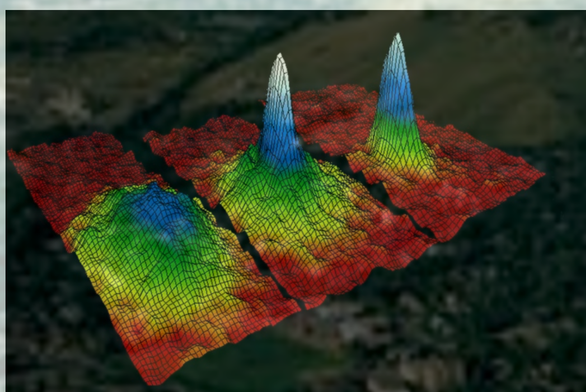
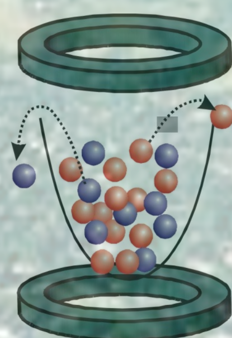
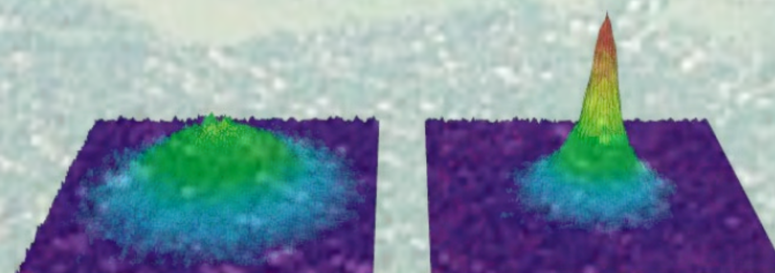
Superfluidity and phase transitions in resonant Bose gas

Boulder

Center for Theory of Quantum Matter
CTQM







with *Jae Park, Peter Weichman, Sungsoo Choi*

for details see, PRL '04, Annals of Physics '08, PRL '09, PRA '11
also Sachdev, et al. PRL '04

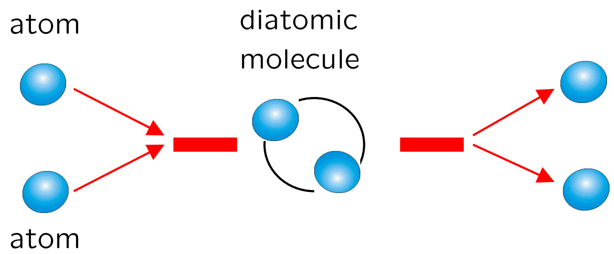
support by: ***NSF Materials Theory, Packard Foundation, Simons Investigator***

Outline

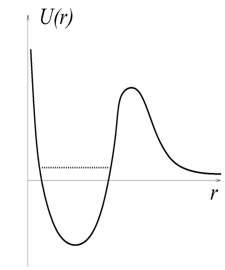
- **S-wave superfluidity**
  - *Feshbach resonant bosonic model*
  - *Atomic SF (ASF) and Molecular SF (MSF)*
  - *Quantum Ising transition*
  - *Half vortices deconfinement transition*
- **P-wave superfluidity**
  - *Feshbach resonant bosonic model*
  - *Finite momentum Atomic-Molecular SF (AMSF)*
  - *Quantum smectic transition*
  - *Phase diagram*

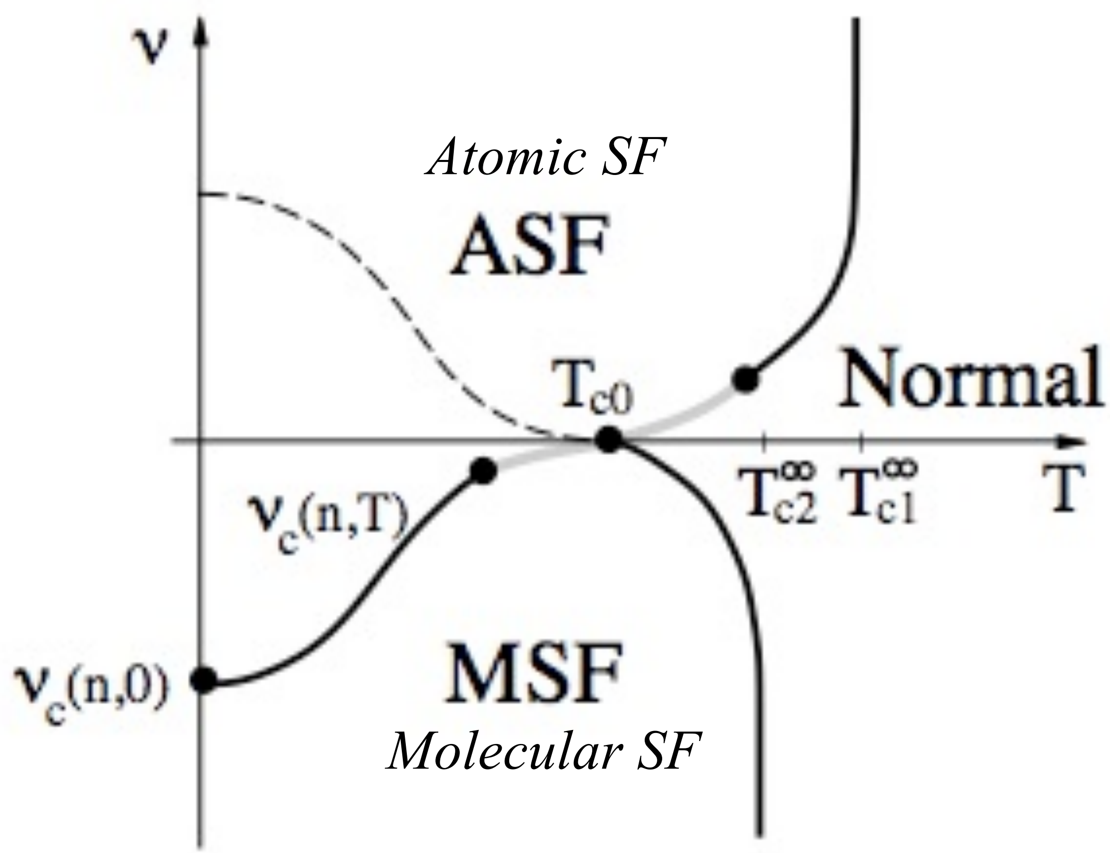
Part I

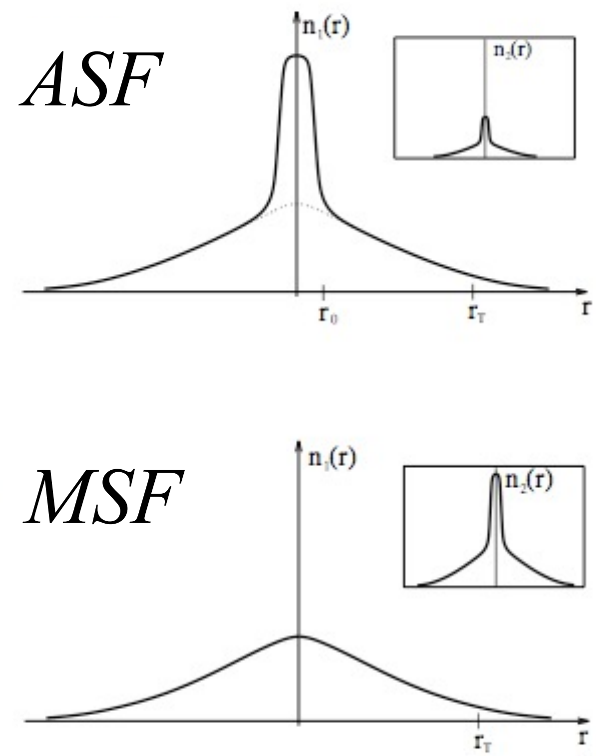
S-wave superfluidity



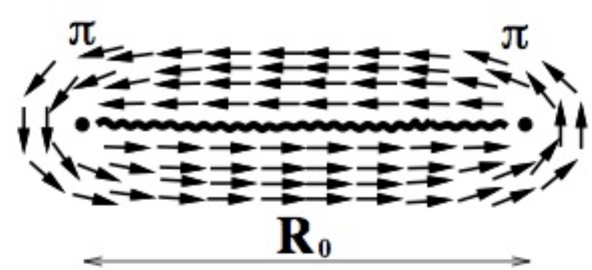
Summary







- atomic (ASF) and molecular (MSF) superfluids
- quantum Ising transition
- π vortex deconfinement



Motivation

- **Bosonic Feshbach resonances exist in e.g., Rb87, Rb85,…**
  $\rightarrow$ ultracold coherent bosonic atom-molecule mixtures

- Resonant fermionic systems led to molecular superfluids, BEC-BCS crossover, strongly interacting superfluid, etc.

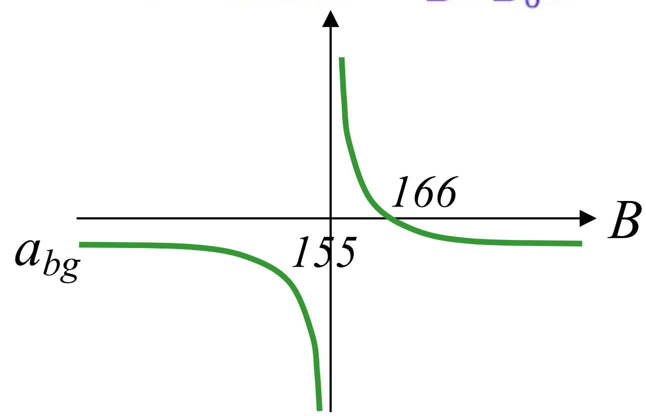
- Allow SF-SF quantum phase transitions (cf. just crossover for fermions)

Feshbach resonance (Bose)

Rb85

$|F=2, m_F=-2\rangle$

$a = a_{bg}(1 - \frac{\Delta B}{B-B_0})$

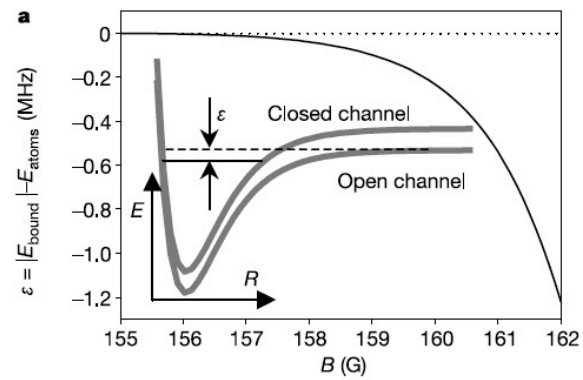


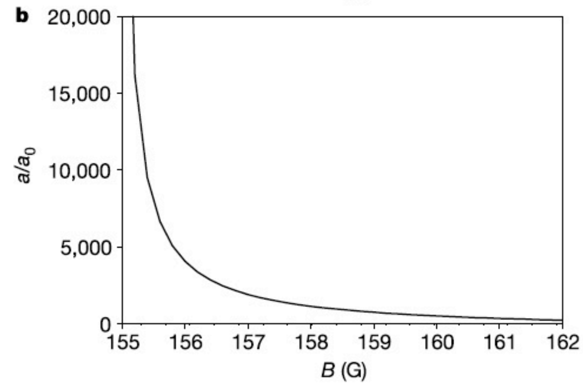
Atom–molecule coherence in a Bose–Einstein condensate

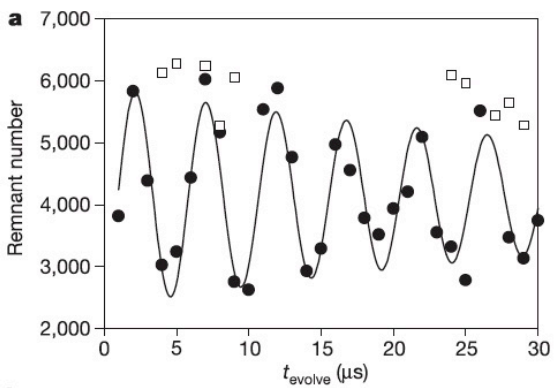
Elizabeth A. Donley, Neil R. Claussen, Sarah T. Thompson & Carl E. Wieman

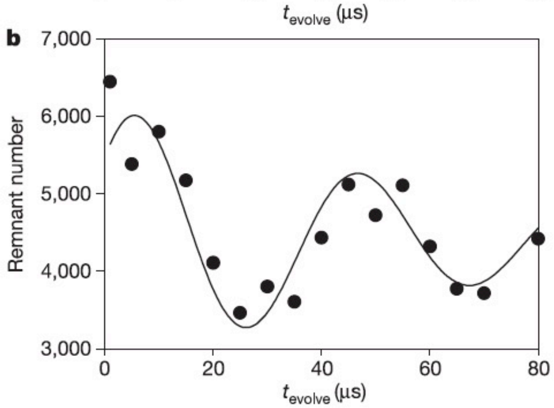
JILA, University of Colorado and National Institute of Standards and Technology, Boulder, Colorado 80309-0440, USA

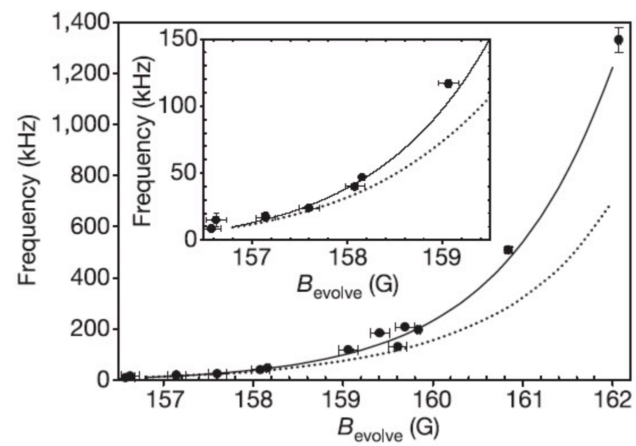
NATURE | VOL 417 | 30 MAY 2002











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Feshbach resonance (Fermi)

Emergence of a molecular Bose–Einstein condensate from a Fermi gas

Markus Greiner[1], Cindy A. Regal[1] & Deborah S. Jin[2]

[1]*JILA, National Institute of Standards and Technology and Department of Physics, University of Colorado,* [2]*Quantum Physics Division, National Institute of Standards and Technology, Boulder, Colorado 80309-0440, USA*

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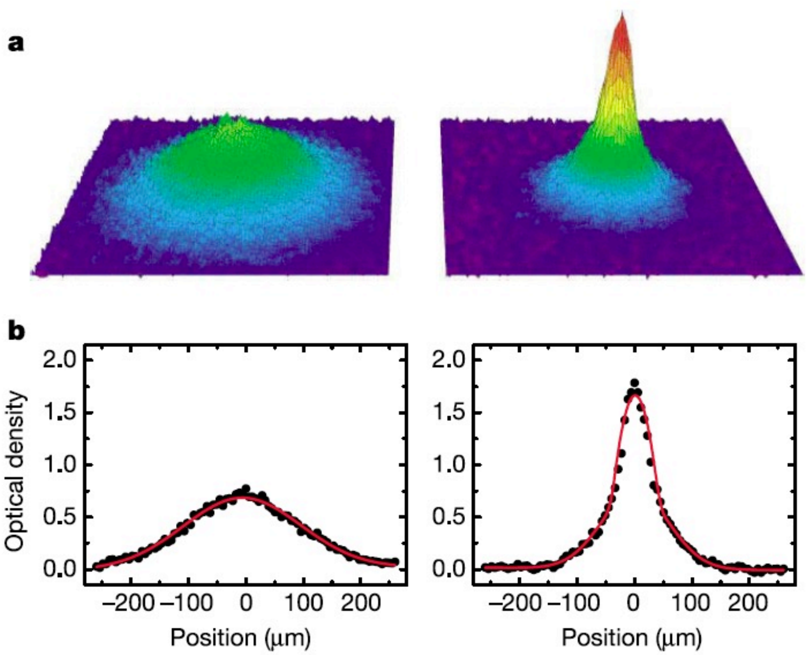
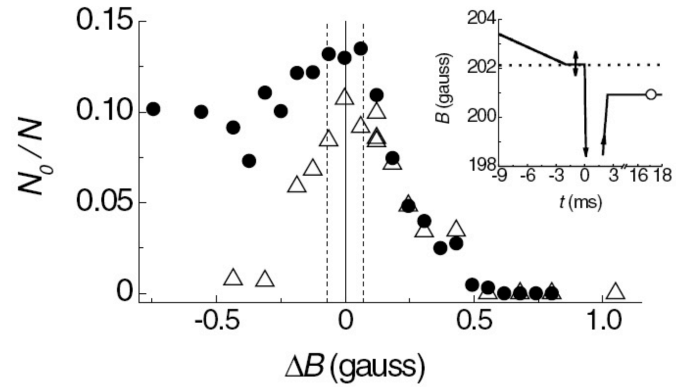
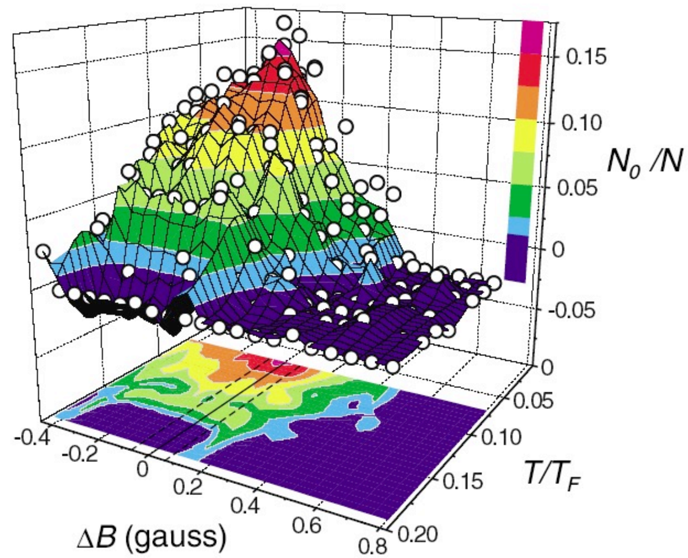


