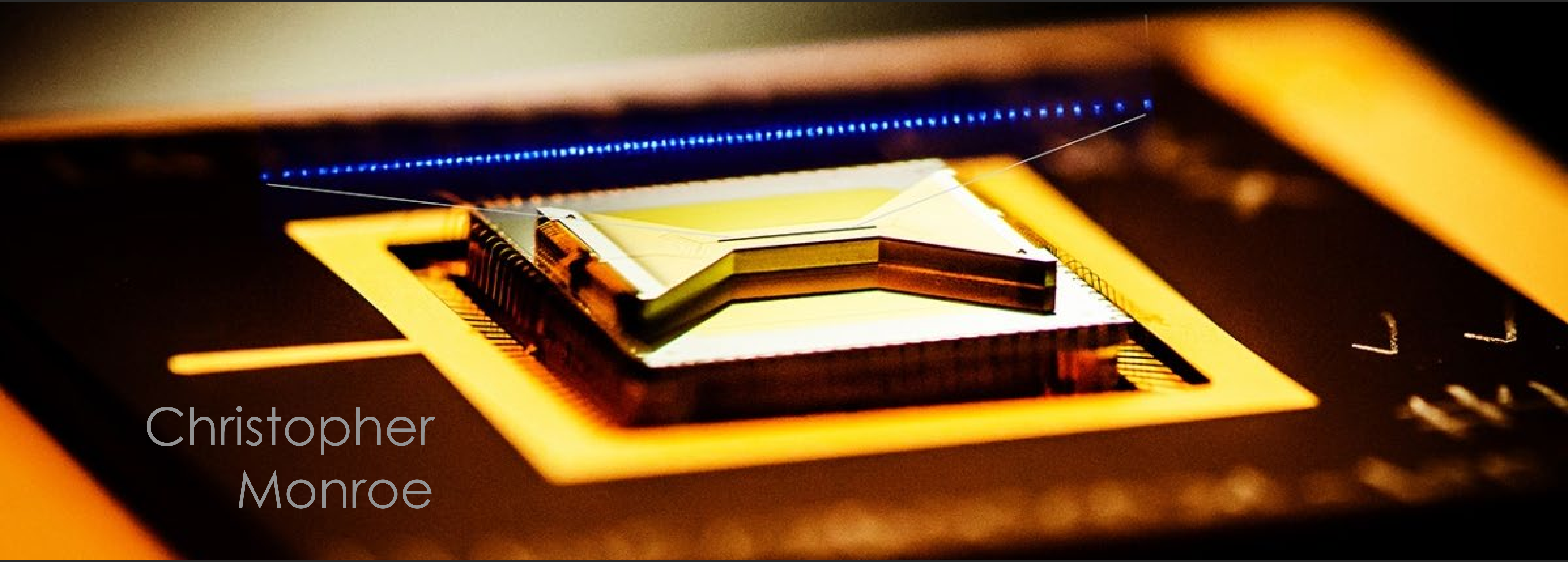
Quantum Simulation and Computing with Atomic Ions



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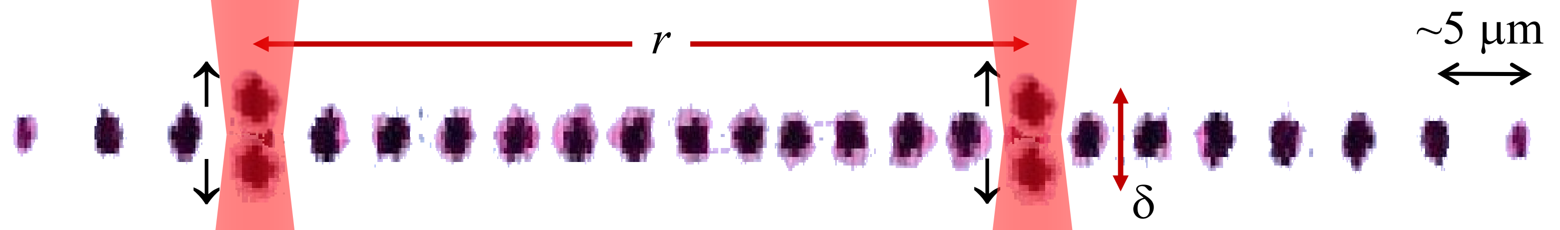
IONQ

Spin-dependent force (simple/fast version)



dipole-dipole coupling $\Delta E = \dfrac{e^2}{\sqrt{r^2+\delta^2}} - \dfrac{e^2}{r} \approx -\dfrac{(e\delta)^2}{2r^3}$

δ ~ 10 nm
eδ ~ 500 Debye

$$\begin{aligned}
|{\downarrow\downarrow}\rangle &\rightarrow |{\downarrow\downarrow}\rangle \\
|{\downarrow\uparrow}\rangle &\rightarrow e^{-i\varphi}|{\downarrow\uparrow}\rangle \\
|{\uparrow\downarrow}\rangle &\rightarrow e^{-i\varphi}|{\uparrow\downarrow}\rangle \\
|{\uparrow\uparrow}\rangle &\rightarrow |{\uparrow\uparrow}\rangle
\end{aligned}
\longrightarrow \quad \varphi = \frac{\Delta E t}{\hbar} = \frac{e^2\delta^2 t}{2\hbar r^3} = \frac{\pi}{2}$$

for full entanglement

Native Ion Trap Operation: "Ising" gate

$$ZZ[\varphi] = e^{-i\sigma_z^{(1)}\sigma_z^{(2)}\varphi}$$

Cirac and Zoller (1995)
Mølmer & Sørensen (1999)
Solano, de Matos Filho, Zagury (1999)
Milburn, Schneider, James (2000)

Spin-motion coupling: 2LS+QHO

$$H = \frac{\hbar\omega_0}{2}\hat{\sigma}_z + \underbrace{\frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2}_{\hbar\omega(a^+a+1/2)} - \hat{\mu}\cdot E(\hat{x})$$

frequency of
applied radiation

Rabi frequency $\hbar g$

$$-\mu_0 \cdot \frac{E_0}{2}(\hat{\sigma}_+ + \hat{\sigma}_-)(e^{ik\hat{x}-i\omega_L t} + e^{-ik\hat{x}+i\omega_L t})$$

interaction frame, “rotating wave approximation”

$$H = \hbar g(\hat{\sigma}_+ e^{ik\hat{x}-i\delta t} + \hat{\sigma}\_e^{-ik\hat{x}+i\delta t})$$

$$\hat{x} = x_0(ae^{-i\omega t} + a^+e^{i\omega t})$$

$\delta = \omega_L - \omega_0 = detuning$

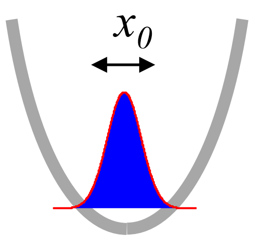
$k = 2\pi/\lambda = wavenumber$

Raman 2-photon configuration

$= (\omega_2 - \omega_1) - \omega_0$

$= (k_2 - k_1)$

$$x_0 = \sqrt{\frac{\hbar}{2m\omega}}$$



$$H = \hbar g\left[\hat{\sigma}_+ e^{ikx_0(ae^{-i\omega t}+a^+e^{i\omega t})-i\delta t} + \hat{\sigma}\_e^{-ikx_0(ae^{-i\omega t}+a^+e^{i\omega t})+i\delta t}\right]$$

$\eta = kx_0$ = "Lamb-Dicke parameter" ~ 0.1

<u>Stationary terms arise in *H* at particular values of δ:</u>

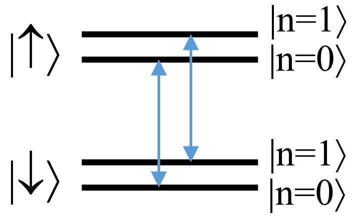
(1) $\delta = 0$

$$H_0 = \hbar g(\hat{\sigma}_+ + \hat{\sigma}_-)\Big\{1 - \frac{\eta^2}{2!}(a^\dagger a + aa^\dagger) + \frac{\eta^4}{4!}(a^\dagger a^\dagger aa + a^\dagger aa^\dagger a + aa^\dagger aa^\dagger + aa^\dagger a^\dagger a) - \eta^6(a^\dagger a^\dagger a^\dagger aaa + \cdots)\Big\}$$

$$\approx \hbar g(\hat{\sigma}_+ + \hat{\sigma}_-)(1 - n\eta^2) \approx \hbar g(\hat{\sigma}_+ + \hat{\sigma}_-)$$

$kx_0\sqrt{n+1} \ll 1$
"Lamb-Dicke limit"

$$\text{"Carrier"}: \langle\downarrow, n|H_0|\uparrow, n\rangle = \hbar g$$



$$H = \hbar g\left[\hat{\sigma}_+ e^{ikx_0(ae^{-i\omega t}+a^+e^{i\omega t})-i\delta t} + \hat{\sigma}\_e^{-ikx_0(ae^{-i\omega t}+a^+e^{i\omega t})+i\delta t}\right]$$

$\eta = kx_0$ = "Lamb-Dicke parameter"

Stationary terms arise in H at particular values of δ:

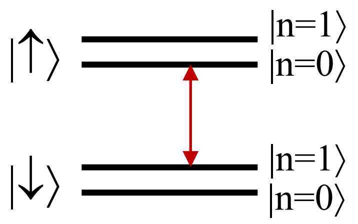
(2) $\delta = -\omega$

$$H_- = \hbar g \hat{\sigma}_+ \left\{\eta a - \frac{\eta^3}{3!}(a^\dagger aa + aa^\dagger a + aaa^\dagger) + \cdots\right\} + h.c.$$

$$\approx \hbar g\left(\hat{\sigma}_+ a + \hat{\sigma}_- a^\dagger\right)$$

$kx_0\sqrt{n+1} \ll 1$
"Lamb-Dicke limit"

