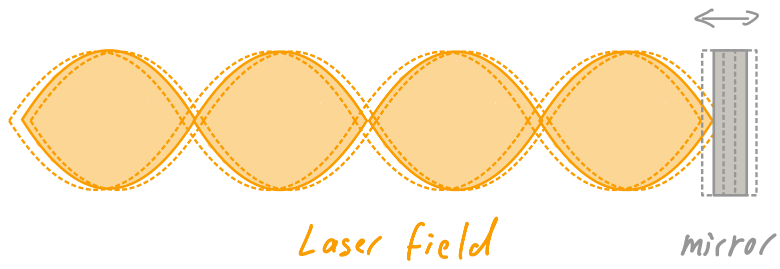
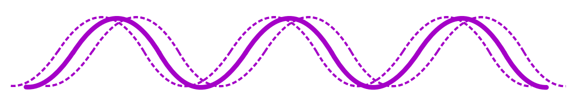
Floquet engineering with optical lattices

Tilman Esslinger, ETH Zurich



What do you associate?

- shaken periodic potential



$V_0 \cos^2[k_{lat}(x - x_m(t))]$

- $x_m(t) = x_0 \cos(\omega t - \phi_0)$

  x_0: amplitude, ω: frequency, ϕ_0: phase

- energy scales: driving frequency: ω

  kinetic energy

  band width : W

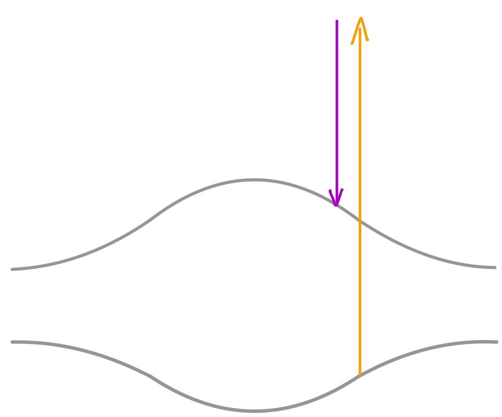
  tunneling : t

  band gap

  Hubbard U

- periodic in time
- switching on, switching off
- "eigenstates" may oscillate in time (phase between eigenstate and drive)
- suitable frames of reference
- frequency spectrum:

- multi-photon transitions between bands



- heating

What else?

- more complicated wave forms, e.g. two-frequencies
- comensurate frequencies
  $\omega_2 = 2\omega_1$
  $\rightarrow$ relative phase
- interfering several driven lattices
- amplitude modulation

Early Experiments I:

simple thought: · modulate at U
· amplitude easiest

Amplitude modulation in 1D Bose-Hubbard model

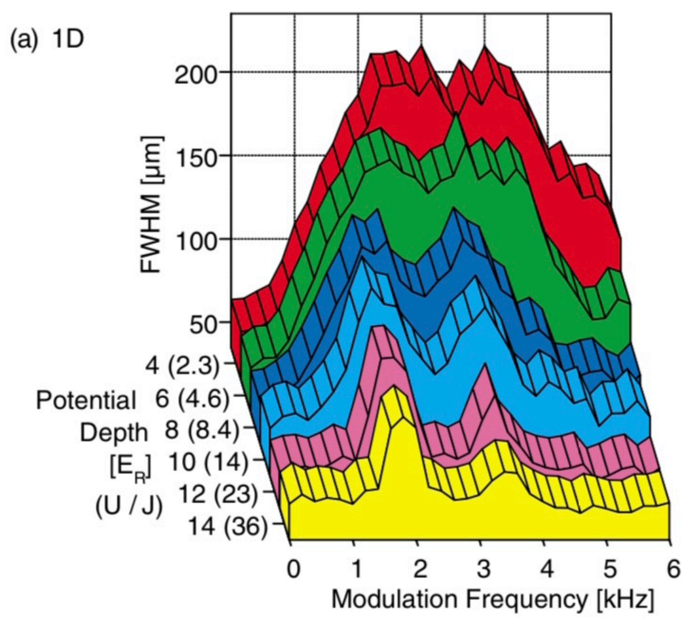


FIG. 2 (color online). The measured excitation spectrum of an array of 1D gases ($V_\perp = 30E_R$) is shown in (a) for different values of $V_{ax,0}$. The interaction ratios U/J given in brackets are calculated numerically using a band structure model in the tight-binding approximation [9]. Spectrum (c) shows the superfluid to Mott insulator transition in the 3D case ($V_\perp = V_{ax,0}$). The crossover region between the one- and the three-dimensional system ($V_\perp = 20E_R$) is shown in (b).

Transition from a Strongly Interacting 1D Superfluid to a Mott Insulator

Thilo Stöferle, Henning Moritz, Christian Schori, Michael Köhl, and Tilman Esslinger
Phys. Rev. Lett. 92, 130403 – 2004

Early Experiments II:

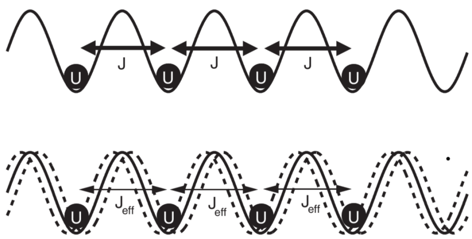
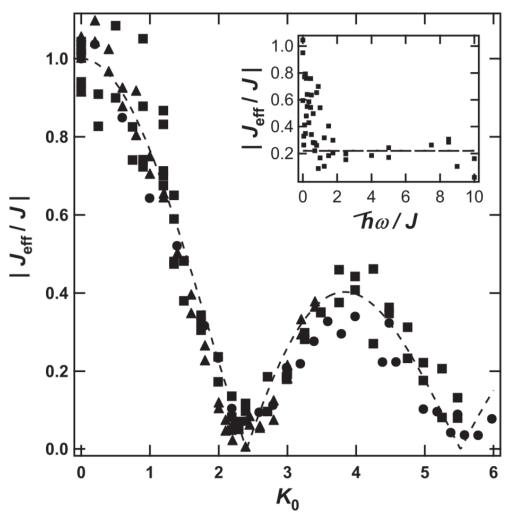