Observation of Resonance Condensation of Fermionic Atom Pairs

C. A. Regal, M. Greiner, and D. S. Jin*

Physical Review Letters 92, (2004)





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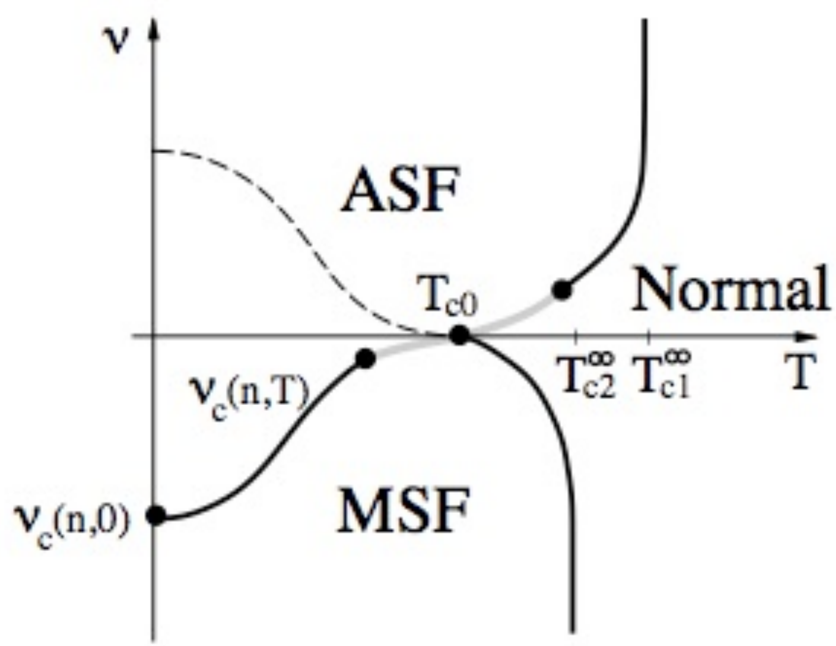
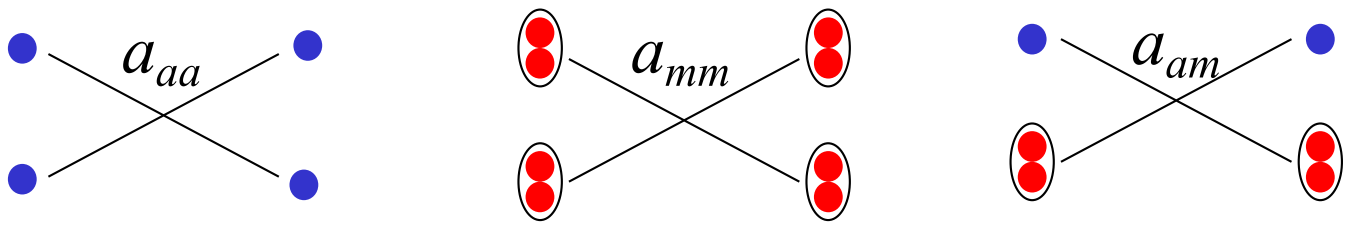


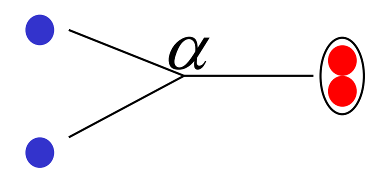
earlier works:
Valatin and Butler '58
Evans and Imry '69
Nozieres and Saint James '82

Resonant model

Interacting bosonic atoms and (diatomic) molecules:



atom-molecule density-density interactions



Feshbach resonance
atom-molecule conversion

$$\hat{H} \;=\; \int_x \sum_{\sigma=1}^{2} \Big[\hat{\psi}_\sigma^\dagger \left(-\frac{\hbar^2}{2m_\sigma}\nabla^2 + \mu_\sigma \right) \hat{\psi}_\sigma + \frac{1}{2} g_\sigma \hat{\psi}_\sigma^\dagger \hat{\psi}_\sigma^2$$

$$+ g_{12} \hat{\psi}_1^\dagger \hat{\psi}_2^\dagger \hat{\psi}_2 \hat{\psi}_1 - \boxed{\frac{1}{2}\alpha \hat{\psi}_1^\dagger \hat{\psi}_1^\dagger \hat{\psi}_2} \Big]$$

$(m_1 = m,\ m_2 = 2m\ ,\ \mu_1 = \mu,\ \mu_2 = 2\mu - \nu)$

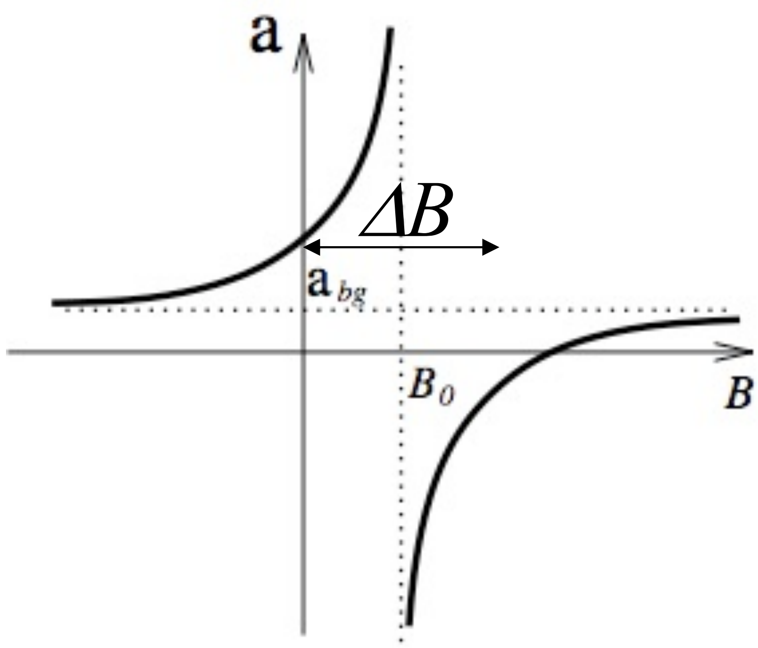
Model parameters (determined by 2-body physics)

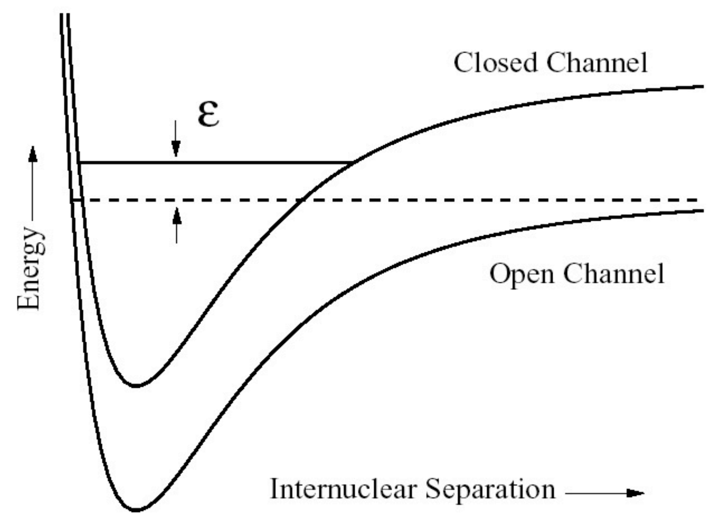
- ***Feshbach interconversion rate and background scattering lengths:***

$$\alpha = \hbar\sqrt{2\pi a_{\mathrm{bg}}\Delta B\Delta\mu_{\mathrm{mag}}}$$

$$g_\sigma = h^2/m\,(a_{aa},\ a_{mm},\ a_{am})$$

$$a(B) = a_{\mathrm{bg}}\left(1 - \frac{\Delta B}{B - B_0}\right)$$



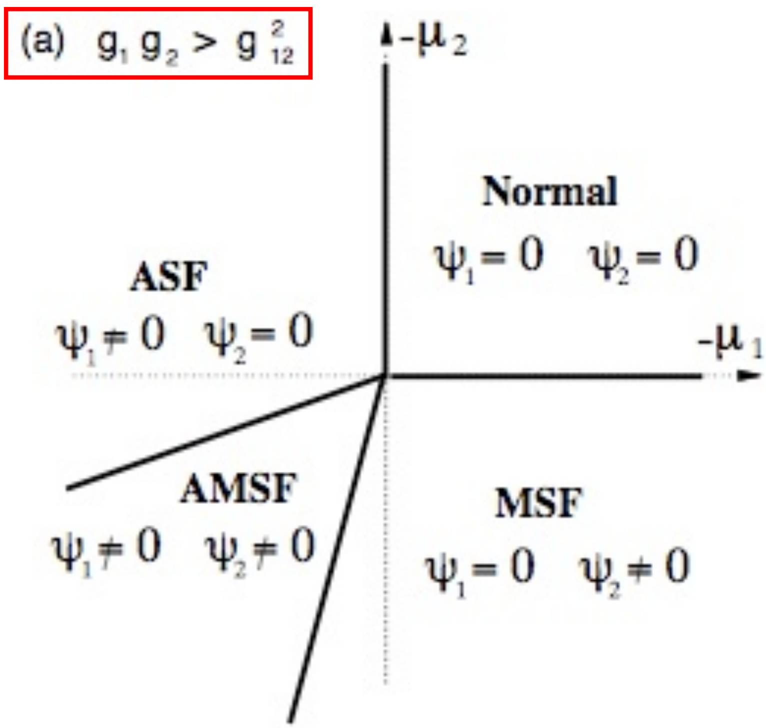


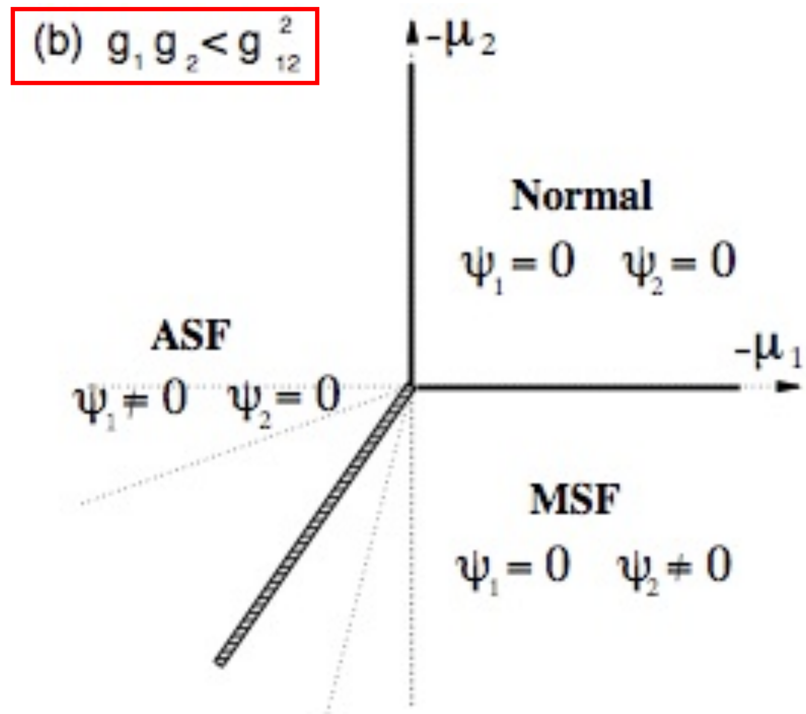
- ***small parameters:*** $\gamma_{\mathrm{FB}} = \sqrt{\dfrac{\Gamma_0}{k_B T_{\mathrm{BEC}}}} = \dfrac{1}{n^{1/3}|r_0|} \sim \dfrac{\alpha^2}{n^{1/3}}, \quad \gamma_\sigma = n a_\sigma^3$

Landau theory

$$
\begin{aligned}
f_{\rm mf} \;=\;& -\mu_1|\Psi_{10}|^2 + \frac{g_1}{2}|\Psi_{10}|^4 - \mu_2|\Psi_{20}|^2 + \frac{g_2}{2}|\Psi_{20}|^4 + g_{12}|\Psi_{10}|^2|\Psi_{20}|^2 \\
& -\alpha \mathrm{Re}[\Psi_{20}^*\Psi_{10}^2]
\end{aligned}
$$

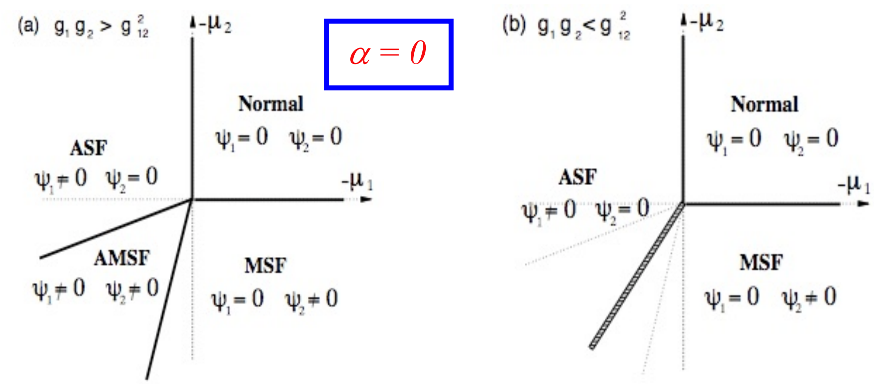
$\boxed{\alpha = 0}$ *(two independent global U(1)'s)*

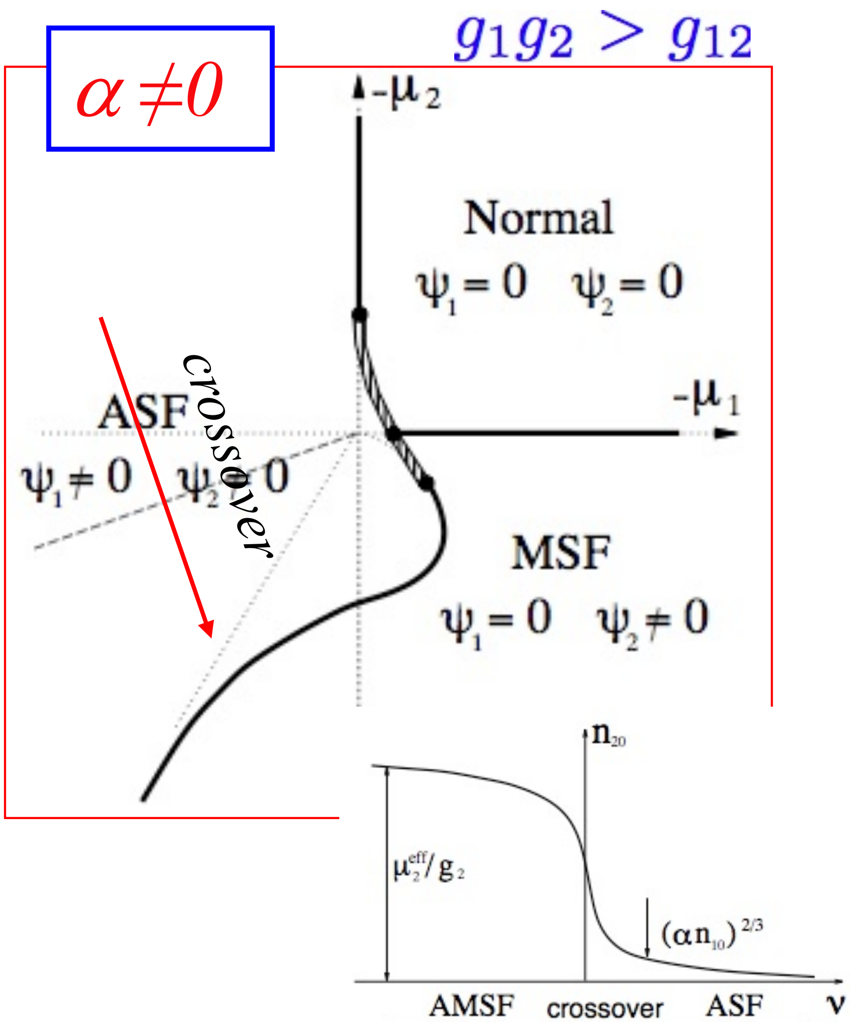


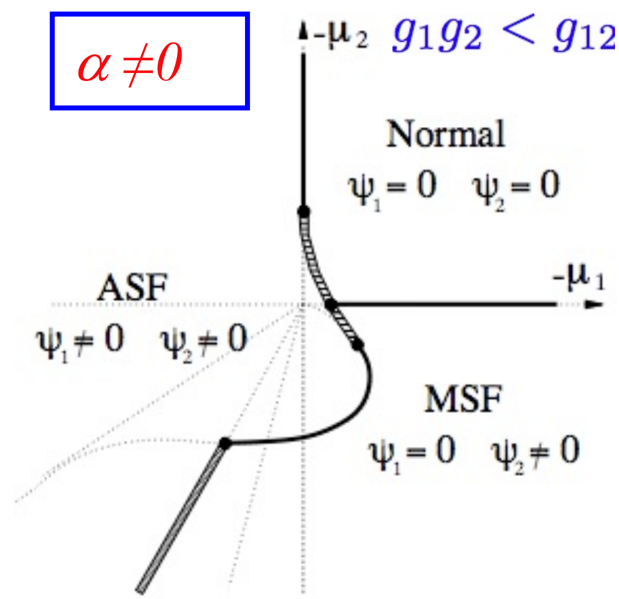


Landau theory

$$f_{\mathrm{mf}} = -\mu_1|\Psi_{10}|^2 + \frac{g_1}{2}|\Psi_{10}|^4 - \mu_2|\Psi_{20}|^2 + \frac{g_2}{2}|\Psi_{20}|^4 + g_{12}|\Psi_{10}|^2|\Psi_{20}|^2$$
$$-\alpha \mathrm{Re}[\Psi_{20}^{*}\Psi_{10}^{2}]$$



