

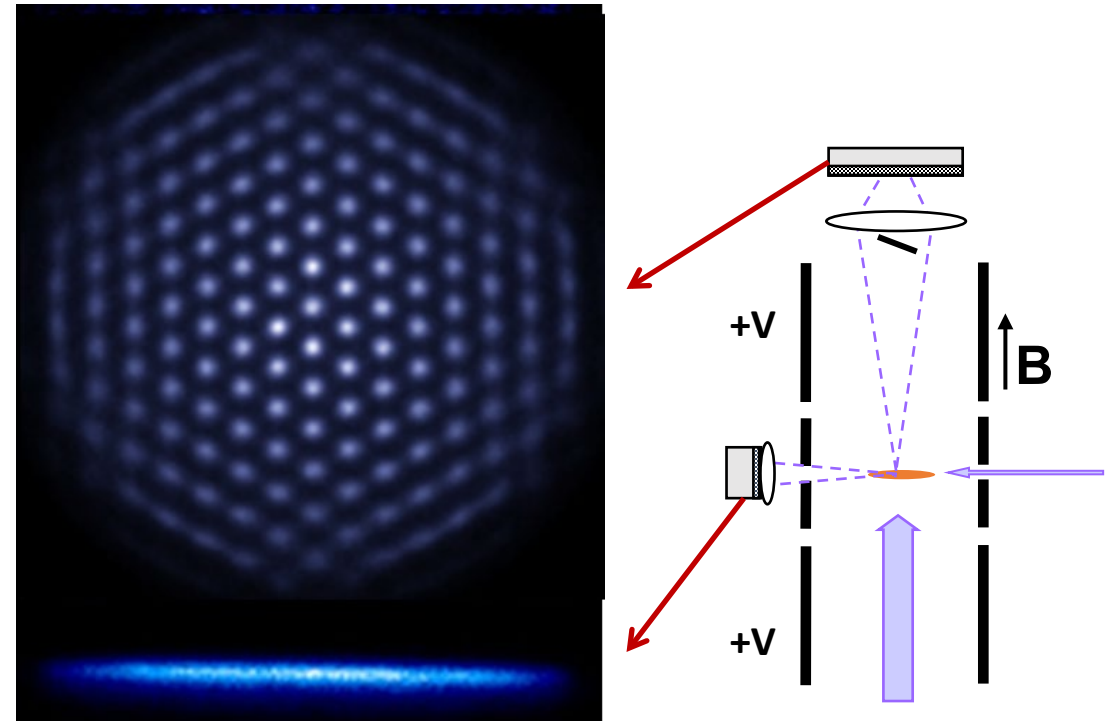
Quantum Simulation and Sensing with Large Trapped-Ion Crystals

John Bollinger
NIST-Boulder
Ion storage group

M. Affolter, J. Lilieholm, B. Bullock, K. Gilmore (Honeywell), E. Jordan (PTB), J. Bohnet (Honeywell), B. Sawyer (GTRI), J. Britton (ARL)

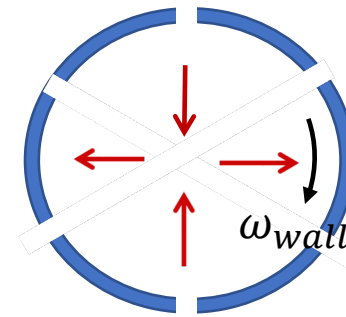
theory –Rey group (JILA/NIST)
Holland group (JILA/NIST)
Freericks group (Georgetown)
Michael Foss-Feig (Honeywell)
Dan Dubin (UCSD)

- Penning trap $\Leftrightarrow$ system for controlling large ion crystals
- global single- and multi-qubit operations
- potential NISQ platform with intro of single σ_z rotations
- quantum sensing of displacements; weak electric fields



Outline:

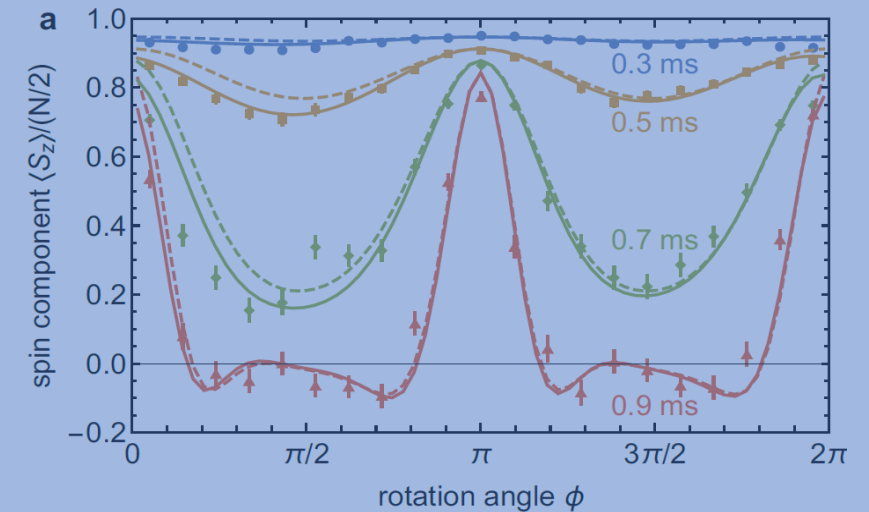
- ion crystals in Penning traps
 - high (4.5 T) magnetic field qubit
 - modes



- engineering tunable Ising interactions

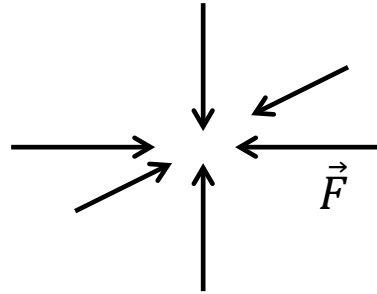
$$H_{\text{Ising}} = \frac{1}{N} \sum_{i < j} J_{i,j} \sigma_i^z \sigma_j^z$$

- benchmarking quantum dynamics, entanglement
 - spin squeezing
 - out-of-time-order correlations (OTOC)



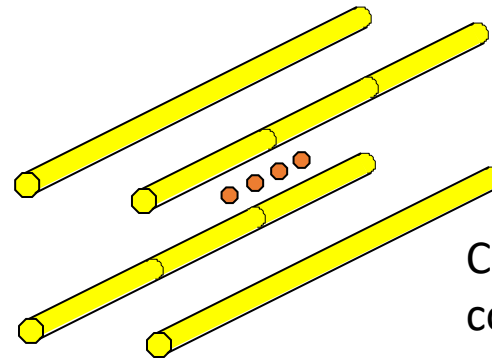
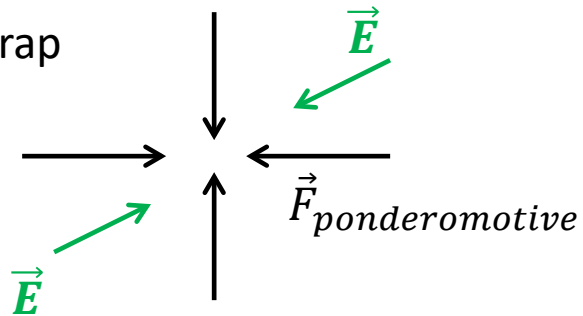
- measuring weak motional excitations and electric fields

rf (Paul) trap vs Penning trap



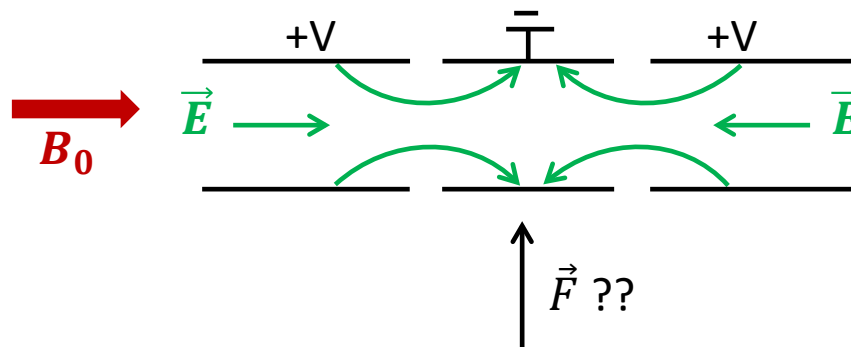
ideal ion trap
desire $\vec{\nabla} \cdot \vec{F} \neq 0$

linear rf trap



Confinement due to
conservation of energy

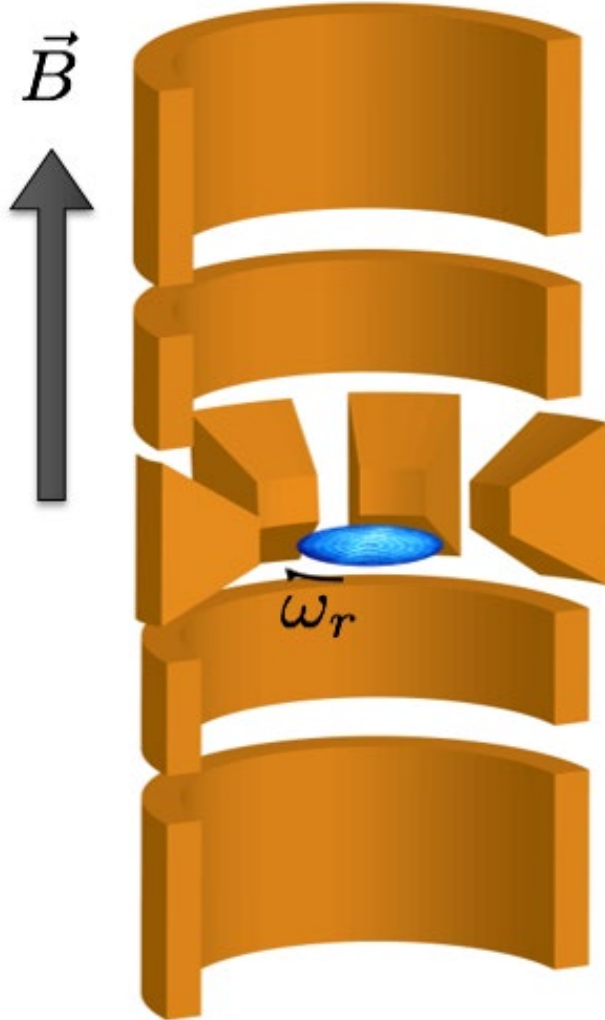
Penning trap



Dubin and O'Neil, RMP 71 (1999)
Radial confinement from conservation
of angular momentum:

$$\rho \sim \exp\left[(H - \omega_r P_\phi)/(k_B T)\right]$$

Penning trap: many particle confinement with static fields

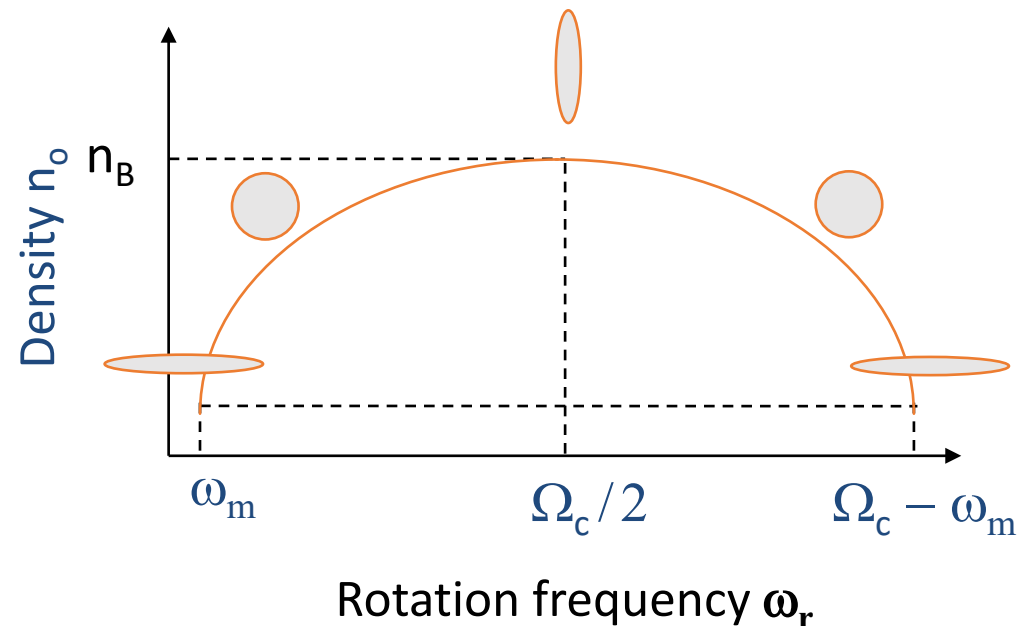


${}^9\text{Be}^+$, $B_0 = 4.5 \text{ T}$

$$\frac{\Omega_c}{2\pi} \sim 7.6 \text{ MHz}, \frac{\omega_z}{2\pi} \sim 1.6 \text{ MHz}, \frac{\omega_m}{2\pi} \sim 160 \text{ kHz}$$

- radial confinement due to rotation –
ion plasma rotates $v_\theta = \omega_r r$ due to $\mathbf{E} \times \mathbf{B}$ fields
Lorentz force from rotation is directed radially inward

$$\phi_{\text{trap}}(r, z) \approx \frac{1}{2} m \omega_z^2 \left(z^2 - \frac{r^2}{2} \right) \xrightarrow{\text{rotating frame}} \phi_{\text{rot}}(r, z) = \frac{1}{2} m \omega_z^2 \left(z^2 + \left(\frac{\omega_r (\Omega_c - \omega_r)}{\omega_z^2} - \frac{1}{2} \right) r^2 \right)$$

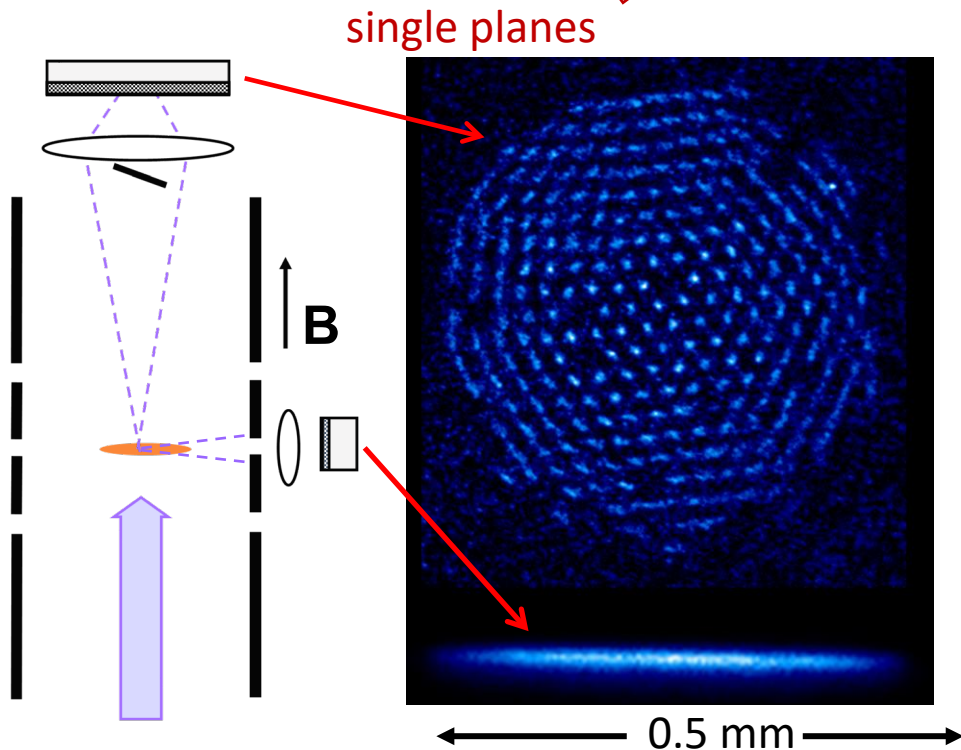
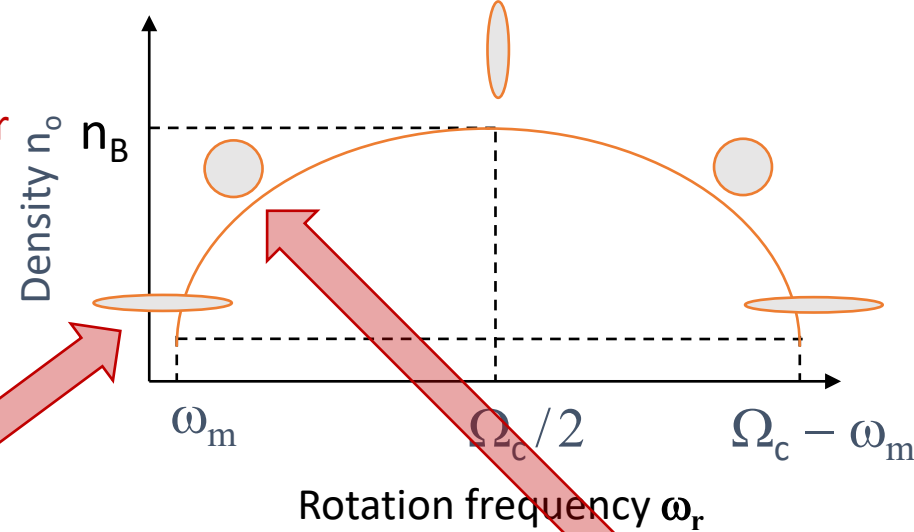


Ion crystals form as a result of minimizing Coulomb potential energy

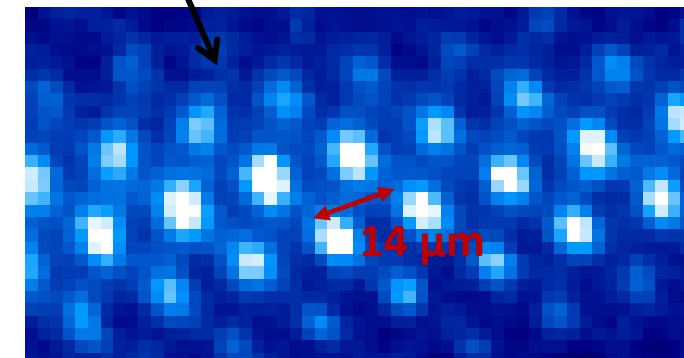
$T \rightarrow 0.4 \text{ mK}$ (Doppler laser cooling) $\Rightarrow q^2/a_{WS} \gg k_B T, 2a_{WS} \sim \text{ion spacing}$

type of crystal, nearest neighbor
ion spacing depend on ω_r

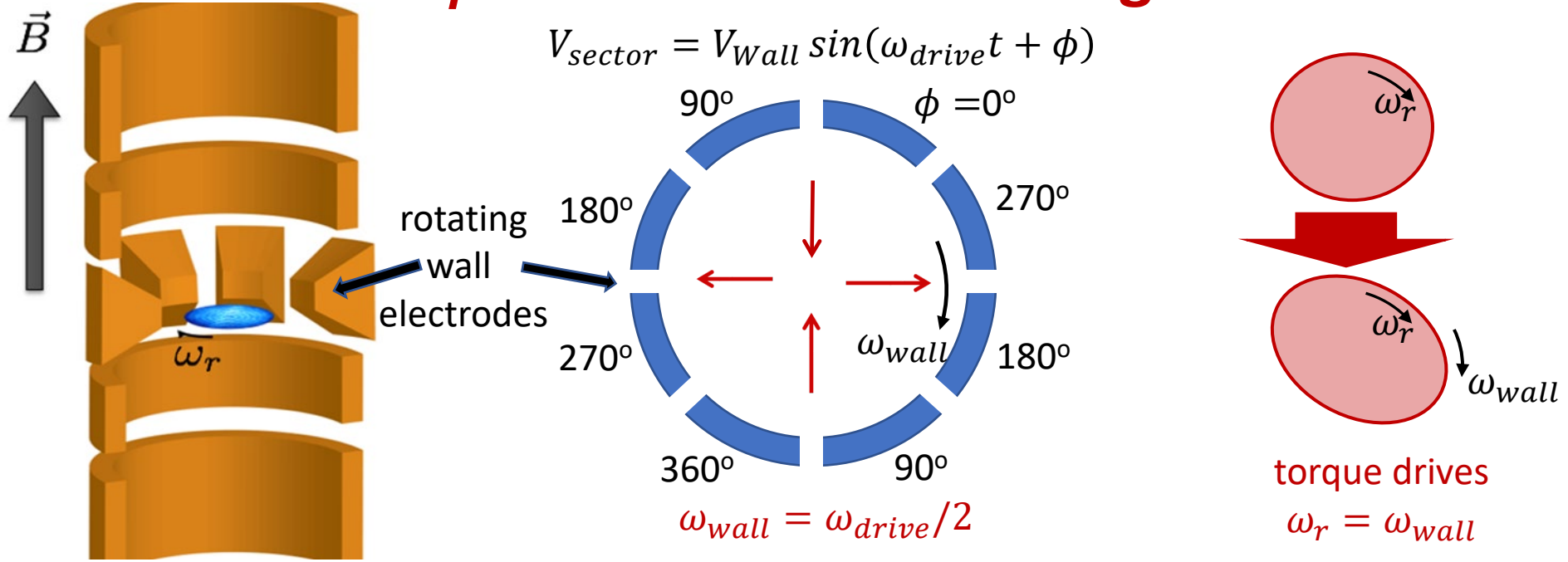
Mitchell et.al., Science (1998)



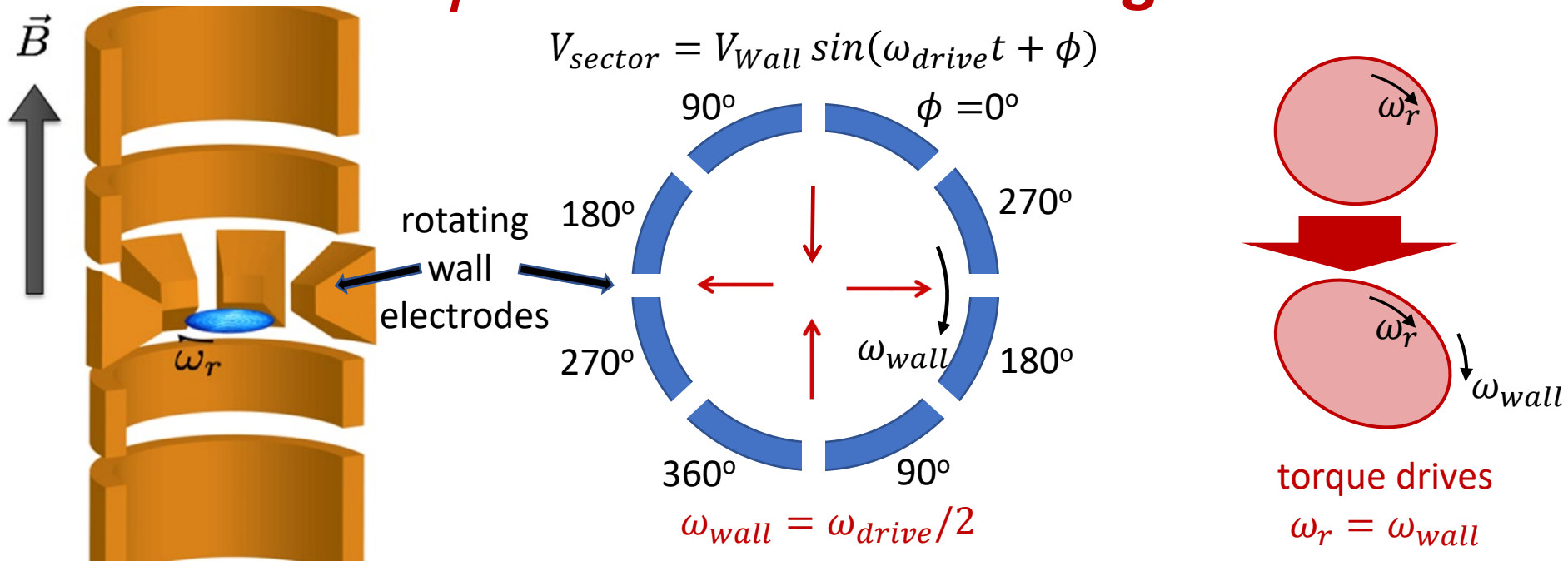
bcc crystals with $N > 100 \text{ k}$
observed with:
Bragg scattering
ion fluorescence imaging



