VORTICES AND QUASIPARTICLES IN SUPERCONDUCTORS : A MODERN PRIMER

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Johns Hopkins

PREREQUISITES:

- GIRVIN I & II
- AMBEGAOKAR I & II
- DORSEY I & II
- SAULS I - IV

RELATED LECTURES:

- SUDBØ I & II
- FRANZ I

(Also, see Balents I & II)

BOULDER 2000

NSF SUMMER SCHOOL ON
CONDENSED MATTER AND MATERIALS [illegible]

VORTICES & QUASIPARTICLES IN SUPERCONDUCTORS : A MODERN PRIMER

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A. Melikyan
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SELECTED REFERENCES:

- M.E. PESKIN, ANN. PHYS. 113, 122 ('78)
- P.R. THOMAS AND M. STONE, NUCL. PHYS. B144, 513 ('78)

- S.M. GIRVIN: "DUALITY IN PERSPECTIVE", SCIENCE 274, 524 ('96)
- C. DASGUPTA AND B.I. HALPERIN, PRL 47, 1556 ('81)
- N.D. ANTUNES et al., PRL 80, 908 ('98)
- → I. HERBUT AND ZBT, PRL 76, 4588 ('96); PRL 78, 980 ('97)
- → ZBT, PR B51, 16204 ('95)
- → ZBT, PRB 59, 6449 ('99)
- → A.K. NGUYEN AND A. SUDBØ, EUROPHYS. LETT. 46, 780 ('99)
- → A.K. NGUYEN AND A. SUDBØ, PRB 60, 15307 ('99)

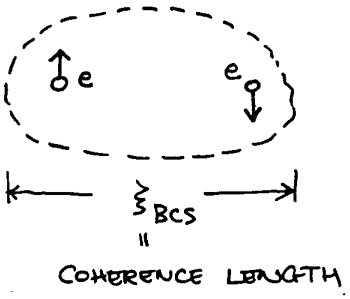
- → M. FRANZ AND ZBT, PRL 84, 554 ('00)
- O. VAFEK, A. MELIKYAN, M. FRANZ AND ZBT, in prep.

- ZT AND P. SACRAMENTO, PRL 80, 1521 ('98)

(BdG eqs. with Landau levels)

● A PRIMER ON SUPERCONDUCTIVITY

● IN SUPERCONDUCTORS ELECTRONS FORM PAIRS:



THESE "COOPER PAIRS" BEHAVE LIKE BOSONS AND ARE DESCRIBED BY A <u>MACROSCOPIC</u> QUANTUM MECHANICAL WAVEFUNCTION: $\underline{\psi(\vec{r})}$

● FREE ENERGY OF A SUPERCONDUCTOR:

$$\boxed{F_{sc} = \int d^3r \left[\alpha|\psi|^2 + \frac{\hbar^2}{2m^*}\left|\left(\frac{\vec{\nabla}}{i} - \frac{e^*}{\hbar c}\vec{A}\right)\psi\right|^2 + \frac{\beta}{2}|\psi|^4\right] + \frac{1}{8\pi}\int d^3r\,(\vec{\nabla}\times\vec{A})^2}$$

(Ginzburg-Landau)

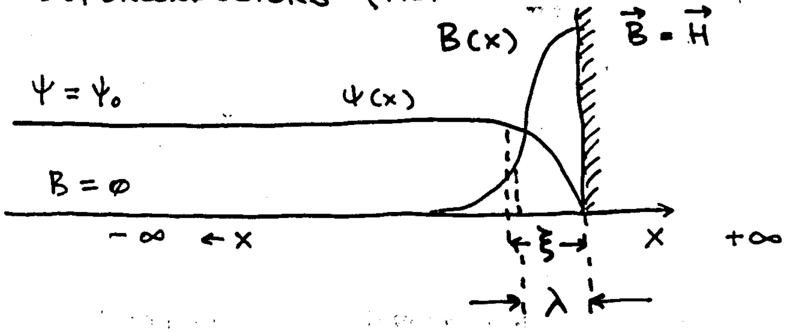
$e^* = 2e$

$\vec{A}$ - EM vector potential: $\vec{B}(\vec{r}) = \vec{\nabla}\times\vec{A}(\vec{r})$

- IN SUPERCONDUCTING STATE $\underline{\langle\psi(\vec{r})\rangle \neq 0}$
- IN NORMAL STATE $\underline{\langle\psi(\vec{r})\rangle = 0}$

○ SUPERCONDUCTORS AND MAGNETIC FIELD

● TYPE-I SUPERCONDUCTORS (MEISSNER EFFECT)



ξ - superconducting coherence length

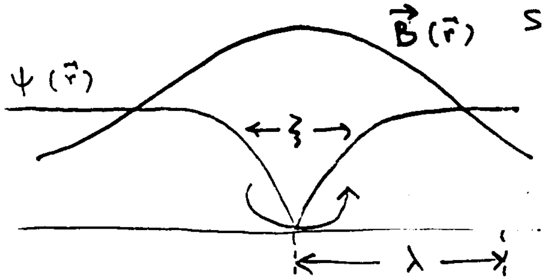
λ - magnetic field penetration depth

$\varkappa \equiv \frac{\lambda}{\xi}$ - Ginzburg parameter $\boxed{\varkappa < \frac{1}{\sqrt{2}}}$

$\Longrightarrow$ * <u>WHAT IF $\varkappa > \frac{1}{\sqrt{2}}$?</u> HTS ($\underline{\varkappa \sim 10^2} >> \frac{1}{\sqrt{2}}$)

$\vec{B} \neq 0$ IN THE INTERIOR

● TYPE-II SUPERCONDUCTO



ABRIKOSOV ~'52

GOR'KOV ~'58

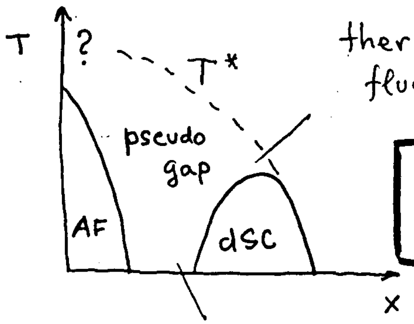
$\Downarrow$

<u>SUPERCONDUCTING VORTEX</u>

● BEYOND MEAN-FIELD THEORY

⇓

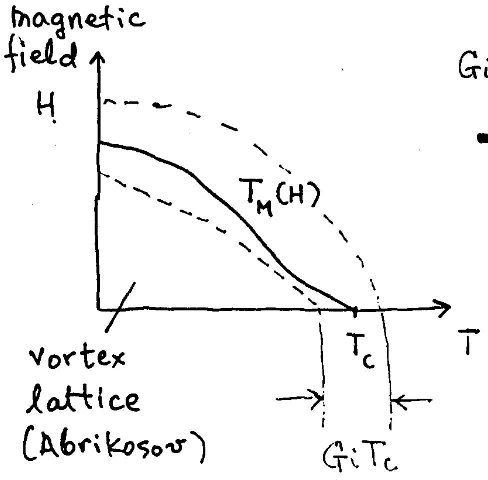
● STRONG (CRITICAL) FLUCTUATIONS IN HIGH TEMPERATURE SUPERCONDUCTORS



thermal critical fluctuations (bosons)

$$\boxed{\text{pseudogap "phase"} = \begin{matrix} \langle \psi^{+}\psi \rangle \neq 0 \\ \langle \psi \rangle = 0 \end{matrix}}$$

quantum critical fluctuations (bosons ⊕ **fermions**)



Gi – GINZBURG NUMBER

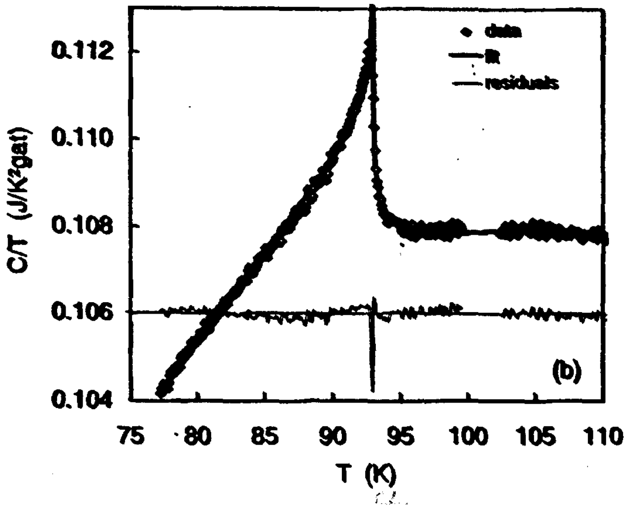
→ $G_i \sim 10^{-2}$ in HTS

(10^{-5} in CSC)

$$G_i \sim \frac{1}{(k_F \xi_{BCS})^{2(4)}}$$

Specific heat of YBCO

(Helium fit)

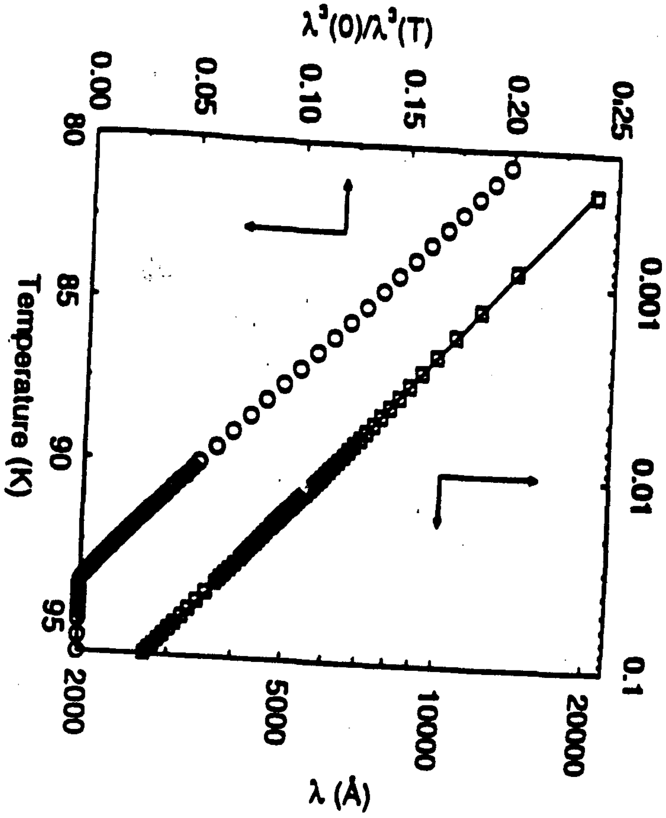


A. Junod et al. ('99)

LT 22, Helsinki

Magnetic field penetration depth of YBCO

S. Kamal et al.



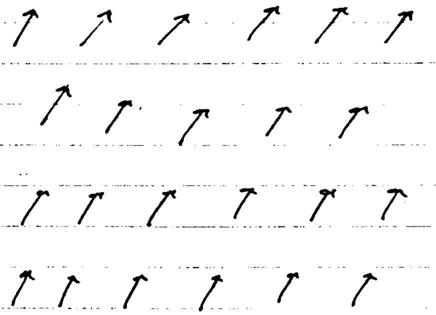
● CRITICAL (FLUCTUATION) BEHAVIOR IN HTS

STEP-BY-STEP PROGRAM:

- ✓ ● VORTICES ARE "ELEMENTARY" OBJECTS (NOT COOPER PAIRS) (TC: TOPOLOGICAL CORRECTNESS)
- ✓ ● CONSTRUCT THEORY OF INTERACTING VORTEX LOOPS (3D) AND LINES (FOR H≠0)
- ✓ ● SOLVE THIS THEORY => HTS PHENOMENOLOGY IN REGION OF THERMAL CRITICAL FLUCTUATIONS
- ✓ ● HTS ARE d-WAVE! QUANTUM CRITICAL THEORY SHOULD CONTAIN BOTH VORTICES AND LOW LYING QUASIPARTICLES
- ? ● CONSTRUCT QUANTUM THEORY OF VORTEX LOOPS INTERACTING WITH d-WAVE QUASIPARTICLES
- ? ● SOLVE THIS THEORY => HTS PHENOMENOLOGY IN REGION OF QUANTUM CRITICAL FLUCTUATIONS

AN ORDERED SUPERCONDUCTOR

girvin I



$\Psi(\bar r)$ Cooper pair wavefunction has **macroscopic** phase coherence.

