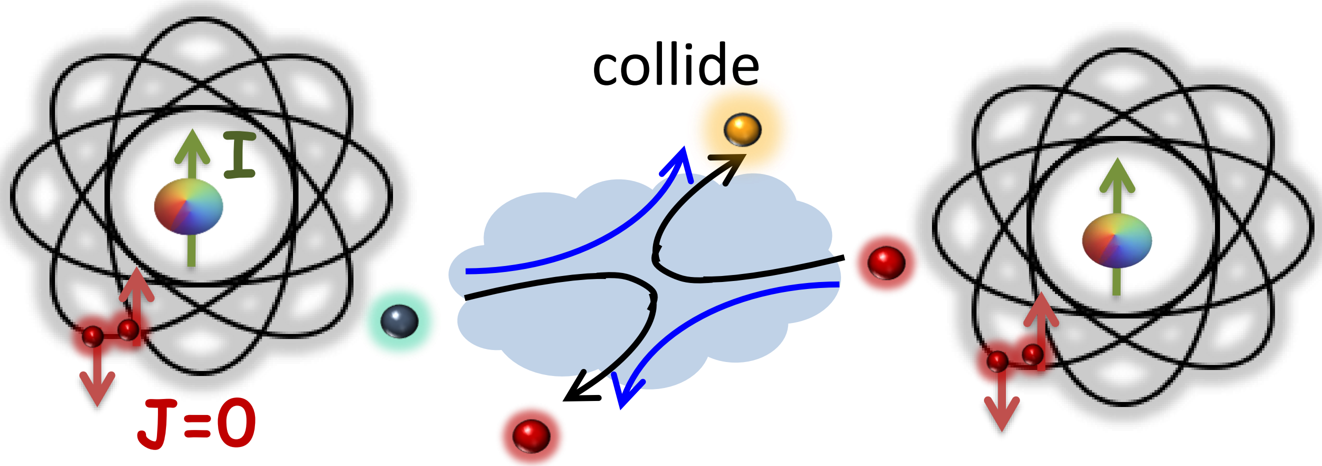
See uploaded by Anjun Chu notes for details

Huang and Yang, Phys. Rev. 105, 767 (1957)
E. Fermi Ricerca Scientifica, 7: 13–52 (1936)
Breit Phys. Rev. 71, 215 (1947),
Blatt and Weisskopf Theoretical Nuclear Physics (Wiley, New York, 1952), pp. 74–75
Idziaszek PRA 79, 062701 (2009)

Alkaline-earth Collisions

See handwritten notes

Alkaline-earth Collisions



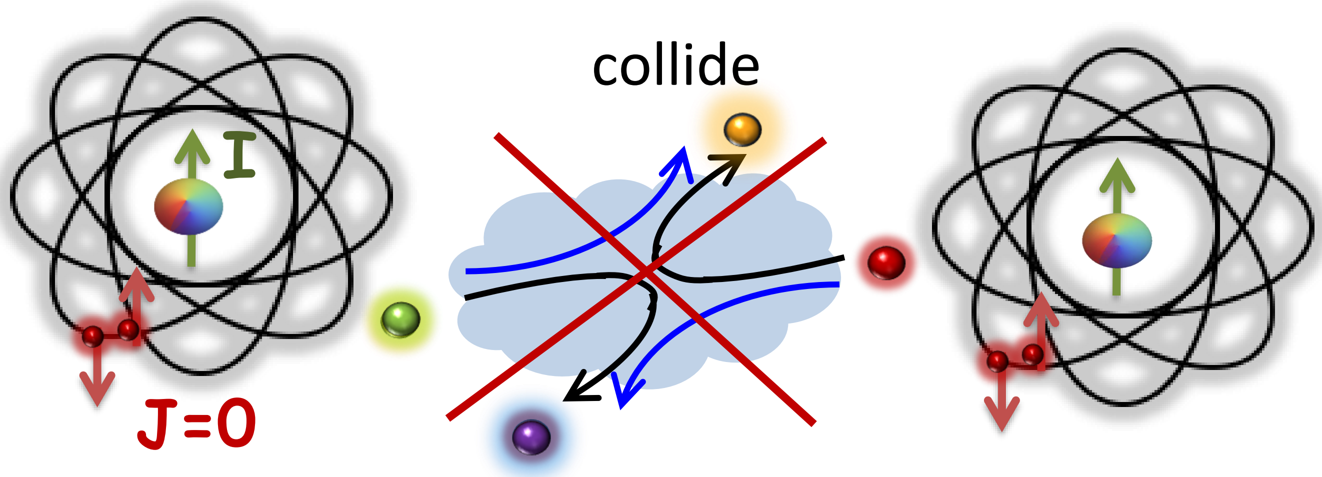
hyperfine coupling $I \cdot J = 0.$

Electrons independent of nuclear spin → collisions independent of nuclear spin. (except via Fermi statistics).

See handwritten notes

M. Cazalilla *and A. M Rey* Reports on Progress in Physics 77, 124401 (2014)

Alkaline-earth Collisions



hyperfine coupling $I \cdot J = 0.$

Electrons independent of nuclear spin → collisions independent of nuclear spin. (except via Fermi statistics).

Up to N=2I+1=10

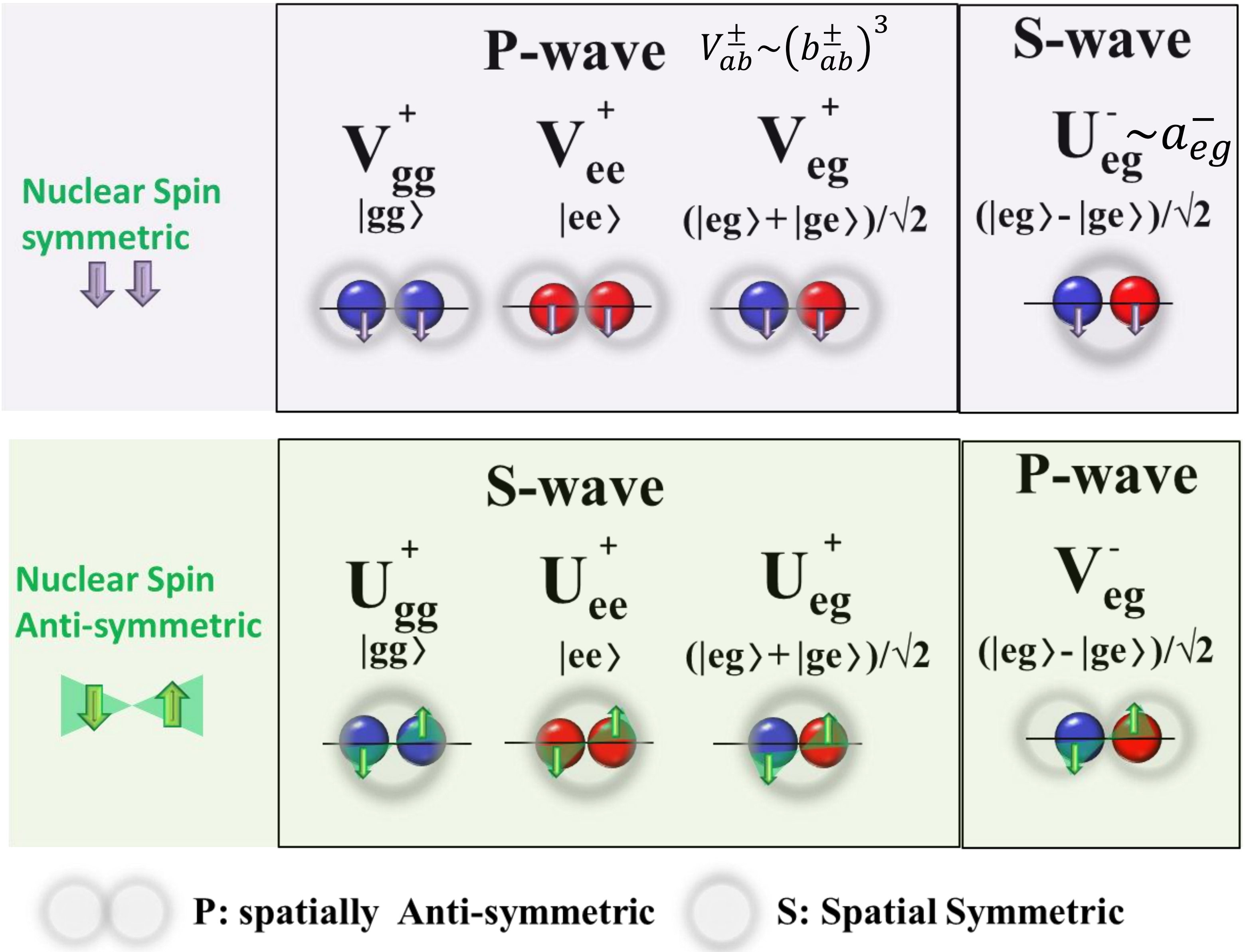
SU(N=2I+1) symmetry:

Nuclear spin independent scattering parameters

No spin changing collisions.

M. Cazalilla *and A. M Rey* Reports on Progress in Physics 77, 124401 (2014)

SU(N) interaction parameters



Many-body Hamiltonian: Spin polarized system

Rey *et al* Annals of Physics 340, 311(2014)

$$\alpha, \beta = e, g$$

$$m = 9/2$$

Fermionic Field operator $\{\hat{\Psi}_{\alpha m}(\boldsymbol{R}), \hat{\Psi}^{\dagger}_{\beta m'}(\boldsymbol{R}')\} = \delta(\boldsymbol{R}-\boldsymbol{R}')\delta_{\alpha\beta}\delta_{m,m'}$