FIG. 1. Suppression of tunneling by strong driving. The dynamics of a Bose-Einstein condensate in a periodic potential is governed by the tunneling matrix element J and the on-site interaction energy U (above). If the potential is strongly shaken, tunneling between the wells is dynamically suppressed, leading to a renormalized tunneling matrix element J_{eff} (below) but leaving the interaction energy U unaffected.



measured from
1D expansion
(Bessel function)

FIG. 2. Dynamical suppression of tunneling in an optical lattice. Shown here is $|J_{\mathrm{eff}}/J|$ as a function of the shaking parameter K_0 for $V_0/E_{\mathrm{rec}} = 6$, $\omega/2\pi = 1$ kHz (squares), $V_0/E_{\mathrm{rec}} = 6$, $\omega/2\pi = 0.5$ kHz (circles), and $V_0/E_{\mathrm{rec}} = 4$, $\omega/2\pi = 1$ kHz (triangles). The dashed line is the theoretical prediction. Inset: $|J_{\mathrm{eff}}/J|$ as a function of ω for $K_0 = 2.0$ and $V_0/E_{\mathrm{rec}} = 9$ corresponding to $J/h = 90$ Hz.

Dynamical Control of Matter-Wave Tunneling in Periodic Potentials
H. Lignier, C. Sias, D. Ciampini, Y. Singh, A. Zenesini, O. Morsch, and E. Arimondo
Phys. Rev. Lett. 99, 220403 – 2007

Superfluid-Insulator Transition in a Periodically Driven Optical Lattice
André Eckardt, Christoph Weiss, and Martin Holthaus
Phys. Rev. Lett. 95, 260404 – 2005

Literature:

[1] Marin Bukov, Luca D'Alessio, and Anatoli Polkovnikov, "Universal high-frequency behavior of periodically driven systems: from dynamical stabilization to Floquet engineering," Advances in Physics **64**, 139–226 (2015).

[2] N. Goldman and J. Dalibard, "Periodically Driven Quantum Systems: Effective Hamiltonians and Engineered Gauge Fields," Physical Review X **4**, 31027 (2014).

[3] Or the 'Floquet-Magnus expansion' in the stroboscopic case.

[4] The analogous expressions for $\mathcal{H}_{\mathrm{F}}$, called 'Floquet-Magnus' expansion, can be found in ref. [1], together with higher-order terms of the expansions.

[5] Jean Dalibard, "Réseaux dépendant du temps," Collège de France, Cours 4 (2013).

[6] Mark S. Rudner and Netanel H. Lindner, *The Floquet Engineer's Handbook*, Vol. 2003.08252 (arXiv, 2020).

[7] Martin Holthaus, "Floquet engineering with quasienergy bands of periodically driven optical lattices," Journal of Physics B: Atomic, Molecular and Optical Physics **49**, 013001 (2016).

[8] H. Lignier, C. Sias, D. Ciampini, Y. Singh, A. Zenesini, O. Morsch, and E. Arimondo, "Dynamical Control of Matter-Wave Tunneling in Periodic Potentials," Physical Review Letters **99** (2007), 10.1103/PhysRevLett.99.220403.

[9] N. R. Cooper, J. Dalibard, and I. B. Spielman, "Topological bands for ultracold atoms," Reviews of Modern Physics **91** (2019), 10.1103/RevModPhys.91.015005.

[10] F. D. M. Haldane, "Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"," Physical Review Letters **61**, 2015–2018 (1988).

[Viebahn 2020]: Lecture Notes

[Görg 2019]: PhD thesis ; ETH Z server

For more literature: Internet

Many experimental groups:

- Hamburg, Munich, Chicago, ...

Floquet approach to time-periodic Hamiltonian

Find solutions for:

$$i\hbar \frac{d}{d\tau} |\psi(\tau)\rangle = \hat{H}(\tau) |\psi(\tau)\rangle$$

$$\text{with } \hat{H}(\tau) = \hat{H}(\tau + T)$$

$$|\psi(\tau)\rangle = U(\tau,\tau_0) |\psi(\tau_0)\rangle$$

Evolution operator:

$$U(\tau_f, \tau_i) = \mathcal{T} \exp\left[-\frac{i}{\hbar} \int_{\tau_i}^{\tau_f} \hat{H}(\tau) d\tau\right] = \prod_{j:\tau_0 \le \tau_j \le \tau_1} e^{-\frac{i}{\hbar}\hat{H}(\tau_j)(\tau_{j+1}-\tau_j)}$$

$\mathcal{T}$: time odering

periodic: $\hat{U}(\tau + T, T) = \hat{U}(\tau, 0)$

can be factorized:

$$\hat{U}(\tau_1 + \tau_2, \tau_0) = \hat{U}(\tau_1 + \tau_2, \tau_1) \hat{U}(\tau_1, \tau_0)$$

periodic evolution:

$$\hat{U}(\tau_0 + nT, \tau_0) = \left[\hat{U}(\tau_0 + T, \tau_0)\right]^n$$

unitary operator $(\hat{U}^\dagger U = \mathbb{1})$

$$\hat{U}(\tau_0 + T, \tau_0) = e^{-i \hat{H}_F[\tau_0] T/\hbar}$$

$\rightarrow$ $\hat{H}_F[\tau_0]$: stroboscopic Floquet Hamiltonian, hermitian $(\hat{H}_F^\dagger = \hat{H}_F)$

depends on choice of τ_0

all stroboscopic Floquet Hamiltonian are related via the gauge transformation:

$$\hat{H}_F[\tau_0'] = \hat{U}(\tau_0', \tau)\, \hat{H}_F[\tau_0]\, \hat{U}(\tau_0, \tau_0')$$