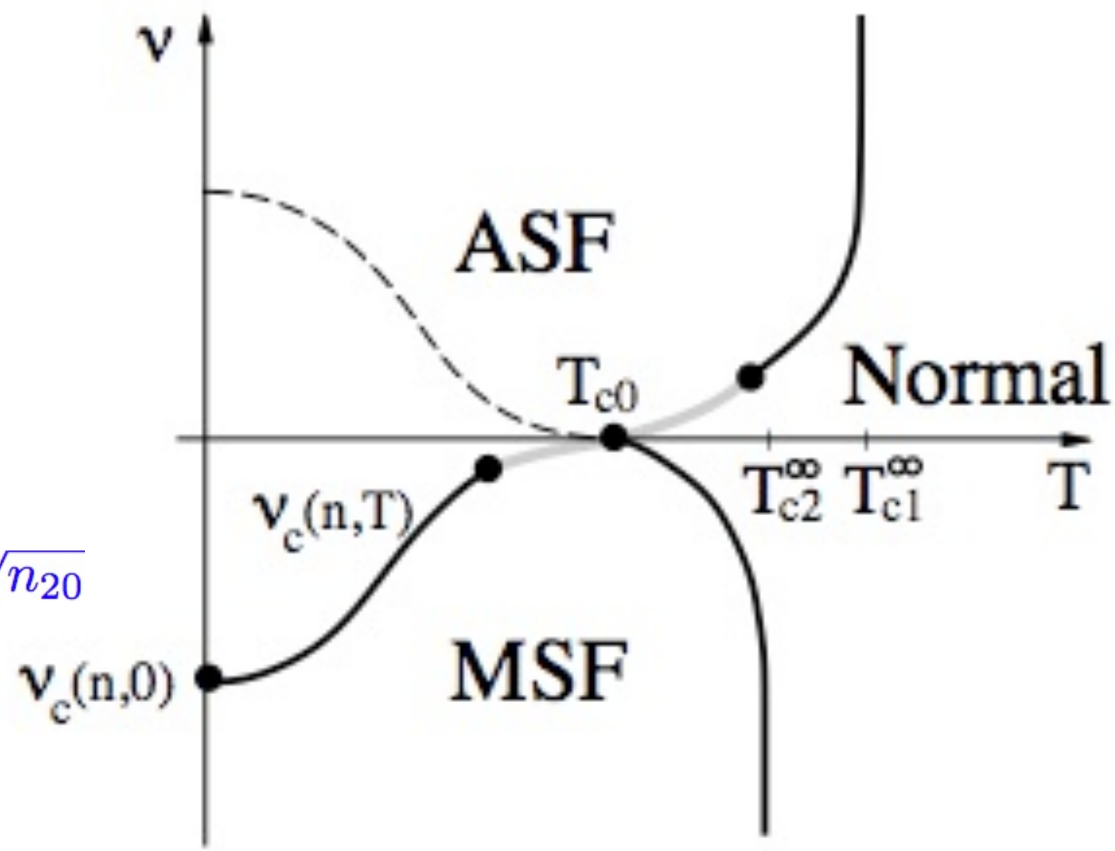




Temperature-detuning phase diagram

$$\langle\hat{\psi}_1^\dagger\hat{\psi}_1\rangle + 2\langle\hat{\psi}_2^\dagger\hat{\psi}_2\rangle = n$$

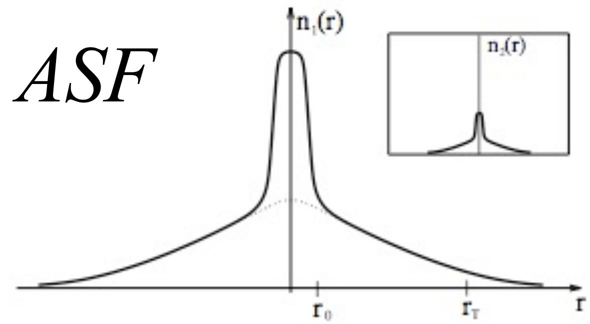
$$\Rightarrow \quad \mu_1 = \mu(n,\nu),\ \ \mu_2 = 2\mu(n,\nu) - \nu$$



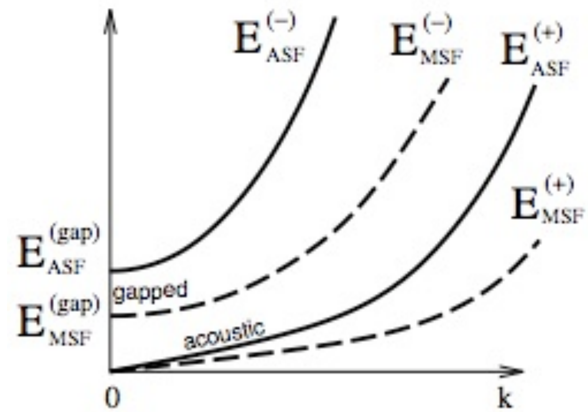
$$\nu_c = -an_{20} - \alpha\sqrt{n_{20}}$$

Atomic and molecular superfluids

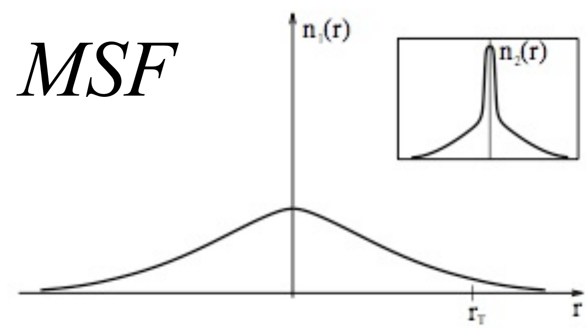
- $$|\mathrm{ASF}\rangle = e^{\Psi_{10}\hat{a}^{\dagger}_{01} + \Psi_{20}\hat{a}^{\dagger}_{02}} \prod_{\sigma,\mathbf{k}} e^{-\chi_{\sigma\mathbf{k}}\hat{a}^{\dagger}_{\sigma,\mathbf{k}}\hat{a}^{\dagger}_{\sigma,-\mathbf{k}}}|0\rangle$$



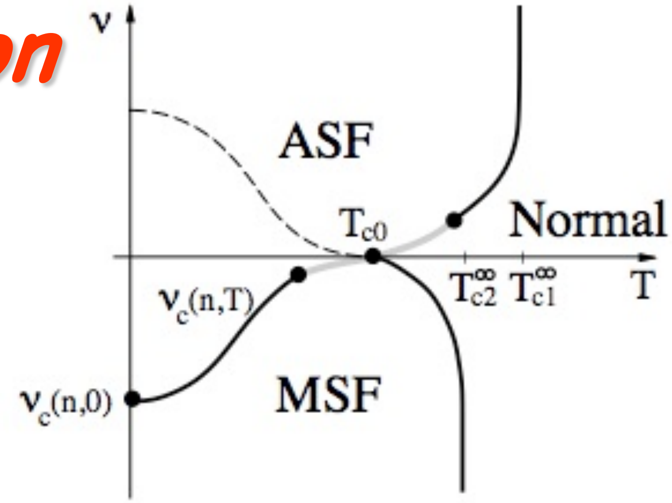
  - ❑ $\psi_1 \neq 0$ (and $\psi_2 \neq 0$)
  - ❑ broken symmetry: U(1)
  - ❑ spectrum:
    - *gapless in-phase Bogoluibov*
    - *gapped out-of-phase θ_1, θ_2*
  - ❑ π-vortices confined



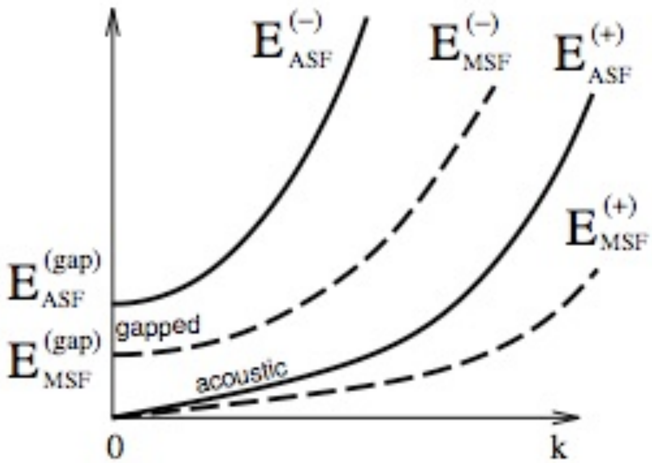
- $$|\mathrm{MSF}\rangle = e^{\Psi_{20}\hat{a}^{\dagger}_{02}} \prod_{\sigma,\mathbf{k}} e^{-\chi_{\sigma\mathbf{k}}\hat{a}^{\dagger}_{\sigma,\mathbf{k}}\hat{a}^{\dagger}_{\sigma,-\mathbf{k}}}|0\rangle$$
  - ❑ $\psi_1 = 0$ (and $\psi_2 \neq 0$)
  - ❑ broken symmetry: $U(1)/Z_2$
  - ❑ spectrum:
    - *gapless Bogoluibov molecules*
    - *gapped atoms*
  - ❑ π-vortices deconfined



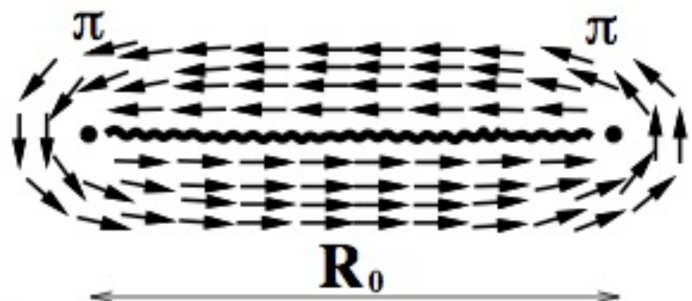
MSF–ASF transition



- atomic gap closing at ν_c:

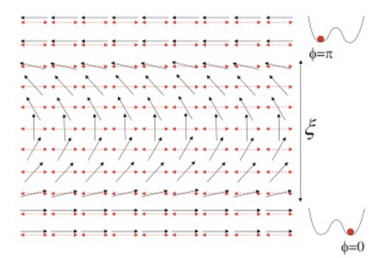


$$\nu_c = -a n_{20} - \alpha\sqrt{n_{20}}$$



- π-vortices deconfinement, $R_0(\nu_c)\to\infty$:

$$\mathcal{E} = (\nabla\theta_1)^2 + (\nabla\theta_2)^2 + K_{12}|\nabla(2\theta_1-\theta_2)|^2 - \alpha n_{10}\sqrt{n_{20}}\cos(2\theta_1-\theta_2)$$

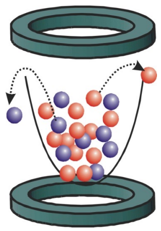


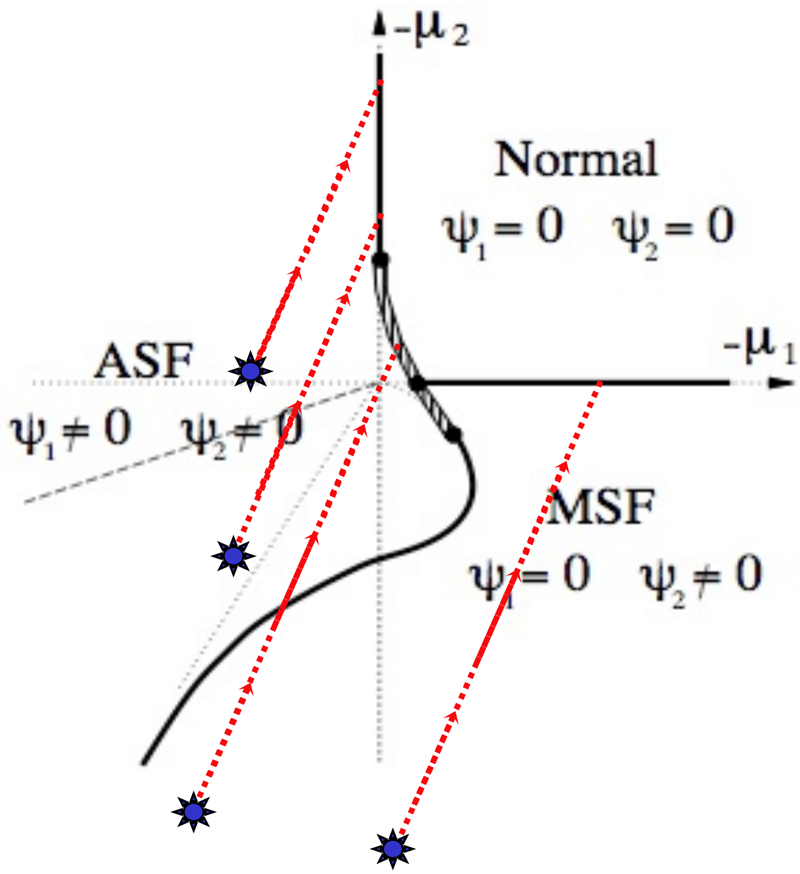
- quantum ("compressible") Ising transition:

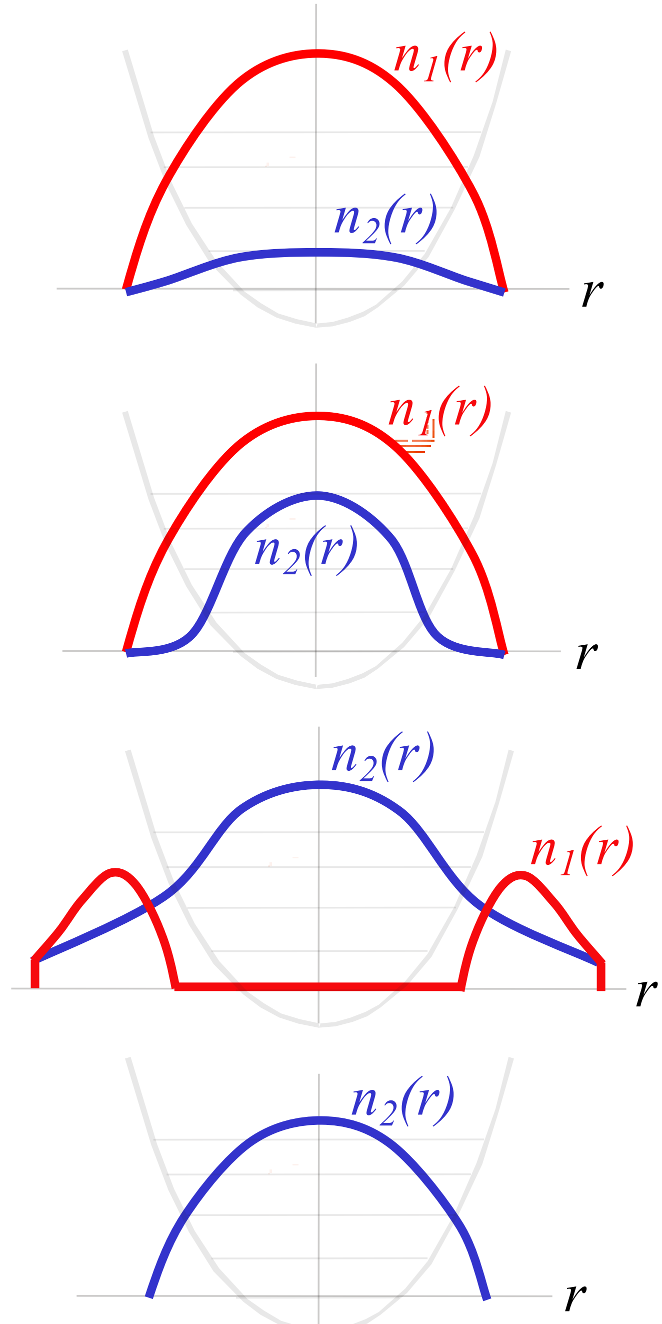
$$S[\Phi,\theta_2] = \int_{\mathbf{x}\tau}\left[(\partial_\mu\theta_2)^2 + i\Phi^2\partial_\tau\theta_2 + (\partial_\mu\Phi)^2 + \Phi^2 + \Phi^4\right]$$

...likely driven 1st order (Halperin, et al; Frey,Balents)

Trapped profiles via LDA

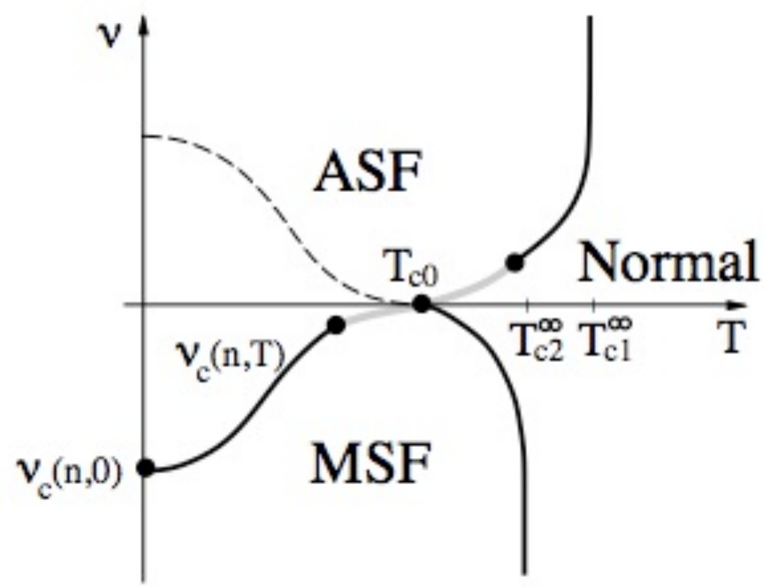






$$\mu \to \mu_{\mathrm{eff}}(r) = \mu - \frac{1}{2} m\omega^2 r^2$$

Summary and conclusions



- resonantly interacting Bose gas:
  - *atomic and molecular superfluids*
  - *quantum Ising transition*
  - *π-vortices*

