Thursday ↓

Let's change to a spin model: spin-$\frac{1}{2}$ chain, L spins

Noninteracting (trivial) MBL system:

$$H_0 = \sum_{n=1}^{L} h_n Z_n$$

static field on spin n (h_n)

Z Pauli operator of spin n (Z_n).

h's are of order one, $|h_n| \neq |h_m|$, no degeneracies.

$$[Z_n, H_0] = 0 = [Z_n, Z_m] \quad : \text{ complete set of conserved operators.}$$

Eigenstates are not thermal (except two of them: ground states of H_0 and $-H_0$).

Now add some general nearest-neighbor interaction:

$$H = H_0 + \underset{\text{interaction strength}}{g} \sum_n J_n(\vec{\sigma}_n, \vec{\sigma}_{n+1})$$

J_n: an order-one two-spin interaction.

Basko, Aleiner, Altshuler 2006 (perturbatively),
Imbrie 2014 (controlling many nonperturbative possibilities):

For small enough g, this system remains MBL:

"Dress" conserved quantities to make "l-bits" Serbyn, Papic, Abanin 2013; H., Nandkishore, Oganesyan 2014

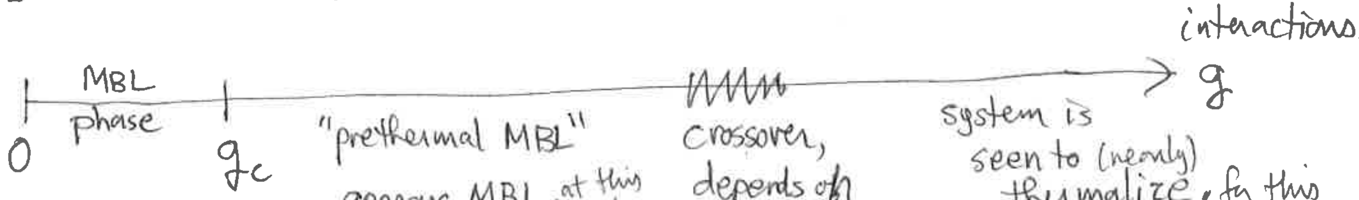
$$\tau_n = Z_n + \left(\text{multisite terms}\right)$$

(dressing includes multisite terms, but exp. decaying w distance from n) ← remain localized operators for small enough g.

$$[\tau_n, H] = 0 = [\tau_n, \tau_m] \quad \text{this can be done for small enough } g.$$

Dynamic phase diagram of MBL (nothing is there in the thermodynamics)

1D with short-enough range interactions:



actual phase transition in limits $L\to\infty$, $t\to\infty$. Is not visible in numerics, experiments.

"prethermal MBL"
appears MBL at this L, t, but is expected to thermalize in $L\to\infty, t\to\infty$ limit.
roughly speaking: here MBL is perturbatively stable, nonperturbatively unstable

crossover, depends on lengths + times accessed, moves to smaller g as L, t are increased.

system is seen to (nearly) thermalize for this L, t.

Longer range interactions or $d>1$, taking standard thermodynamic limit: $g_c\to 0$

(no true MBL phase in this limit)

We do not have a strong ~~good~~ theoretical understanding of the thermal / prethermal MBL crossover that is seen in numerics + experiments. ~~(But see~~ Crowley + Chandran 2012.14393: Morningstar ... H, 2107.05642

Rough description: It is where most eigenstates get involved in many-body resonances.

Instabilities of the MBL phase &, prethermal MBL regime: (or ∞-lived)

MBL is coherence; long-lived localized excitations

Thermalization is decoherence.

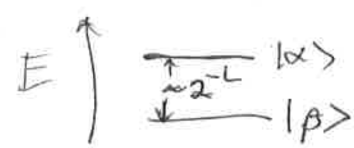
So Thermalization very much has the upper hand in this competition. ~~The~~ MBL is "fragile", like all quantum coherence.

Many-body resonances:

In MBL phase + prethermal MBL regime, of finite-L systems, almost all eigenstates are localized near particular spin patterns.

~~Next~~ Energy difference between adjacent in energy eigenstates $\sim 2^{-L}$

Typically these two eigenstates differ on $\sim L/2$ of the spins.



$$\uparrow\ \uparrow\ \downarrow\ \uparrow\ \downarrow\ \downarrow\ \uparrow\ \downarrow\ = |\alpha\rangle$$

$$\uparrow\ \downarrow\ \downarrow\ \downarrow\ \uparrow\ \downarrow\ \downarrow\ \uparrow\ = |\beta\rangle$$

Change H a little to H' to ~~bring~~ attempt to bring these two eigenstates to degeneracy. $\langle\alpha|H'|\alpha\rangle = \langle\beta|H'|\beta\rangle$

There are no selection rules so H' does have nonzero matrix elements $\langle\alpha|H'|\beta\rangle \neq 0$.

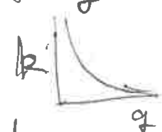
Eigenstates of H' will be $\frac{1}{\sqrt{2}}(|\alpha\rangle \pm |\beta\rangle)$ with some energy splitting $= 2\langle\alpha|H'|\beta\rangle$

There are no selection rules preventing hybridization and energy level repulsion between these eigenstates, even deep in MBL phase.
(conservation laws are "emergent", not like usual symmetries)

Thinking perturbatively in the interaction, for H to couple $|\alpha\rangle$ to $|\beta\rangle$ requires going to order $\sim L$ in in interaction (to flip $\sim L/2$ spins).