→ i.e. spectrum is unaffected

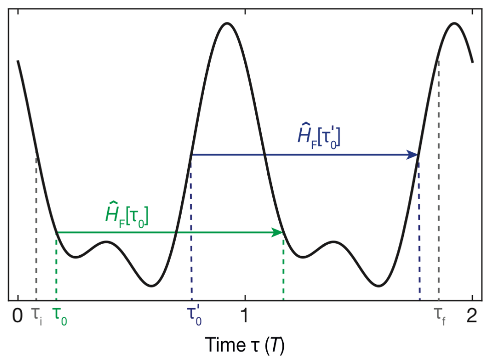
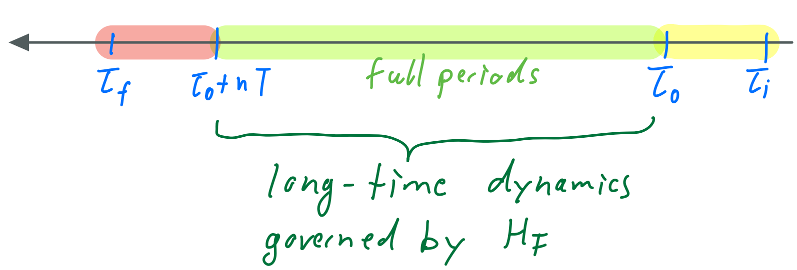


Figure 3.1: Stroboscopic time evolution in a periodically modulated system. The stroboscopic Floquet Hamiltonian $\hat{H}_{\mathrm{F}}[\tau_0]$ evolves the system over an integer multiple of the driving cycle nT. It explicitly depends on the choice of the initial time τ_0, e.g. the Hamiltonian $\hat{H}_{\mathrm{F}}[\tau_0']$ corresponds to a different Floquet gauge. Additional evolution operators are required to evolve the system from the initial time τ_{i} to τ_0 and from $\tau_0 + nT$ to the final time τ_{f} (see Eq. (3.8)). Görg 2019

Evolution:

$$\hat{U}(\tau_f, \tau_i) = \hat{U}(\tau_f, \tau_0 + nT)\, e^{-i \hat{H}_F[\tau_0]\, nT/\hbar}\, \hat{U}(\tau_0, \tau_i)$$



gauge choice

Define stroboscopic kick operator $\hat{K}_F[\tau_0](\tau)$:

$$e^{-i\hat{K}_F[\tau_0](\tau)} = \hat{U}(\tau,\tau_0)\, e^{i\hat{H}_F[\tau_0](\tau-\tau_0)/\hbar}$$

- periodic: $\hat{K}_F[\tau_0](\tau+T) = \hat{K}_F[\tau_0](\tau+T)$
- zero at stroboscopic times:

$$\hat{K}_F[\tau_0](\tau_0+nT) = 0$$

General evolution operator:

$$\hat{U}(\tau_f,\tau_i) = e^{-i\hat{K}_F[\tau_0](\tau_f)}\, e^{-i\hat{H}_F[\tau_0](\tau_f-\tau_i)/\hbar}\, e^{i\hat{K}_F[\tau_0](\tau_i)}$$

(Floquet's theorem for stroboscopic Floquet Hamiltonian and kick operator)

Effective Hamiltonian for the non-stroboscobic evolution:

- time-independent Hamiltonian without explicit dependence on τ_0
- absorb gauge-dependent terms in kick operators

$$\hat{H}_{eff} = e^{i\hat{K}(\tau_0)}\, \hat{H}_F[\tau_0]\, e^{-i\hat{K}(\tau_0)}$$

- new kick operators:

$$e^{-i\hat{K}_F[\tau_0](\tau)} = e^{-i\hat{K}(\tau)}\, e^{i\hat{K}(\tau_0)}$$

Floquet theorem in its gauge-independent form:

$$\hat{U}(\tau_f,\tau_i) = \underbrace{e^{-i\hat{K}(\tau_f)}}_{\substack{\text{transform back}\\ \to \text{dependence on micromotion}\\ \text{(final time } \tau_f \text{ within period } T)}} \underbrace{e^{-i\hat{H}_{eff}(\tau_f-\tau_i)/\hbar}}_{\substack{\text{evolve under}\\ \text{effective Hamiltonian}}} \underbrace{e^{i\hat{K}(\tau_i)}}_{\substack{\text{transform into}\\ \text{effective frame}}}$$

- differential form:

$$\hat{H}_{eff} = e^{i\hat{K}(\tau)}\hat{H}(\tau)e^{-i\hat{K}(\tau)} - i\hbar e^{i\hat{K}(\tau)} \frac{\partial}{\partial t}\left[e^{-i\hat{K}(\tau)}\right]$$

Floquet states and quasienergies:

- evolution of state by one driving period

$$\hat{U}(\tau_0+T,\tau_0) = e^{-i\hat{H}_F \times T/\hbar}$$

$\hat{U}$: unitary

- eigenvalues / Floquet multiplier:

$$\left\{ e^{-i\epsilon_n T/\hbar} \right\}$$

- multi-valued "quasienergies" ϵ_n

$$\hat{H}_F |n\rangle = \epsilon_n |n\rangle$$

really $|n[\tau_0]\rangle$

ϵ_n define mod $\hbar\omega$ with $\omega = \frac{2\pi}{T}$

- take arbitrary initial state $|\Psi(\tau_0)\rangle$
  and expand in eigen basis $\mathcal{U}(\tau_0+T,\tau_0)$

$$|\Psi(\tau_0)\rangle = \sum_n |n\rangle \underbrace{\langle n|\Psi(\tau_0)\rangle}_{\equiv a_n}$$

$$= \sum_n a_n |n\rangle$$

- evolve state in time:

$$|\Psi(\tau)\rangle = \hat{\mathcal{U}}(\tau,\tau_0)\,|\Psi(\tau_0)\rangle$$

$$= \sum_n a_n\, e^{-i\hat{k}_F[\tau_0](\tau)}\, e^{-i\hat{H}_F(\tau-\tau_0)/\hbar}\,|n\rangle$$

see definition of kick-operator

$$= \sum_n a_n\, e^{-i\hat{k}_F[\tau_0](\tau)}\, e^{-i\epsilon_n(\tau-\tau_0)/\hbar}\,|n\rangle$$

$$= \sum_n a_n\, e^{-i\epsilon_n(\tau-\tau_0)/\hbar}\,|u_n(\tau)\rangle$$

time independent coefficients!