● SUPERCONDUCTING STATE AND HOW TO GET RID OF IT

"Cooper pair" field $\psi^+(\bar r) \leftrightarrow c^+_\uparrow(\bar r)\, c^+_\downarrow(\bar r)$

$\lim_{|\vec r|\to\infty} \langle \psi(0)\, \psi^+(\bar r)\rangle \rightarrow |\psi|^2$; ψ - order parameter

ODLRO - off-diagonal long-range order

$\Downarrow$

breaking of U(1) symmetry

$\hat\varphi \Longleftrightarrow \hat N = \frac{\hbar}{i}\frac{\partial}{\partial\hat\varphi}$ (P.W. Anderson, "Concepts in Solids")

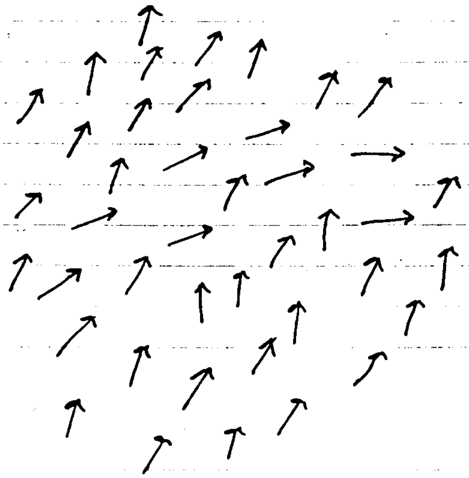
✱ Nature of fluctuations in $\underline{\psi(\bar r) = |\psi(\bar r)| e^{i\varphi}}$

- Amplitude fluctuations $|\psi(\bar r)|$: "**frozen**" in HTS
- Longitudinal **phase** fluctuations : "**inert**"
- Transverse **phase** fluctuations :

VORTICES <=> TOPOLOGICAL DEFECTS OF U(1) BROKEN SYMMETRY

● LONGITUDINAL PHASE FLUCTUATIONS

("spin-waves")



$\nabla \times \nabla \varphi(\vec{r}) = 0$

INEFFICIENT IN DISORDERING

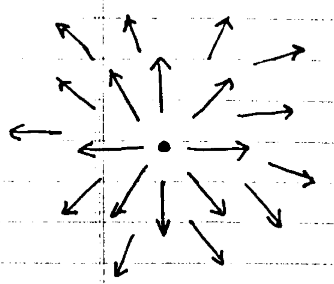
$\langle \psi(0)\, \psi^{+}(\vec{r}) \rangle$

THE PHASE ORDER REMAINS UP TO

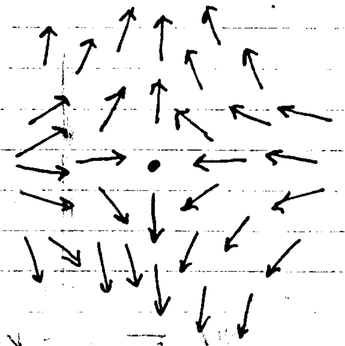
$T \cong \infty$

● TRANSVERSE PHASE FLUCTUATIONS

(Vortices)



VORTEX (+1)



ANTIVORTEX (−1)

SUPERCURRENT $\sim |\psi|^2 (\nabla \varphi)$

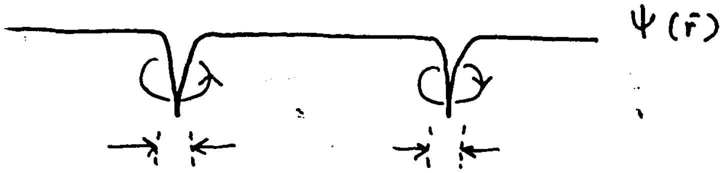
$\nabla \times \nabla \varphi = 2\pi\, \delta(\vec{r})$ | $\nabla \times \nabla \varphi = -2\pi\, \delta(\vec{r})$

VERY EFFICIENT IN DISORDERING

$\langle \psi(0)\, \psi^{+}(\vec{r}) \rangle$

VERTICES $\Longleftrightarrow \overset{\text{IFF UNBOUND!}}{\text{TOPOLOGICAL DEFECTS}}$

- **VORTICES IN 2D:**



Vortex core size – finite at T_c

VORTEX-(ANTI) VORTEX INTERACTIONS:

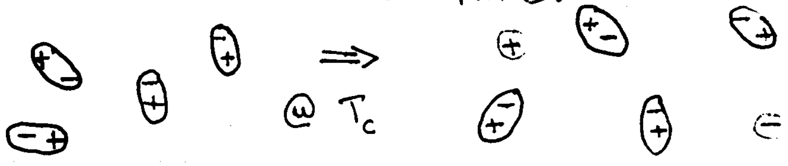
$$\sim - \sum_{i<j} q_i q_j \ln |\vec{r}_i - \vec{r}_j|$$

q_i – vortex charge ($q_i = \pm 1$)

- SUPERCONDUCTING TRANSITION IN 2D

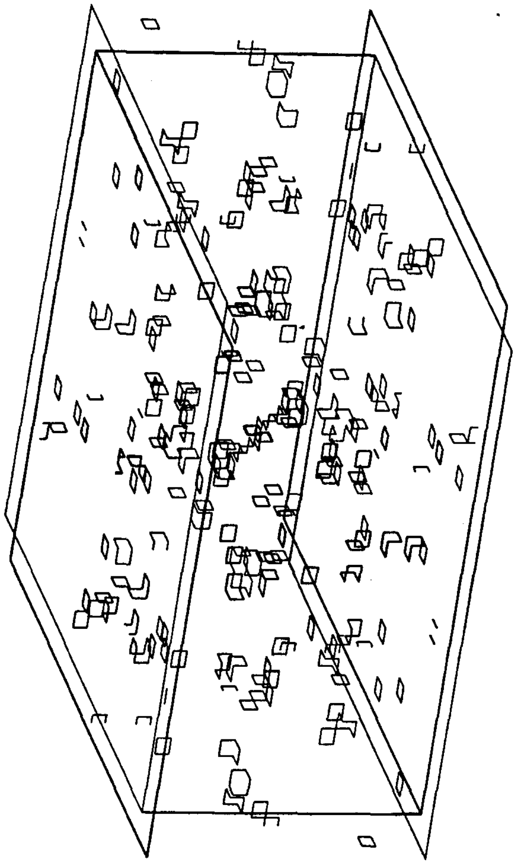
$\Downarrow$ via 2D Coulomb plasma

- KOSTERLITZ-THOULESS UNBINDING OF VORTEX-ANTIVORTEX PAIRS:



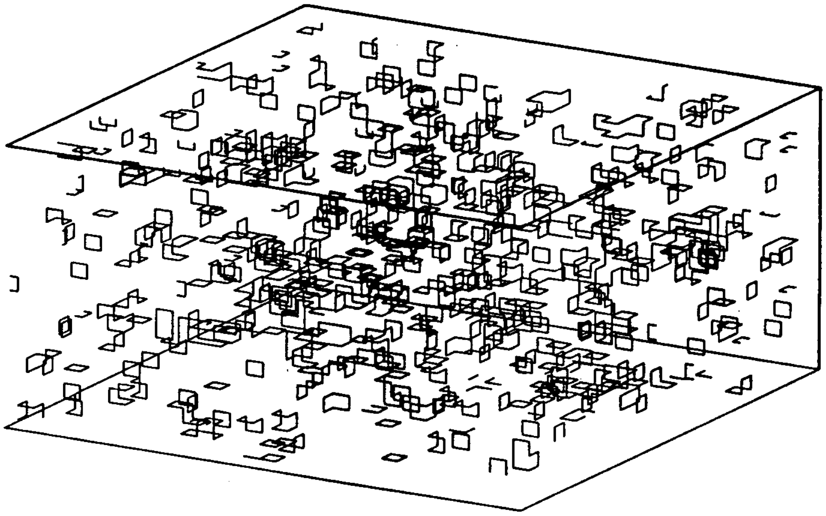
MONTE-CARLO SIMULATIONS OF A HIGH TEMP. SUPERCONDUCTOR

(Nguyen & Sudbø)



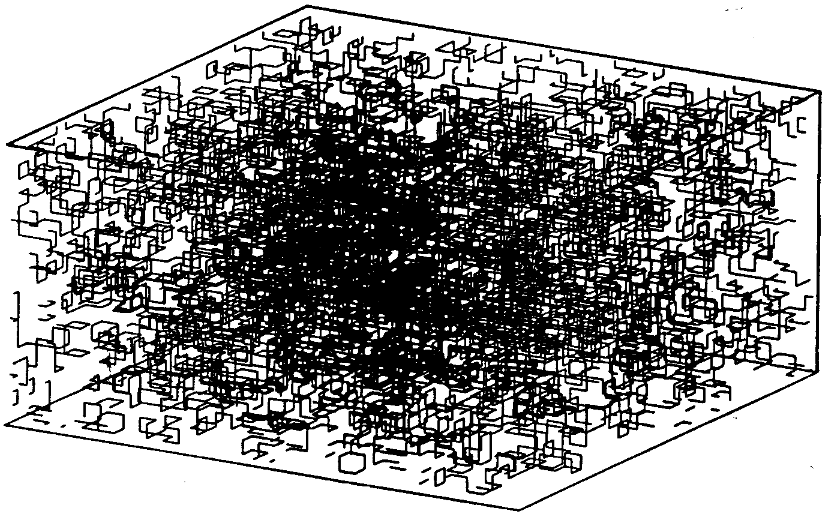
THERMALLY EXCITED VORTEX LOOPS

$T \ll T_c$



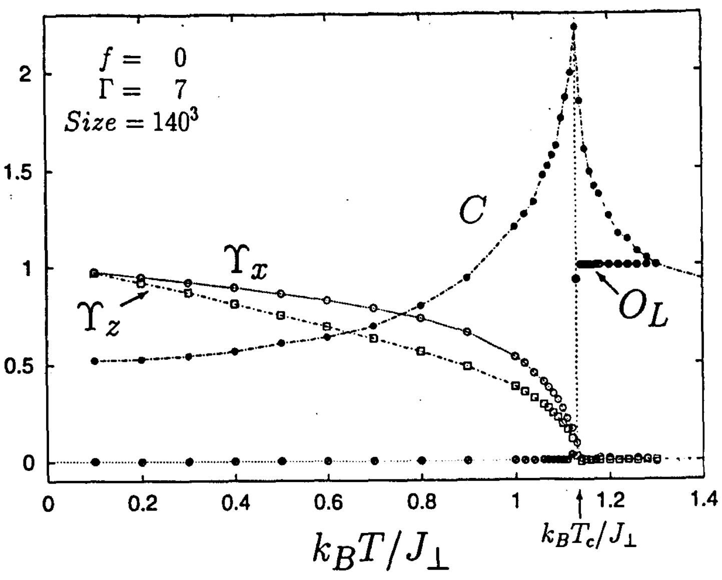
T < T_c

Onsager, '49
Feynman, '60's



T > T_c

(17)