- …but: *expect short lifetime due to 3-body instabilities (Efimov states)*

- fixes:
  - *optical lattice?*
  - *avoid immediate vicinity of the FBR?*
  - *spinor condensate with on average repulsive interactions?*

- p-wave resonance generalization: *periodic ASF, orbital condensates, etc…*

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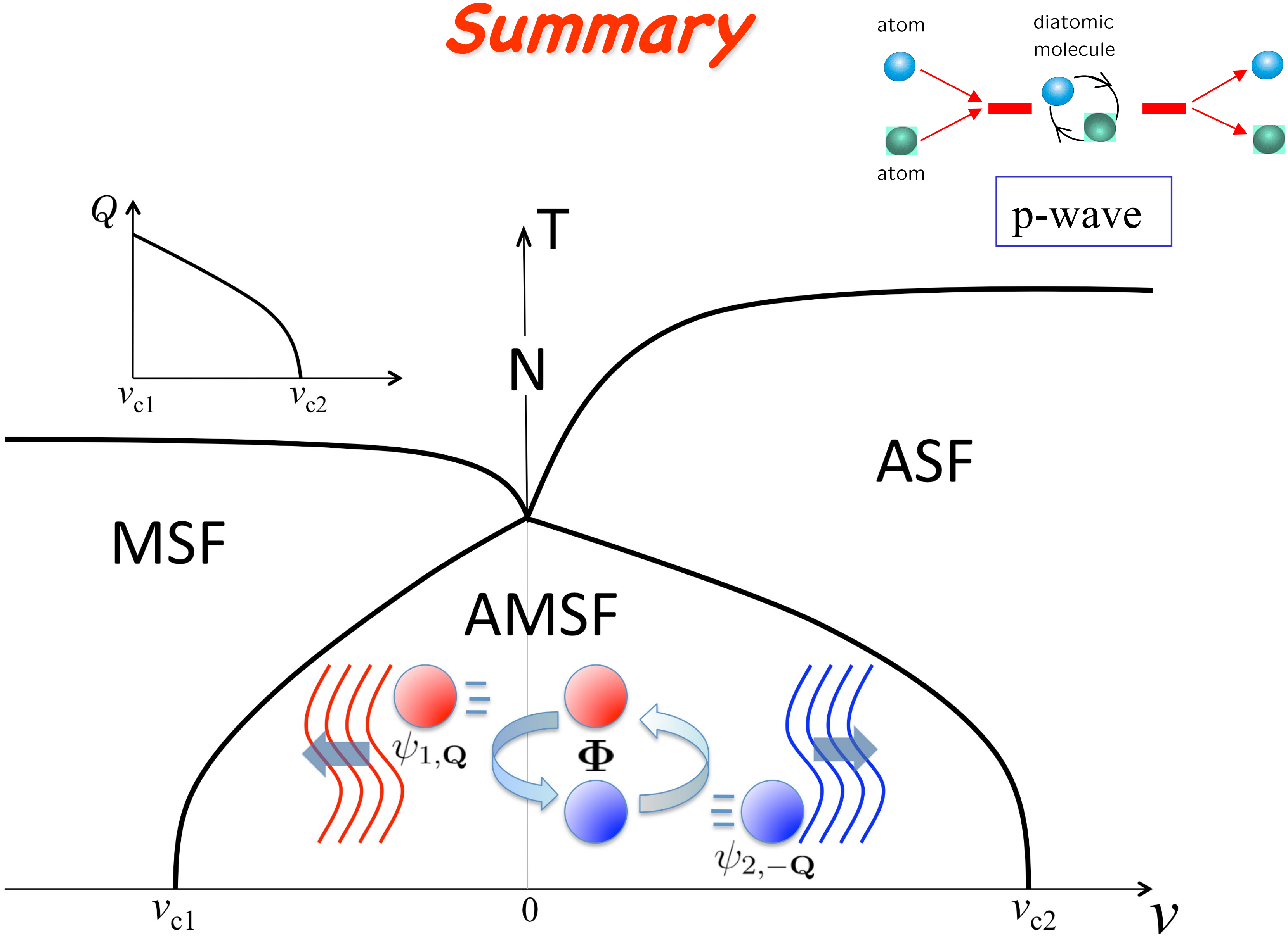
Part II

P-wave superfluidity

Summary



- atomic (ASF) and spinor-molecular (MSF) superfluids
- atomic-molecular superfluid (AMSF) with ***finite momentum*** atomic BEC
- quantum and thermal phase transitions

Motivation

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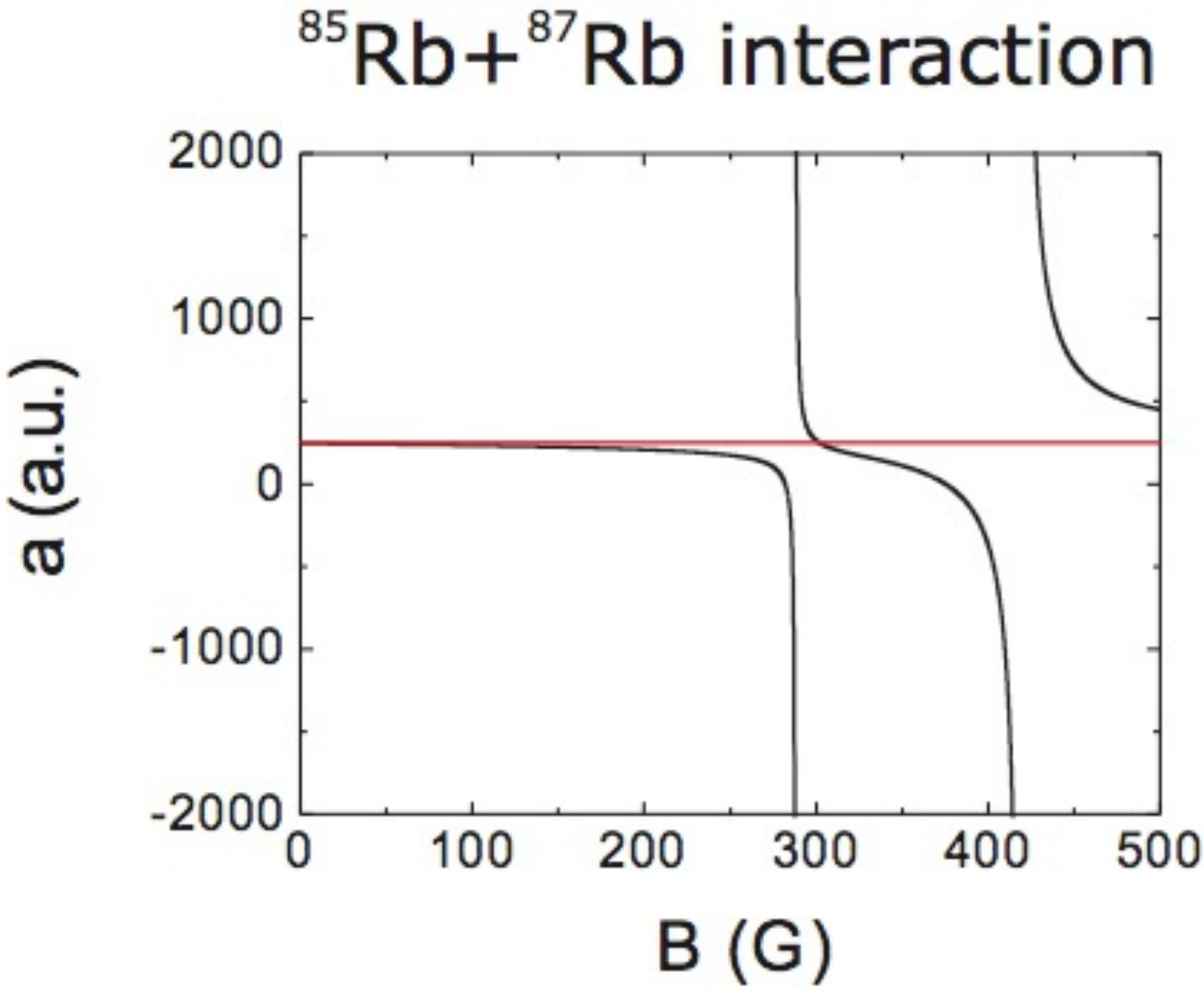
  → ultracold coherent bosonic atom-molecule mixtures

- Resonant fermionic systems led to molecular superfluids, BEC-BCS crossover, strongly interacting superfluid, etc.

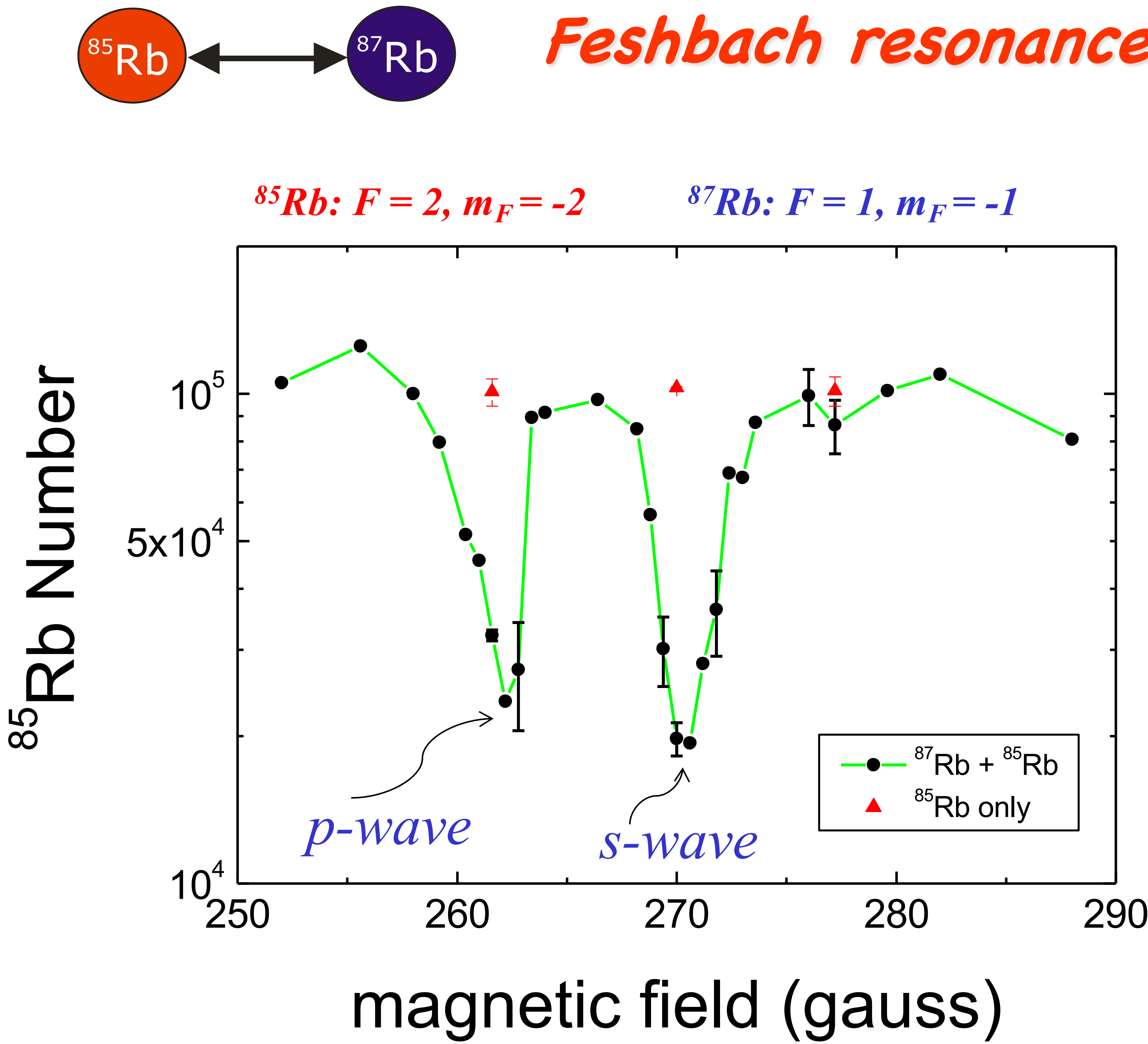
- Allow SF-SF quantum phase transitions (cf. just crossover for fermions)