- with Floquet modes:

$$|u_n(\tau)\rangle \equiv e^{-i\hat{k}_F[\tau_0](\tau)}\,|n\rangle$$

$$|u_n(\tau)\rangle = |u_n(\tau+T)\rangle$$

are gauge dependent

- Floquet states:

$$|\Psi_n(\tau)\rangle = e^{-i\epsilon_n(\tau-\tau_0)/\hbar}\,|u_n(\tau)\rangle$$

stationary states of time-dependent Schrödinger equation

Observables

Time-dependent expectation value of $\hat{O}$ in a Floquet eigenstate

$$\langle \hat{O} \rangle_n (\tau) = \langle \psi_n(\tau) | \hat{O} | \psi_n(\tau) \rangle$$
$$= \langle u_n | e^{i\hat{K}(\tau)} \hat{O} e^{-i\hat{K}(\tau)} | u_n \rangle$$

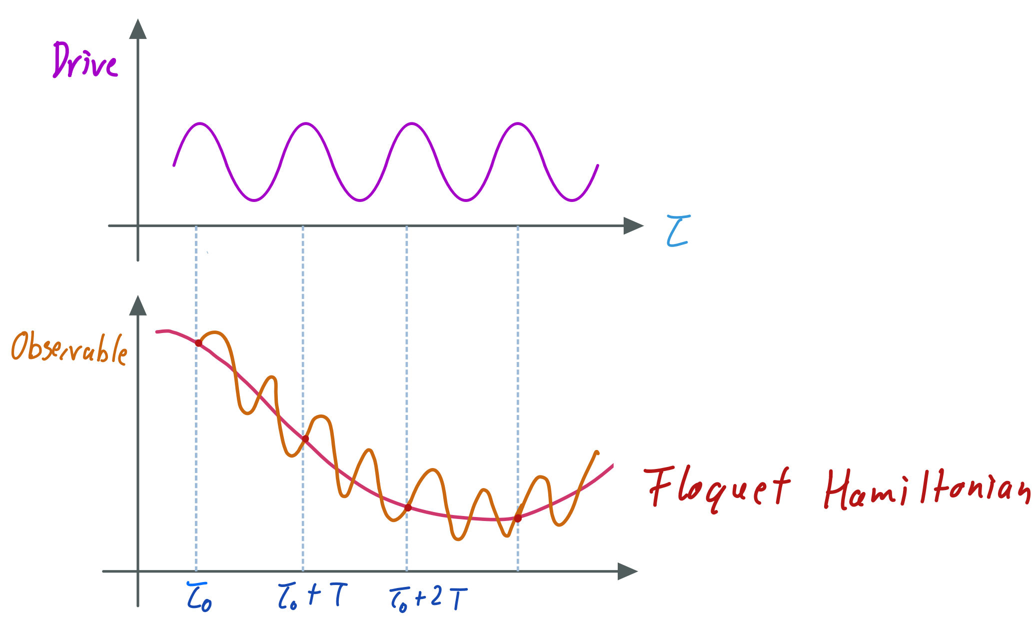
→ oscillates at frequency of drive + higher harmonics

→ micro motion

- in general:

$$\overline{\langle \hat{O} \rangle_n (\tau)} = \frac{1}{T} \int_0^T \langle \hat{O} \rangle_n (\tau)\, d\tau \neq \langle u_n | \hat{O} | u_n \rangle$$

→ since $\overline{e^{i\hat{K}(\tau)} \hat{O} e^{-i\hat{K}(\tau)}} \neq \hat{O}$



High-frequency expansion:

- often $\hat{H}_F$ or $\hat{H}_{eff}$ cannot be calculated in closed form
- if there is a separation of energy scales, i.e. $\omega \gg$ other energy scales
  $\rightarrow$ expand in powers of ω

$$\hat{H}_{eff} = \sum_{n=0}^{\infty} \hat{H}_{eff}^{(n)} \quad ; \text{ with } \quad \hat{H}_{eff}^{(n)} \sim \omega^{-n}$$

$$\hat{K}(\tau) = \sum_{n=0}^{\infty} \hat{K}^{(n)}(\tau) \quad ; \text{ with } \quad \hat{K}^{(n)}(\tau) \sim \omega^{-n}$$