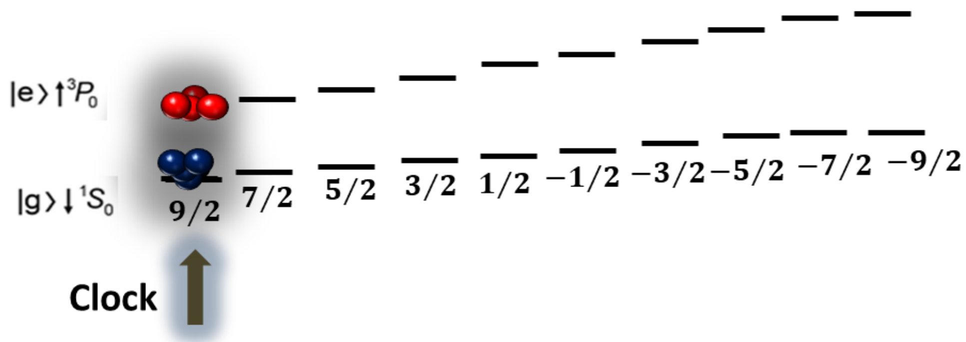
$$\hat{H} = \hat{H}_0 + \hat{H}_1,$$

$$\hat{H}_0 = \sum_{\alpha} \int d^3\mathbf{R}\hat{\Psi}^{\dagger}_{\alpha}(\mathbf{R})\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(\mathbf{R})\right)\hat{\Psi}_{\alpha}(\mathbf{R})$$

$$+\frac{4\pi\,\hbar^2\,a^{-}_{eg}}{m}\int d^3\mathbf{R}\hat{\Psi}^{\dagger}_{e}(\mathbf{R})\hat{\Psi}_{e}(\mathbf{R})\hat{\Psi}^{\dagger}_{g}(\mathbf{R})\hat{\Psi}_{g}(\mathbf{R})$$

$$+\frac{3\pi\,\hbar^2}{2m}\sum_{\alpha,\beta} b^3_{\alpha\beta}\int d^3\mathbf{R}\Big[\left(\vec{\nabla}\hat{\Psi}^{\dagger}_{\alpha}(\mathbf{R})\right)\hat{\Psi}^{\dagger}_{\beta}(\mathbf{R}) - \hat{\Psi}^{\dagger}_{\alpha}(\mathbf{R})\left(\vec{\nabla}\hat{\Psi}^{\dagger}_{\beta}(\mathbf{R})\right)\Big]$$

$$\cdot\Big[\hat{\Psi}_{\beta}(\mathbf{R})\left(\vec{\nabla}\hat{\Psi}_{\alpha}(\mathbf{R})\right) - \left(\vec{\nabla}\hat{\Psi}_{\beta}(\mathbf{R})\right)\hat{\Psi}_{\alpha}(\mathbf{R})\Big] + \frac{1}{2}\hbar\,\delta\int d^3\mathbf{R}\Big[\hat{\rho}_e(\mathbf{R}) - \hat{\rho}_g(\mathbf{R})\Big],$$



$$\hat{H}_1 = \frac{\hbar\Omega_0}{2}\int d^3\mathbf{R}\Big[\hat{\Psi}^{\dagger}_{e}(\mathbf{R})e^{-i(\omega_L t - \boldsymbol{k}\cdot\mathbf{R})}\hat{\Psi}_{g}(\mathbf{R}) + \text{h.c.}\Big].$$

$$\hat{\rho}_{\alpha}(\mathbf{R}) = \hat{\Psi}^{\dagger}_{\alpha}(\mathbf{R})\hat{\Psi}_{\alpha}(\mathbf{R})$$

Many-body Hamiltonian

Rey *et al* Annals of Physics 340, 311(2014)

$$\hat{\Psi}_\alpha(\mathbf{R}) = \phi_0^Z(Z)\sum_{\mathbf{n}} \hat{c}_{\alpha\mathbf{n}}\phi_{n_X}(X)\phi_{n_Y}(Y), \quad \phi_n(x) = \frac{1}{\sqrt{2^n\, n!}}\cdot\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\cdot e^{-\frac{m\omega x^2}{2\hbar}}\cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right)$$

$$\hat{H} = \sum_{\alpha,\beta,\mathbf{n},\mathbf{n}',\mathbf{n}'',\mathbf{n}'''} \frac{\hbar}{4}\left(v^{\alpha,\beta} P_{\mathbf{n}\mathbf{n}'\mathbf{n}''\mathbf{n}'''}\right)\hat{c}^\dagger_{\alpha\mathbf{n}}\hat{c}^\dagger_{\beta\mathbf{n}'}\hat{c}_{\beta\mathbf{n}''}\hat{c}_{\alpha\mathbf{n}'''}, \qquad v^{\alpha,\beta} = \frac{6}{\sqrt{2\pi}}\sqrt{\omega_Z\omega_R}\frac{b^3_{\alpha,\beta}}{a^{R\ 3}_{ho}}.$$

$$P_{\mathbf{n}\mathbf{n}'\mathbf{n}''\mathbf{n}'''} = s(n_X, n'_X, n''_X, n'''_X)p(n_Y, n'_Y, n''_Y, n'''_Y) + p(n_X, n'_X, n''_X, n'''_X)s(n_Y, n'_Y, n''_Y, n'''_Y),$$

$$s(n, n', n'', n''') \equiv \frac{\int d\xi\, e^{-2\xi^2} H_n(\xi)H_{n'}(\xi)H_{n''}(\xi)H_n'''(\xi)d\xi}{\pi\sqrt{2^{n+n'+n''+n'''}n!n'!n''!n'''!}},$$

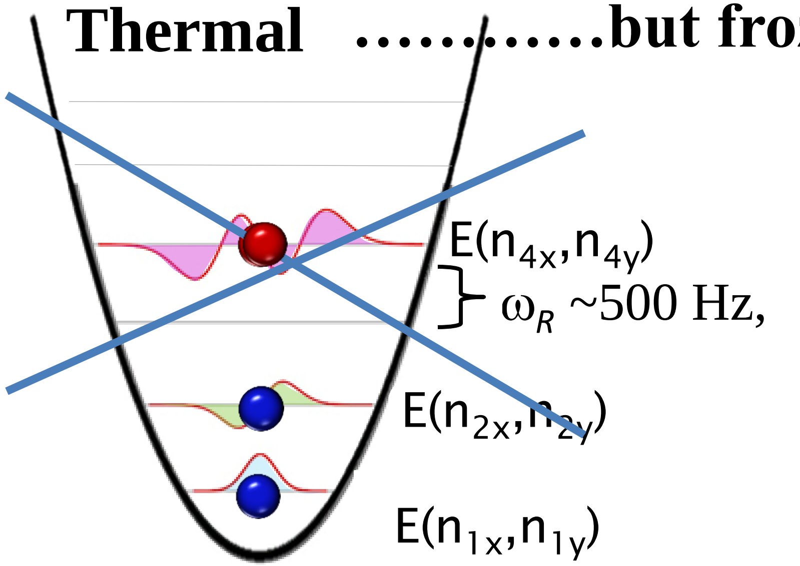
$$p(n, n', n'', n''')$$

$$= \frac{\int d\xi\, e^{-2\xi^2}\left[\left(\frac{d}{d\xi}H_n(\xi)\right)H_{n'}(\xi) - H_n(\xi)\left(\frac{d}{d\xi}H_{n'}(\xi)\right)\right]\left[\left(\frac{d}{d\xi}H_{n''}(\xi)\right)H_n'''(\xi) - H_{n''}(\xi)\left(\frac{d}{d\xi}H_n'''(\xi)\right)\right]}{\pi\sqrt{2^{n+n'+n''+n'''}n!n'!n''!n'''!}}.$$

1D lattice clock: A large spin simulator

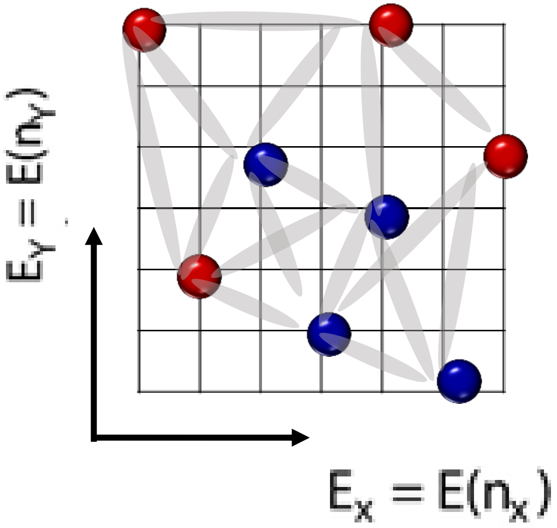
$^3P_0$ (e) $^1S_0$ (g) **Interaction Energy~ Hz** **Weak interactions simplify physics**

Thermal ……………but frozen motional levels



NO mode changing collisions

Energy lattice:



$$\hat{c}^\dagger_{\alpha\mathbf{n}}\hat{c}^\dagger_{\beta\mathbf{n}'}\hat{c}_{\beta\mathbf{n}'}\hat{c}_{\alpha\mathbf{n}} \quad \hat{c}^\dagger_{\alpha\mathbf{n}}\hat{c}^\dagger_{\beta\mathbf{n}'}\hat{c}_{\beta\mathbf{n}}\hat{c}_{\alpha\mathbf{n}'} \quad \vec{S}_{\mathbf{n}_j} = \frac{1}{2}\sum_{\alpha,\beta}\hat{c}^\dagger_{\alpha\mathbf{n}_j}\vec{\sigma}_{\alpha\beta}\hat{c}_{\beta\mathbf{n}_j},$$

$$\hat{S}^z_{\mathbf{n}}\hat{S}^z_{\mathbf{n}'} \quad \hat{S}^+_{\mathbf{n}}\hat{S}^-_{\mathbf{n}'}$$

Hamiltonian can be reduced to a Spin model with long range couplings

Two spin polarized atoms - two modes

Singlet State

Interaction

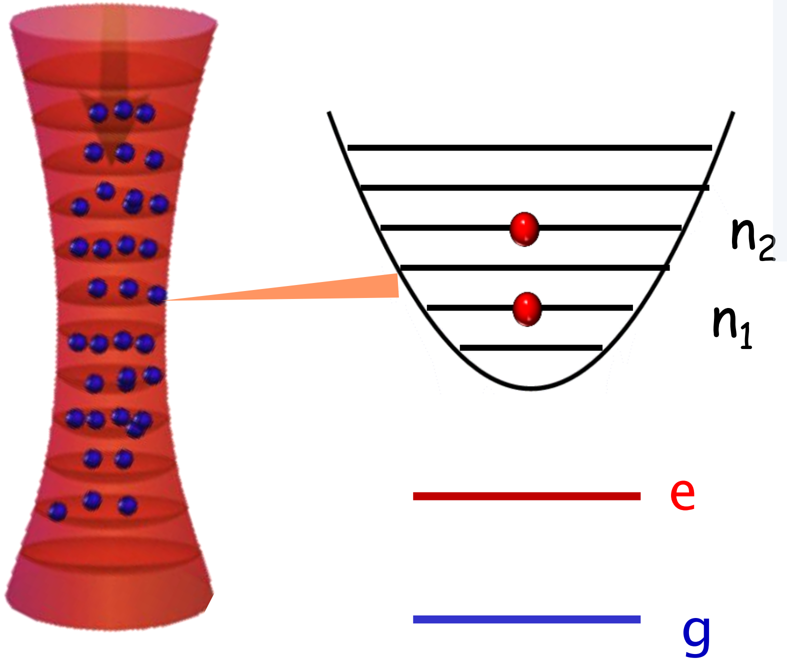
$$|s\rangle = \frac{|{\color{blue}\bullet}{\color{red}\bullet}\rangle - |{\color{red}\bullet}{\color{blue}\bullet}\rangle}{\sqrt{2}} \frac{|n_1\, n_2\rangle + |n_2\, n_1\rangle}{\sqrt{2}}$$

Triplet States

$$|t^+\rangle = |{\color{red}\bullet}{\color{red}\bullet}\rangle \frac{|n_1\, n_2\rangle - |n_2\, n_1\rangle}{\sqrt{2}}$$

$$|t^-\rangle = |{\color{blue}\bullet}{\color{blue}\bullet}\rangle \frac{|n_1\, n_2\rangle - |n_2\, n_1\rangle}{\sqrt{2}}$$

$$|t^0\rangle = \frac{|{\color{blue}\bullet}{\color{red}\bullet}\rangle + |{\color{blue}\bullet}{\color{red}\bullet}\rangle}{\sqrt{2}} \frac{|n_1\, n_2\rangle - |n_2\, n_1\rangle}{\sqrt{2}}$$



$$\begin{array}{cc} & \begin{array}{cccc} |t^-\rangle & |t^+\rangle & |t^0\rangle & |s\rangle \end{array} \\ H_{S+P} = & \begin{pmatrix} \delta + V_{gg} & 0 & \bar{\Omega}/\sqrt{2} & 0 \\ 0 & -\delta + V_{ee} & \bar{\Omega}/\sqrt{2} & 0 \\ \bar{\Omega}/\sqrt{2} & \bar{\Omega}/\sqrt{2} & +V_{eg} & 0 \\ 0 & 0 & 0 & U_{eg} \end{pmatrix} \end{array}$$

Only p-wave interactions relevant.