Monte-Carlo simulations of 3D XY model

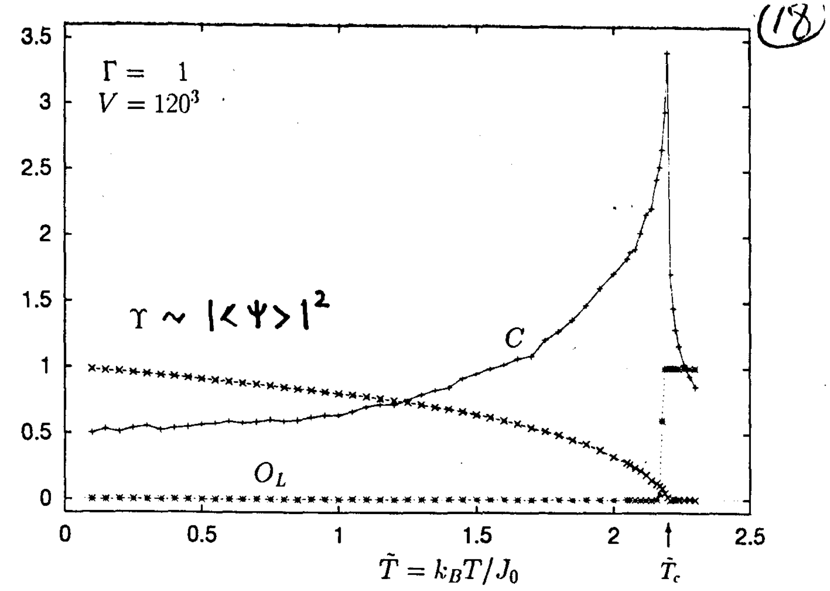
Figure 1

Nguyen & Sudbø

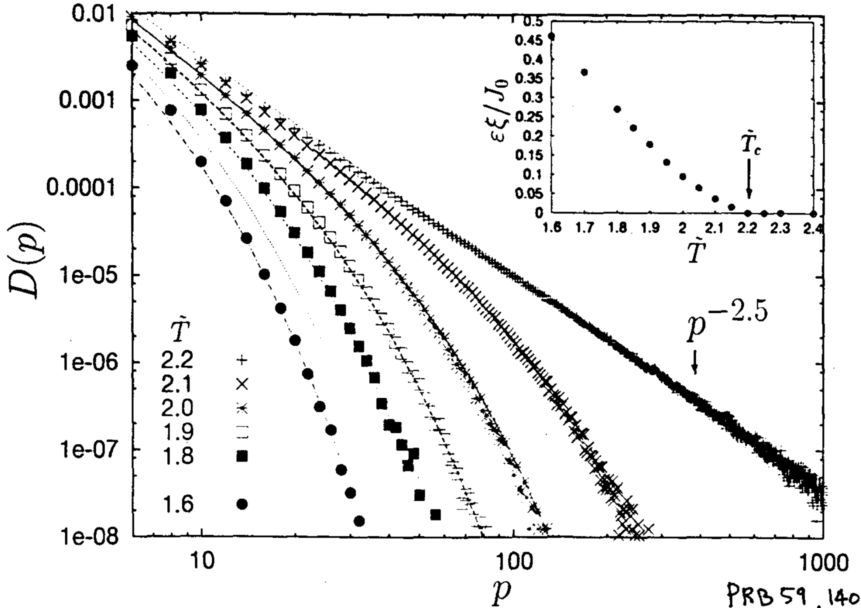
Phys. Rev. B $\underline{60}$, 15307 ('99).

Europhys. Lett. $\underline{46}$, 780 ('99).

(18)

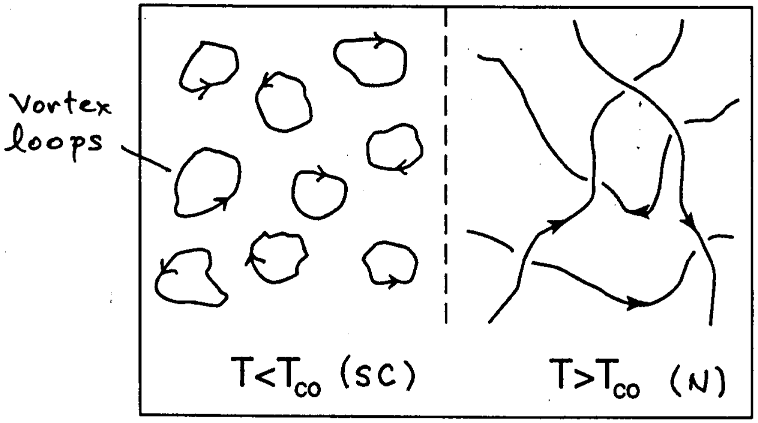


$\Upsilon \sim |\langle\Psi\rangle|^2$



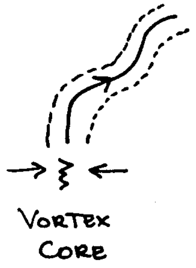
PRB $\underline{59}$, 14017(

S-K. Chiu, AK Nguyen, A. Sudbø; ~~unpublish~~



ZT
Cond-mat/9801306, Phys. Rev. B $\underline{59}$, 6449 ('99).

VORTICES AS TOPOLOGICAL DEFECTS (3D)



$$\psi(\vec r) = \psi_0 e^{i\varphi(\vec r)}$$

- AMPLITUDE ψ_0 UNIFORM OUTSIDE CORE REGION
- $\varphi(\vec r)$ WINDS BY $\pm 2\pi$ AROUND THE VORTEX

$$\underline{\vec\nabla\cdot\vec\nabla\varphi = 0} \quad ; \quad \underline{\vec\nabla\times(\vec\nabla\varphi) = 2\pi\,\vec n(\vec r)}$$

$$\vec n(\vec r) = \sum_i q_i \int_l d\vec r_i\, \delta(\vec r - \vec r_i) \qquad \underline{q_i = \pm 1}$$

$$F_{sc}^{Vor} = \frac{\hbar^2|\psi_0|^2}{2m^*}\int_c d^3r\,(\vec\nabla\varphi)^2 \quad + \quad \text{Core terms}$$

- MAGNETOSTATIC ANALOGY:

$$\underline{\vec\nabla\cdot\vec B = 0} \quad ; \quad \underline{\vec\nabla\times\vec B(\vec r) = \frac{4\pi}{c}\vec j(\vec r)}$$

$$F^{mag} = \frac{1}{8\pi}\int d^3r\,\vec B^2(\vec r)$$

☆ $\vec B(\vec r) \leftrightarrow \nabla\varphi(\vec r)$ $\qquad$ $\underset{\text{CURRENT LOOPS}}{\vec j(\vec r)} \leftrightarrow \underset{\text{VORTEX LOOPS}}{\vec n(\vec r)}$

Jackson, Classical ED

Prob. 6 **Time-Varying Fields, Maxwell Equations, Conservation Laws** 261

The derivation of the macroscopic Maxwell equations from a statistical-mechanical point of view has long been the subject of research for a school of Dutch physicists. Their conclusions are contained in two comprehensive books,
de Groot,
de Groot and Suttorp.

A treatment of the energy, momentum and Maxwell stress tensor of electromagnetic fields somewhat at variance with these authors and Brevik (*op. cit.*) is given by
Penfield and Haus.

For the reader who wishes to explore the detailed quantum-mechanical treatment of dielectric constants and macroscopic field equations in matter, the following are suggested:
S. L. Adler, *Phys. Rev.* **126**, 413 (1962),
B. D. Josephson, *Phys. Rev.* **152**, 21 (1966),
G. D. Mahan, *Phys. Rev.* **153**, 983 (1967).

Symmetry properties of electromagnetic fields under reflection and rotation are discussed by
Argence and Kahan.

The subject of magnetic monopoles has an extensive literature. We have already cited the review by Amaldi, and the papers by Carrigan and Goldhaber, as well as the fundamental papers of Dirac. The relevance of monopoles to particle physics is discussed by
J. Schwinger, *Science* **165**, 757 (1969).

Some experimental searches are described in
Hart et al., *Phys. Rev.* **184**, 1393 (1969),
Fleischer, Price and Woods, *Phys. Rev.* **184**, 1398 (1969),
Alvarez et al., *Science* **167**, 701 (1970).

The mathematical topics in this chapter center around the wave equation. The initial-value problem in one, two, three, and more dimensions is discussed by
Morse and Feshbach, pp. 843–847,
and, in more mathematical detail, by
Hadamard.

PROBLEMS

6.1 (*a*) Show that for a system of current-carrying elements in empty space the total energy in the magnetic field is

$$W=\frac{1}{2c^2}\int d^3x\int d^3x'\,\frac{\mathbf{J}(\mathbf{x})\cdot\mathbf{J}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}$$

where $\mathbf{J}(\mathbf{x})$ is the current density.

(*b*) If the current configuration consists of n circuits carrying currents $I_1, I_2, \ldots, I_n$, show that the energy can be expressed as

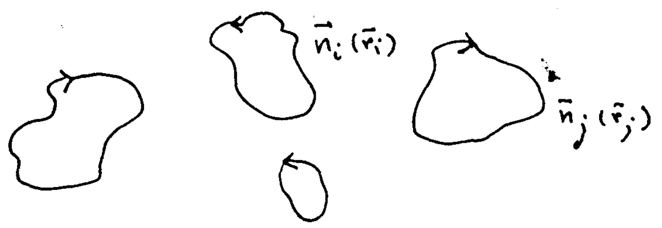
$$W=\frac{1}{2}\sum_{i=1}^{n} L_iI_i^2+\sum_{i=1}^{n}\sum_{j>i}^{n} M_{ij}I_iI_j$$

Exhibit integral expressions for the self-inductances (L_i) and the mutual inductances (M_{ij}).

6.2 A two-wire transmission line consists of a pair of nonpermeable parallel wires of radii a and b separated by a distance $d>a+b$. A current flows down one wire and back

iot-Savart interactions between current loops.

- BIOT-SAVART INTERACTIONS BETWEEN VORTEX LOOPS



$$\rightarrow F_{SC}^{vor} = \frac{\hbar^2|\psi_0|^2}{2m^*}(2\pi)^2\int_C d^3r\int_C d^3r'\;\frac{\vec{n}(\vec{r})\cdot\vec{n}(\vec{r}')}{|\vec{r}-\vec{r}'|}$$

$$+\sum_i E_c\int_\ell ds_i \qquad \text{— CORE ENERGY}$$

$$+\frac{g}{2}\sum_i\sum_j\int_\ell ds_i\int_\ell ds_j\;\delta\left(\vec{r}_i[s_i]-\vec{r}_j[s_j]\right)$$

— CONTACT REPULSION

- STILL, VORTEX LOOPS ARE <u>NOT</u> ORDINARY CURRENT LOOPS: THEY ARE <u>QUANTIZED</u> !

$$\boxed{\oint d\vec{r}\cdot(\vec{\nabla}\varphi)_{vor} = 2\pi n \quad ; \quad n=\pm1,\pm2,\ldots}$$

○ WHY DO LOOPS BLOW UP ?

ENERGY: $E = \varepsilon_\ell L$

ENTROPY: $S = L\ln(m)$

$(m \sim 5) \rightarrow$

→ FREE ENERGY OF A VERY LONG VORTEX LOOP:

$$F = E - TS = (\varepsilon_\ell - T\ln m)L \equiv f_\ell L$$

FOR $\boxed{T > T_c = \frac{\varepsilon_\ell}{\ln m}}$ $\boxed{f_\ell < 0}$ →

→ $\boxed{\text{PROLIFERATION OF LONG LOOPS !}}$

FOR $T \to T_c$

$L \sim \frac{1}{|t|^\gamma}$ $t \equiv 1 - \frac{T}{T_c}$

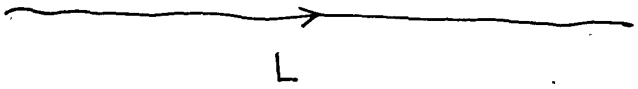
loop size in real space → $R \sim \frac{1}{|t|^\nu}$

γ, ν – CRITICAL EXPONENTS

↓

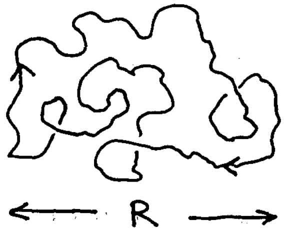
$$\gamma = (2-\eta)\nu$$

● STATISTICAL PHYSICS OF VORTEX LOOPS:



NO INTERACTION → RANDOM WALKS

$\underline{R^2 \sim L}$



SHORT RANGE REPULSION →

SELF-AVOIDING RANDOM WALKS

$\underline{R^{2-\eta} \sim L \; ; \; \eta > 0}$

SHORT RANGE REPULSION +

+ LONG RANGE BIOT-SAVART INT. →

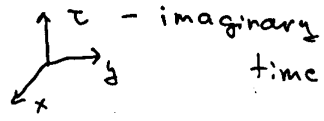
SELF-AVOIDING + <u>SELF-SCREENING</u>

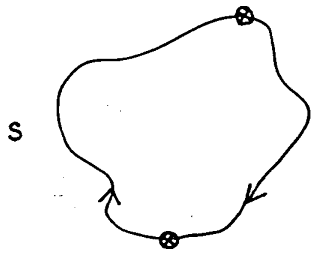
RANDOM WALKS

$\underline{R^{2-\eta} \sim L \; ; \; \eta < 0}$

RELATIVISTIC VORTEX BOSONS:

(see Appendix A)





particle - antiparticle creation and annihilation processes in vacuum

Schwinger proper time s

CONNECTIONS:

VORTEX LOOPS

$$\Downarrow$$

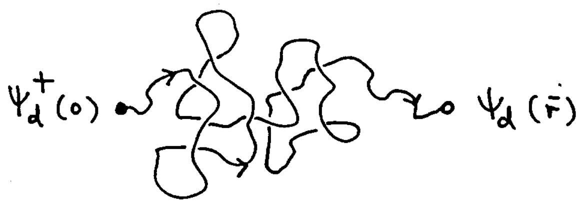
VIA PATH-INTEGRAL QM
IN PROPER TIME

RELATIVISTIC FIELD-THEORY FOR

$\psi_d(\vec{r})$ – VORTEX BOSON FIELD OPERATOR

★ $\psi_d(\vec{r})$ is DUAL OF $\psi(\vec{r})$

● CONNECTION BETWEEN LOOPS AND $\psi_d(\vec{r}), \vec{A}_d(\vec{r})$



$$\boxed{\langle \psi_d(\vec{r})\, \psi_d^+(0) \rangle \sim \frac{e^{-r/\xi_d}}{r^{D-2+\eta}}}$$

CORRELATION FUNCTION

$\xi_d \sim R$ $\quad m_d^2 \sim 1/L$ $\quad$ ★ η - ANOMALOUS DIMENSION OF ψ_d

CORRELATION LENGTH

$$\boxed{\langle \vec{A}_d(\vec{r})\, \vec{A}_d(0) \rangle \sim \frac{e^{-r/\lambda_d}}{r^{D-2+\eta_A}}}$$

λ_d – DUAL MAGNETIC FIELD PENETRATION DEP.