- with Fourier components

$$\hat{H}(\tau) = \sum_{l=-\infty}^{l=\infty} \hat{H}_l e^{il\omega\tau}$$

- one finds:

$$\hat{H}_{eff}^{(0)} = \frac{1}{T}\int_0^T \hat{H}(\tau)\, d\tau \equiv H_0$$

static part

$$\hat{H}_{eff}^{(1)} = \frac{1}{\hbar\omega}\sum_{l=1}^{\infty}\frac{1}{l}\left[\hat{H}_l, \hat{H}_{-l}\right]$$

$$\hat{H}_{eff}^{(2)} = \frac{1}{(\hbar\omega)^2}\Bigg[\sum_{l=1}^{\infty}\frac{1}{2l^2}\left(\left[\left[\hat{H}_l, \hat{H}_0\right], \hat{H}_{-l}\right] + \left[\left[\hat{H}_{-l}, \hat{H}_0\right], \hat{H}_l\right]\right)$$

$$+ \sum_{l,k=1}^{\infty}\left(\left[\hat{H}_l, \left[\hat{H}_k, \hat{H}_{-l-k}\right]\right] - 2\left[\hat{H}_l, \left[\hat{H}_{-k}, \hat{H}_{k-l}\right]\right] + h.c.\right)\Bigg]$$

and

$$\hat{K}^{(0)} = 0$$

$$\hat{K}^{(1)} = \frac{1}{i\hbar\omega} \sum_{l=1}^{\infty} \frac{1}{l} \left(\hat{H}_l e^{il\omega\tau} - \hat{H}_{-l} e^{-il\omega\tau} \right)$$

$$\hat{K}^{(2)} = \dots$$

Extended Hilbert space (τ_0 not explicit)

- method of obtaining quasi-energy spectrum [5,6]
- use periodicity of Floquet modes:
  $|u_n(\tau)\rangle = |u_n(\tau + T)\rangle$
- write Floquet modes as Fourier series of discrete frequencies $\omega = \frac{\epsilon_n}{\hbar}$

$$|\psi_n(\tau)\rangle = e^{-i\epsilon_n \tau/\hbar} |u_n(\tau)\rangle$$

$$= e^{-i\epsilon_n \tau/\hbar} \sum_m e^{-im\omega\tau} \underbrace{|n,m\rangle}_{m_{th} \text{ Fourier coefficient of } |u_n(\tau)\rangle}$$

- into time-dependent Schrödinger equation:

$$(\epsilon_n + m\hbar\omega)|n,m\rangle = \sum_{m'} \hat{H}_{m-m'} |n,m'\rangle$$

$\hat{H}_l$: see above

- ϵ_n defined mod $\hbar\omega$ → extended Hilbertspace | overcomplete
  → solutions $\hbar\omega$ periodic

- for example:

$$\hat{H}(\tau) = \hat{H}_0 + V e^{i\omega\tau} + V^\dagger e^{-i\omega\tau}$$

$$\begin{pmatrix} \ddots & V & 0 & & \\ V^\dagger & H_0 + \hbar\omega & V & 0 & \\ 0 & V^\dagger & H_0 & V & 0 \\ & 0 & V^\dagger & H_0 - \hbar\omega & \\ & & 0 & V^\dagger & \ddots \end{pmatrix}$$

→ nearest-neighbour hopping
→ evaluate numerically

- weak driving regime / high-frequency regime

$$\hbar\omega \gg \langle V \rangle$$

→ single block, see rotating wave approximation

$$\begin{pmatrix} H_0 + \hbar\omega & V \\ V^\dagger & H_0 \end{pmatrix}$$

→ PPT

Lattice shaking

Laboratory frame:

$$\hat{H}_{lab} = \frac{\hat{p}^2}{2m} + V_{lat}[\hat{x} - x_m(\tau)]$$

lattice potential — V_{lat}

periodic motion of lattice position — $x_m(\tau)$

general Hamiltonian including higher bands

Separate into static and time-dependent parts:

$H = H_0 + V(t)$

- apply unitary transformation into rotating frame

$$\hat{R}_0(\tau) = e^{-i\hat{p}x_m(\tau)/\hbar}$$

i.e. $\hat{H}_{rot}(\tau) = \hat{R}_0^\dagger(\tau)\hat{H}_{lab}\hat{R}_0(\tau) - i\hbar\hat{R}_0^\dagger(\tau)\frac{\partial}{\partial\tau}\hat{R}_0(\tau)$

$$\hat{H}_{rot}(\tau) = \frac{[\hat{p} - A(\tau)]^2}{2m} + V_{lat}(\hat{x})$$

$e^A B e^{-A} = B + [A,B] + \frac{1}{2!}[A,[A,B]] + \dots$

- describes particle in a time dependent "vector potential"

$$A(\tau) = m\dot{x}_m(\tau)$$

- transform into co-moving frame with shaken lattice

$$R_1(\tau) = e^{-i\hat{x}A(\tau)/\hbar}$$

$$\hat{H}_{cm}(\tau) = \frac{p^2}{2m} + V_{lat}(\hat{x}) - \overbrace{F(\tau)\hat{x}}^{\text{time dependent}}$$

with $F(\tau) = -m\ddot{x}_m(\tau) = -\dot{A}(\tau)$

Tight-binding approximation:

- restrict to lowest band
- localized states on sites j
- hopping between neighboring sites
- assume no coupling to higher bands
- in co-moving frame:

in one dimension

$$\hat{H}_{cm} = \underbrace{-\underset{\uparrow\ \text{tunnel coupling}}{t}\sum_j \big(\hat{c}^\dagger_{j+1} \underset{\uparrow\ \text{annihilation on site } j}{\hat{c}_j} + h.c.\big)}_{\text{static part } (\hat{H}_{0,cm})} - F(\tau)\, \underbrace{\underset{\uparrow\ \text{lattice spacing}}{d}\sum_j j \underset{\uparrow\ =\hat{c}_j^\dagger \hat{c}_j}{\hat{n}_j}}_{\hat{x}}$$

- in rotating frame:

$$\hat{H}_{rot} = -t\sum_j \big(\underbrace{e^{i d A(\tau)/\hbar}}_{\text{oscillating phase of tunneling matrix element!}} \hat{c}^\dagger_{j+1}\hat{c}_j + h.c\big)$$

→ engineer complex tunneling phases

---

use: $\hat{H}_{rot}(\tau) = \hat{R}_1^\dagger(\tau)\,\hat{H}_{cm}\,\hat{R}_1(\tau) - i\hbar\,\hat{R}_1^\dagger(\tau)\frac{\partial}{\partial t}\hat{R}_1(\tau)$

with $\hat{R}_1(\tau) = \exp\left[-\frac{i}{\hbar}\int F(\tau)d\tau \; d\sum_j j\hat{n}_j\right]$