No coupling to singlet

What happens if $\Omega_1 \neq \Omega_2$?

Rey *et al* PRL (2009), Gibble PRL (2009), Yu *et al* PRL (2010)

Two atom - two modes

Singlet State

Interaction

$U_{eg}^{-} \propto a_{eg}^{-}$

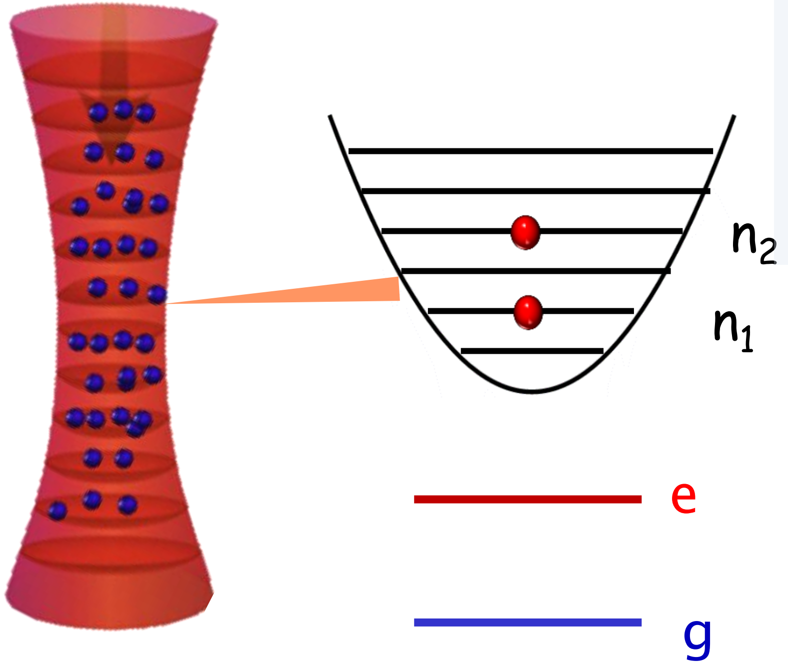
$$|s\rangle = \frac{|{\color{blue}\bullet}{\color{red}\bullet}\rangle - |{\color{red}\bullet}{\color{blue}\bullet}\rangle}{\sqrt{2}} \frac{|n_1\, n_2\rangle + |n_2\, n_1\rangle}{\sqrt{2}}$$

Triplet States

$V_{gg}^{+} \propto b_{gg}^{3}$
$$|t^{+}\rangle = |{\color{red}\bullet}{\color{red}\bullet}\rangle \frac{|n_1\, n_2\rangle - |n_2\, n_1\rangle}{\sqrt{2}}$$

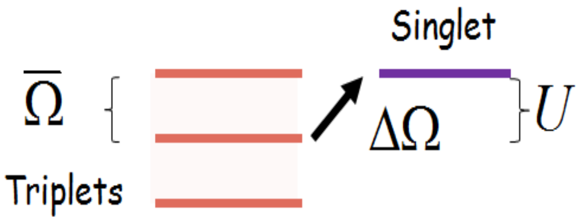
$V_{ee}^{+} \propto b_{ee}^{3}$
$$|t^{-}\rangle = |{\color{blue}\bullet}{\color{blue}\bullet}\rangle \frac{|n_1\, n_2\rangle - |n_2\, n_1\rangle}{\sqrt{2}}$$

$V_{eg}^{+} \propto b_{eg}^{3}$
$$|t^{0}\rangle = \frac{|{\color{blue}\bullet}{\color{red}\bullet}\rangle + |{\color{blue}\bullet}{\color{red}\bullet}\rangle}{\sqrt{2}} \frac{|n_1\, n_2\rangle - |n_2\, n_1\rangle}{\sqrt{2}}$$



$$H_{S+P} = \begin{array}{c} \begin{array}{cccc} |t^{-}\rangle & |t^{+}\rangle & |t^{0}\rangle & |s\rangle \end{array} \\ \begin{pmatrix} \delta + V_{gg} & 0 & \bar{\Omega}/\sqrt{2} & \Delta\Omega/\sqrt{2} \\ 0 & -\delta + V_{ee} & \bar{\Omega}/\sqrt{2} & -\Delta\Omega/\sqrt{2} \\ \bar{\Omega}/\sqrt{2} & \bar{\Omega}/\sqrt{2} & +V_{eg} & 0 \\ \Delta\Omega/\sqrt{2} & -\Delta\Omega/\sqrt{2} & 0 & U_{eg}^{-} \end{pmatrix} \end{array}$$

What happens if $\Omega_1 - \Omega_2 = \Delta\Omega \neq 0$?



Initially thought to be the mechanism leading to density shifts but was ruled out later

Rey *et al* PRL (2009), Gibble PRL (2009), Yu *et al* PRL (2010)

Understanding collisions in the clock

Atoms as a quantum magnet: frozen in energy space

M. Martin *et al*, Science 341, 632 (2013), Rey *et al* Annals of Physics 340, 311(2014)

$$\hat{H} \approx -\delta \sum_{j=1}^{N} \hat{S}^z_{\mathbf{n}_j} + \sum_{j\neq j'}^{N} \left[J^{\perp}_{\mathbf{n}_j,\mathbf{n}_{j'}} (\vec{S}_{\mathbf{n}_j} \cdot \vec{S}_{\mathbf{n}_{j'}}) + \chi_{\mathbf{n}_j,\mathbf{n}_{j'}} \hat{S}^z_{\mathbf{n}_j} \hat{S}^z_{\mathbf{n}_{j'}} \right] + \sum_{j\neq j'}^{N} \left[\frac{C_{\mathbf{n}_j,\mathbf{n}_{j'}}}{2} (\hat{S}^z_{\mathbf{n}_j} I_{\mathbf{n}_{j'}} + \hat{S}^z_{\mathbf{n}_{j'}} I_{\mathbf{n}_j}) + \frac{K_{\mathbf{n}_j,\mathbf{n}_{j'}}}{4} I_{\mathbf{n}_j} I_{\mathbf{n}_{j'}} \right]$$

Delocalized modes: Long range interactions

$$J^{\perp}_{\mathbf{n}_j,\mathbf{n}_{j'}} = \frac{V^{eg}_{\mathbf{n}_j,\mathbf{n}_{j'}} - U^{eg}_{\mathbf{n}_j,\mathbf{n}_{j'}}}{2},$$

$$\chi_{\mathbf{n}_j,\mathbf{n}_{j'}} = \frac{V^{ee}_{\mathbf{n}_j,\mathbf{n}_{j'}} + V^{gg}_{\mathbf{n}_j,\mathbf{n}_{j'}} - 2V^{eg}_{\mathbf{n}_j,\mathbf{n}_{j'}}}{2},$$

$$C_{\mathbf{n}_j,\mathbf{n}_{j'}} = \frac{(V^{ee}_{\mathbf{n}_j,\mathbf{n}_{j'}} - V^{gg}_{\mathbf{n}_j,\mathbf{n}_{j'}})}{2}$$

$$K_{\mathbf{n}_j,\mathbf{n}_{j'}} = \frac{(V^{ee}_{\mathbf{n}_j,\mathbf{n}_{j'}} + V^{gg}_{\mathbf{n}_j,\mathbf{n}_{j'}} + V^{eg}_{\mathbf{n}_j,\mathbf{n}_{j'}} + U^{eg}_{\mathbf{n}_j,\mathbf{n}_{j'}})}{2}.$$

Many-body Hamiltonian

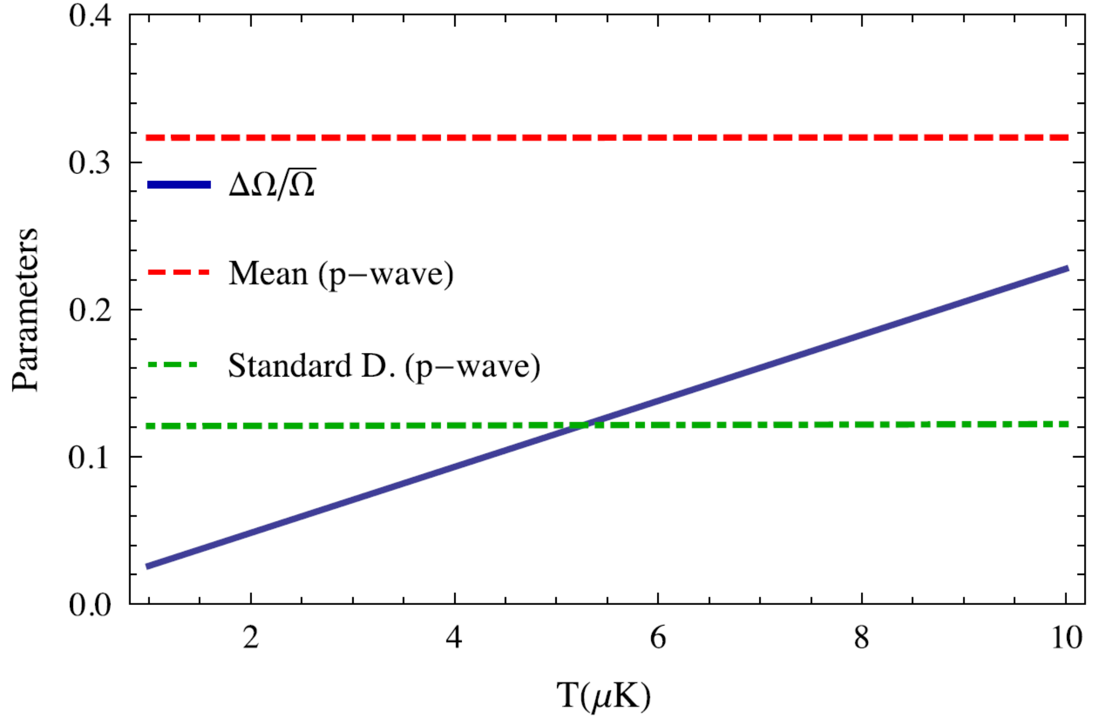
Rey *et al* Annals of Physics 340, 311(2014)