"Red Sideband": $\langle \uparrow, n-1 | H_- | \downarrow, n \rangle = \hbar g \sqrt{n}$



$$H = \hbar g\left[\hat{\sigma}_+ e^{ikx_0(ae^{-i\omega t}+a^+e^{i\omega t})-i\delta t} + \hat{\sigma}\_e^{-ikx_0(ae^{-i\omega t}+a^+e^{i\omega t})+i\delta t}\right]$$

$\eta = kx_0$ = "Lamb-Dicke parameter"

Stationary terms arise in *H* at particular values of *d* :

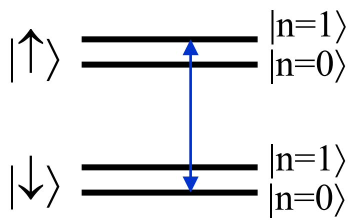
(3) $\delta = \omega$

$$H_+ = \hbar g\hat{\sigma}_+\left\{\eta a^\dagger - \frac{\eta^3}{3!}\left(a^\dagger a^\dagger a + aa^\dagger a^\dagger + a^\dagger a a^\dagger\right) + \cdots\right\} + h.c.$$

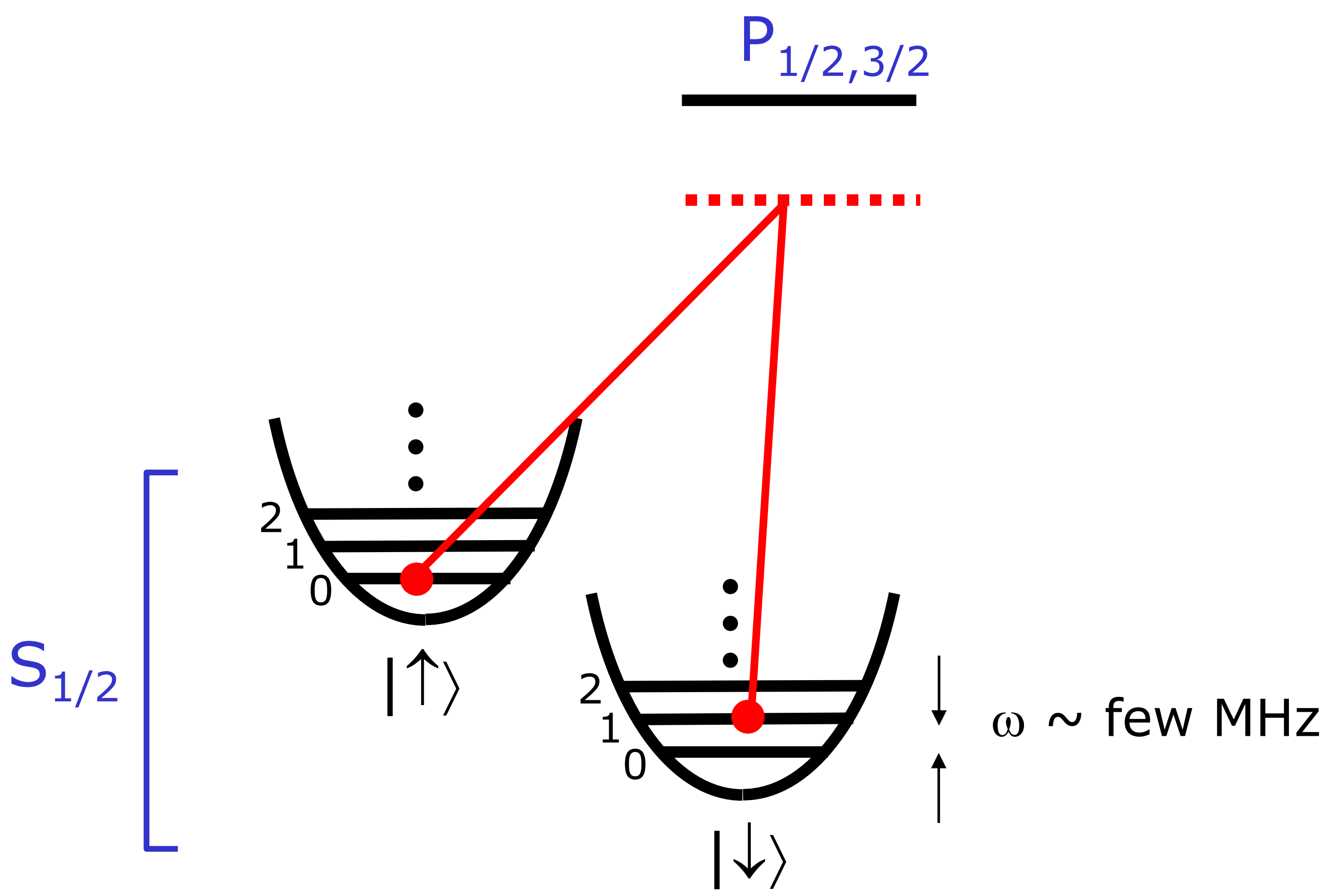
$$\approx \hbar g\left(\hat{\sigma}_+ a^\dagger + \hat{\sigma}\_a\right)$$

$kx_0\sqrt{n+1} \ll 1$
"Lamb-Dicke limit"

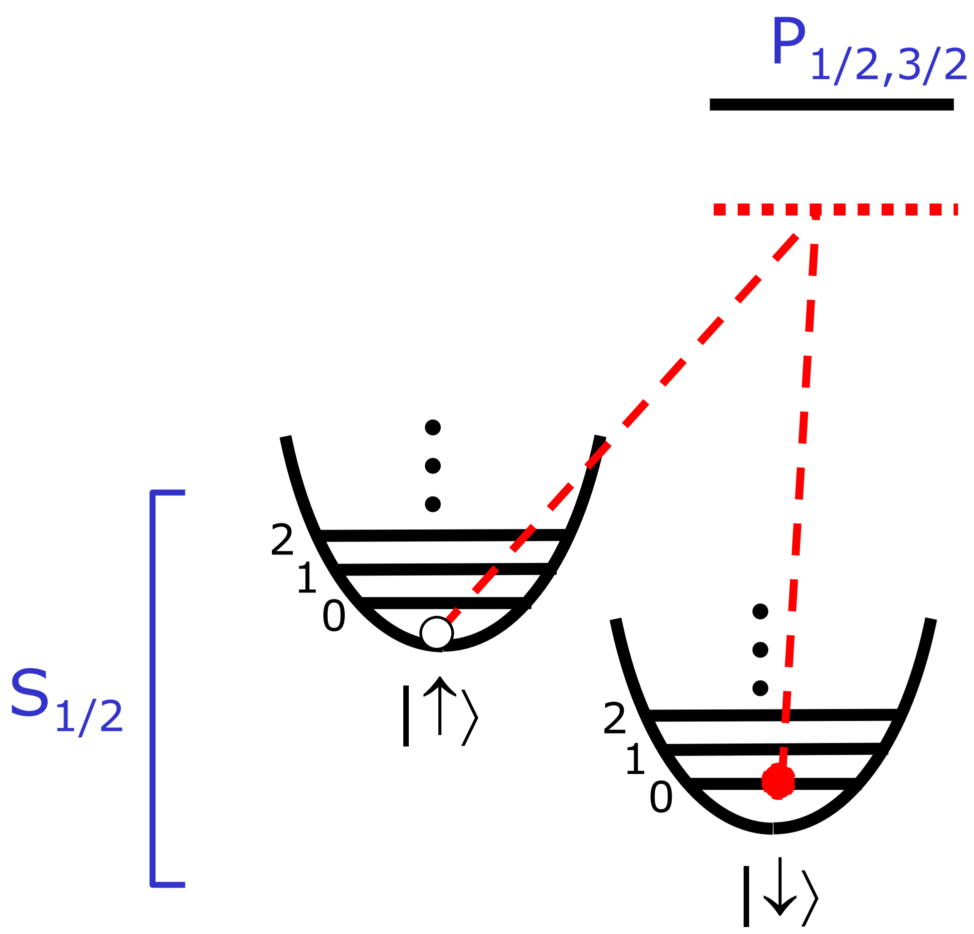
"*Blue Sideband*": $\langle\uparrow, n+1|H_+|\downarrow, n\rangle = \hbar g\sqrt{n+1}$