So typical level repulsion $\sim k(g)^{-L}$ is exponentially small in L

for $g \to 0$ $k \to \infty$



crossover: prethermal MBL/thermal is when

~~typical~~ $k(g) \cong 2$ so typical adjacent ~~levels~~ eigenstates do hybridize substantially. (Basko, Aleiner, Altshuler 2006).

Really we find (in numerics) strong finite-size effects so typical level repulsion $\sim k(g,L)^{-L}$

k decreases with $\uparrow g$

mechanism for this is not yet clear; likely is nonperturbative. $\longrightarrow$ k decreases with $\uparrow L$

Many body resonances are eigenstates that are linear superp. with significant amplitude of two or more very different localized states: e.g.

$$\frac{1}{\sqrt{2}}\left(|\alpha\rangle \pm |\beta\rangle\right)$$

due to H coupling $|\alpha\rangle$ to $|\beta\rangle$

If only two: Schr. cat-like eigenstate

If very many $\mapsto$ Thermal state.

MBL phase/prethermal regime do have such resonances, but only in a very small fraction of eigenstates, The other known instability of the MBL <u>phase</u>:

The "avalanche". De Roeck + Huveneers 2017

Assume H has randomness. ~~Will by chance have~~

Even in MBL phase/prethermal regime it will have "patches" (of all sizes) with low randomness/strong interactions that locally become highly-entangled, <u>locally</u> thermal, just "by chance":

outside is all MBL

thermal patch, contains N spins

1) ~~Cut~~ Turn off all couplings between thermal patch + MBL region.

Diagonalize: get entangled thermal-like eigenstates in thermal patch, get localized l-bits outside. l-bit-to-l-bit interactions do <u>not</u> flip l-bits.

2) Turn on couplings, ask: does this small bath relax/entangle with the previously MBL spins/l-bits? (flip them)

Neighboring spins: coupling $= g$, level spacing of the putative bath $\sim 2^{-N}$ so for large enough N, $g \gg 2^{-N}$ and these nearby spins do get entangled with the bath, joining it. This is the start of the avalanche.

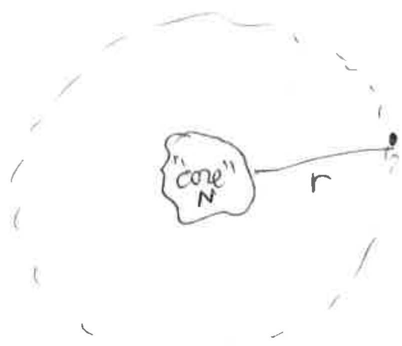
spin at distance r :

If bath is large enough to relax it, this spin relaxes at rate $\Gamma(r) \sim 2^{-2r/\xi}$ ← decay length.

exponential in distance from the core.

If level spacing of bath is $\ll \Gamma$, it "looks like" a continuum to this spin.
If level spacing of bath is $\gg \Gamma$, spin will not relax, it remains MBL.

Let avalanche proceed to distance r:



Now bath has $\sim (N^{1/d} + ar)^d$ spins. (a: a constant.)

in 1D: $(N+2r)$ spins.

So bath's level spacing $\delta \sim 2^{-(N^{1/d}+ar)^d}$

$\sim 2^{-(N+2r)}$ in 1D.

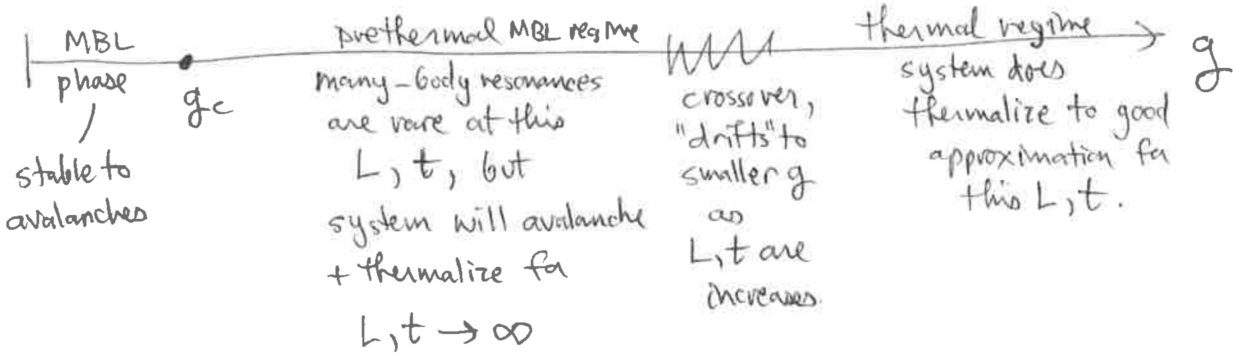
compare to $\Gamma \sim 2^{-2r/\xi}$

For $d>1$ and large N, always have $\delta \ll \Gamma$: spin sees bath as a continuum and does relax, get entangled, but slowly, avalanche never stops.

For $d=1$: for $\xi<1$ get to $\Gamma \simeq \delta$ and avalanche stops: MBL phase is stable to these locally thermal patches. MBL transition occurs where $\xi = \xi_c = 1$.

summary of phase diagram for MBL:

1D, short-enough range interactions:



longer range interactions ~~of~~ or $d>1$:

$g_c \to 0$, no MBL phase at $g>0$ in limit $L, t \to \infty$.

1D: avalanche transition at g_c is ∞-order
"Kosterlitz-Thouless-like"

strong-randomness RG treatment: Morningstar, H, Imbrie (2020)