$\hat{H}_{rot}(\tau) = \hat{R}_1^\dagger(\tau)\,\hat{H}_{0,cm}\,\hat{R}_1(\tau)$ since $-i\hbar\,\hat{R}_1^\dagger(\tau)\frac{\partial}{\partial\tau}\hat{R}_1(\tau) = -\hat{V}_{cm}(\tau)$

Summary:

	Continuum Hamiltonian	Tight-binding Hamiltonian
Lab frame	$\frac{\hat{p}^2}{2m} + V[\hat{x} - x_m(\tau)]$	
Rotating frame	$\frac{(\hat{p} - A(\tau))^2}{2m} + V_{lat}(\hat{x})$	$-t\sum_j \left(e^{idA(\tau)/\hbar}\hat{c}^{\dagger}_{j+1}\hat{c}_j + h.c.\right)$
Co-moving frame	$\frac{\hat{p}^2}{2m} + V_{lat}(\hat{x}) - F(\tau)\hat{x}$	$-t\sum_j \left(\hat{c}^{\dagger}_{j+1}\hat{c}_j + h.c.\right) - F(\tau)d\sum_j j\hat{n}_j$

Sinusoidal lattice shaking:

- sinusoidal modulation of lattice position

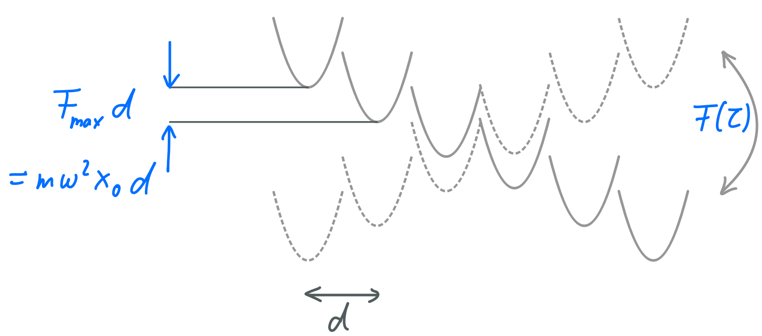
$$x_m(\tau) = x_0 \cos(\omega \tau + \varphi)$$

$$\rightsquigarrow \quad A(\tau) = -m\omega x_0 \sin(\omega\tau + \phi) = -\frac{\hbar K_0}{d} \sin(\omega\tau + \phi)$$

$$\mathcal{F}(\tau) = m\omega^2 x_0 \cos(\omega\tau + \phi) = \frac{\hbar\omega}{d} K_0 \cos(\omega\tau + \phi)$$

- dimensionless shaking amplitude characterizing modulation strength

$$K_0 = \frac{m\omega x_0 d}{\hbar} = \frac{m\omega^2 x_0 d}{\hbar\omega} = \frac{\text{energy offset between adjacent sites}}{\text{driving energy}}$$



Floquet-Hubbard Hamiltonians

- additional static terms that are sums over local onsite Hamiltonians can be added
  - energy offsets between neighboring sites
  - onsite interaction
- local term commutes with $\hat{R}_1$

no additional differences between frames

driven Fermi-Hubbard in co-moving frame:

$$\hat{H}_{dFH} = -\sum_{j,\sigma} \left(\hat{c}^{\dagger}_{j+1,\sigma} \hat{c}_{j\sigma} + h.c. \right) + U \sum_{j} \hat{n}_{j\uparrow} \hat{n}_{j\downarrow} - F(\tau) d \sum_{j,\sigma} j \hat{n}_{j\sigma}$$

Off-resonant driving:

- modulation frequency larger than other energy scales, i.e. $\hbar\omega \gg t, U$

- lowest order of effective Hamiltonian

  easy in rotating frame!

  $\rightarrow$ time average of $\hat{H}^{(n=0)}_{\text{eff, rot}}$

- tunneling matrix element replaced by effective tunneling matrix element.

$$t^{(0)}_{\text{eff}} = t \, \frac{1}{T} \int_0^T e^{-iK_0 \sin(\omega\tau + \phi)} \, d\tau = t \, \mathcal{J}_0(K_0)$$

zeroth order Bessel function

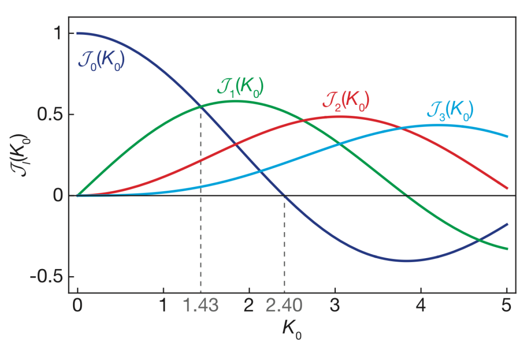
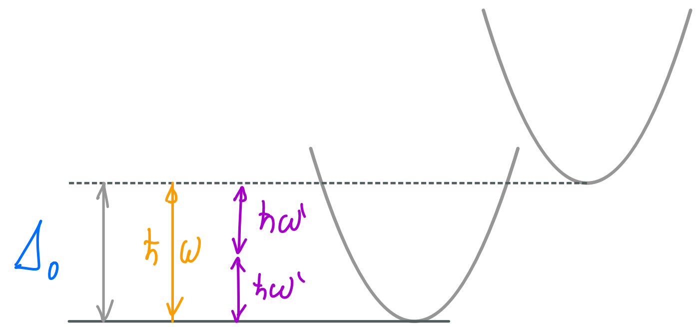