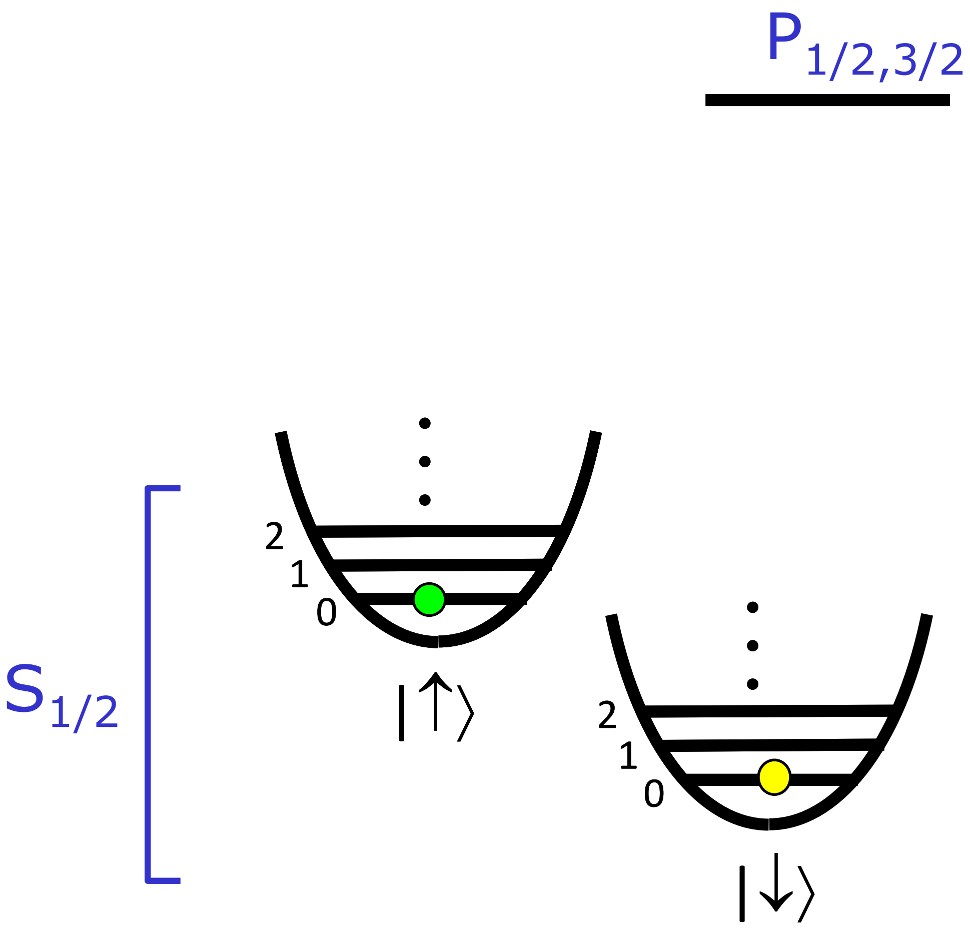


excitation on 1st lower ("red") motional sideband (n=0)

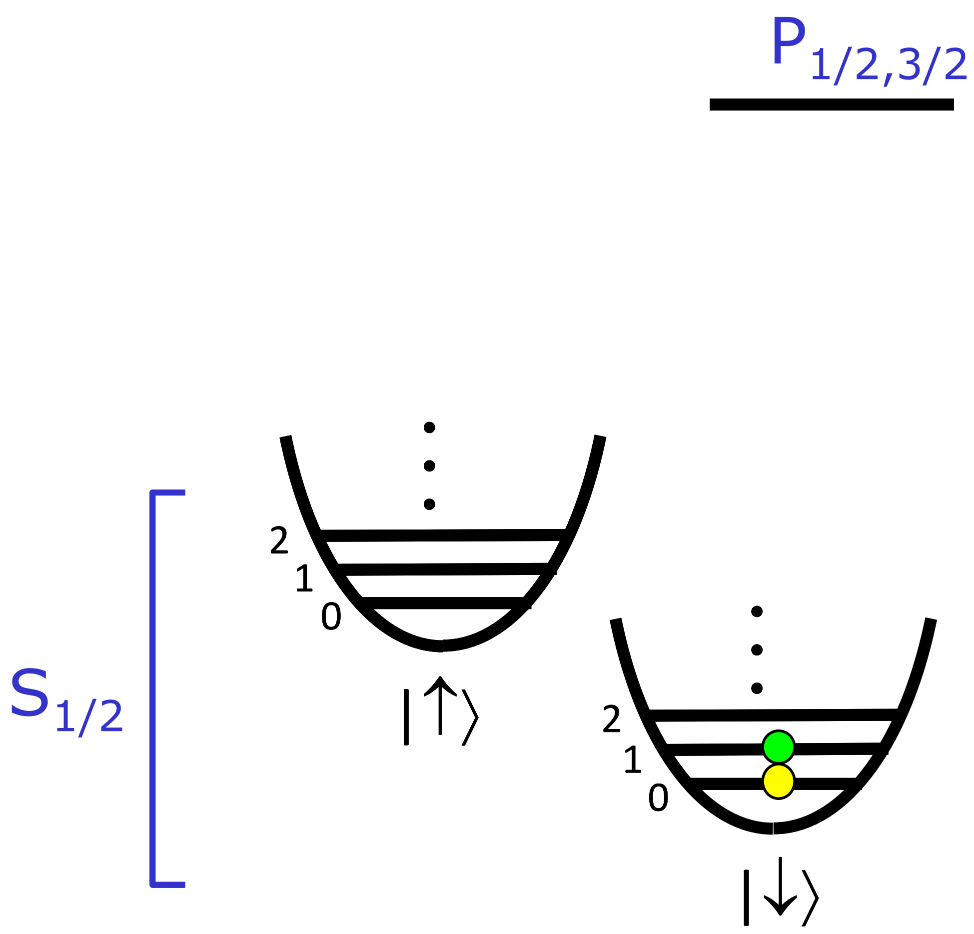


excitation on 1st lower ("red") motional sideband (n=0)



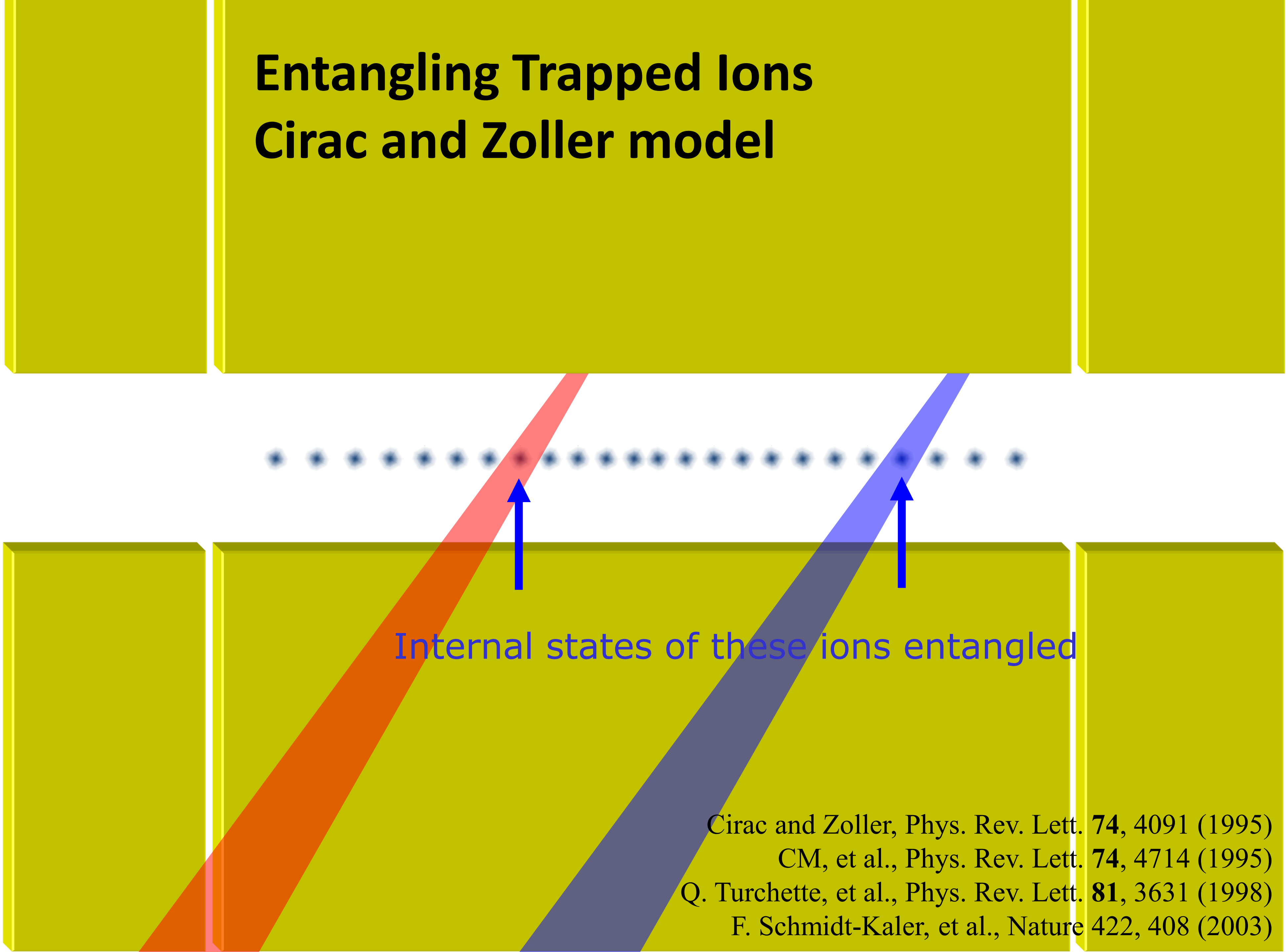


Mapping: $(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)\,|0\rangle_m \;\rightarrow\; |\downarrow\rangle\,(\alpha|0\rangle_m + \beta|1\rangle_m)$



Mapping: $(\alpha|\downarrow\rangle + \beta|\uparrow\rangle)\,|0\rangle_m \;\rightarrow\; |\downarrow\rangle(\alpha|0\rangle_m + \beta|1\rangle_m)$

Entangling Trapped Ions
Cirac and Zoller model



Cirac and Zoller, Phys. Rev. Lett. **74**, 4091 (1995)
CM, et al., Phys. Rev. Lett. **74**, 4714 (1995)
Q. Turchette, et al., Phys. Rev. Lett. **81**, 3631 (1998)
F. Schmidt-Kaler, et al., Nature 422, 408 (2003)