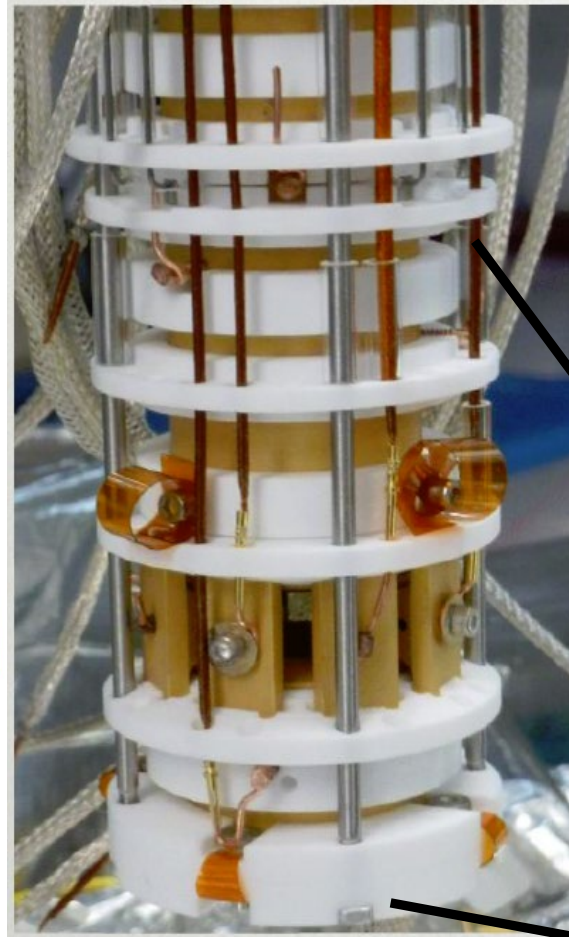
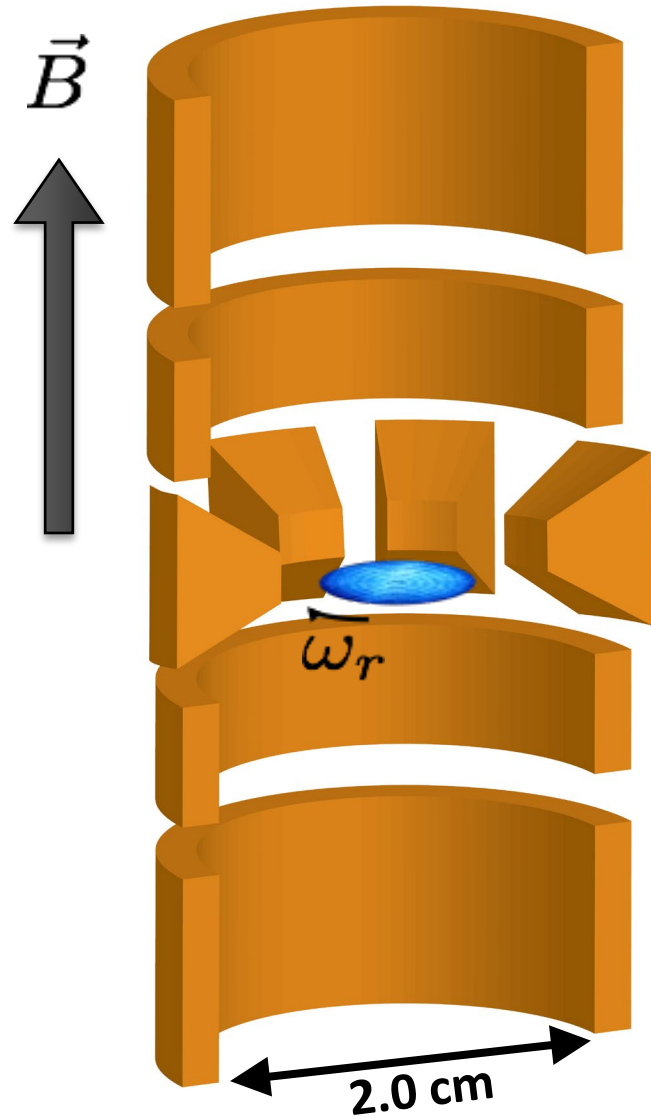
Precise ω_r control with a rotating electric field



Precise ω_r control with a rotating electric field

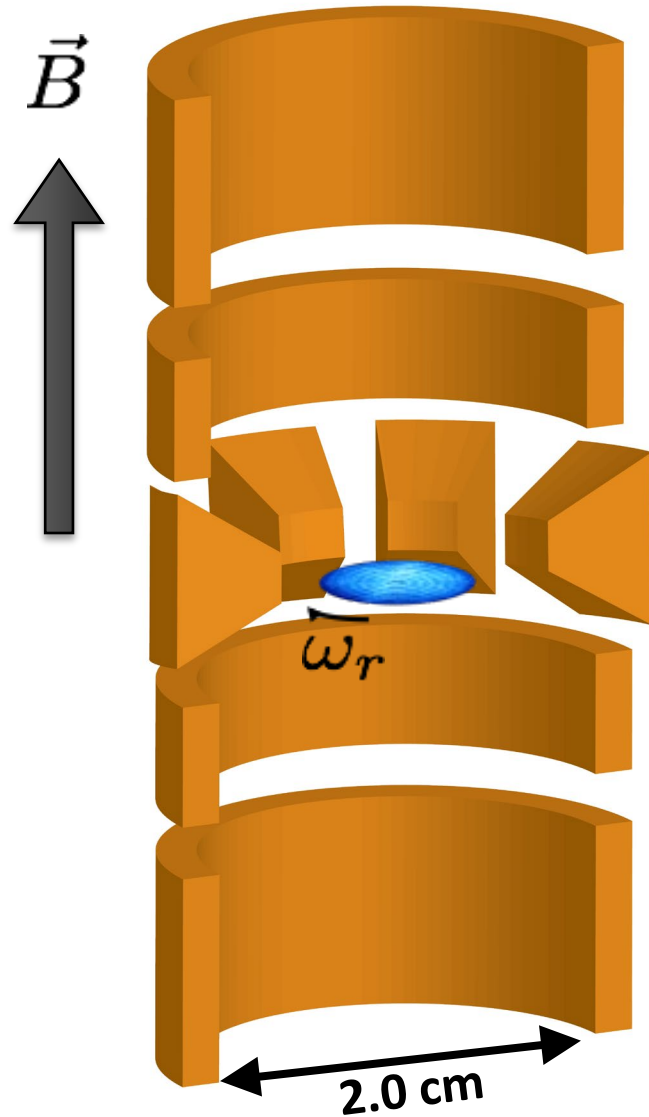


NIST Penning trap



$${}^9\text{Be}^+, B_0 = 4.5 \text{ T}, \frac{\Omega_c}{2\pi} \sim 7.6 \text{ MHz}, \frac{\omega_z}{2\pi} \sim 1.6 \text{ MHz}, \frac{\omega_m}{2\pi} \sim 160 \text{ kHz}$$

NIST Penning trap



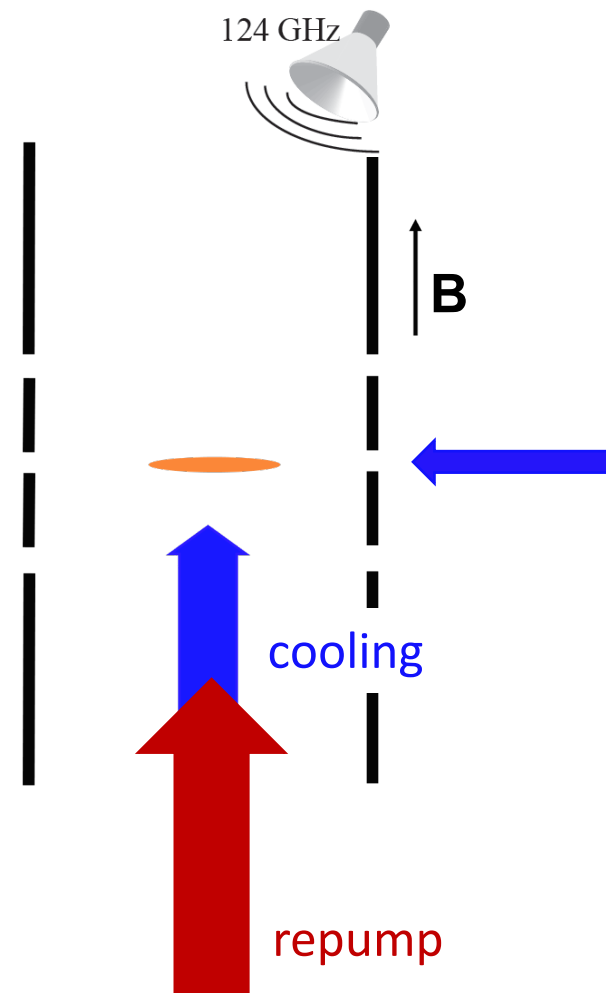
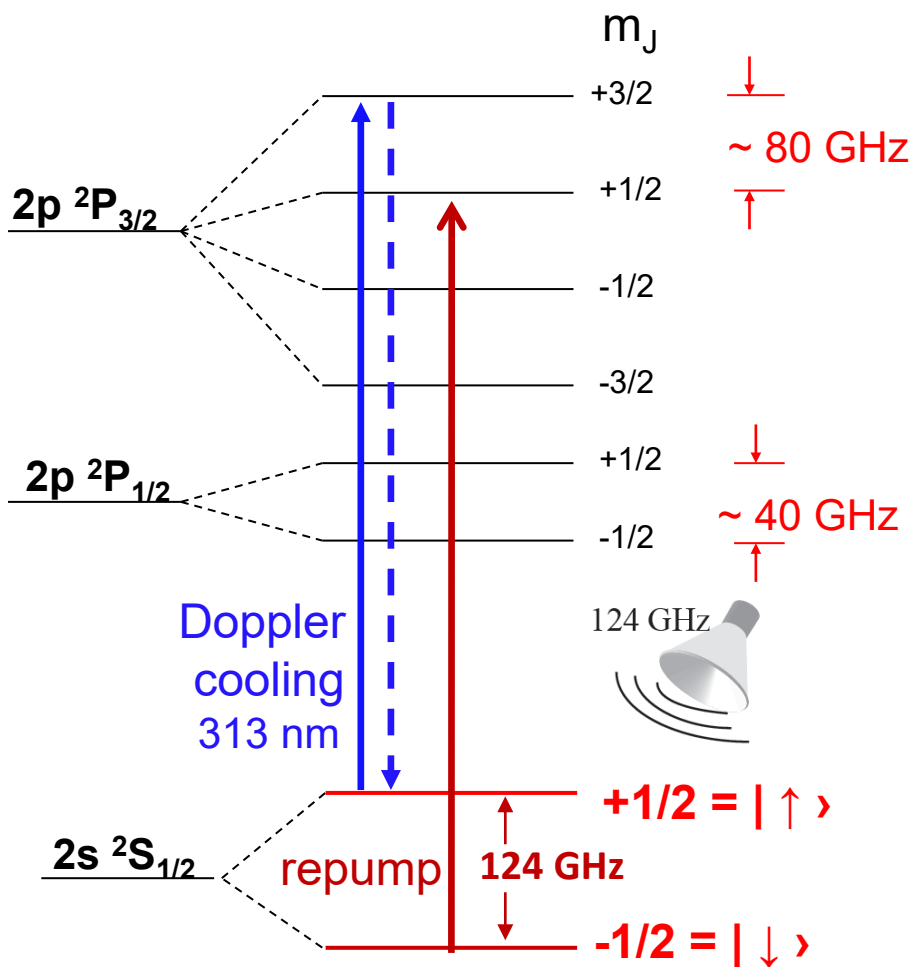
4.5 Tesla
superconducting
solenoid

Be⁺ high magnetic field qubit

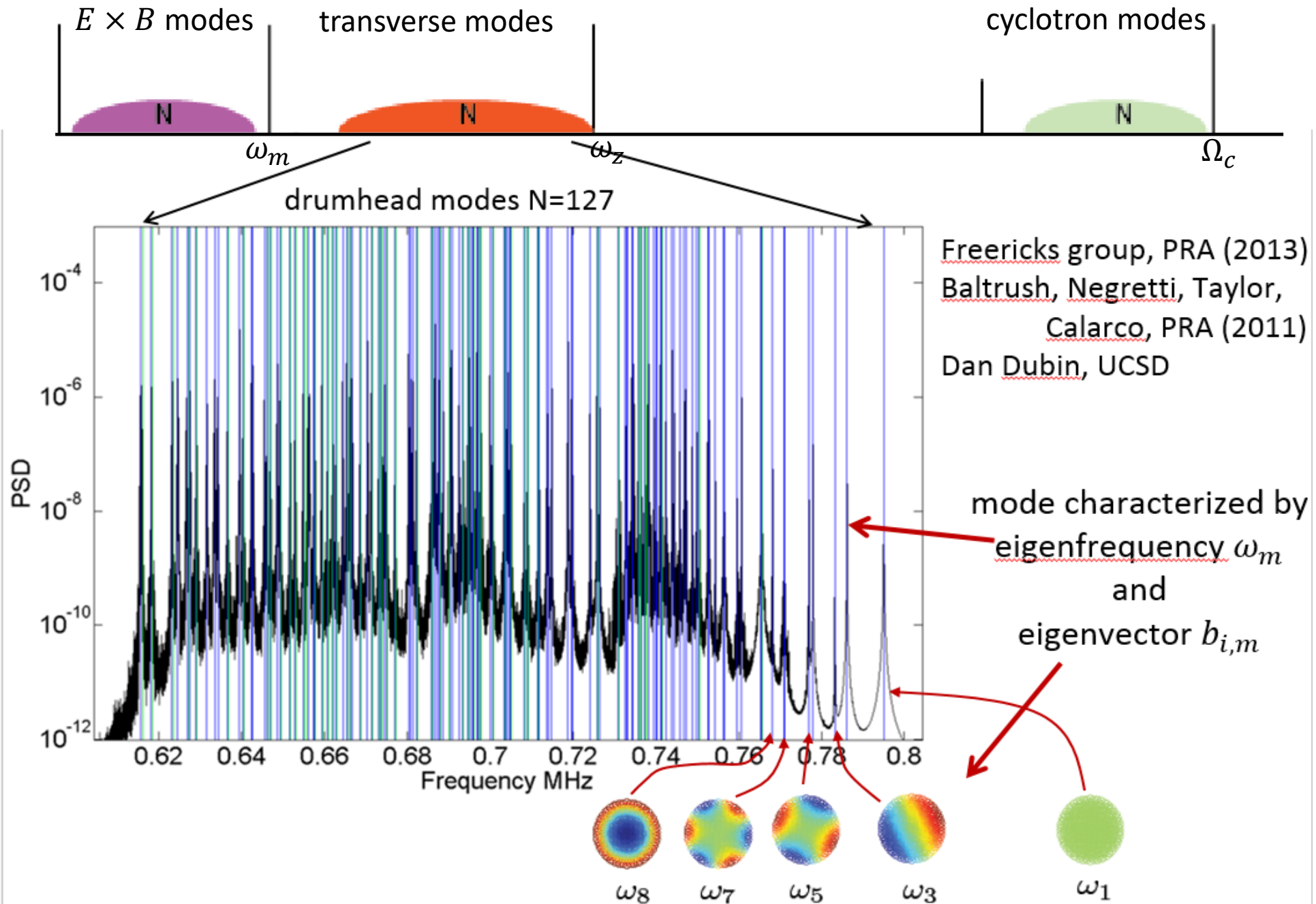
⁹Be⁺, B ~ 4.5 T, $\omega_o / 2\pi \sim 124.1$ GHz

$$H_{\mu W} = \sum_i B_{\perp} \hat{\sigma}_i^x,$$

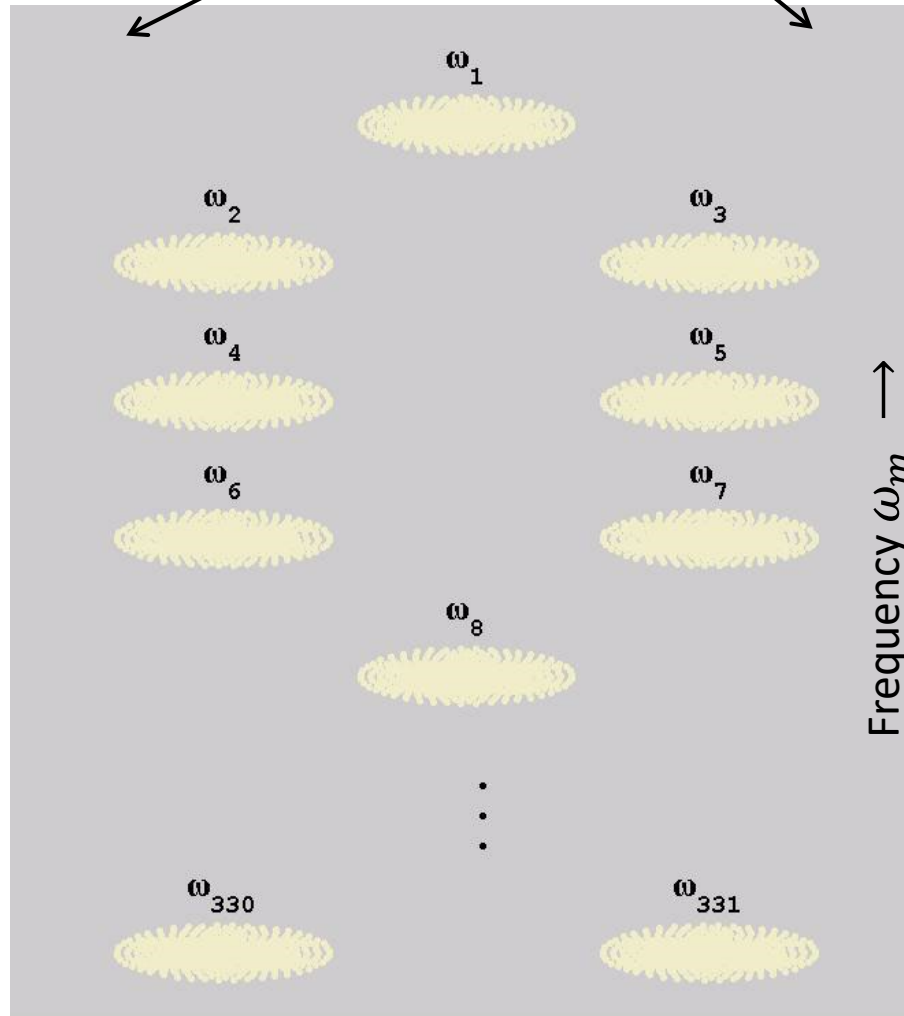
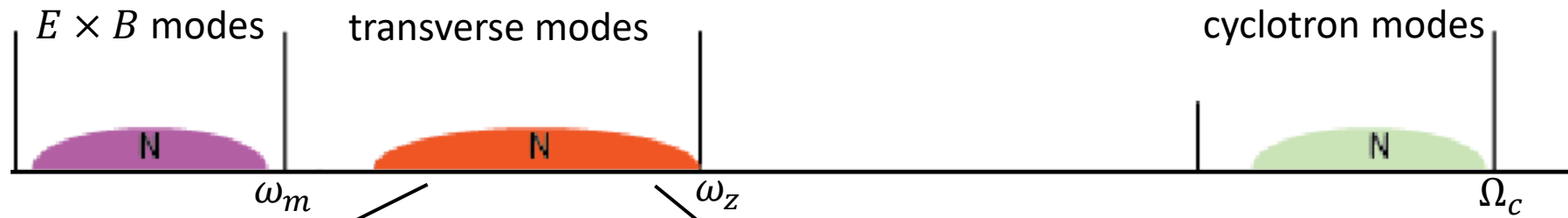
$$B_{\perp} > 10 - 15 \text{ kHz}$$



Transverse (drumhead) modes



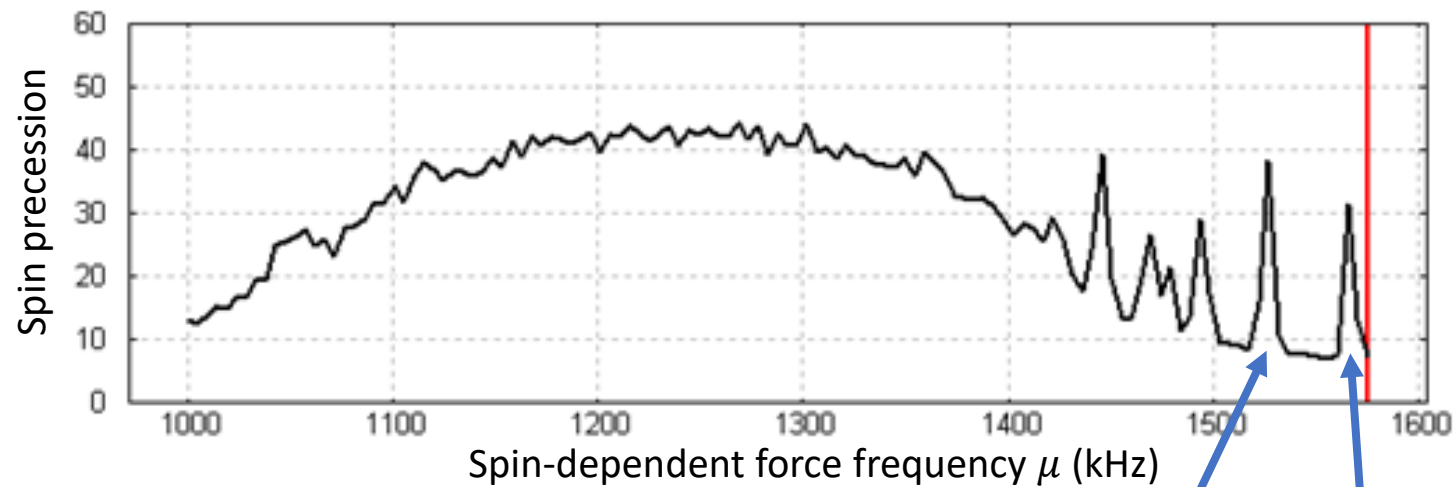
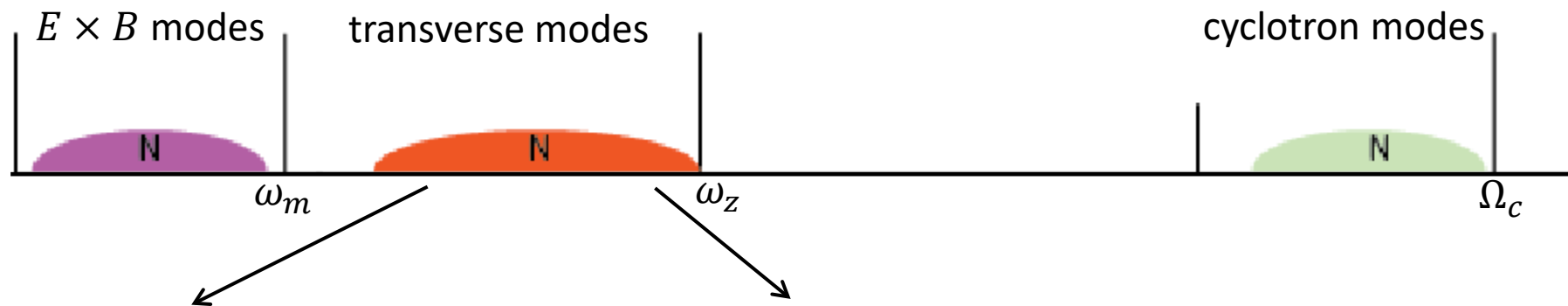
Transverse (drumhead) modes



Freericks group, PRA (2013)
Baltrush, Negretti, Taylor,
Calarco, PRA (2011)
Dubin, UCSD

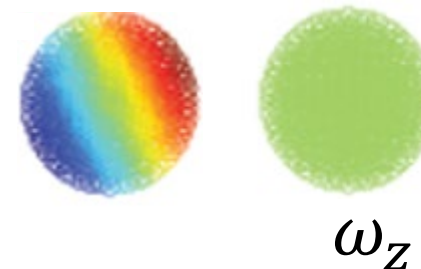
Modes characterized by
eigenfrequency ω_m
and eigenvector $b_{i,m}$