Rb85-Rb87 Feshbach resonances



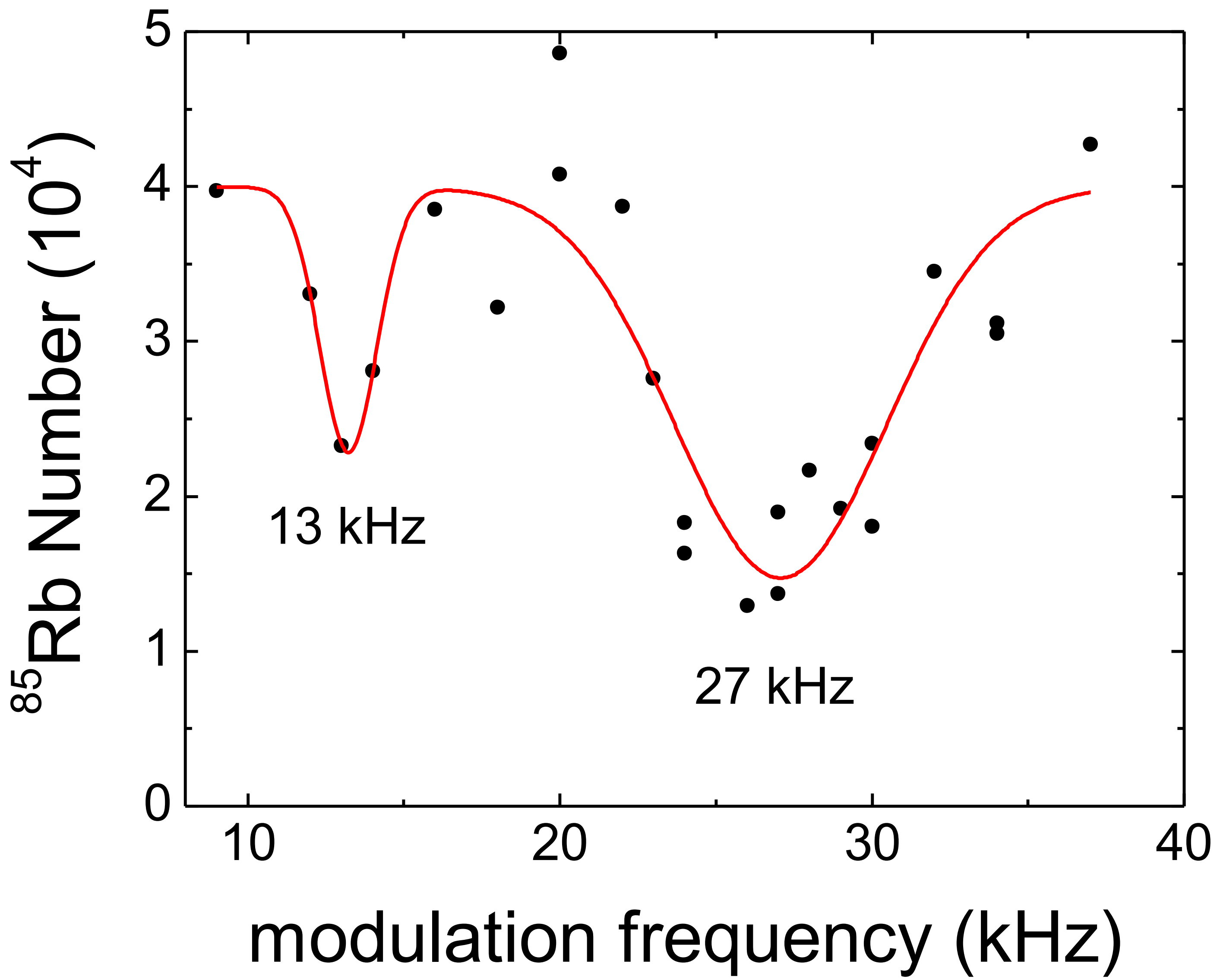
Papp, Pino, Wieman

Feshbach resonances



Papp, Pino, Wieman

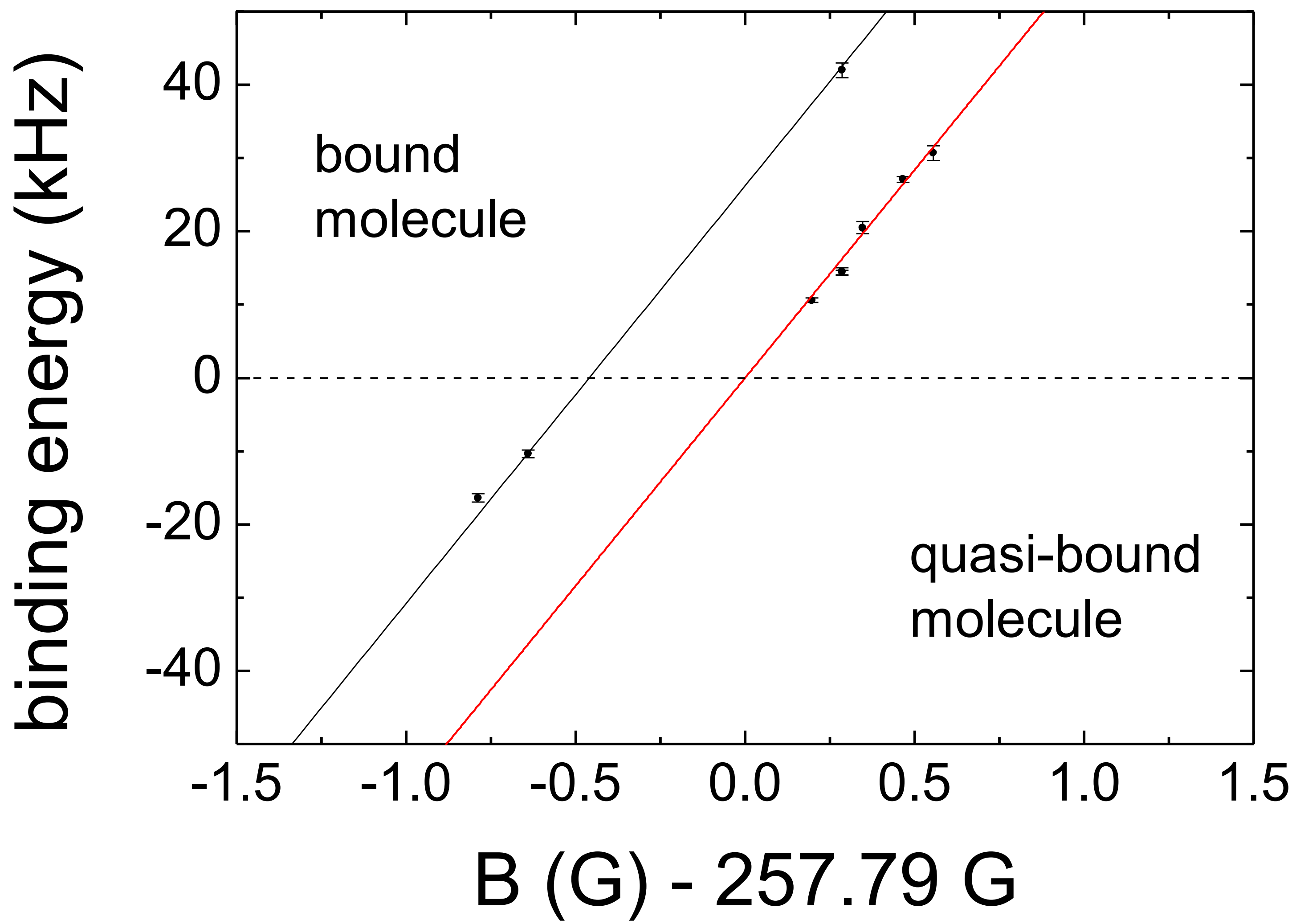
p-wave resonant modulation



Papp, Pino, Wieman

Rb85-Rb87 p-wave molecules

two m_l levels



Papp, Pino, Wieman

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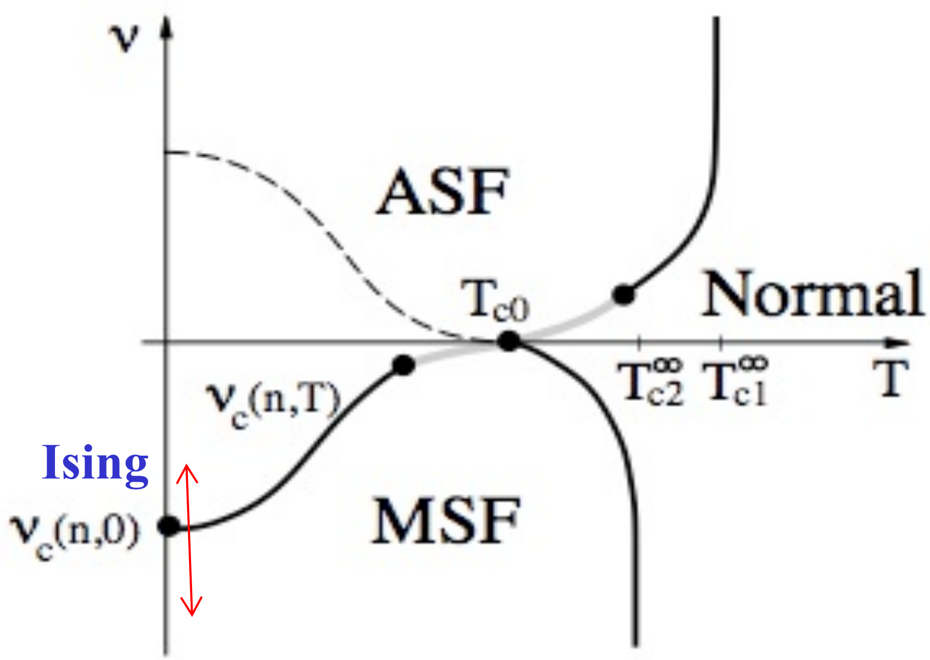
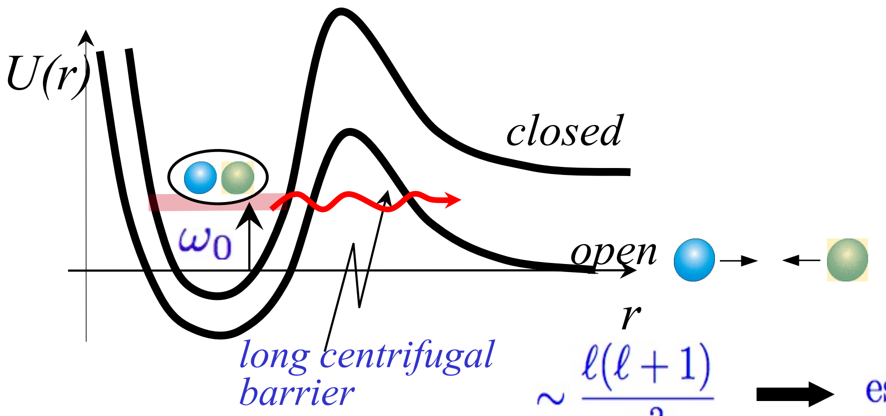
- **Allow SF-SF quantum phase transitions (cf. just crossover for fermions) even for s-wave resonance**

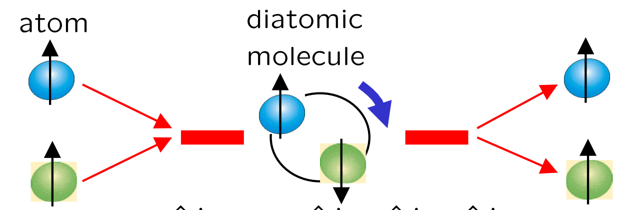


original proposal in continuum:
LR, Park, Weichman, PRL '04
Romans, et al, PRL '04

more recently on lattice:
- *Diehl, et al, 2010*
- *Ejima, et al, 2011 (DMRG)*
- *Bonnes, Wessel, 2011 (QMC)*

P-wave Feshbach resonant scattering



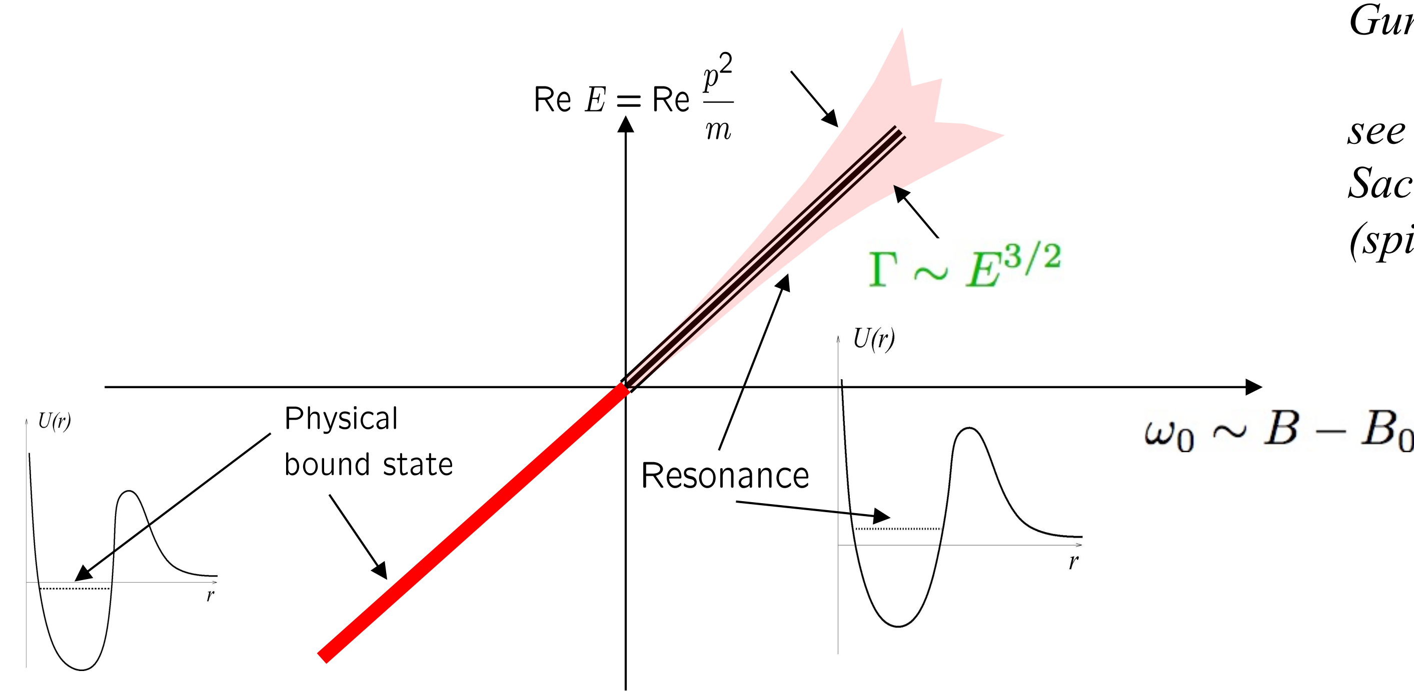


$$\hat{\phi}^\dagger = (\hat{\phi}^\dagger_x, \hat{\phi}^\dagger_y, \hat{\phi}^\dagger_z)$$

$$\sim \frac{\ell(\ell+1)}{r^2} \longrightarrow \text{escape (molecular life) time } \tau \sim \Gamma^{-1} \sim E^{-\frac{3}{2}} \gg E^{-1}, \text{ for } E \to 0$$

$$\boxed{H = \psi^\dagger_\sigma \frac{\hat{p}^2}{2m}\psi_\sigma + \vec{\phi}^\dagger \cdot \left(\frac{\hat{p}^2}{4m} + \nu_0\right)\vec{\phi} - i\alpha\vec{\phi}^\dagger \cdot \hat{\psi}_1 \vec{\nabla}\psi_2 + h.c.}$$

Gurarie, L.R., AOP '09



see also:
Sachdev + Read '91
(spin liquids)

p-wave resonant Bose model

- **two distinguishable open-channel bosonic atoms:** $\hat{\psi}^\dagger_\sigma = \left(\hat{\psi}^\dagger_1, \hat{\psi}^\dagger_2\right)$
- **p-wave closed-channel molecule:** $\hat{\vec{\phi}}^\dagger = (\hat{\phi}^\dagger_x, \hat{\phi}^\dagger_y, \hat{\phi}^\dagger_z)$
- **model:** $H = \boxed{H_a} + \boxed{H_m} + \boxed{H_{am}} + \boxed{H_{FR}}$

two species BEC: $\boxed{H_a = \sum_{\sigma=1,2} \left(\psi^\dagger_\sigma (-\frac{\nabla^2}{2m} - \mu_\sigma)\psi_\sigma + \frac{\lambda_\sigma}{2}\psi^{\dagger 2}_\sigma \psi^2_\sigma \right) + \lambda_{12}\psi^\dagger_1\psi^\dagger_2\psi_2\psi_1}$

spinor=1 condensates: $\boxed{H_m = \vec{\phi}^\dagger(-\frac{\nabla^2}{4m} - \mu_m)\vec{\phi} + \frac{g_1}{2}|\vec{\phi}^\dagger \cdot \vec{\phi}|^2 + \frac{g_2}{2}|\vec{\phi} \cdot \vec{\phi}|^2}$

$\mu_m = \mu_1 + \mu_2 - \nu$

ν *- detuning*

nonresonant interaction: $\boxed{H_{am} = g_{am}\psi^\dagger_\sigma\psi_\sigma\vec{\phi}^\dagger \cdot \vec{\phi}}$

Feshbach resonant interaction:

$$\boxed{H_{FR} = -i\frac{\alpha}{2}\left[\vec{\phi}^\dagger \cdot (\psi_1\vec{\nabla}\psi_2 - \psi_2\vec{\nabla}\psi_1) + h.c.\right]}$$

Landau theory

$$F \approx -\mu_m|\vec{\Phi}|^2 + \frac{g_1}{2}(\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2}|\vec{\Phi} \cdot \vec{\Phi}|^2$$

$$+(\frac{Q^2}{2m} - \mu_\sigma)|\Psi_{Q,\sigma}|^2 + \frac{\lambda}{2}|\Psi_{Q,\sigma}|^4 + \ldots$$

$$+\alpha(\vec{\Phi}^* \cdot \vec{Q})\Psi_{Q,1}\Psi_{-Q,2} + c.c.$$

conserved: $n_1 + n_m$

$n_2 + n_m$

$$\mu_m = \mu_1 + \mu_2 - \nu$$

Landau theory

*large **negative** detuning* $\longrightarrow$ $\mu_\sigma < 0, \quad \mu_m > 0$

$$\boxed{F \approx -\mu_m |\vec{\Phi}|^2 + \frac{g_1}{2}(\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2}|\vec{\Phi} \cdot \vec{\Phi}|^2}$$

$$+\left(\frac{Q^2}{2m} - \mu_\sigma\right)|\Psi_{Q,\sigma}|^2 + \frac{\lambda}{2}|\Psi_{Q,\sigma}|^4 + \ldots$$

$$+\alpha(\vec{\Phi}^* \cdot \vec{Q})\Psi_{Q,1}\Psi_{-Q,2} + c.c.$$

conserved: $n_1 + n_m$

$n_2 + n_m$

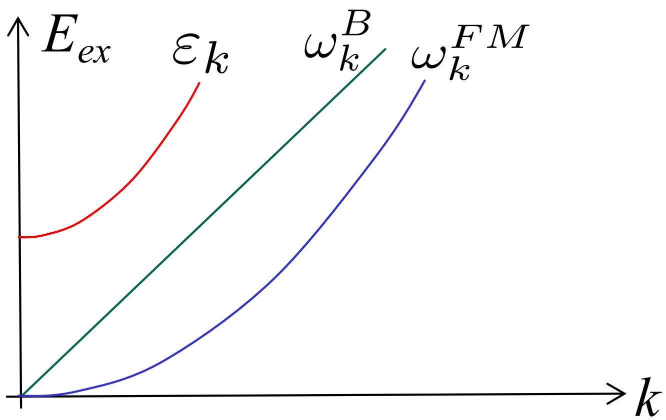
$$\mu_m = \mu_1 + \mu_2 - \nu$$

L=1 molecular superfluid (MSF)

$$\boxed{\vec{\Phi} \neq 0, \quad \Psi_\sigma = 0}$$

large ***negative*** *detuning* $\longrightarrow$ $\mu_\sigma < 0, \quad \mu_m > 0$

$$\boxed{F_{MSF} \approx -\mu_m |\vec{\Phi}|^2 + \frac{g_1}{2}(\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2}|\vec{\Phi} \cdot \vec{\Phi}|^2}$$



g_2

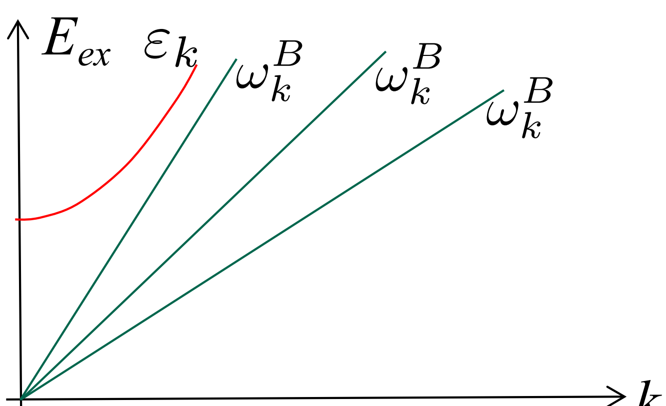
$MSF_{FM}\,(l_z = 1)$

$$\vec{\Phi} = \vec{u} + i\vec{v}$$

0

ν_{c1}

ν



$MSF_{Polar}\,(l_z = 0)$

$$\vec{\Phi} = \vec{u}$$

Ho, Yip, Zhou, Machida, Mukerjee, Demler, et al.