Cirac-Zoller: need pure ("Fock") states of motion

- extreme cooling to $|n = 0\rangle$: possible, but gate error $P(n > 0) \sim \bar{n}$
- not scalable: for large # qubits, cooling harder and modes overlap

$h \times 1\ MHz$

Better: "spin-dependent displacements"

- only requires cooling to the Lamb-Dicke limit $kx_{rms} \ll 1 \ \ or \ \ \bar{n} \ll \frac{\hbar\omega}{E_R}$
- "virtual" coupling to phonons possible (detuned force)

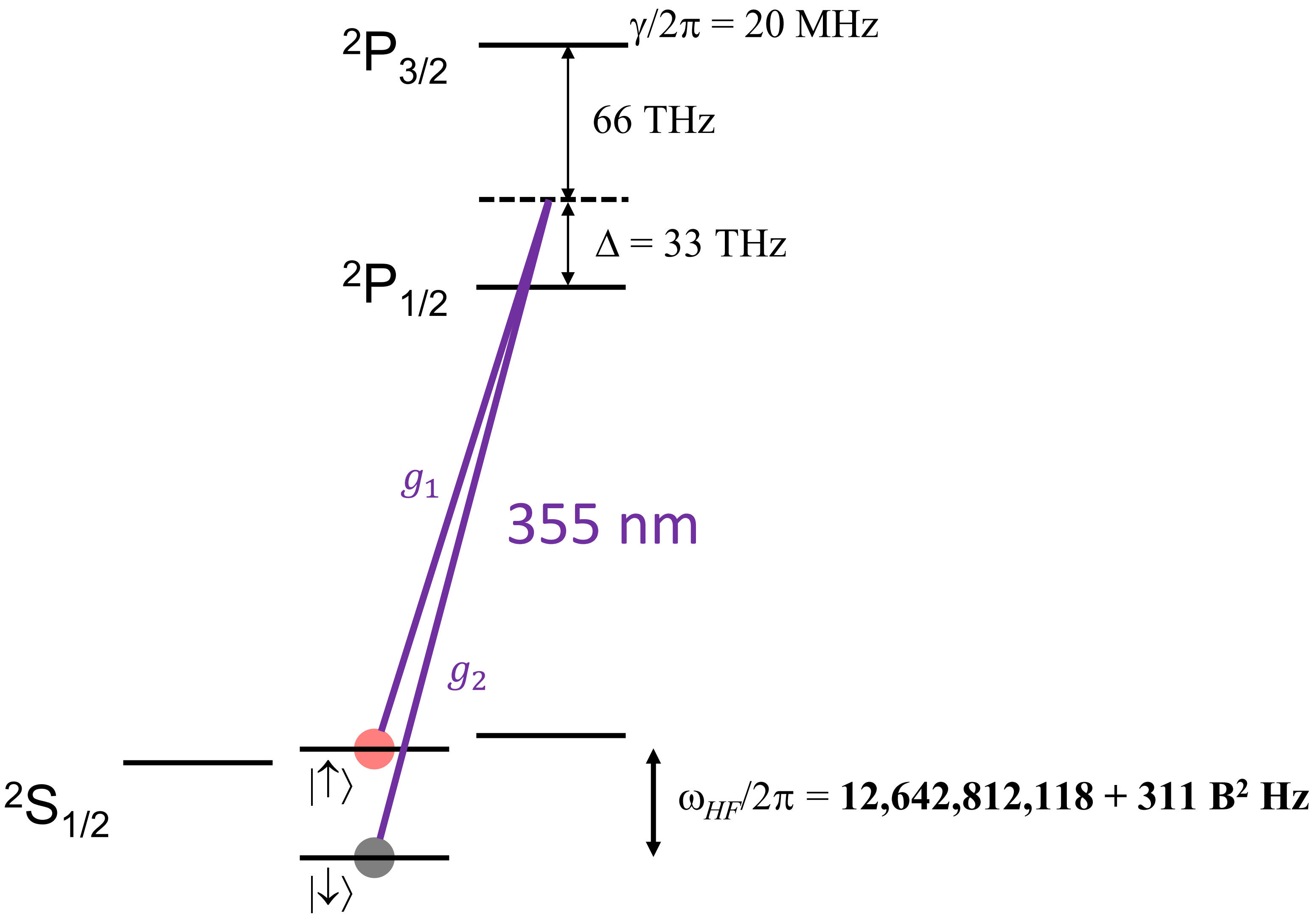
$\frac{\hbar^2 k^2}{2m} = h \times 20\ kHz$ (^{171}Yb^{+})

Mølmer & Sørensen (1999)
Solano, de Matos Filho, Zagury (1999)
Milburn, Schneider, James (2000)

$\bar{n} \ll 50$

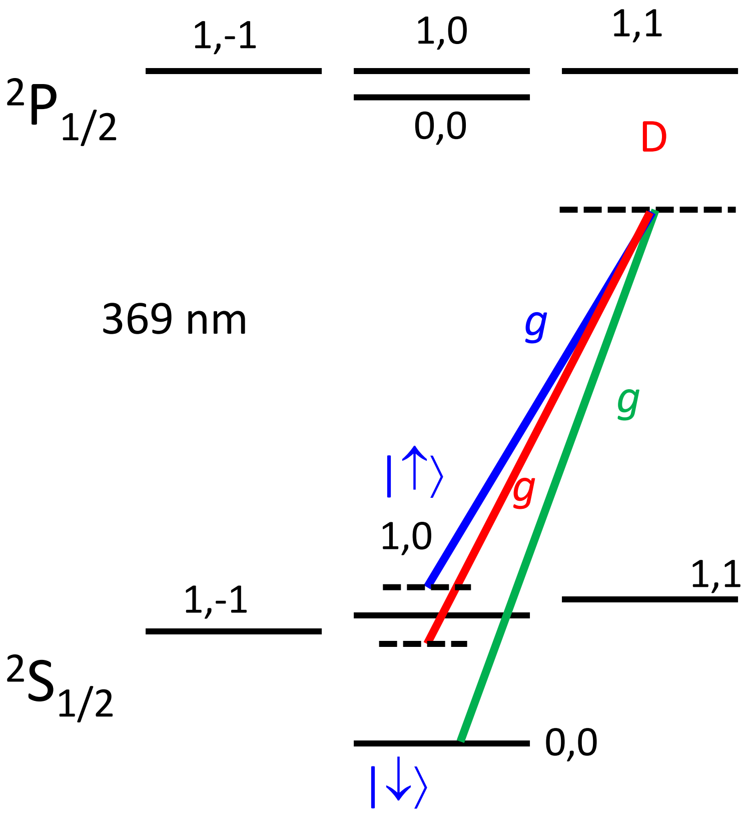
$\bar{n}_{Doppler} \sim \frac{\gamma}{2\omega} \sim 10$

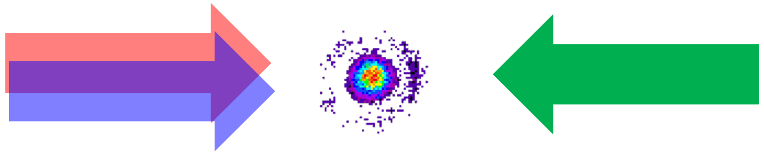
Recall: $^{171}Yb^+$ Qubit Manipulation



D. Hayes et al., PRL 104, 140501 (2010)

Spin-dependent *resonant* force (single ion)





Red+blue sideband applied *simultaneously*

$$H = \eta\Omega\left(\sigma_+ a + \sigma_- a^\dagger\right) + \eta\Omega\left(\sigma_- a + \sigma_+ a^\dagger\right)$$

$$= \eta\Omega\, \sigma_x\left(a + a^\dagger\right)$$

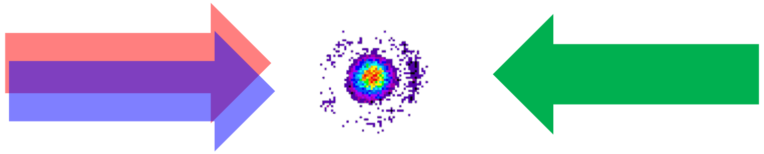
$$= \Omega\, \sigma_x(\Delta k \cdot \hat{x})$$

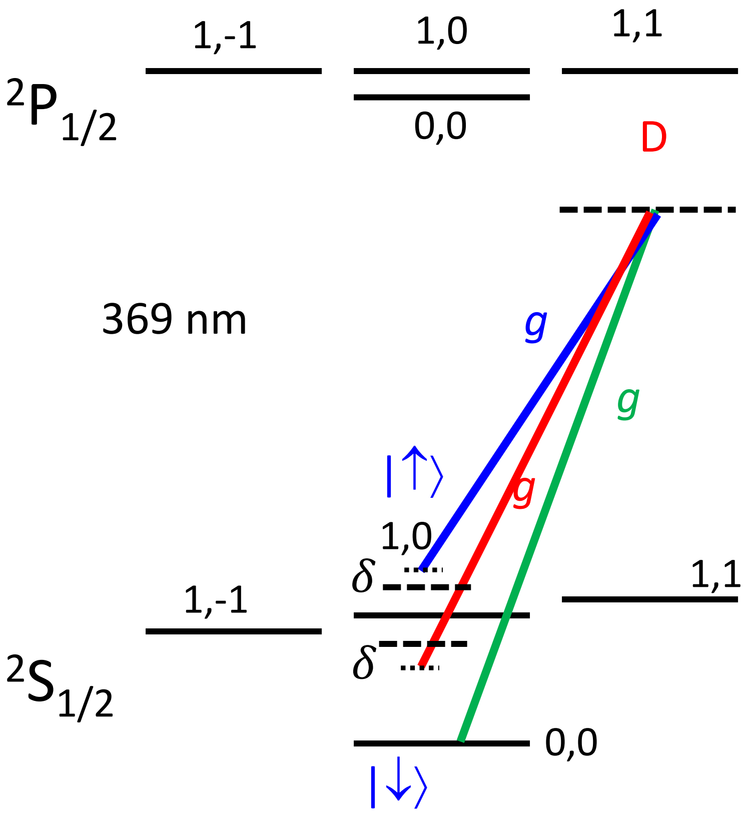
Lamb-Dicke parameter $\eta = \Delta k x_0$

$$\Omega = \frac{g^2}{2\Delta}$$

K. Molmer and A. Sorenson, PRL **82**, 1835 (1999)

Spin-dependent *detuned* force (single ion)





