Fermions: even protons and odd number of neutrons

two valence electrons ⇒ even protons ⇒ odd neutrons

Bosons: even protons, even neutrons ⇒ $I=0$

Remember $^{2S+1}L_J$

Ground $^1S_0$
1) Singlet
2) $L=0$
3) $J=0$

$^3P_0$:
1) Triplet
2) $L=1$
3) $J=0$

mHz vs MHz : 9 orders of magnitud narrower

Why: $J=0 \nLeftrightarrow J=0$ forbidden selection rules.

$S=0 \nLeftrightarrow S=1$ spin forbidden.

Then how is that possible?

Nuclear spin: in Fermionic Isotopes

Hyperfine structure:

$H = A\, I \cdot J$

$^3P_0,\ ^3P_1,\ ^3P_2\ ,\ ^1P_1$

$\Downarrow \qquad \Downarrow \qquad \swarrow$

$F = 9/2 \quad F = 9/2,\ 11/2,\ 7/2$

- Ramsey Spectroscopy

$H = \hbar\Omega S_x + \hbar\delta S_z \quad | \quad \Omega \gg \delta$

$\langle\vec{S}\rangle = \frac{1}{2}\{0, \sin\theta, -\cos\theta\} \quad \theta = \Omega t$

pulse area

(1) Rotating frame:

$H = \hbar\omega_0 |e\rangle\langle e| + \frac{\hbar\Omega}{2}(\sigma^+ e^{-i\omega_L t} + \sigma^- e^{i\omega_L t})$

$\hat{U} = e^{-i\omega_L t\, \sigma_z/2}$

$U^\dagger \sigma^\pm \hat{U} = \sigma^\pm e^{\pm i\omega_L t}$

$$\underbrace{\frac{\partial\Psi}{\partial t} = U^\dagger H U + i\hbar\left(\frac{\partial U^\dagger}{\partial t}\right)U}_{H_{eff}}$$

$H_{eff} = \hbar(\omega_0 - \omega_L)|e\rangle\langle e| + \frac{\hbar\Omega}{2}(\sigma^+ + \sigma^-)$

$= -\hbar\delta/2\ \tilde{\sigma}_z + \hbar\Omega S_x \quad = -\hbar\delta S_z + \hbar\Omega S_x$

2) Dark time

$\langle S^- \rangle = -\frac{i}{2} \sin\theta\, e^{-i\delta t}$ → phase

$|\langle S^- \rangle| =$ contrast

3) Detected by $\pi/2$ rotation, measure population

$\langle S_y \rangle \Rightarrow \langle S_z \rangle = \frac{1}{2} \sin\theta \cos\delta t$

⇒ Optical Clocks : optical frequencies

Microwave clocks : GHz frequencies

1D lattice clock

Signal $\propto N$

Noise $\propto \sqrt{N}$

$\frac{\text{Signal}}{\text{Noise}} \propto \sqrt{N}$

SU(N) symmetry

Let's start with SU(2) e.g. Coulomb interactions between electrons

$V \propto \frac{1}{r}$ Independent of spin

However singlet and triplet states can have different energies due to quantum statistics

Imagine the case of contact interactions

$$|\psi\rangle = \underbrace{\frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}}\ \underbrace{\psi(r_1, r_2)}_{\text{symmetric}}$$

$$|\psi\rangle = \frac{(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)}{\sqrt{2}}\ \underbrace{\tilde{\psi}(r_1, r_2)}_{\text{anti-symmetric}}$$

also $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$

s-wave interactions are described by

$$H = -V(r_1, r_2) \times (\vec{S}_1 \cdot \vec{S}_2 - 1/4)$$ → only singlets interact via s-wave

H is SU(2) symmetric

commutes with $\hat{S}_x = \frac{1}{2}(\sigma_{x1} + \sigma_{x2})$

$\hat{S}_y = \frac{1}{2}(\sigma_{y1} + \sigma_{y2})$ — 3 generators of SU(2)

$S_z = 1/2\ (\sigma_{z1} + \sigma_{z2})$

SU(N) symmetry $N = 2I+1$

$N = 10$ for ^{87}Sr.

$$H_{int} = -\sum_{i,j} V(r_i, r_j)\, S^m_n(i)\, S^n_m(j)$$

$S^m_n(i) \equiv |n\rangle_i \langle m|$ $\quad N^2-1$ SU(N) generators.

$$S^m_n = \sum_i S^m_n(i)$$

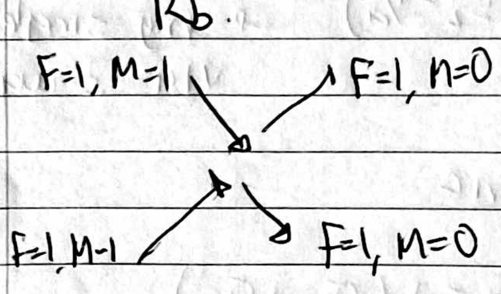
$$[H, S^m_n] = 0 \quad \forall\, n, m$$

→ No mode changing collisions

→ Equal scattering lenghts.

Not the case in alkali atoms

Rb.



Rb. $a_{F=0}$ $a_{F=2}$

Two scattering lenghts

This is forbidden in alkaline-earth atoms

All a are the same

$$\frac{\delta a_s}{a_s} \simeq \Delta V \, \delta t$$

δt: time in short range potential $\sim$ 1 ps

ΔV: different potential experiensed by nuclear spins

$^1S_0$

$$\Delta V \sim \frac{H_{HF}^2}{\Delta E} \simeq \frac{(300\ \text{MHz})^2}{(400\ \text{THz})} \sim 200\ \text{Hz}$$

$$\frac{\delta a_s}{a_s} \sim 10^{-9}$$

$^3P_0 - {}^3P_0$, $^1S_0 - {}^3P_0$

$$\Delta V \sim H_{HF} \simeq 300\ \text{MHz} \rightarrow \frac{\delta a_s}{a_s} \sim 10^{-3}$$

first order.

Collisions $\quad U(1) \times SU(N)$

$\Downarrow$

N_e conserved $\quad$ up to ee losses

N_g conserved