Landau theory

*large **positive** detuning* $\longrightarrow$ $\mu_\sigma > 0, \quad \mu_m < 0$

$$F \approx -\mu_m |\vec{\Phi}|^2 + \frac{g_1}{2}(\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2}|\vec{\Phi} \cdot \vec{\Phi}|^2$$

$$\boxed{+(\frac{Q^2}{2m} - \mu_\sigma)|\Psi_{Q,\sigma}|^2 + \frac{\lambda}{2}|\Psi_{Q,\sigma}|^4 + \ldots}$$

$$+\alpha(\vec{\Phi}^* \cdot \vec{Q})\Psi_{Q,1}\Psi_{-Q,2} + c.c.$$

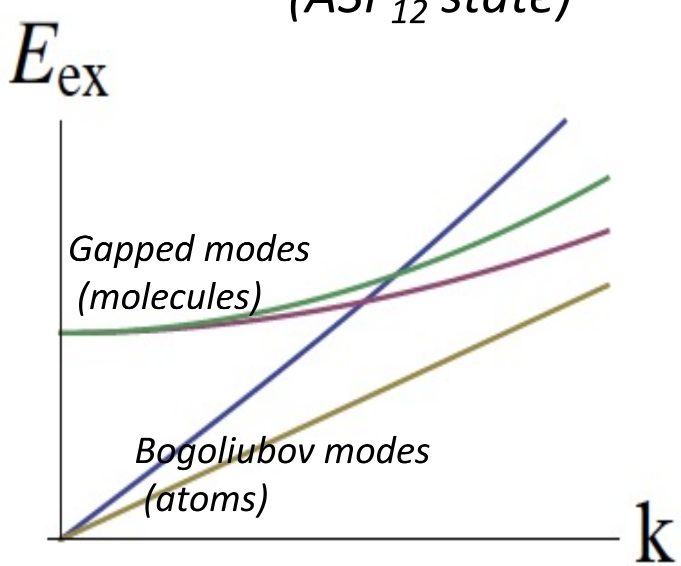
Atomic superfluid (ASF)

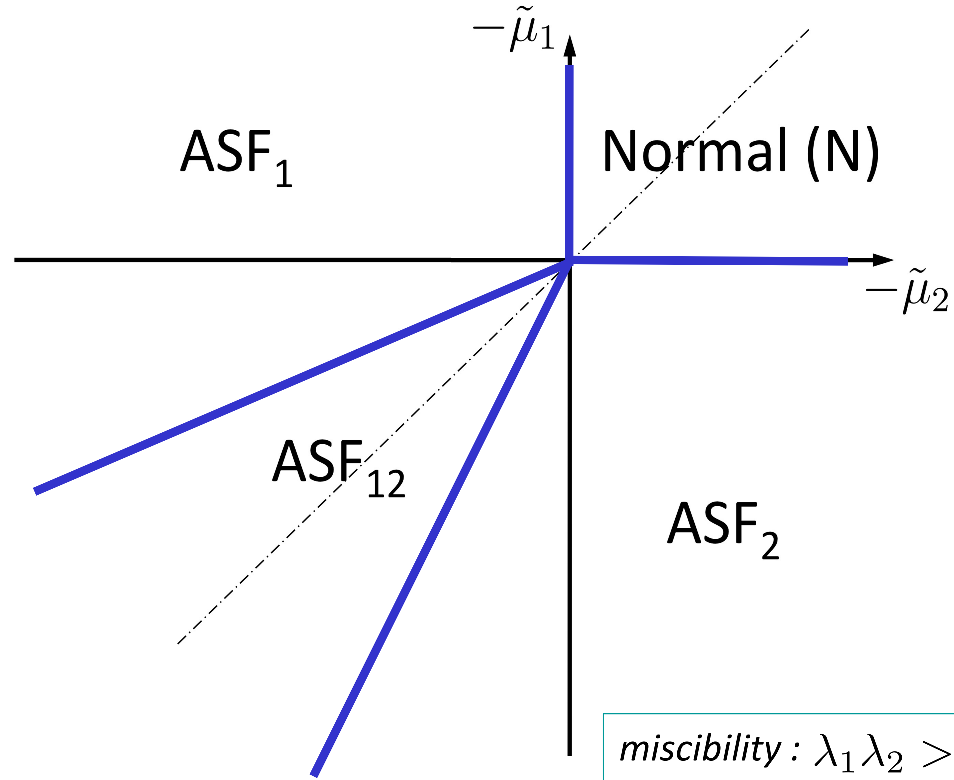
$\boxed{\vec{\Phi} = 0\ , \quad \Psi_\sigma \neq 0}$

large ***positive*** *detuning* $\longrightarrow$ $\mu_\sigma > 0, \quad \mu_m < 0$

$$\boxed{F_{ASF} \approx -\mu_\sigma |\Psi_\sigma|^2 + \frac{\lambda_\sigma}{2}|\Psi_\sigma|^4 + \frac{\lambda_{12}}{2}|\Psi_1|^2|\Psi_2|^2}$$

(ASF_{12} state)





miscibility : $\lambda_1 \lambda_2 > \lambda_{12}^2$

Esry, Greene '97

Landau theory

intermediate detuning $\nu_{c1} < \nu < \nu_{c2} \longrightarrow \mu_\sigma < 0, \quad \mu_m > 0$

$$F \approx -\mu_m |\vec{\Phi}|^2 + \frac{g_1}{2}(\vec{\Phi}^* \cdot \vec{\Phi})^2 + \frac{g_2}{2}|\vec{\Phi} \cdot \vec{\Phi}|^2$$

$$+(\frac{Q^2}{2m} - \mu_\sigma)|\Psi_{Q,\sigma}|^2 + \frac{\lambda}{2}|\Psi_{Q,\sigma}|^4 + \ldots$$

$$\boxed{+\alpha(\vec{\Phi}^* \cdot \vec{Q})\Psi_{Q,1}\Psi_{-Q,2} + c.c.}$$

Atomic-molecular superfluid (AMSF)

$\boxed{\vec{\Phi} \neq 0, \quad \Psi_{Q,\sigma} \neq 0}$

intermediate detuning $\nu_{c1} < \nu < \nu_{c2} \longrightarrow \mu_\sigma < 0, \quad \mu_m > 0$

$$F_{AMSF} \approx \left(\frac{Q^2}{2m} - \mu_\sigma\right)|\Psi_{Q,\sigma}|^2 + \alpha(\vec{\Phi}^* \cdot \vec{Q})\Psi_{Q,1}\Psi_{-Q,2} + \ldots$$

$$\approx -\frac{\mu_+}{2}|\Psi_Q^{(+)}|^2 - \frac{\mu_-}{2}|\Psi_Q^{(-)}|^2 + \ldots$$

$$\mu_\pm = -\left(\frac{Q^2}{2m} - \mu\right) \pm \alpha|\vec{\Phi} \cdot \vec{Q}|$$

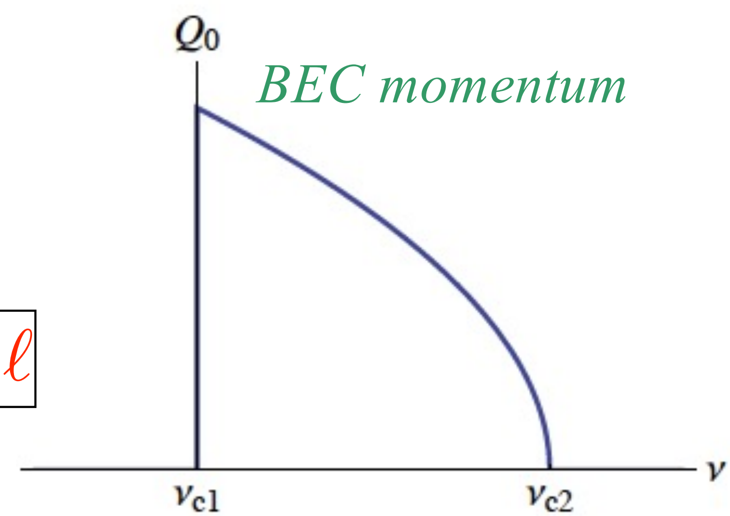
$\longrightarrow$ *transition to AMSF at* ν_{c1} $(\mu_+ = 0 > \mu_-)$

$$\Psi_\pm = \Psi_{Q,1} \pm e^{i\varphi}\Psi^*_{-Q,2}$$

<u>physics of Q ≠ 0:</u> $\frac{Q^2}{2m} \sim \alpha\vec{Q} \cdot \vec{\Phi}$

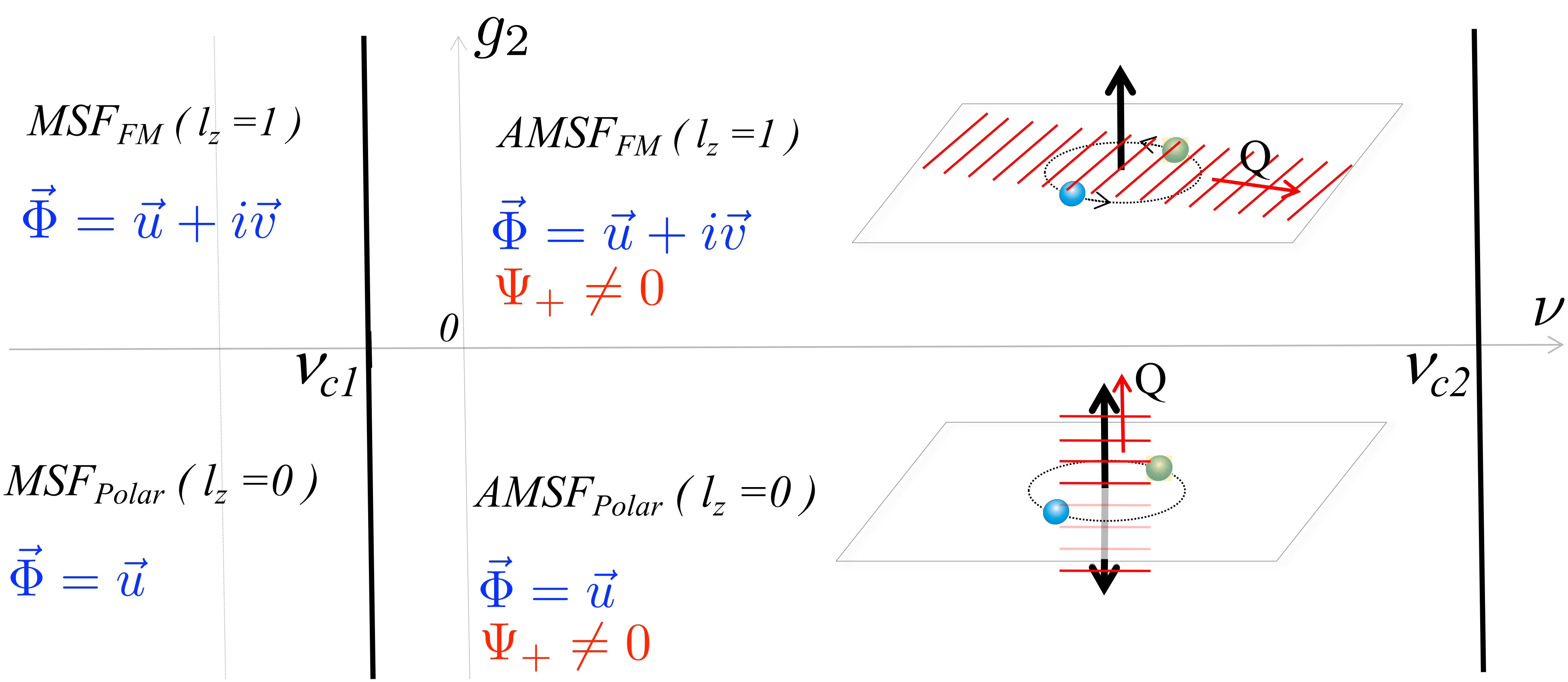
$\longrightarrow$ $\boxed{Q \approx \alpha m\sqrt{n_m} \sim \sqrt{\gamma_p \ell n_m} \lesssim \sqrt{\gamma_p}/\ell}$

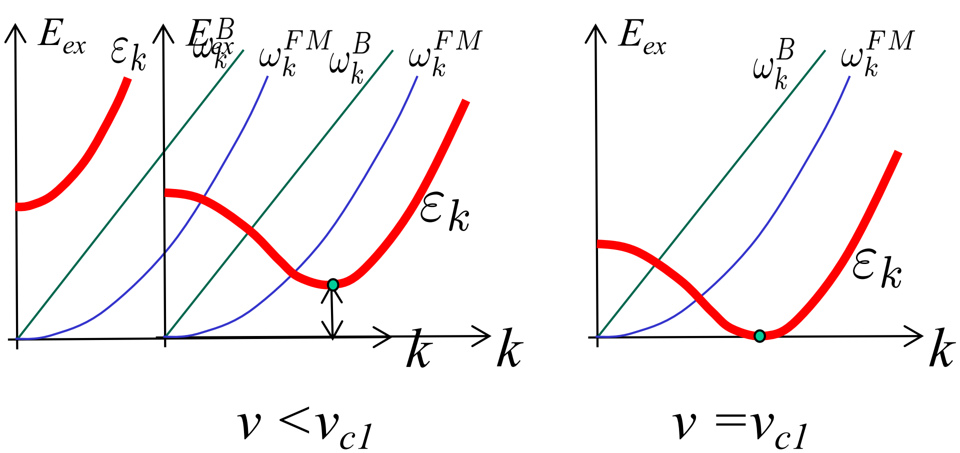
(tunable with ν)



$\nu_{c1} = -(m\alpha^2 + g)n_m \qquad \nu_{c2} = (2\lambda - g_{am})n_a$

Near MSF-AMSF transition



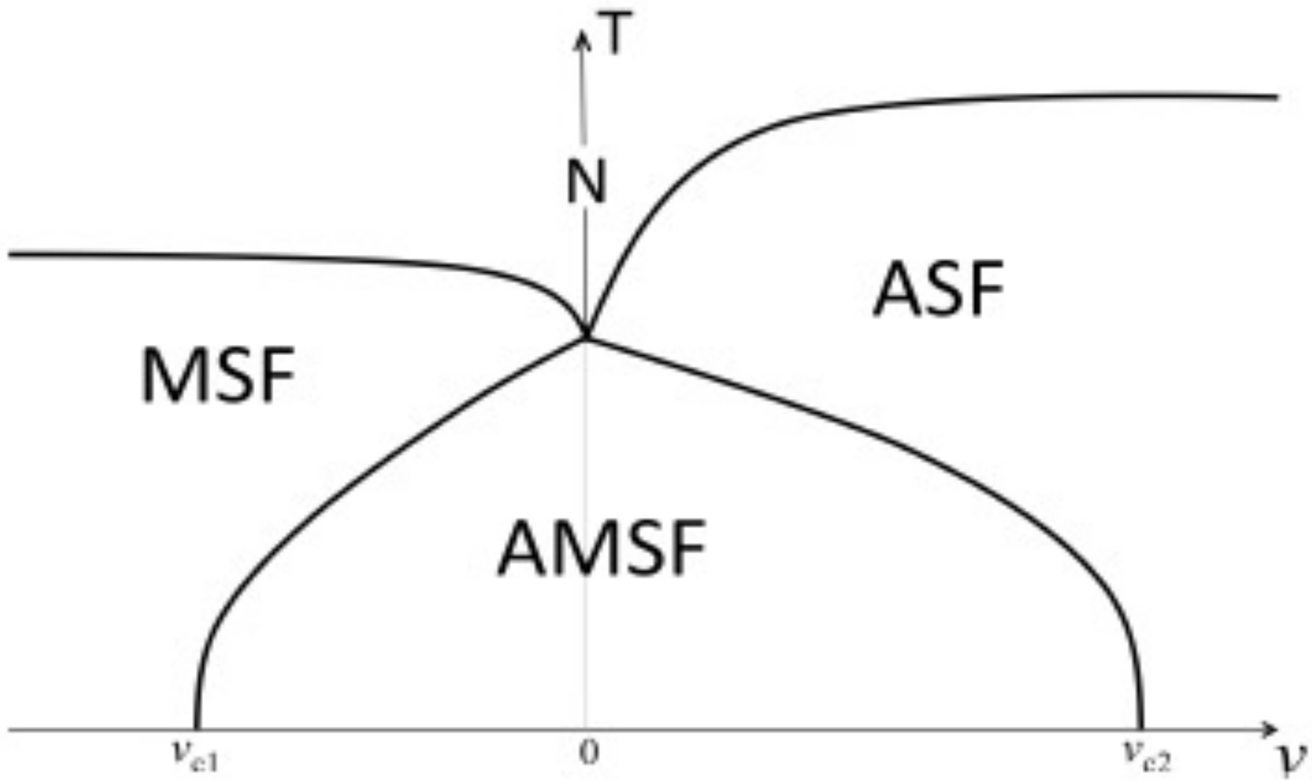


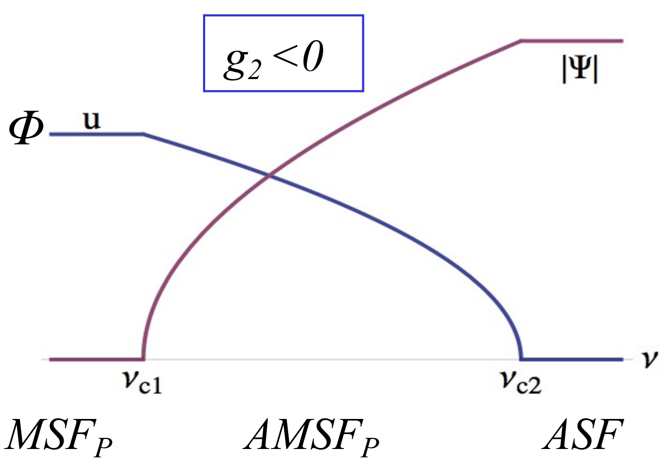
$$H_a^{MSF} = \begin{pmatrix} \frac{k^2}{2m} - \mu & \alpha\vec{\Phi}\cdot\vec{k} \\ \alpha\vec{\Phi}^*\cdot\vec{k} & \frac{k^2}{2m} - \mu \end{pmatrix}$$