Red+blue sideband applied *simultaneously*, symmetrically detuned

$$H = \eta\Omega\left(\sigma_+ a e^{-i\delta t} + \sigma_- a^\dagger e^{i\delta t}\right) + \eta\Omega\left(\sigma_- a e^{-i\delta t} + \sigma_+ a^\dagger e^{i\delta t}\right)$$

$$= \eta\Omega\,\sigma_x\left(a e^{-i\delta t} + a^\dagger e^{i\delta t}\right)$$

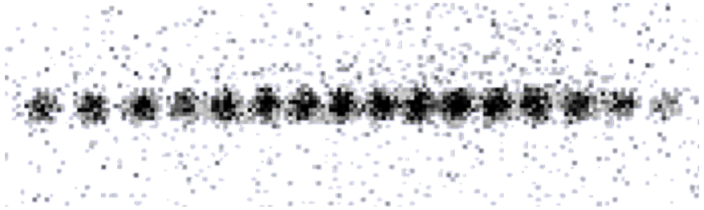
Lamb-Dicke parameter $\eta = \Delta k x_0$

$$\Omega = \frac{g^2}{2\Delta}$$

K. Molmer and A. Sorenson, PRL **82**, 1835 (1999)

Many ions: many phonon modes

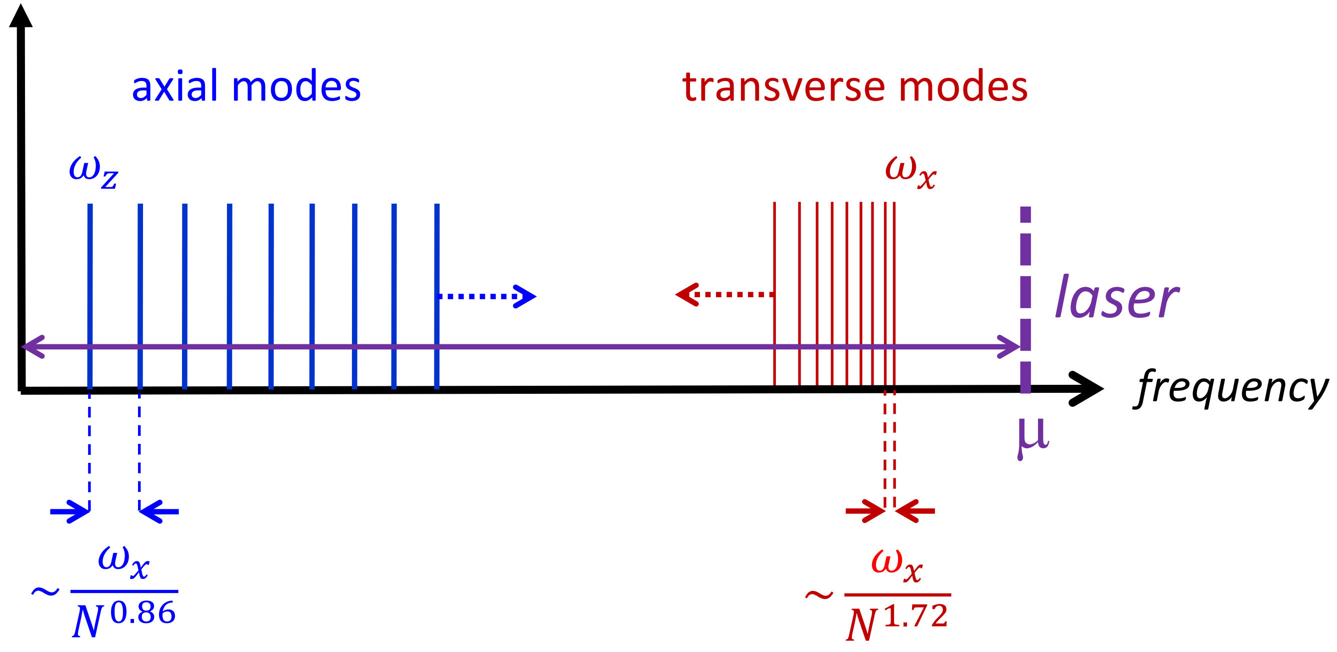
***N* ions in a line**



transverse trap frequency $\omega_{x,y}$ = high as possible

axial trap frequency $\omega_z < \frac{\omega_x}{N^{0.86}}$

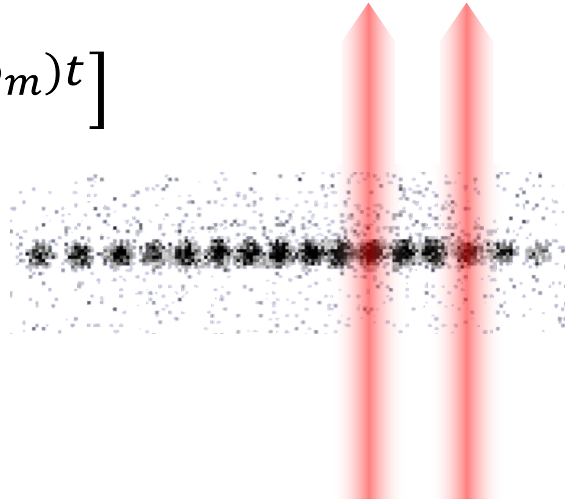
A. Steane, Appl. Phys. B 64, 623 (1997)



laser (beatnote) detuned from sidebands!

Many ions: many phonon modes

$$H = \sum_{i,m} \eta_{im} \Omega_i \sigma_x^i \left[a_m e^{-i(\mu-\omega_m)t} + a_m^\dagger e^{i(\mu-\omega_m)t} \right]$$



ion i
mode m

$$\eta_{im} = \sqrt{\frac{\hbar k^2}{2m\omega_m}}\, b_{i,m}$$

displacement eigenvector mode m with ion i

$$\sum_{i=1}^{N} b_{i,m} b_{i,n} = \delta_{mn} \qquad \sum_{m=1}^{N} b_{i,m} b_{j,m} = \delta_{ij}$$

evolution operator (Magnus expansion)

$$U(\tau) = \exp\left[-i\int_0^\tau dt H(t) - \frac{1}{2}\int_0^\tau dt_2 \int_0^{t_2} dt_1 [H(t_2), H(t_1)] - \frac{i}{6}\int_0^\tau dt_3 \int_0^{t_3} dt_2 \int_0^{t_2} dt_1 [H(t_3),[H(t_2), H(t_1)]] + \ldots \right]$$

$$U(\tau) = \exp\left[\sum_i \hat{\zeta}_i(\tau)\sigma_x^{(i)} + i\sum_{i,j} \chi_{i,j}(\tau)\sigma_x^{(i)}\sigma_x^{(j)} \right]$$

Many ions: many phonon modes

$$U(\tau) = \exp\left[\sum_i \hat{\zeta}_i(\tau)\sigma_x^{(i)} - i\sum_{i,j}\chi_{i,j}(\tau)\sigma_x^{(i)}\sigma_x^{(j)}\right]$$

phonons

$$\hat{\zeta}_i(\tau) = \sum_m [\alpha_i^m(\tau)a_m^\dagger - \alpha_i^{m*}(\tau)a_m]$$

Soln 1: Modulate Laser → **<u>Quantum Computer</u>**

$$\alpha_i^m(\tau) = \frac{-i\eta_{i,m}\boxed{\Omega_i}}{2\boxed{(\mu-\omega_m)}}\left[1 - e^{-i(\boxed{\mu}-\omega_m)\tau}\right]$$

Soln 2: Detune far → **<u>Quantum Simulator</u>**

$\alpha_i^m(\tau)$ is a circle in phase space

$x_i^m \sim Re\ \alpha_i^m(\tau)$

$p_i^m \sim Im\ \alpha_i^m(\tau)$

interaction between qubits (entanglement)

$$\chi_{i,j}(\tau) = \Omega_i\Omega_j\omega_R \sum_m \frac{b_{i,m}b_{j,m}}{2\omega_m(\mu-\omega_m)}\left[\tau - \frac{\sin(\mu-\omega_m)\tau}{\mu-\omega_m}\right]$$

$$\omega_R = \frac{\hbar k^2}{2m}$$

“recoil frequency”

Solution 1: Individual addressing of ions.
UNIVERSAL QUANTUM GATES

Segment pulse in time $\Omega_i = \Omega_i(t)$

$$\alpha_i^m(\tau) = \frac{-i\eta_{i,m}\Omega_i(t)}{2(\mu-\omega_m)}\left[1 - e^{-i(\mu-\omega_m)\tau}\right]$$

for all modes m

2N motional conditions

$$\chi_{i,j}(\tau) = \Omega_i\Omega_j\omega_R \sum_m \frac{b_{i,m}b_{j,m}}{2\omega_m(\mu-\omega_m)}\left[\tau - \frac{\sin(\mu-\omega_m)\tau}{\mu-\omega_m}\right] = \frac{\pi}{4}$$