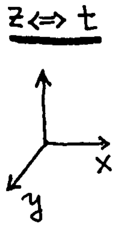
η_A – ANOMALOUS DIMENSION OF $\vec{A}_d$

• DUALITY IN 3D : (3D is special!)

$F_d \Longleftrightarrow F_{sc}$

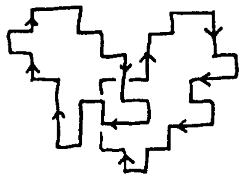
$\psi_d(\bar r) \Longleftrightarrow \psi(\bar r)$

$\langle\psi(\bar r)\rangle \neq 0$ $\langle\psi_d(\bar r)\rangle = 0$	$\langle\psi(\bar r)\rangle = 0$ $\langle\psi_d(\bar r)\rangle \neq 0$
broken U(1) symmetry	restored U(1) symmetry
restored U(1) dual symmetry	broken U(1) dual symmetry



• VORTEX LOOP GAS

(H=0)



BUILDING BLOCKS ⇒

⇒ Coulomb "stick" charges:

↑↓ ⇄

• SELF-AVOIDING RANDOM WALKS ON A 3D DUAL LATTICE WITH "BIOT-SAVART" INTERACTION ($\varkappa \to \infty$)

$$F_d \to \int d^3r \left\{ m_d^2|\psi_d|^2 + \frac{g}{2}|\psi_d|^4 + |(\vec\nabla - i\vec A_d)\psi_d|^2 + \frac{1}{2e_d^2}(\vec\nabla \times \vec A_d)^2 \right\}$$

– dual theory in 3D

$m_d \sim \frac{1}{\Lambda}$

Λ – average loop size

g – short-range repulsion (SARW)

$e_d^2 \propto \langle|\psi|^2\rangle$ – dual charge

$\vec A_d$ – dual gauge field ⇒ "Biot-Savart" interactions

masslesss ($\varkappa \to \infty$)

$\psi_d(\bar r)$ – dual (loop) order parameter $\Longleftrightarrow \psi(\bar r)$

↓

original superconducting order parameter!

● DUALITY IN 3D (NOT SELF-DUAL !!)

● $\alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + |\bar{\nabla}\psi|^2$ EXTREME TYPE-II SUPERCONDUCTOR

$$\Downarrow \text{ duality}$$

● $m_d^2|\psi_d|^2 + \frac{g}{2}|\psi_d|^4 + |(\bar{\nabla} - i\bar{A}_d)\psi_d|^2 + \frac{1}{2e_d^2}(\bar{\nabla}\times\bar{A}_d)^2$

"DUAL" SUPERCONDUCTOR

$\psi(\bar{r})$ - Cooper pair field => $\psi_d(\bar{r})$ - loop field of vortices

○ If we include the physical EM field $\vec{A}$:

● $\alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + |(\bar{\nabla} - i\bar{A})\psi|^2 + \frac{1}{2e^2}(\bar{\nabla}\times\bar{A})^2$

$$\Downarrow \text{ duality}$$

● $m_d^2|\psi_d|^2 + \frac{g}{2}|\psi_d|^4 + |(\bar{\nabla} - i\bar{A}_d)\psi_d|^2 + \frac{1}{2e_d^2}(\bar{\nabla}\times\bar{A}_d)^2$

$+ \quad M_d^2 A_d^2$

L mass of dual gauge field

$M_d^2 = e^2$

○ DUALITY AT WORK:

- SUPERCONDUCTOR AND "DUAL" SUPERCONDUCTOR MUST HAVE SAME THERMODYNAMICS

$$\underline{\nu} = \nu_{SC} = \nu_{3DXY} \cong \underline{\frac{2}{3}}$$

"DUAL" SUPERCONDUCTOR IS IN "INVERTED 3DXY" UNIVERSALITY CLASS

(Dasgupta & Halperin)

- $(\text{DUAL})^2 = 1 \quad => \quad \underline{\lambda \sim \xi} \sim \frac{1}{|t|^\nu}$

  $\underline{\eta_A = 1}$

  (Herbut & Tešanović

  PRL 76, 4588 ('96); 78, 980 ('97

- DUALITY + RENORMALIZATION GROUP =>

  SOLUTION FOR $\nu = \nu_{3DXY}$ =>

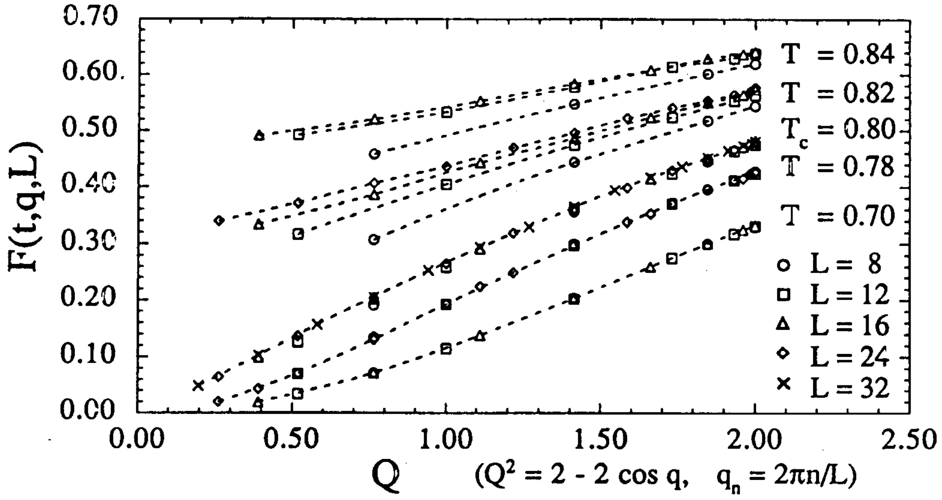
SUPERCONDUCTOR: $\nu_{sc} = 0.67 \quad \eta_{sc} = +0.03$

"DUAL" SUPERCONDUCTOR: $\nu_d = 0.67 \quad \underline{\eta_d \cong -0.16}$

(see Appendix B) (Herbut & Tešanović)

㉛

Magnetic field fluctuations: $F(t,q,L) \sim \langle b(q)b(-q)\rangle$ vs. q



$$\text{as } q \to 0 \begin{cases} F(q) \sim q^2 & \text{for } T < T_c \\ F(q) \sim q & \text{for } T = T_c \\ F(q) \sim \text{const.} & \text{for } T > T_c \end{cases}$$

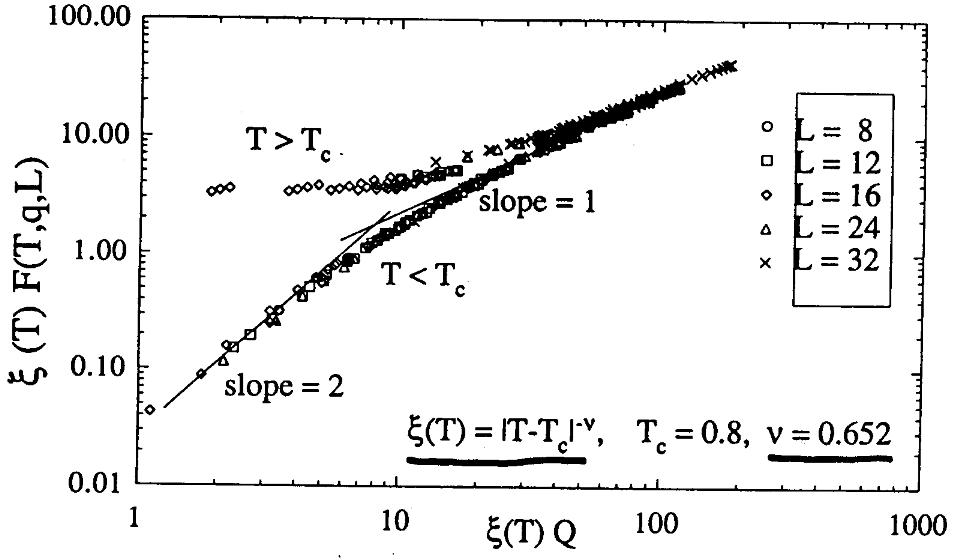
anomalous dimension
(Herbut and Tesanovic)

(Note: virtually no finite size effects for $T < T_c$)

P. Olsson and S. Teitel, PRL <u>80</u>, 1964 ('98)

㊳

Scaling Collapse: $\xi F(t,q,L)$ vs. ξq (large L limit, no finite size effects in data)



P. Olsson and S. Teitel, PRL <u>80</u>, 1964 ('98)

Conclusions

P. Olsson and S. Teitel, PRL 80, 1964(

- Scaling collapse of L F(t,2π/L,L) vs. $tL^{1/\nu}$

  determines: x = 1, ν = 0.65 ± 0.03, $T_c = 0.80$

  $\xi \sim |T-T_c|^{-\nu}$ where ν is consistent with inverted 3D XY behavior $\nu_{XY} \approx 2/3$

  $\nu_e \approx 0.667$

- Scaling collpse of ξF(t,q,L) vs. qξ

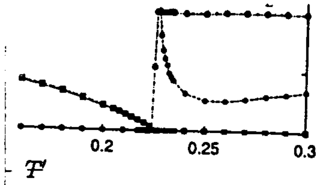
  demonstrates that ξ is the only length scale of magnetic field fluctuations, i.e. ξ ~ λ the magnetic penetration length

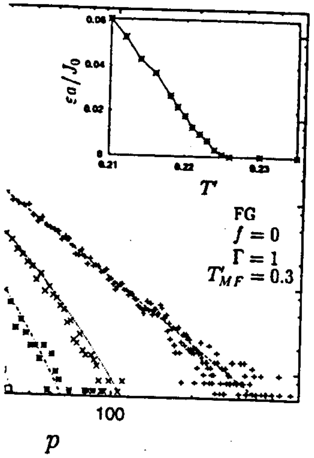
- F(q) ~ q at T = T_c $\eta_A = 1$

  demonstrates anomalous dimension of magnetic field fluctuations confirms primary prediction of Herbut and Tesanovic

(33)

33





5, with $T'_{MF} = 0.3$, and $/k_BT$).

of the properties of thermally
now focus on the vortex-loop
as a function of vortex-loop
peratures. These are shown
re clearly well approximated
$e^{-\varepsilon(T)p/k_BT}$ for all tempera-
$\varepsilon(T)$ is the only temperature-
r in all plots. The effective
of vortex loops is finite be-
$\geq T_c$. The physical picture
is follows. Below T_c, $\varepsilon(T)$ is
th scale for the vortex loops,
p) is dominated by an expo-

vortex line tension vanishes as

$$\underline{\varepsilon(T) \sim |T-T_c|^{\gamma}; \quad \gamma = 1.45 \pm 0.05} \tag{30}$$

The numerical value of the exponent γ has been extracted from the systems with the largest critical regions, i.e. Figs. 3, 4, and 5. The system shown in Fig. 6 does not allow a very precise value for γ to be obtained, although the qualitative aspects of the results are clearly precisely the same as those for the 3*DXY*-model and the FG-approximation of the GL-model with $T'_{MF} = 1.0$. This implies that the typical vortex-loop perimeter diverges when T_c is approached from below, using Eq. 29, as

$$\underline{L_0(T) \sim |T-T_c|^{-\gamma},} \tag{31}$$

such that $L_0(T)$ is a power of the correlation length ξ of the 3*DXY* model. It is natural, within the formulation of the problem given in Section IIF, to associate the proliferation of unbound vortex-loops with a vortex-loop susceptibility, or equivalently a susceptibility for the φ-field of Section IIF. The field φ has a correlation length exponent which should be $\nu = 2/3$ since it is in the same usiversality class as the GL theory. The standard scaling relation between ν and γ is given by

$$\underline{\gamma = (2-\eta_\phi)\,\nu} \tag{32}$$

where η_ϕ is the anomalous dimension of the φ-field. Using our estimate $\gamma = 1.45 \pm 0.05$ with $\nu = 2/3$ gives $\eta_\phi = -0.18 \mp 0.07$ in close agreement with previous renormalization group calculations[82].