Transverse (drumhead) modes



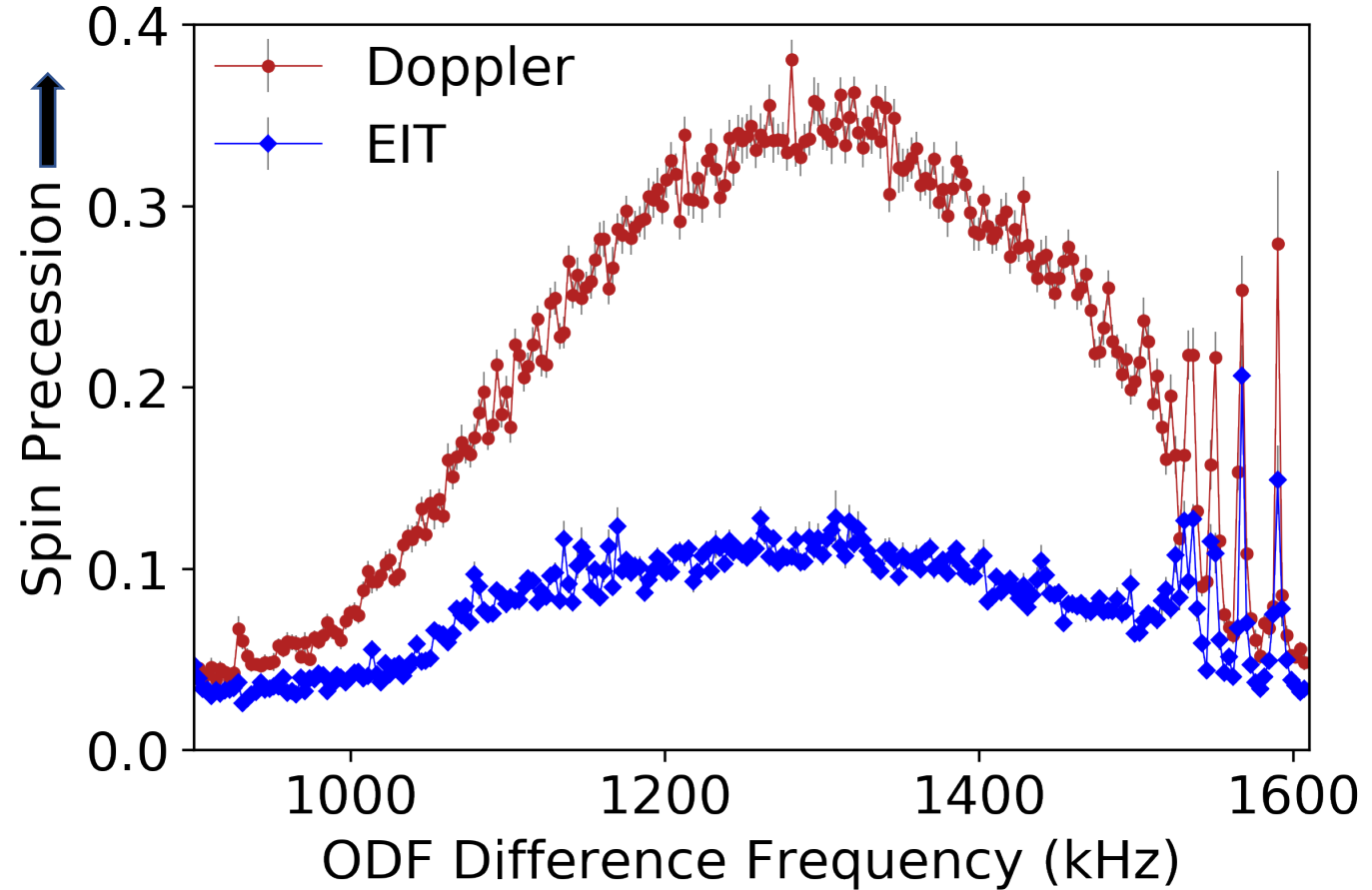
Measure mode spectrum with spin-dependent force

tilt mode COM mode



Ground-state cooling of the drumhead modes

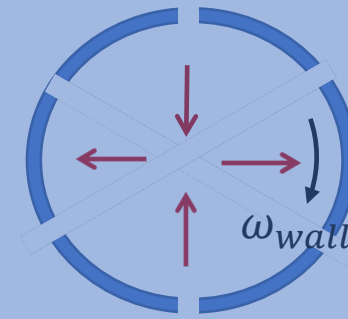
- $N \approx 160$ ions
- Simultaneously cool all drumhead modes
- $200 \mu\text{s}$ EIT cooling
- $\bar{n}_{COM} \approx 0.3$ (0.2)



Elena Jordan, et al., arXiv:1809.06346
Phys. Rev. Lett. 122, 053603 (2019)

Outline:

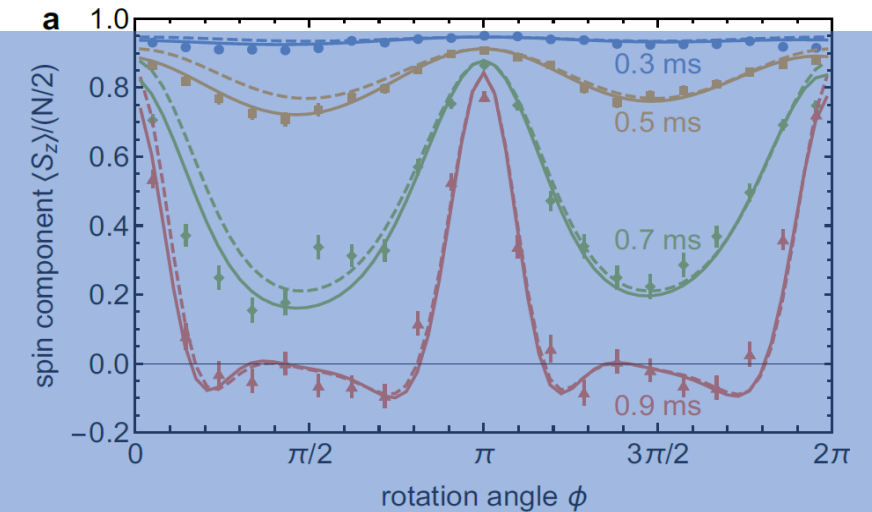
- ion crystals in Penning traps
 - high (4.5 T) magnetic field qubit
 - modes



- engineering tunable Ising interactions

$$H_{\text{Ising}} = \frac{1}{N} \sum_{i < j} J_{i,j} \sigma_i^z \sigma_j^z$$

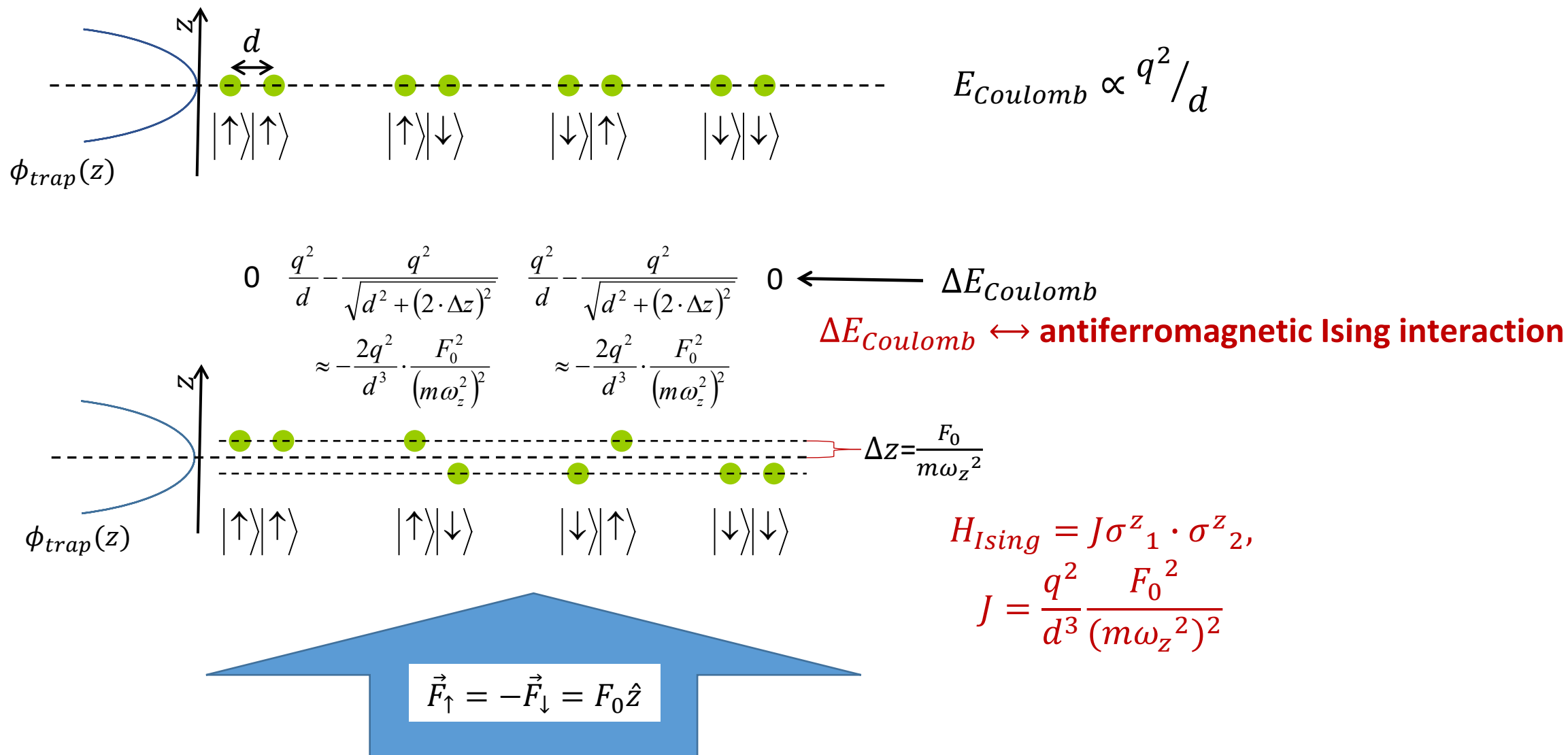
- benchmarking quantum dynamics, entanglement
 - spin squeezing
 - out-of-time-order correlations (OTOC)



- measuring weak motional excitations and electric fields

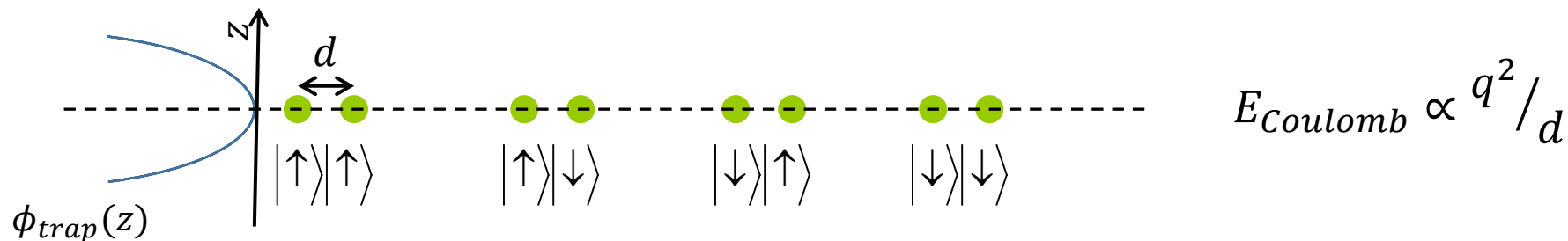
Generating entangled states with spin-dependent forces

Simple example – adiabatic spin-dependent force Calarco, Cirac, Zoller, PRA (2001)

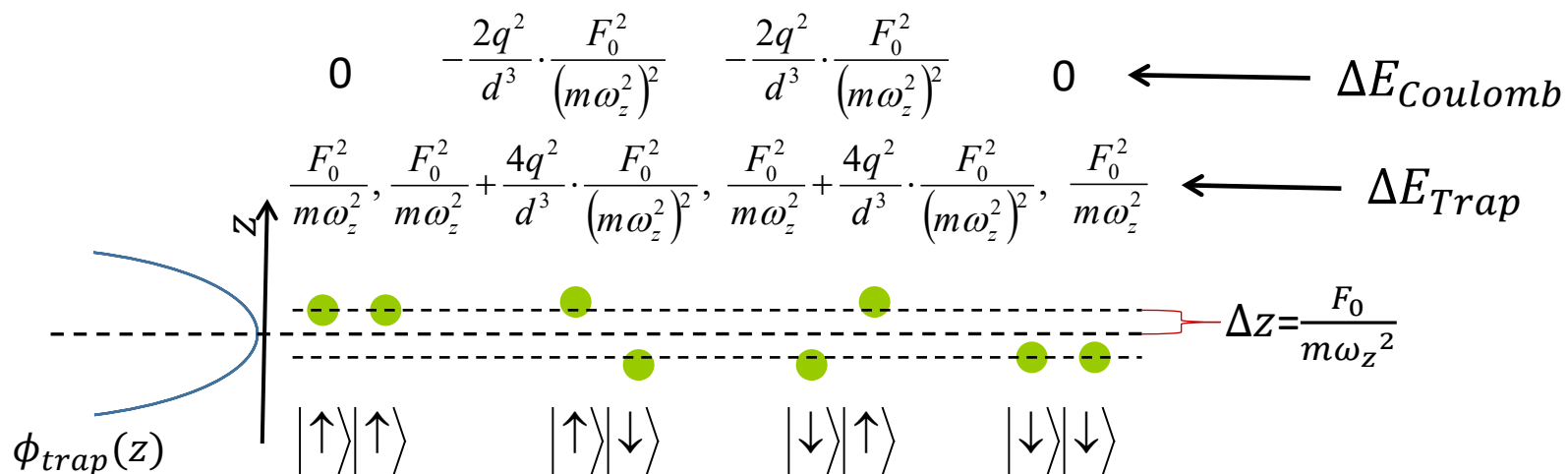


Generating entangled states with spin-dependent forces

Simple example – adiabatic spin-dependent force



Spin-dependent push can be enhanced by the remaining ion!



$\Delta E_{Coulomb} + \Delta E_{Trap} \Rightarrow$
ferromagnetic interaction

$$H_{Ising} = J \sigma^z_1 \cdot \sigma^z_2,$$

$$J = -\frac{q^2}{d^3} \frac{F_0^2}{(m\omega_z^2)^2}$$

$$\vec{F}_{\uparrow} = -\vec{F}_{\downarrow} = F_0 \hat{z}$$

Generating entangled states with spin-dependent forces

Oscillating spin-dependent force: $\vec{F}_{\uparrow}(t) = -\vec{F}_{\downarrow}(t) = F_0 \cos(\mu t) \hat{z}$

- $\mu < \omega_z$, ion oscillation, $\vec{F}_{\uparrow,\downarrow}(t)$ in phase

$|\uparrow\rangle|\downarrow\rangle, |\downarrow\rangle|\uparrow\rangle$ have larger oscillation amplitude and energy

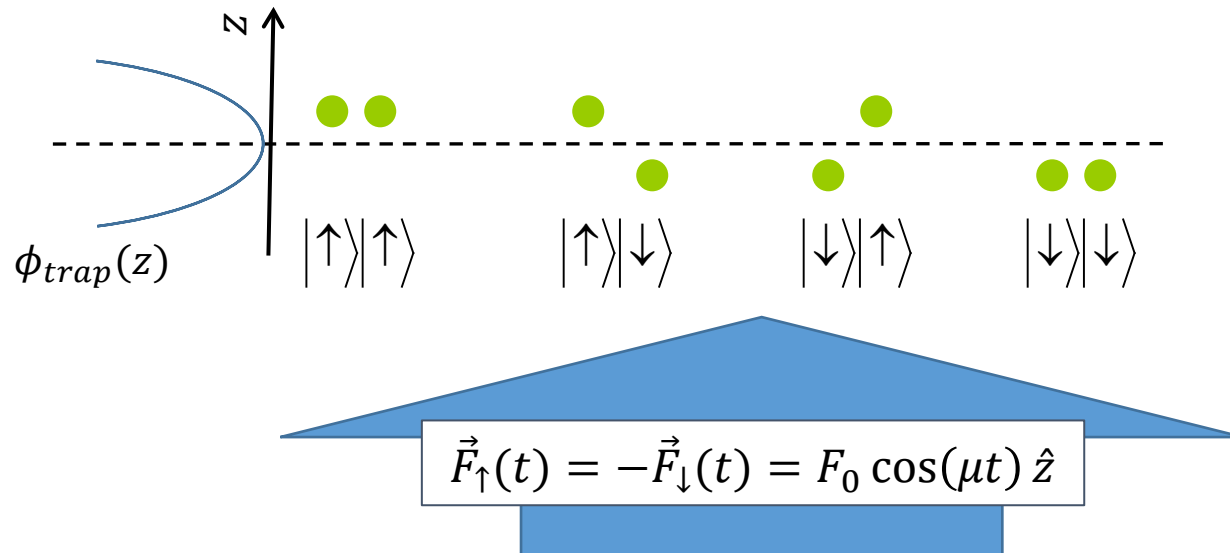
⇒ ferromagnetic interaction

- $\mu > \omega_z$, ion oscillation, $\vec{F}_{\uparrow,\downarrow}(t)$ 180° out of phase

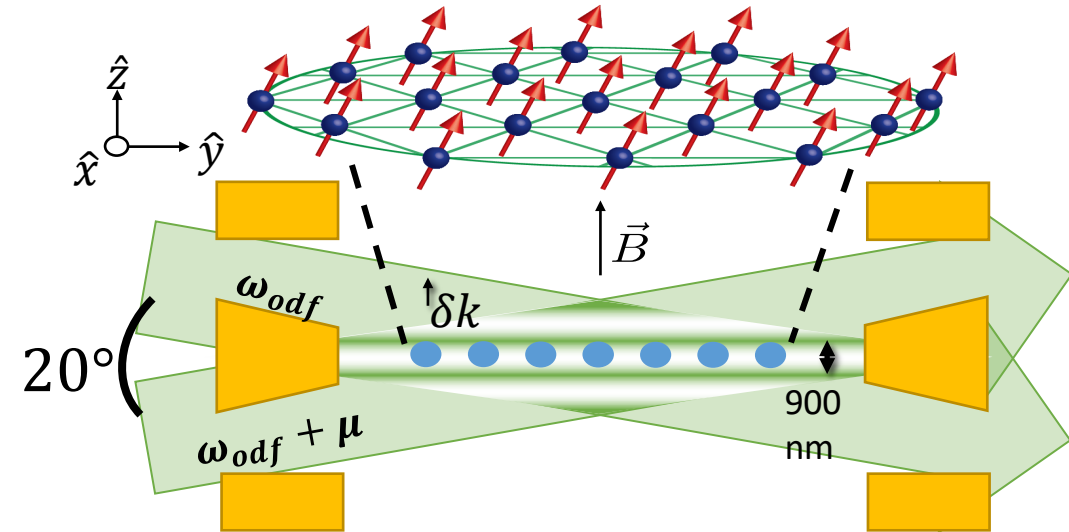
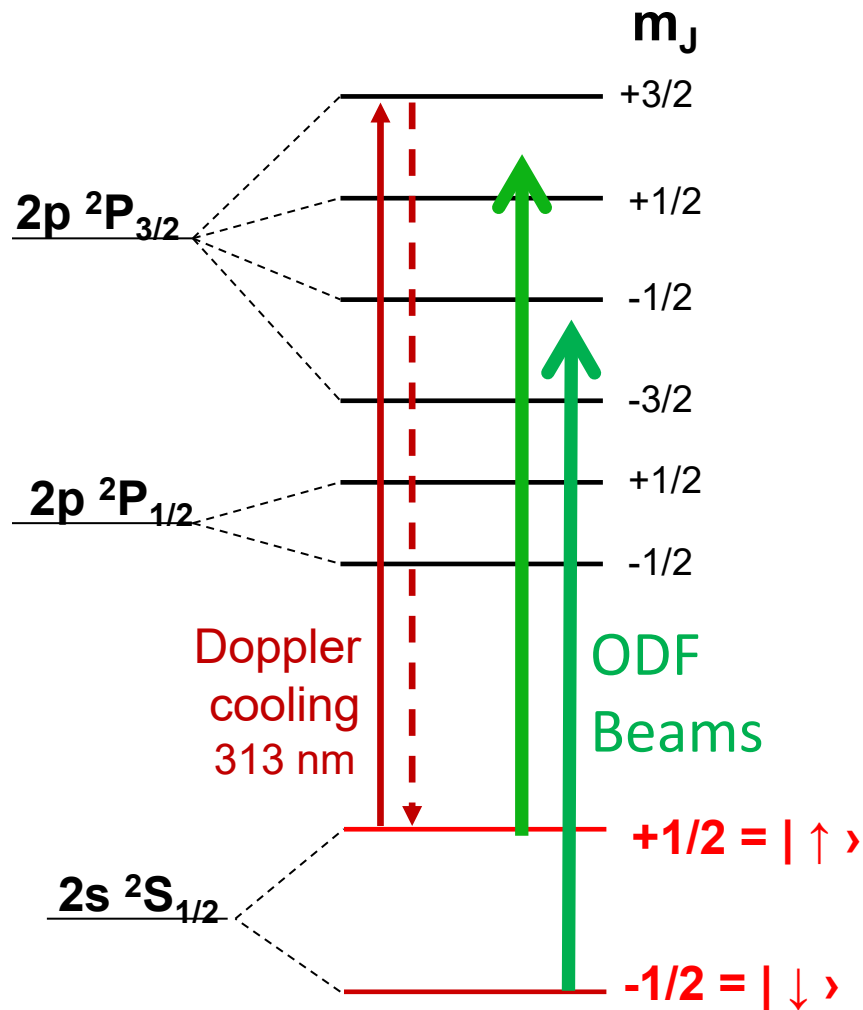
Coulomb force opposes $\vec{F}_{\uparrow,\downarrow}(t)$ for $|\uparrow\rangle|\downarrow\rangle, |\downarrow\rangle|\uparrow\rangle$ states,

$|\uparrow\rangle|\downarrow\rangle, |\downarrow\rangle|\uparrow\rangle$ have smaller oscillation amplitude and energy

⇒ anti-ferromagnetic interaction



Engineering quantum magnetic couplings with spin-dependent forces



- $F_{\uparrow}(t) = -F_{\downarrow}(t)$
 $F_{\uparrow}(t) = F_0 \cos(\mu t)$
- alignment of 1D lattice and ion plane

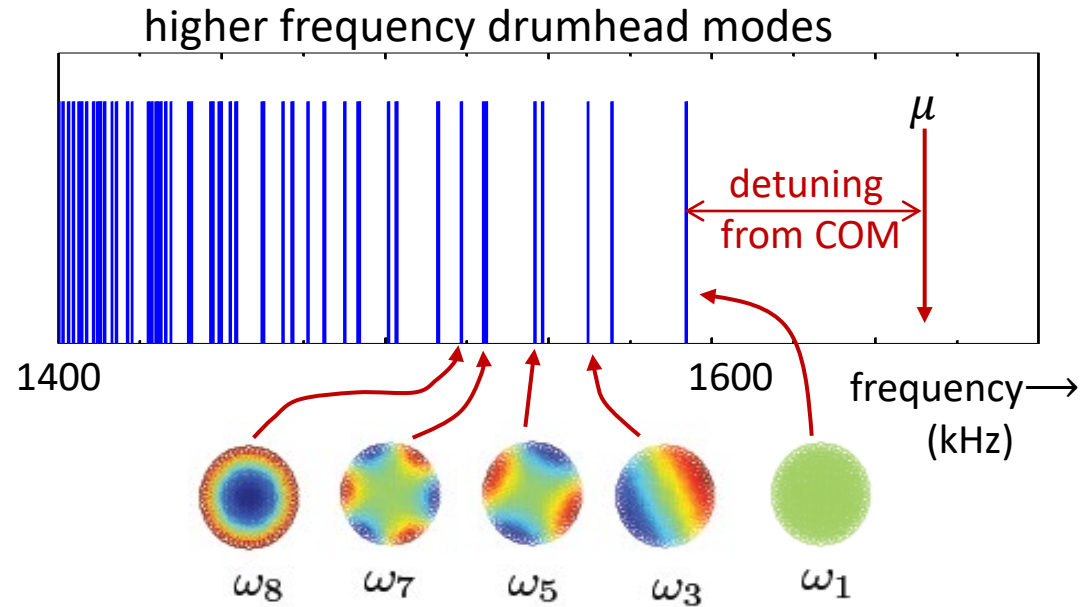
Leibfried et al., Nature **422**, (2003) - quantum gates through spin-dependent forces
 Sorensen and Molmer, PRL (1999) with small numbers of ion in rf traps