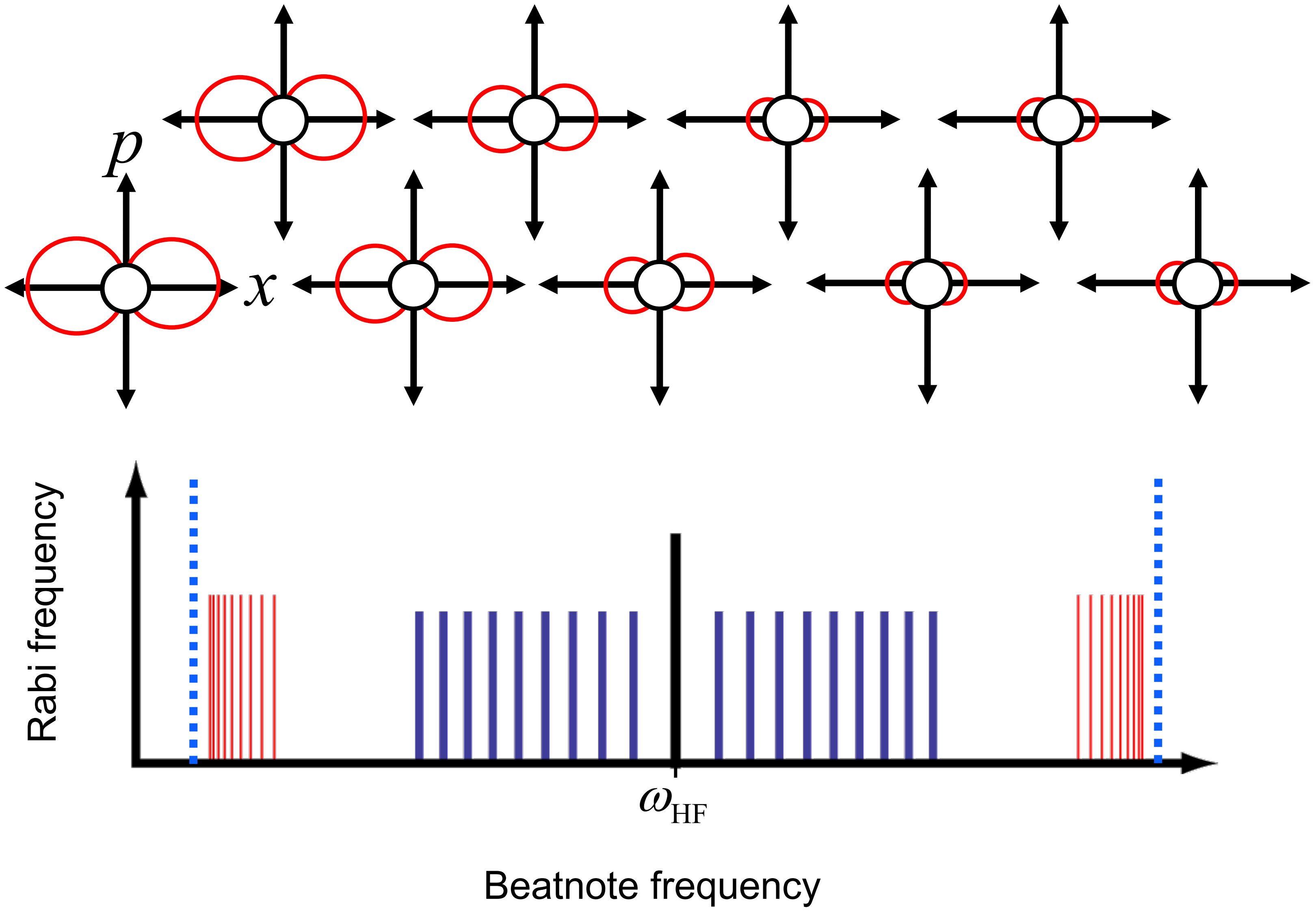
1 gate condition

2N+1 constraints

→ *break $\Omega(t)$ into 2N+1 segments*

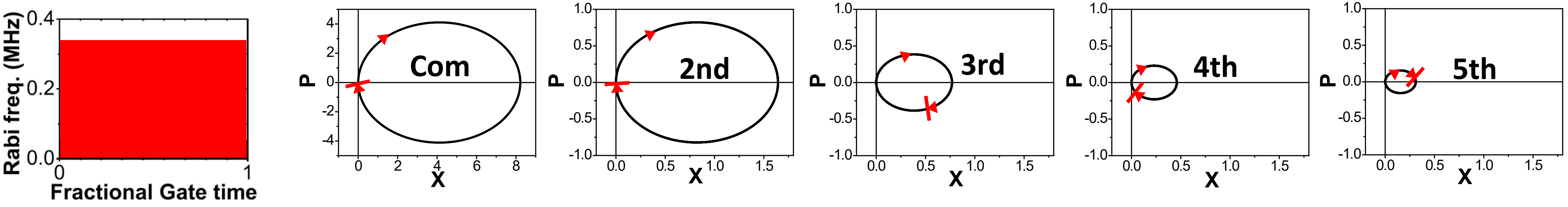
S.-L. Zhu, et al., Europhys Lett. 73 (4), 485 (2006)

Phase Space evolutions

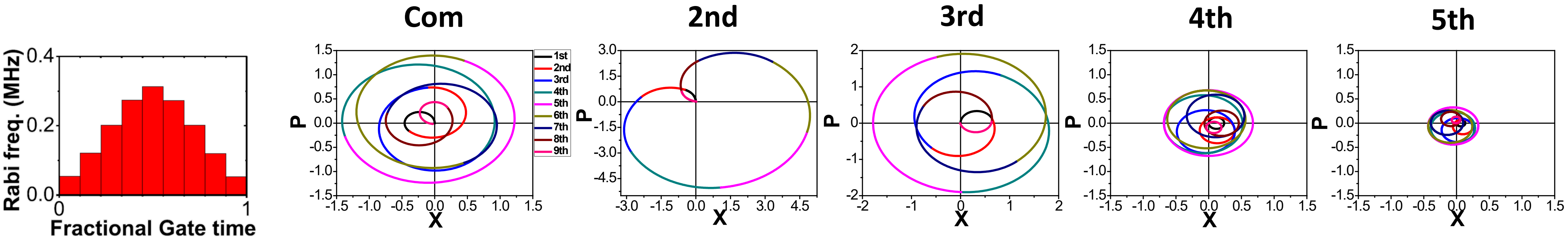


Closing Phase Space(s) N=5 ions: Addressing ions #2 & #5

constant pulse (3rd, 4th, 5th modes are not closed) tuned blue of COM



Modulated pulse (9 segments AM) (all modes are “mostly” closed)



$$XX_{2,5}[\varphi] = e^{-i\sigma_x^{(2)}\sigma_x^{(5)}\varphi}$$

S.-L. Zhu, et al., Europhys Lett. 73 (4), 485 (2006)
T. Choi, et al., Phys. Rev. Lett. 112, 19502 (2014)
S. Debnath, et al., Nature 536, 63 (2016)

Spin-dependent force (simple/fast version)

dipole-dipole coupling $\Delta E = \frac{e^2}{\sqrt{r^2+\delta^2}} - \frac{e^2}{r} \approx -\frac{(e\delta)^2}{2r^3}$

δ ~ 10 nm
$e\delta$ ~ 500 Debye

$$\begin{aligned} |\downarrow_x\downarrow_x\rangle &\rightarrow |\downarrow_x\downarrow_x\rangle \\ |\downarrow_x\uparrow_x\rangle &\rightarrow e^{-i\varphi}|\downarrow_x\uparrow_x\rangle \\ |\uparrow_x\downarrow_x\rangle &\rightarrow e^{-i\varphi}|\uparrow_x\downarrow_x\rangle \\ |\uparrow_x\uparrow_x\rangle &\rightarrow |\uparrow_x\uparrow_x\rangle \end{aligned} \longrightarrow \varphi = \frac{\Delta E t}{\hbar} = \frac{e^2\delta^2 t}{2\hbar r^3} = \frac{\pi}{2}$$