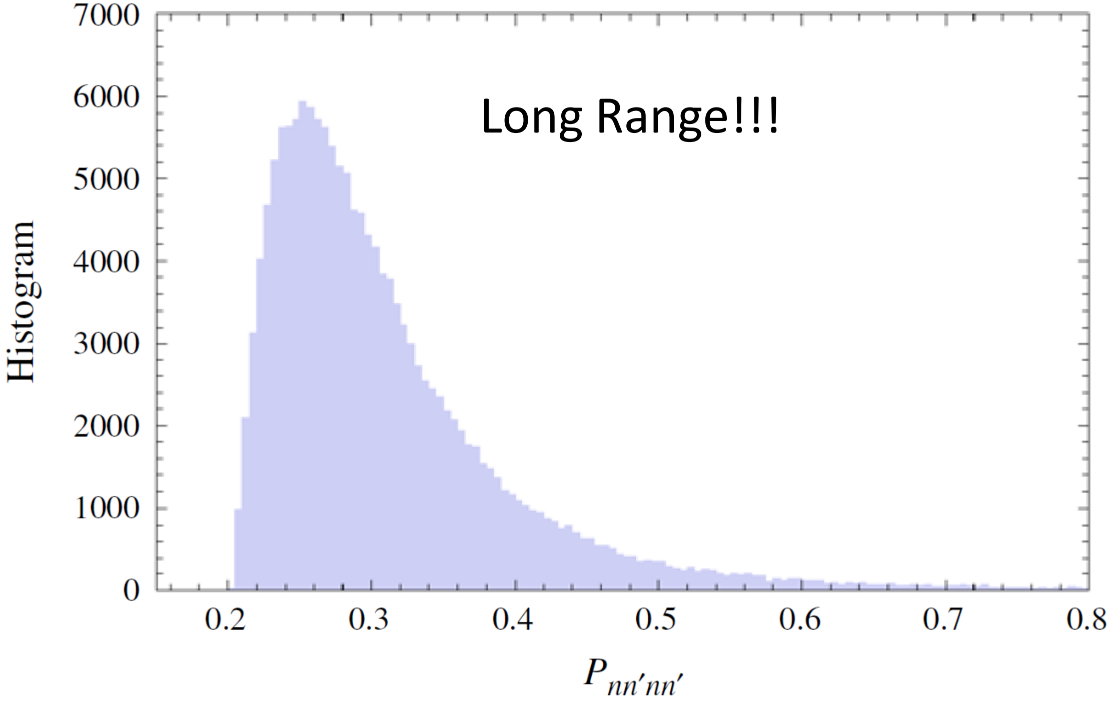
$$\hat{\Psi}_\alpha(\mathbf{R}) = \phi_0^Z(Z)\sum_{\mathbf{n}} \hat{c}_{\alpha\mathbf{n}}\phi_{n_X}(X)\phi_{n_Y}(Y), \quad \phi_n(x) = \frac{1}{\sqrt{2^n\, n!}}\cdot\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\cdot e^{-\frac{m\omega x^2}{2\hbar}}\cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right)$$

$$\hat{H} = \sum_{\alpha,\beta,\mathbf{n},\mathbf{n}',\mathbf{n}'',\mathbf{n}'''} \frac{\hbar}{4}\left(\boxed{v^{\alpha,\beta}P_{\mathbf{n}\mathbf{n}'\mathbf{n}''\mathbf{n}'''}}\right)\hat{c}^\dagger_{\alpha\mathbf{n}}\hat{c}^\dagger_{\beta\mathbf{n}'}\hat{c}_{\beta\mathbf{n}''}\hat{c}_{\alpha\mathbf{n}'''}, \qquad v^{\alpha,\beta} = \frac{6}{\sqrt{2\pi}}\sqrt{\omega_Z\omega_R}\frac{b^3_{\alpha,\beta}}{a^{R\ 3}_{ho}}.$$

$$\langle \boldsymbol{k}|\boldsymbol{V}_{\boldsymbol{ps}}^{\ell=1}|\boldsymbol{k}\rangle \propto \frac{b^3}{\text{Volume}}\frac{\hbar^2k^2}{2m} \propto \frac{b^3}{\text{Area }a^Z_{ho}}k_BT \propto \frac{b^3}{a^Z_{ho}}\frac{\hbar\,\omega_R}{(a^R_{ho})^2k_BT}k_BT$$

Temperature Independent





Understanding collisions in the clock

Atoms as a quantum magnet: frozen in energy space

M. Martin *et al,* Science 341, 632 (2013), Rey *et al* Annals of Physics 340, 311(2014)

$$\hat{H} \approx -\delta \sum_{j=1}^{N} \hat{S}^z_{\mathbf{n}_j} + \sum_{j \neq j'}^{N} \left[\chi_{\mathbf{n}_j,\mathbf{n}_{j'}} \hat{S}^z_{\mathbf{n}_j} \hat{S}^z_{\mathbf{n}_{j'}} \right] + \sum_{j \neq j'}^{N} \left[\frac{C_{\mathbf{n}_j,\mathbf{n}_{j'}}}{2} (\hat{S}^z_{\mathbf{n}_j} + \hat{S}^z_{\mathbf{n}_{j'}}) \right]$$

Delocalized modes: Long range interactions

Collective spin model

$$\hat{H} \approx -\delta \hat{S}^z + \bar{\boldsymbol{\chi}} (\hat{S}^z)^2 + \bar{\boldsymbol{C}} N \hat{S}^z$$

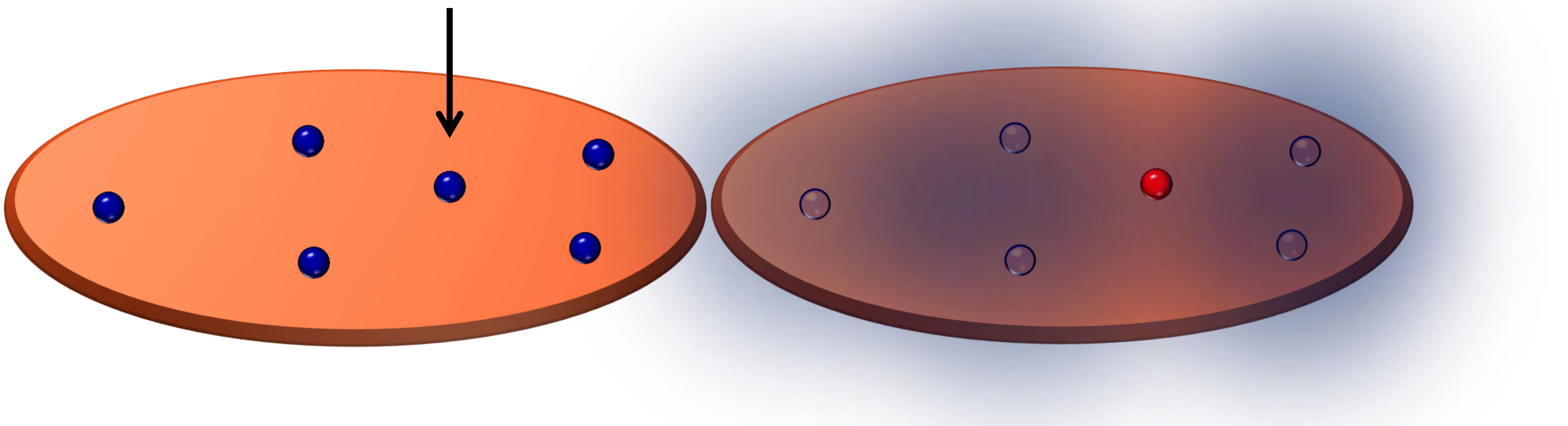
$\bar{\boldsymbol{\chi}}$ and $\bar{\boldsymbol{C}}$: Mean P-wave Interaction parameters

$$\hat{S}^\alpha = \sum_{j=1}^{N} \hat{S}^\alpha_{\boldsymbol{n}_j}$$

$$\bar{\boldsymbol{C}} = (\bar{\boldsymbol{V}}^+_{ee} - \bar{\boldsymbol{V}}^+_{gg})/2$$

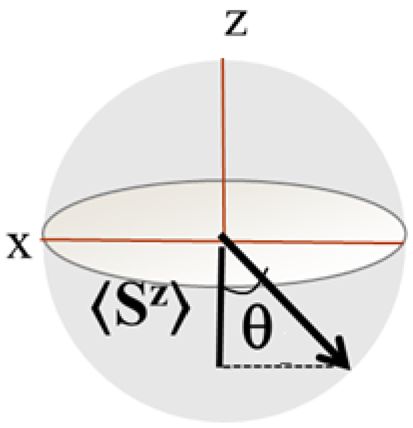
$$\bar{\boldsymbol{\chi}} = (\bar{\boldsymbol{V}}^+_{ee} - 2\bar{\boldsymbol{V}}^+_{eg} + \bar{\boldsymbol{V}}^+_{gg})/2$$

Mean Field: Phase shift



Treat other surrounding atoms as an average

$$-\delta\hat{S}^z + \bar{\chi}\,(\hat{S}^z)^2 + \bar{C}\,N\hat{S}^z \rightarrow -\delta\hat{S}^z + 2\bar{\chi}\langle\hat{S}^z\rangle\hat{S}^z + \bar{C}\,N\hat{S}^z$$



Density Shift $\qquad \delta \rightarrow \delta - 2\bar{\chi}\langle\hat{S}^z\rangle + \bar{C}\,N$