Engineering quantum magnetic couplings

$$\hat{H}_{ODF}(t) = -F_0 \cos(\mu t) \sum_{j=1}^N \hat{z}_j \cdot \hat{\sigma}_j^z =$$

$$\sum_{m=1}^N b_{jm} \sqrt{\frac{\hbar}{2M\omega_m}} (\hat{a}_m^\dagger e^{i\omega_m t} + \hat{a}_m e^{-i\omega_m t})$$

N drumhead eigenvalues ω_m and eigenvector $\vec{b}_m$



$$\hat{U}(t) = \hat{U}(t) \hat{U}(t)$$

Infinite range $\Rightarrow$ Single axis twisting

$$H_{Ising} = \frac{J}{N} \sum_{i < j} \sigma_i^z \sigma_j^z = \frac{2J}{N} S_z^2$$

where $S_z = \sum_i \frac{\sigma_i^z}{2}$

generates a “cat state” $\frac{1}{\sqrt{2}} \{ |\uparrow\uparrow\uparrow\cdots\uparrow\rangle_x + |\downarrow\downarrow\downarrow\cdots\downarrow\rangle_x \}$

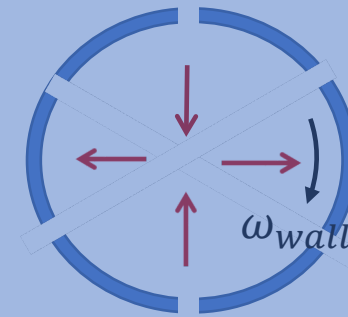
at long times τ , such that $\frac{2J}{N} \tau = \frac{\pi}{2}$

Produces spin

- useful metrology
- source of decoherence

Outline:

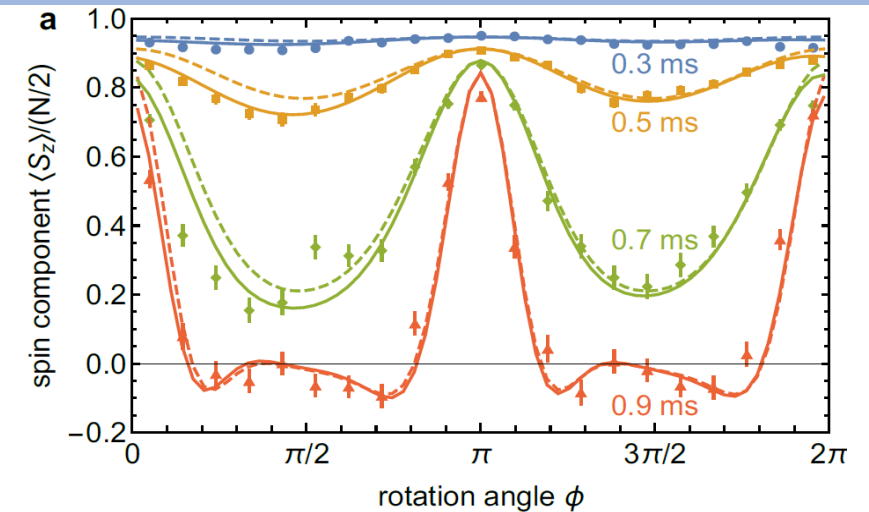
- ion crystals in Penning traps
 - high (4.5 T) magnetic field qubit
 - modes



- engineering tunable Ising interactions

$$H_{\text{Ising}} = \frac{1}{N} \sum_{i < j} J_{i,j} \sigma_i^z \sigma_j^z$$

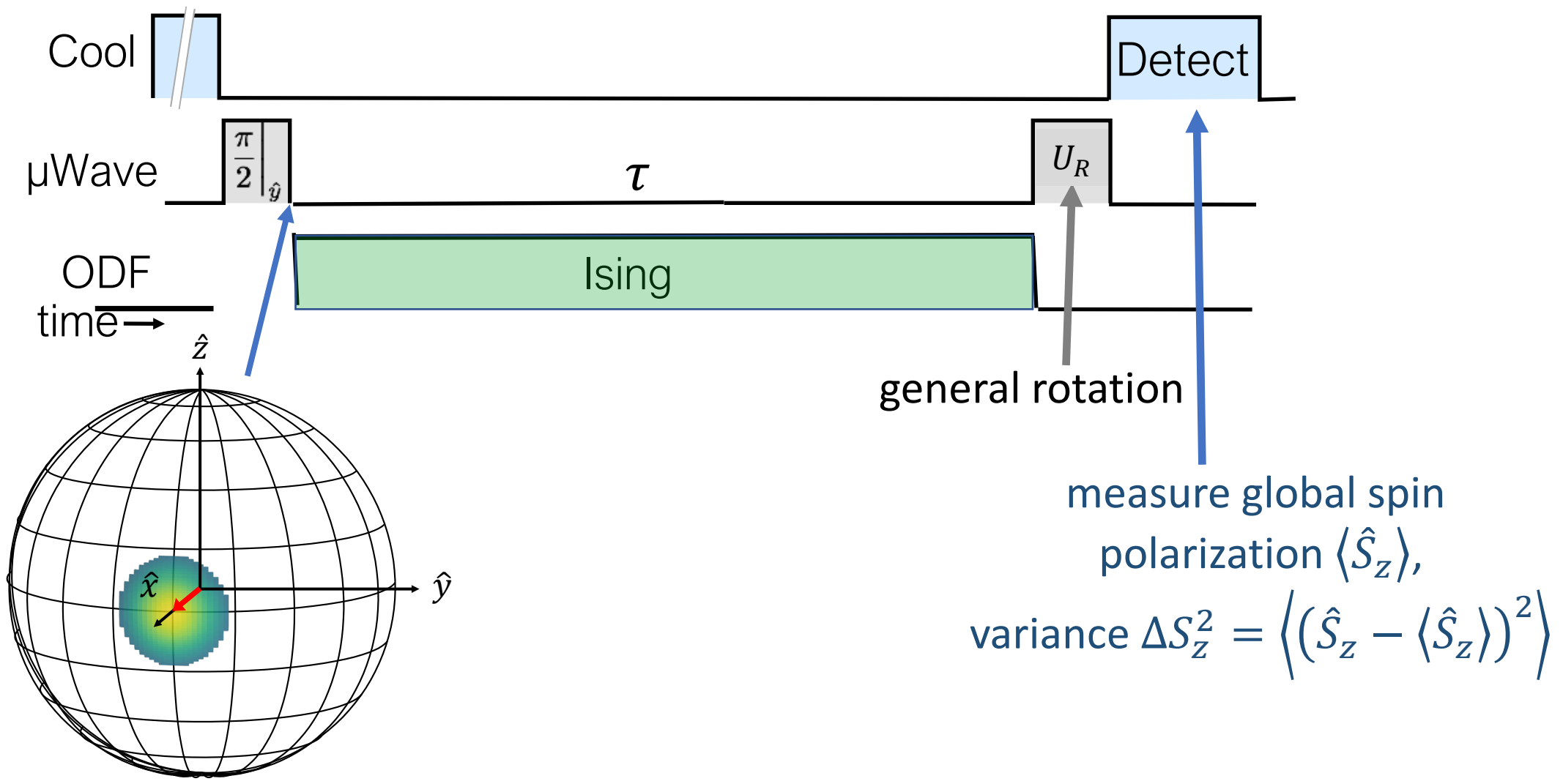
- benchmarking quantum dynamics, entanglement
 - spin squeezing
 - out-of-time-order correlations (OTOC)



- measuring weak motional excitations and electric fields

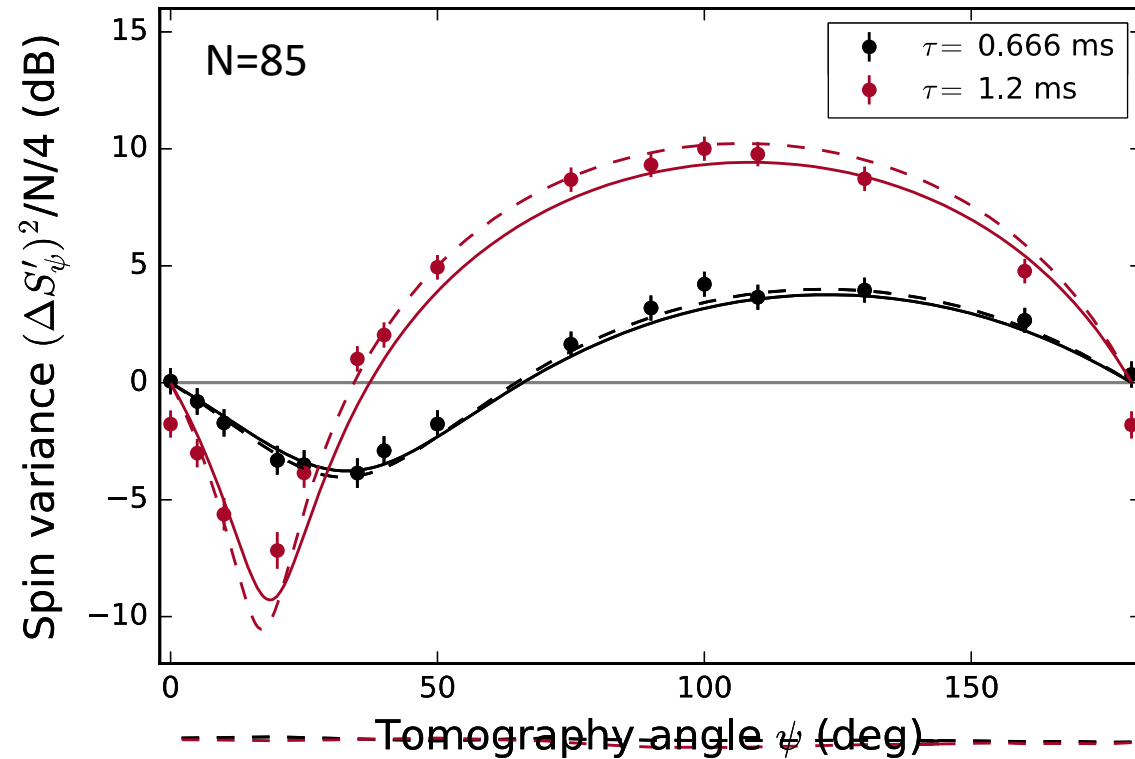
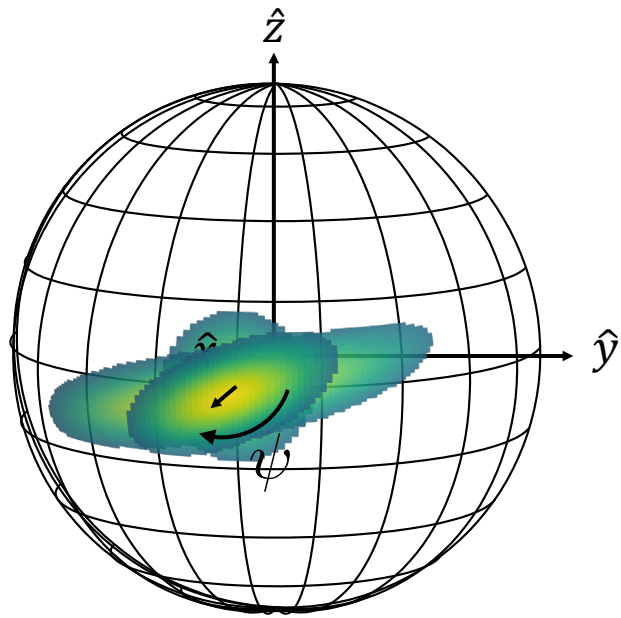
Benchmarking quantum dynamics

- employ infinite range interactions $H_{Ising} \approx \frac{2J}{N} S_z^2$, $S_z \equiv \sum_i \sigma_i^z / 2$
- prepare eigenstate of $H_{\perp} = \sum_i B_{\perp} \hat{\sigma}_i^x$, turn on H_{Ising}



Benchmarking quantum dynamics

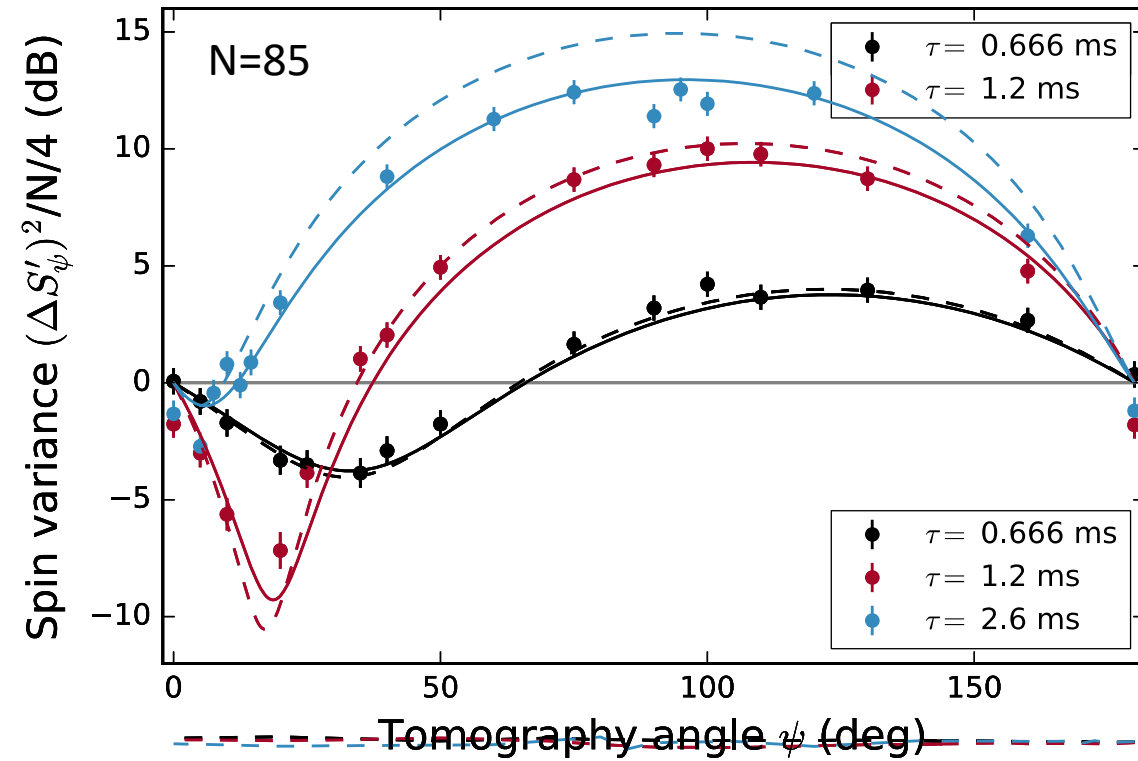
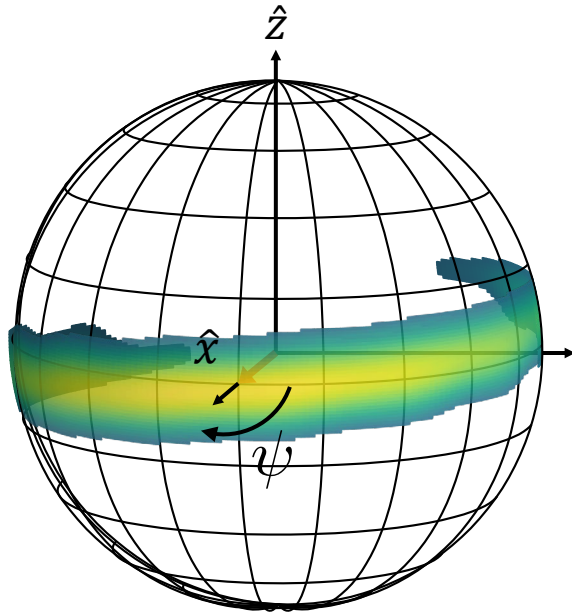
Bohnet *et al.*, *Science* 352, 1297 (2016)



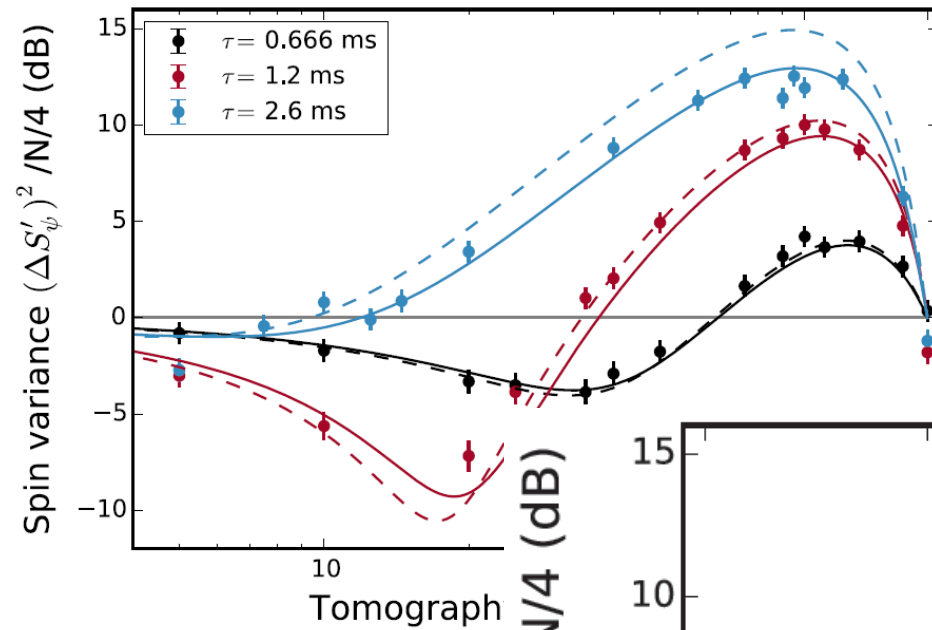
- Measurements of Ramsey squeezing parameter $\Rightarrow$
prove entanglement for $25 < N < 220$
- Largest inferred squeezing: -6.0 dB

Benchmarking quantum dynamics

Bohnet *et al.*, *Science* 352, 1297 (2016)



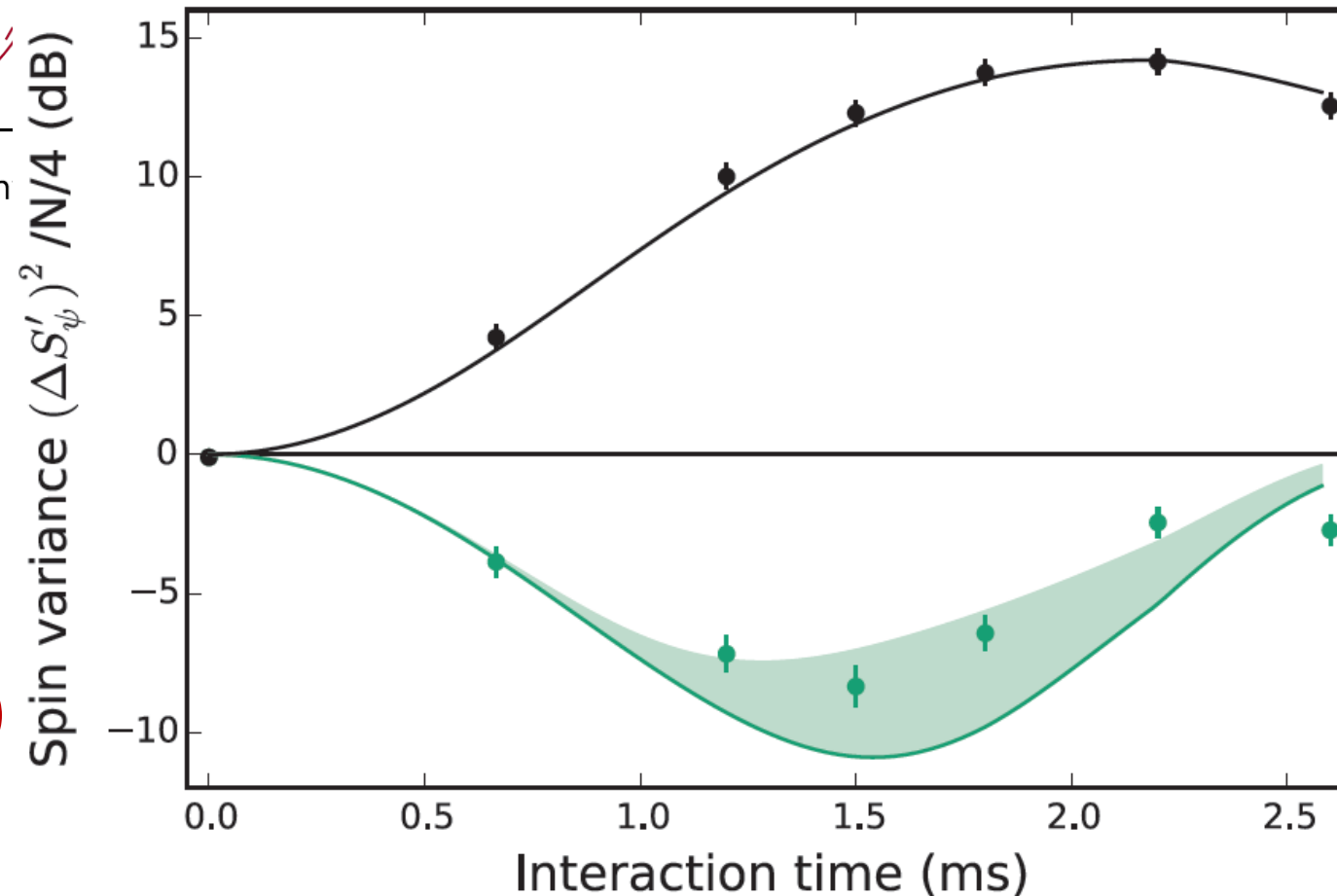
Benchmarking quantum dynamics



N = 86

Time dependence of squeezed and anti-squeezed variance

**Bohnet *et al.*,
Science352 (2016)**



Out-of-time-order correlation functions

$$F(t) \equiv \langle \psi | W(t)^\dagger V^\dagger W(t) V | \psi \rangle \text{ where } W(t) = e^{iHt} W(0) e^{-iHt}, \quad [V, W(0)] = 0$$

$$\text{Re}[F(t)] = 1 - \langle |[W(t), V]|^2 \rangle / 2$$

$\Rightarrow$ measures failure of initially commuting operators to commute at later times

$\Rightarrow$ quantifies spread or scrambling of quantum information across a system's degrees of freedom

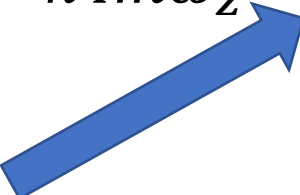
Swingle et al., arXiv:1602.06271; Shenker et al., arXiv:1306.0622; Kitaev (2014)

Difficult to measure $\Leftrightarrow$ requires time-reversal of dynamics

time reversal is possible in many quantum simulators!