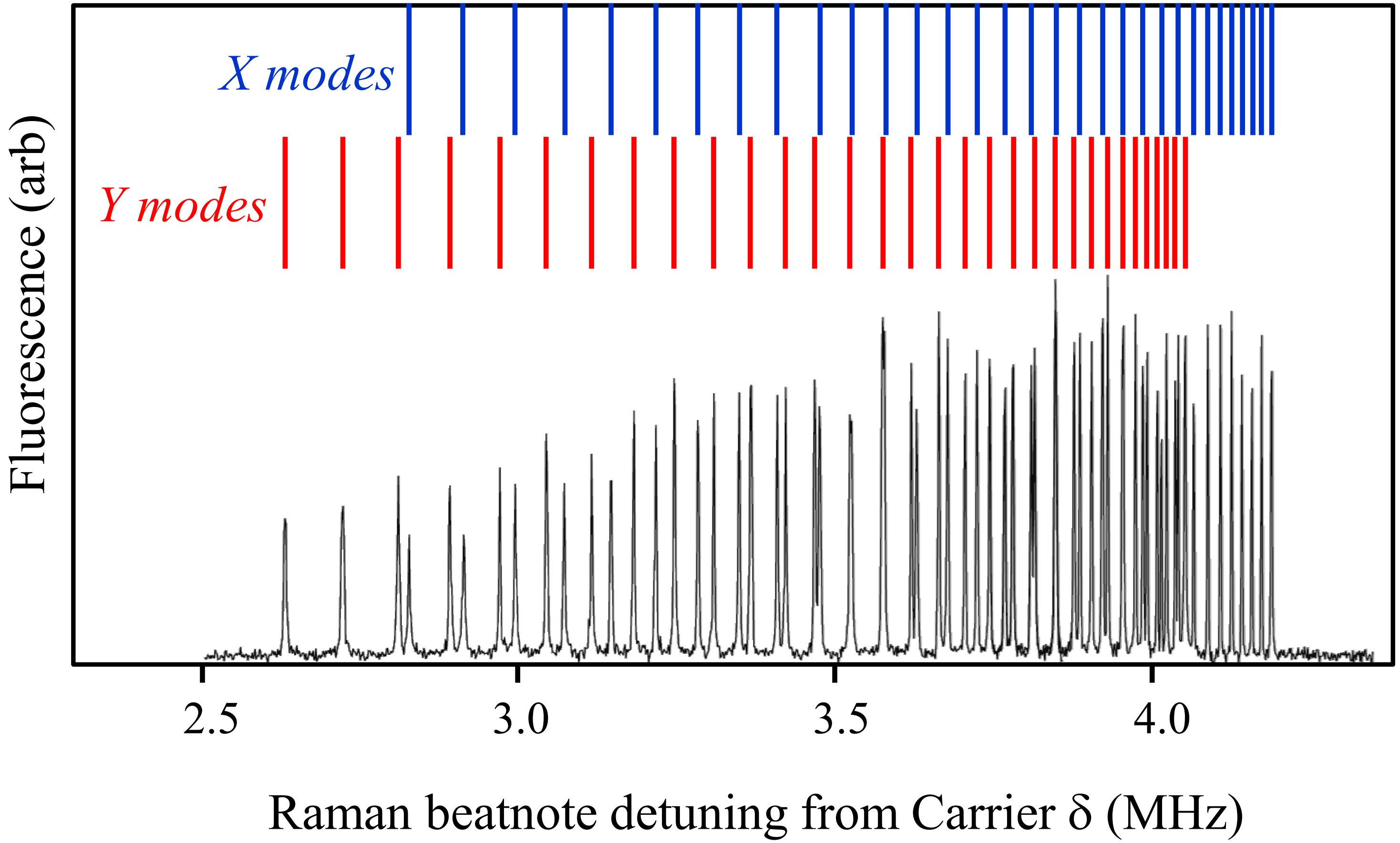
for full entanglement

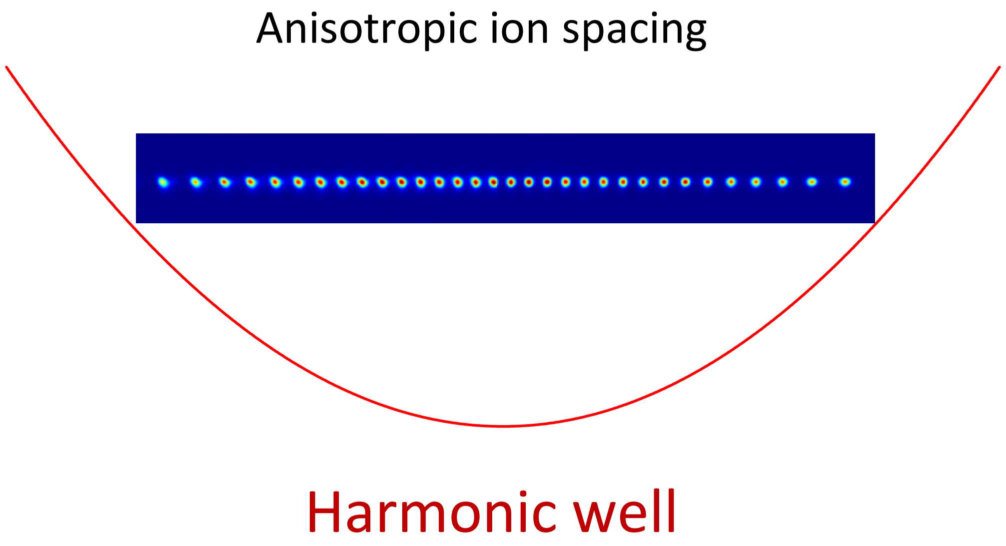
Native Ion Trap Operation: “Ising” gate

$$XX[\varphi] = e^{-i\sigma_x^{(1)}\sigma_x^{(2)}\varphi}$$

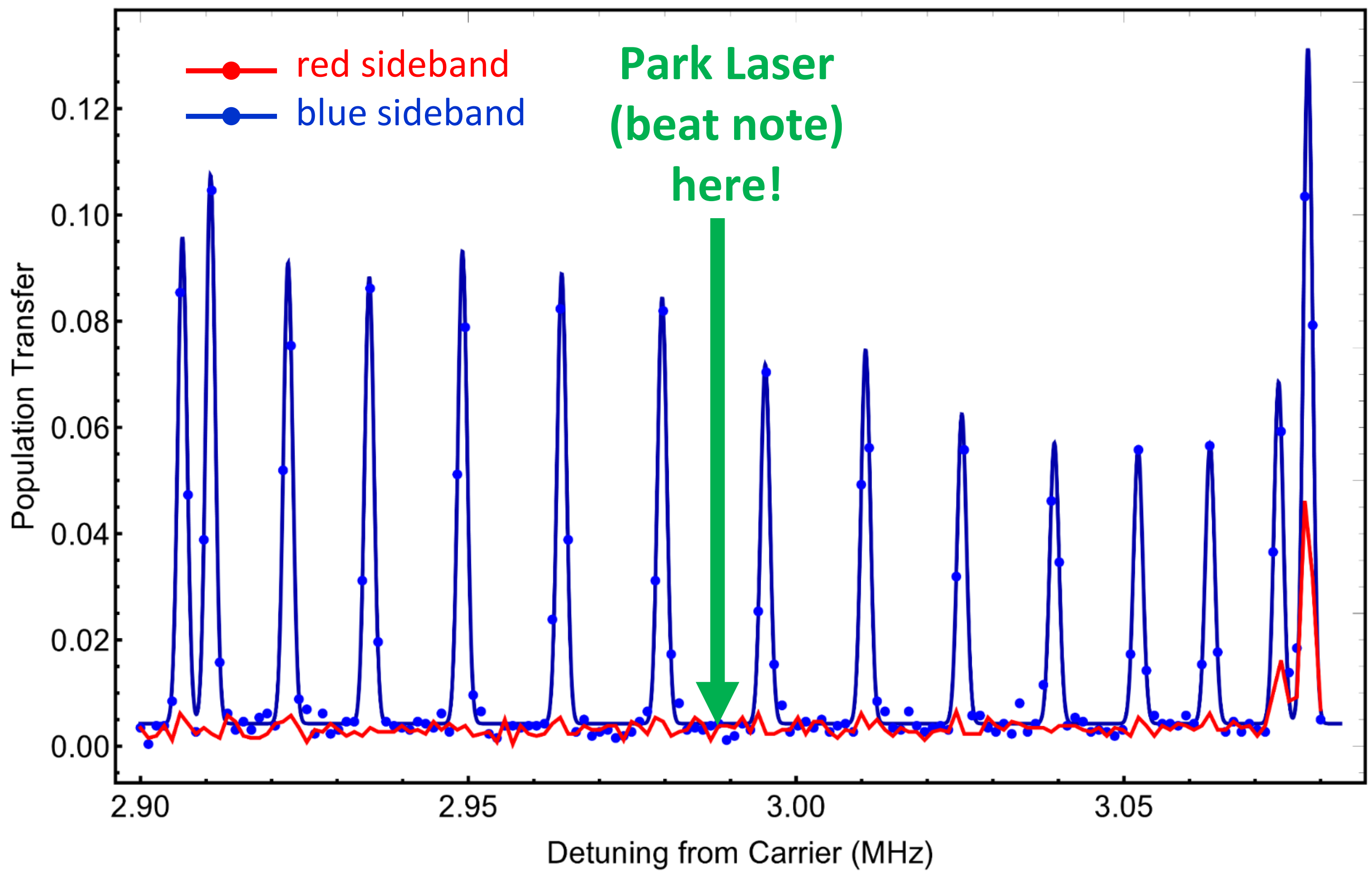
Cirac and Zoller (1995)
Mølmer & Sørensen (1999)
Solano, de Matos Filho, Zagury (1999)
Milburn, Schneider, James (2000)

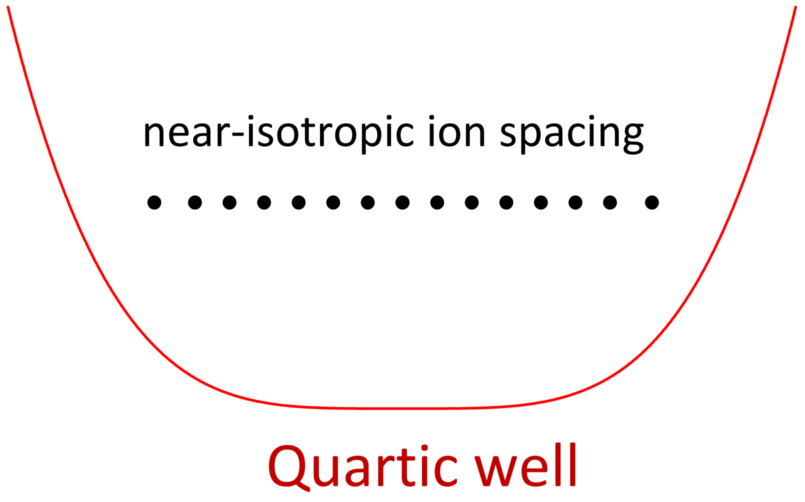
Raman Sideband Spectrum of 32 $^{171}Yb^{+}$ ions (Harmonic trap)