At and below T_c, the order field $< \psi'(\mathbf{r}) >$ develops an expectation value, and explicitly breaks the global $U(1)$ symmetry of the GL-theory. In contrast to the order field picture, in a description using only topological excitations, *the global $U(1)$ symmetry is hidden*. There does not appear to be any symmetry operation involving the phase of a local field, that will leave the effective action Eq. 13 or vortex Hamiltonian Eq. 14 invariant. Therefore, there is also no obvious local quantity that develops an expectation value in the non-symmetric phase. Nevertheless, it is possible to define a global quantity that implicitly probes the breaking of the global $U(1)$ symmetry, namely O_L. Let N_μ denote the number of "vortex

Hove, Nguyen & Sudbø, LT22 ('99
Nguyen & Sudbø, preprint ('99

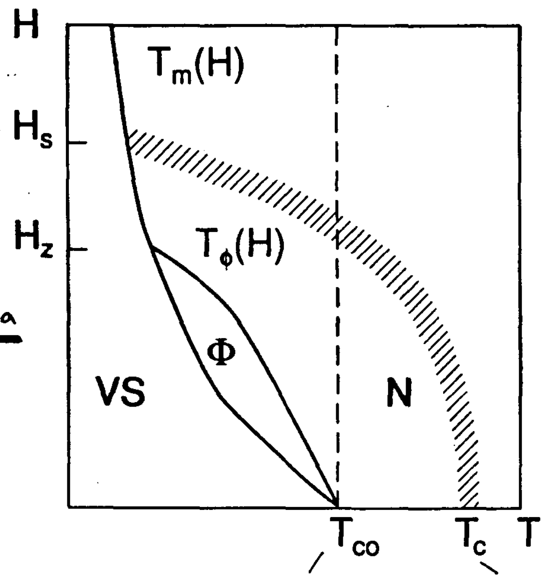
(34)

ZT, PRB 51, 16204 ('95) short paper

ZT, xxx.lanl.gov/cond-mat/9801306,
Phys. Rev. B59, 6499 ('99) long paper

$T_m(H)$ - Vortex lattice melting line
(Bishop, Gammel, ...)
Safar

$Gi\ H_{c_2}(0) \cong H_s$
$\wr\wr$
1-2 Tesla
or
10-20 Tesla

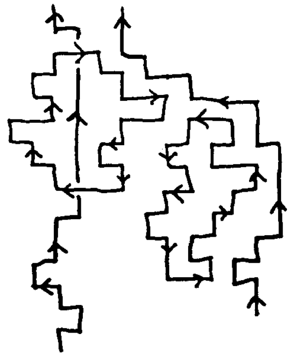


true / mean-field
transition temperature

$T_c - T_{co} \cong Gi\ T_c \cong$
$\cong$ 2-3 K
or
8 - 10 K

=> there is still much uncertainty concerning the width of the critical region in HTS.

● VORTEX LOOP GAS $(H \neq 0)$



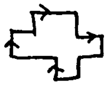
Finite H acts like uniform charge background in the dual model.

$$\boxed{n_\uparrow - n_\downarrow = n_\phi = \frac{1}{2\pi \ell^2}}$$

$\ell = \sqrt{\frac{c}{e^* H}}$ - magnetic length

★ Minimum N_ϕ vortex lines in each configuratio

Loops and lines are topologically distinct:



$\Longrightarrow \underline{\Phi_d(\bar{r})}$ (analogous to $\psi_d(\bar{r})$ for $H=0$)

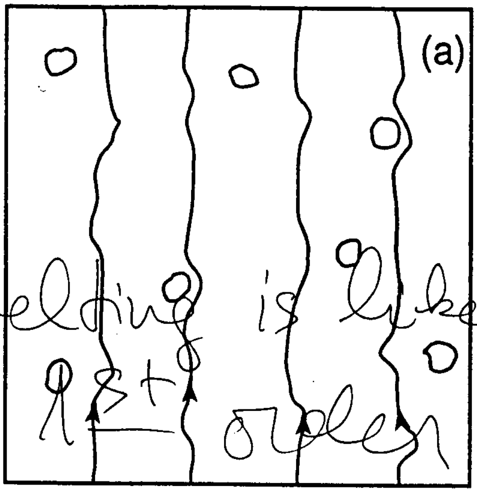
$\Longrightarrow \underline{\psi_s(\bar{r})} \longrightarrow$

$$\underline{\underline{F_d}} = \int d^3r \left\{ \frac{1}{2\underline{e_d}^2}(\vec{\nabla} \times \underline{\vec{A}_d})^2 + \right.$$
$$+ m_\phi^2 |\Phi_d|^2 + \frac{g}{2}(\Phi_d)^4 + |(\vec{\nabla} - i\underline{\vec{A}_d})\Phi_d|^2 +$$
$$+ \psi_s^*(\partial_z - i\underline{A_{dz}})\psi_s + \frac{1}{2m_s}|(\vec{\nabla}_\perp - i\underline{\vec{A}_{d\perp}})\psi_s|^2 + \frac{g}{2}|\psi_s|^4$$
$$\left. + g|\Phi_d|^2|\psi_s|^2 \right\}$$

"line solid"
or
Abrikosov
vortex lattice

ψ-order $\neq 0$

ϕ-order $\neq 0$



$T < T_m(H)$

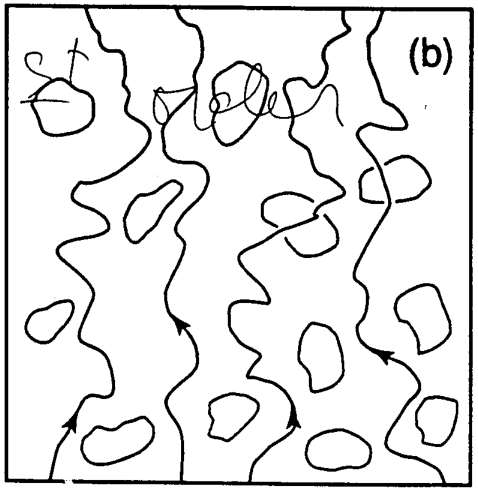
ϕ_d-order $= 0$

Melting is likely 1st order

ϕ could be

"line liquid"

1st order



$T_m(H) < T < T_\phi(H)$

Cond-mat/9801306, Phys. Rev B $\underline{59}$, 6449 (199).

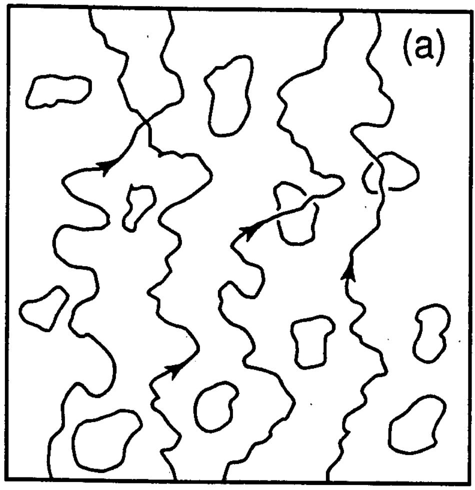
"line liquid"

ψ- order $= 0$

ϕ- order $\neq 0$

$\tau \neq 0$

finite line tension



$T_m(H) < T < T_\phi($

confined "vortex liquid"

ϕ_d- order =

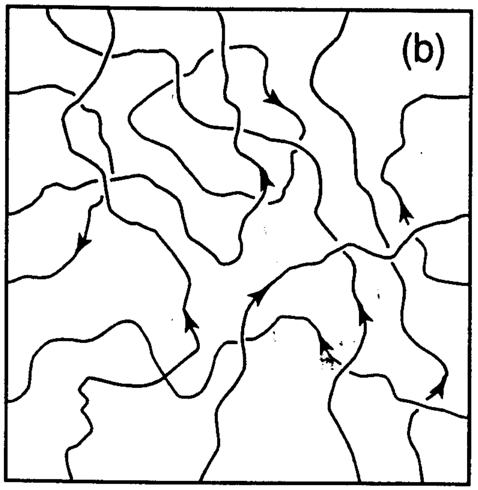
true normal state

ψ-order $= 0$

ϕ -order $= 0$

$\tau = 0$

line tension vanishes



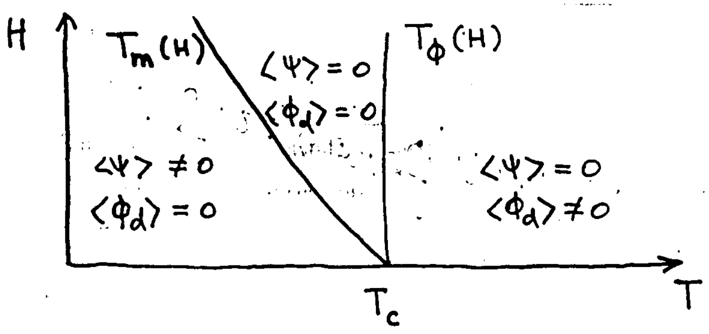
$T_\phi(H) < T$

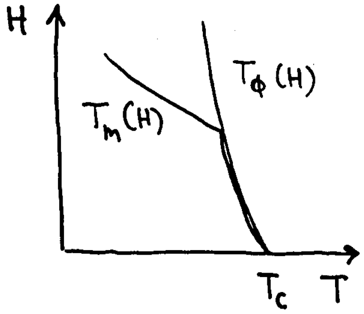
deconfined "vortex gas"

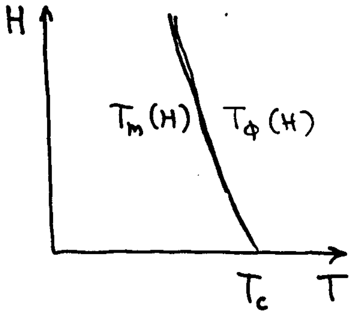
ϕ_d-order $\neq 0$

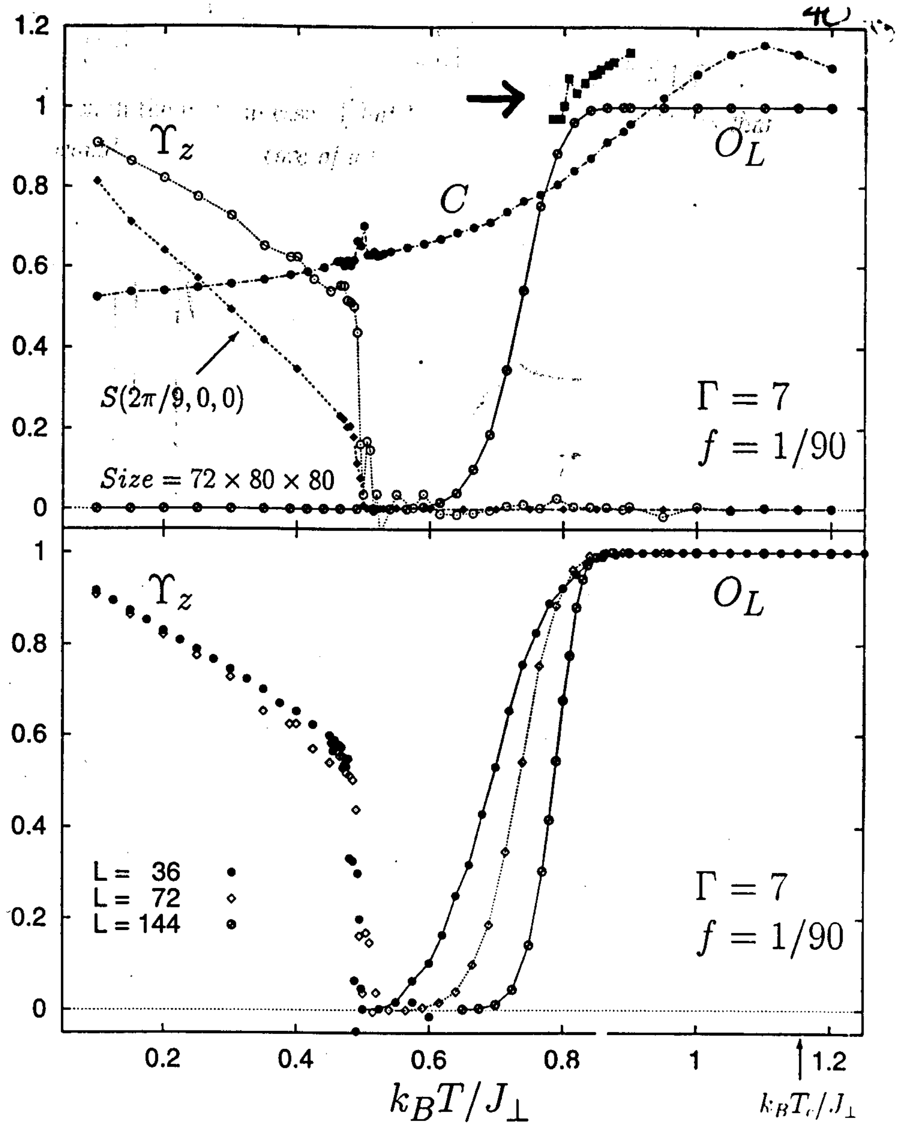
Cond-mat/9801306, Phys. Rev. B $\underline{59}$, 6449 (199).