Time reversal of the Ising dynamics

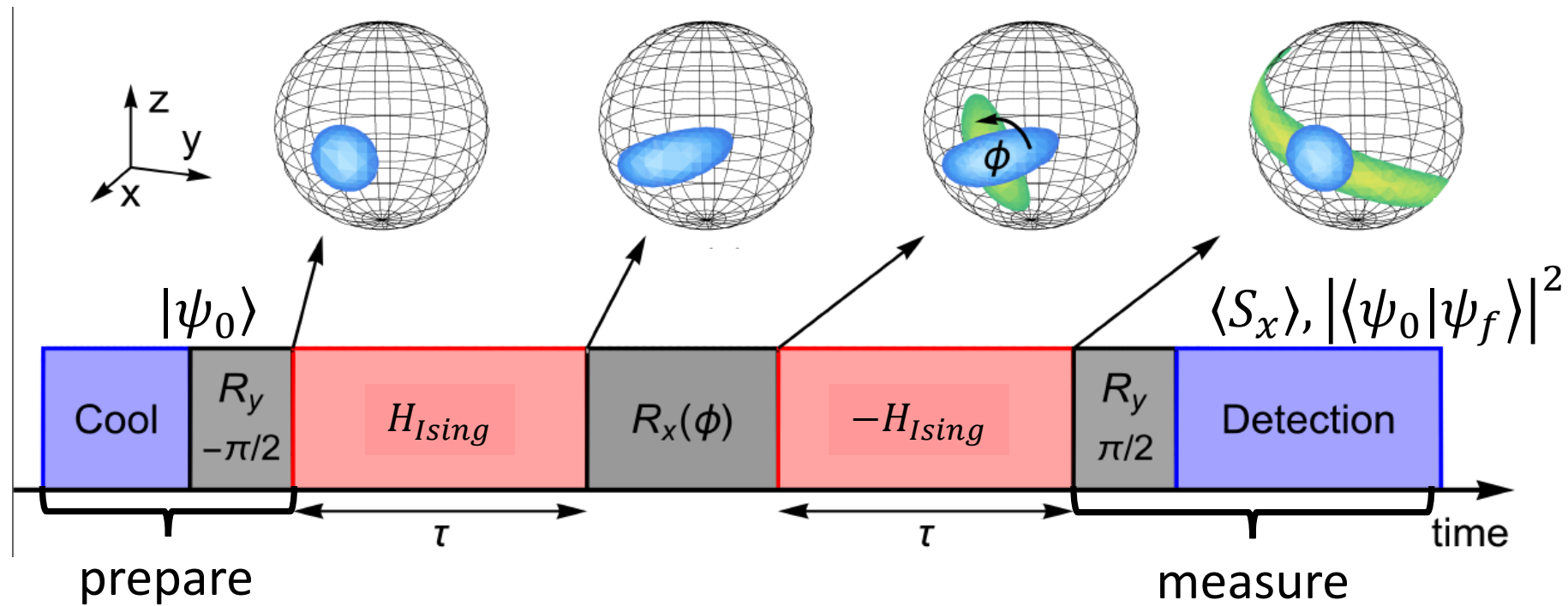
$$H_{Ising} = \frac{J}{N} \sum_{i < j} \hat{\sigma}_i^z \hat{\sigma}_j^z, \quad \frac{J}{N} \cong \frac{F_0^2}{\hbar 4m\omega_z} \cdot \frac{1}{\mu - \omega_z}$$



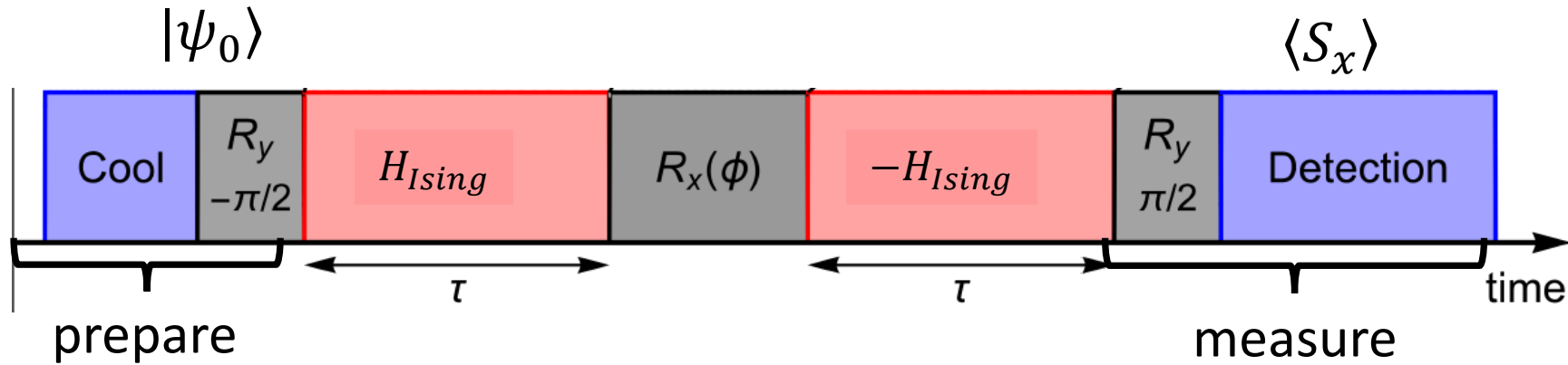
Change $\mu = \omega_z + \delta$ (antiferromagnetic)
to $\mu = \omega_z - \delta$ (ferromagnetic)

Multiple quantum coherence protocol

- Probe higher-order coherences and correlations (Pines group, 1985)



Multiple quantum coherence protocol



$$\begin{aligned}
 \langle S_x \rangle &= \langle \Psi_0 | e^{iH_{Ising}\tau} e^{i\phi S_x} e^{-iH_{Ising}\tau} S_x e^{iH_{Ising}\tau} e^{-i\phi S_x} e^{-iH_{Ising}\tau} | \Psi_0 \rangle \\
 &= \frac{2}{N} \langle \Psi_0 | \underbrace{e^{iH_{Ising}\tau} W^\dagger}_{W^\dagger(t)} \underbrace{e^{-iH_{Ising}\tau} V^\dagger}_{V^\dagger(0)} \underbrace{e^{iH_{Ising}\tau} W}_{W(t)} \underbrace{e^{-iH_{Ising}\tau} V}_{V(0)} | \Psi_0 \rangle
 \end{aligned}$$

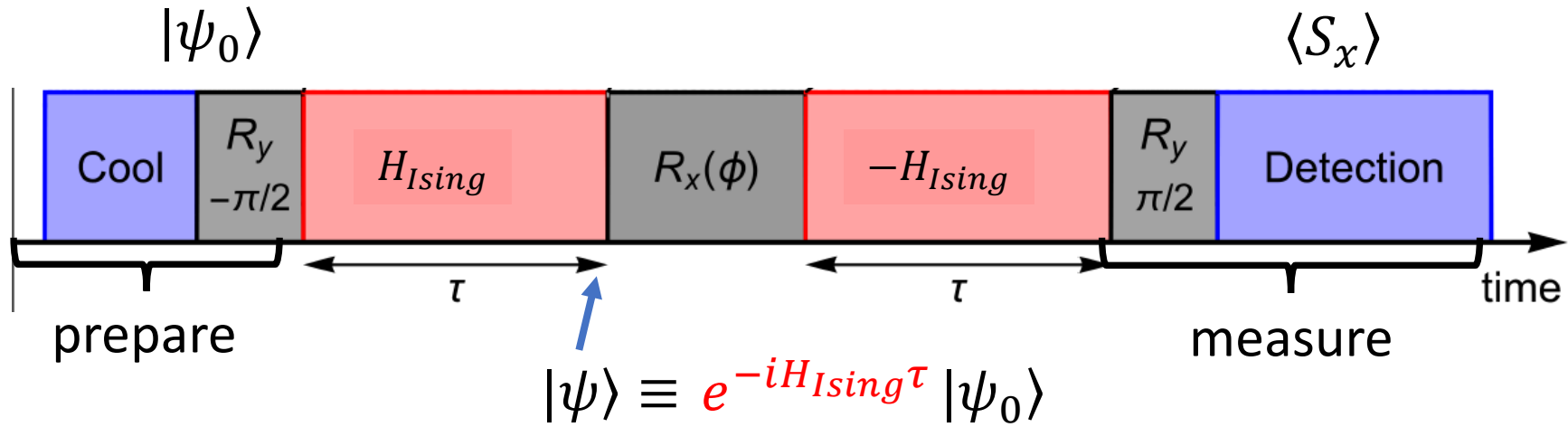
Out-of-time-order correlation (OTOC) function

$\Rightarrow$ quantifies spread or scrambling of quantum

information across a system's degrees of freedom

Swingle et al., arXiv:1602.06271; Shenker et al., arXiv:1306.0622; Kitaev (2014)

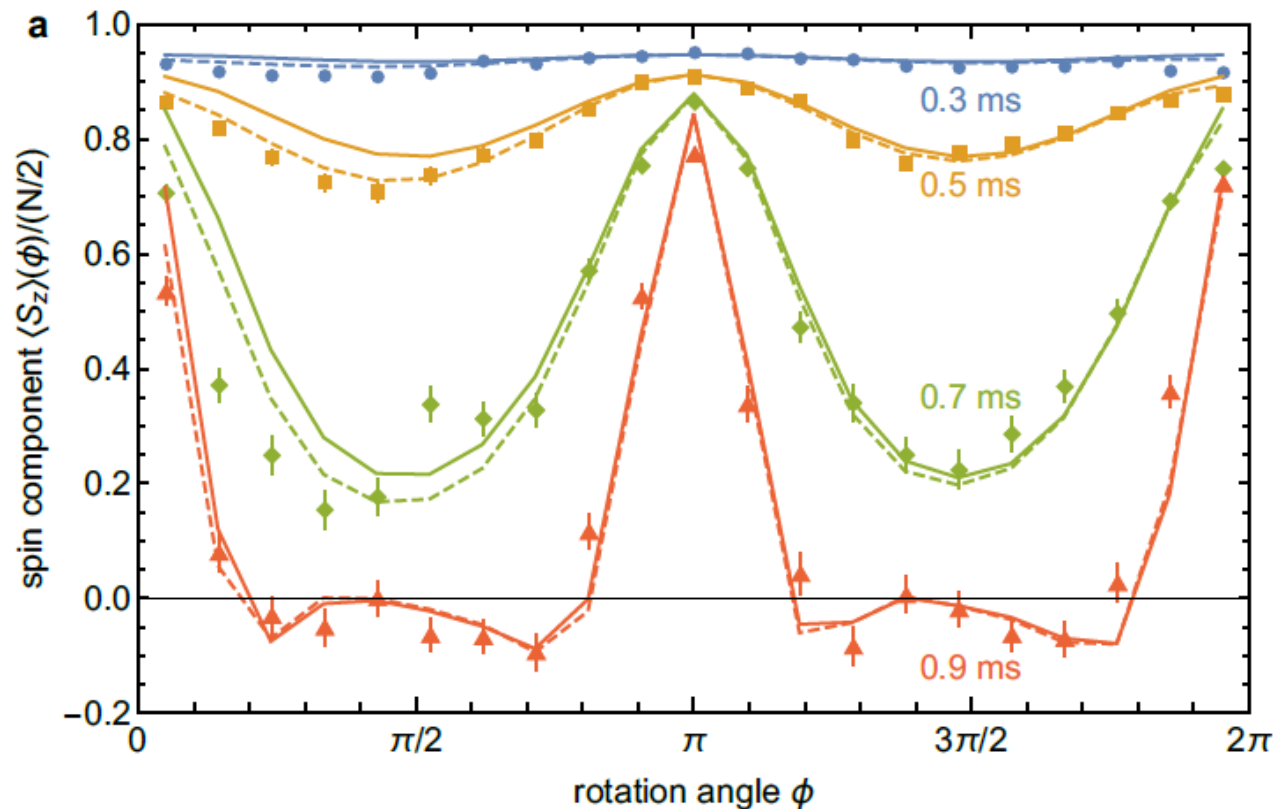
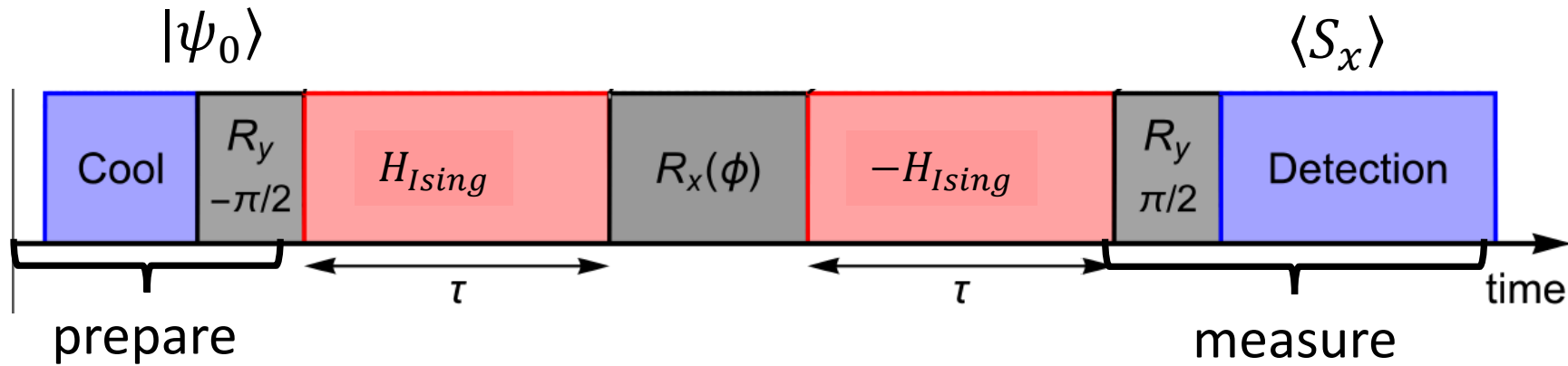
Multiple quantum coherence protocol



$$\begin{aligned}
 \langle S_x \rangle &= \langle \Psi_0 | e^{iH_{Ising}\tau} e^{i\phi S_x} e^{-iH_{Ising}\tau} S_x e^{iH_{Ising}\tau} e^{-i\phi S_x} \underbrace{e^{-iH_{Ising}\tau} |\Psi_0\rangle}_{\equiv |\Psi\rangle} \\
 &= \sum_m \langle \Psi | C_m | \Psi \rangle e^{i\phi m} \quad C_m = \sum \underbrace{\sigma_1^z \sigma_4^y \dots \sigma_k^z}_{\text{At least m terms}}
 \end{aligned}$$

m^{th} order Fourier coefficient $\langle \Psi | C_m | \Psi \rangle$ indicates $|\Psi\rangle$ has correlations of at least order m

MQC protocol – $\langle S_x \rangle$ measurement



$$H_{Ising} = J/N \sum_{i < j} \sigma_i^z \sigma_j^z$$

$$J \lesssim 5 \text{ kHz}$$

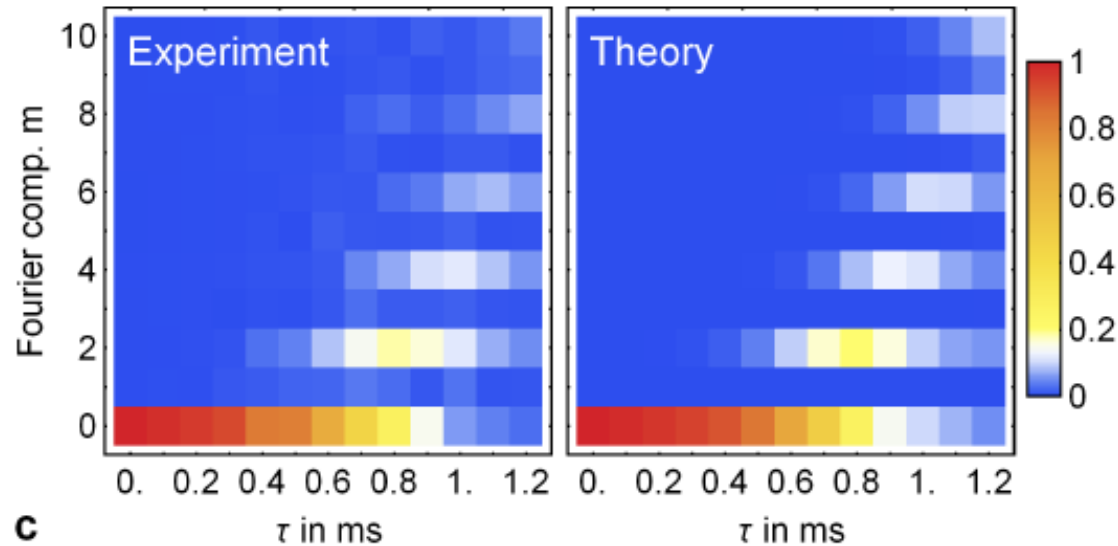
$$N = 111$$

$$\Gamma = 93 \text{ Hz}$$

[Gärtner, Bohnet et al.
Nature Physics 2017]

Fourier transform of magnetization

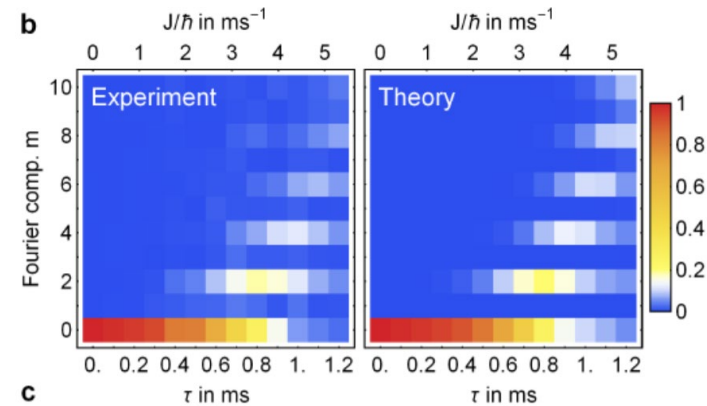
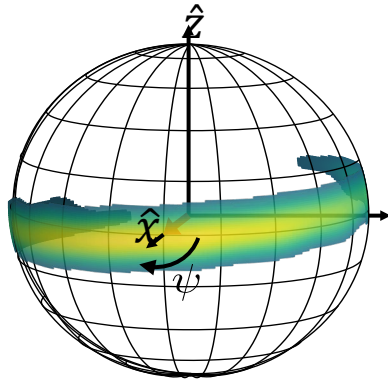
[Gärttner, Bohnet et al. Nature Physics 2017]



- Measure build-up of 8-body correlations
- Only global spin measurement
- Illustrates how OTOCs measure spread of quantum information

Summary:

- Penning traps good for controlling large ion crystals (> 200 ; 3D, $> 10^5$)
- employed spin-squeezing, OTOCs to benchmark quantum dynamics with long range Ising interactions



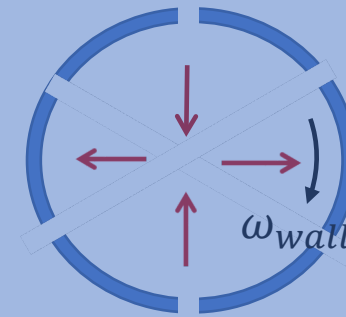
Future directions $\Leftrightarrow$ increase coherence, complexity!

- increase entangling operation coherence with parametric amplification (W. Ge, et al., PRL (2019))
- implement single-site σ_z - rotations (potential NISQ platform)
- transverse field, spin-phonon models

$$\text{Dicke model } \delta a^\dagger a + \frac{2g}{\sqrt{N}} (a + a^\dagger) S_z + B_\perp S_x$$

Outline:

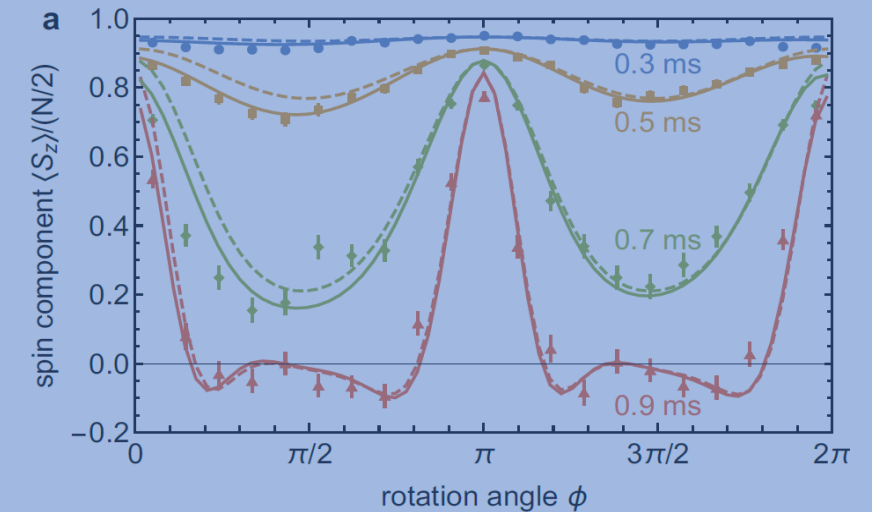
- ion crystals in Penning traps
 - high (4.5 T) magnetic field qubit
 - modes



- engineering tunable Ising interactions

$$H_{\text{Ising}} = \frac{1}{N} \sum_{i < j} J_{i,j} \sigma_i^z \sigma_j^z$$

- benchmarking quantum dynamics, entanglement
 - spin squeezing
 - out-of-time-order correlations (OTOC)



- measuring weak motional excitations and electric fields

Motional amplitude sensing / Trapped ions as sensitive $\vec{E}$ -field and force detectors

Maiwald, *et al.*, Nature Physics 2009 – $1 \text{ yN Hz}^{-1/2}$

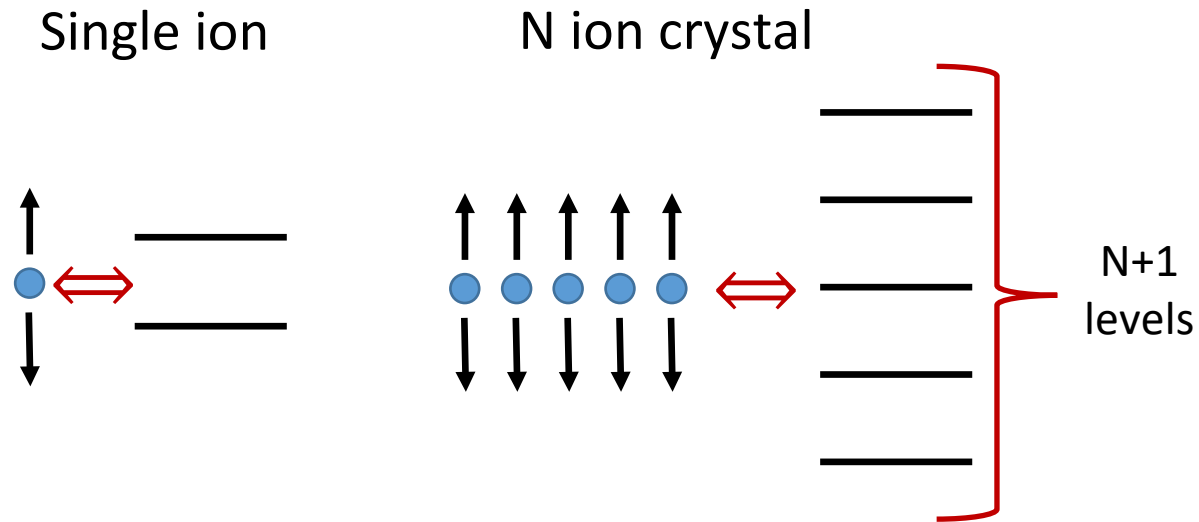
Hempel *et al.*, Nature Photonics 2013 – detect single photon recoil

Shaniv, Ozeri, Nature Communications, 2017 – high sensitivity ($\sim 28 \text{ zN Hz}^{-1/2}$) at low frequencies

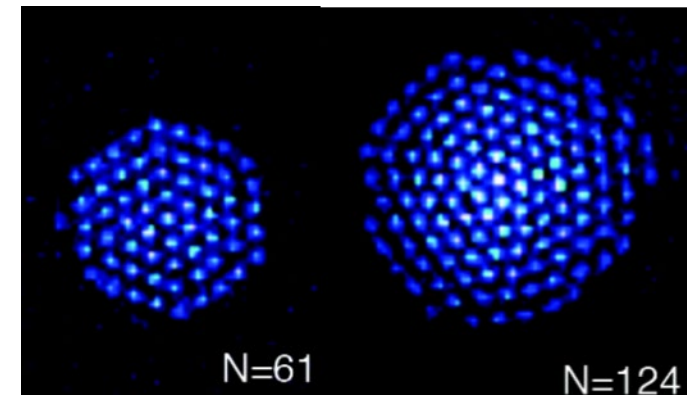
⋮

Biercuk *et al.*, Nature Nanotechnology, 2010 – 100-ion crystal ($400 \text{ yN Hz}^{-1/2}$)

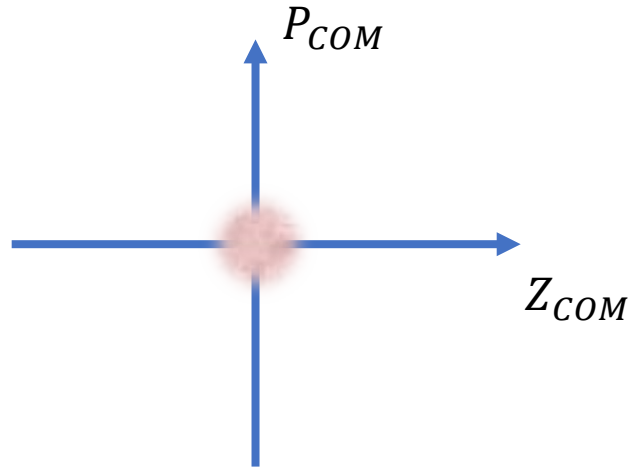
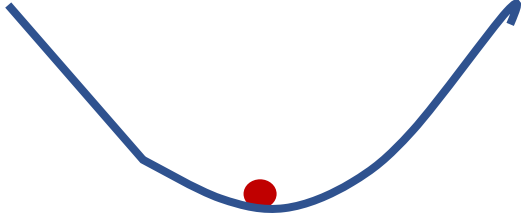
Basic idea: map motional amplitude onto spin precession