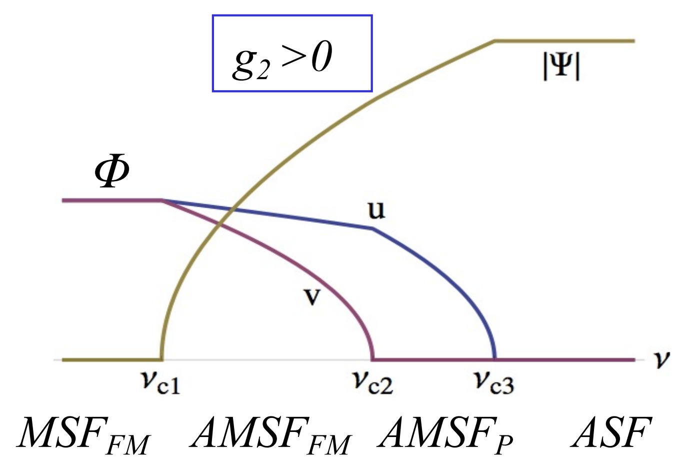
$$\begin{aligned} E_{ex} &= \sqrt{\mu_+\mu_-} \\ &= \sqrt{\left(\frac{k^2}{2m} - \mu\right)^2 - |\alpha\vec{\Phi}\cdot\vec{k}|^2} \end{aligned}$$

→ co-linear (FF) for MSF_{Polar}

Global phase diagram

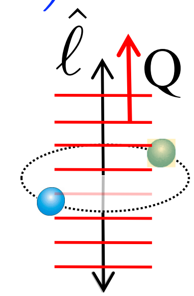






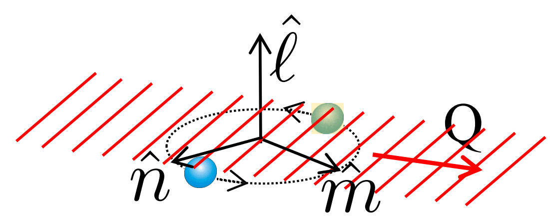
Symmetries, order parameters, Goldstone modes

- $\mathrm{AMSF}_{\mathrm{Polar}}$
  - *OP:* $\vec{\Phi} = e^{i\phi}\hat{\ell}, \quad \Psi = \sum_Q \Psi_Q e^{i\theta_Q + i\vec{Q}\cdot\vec{r}} \begin{pmatrix} e^{i\phi/2} \\ e^{-i\phi/2} \end{pmatrix} \approx e^{i\theta} \begin{pmatrix} e^{i\phi/2} \\ e^{-i\phi/2} \end{pmatrix} \cos(\vec{Q}\cdot\vec{r} + Qu)$
  - *breaks:* $U_N(1) \times U_{\Delta N}(1) \times SO(3) \times T_{\varepsilon\hat{\ell}}$
  - *GM*: θ_1 , θ_2 , φ , $\hat{\ell}$ → Higgs' ed: θ_c , θ_s



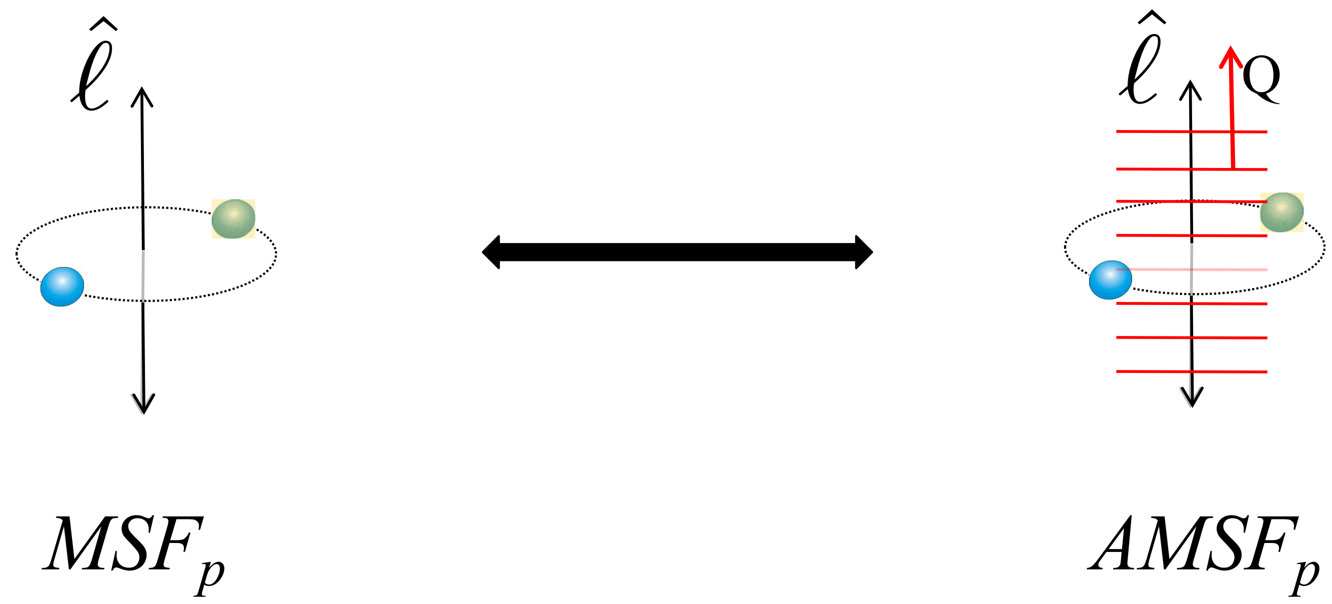
$$\mathcal{L}_p = \frac{n_c}{2}(\partial_\mu \theta_c)^2 + \frac{\chi_s}{2}(\partial_\tau \theta_s)^2 + \frac{n_s}{2}(\partial_{||}\theta_s)^2 + \frac{K}{2}(\nabla_\perp^2 \theta_s)^2$$

- $\mathrm{AMSF}_{\mathrm{FM}}$
  - *OP:* $\vec{\Phi} = \hat{n} + i\hat{m}, \quad \Psi = \sum_Q \Psi_Q e^{i\theta_Q + i\vec{Q}\cdot\vec{r}} \begin{pmatrix} e^{i\phi/2} \\ e^{-i\phi/2} \end{pmatrix} \approx e^{i\theta} \begin{pmatrix} e^{i\phi/2} \\ e^{-i\phi/2} \end{pmatrix} \cos(\vec{Q}\cdot\vec{r} + Qu)$
  - *breaks:* $U_N(1) \times U_{\Delta N}(1) \times SO(3) \times T_{\varepsilon\hat{\ell}} \times \Theta$
  - *GM*: θ_1 , θ_2 , φ , $\hat{n}$, $\hat{m}$ → Higgs' ed: θ_c , θ_s , γ



$$\mathcal{L}_{fm} = \frac{n_c}{2}(\partial_\mu \theta_c)^2 + \frac{\chi_s}{2}(\partial_\tau \theta_s)^2 + \frac{n_s}{2}(\partial_{||}\theta_s)^2 + \frac{K}{2}(\nabla_\perp^2 \theta_s)^2 + i\kappa \partial_y \theta_s \partial_\tau \gamma + \frac{J}{2}(\nabla\gamma)^2$$

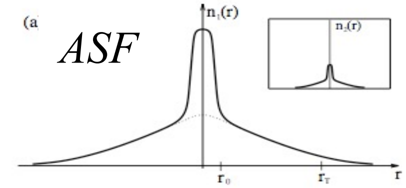
MSF - AMSF transition

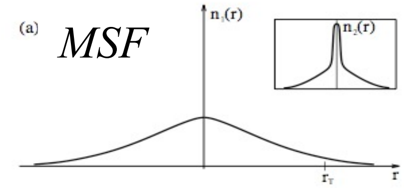


Abelian Higgs (quantum de Gennes) model:

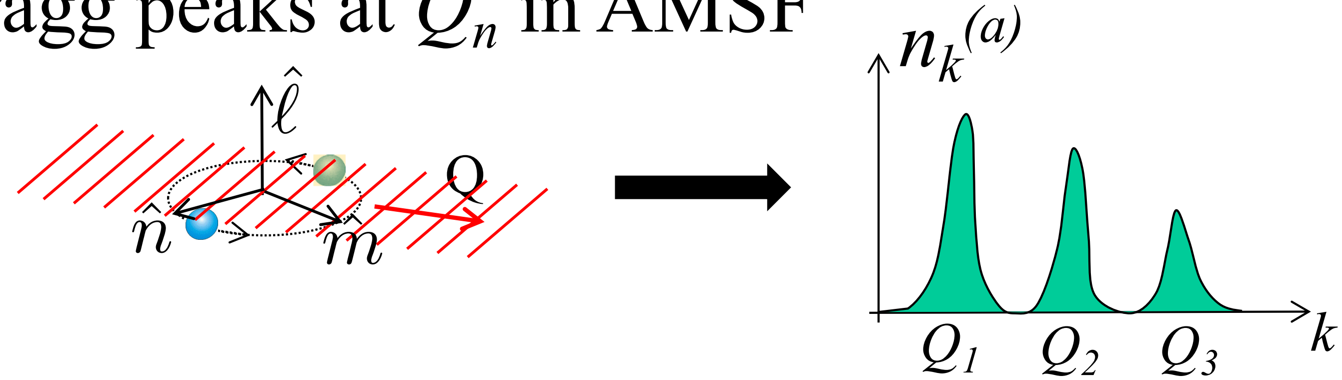
$$\mathcal{L}_p = |\partial_\tau \psi|^2 + \frac{1}{2m}|\left(i\nabla - Q\delta\hat{\ell}\right)\psi|^2 + \epsilon_+|\psi|^2 + \frac{\lambda}{2}|\psi|^4 + \frac{1}{2g_\ell}(\partial_\mu \hat{\ell})^2 + \frac{1}{2g_\varphi}(\partial_\mu \varphi)^2$$

Experimental signatures

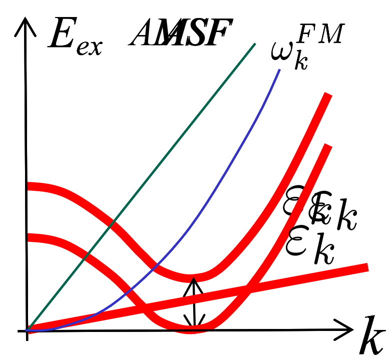




- momentum distributions $n_k^{(a)}$, $n_k^{(m)}$
- Bragg peaks at Q_n in AMSF

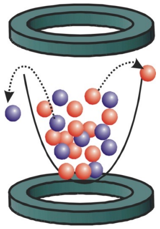


- thermodynamic singularities at transitions
- excitation spectra (phonons, Bogoluibov and spin-wave modes)
  via Bragg spectroscopy

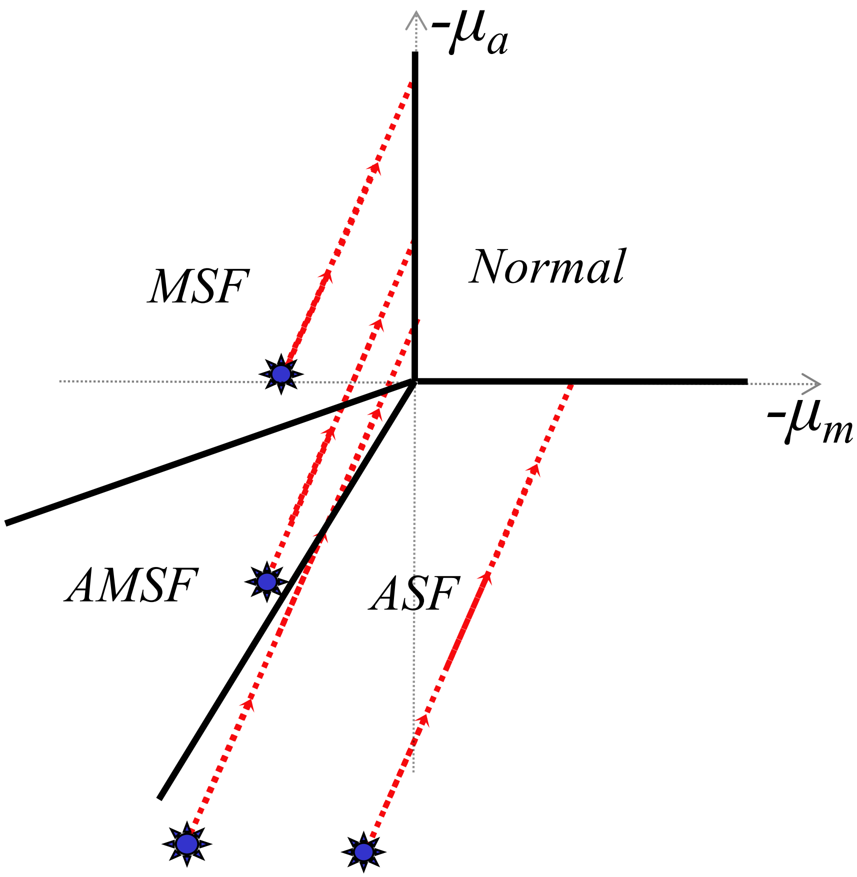


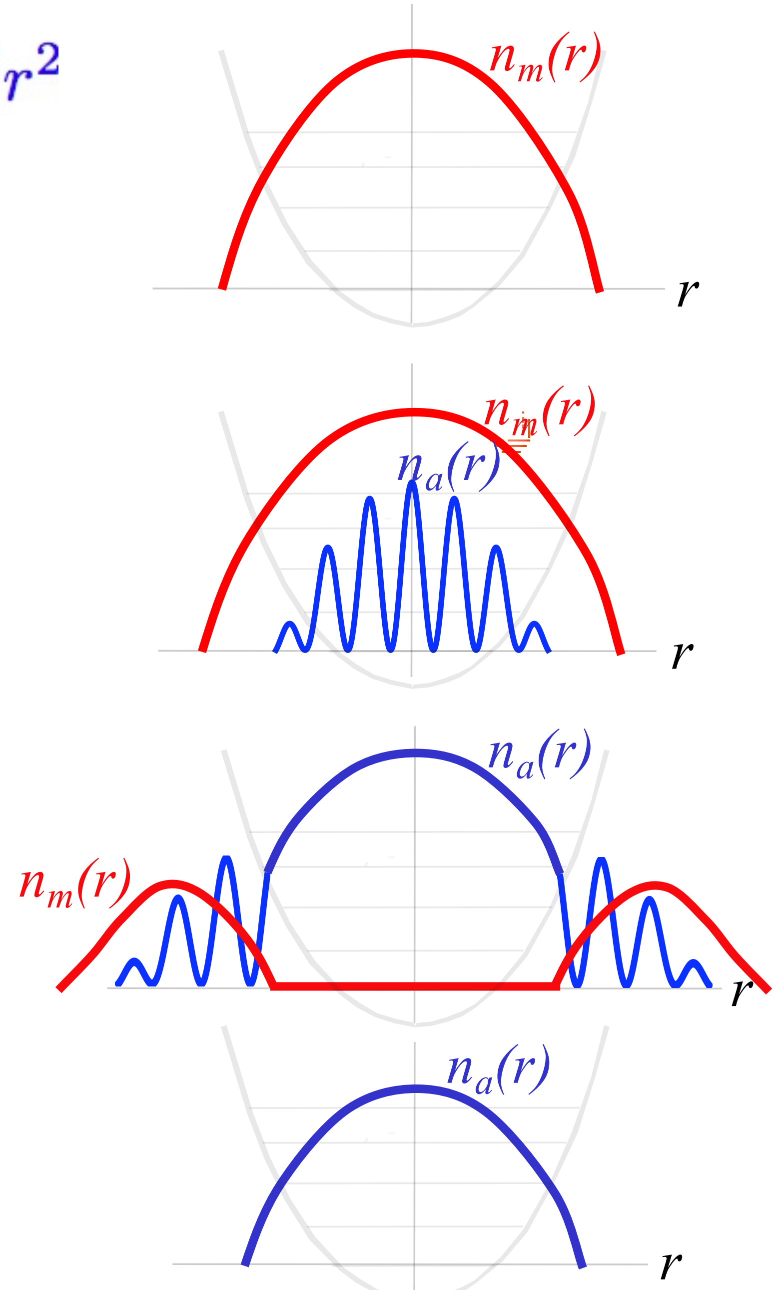
- novel vortices and dislocations

Trapped profiles via LDA

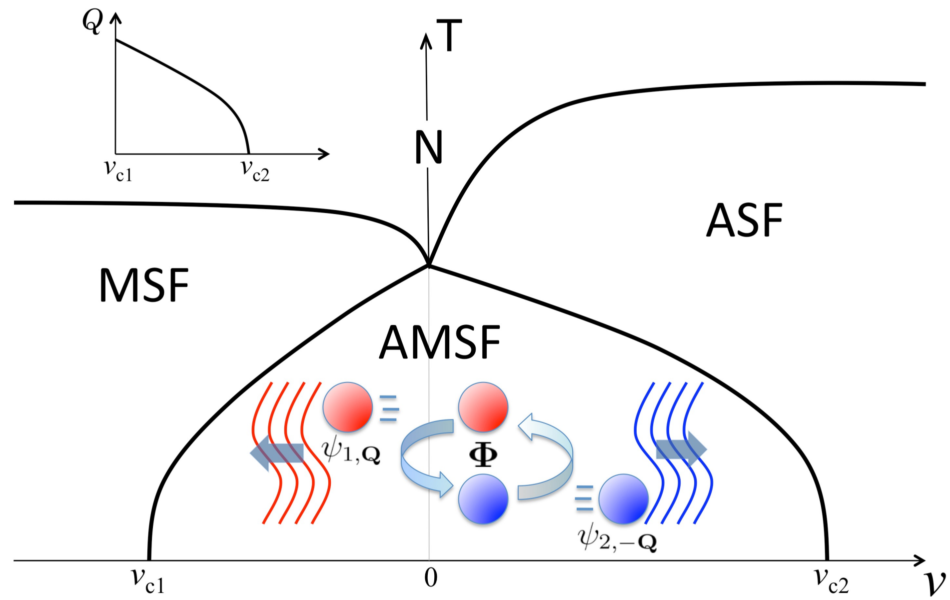


$$\mu \to \mu_{\text{eff}}(r) = \mu - \frac{1}{2} m\omega^2 r^2$$





Summary and conclusions

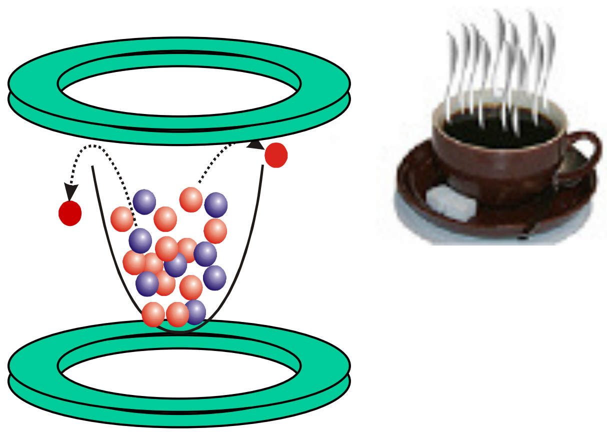


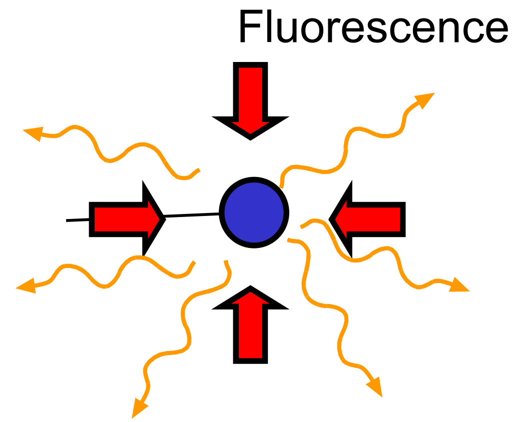
- resonantly interacting Bose gas:
  - *atomic and molecular superfluids*
  - *atomic supersolid, tunable Q(v)*
  - *quantum, thermal transitions*
  - *topological defects…*