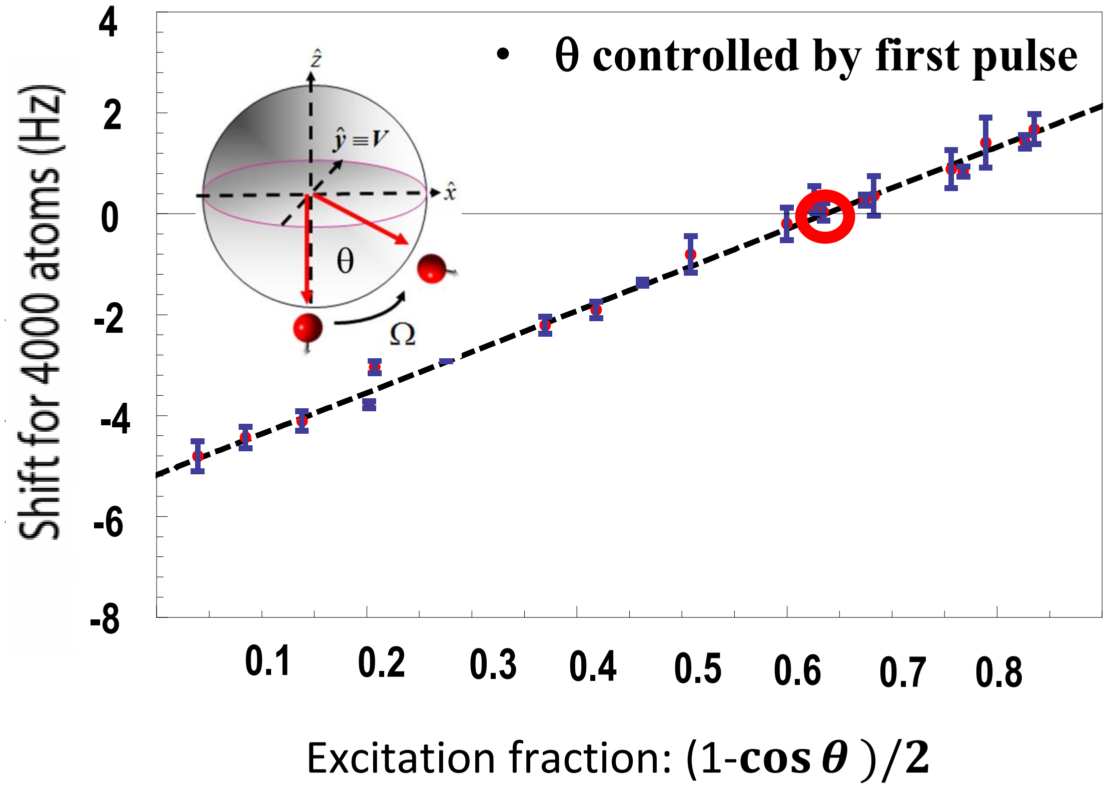
Spin precesses at a rate that depends on atom number and excitation fraction

Controlled by pulse area θ

P-wave interactions: 1D lattice clock

M. Martin *et al*, Science 341, 632 (2013), Rey *et al* Annals of Physics 340, 311(2014)

Theory vs experiment



$$\langle \hat{S}^-(\tau)\rangle = \boxed{|\langle \hat{S}^-(\tau)\rangle|}\,\boxed{e^{-i(\delta+\Delta\delta)\tau}}$$

Contrast　　Phase

$$\boxed{\Delta\delta \sim N\,(\bar{C} - \bar{\chi}\cos\theta\ )}$$

- **Determine p-wave interaction parameters**
- **Operate sweet spot: no density shift**

In Yb: Lemke *et al* *PRL* 107, 103902 (2011) Ludlow *et al*, Phys. Rev. A 84, 052724 (2011)

Quantum correlations – beyond mean field

M. Martin *et al*, Science 341, 632 (2013), Rey *et al* Annals of Physics 340, 311(2014)

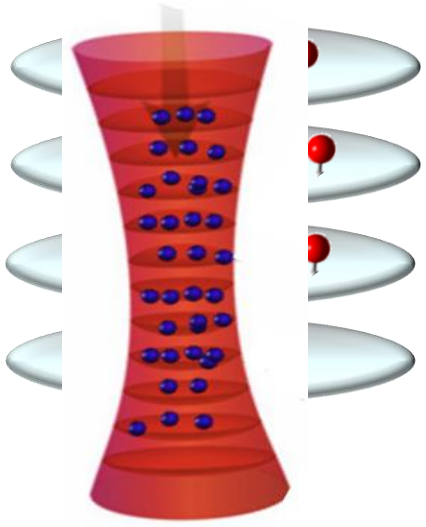
$$\langle \hat{S}^-(\tau)\rangle = \boxed{|\langle \hat{S}^-(\tau)\rangle|}\,\boxed{e^{-i(\delta+\Delta\delta)\tau}}$$

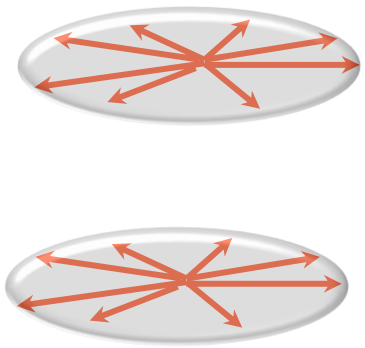
Contrast **Phase**

- At the mean field level interaction only affect the precession rate.

  But….. in the experiments there are many pancakes with different atom number.

  Mean-field interactions causes the pancakes with more atoms to precess faster.





Signal adds → contrast decay due to dephasing

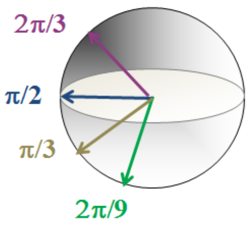
- Atom number decay also leads to decay of the amplitude

Comparisons with experiment

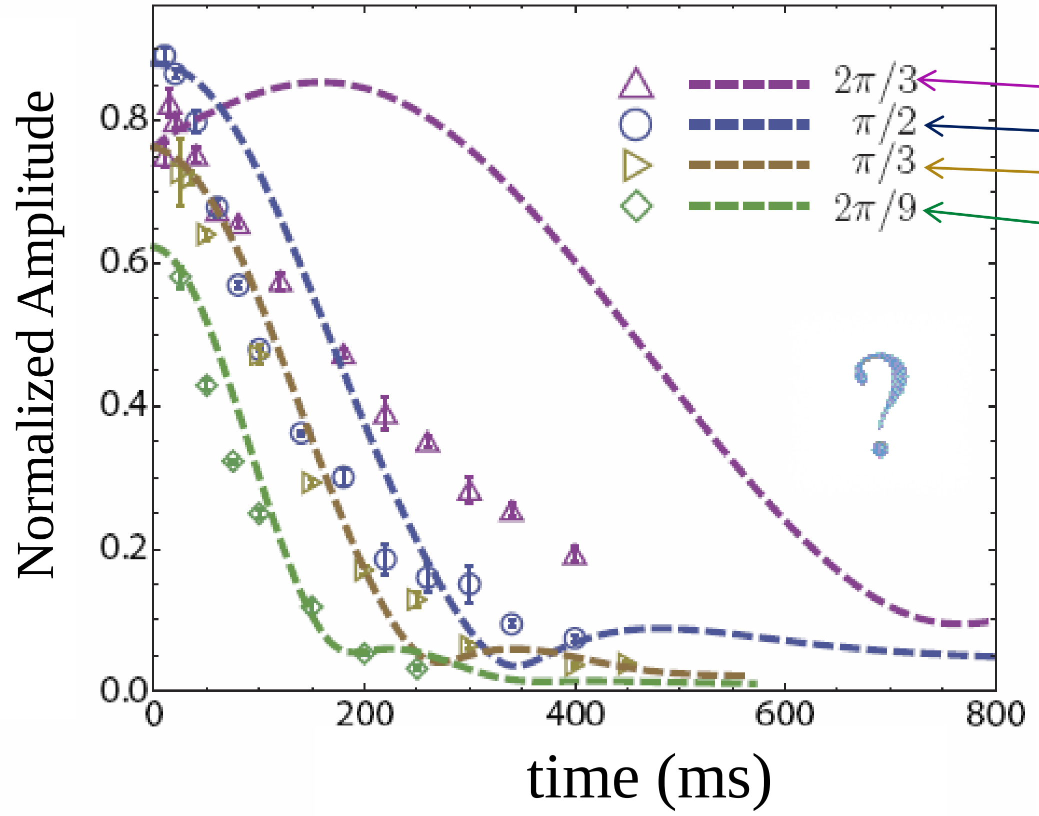
M. Martin *et al*, Science **341,** 632 (2013)

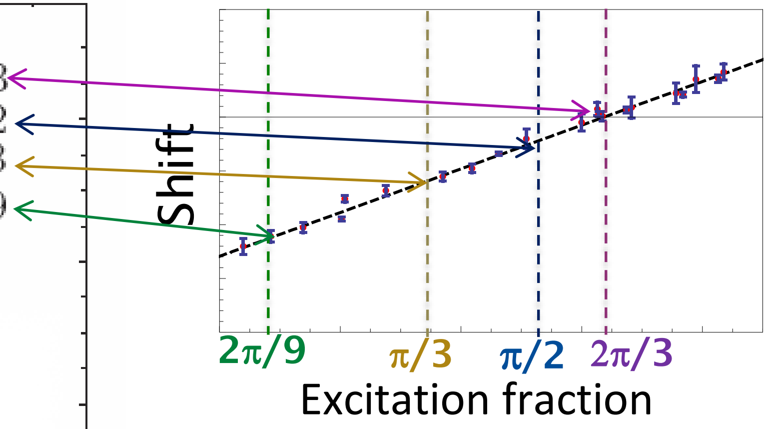
Ramsey fringe decay vs. the spin tipping angle

To eliminate the effect of decay we normalize the amplitude with atom number



Symbols: Exp data lines: mean field



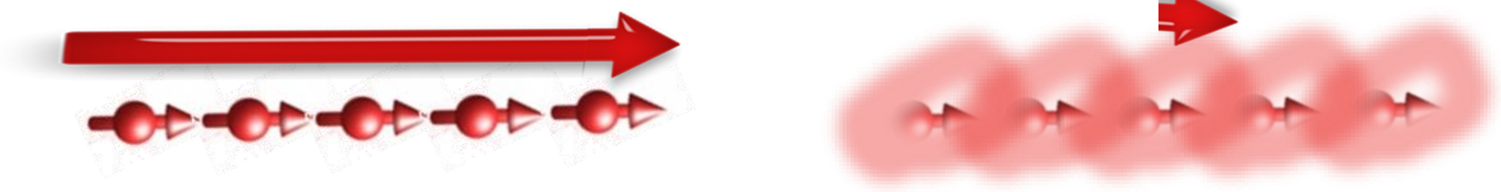


Failure of mean field theory

Quantum correlations – beyond mean field

M. Martin *et al,* Science 341, 632 (2013), Rey *et al* Annals of Physics 340, 311 (2014)

Contrast: $\sqrt{\langle S^x\rangle^2 + \langle S^y\rangle^2}$



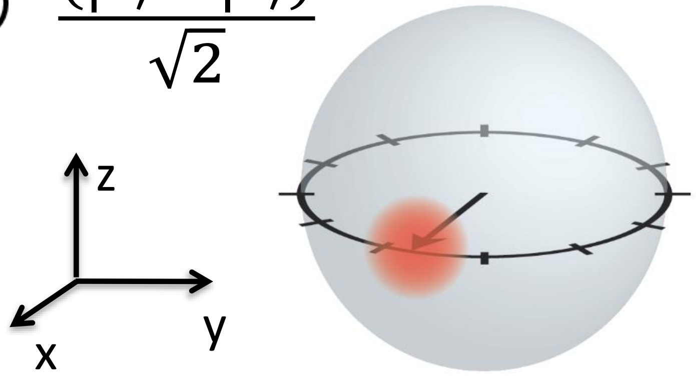
Quantum correlations induce faster decay of the amplitude