- N ion crystal
- Less projection noise
 - Smaller zero-point motion, $z_{zpt} \approx 2 \text{ nm}$ for $N=100$
- $\sim \frac{1}{\sqrt{N}}$

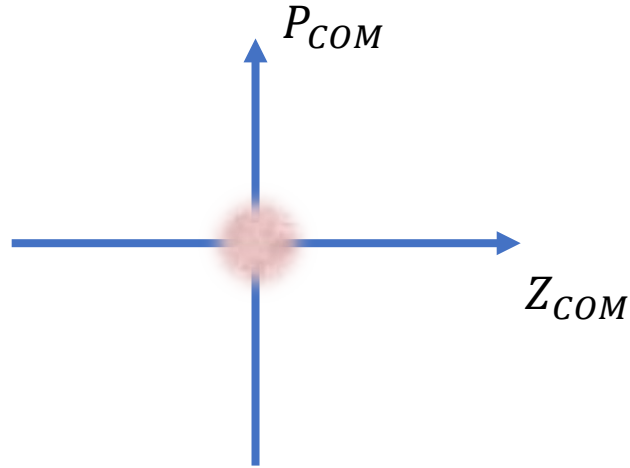
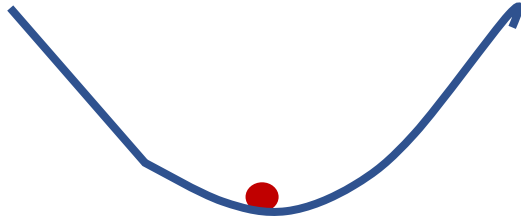


Conventions

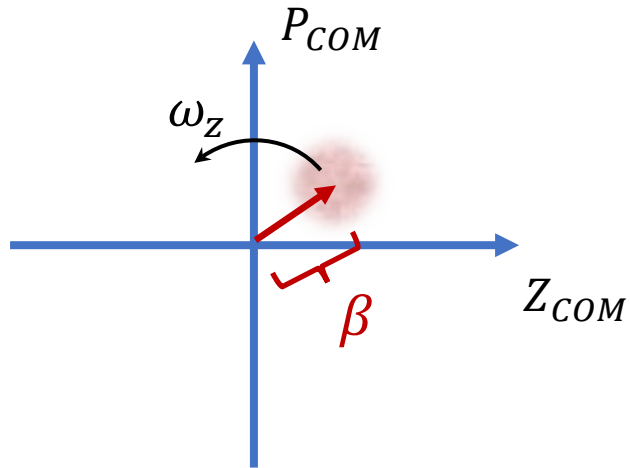
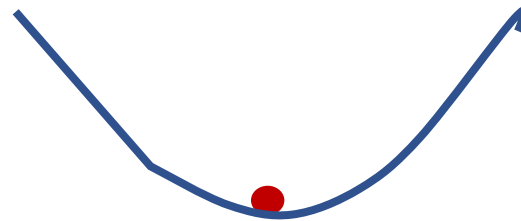



$$z_{zpt} = \frac{1}{\sqrt{N}} \sqrt{\frac{\hbar}{2m\omega_z}}$$

Conventions



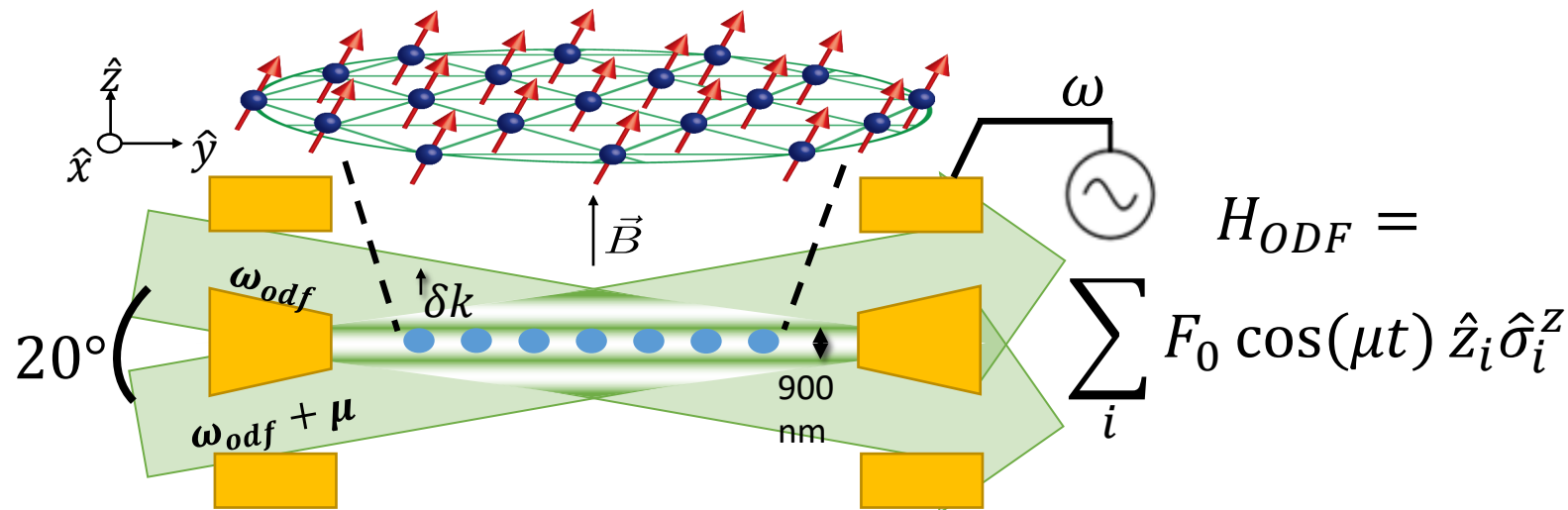
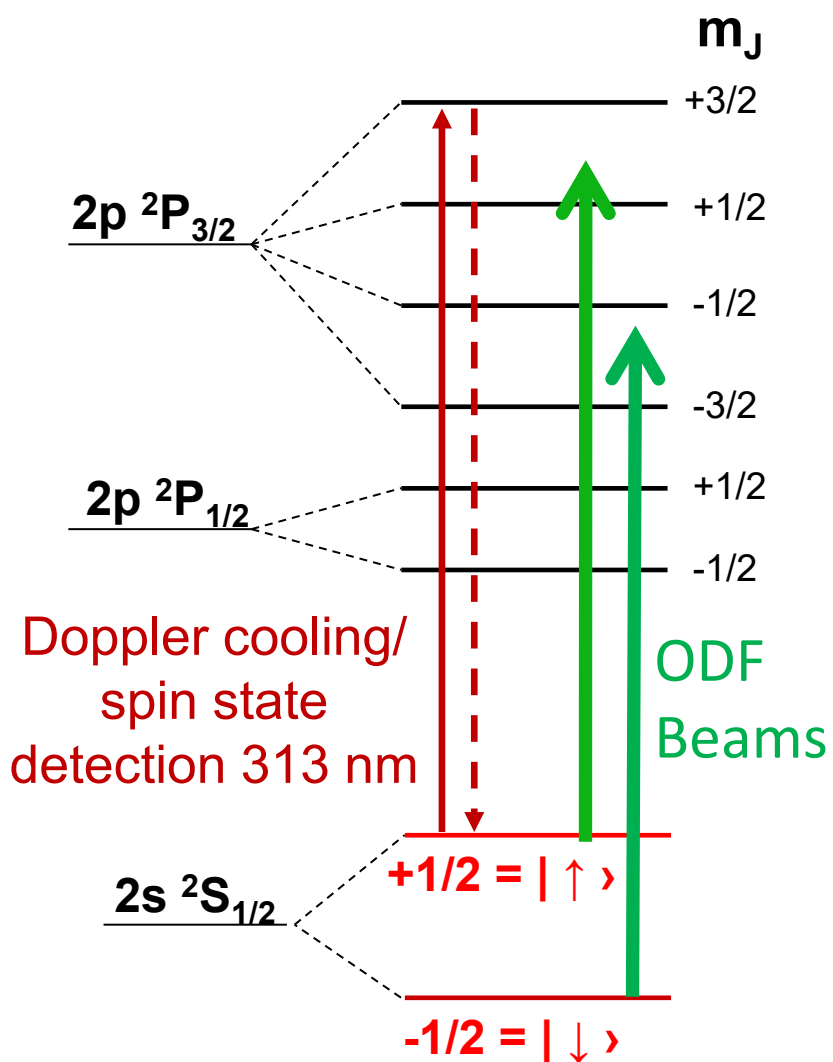
$$\updownarrow z_{zpt} = \frac{1}{\sqrt{N}} \sqrt{\frac{\hbar}{2m\omega_z}}$$



motional coherent state $|\beta\rangle$
 $\beta \equiv$ dimensionless displacement
 amplitude
 in dimensionless units $z_{zpt} = 1/2$

The standard quantum limit (SQL) for sensing (or measuring) β in a single measurement is $1/2$

Motional sensing through spin precession



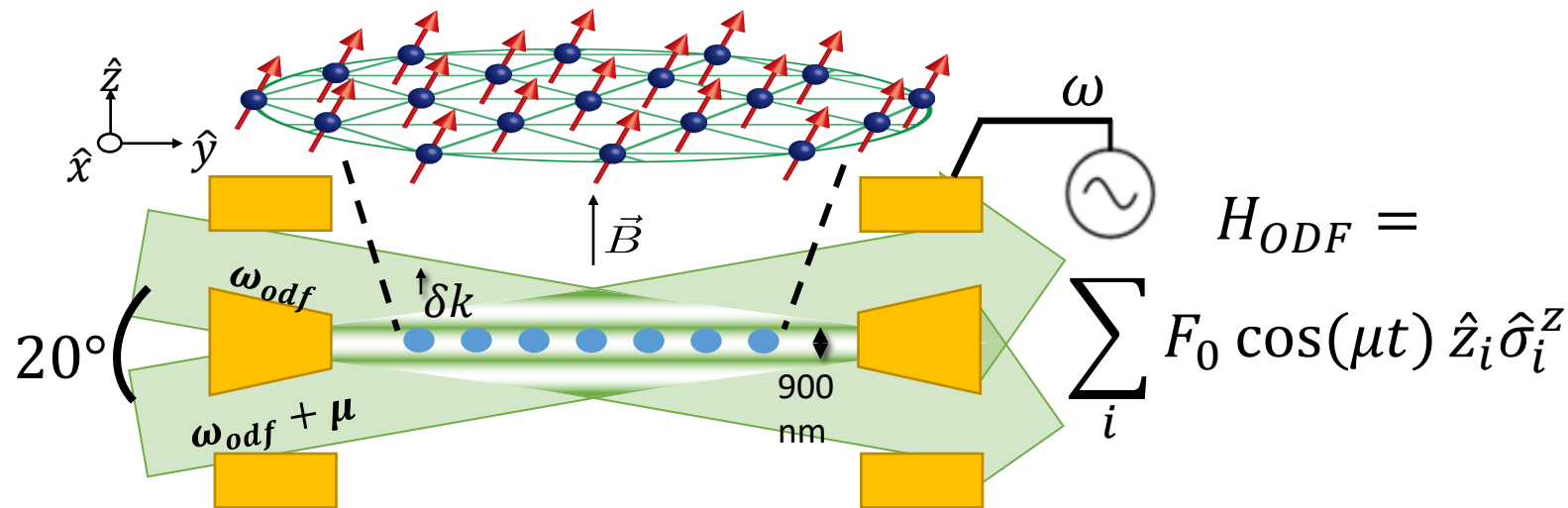
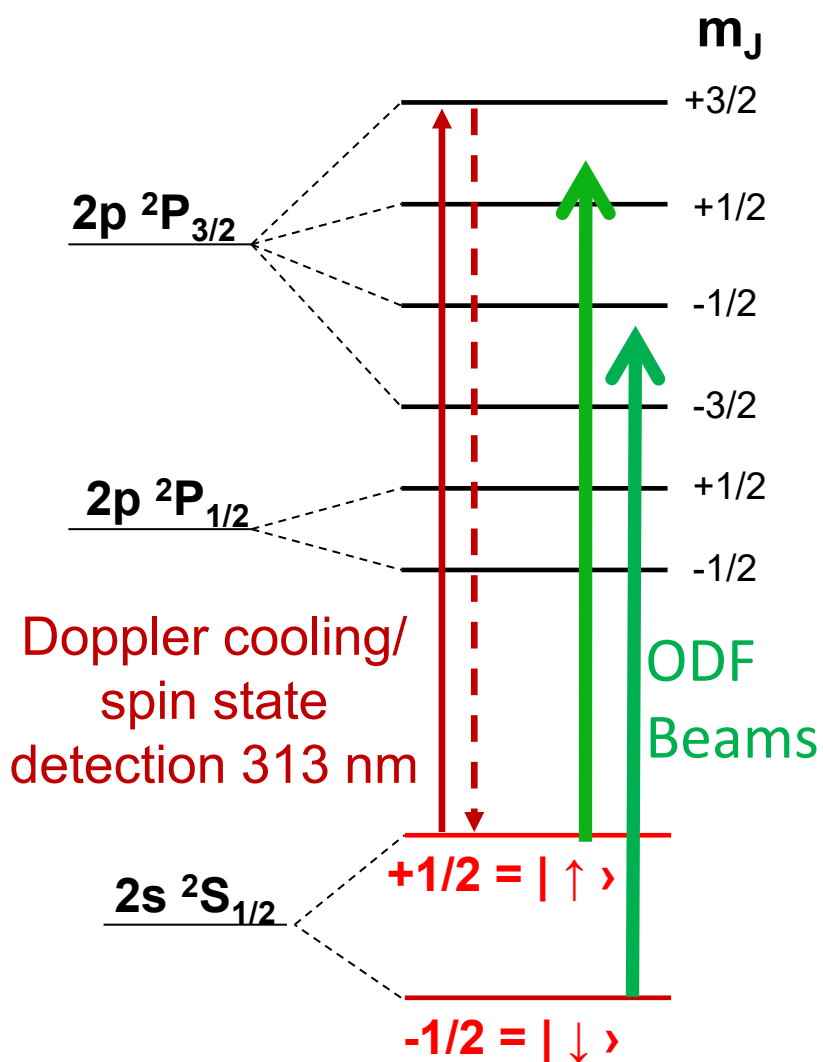
Implement classical COM oscillation: $\hat{z}_i \rightarrow \hat{z}_i + Z_c \cos(\omega t + \phi)$

$$H_{ODF} \cong F_0 \cdot Z_c \cos[(\omega - \mu)t + \phi] \sum_i \frac{\hat{\sigma}_i^z}{2}$$

$$= F_0 \cdot Z_c \cos[(\omega - \mu)t + \phi] \hat{S}_z$$

For $\mu = \omega$, produces spin precession with rate $\propto F_0 \cdot Z_c \cos(\phi)$

Motional sensing through spin precession

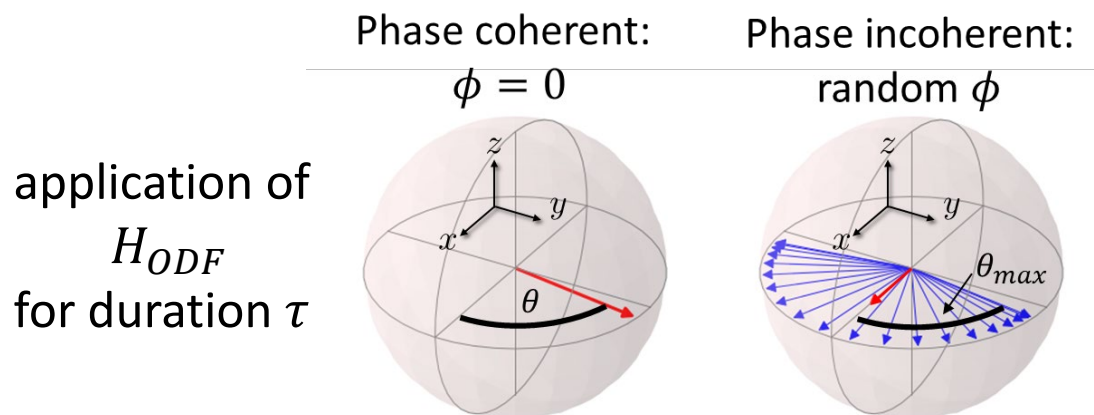


$$H_{ODF} = \sum_i F_0 \cos(\mu t) \hat{z}_i \hat{\sigma}_i^z$$

Implement classical COM oscillation: $\hat{z}_i \rightarrow \hat{z}_i + Z_c \cos(\omega t + \phi)$

$$H_{ODF} \cong F_0 \cdot Z_c \cos[(\omega - \mu)t + \phi] \hat{S}_z$$

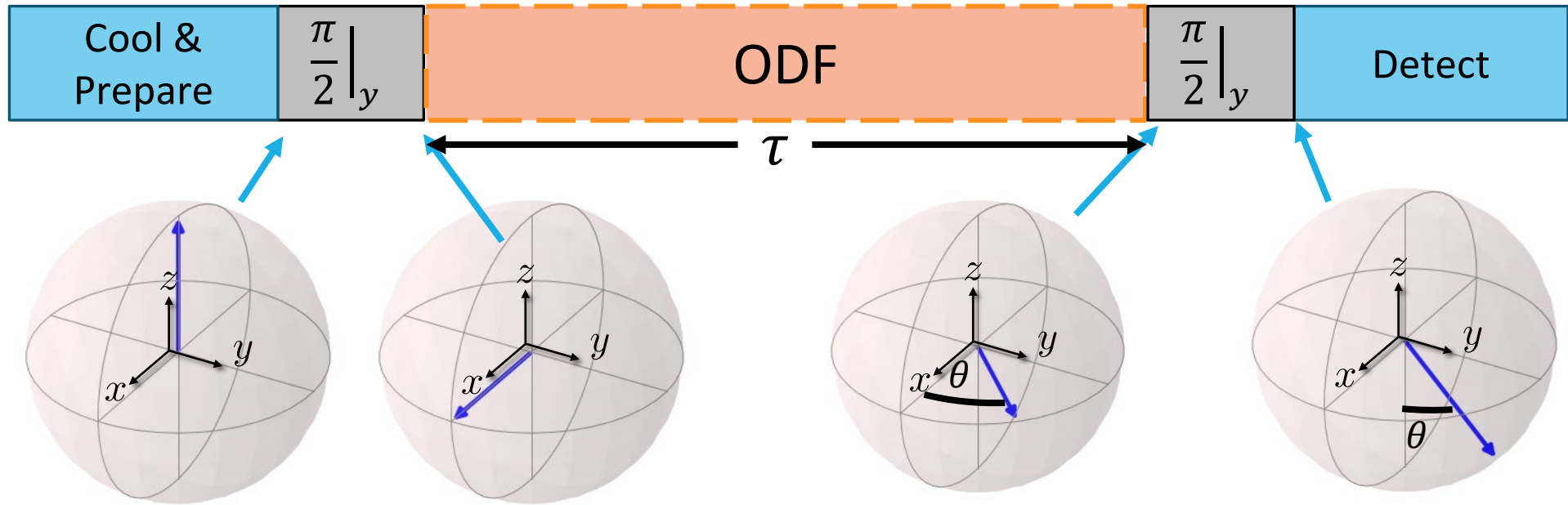
For $\mu = \omega$, produces spin precession with rate $\propto F_0 \cdot Z_c \cos(\phi)$



application of H_{ODF} for duration τ

$$\theta_{max} = F_0 Z_c \tau$$

Measuring spin precession (phase incoherent)



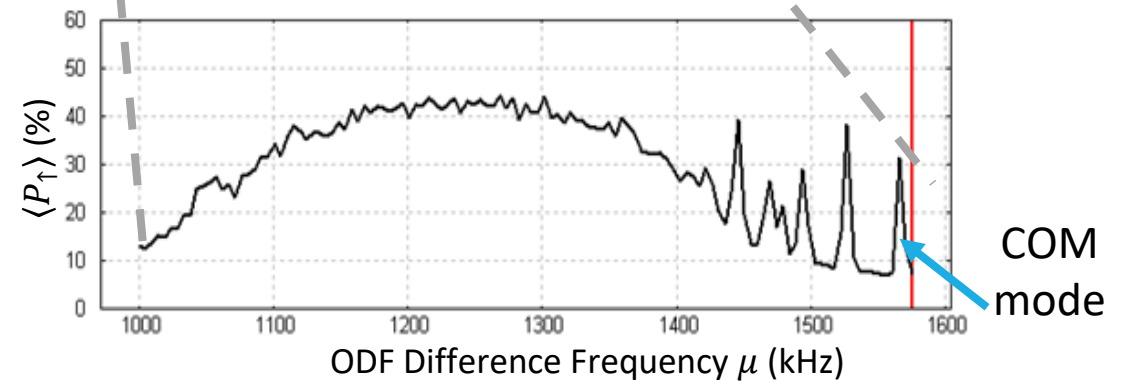
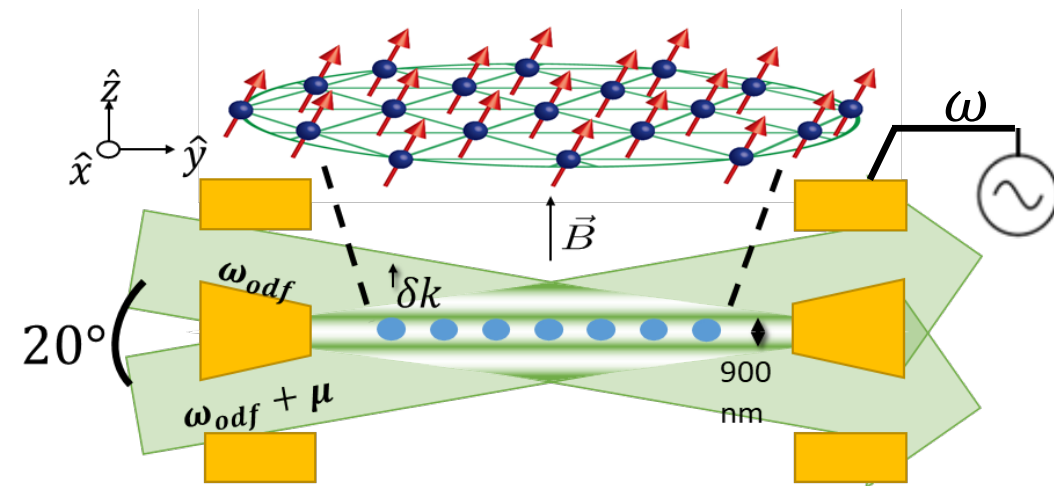
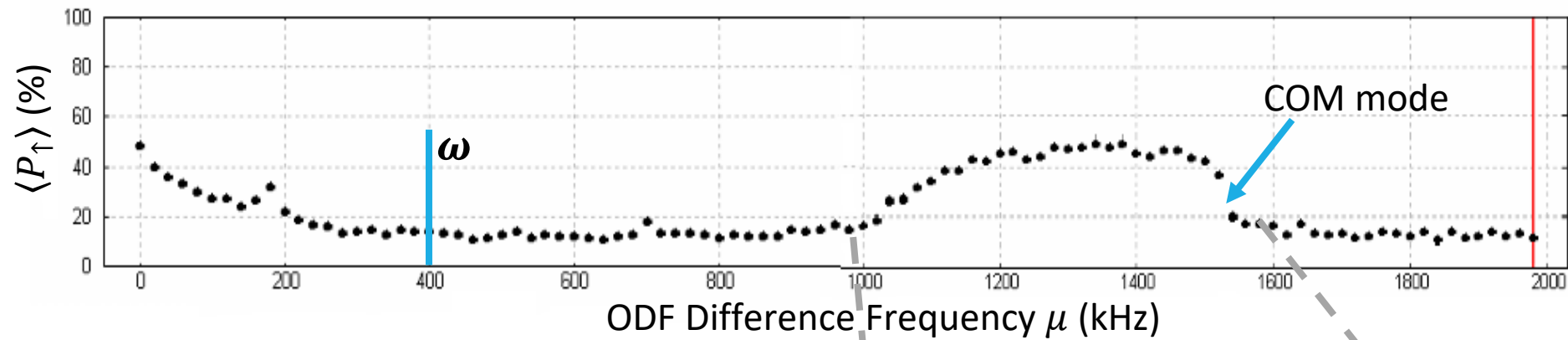
Precession θ ,

$$\theta = \frac{F_0}{\hbar} Z_c \tau \cos(\phi)$$
$$-\frac{F_0}{\hbar} Z_c \tau < \theta < \frac{F_0}{\hbar} Z_c \tau$$