VORTEX MELTING vs. VORTEX LOOP UNBINDING









Europhys. Lett. 46, 780 ('99). Figure 2 Cond-mat/9811149

↖ Nguyen & Sudbø, preprint (Aug '98)

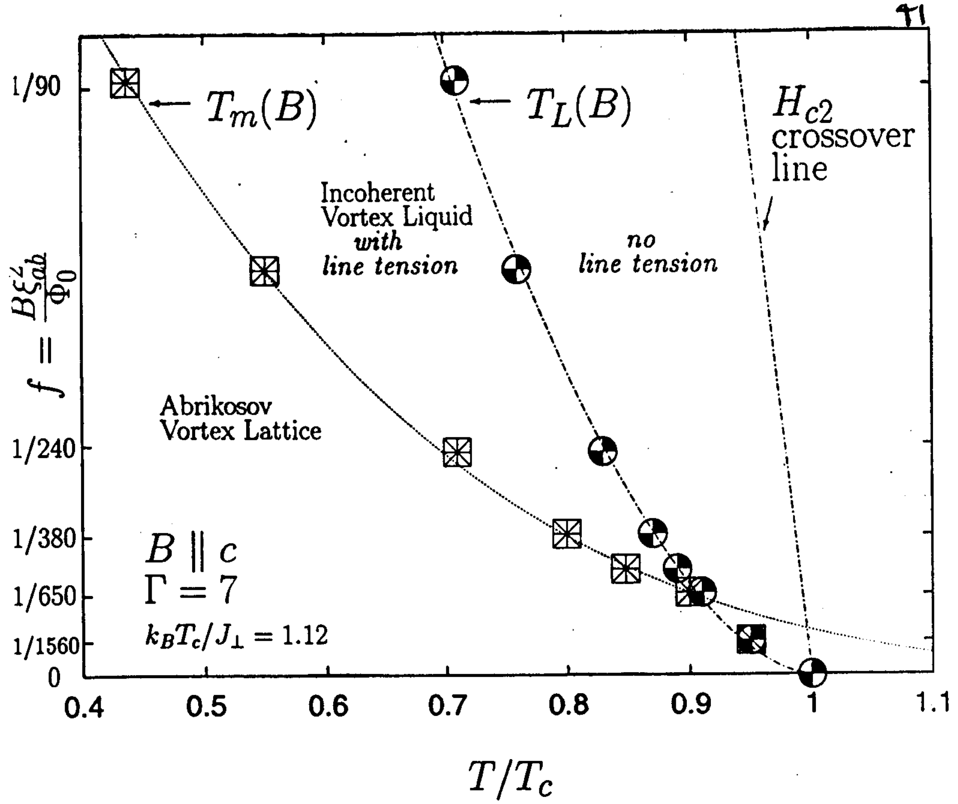
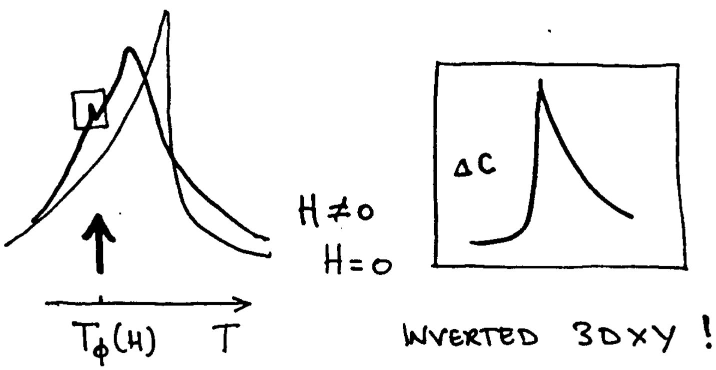


Figure 3

Nguyen & Sudbø, Aug '98
cond-mat/9811149, preprint
↓
Europhys. Lett. 46, 780 ('99).

● SPECIFIC HEAT AT $T_\phi(H)$



INVERTED 3DXY !

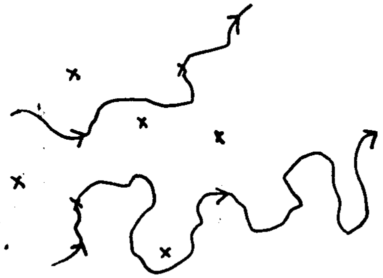
★ ΔC HAS A "WRONG" SIGN AT $T_\phi(H)$! ★

$\langle \Phi_d \rangle = 0 \longrightarrow \langle \Phi_d \rangle \neq 0$

● PINNING AT $T_\phi(H)$

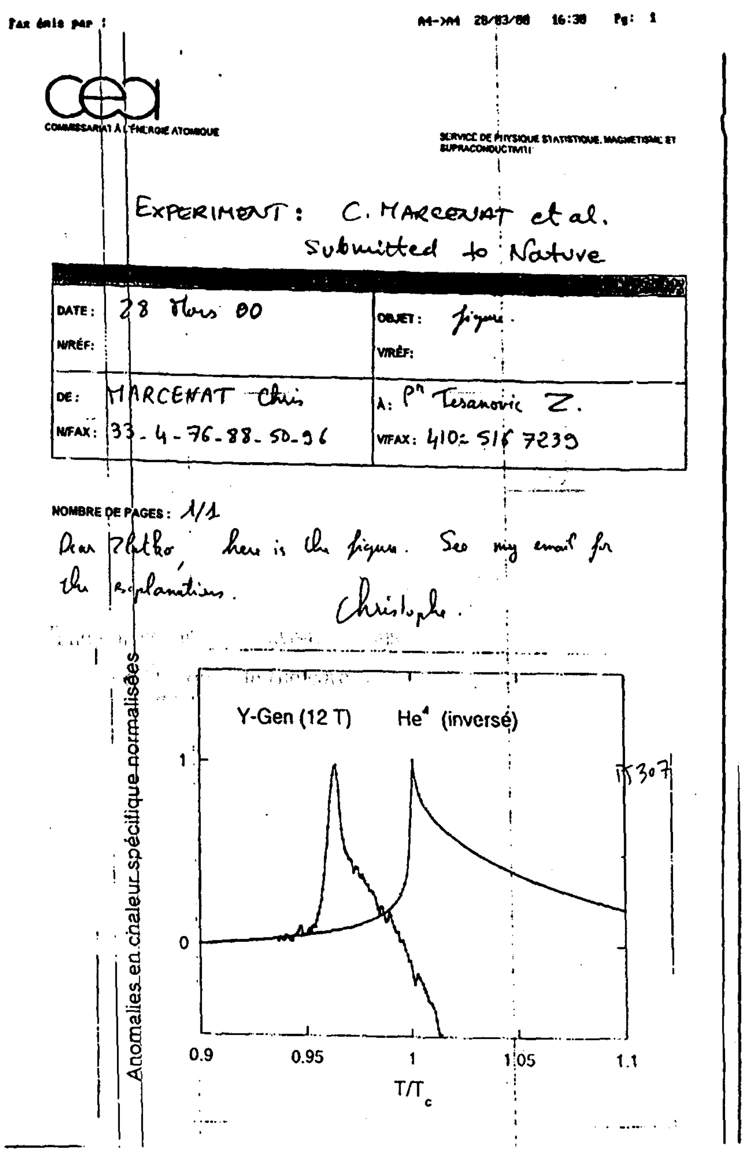
$T < T_\phi(H)$

LINE TENSION $\tau > 0$



$T > T_\phi(H)$

$\underline{\tau = 0}$ $\underline{\text{NO PINNING}}$



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COMMISSARIAT À L'ÉNERGIE ATOMIQUE

SERVICE DE PHYSIQUE STATISTIQUE, MAGNETISME ET SUPRACONDUCTIVITE

EXPERIMENT : C. MARCENAT et al.
submitted to Nature

DATE : 28 Mars 00	OBJET : figure.
N/RÉF:	V/RÉF:
DE : MARCENAT Chris	À : Pr Tesanovic Z.
N/FAX : 33-4-76-88-50-96	V/FAX : 410-516 7239

NOMBRE DE PAGES : 1/1

Dear Zlatko, here is the figure. See my email for the explanations.

Christophe.

Nature Of Quasiparticle States in the Vortex Lattice of Unconventional Superconductors

Oskar Vafek
Ashot Melikyan
M. Franz
Z. Tesanovic　(Johns Hopkins U.)

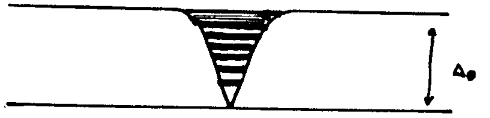
SOLUTION FOR A SINGLE VORTEX

Caroli, de Gennes and Matricon (1964) solved the above problem for a single isolated vortex with order parameter of the form

$$\boxed{\Delta(\mathbf{R}) = \Delta(R)e^{-i\theta},}$$

by matching the wave-functions in two asymptotic regions. They found a set of ***discrete bound states*** in the core labeled by the angular momentum quantum number μ with low lying energy levels of the form:

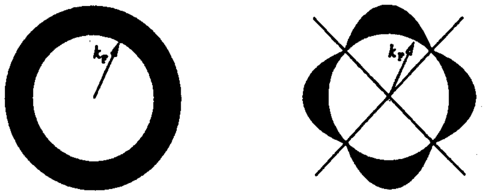
$$E_\mu \simeq \mu\Delta_0\left(\frac{\Delta_0}{E_F}\right), \quad \mu = \frac{1}{2}, \frac{3}{2}, \cdots$$



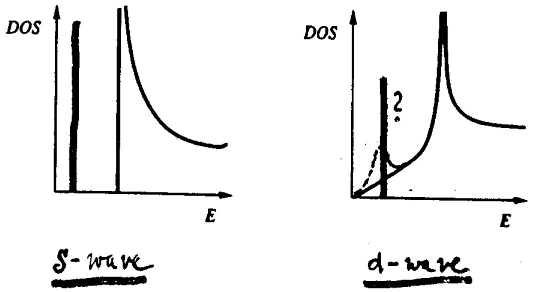
The corresponding wave-functions are peaked near $\underline{\mu k_F^{-1}}$ and decay as $\sim \underline{e^{-R/\xi}}$ outside the core.

VORTEX CORE IN A *D*-WAVE SUPERCONDUCTOR

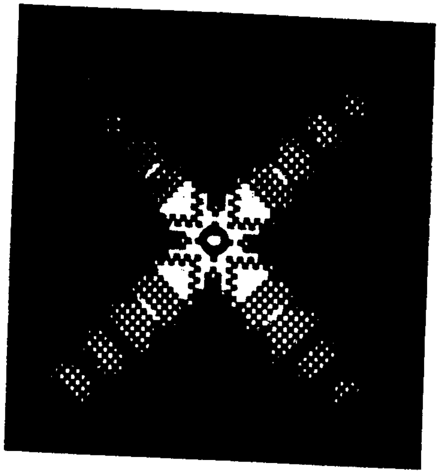
In *d*-wave superconductors, such as high-T_c cuprates, the order parameter has a nontrivial dependence on the ***relative*** coordinate of the Cooper pair: it vanishes along the four nodes on the Fermi surface.