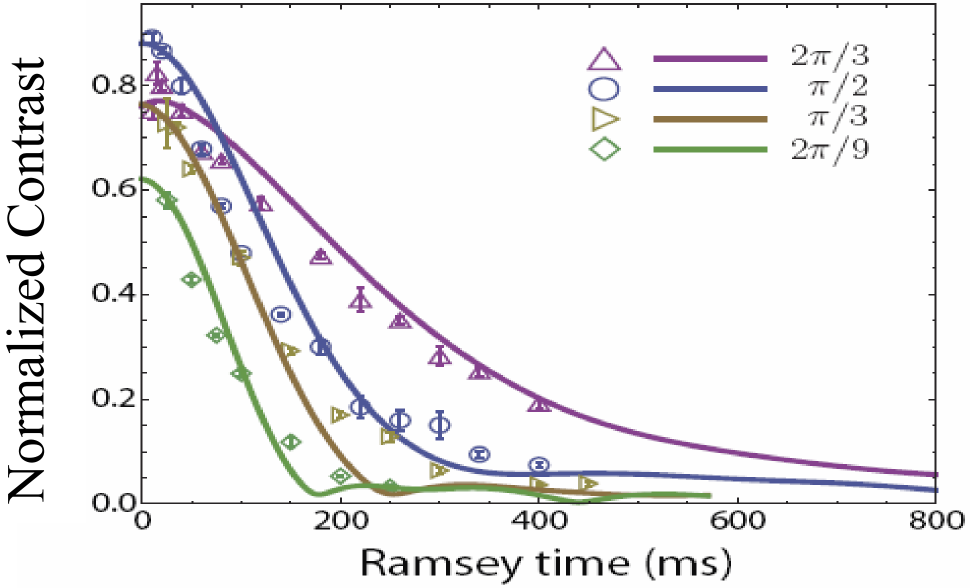
We can solve for the full many body solution using the Truncated Wigner Approximation: Average over random initial conditions that account for quantum noise distribution

Coherent Spin states $\otimes \dfrac{(|\uparrow\rangle + |\downarrow\rangle)}{\sqrt{2}}$

$\langle \hat{S}_x \rangle = N/2 \qquad \langle \hat{S}_{y,z} \rangle = 0$

$\langle \Delta \hat{S}_{y,z} \rangle = \sqrt{\frac{N}{4}}$





See: Polkovnikov Annals of Physics 325,1790 (2010), J. Schachenmayer *et al* PRX 5, 011022 (2015)

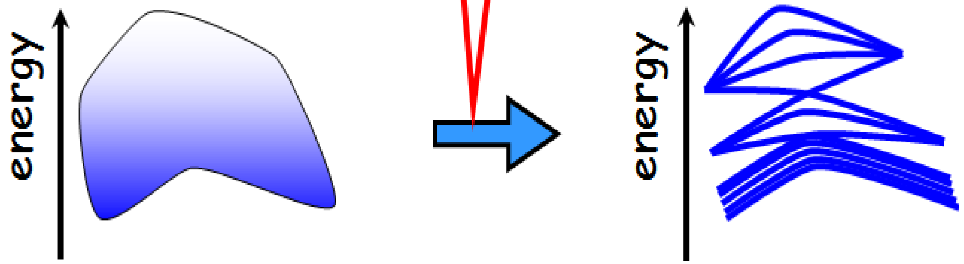
Why alkaline-earth (like) atoms?

Unique atomic properties

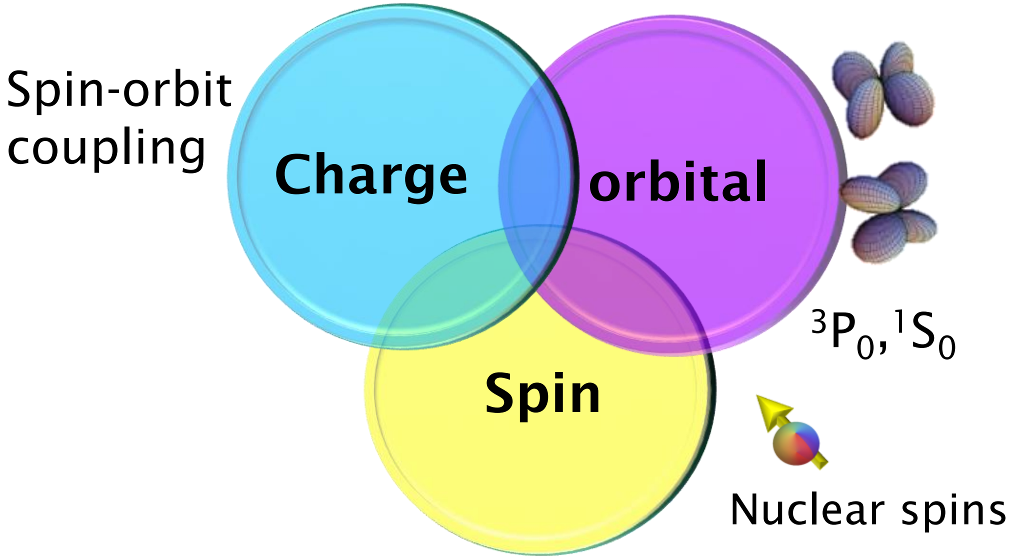
Atomic clock experiments

Many-Body Physics

Clock precision = 



Strongly correlated materials

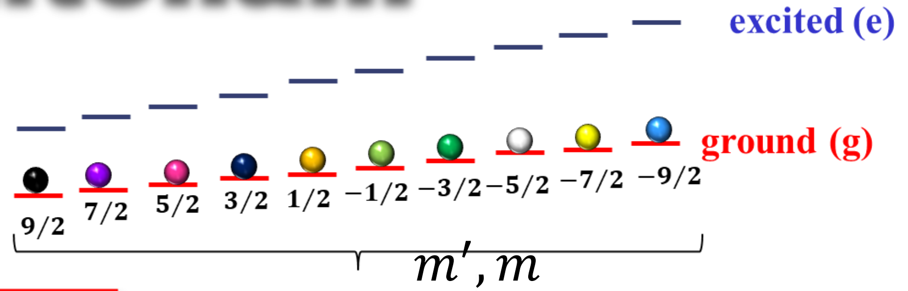


S-Wave Many body Hamiltonain

$\sigma, \sigma' = e, g$

Fermionic Field operator

$$\{\hat{\psi}_{\sigma m}(\boldsymbol{r}), \hat{\psi}^{\dagger}_{\sigma' m'}(\boldsymbol{r}')\} = \delta(\boldsymbol{r}-\boldsymbol{r}')\delta_{\sigma,\sigma'}\delta_{m,m'}$$



$$H_s = \frac{2\pi\hbar^2 a_{gg}}{M}\sum_{mm'}\int \mathrm{d}^3\mathbf{r}\,\psi^{\dagger}_{gm}(\mathbf{r})\psi^{\dagger}_{gm'}(\mathbf{r})\psi_{gm'}(\mathbf{r})\psi_{gm}(\mathbf{r})$$

Ground-ground

$$+\frac{2\pi\hbar^2 a_{ee}}{M}\sum_{mm'}\int \mathrm{d}^3\mathbf{r}\,\psi^{\dagger}_{em}(\mathbf{r})\psi^{\dagger}_{em'}(\mathbf{r})\psi_{em'}(\mathbf{r})\psi_{em}(\mathbf{r})$$

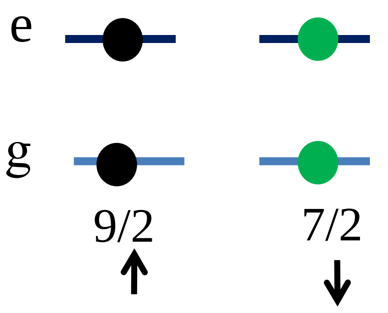
Excited-Excited

$$+\frac{2\pi\hbar^2 (a^{-}_{eg}+a^{+}_{eg})}{M}\sum_{mm'}\int \mathrm{d}^3\mathbf{r}\,\psi^{\dagger}_{gm}(\mathbf{r})\psi^{\dagger}_{em'}(\mathbf{r})\psi_{em'}(\mathbf{r})\psi_{gm}(\mathbf{r})$$

Direct

$$+\frac{2\pi\hbar^2 (a^{-}_{eg}-a^{+}_{eg})}{M}\sum_{mm'}\int \mathrm{d}^3\mathbf{r}\,\psi^{\dagger}_{gm}(\mathbf{r})\psi^{\dagger}_{em'}(\mathbf{r})\psi_{em}(\mathbf{r})\psi_{gm'}(\mathbf{r}).$$

Exchange



P-wave Many body Hamiltonain

$$H_p = \boxed{\frac{3\pi\hbar^2 b_{gg}^3}{2M}\sum_{mm'}\int \mathrm{d}^3\mathbf{r}\,[(\nabla\psi_{gm}^\dagger)\psi_{gm'}^\dagger - \psi_{gm}^\dagger(\nabla\psi_{gm'}^\dagger)]\cdot[\psi_{gm'}(\nabla\psi_{gm}) - (\nabla\psi_{gm'})\psi_{gm}]}$$

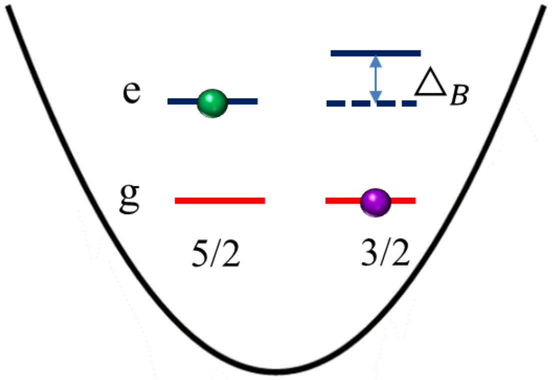
$$+ \boxed{\frac{3\pi\hbar^2 b_{ee}^3}{2M}\sum_{mm'}\int \mathrm{d}^3\mathbf{r}\,[(\nabla\psi_{em}^\dagger)\psi_{em'}^\dagger - \psi_{em}^\dagger(\nabla\psi_{em'}^\dagger)]\cdot[\psi_{em'}(\nabla\psi_{em}) - (\nabla\psi_{em'})\psi_{em}]}$$

$$+ \boxed{\frac{3\pi\hbar^2[(b_{eg}^+)^3 + (b_{eg}^-)^3]}{2M}\sum_{mm'}\int \mathrm{d}^3\mathbf{r}\,[(\nabla\psi_{gm}^\dagger)\psi_{em'}^\dagger - \psi_{gm}^\dagger(\nabla\psi_{em'}^\dagger)]\cdot[\psi_{em'}(\nabla\psi_{gm}) - (\nabla\psi_{em'})\psi_{gm}]}$$

$$+ \boxed{\frac{3\pi\hbar^2[(b_{eg}^+)^3 - (b_{eg}^-)^3]}{2M}\sum_{mm'}\int \mathrm{d}^3\mathbf{r}\,[(\nabla\psi_{gm}^\dagger)\psi_{em'}^\dagger - \psi_{gm}^\dagger(\nabla\psi_{em'}^\dagger)]\cdot[\psi_{em}(\nabla\psi_{gm'}) - (\nabla\psi_{em})\psi_{gm'}]}$$