Probability of measuring spin up:

$$\langle P_{\uparrow} \rangle = \frac{1}{2} (1 - e^{-\Gamma \tau} \langle \cos \theta \rangle)$$
$$= \frac{1}{2} \left(1 - e^{-\Gamma \tau} J_0 \left(\frac{F_0}{\hbar} Z_c \tau \right) \right)$$

Measuring spin precession (phase incoherent)

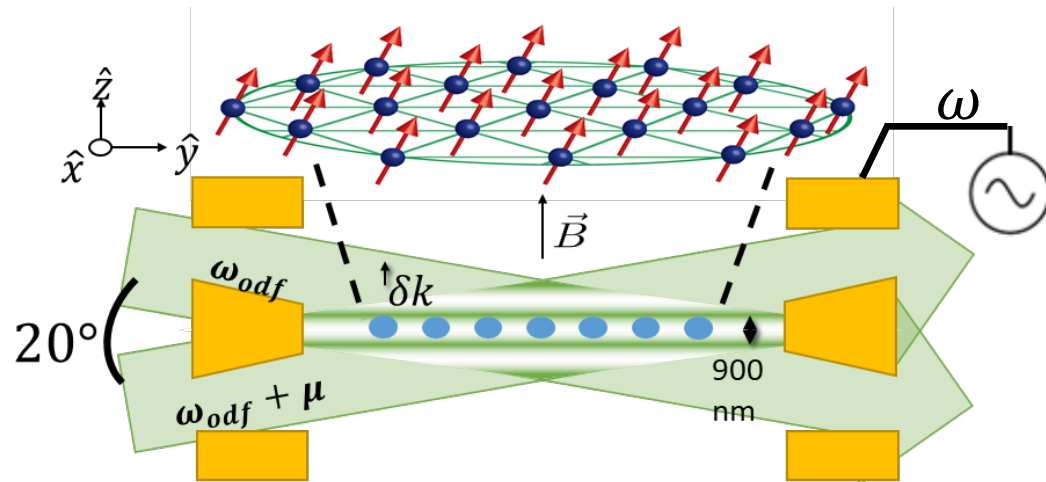
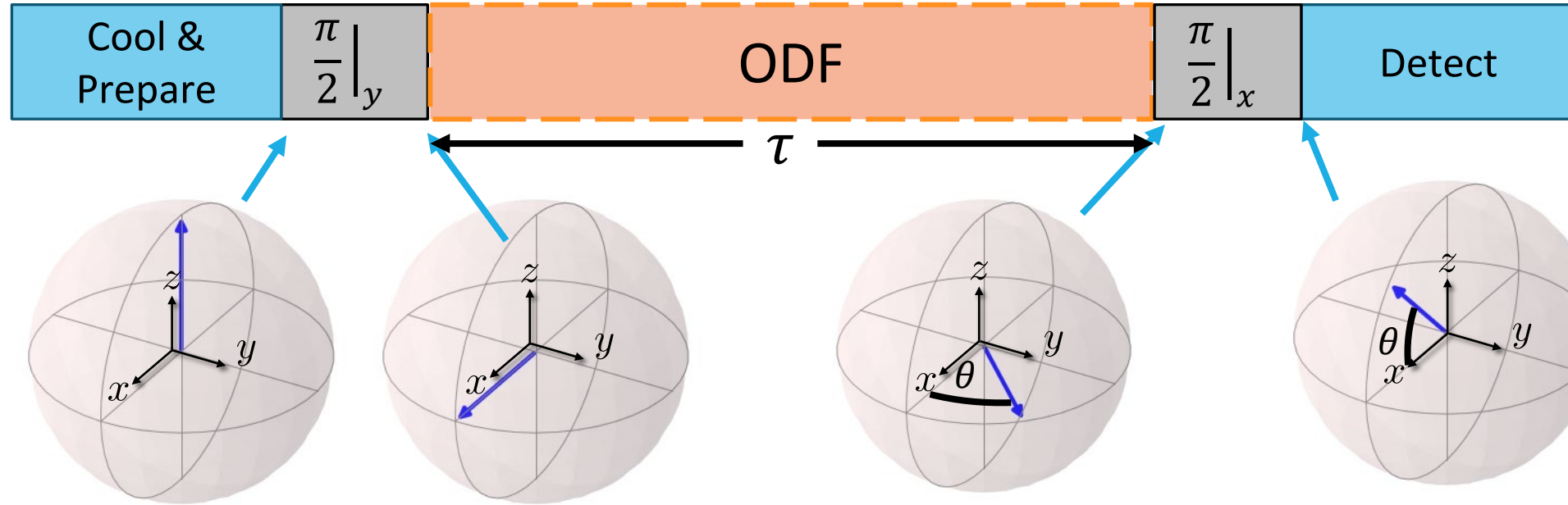


Gilmore et al.,
PRL 2017

$$\left. \frac{Z_c^2}{\delta Z_c^2} \right|_{\text{limiting}} \approx \left[\frac{Z_c}{0.2 \text{ nm}} \right]^2$$

$\omega \neq \omega_z$
 ϕ random

Measuring spin precession (phase coherent)



Phase coherent sensing
first order sensitive to Z_c !

Affolter et al.,
PRA 2020

$$\left. \frac{Z_c}{\delta Z_c} \right|_{\text{limiting}} \approx \left[\frac{Z_c}{0.050 \text{ nm}} \right]$$

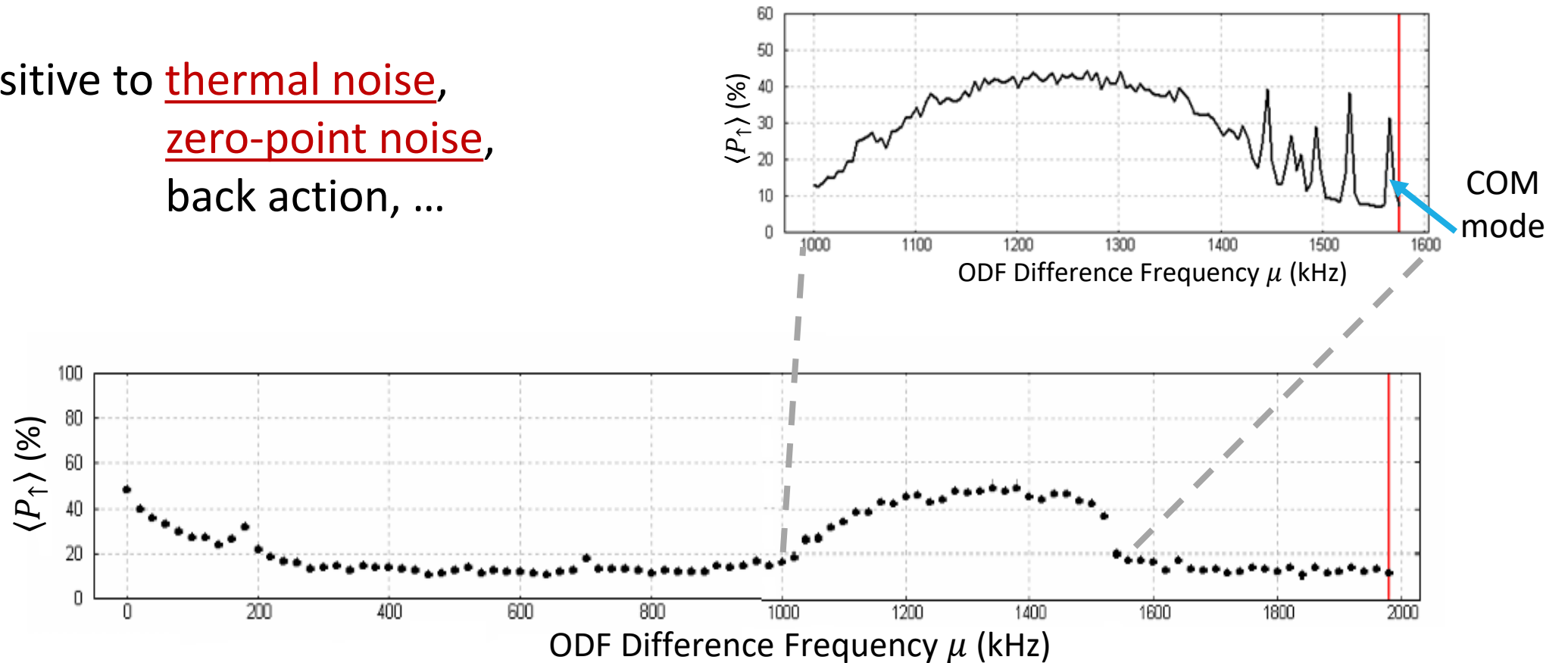
$\omega \neq \omega_z$
 ϕ fixed

40x smaller than $z_{zpt} \approx 2 \text{ nm}$

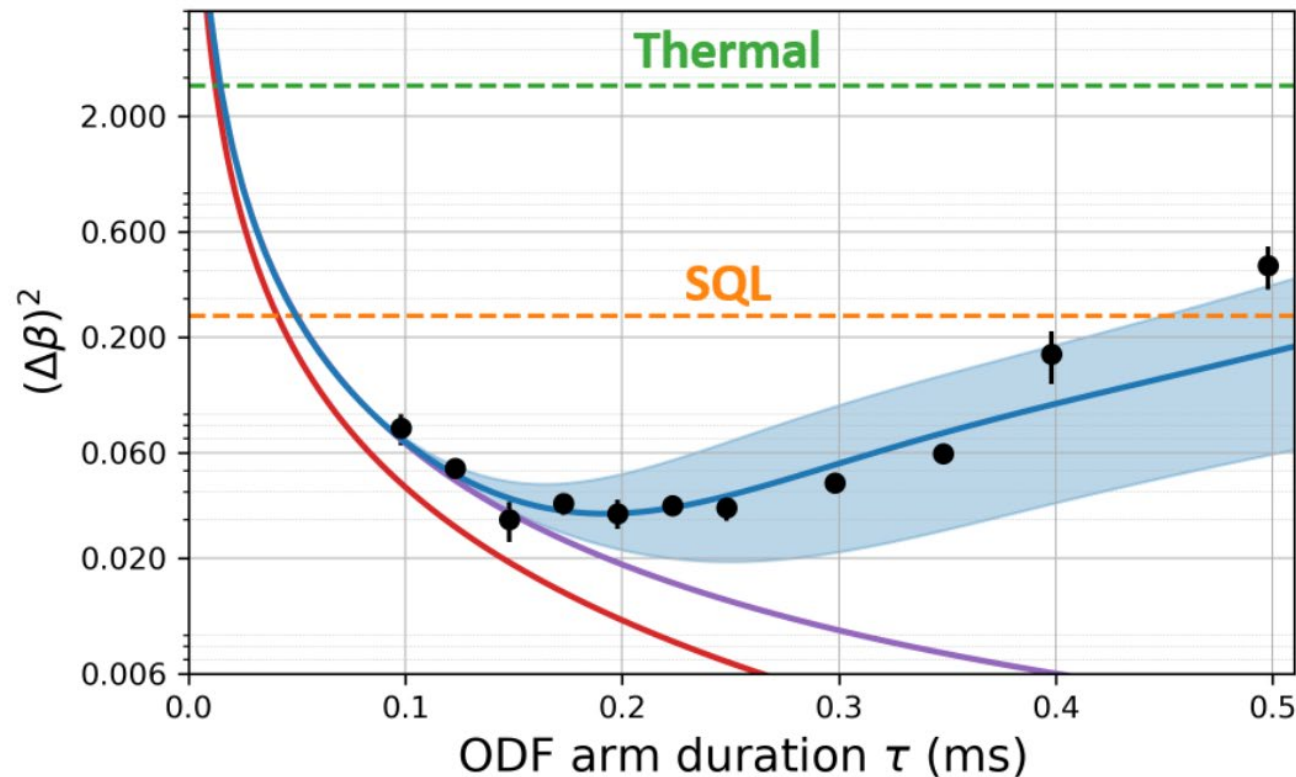
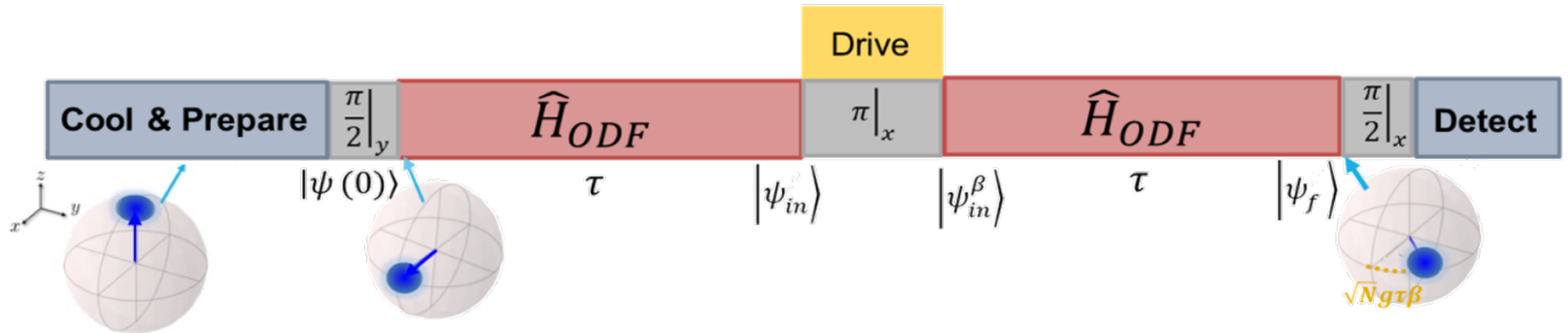
On resonance motional sensing: sensitivity to noise

Sensing motion resonant with ω_z much more sensitive to weak forces and electric fields
BUT

now also sensitive to thermal noise,
zero-point noise,
back action, ...



On resonance motional sensing: use time reversal



$$\Delta\beta_{Th}^2 = \frac{(2\overline{n(0)} + 1)}{4}$$

$$\Delta\beta_{SQL}^2 = \frac{1}{4}$$

See:

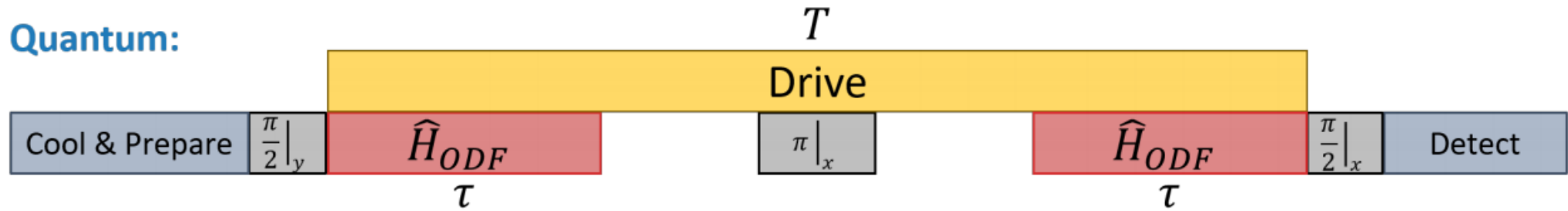
Hempel, et al., Nat. Photonics 7 (2013)

Toscano, et al., PRA 94 (2006)

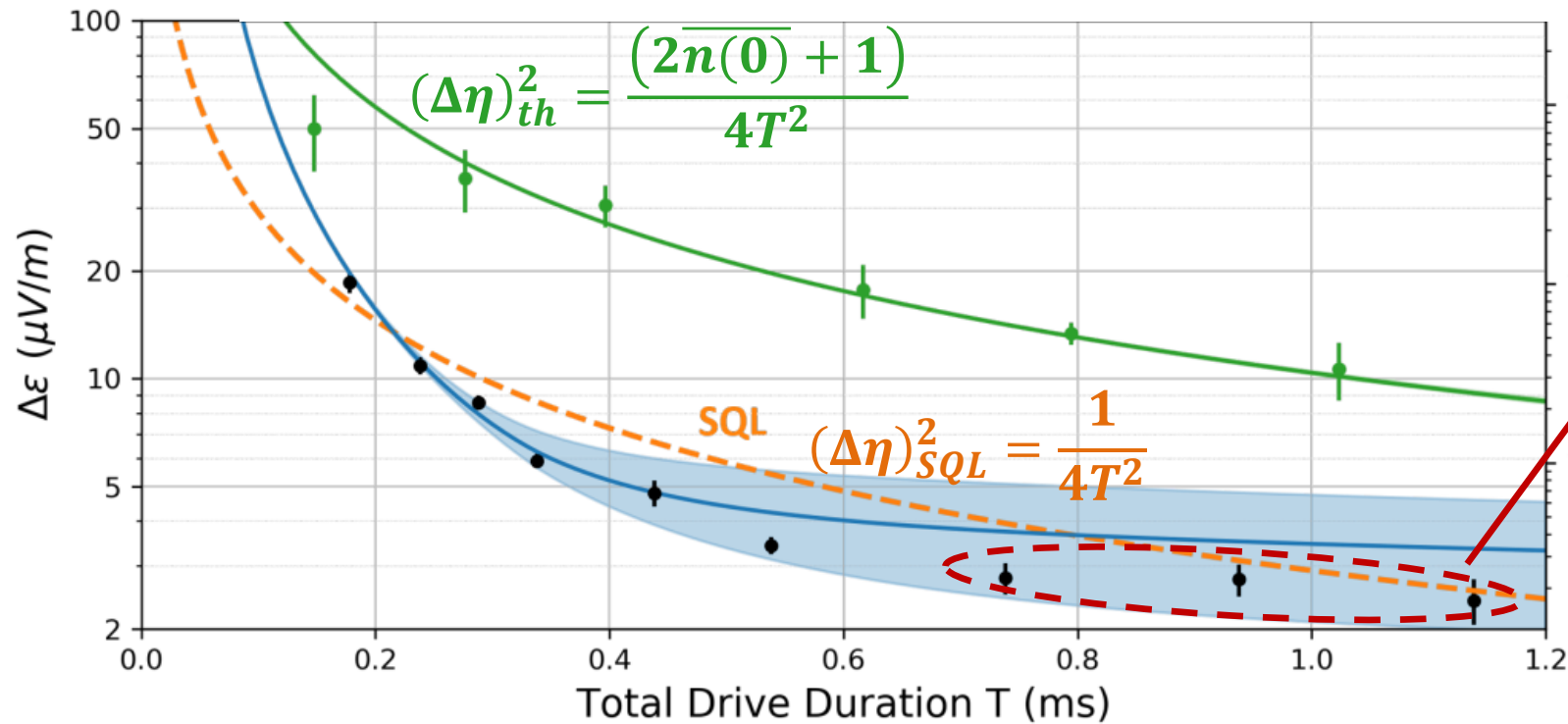
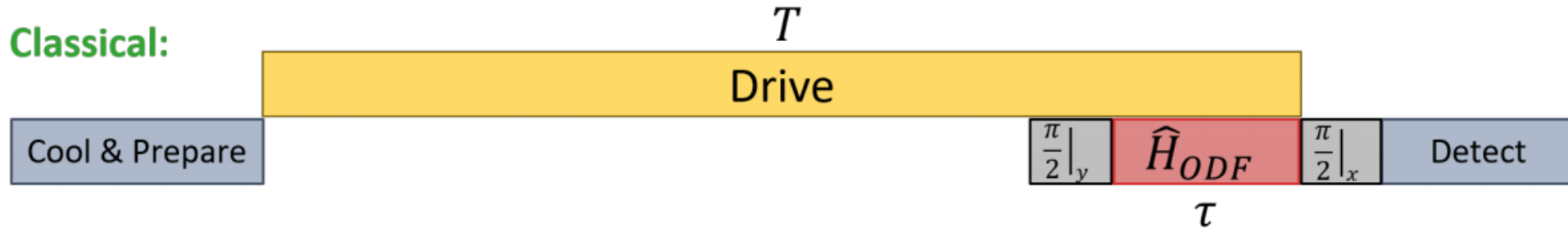
Gilmore et al., arXiv:2103.08690,
Science (to appear)

On resonance $\vec{E}$ -field sensing: increase drive duration T

Quantum:



Classical:



$$(240 \pm 10) \frac{(n\text{V/m})}{\sqrt{\text{Hz}}} @ 1.6 \text{ MHz}$$

Favorable comparison
with state-of-the-art
Rydberg sensors

$$\Delta E \sim 5.5 \frac{(\mu\text{V/m})}{\sqrt{\text{Hz}}} @ 10 \text{ GHz}$$

Gilmore et al., arXiv:2103.08690

Summary

- ❑ Demonstrated quantum-enhanced sensor of mechanical displacements and weak electric fields in a crystal composed of ~ 150 trapped ions.

Sensitivities @ 1.6 MHz:

- ✓ Displacement: $36 \pm 1.5 \frac{\text{pm}}{\sqrt{\text{Hz}}}$ [8.8 ± 0.4 dB below SQL (19 dB below thermal bound)]
- ✓ Electric Field: $240 \pm 10 \frac{\text{nV/m}}{\sqrt{\text{Hz}}}$ [4 ± 0.5 dB below SQL (14 dB below thermal bound)]

- ❑ Limitations: COM fluctuations $\sigma(\delta) \sim (20 - 60) \text{ Hz}$
COM Doppler cooled $\bar{n} \sim 4.5$

- ❑ Improvements:

Reduce COM fluctuations to $\sigma(\delta) \sim 1 \text{ Hz}$	} $\Rightarrow 10 \frac{(nV/m)}{\sqrt{\text{Hz}}}$
Near gnd state cooling of COM	

- ❑ Further improvements: ion number $N \sim 10^6$; employ spin squeezed states

Detection of hidden photons and axions @ frequencies [10 kHz to 10 MHz] ??