Raman Sideband Spectrum of 15 $^{171}Yb^{+}$ ions (Quartic trap)

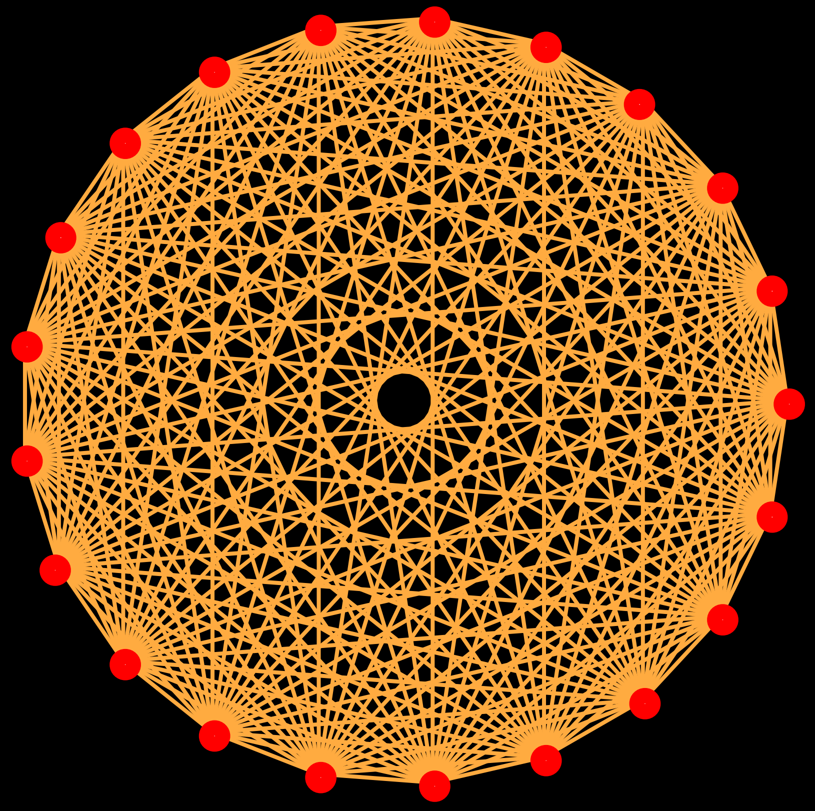




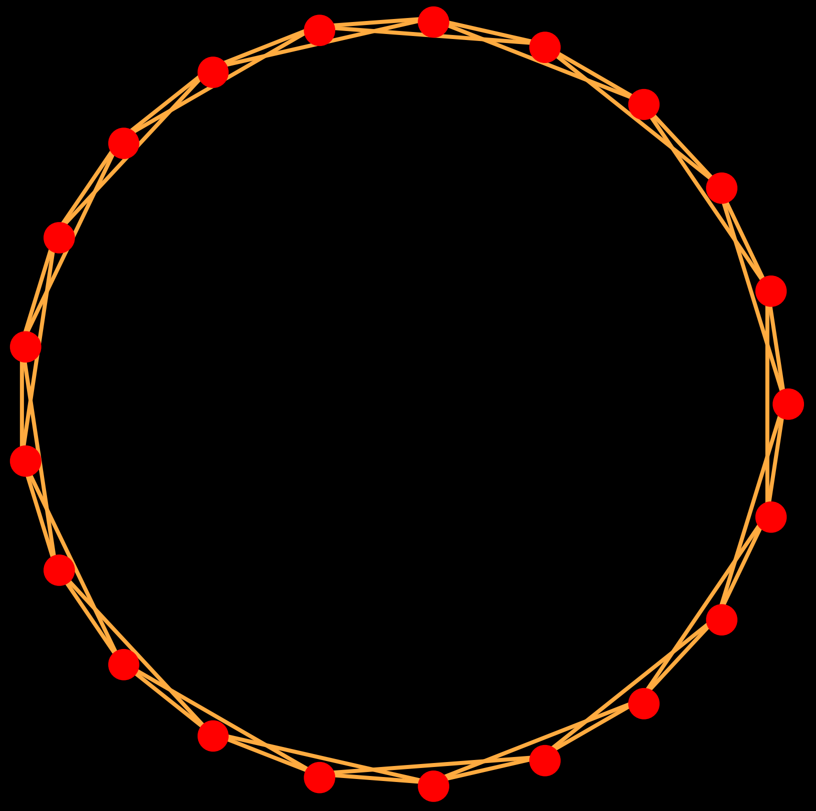
- surprise: nearly equally spaced modes
- easy to close all phase spaces by tuning in between modes: detunings are commensurate ($\pm\delta, \pm 3\delta, \pm 5\delta, \ldots$)

Fully Connected Graphs are Nice!

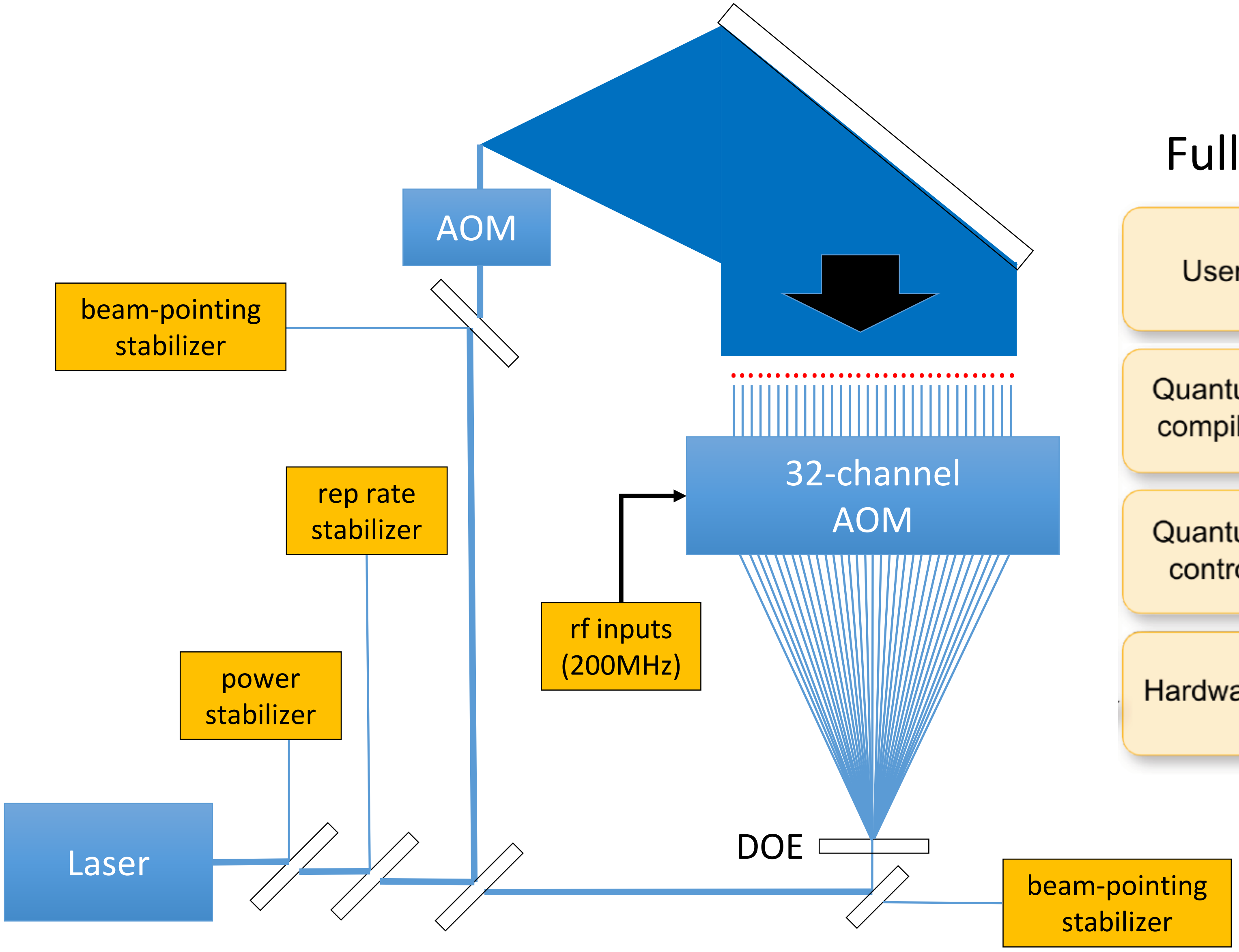
21 qubits
Fully-Connected



21 qubits
Nearest-Neighbor Connected



Programmable/Reconfigurable Quantum Computer



Full "Quantum Stack" architecture

User	**Quantum Algorithms:** *Deutsch-Jozsa, QFT, etc.*
Quantum compiler	**Universal gates:** *Hadamard, C-NOT, C-Phase, etc.* **Native gates:** *XX-Gates, R-gates*
Quantum control	**Pulse shaping:** *Optimization of XX- and R-Gates*
Hardware	**Optical addressing:** *Qubit manipulation/ detection* **Ion trap:** *Linear ion-chain, optical access, etc.*

S. Debnath, et al., *Nature* **536**, 63 (2016)
N. Linke, et al., *PNAS* **114**, 13 (2017)