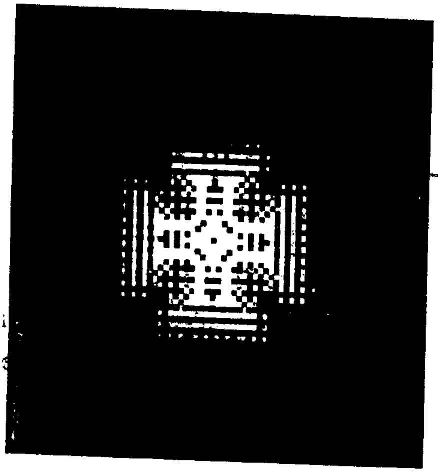


Correspondingly, there exist low-lying excitations near the nodes even in the uniform case. ***What happens to the bound states that would exist in the core of a conventional s-wave vortex?***



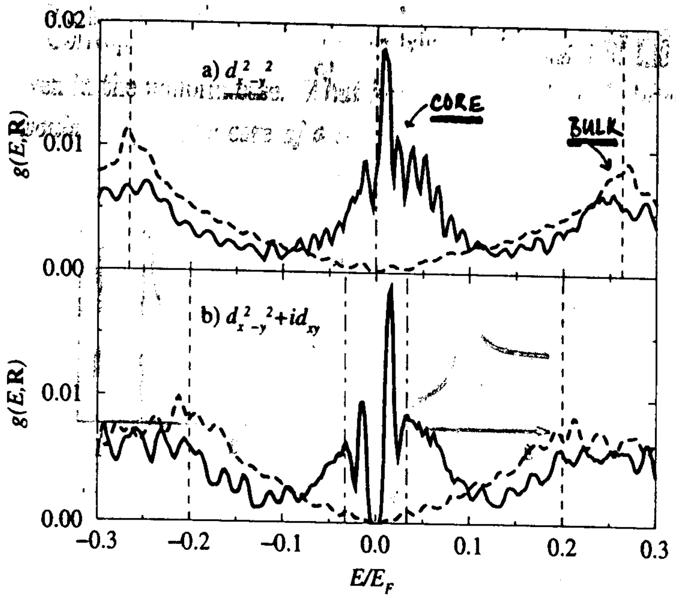
Quasiparticle wave-functions





⊙ TUNNELING CONDUCTANCE

[M. FRANZ & Z. TEŠANOVIĆ
PRL 60, 4763 (1998)]



○ BSCCO

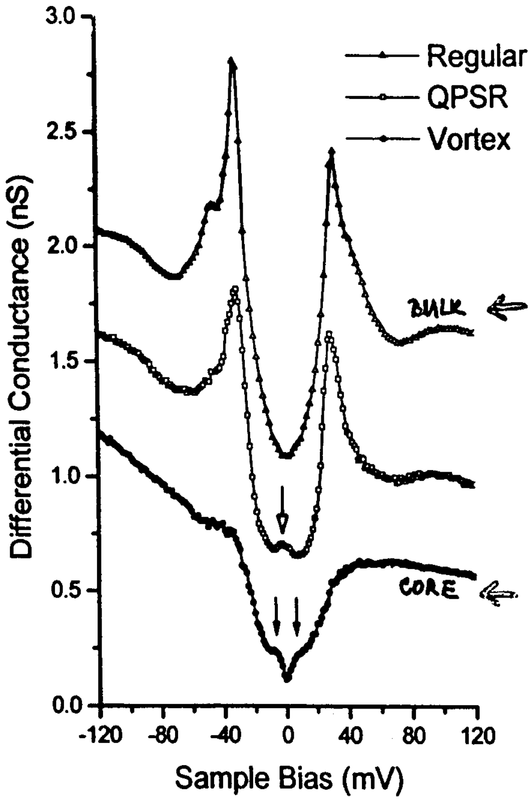
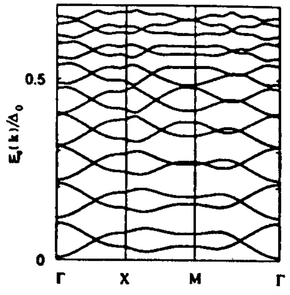
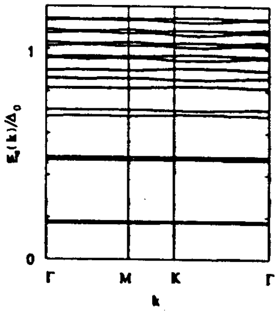


Figure 2
J.C. Davis



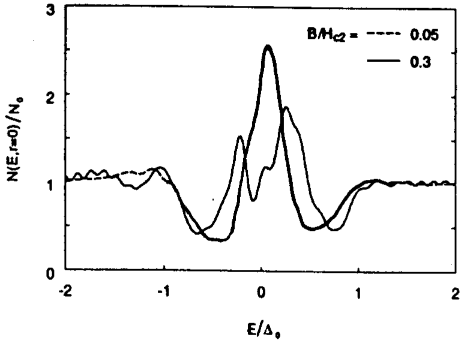
● d-wave



● s-wave

K. YASUI & T. KITA
[COND-MAT/9905067]
PRL 83, 4168 (1999)

● CORE SPECTRA



Bogoliubov-de Gennes Hamiltonian

$$\mathcal{H} = \begin{pmatrix} \hat{\mathcal{H}}_e & \hat{\Delta} \\ \hat{\Delta}^* & -\hat{\mathcal{H}}_e^* \end{pmatrix}$$

$$\hat{\Delta} \equiv \hat{\mathbb{D}}_{x^2-y^2} \Delta(\vec{r})$$

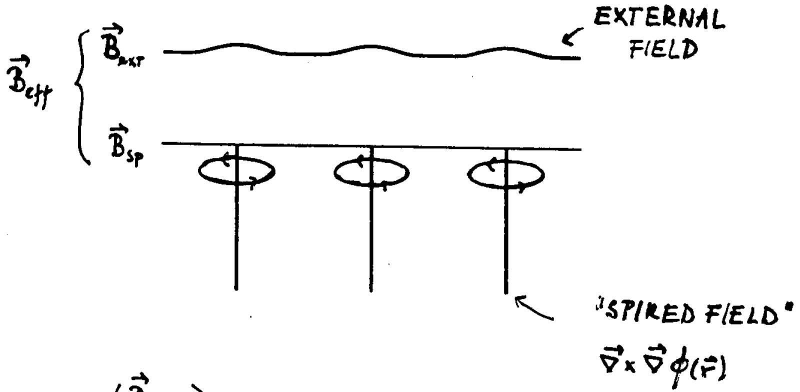
with $\hat{\mathcal{H}}_e = (\mathbf{p} - e\mathbf{A}/c)^2/2m - \epsilon_F$ and $\hat{\Delta}$ the pairing operator with pairing function of the form

$$\underline{\Delta(\mathbf{r}) \simeq \Delta_0 e^{i\phi(\mathbf{r})}}.$$

The phase $\phi(\mathbf{r})$ acts as an "gauge field" coupled to quasiparticles. In the vortex state $\phi(\mathbf{r})$ is <u>*not*</u> a pure gauge, however,

$$\longrightarrow \quad \nabla \times \nabla\phi(\mathbf{r}) = 2\pi\hat{z}\sum_i \delta(\mathbf{r} - \mathbf{R}_i) \quad \longleftarrow$$

and correspond to a "spiked" magnetic field concentrated at vortex cores which on average *exactly cancels* the applied field **B**.



$$\langle \vec{B}_{eff} \rangle = 0$$

<u>EXACT</u> CONSEQUENCE OF FLUX QUANTIZATION

⊙ GAUGE INVARIANT SIMON-LEE HAMILTONIAN (FRANZ & ZT):

$$\hat{\Delta} \equiv \hat{\mathbb{D}}_{x^2-y^2}\Delta(\bar{r}) = \left\{\frac{\partial}{\partial x}, \left\{\frac{\partial}{\partial y}, \Delta\right\}\right\} +$$

$$\underline{\underline{+\, i\left(\frac{\partial^2\phi}{\partial x\partial y}\right)\Delta(\bar{r})}}$$

$\phi(\bar{r})$ - phase of $\Delta(\bar{r})$ in a particular gauge

● FT GAUGE TRANSFORMATION*

$$\phi(\vec r) = \phi^A(\vec r) + \phi^B(\vec r)$$

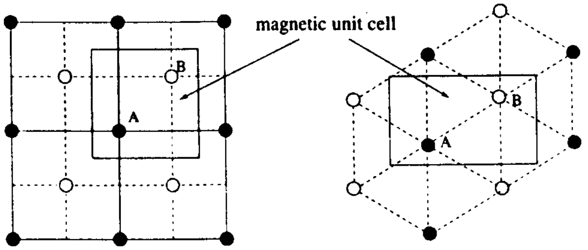


Figure 1: Two sublattices A and B of the square (a) and triangular (b) vortex lattice.

* FRANZ & TESANOVIC
PRL $\underline{84}$, 554 (2000)
MARINELLI, HALPERIN & SIMON
Cond-mat/0001406

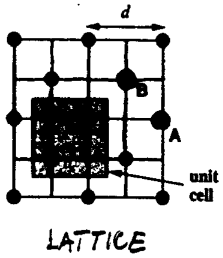
We still have considerable freedom to choose $\phi_A(\mathbf{r})$ and $\phi_B(\mathbf{r})$, subject to the constraint

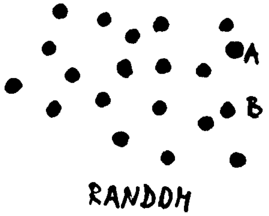
$$\Longrightarrow \quad \phi_A(\mathbf{r}) + \phi_B(\mathbf{r}) = \phi(\mathbf{r}).$$

This corresponds to "trading" the spiked field between electron and hole sectors of $\mathcal{H}'$.

We require that:

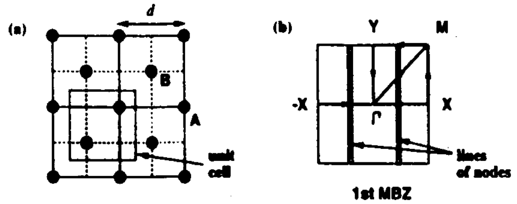
- $\phi_A(\mathbf{r})$ and $\phi_B(\mathbf{r})$ contain no additionial singularities beyond those already present in $\phi(\mathbf{r})$
- Transformation U is single valued
- The effective field $\mathbf{B}^\mu_{\rm eff} = -\frac{mc}{e}(\nabla \times \mathbf{v}^\mu_s)$ vanishes on average

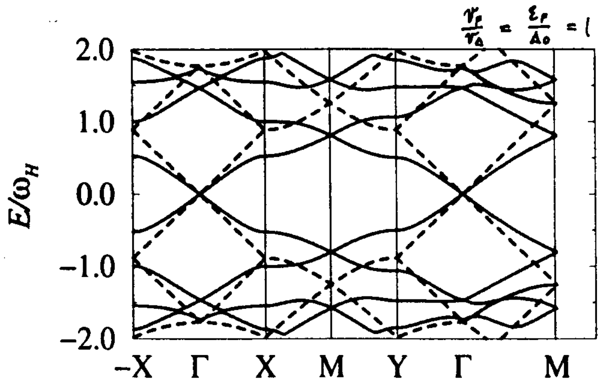


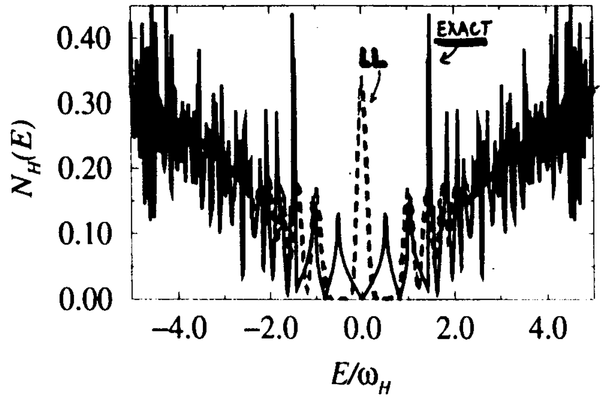


Note that with this choice U is single valued and

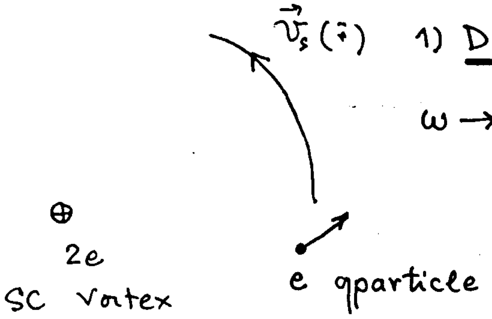
$$\longrightarrow \quad \underline{\bar{\mathbf{B}}^\mu_{\rm eff} = -\frac{mc}{e}\langle\nabla \times \mathbf{v}^\mu_s\rangle = 0.}$$