Axion coupling constant sensitivity estimate

The Axion-EM field interaction modifies Maxwell's equations:

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} - g_{a\gamma\gamma} \left(\mathbf{E} \times \nabla a - \frac{\partial a}{\partial t} \mathbf{B} \right) \quad a(t) = \frac{\sqrt{2\rho_{DM}}}{m_a} \sin(m_a t),$$

m_a : Axion mass

ρ_{DM} : 0.3 GeV/cm³

$g_{a\gamma\gamma}$: axion – photon coupling

For COM frequencies $\omega_z/(2\pi) < 10$ MHz $\Rightarrow \lambda_{\text{Compton}}^{\text{axion}} \gg R$ (size of B_0 field region) $\Rightarrow$
axion electric field $\mathbf{E}_0$ suppressed to 2nd order in $(m_a R)^2$ (natural units)

$$E_0(t) \sim (m_a R)^2 g_{a\gamma\gamma} a(t) B_0$$

$$\delta g_{a\gamma\gamma} = (\delta E)(m_a) \left(\frac{1}{B_0} \right) \left(\frac{1}{\sqrt{2\rho_{DM}}} \right) \left(\frac{1}{\omega_z} \right)^2$$

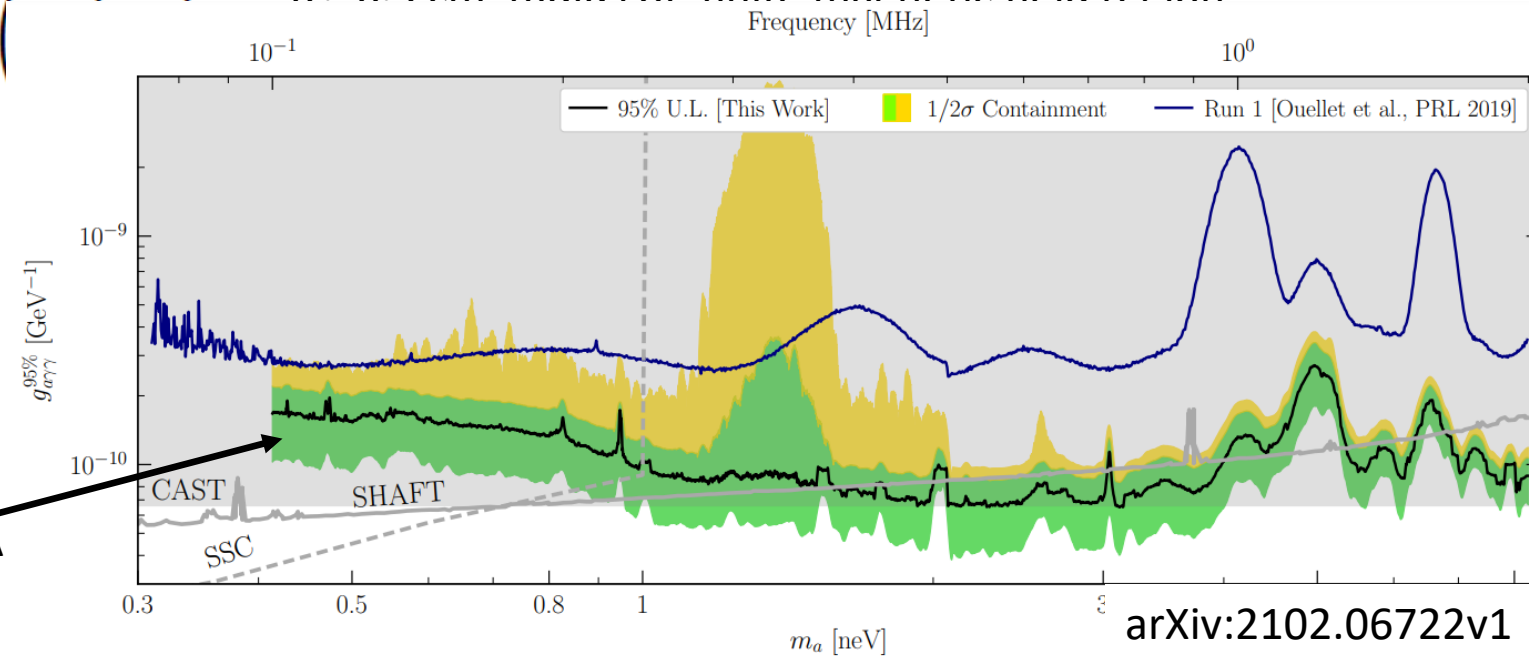
$$\delta g_{a\gamma\gamma} = 4.80 \times 10^{-9} \text{ GeV}^{-1} \times \sqrt{\frac{1 \text{ s}}{t_{\text{avg}}}}$$

$$\delta g_{a\gamma\gamma}(1 \text{ day}) = 1.6 \times 10^{-11} \text{ GeV}^{-1}.$$

B_0 : External magnetic field ~ 4.5 T

R : 6.5 cm radius of finite magnetic field region

Compatible with state-of-the-art ABRACADABRA
and BASE collaboration limits (PRL 126 (2021))

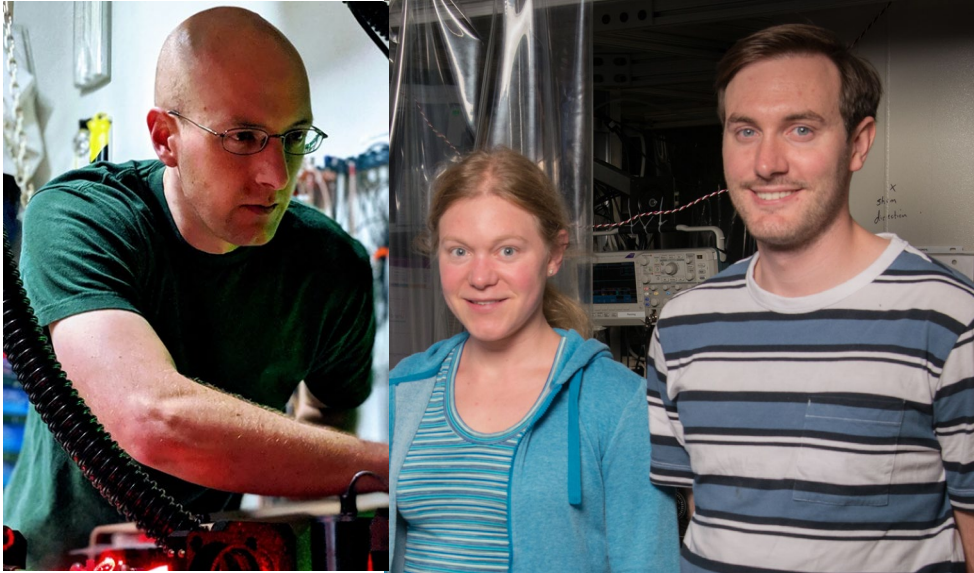


NIST ion storage group (2019)



John Bollinger; James Chou; David Hume; Dietrich Leibfried; David Leibbrandt; Daniel Slichter; Andrew Wilson; David J. Wineland;
Matt Affolter; James Bergquist; Kevin Boyce; Shaun Burd; Dalton Chaffee; Ethan Clements; Daniel Cole; Alejandra Collopy; Kaifeng Cui;
Robert Drullinger; Stephen Erickson; Kevin Gilmore; Panyu Hou; Wayne Itano; Elena Jordan; Jonas Keller; May Kim; Hannah Knaack;
Felix Knollman; Justin Niedermeyer; Julian Schmidt; Raghavendra Srinivas; Laurent Stephenson; Susana Todaro; Jose Valencia; Jenny Wu

Experiment



Matt Affolter Elena Jordan Kevin Gilmore

Justin Bohnet



Jennifer Lilieholm



Bryce Bullock

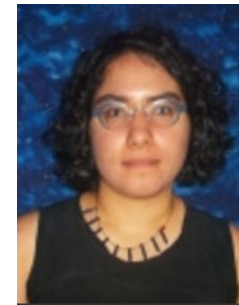
Theory



Ana Maria
Rey



Martin
Gärttner



Arghavan
Safavi-Naini



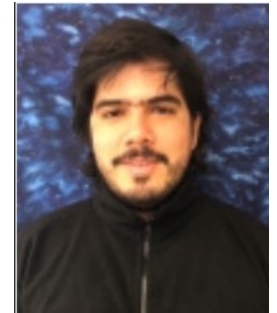
Robert Lewis-
Swan



Murray
Holland



Athreya Shankar
Holland group



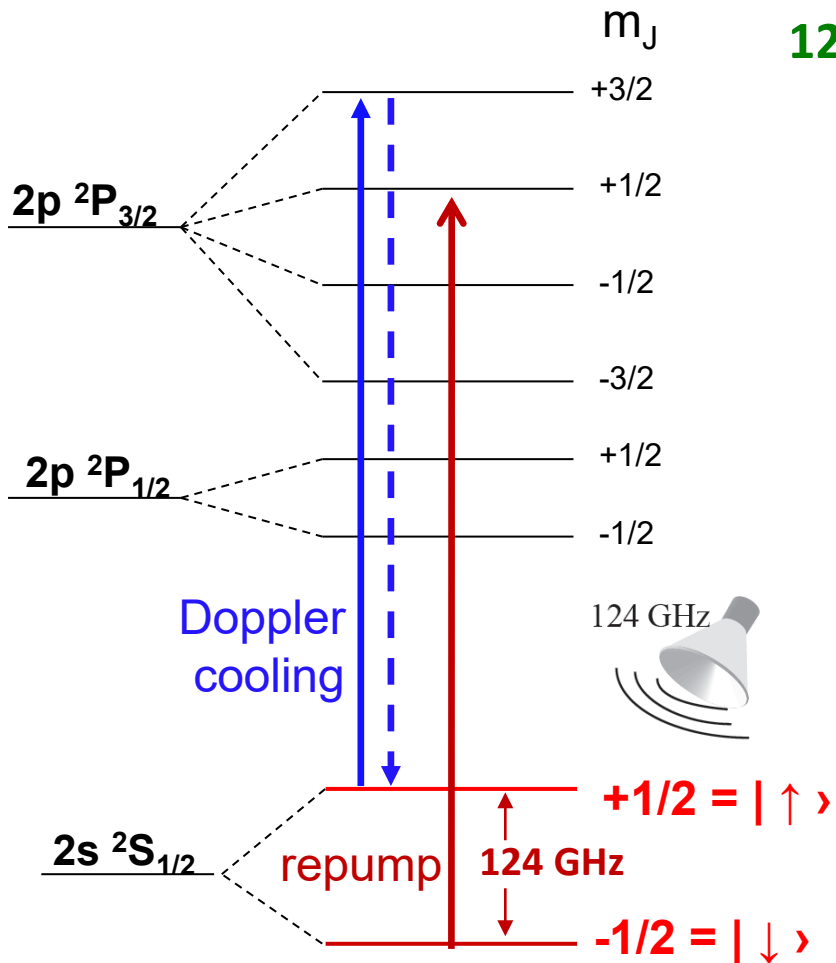
Diego Barberena



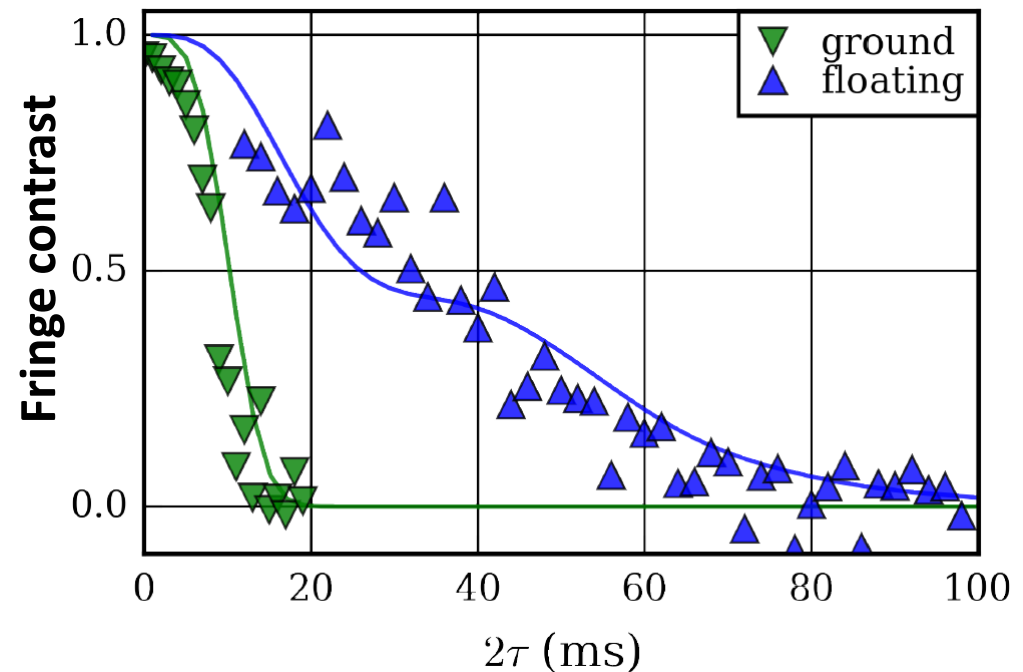
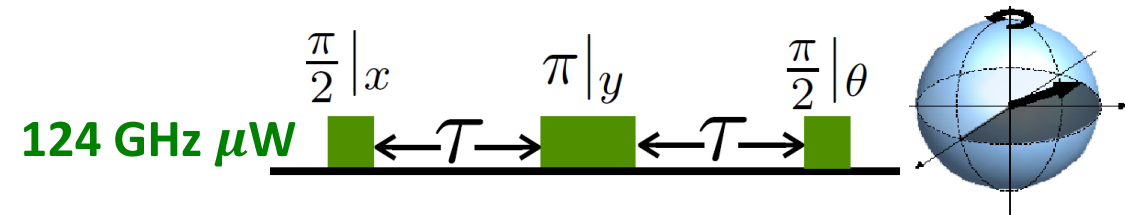
Extra Slides

Be⁺ high magnetic field qubit

⁹Be⁺, B ~ 4.5 T, $\omega_0/2\pi \sim 124.1$ GHz



T2 coherence

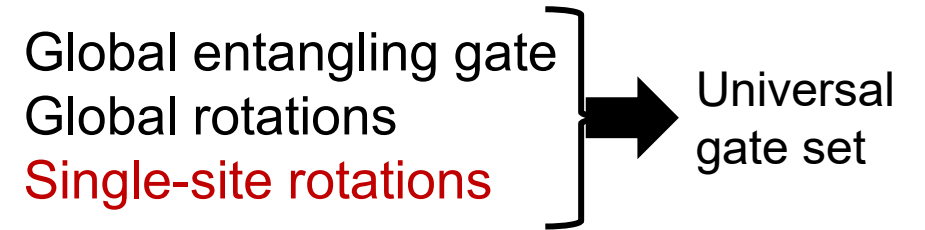
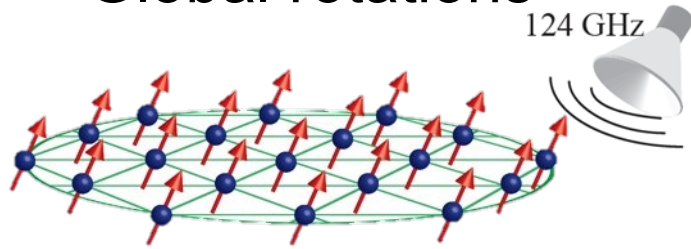


Spin-echo precession time →

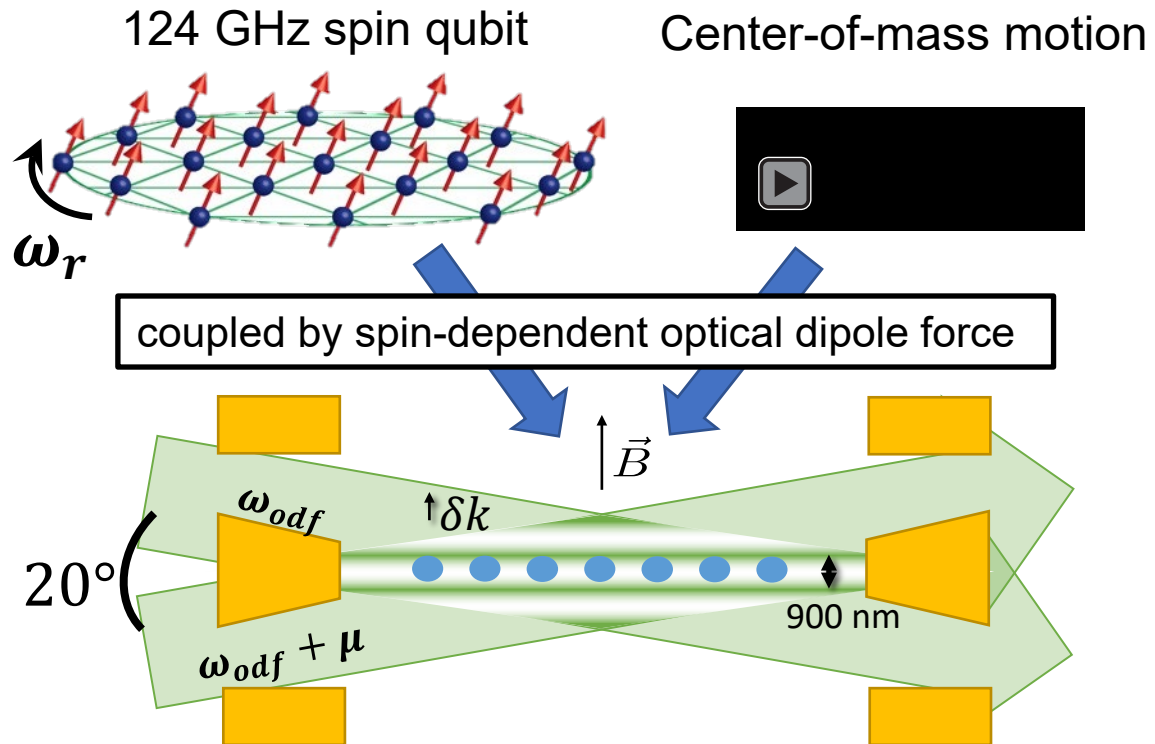
Britton et al., PRA (2016)
arXiv_1512.00801

Implementing single-site rotations

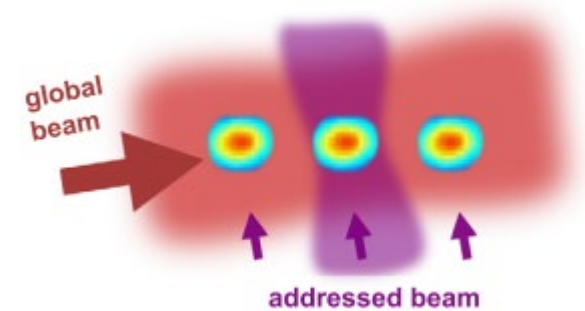
Global rotations



Global entangling gate



With linear arrays, single-site σ_z -rotations with AC Stark shifts from focused laser beams



From Schindler et al 2013 New J. Phys. 15 123012

Implementing single-site rotations - σ_z rotations with patterned AC Stark shifts

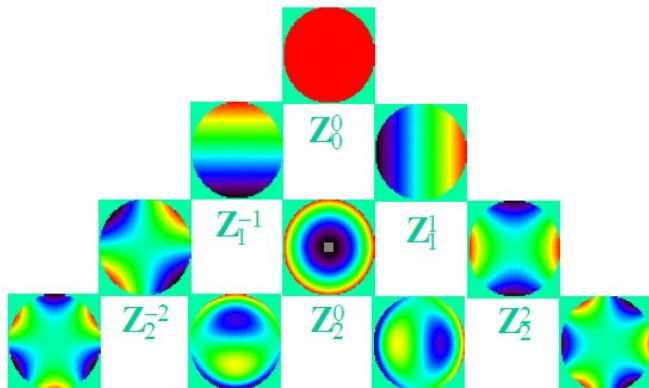
$$\hat{H}_{ODF} = U \sum_i \sin(\delta k \hat{z}_i - \mu t + \psi) \hat{\sigma}_i^z. \quad \xRightarrow{\text{DM}} \quad \hat{H}_{ODF} = U \sum_i \sin[\delta k \hat{z}_i - \mu t + \psi + \delta(x_i, y_i)] \hat{\sigma}_i^z.$$

$$\delta(x, y) \equiv \delta(\rho, \phi^{lab}) := P^m(\rho) \cos(m\phi^{lab}) \quad \begin{matrix} \mu = m\omega_r \\ \phi_i^{lab} \xRightarrow{\quad} \phi_i^{rot} - \omega_r t \end{matrix}$$

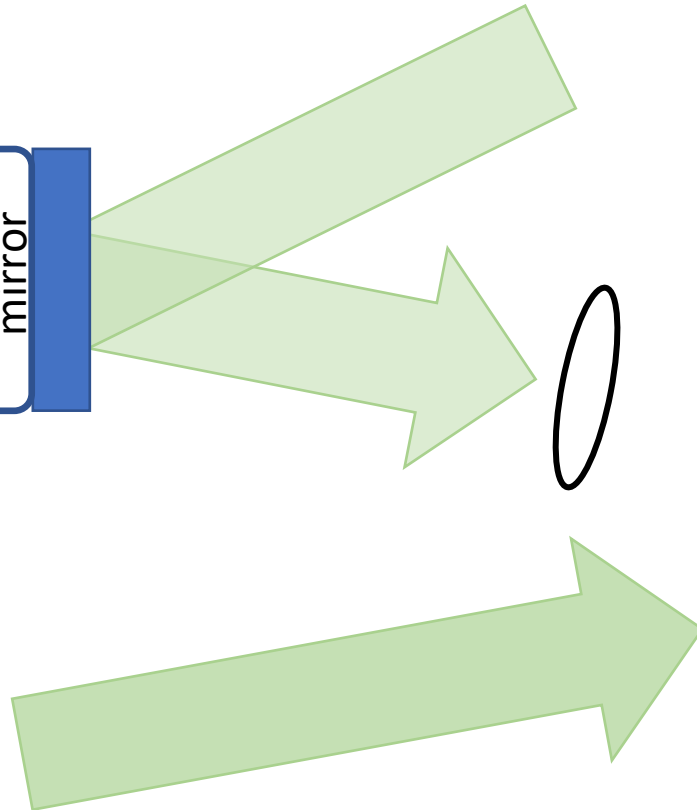
$$\begin{aligned} \hat{H}_{ODF} &= U \sum_i J_1[P^m(\rho_i)] \sin(m\phi_i^{rot} - \psi) \hat{\sigma}_i^z \\ &\approx \frac{U}{2} \sum_i P^m(\rho_i) \sin(m\phi_i^{rot} - \psi) \hat{\sigma}_i^z \end{aligned}$$

Zernike polynomials

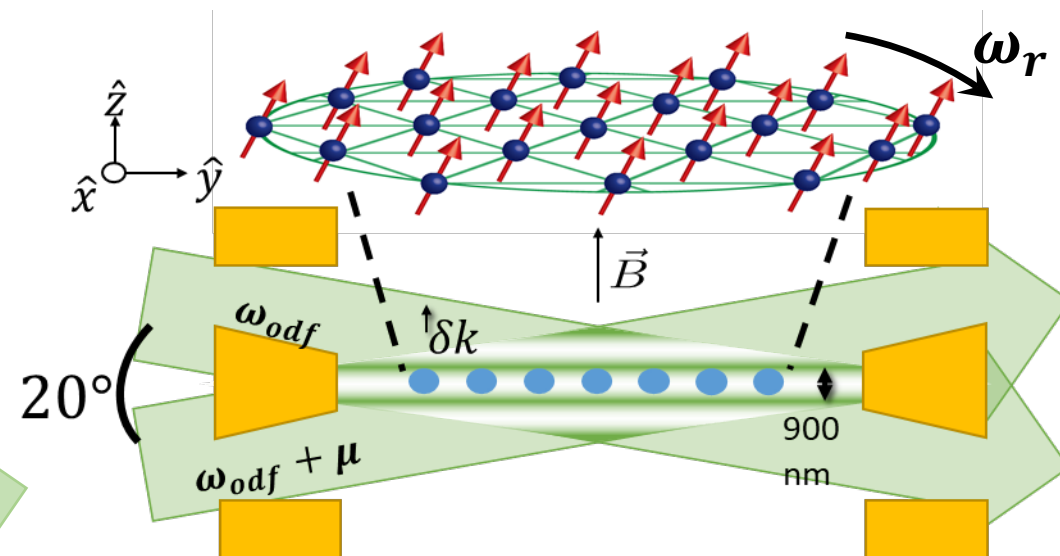
$$Z_n^m(\rho, \phi) = \begin{cases} N_n^m R_n^{|m|}(\rho) \cos(m\phi), \\ N_n^m R_n^{|m|}(\rho) \sin(m\phi), \end{cases}$$



deformable
mirror



Time independent in the rotating frame !!



Implementing single-site rotations - σ_z rotations with patterned AC Stark shifts

$$\hat{H}_{ODF} = U \sum_i \sin(\delta k \hat{z}_i - \mu t + \psi) \hat{\sigma}_i^z. \xrightarrow{\text{DM}} \hat{H}_{ODF} = U \sum_i \sin[\delta k \hat{z}_i - \mu t + \psi + \delta(x_i, y_i)] \hat{\sigma}_i^z.$$

$$\delta(x, y) \equiv \delta(\rho, \phi^{lab}) := P^m(\rho) \cos(m\phi^{lab}) \quad \begin{matrix} \mu = m\omega_r \\ \phi_i^{lab} \xrightarrow{\quad} \phi_i^{rot} - \omega_r t \end{matrix}$$

$$\begin{aligned} \hat{H}_{ODF} &= U \sum_i J_1[P^m(\rho_i)] \sin(m\phi_i^{rot} - \psi) \hat{\sigma}_i^z \\ &\approx \frac{U}{2} \sum_i P^m(\rho_i) \sin(m\phi_i^{rot} - \psi) \hat{\sigma}_i^z \end{aligned}$$

Zernike polynomials

$$Z_n^m(\rho, \phi) = \begin{cases} N_n^m R_n^{|m|}(\rho) \cos(m\phi), \\ N_n^m R_n^{|m|}(\rho) \sin(m\phi), \end{cases}$$

nale
ror

Simple protocol for small argument regime:

1. Set DM once
2. Cycle ODF beat note $\mu = m\omega_r$ through harmonics of ω_r

Qubit rotations are conducted in parallel

Time independent in the rotating frame !!