Probing Exchange Interactions: Two ^{173}Yb Atoms

G. Cappellini *et al* PRL 113, 120402 (2014) && F. Scazza et al Nature Physics 10, pages 779 (2014)



Lattice site

Eigenstates of Interaction

$$U_{eg}^{\pm} \propto \frac{4\pi\hbar^2}{m} a_{eg}^{\pm}$$

$$|+\rangle = \frac{|eg\rangle + |ge\rangle}{\sqrt{2}} \otimes \frac{|\uparrow\downarrow\rangle - |\uparrow\downarrow\rangle}{\sqrt{2}}$$

$$|-\rangle = \frac{|eg\rangle - |ge\rangle}{\sqrt{2}} \otimes \frac{|\uparrow\downarrow\rangle + |\uparrow\downarrow\rangle}{\sqrt{2}}$$

$$H_{eg} = \begin{pmatrix} U_{eg}^{+} & \Delta_B \\ \Delta_B & U_{eg}^{-} \end{pmatrix}$$

$$|\psi(0)\rangle = |e\downarrow, g\uparrow\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}}$$

At $\Delta_B = 0$

$$P(|g\uparrow\rangle)(t) = \frac{1}{2}\left[1 + \cos\left(\frac{2V_{\text{ex}}}{\hbar}t\right)\right].$$

Exchange $\boxed{V_{ex} = \frac{U_{eg}^{+} - U_{eg}^{-}}{2}}$

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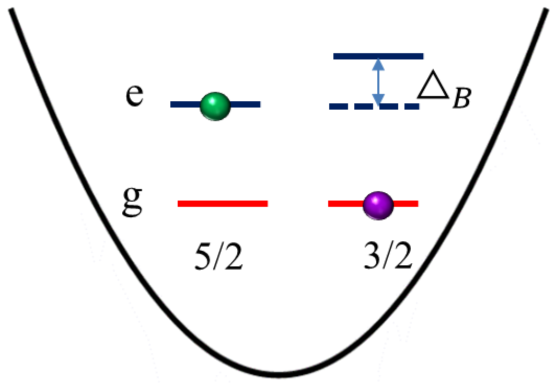

Lattice site

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$$U_{eg}^{\pm} \propto \frac{4\pi\hbar^2}{m} a_{eg}^{\pm}$$

$$|+\rangle = \frac{|eg\rangle + |ge\rangle}{\sqrt{2}} \otimes \frac{|\uparrow\downarrow\rangle - |\uparrow\downarrow\rangle}{\sqrt{2}}$$

$$|-\rangle = \frac{|eg\rangle - |ge\rangle}{\sqrt{2}} \otimes \frac{|\uparrow\downarrow\rangle + |\uparrow\downarrow\rangle}{\sqrt{2}}$$

$$H_{eg} = \begin{pmatrix} U_{eg}^{+} & \Delta_B \\ \Delta_B & U_{eg}^{-} \end{pmatrix}$$

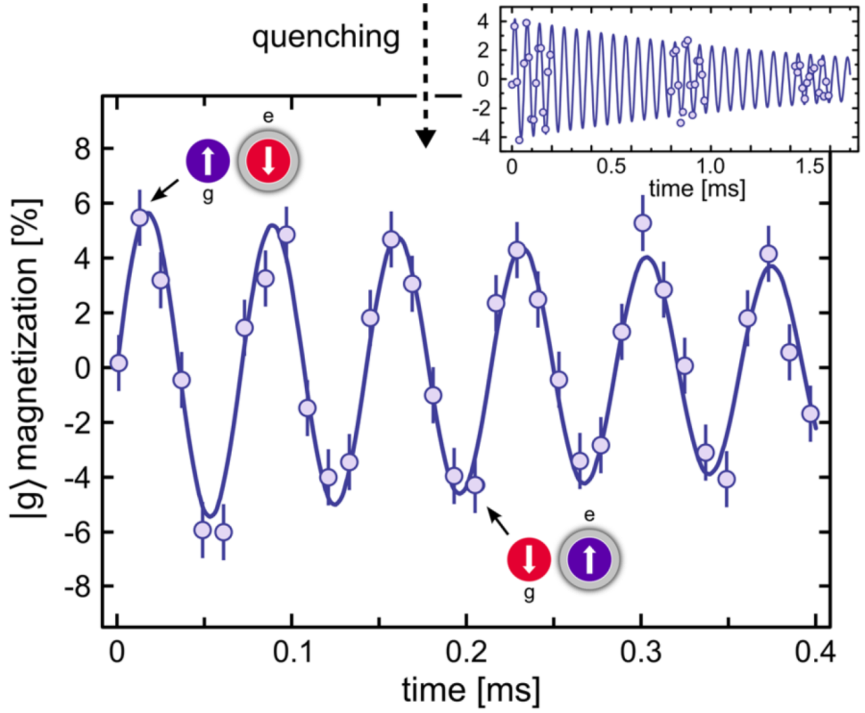
$$a_{eg}^{-} = 219.5(29)a_0$$
$$a_{eg}^{+} = 3300(300)a_0$$

$$|\psi(0)\rangle = |e\downarrow, g\uparrow\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}}$$

At $\Delta_B = 0$

$$P(|g\uparrow\rangle)(t) = \frac{1}{2}\left[1 + \cos\left(\frac{2V_{\rm ex}}{\hbar} t\right)\right].$$

Exchange $\boxed{V_{ex} = \dfrac{U_{eg}^{+} - U_{eg}^{-}}{2}}$



Depends on magnetic field

Orbital Feshbach Resonance

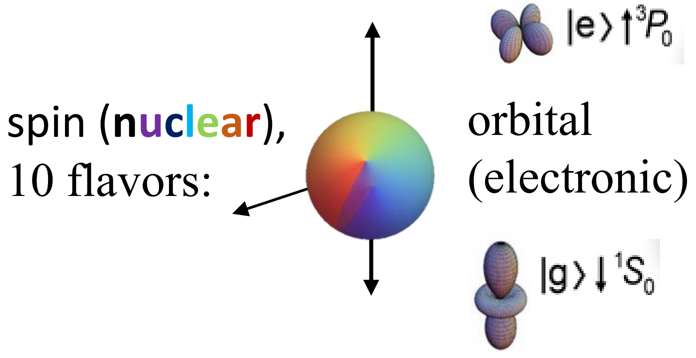
PRL 115, 135301(2015)

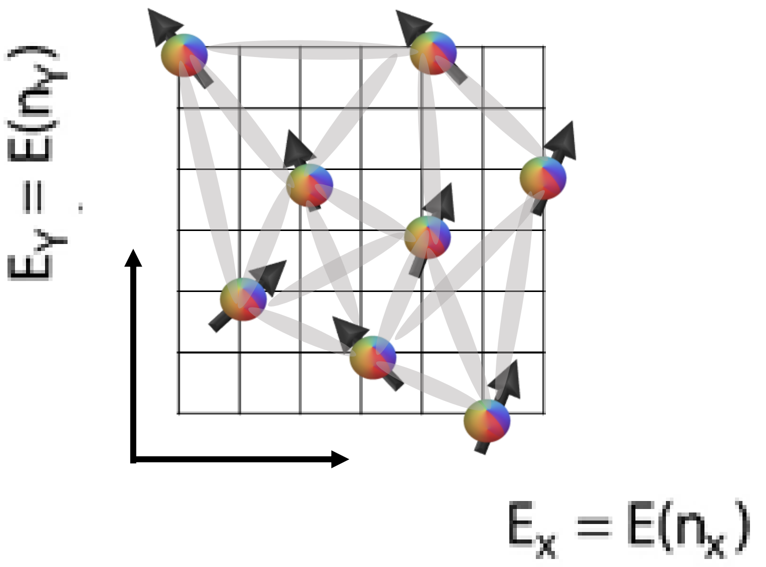
PRL 115, 265302 (2015)

PRL 115, 265301 (2015)

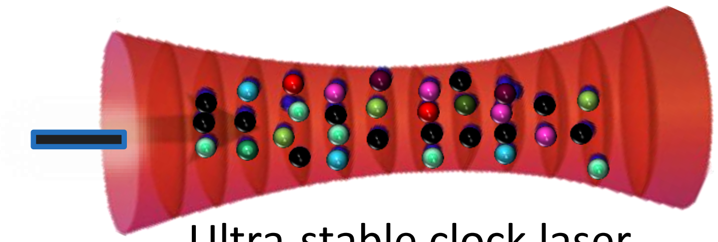
SU(N) orbital magnetism in a 1D lattice Sr Clock

Zhang et al Science, 345,1467 (2014)