- questions:
  - *nature of the AMSF solidity: vortex lattice? 3d crystal?*
  - *stability? expect short lifetime due to 3-body instabilities*
  - …

- fixes:
  - *optical lattice?*
  - *avoid immediate vicinity of FBR?*

Laser cooling, trapping and imaging





1997 *Chu, Cohen-Tannoudji, Phillips*

Evaporative cooling

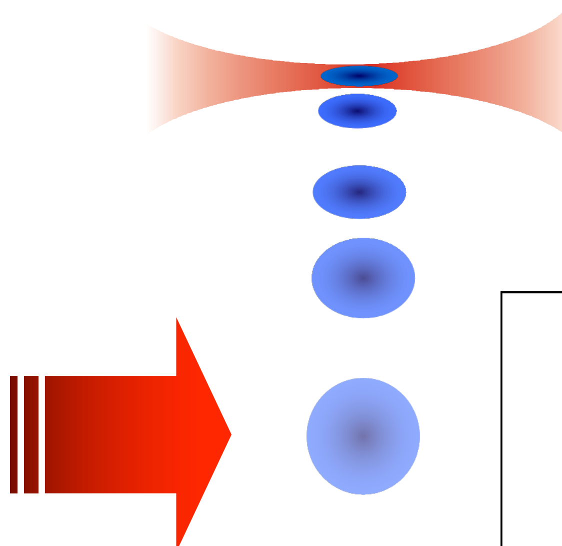
1 mK to 1 μK
~$10^8 \rightarrow 10^6$ atoms

Laser (Doppler) cooling

300 K to 1 mK
~10^9 atoms

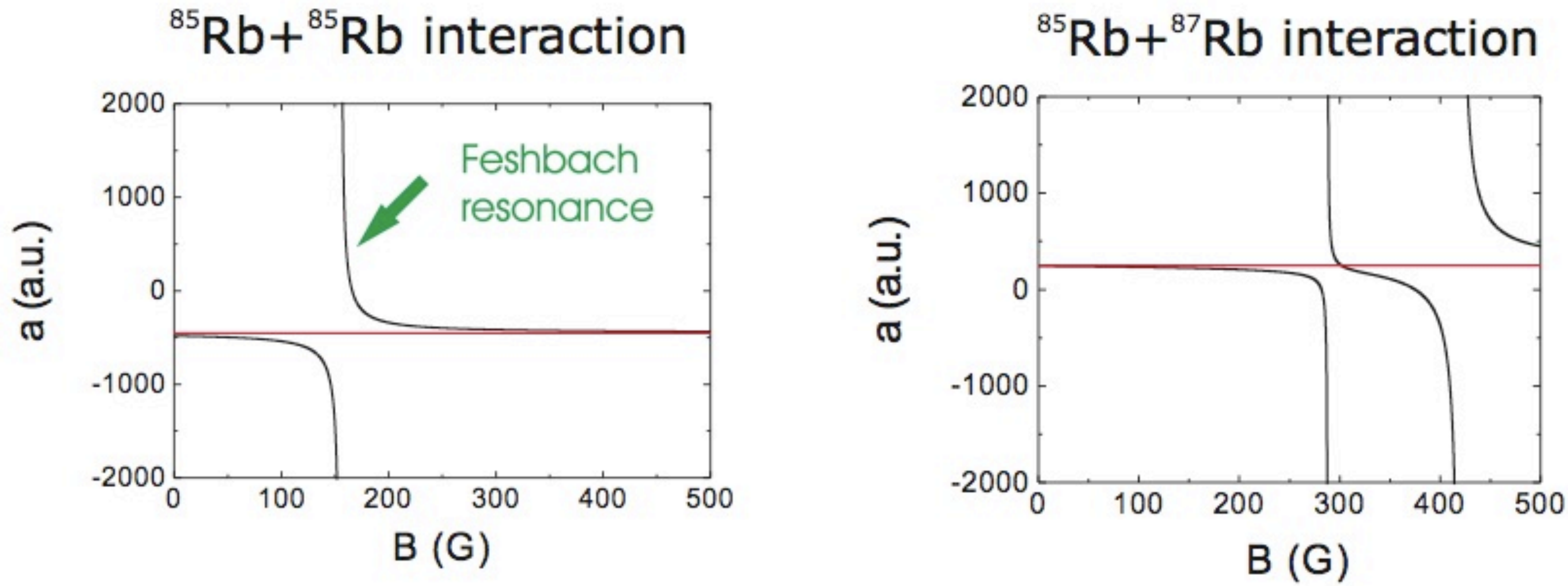
T

probing w/ resonant laser



shadow image

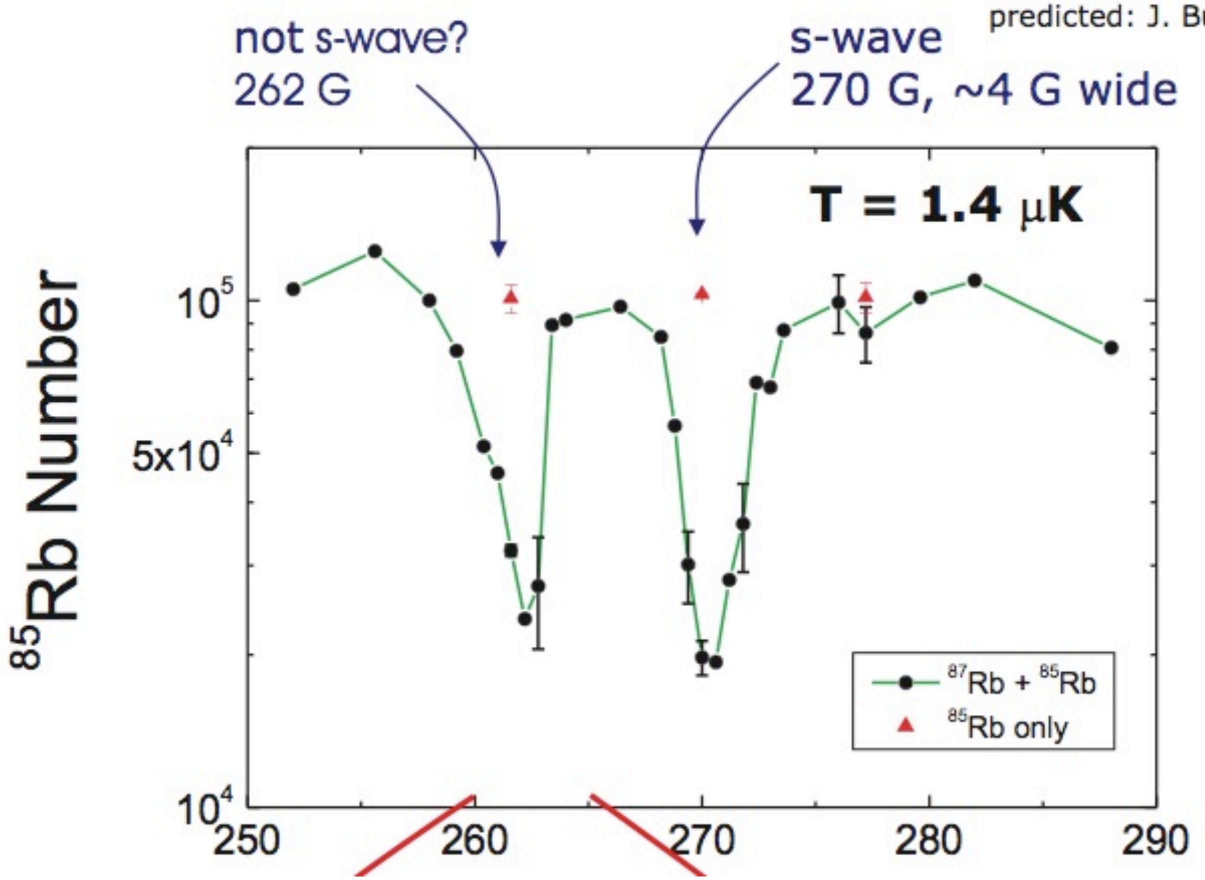
$$n(r,t) \approx \tilde{n}(\hbar k = mr/t)$$

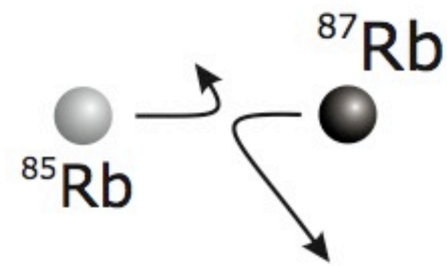


^{85}Rb+^{85}Rb interaction
2000
1000
0
-1000
-2000
a (a.u.)
0
100
200
300
400
500
B (G)
Feshbach
resonance
^{85}Rb+^{87}Rb interaction
2000
1000
0
-1000
-2000
a (a.u.)
0
100
200
300
400
500
B (G)

Interspecies Feshbach resonances

predicted: J. Burke *et al.* PRL **80**, 2097 (1998)

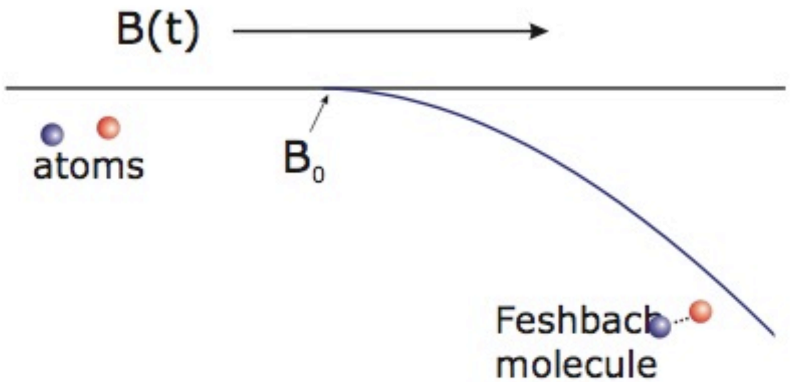


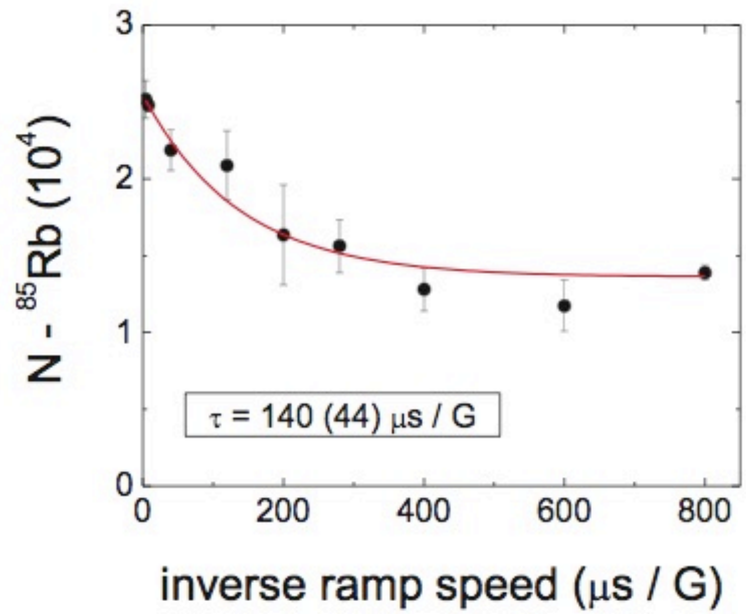


loss occurs during
1 second hold

Production of heteronuclear molecules

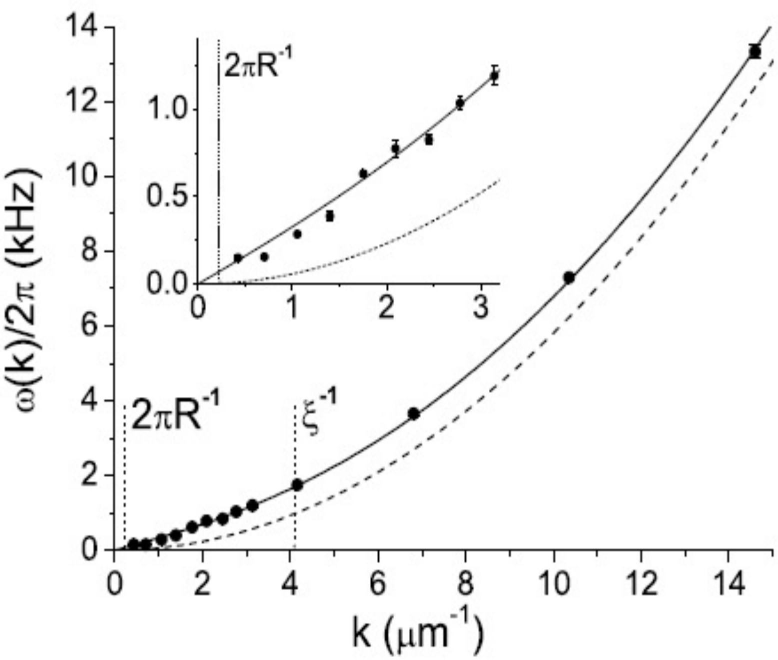
Use adiabatic magnetic field ramps to produce molecules





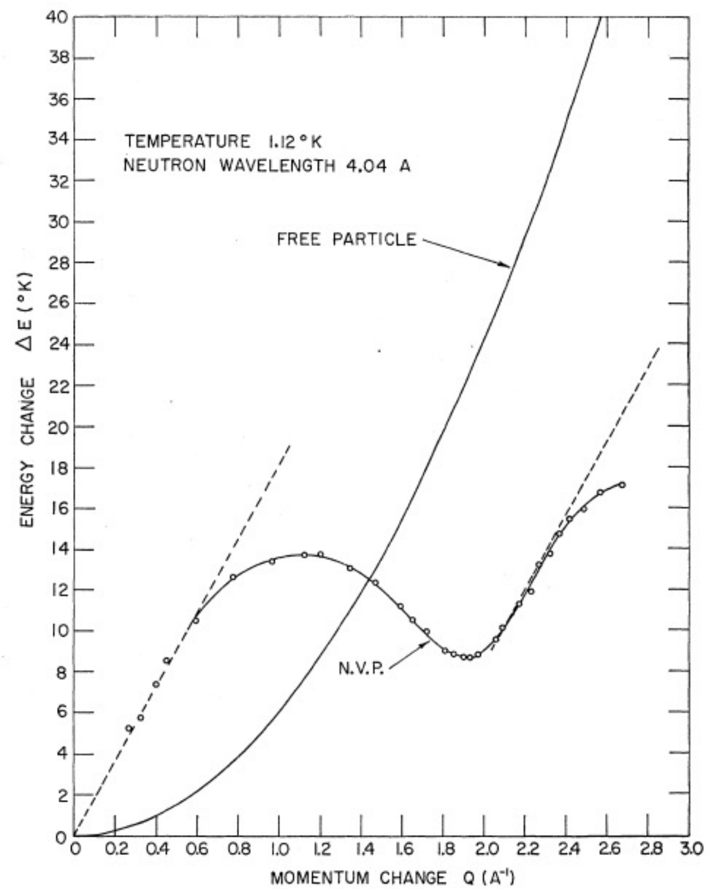
BEC excitation spectrum

BEC



Steinhauer et al., *PRL* **88**, 2002

Helium



Photoassociation (Bose)

Molecules in a Bose-Einstein Condensate

Roahn Wynar, R. S. Freeland, D. J. Han, C. Ryu, D. J. Heinzen*

11 FEBRUARY 2000 VOL 287 SCIENCE