Figure 3.2: Bessel functions. Dependence of the Bessel functions $\mathcal{J}_l(K_0)$ on the driving amplitude K_0. For small arguments, the functions can be approximated by $\mathcal{J}_l(K_0) \approx (K_0/2)^l/l!$. The envelope of the oscillating functions decays like $\sqrt{2/(\pi K_0)}$ for large values of K_0 independent of l. Görg 2019

Resonant driving in a Hubbard system with gradient:



Driving frequency:

$$\ell \hbar\omega = \Delta_0$$

integer

Transformation

$$\hat{R}_{\Delta}^{(\ell)}(\tau) = \exp\left[i \ell \omega \tau \sum_j j \hat{n}_j \right]$$

Effective Hamiltonian:

$$\hat{H}^{(\ell)}_{eff,rot} = -\sum_j \left(t^{(\ell)}_{eff} \hat{c}^{\dagger}_{j+1} \hat{c}_j + h.c. \right) - \left(\Delta_0 - \ell\hbar\omega \right) \sum_j j \hat{n}_j$$

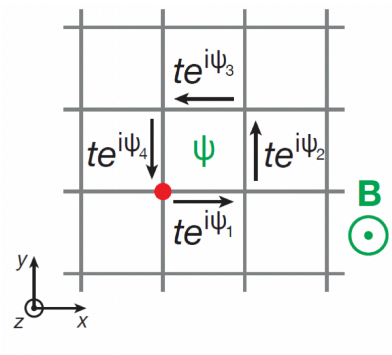
Effective tunneling:

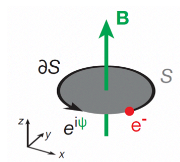
$$t^{(\ell)}_{eff} = t \frac{1}{T} \int_0^T e^{-i[\ell\omega\tau + K_0 \sin(\omega\tau + \phi)]} d\tau$$

$$= t (-1)^{\ell} J_{\ell}(K_0) e^{i\ell\phi}$$

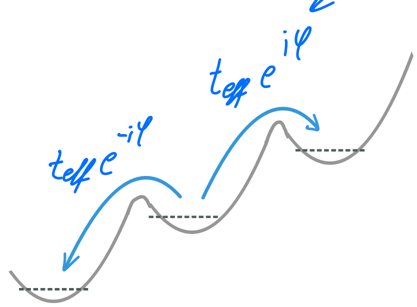
Artificial magnetic fields in shaken lattices:

$$\psi = \psi_{\text{ext}}$$

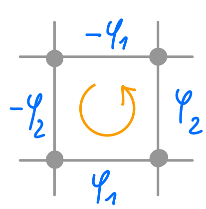




Peierls phase



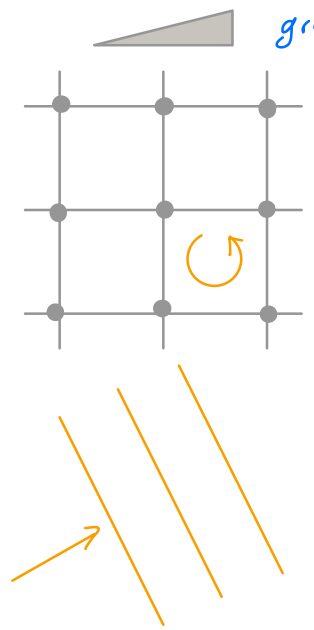
Beware!



$$\sum_i \varphi_i = 0$$

Solution :

gradient



$\rightarrow \sum_i \varphi_i \neq 0$

+ moving lattice

$\rightarrow$ periodic modulation of on-site energies of lattice

cannot be "gauged" away

D Jaksch and P Zoller 2003 New J. Phys. 5, 56 (2003)

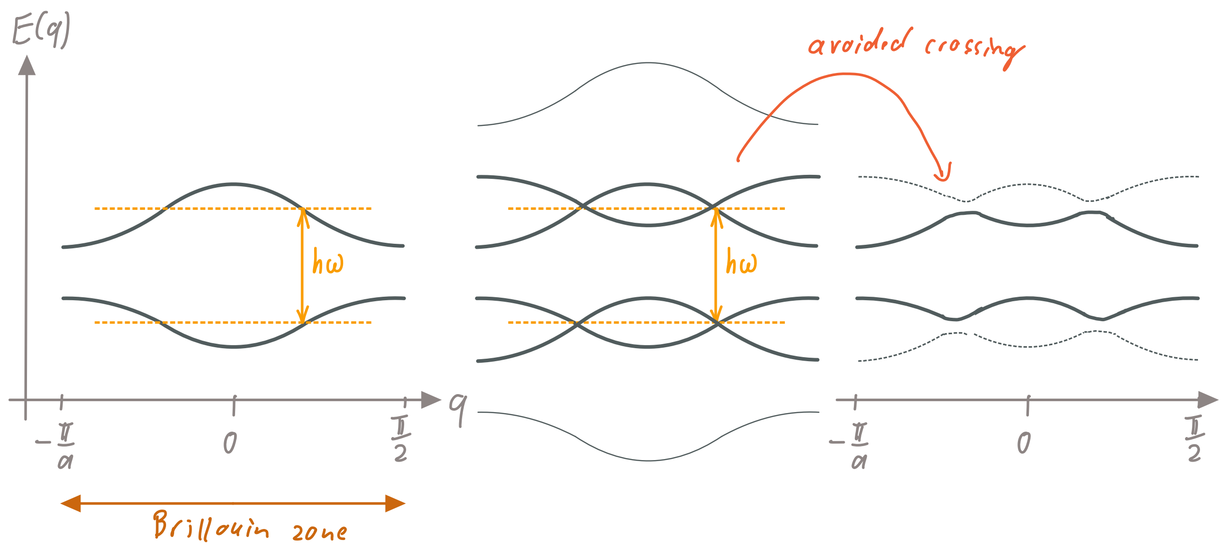
I Spielman, NIST

M Aidelsburger, Munich

...