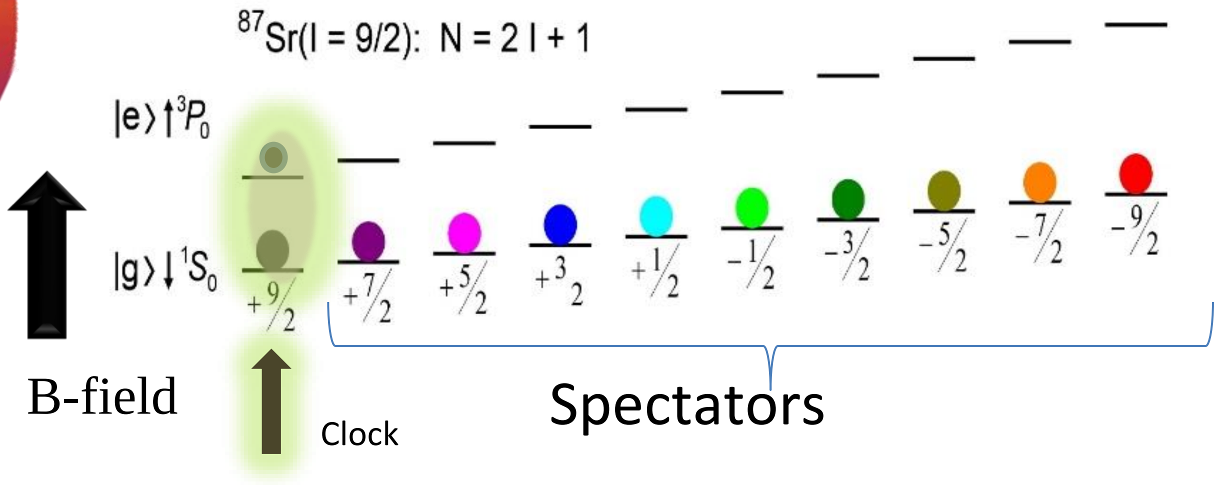


Atoms trapped in array of disk-shaped pancake



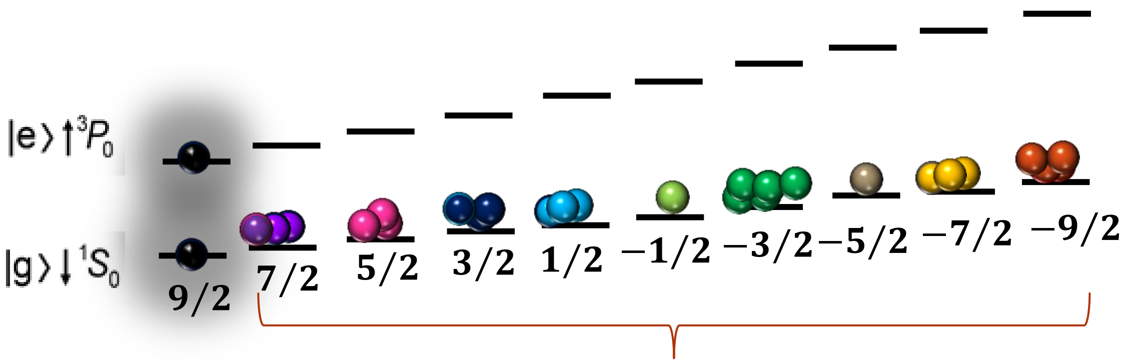
Ultra-stable clock laser

Statistical mixture



Density shifts and SU(N) symmetry

Zhang et al Science, 345,1467 (2014)



$\mathcal{N}_I$

Interrogated

$\mathcal{N}_S$: **Spectators**

- SU(N): density shift only depends on the total number of spectators not on its distribution

$$\Delta\nu = \Delta\nu^I + \Delta\nu^S$$

Interrogated atoms p-wave shift

$$\Delta\nu^I \sim \mathcal{N}_I\,(\bar{C} - \cos\theta \;\; \bar{\chi})$$

Spectators generate a density shift

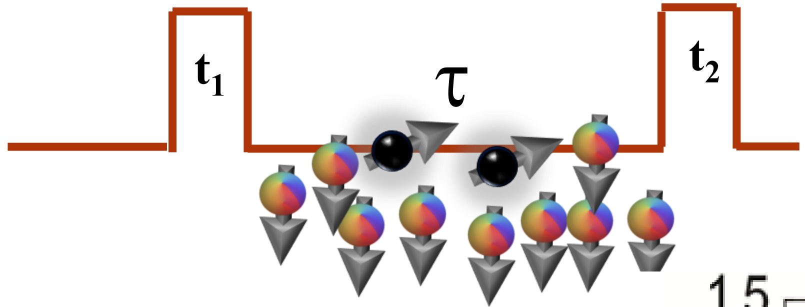
$$\Delta\nu^S = \bar{\Lambda}\mathcal{N}_S$$

↓

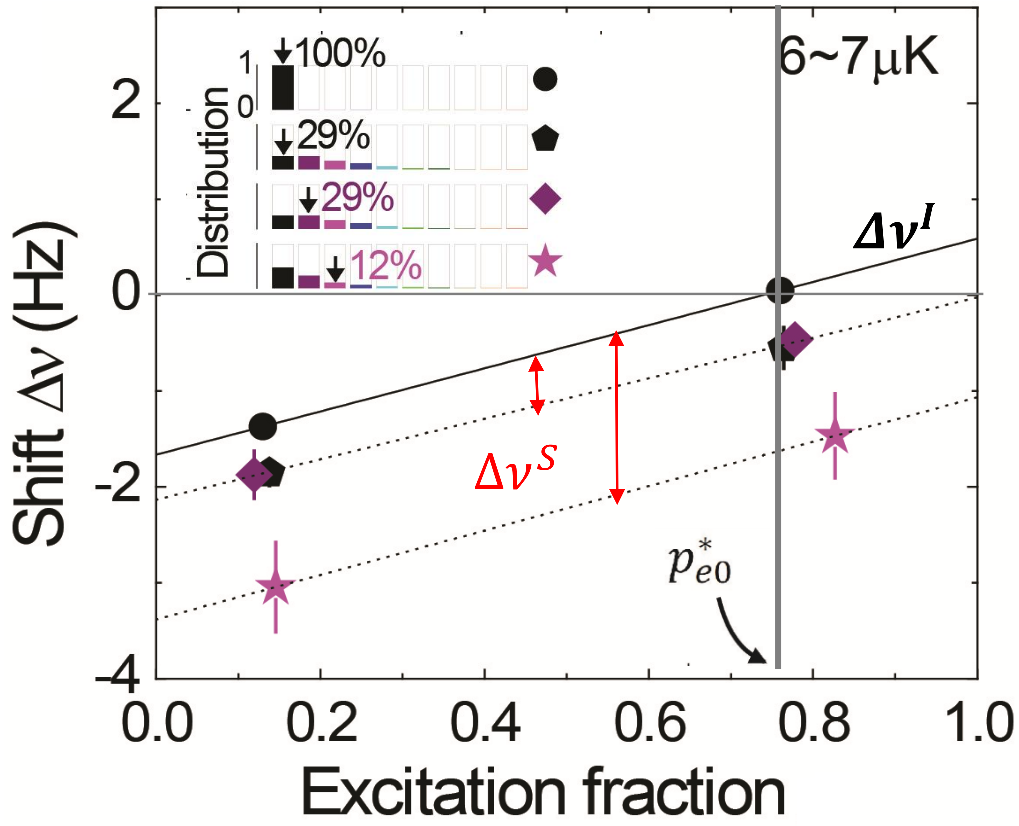
Both *s* and *p*-wave contributions

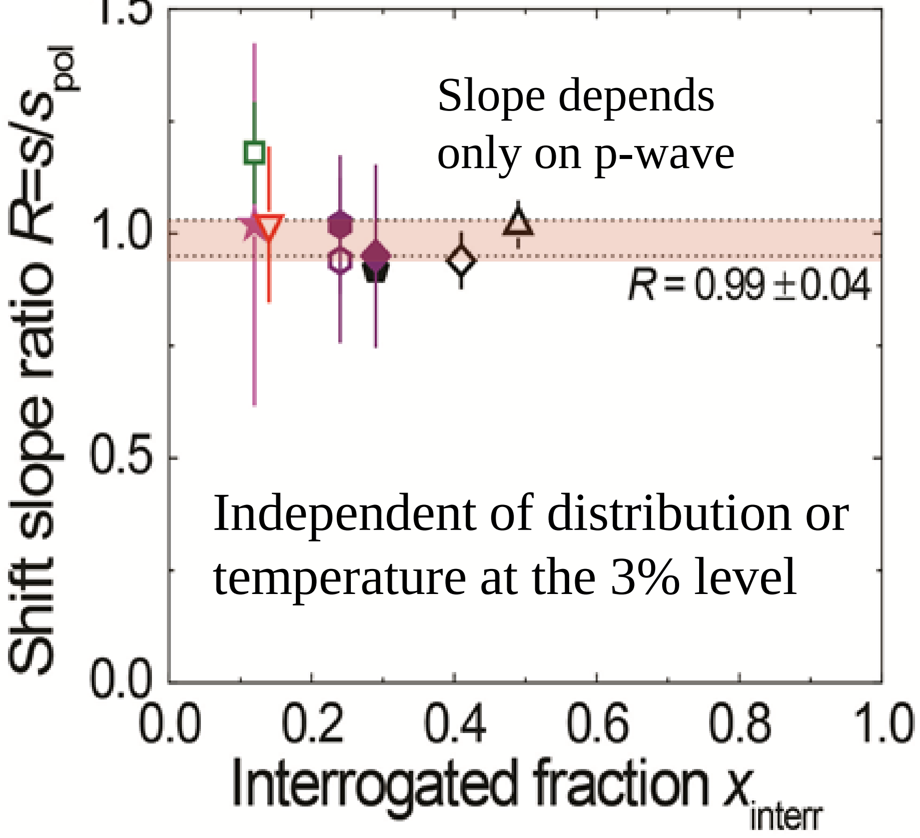
Density shifts and SU(N) symmetry

Zhang et al Science, 345,1467 (2014)



Normalized to 4000 interrogated atoms





Slope depends only on p-wave

Independent of distribution or temperature at the 3% level

Determination of scattering parameters

Use: Relation between p and s-wave through Van-der-Waals coefficient

Z. Idziaszek, P. S. Julienne, Phys. Rev. Lett. 104, 113202 (2010).

Channel	*S*-wave(ao)	*P*-wave(ao)	Determination
gg	**96.2(1)**	**74(2)**	**[S-wave] Two-photon photo-associative [P-wave] Analytic relation**
eg⁺	**169(8)**	**-169(23)**	**[S-wave] Analytic relation [P-wave] Density shift in a polarized sample**
eg⁻	**68(22)**	-42^{+103}_{-22}	**[S-wave] Density shift in a spin mixture at different temperatures [P-wave] Analytic relation**
***ee* (elastic)**	**176(11)**	**-119(18)**	**[S-wave] Analytic relation [P-wave] Density shift in a polarized sample**
***ee* (inelastic)**	$\tilde{a}_{ee}$ **= 46(19)**	$\tilde{b}_{ee}$ **= 125(15)**	**Two-body loss measurement**

$$\frac{A_\eta}{\bar{a}} = 1 + \left(\frac{B_\eta}{\bar{a}}\right)^3 \left[\left(\frac{B_\eta}{\bar{a}}\right)^3 + 2.128\right]^{-1}$$

$$\bar{a} = \frac{2\pi}{\Gamma\left(\frac{1}{4}\right)^2}\left(\frac{2\mu C_6}{\hbar^2}\right)^{1/4}, \Gamma\left(\tfrac{1}{4}\right) \approx 3.626$$

www.idigitaltimes.com/big-bang-theory-season-8-premiere-spoilers-will-sheldons-spin-symmetry-theory-paper-pass-peer-review

Getting Started

iDigitalTimes SHARE THIS STORY

According to scientists working on the big bang theory it all began with Sheldon's participation in a JILA research team hoping to confirm via direct observation the spin symmetry theory in quantum physics. Sheldon, used to working in the theoretical realm, found his colleagues alienating, like Penny in Seasons 1 through 3 of "The Big Bang Theory." However, under the tutelage of Ana Maria Rey and Jun Ye, Sheldon has learned how best to unify the two fields, resulting in their remarkable new findings.