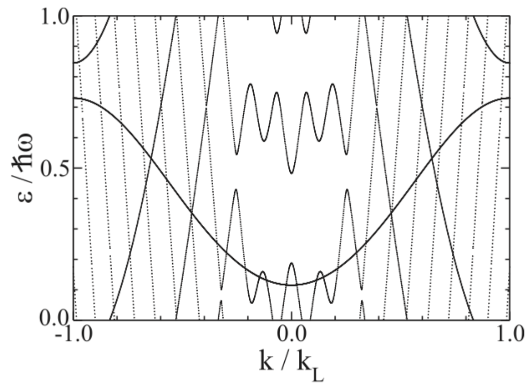
Driven two-band model:

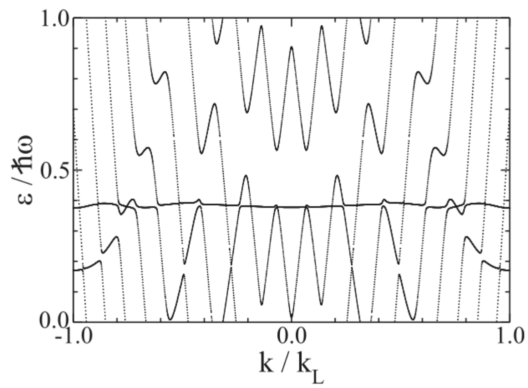
Static no-coupling driven



Multiband Model:

Martin Holthaus





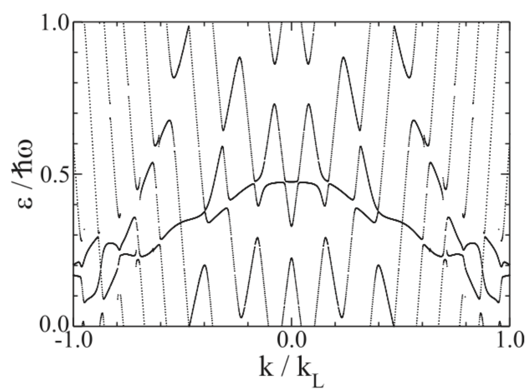
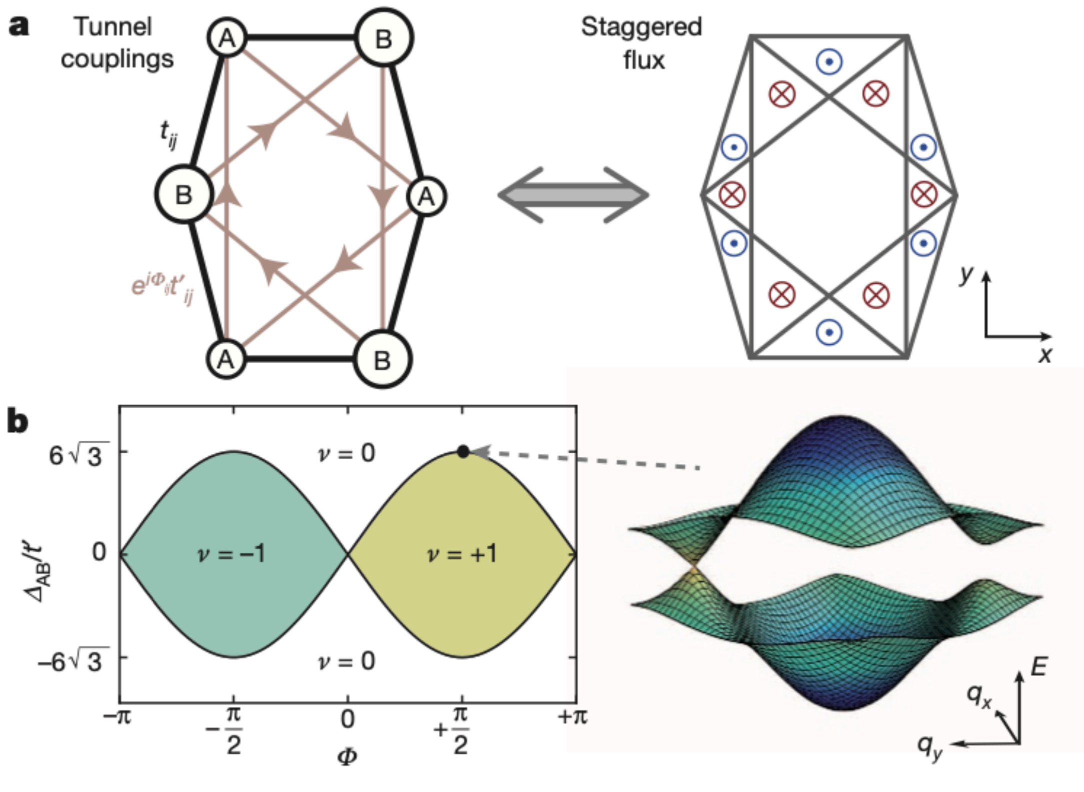


Figure 8. Quasienergy dispersion relations for $V_0/E_R = 4.0$ and $\hbar\omega/E_R = 0.5$, implying that the gap Δ_{01} between the lowest and the first excited unperturbed energy band is a bit smaller than 4 $\hbar\omega$ (see table 1). For scaled driving amplitude $\beta = 0.20$ the quasienergy band originating from the lowest energy band still is well described by the single-band approximation (132) (upper panel). For $\beta = 0.74$, corresponding to the 'collapse point' observed in figure 7, the band is not perfectly flat, but rather disrupted by several small anticrossings (middle panel). For $\beta = 1.21$, for which an isolated band (132) would be maximally inverted, strong hybridization occurs (lower panel), forecasting rapid heating.

The Haldane model



D. Haldane, PRL 61, 2015 (1988)

T. Oka and H. Aoki, PRL 79, 081406 (2009)

G. Jotzu et al., Nature 515, 237 (2014)

→ PPT