⊙ INTERACTION OF VORTICES AND QUASIPARTICLES (1) + 2))



1) **DOPPLER SHIFT**:

$$\omega \to \omega' = \omega - \underline{\underline{\vec{v}_s(\vec{r})}} \cdot \vec{k}$$

Volovik ('93)
Sauls ('92)

★ DOPPLER SHIFT LEADS TO $\underline{\underline{\Delta\varphi = \pi}}$ AS QUASIPARTICLE GOES AROUND THE SUPERCONDUCTING (2e) VORTEX ⇒

2) **TOPOLOGICAL TERM**:

$$\vec{k} \to \vec{k}' = \vec{k} - \underline{\underline{\vec{D}(\vec{r})}}$$

$$\underline{\vec{D}(\vec{r}) = \frac{1}{2}\left(\vec{\nabla}\varphi_A - \vec{\nabla}\varphi_B\right)}$$

ACTS AS A VECTOR POTENTIAL OF A DIRAC QUASIPARTICLE AND UNWINDS THE PHASE FOR GLOBALLY CURVED SUPERFLOW (IE, IN PRESENCE OF VORTICES)

Franz-Tešanović
PRL 84, 554 ('00)

ORIGIN OF VORTEX-QPARTICLE INTERACTION

$$\int d\vec{r}\ \hat{\mathbb{D}}_{x^2-y^2}\,\Delta(\vec{r})\,u^*(\vec{r})\,v(\vec{r})$$

If $\Delta(\vec{r}) \sim e^{i\phi(\vec{r})}$ remove this phase by

GT : $u(r) \to e^{-\frac{i\phi}{2}}\,u(r)$

$v(r) \to e^{\frac{i\phi}{2}}\,v(r)$

★ IN PRESENCE OF VORTICES SUCH GT IS NOT SINGLE-VALUED. WE MUST USE DIFFERENT GT:

$u(r) \to e^{-i\phi_A}\,u(r)$ where $\phi_A(r) + \phi_B(r) = \phi(r)$

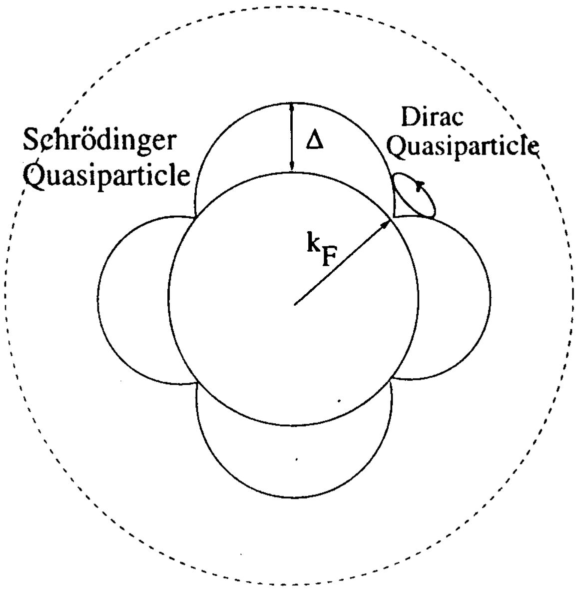
$v(r) \to e^{i\phi_B}\,v(r)$ (Franz-Tešanović) $\Rightarrow$

LOW ENERGY QPARTICLE HAMILTONIAN :

$$\hat{\mathcal{H}}_D \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} v_F(\hat{p}_x + D_x) & +v_\Delta(\hat{p}_y + D_y) \\ +v_\Delta(\hat{p}_y + D_y) & -v_F(\hat{p}_x + D_x) \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = (E - \mu_D) \begin{pmatrix} u \\ v \end{pmatrix}$$

$\mu_D \equiv v_F\, v_{sx}(\vec{r})$ - DOPPLER SHIFT IS SCALAR POTENTIAL

$\vec{D} \equiv \frac{1}{2}(\vec{\nabla}\phi_A - \vec{\nabla}\phi_B)$ - TOPOLOGICAL TERM IS VECTOR POTENTIAL



• $d_{x^2-y^2}$ wave (square vortex lattice)

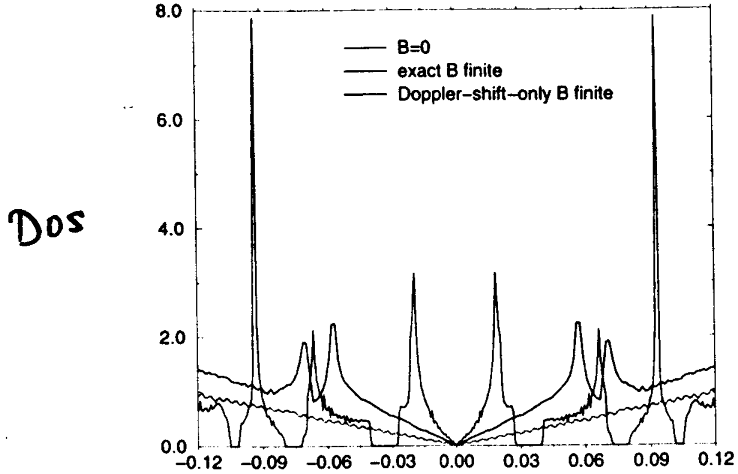


Figure 7: Comparison of the low energy part of the quasiparticle density of states for an d-wave superconductor with square arrangement of vortices with the DOS obtained from the Doppler-shift-only approximation. Plotted on arbitrary scale, the energy is in units of t. The parameters are $\mu = 0$, $\alpha_D = \frac{t}{\Delta_0} = 4$.

APPENDIX A: FIELD THEORY OF ORIENTED LOOP GAS

CONSIDER FREE COMPLEX FIELD THEORY:

$$H = \int d^3x \left(|\nabla\phi|^2 + m^2|\phi|^2 \right)$$

GREEN FUNCTION $G(\vec{x})$ IS:

$$\langle \phi(0)\phi^*(\vec{x}) \rangle = G(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{x}}}{k^2+m^2}$$

SCHWINGER PROPER TIME (S) REPRESENTATION:

$$G(\vec{x}) = \int_0^\infty ds\, e^{-sm^2} \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x} - sk^2} =$$

$$= \int_0^\infty ds\, e^{-sm^2} \left(\frac{1}{4\pi s}\right)^{3/2} e^{-\frac{1}{4}\frac{x^2}{s}}$$

* USING FEYNMAN PATH INTEGRALS:

$$\boxed{\left(\frac{1}{4\pi s}\right)^{3/2} e^{-\frac{1}{4}\frac{x^2}{s}} = \int_{\vec{x}(0)=0}^{\vec{x}(s)=\vec{x}} \mathcal{D}\vec{x}(s')\, \exp\left[-\frac{1}{4}\int_0^s ds'\, \dot{\vec{x}}^2(s')\right]}$$

WE USE FEYNMAN REPRESENTATION TO WRITE (UP TO A CONSTANT):

$$\mathrm{Tr}\ln\left[-\nabla^2+m^2\right]=\int\frac{d^3k}{(2\pi)^3}\ln\left(k^2+m^2\right)=$$

$$=-\int_0^\infty\frac{ds}{s}\,e^{-sm^2}\int\frac{d^3k}{(2\pi)^3}\,e^{-sk^2}=$$

$$=-\int_0^\infty\frac{ds}{s}\,e^{-sm^2}\oint\mathcal{D}\vec{x}(s')\,\exp\left[-\frac{1}{4}\int_0^s ds'\,\dot{x}^2(s')\right]$$

$$\downarrow$$

CLOSED LOOPS STARTING AND ENDING AT ϕ.

NOW USE: $\det^{-1}\left(-\nabla^2+m^2\right)\equiv\exp(-\Gamma)$ AND

$$\mathrm{Tr}\ln\left(-\nabla^2+m^2\right)=\ln\det\left(-\nabla^2+m^2\right)\quad\Rightarrow$$

$$\boxed{e^{-\Gamma}=\sum_{N=0}^{\infty}\frac{1}{N!}\prod_{\ell=1}^{N}\left[\int_0^\infty\frac{ds_\ell}{s_\ell}\,e^{-m^2 s_\ell}\oint\mathcal{D}\vec{x}(s_\ell')\right]\exp\left[-\frac{1}{4}\sum_{\ell}^{N}\int_0^{s_\ell}ds_\ell'\,\dot{x}^2(s_\ell')\right]}$$

FREE LOOP GAS

☆ WITH SHORT RANGE REPULSION:

$$e^{-\frac{g}{4}|\phi|^4}=\int\mathcal{D}\lambda(\vec{x})\,e^{-i\lambda(\vec{x})|\phi|^2-\frac{|\lambda(\vec{x})|^2}{g}}$$

$\Rightarrow$ SELF-AVOIDING LOOP GAS

APPENDIX B: RG SOLUTION OF DUAL THEORY

$$\boxed{\begin{array}{l}\text{I. F. HERBUT \& ZT, PRL \underline{76}, 4588 ('96)}\\ \qquad\qquad\quad\text{PRL \underline{78}, 980 ('97)}\end{array}}$$

$$m_\phi^2|\phi|^2+|(\nabla-\hat{A}_d)\phi|^2+\frac{g}{4}|\phi|^4+\frac{1}{2e_d^2}(\nabla\times\breve{A}_d)^2$$

$e_d^2\sim\langle|\psi|^2\rangle$ FLUCTATING DENSITY OF COOPER PAIRS $\hookrightarrow$

$$\beta_e(g,e_d)\equiv\frac{de_d^2}{d\log(p)}=-e_d^2+\frac{\textcircled{C}}{16}e_d^4+\mathcal{O}\left(e_d^6,ge_d^4\right)$$

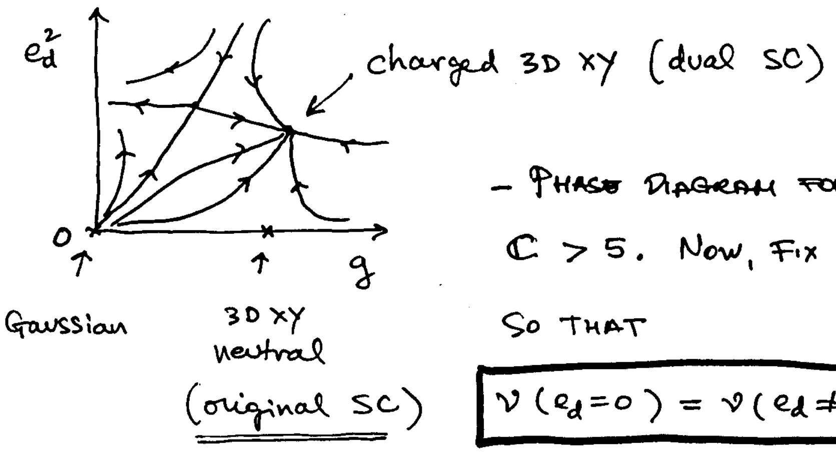
$$\beta_g(g,e_d)\equiv\frac{dg}{d\log(p)}=-g+\frac{2\sqrt{2}+1}{8}g^2-\frac{1}{2}ge_d^2+\frac{1}{2\sqrt{2}}e_d^4+$$
$$+\mathcal{O}\left(g^3,e_d^6,ge_d^4,g^2e_d^2\right)$$

e_d – dimensionless <u>dual</u> charge (Biot-Savart)

g – short range repulsion between vortex loop segments

$\mathbb{C}$ – treshold function of non-perturbative RG

PHASE DIAGRAM:



– PHASE DIAGRAM FOR $\mathbb{C} > 5$. NOW, FIX $\mathbb{C}$ SO THAT

$$\boxed{\nu(e_d=0) = \nu(e_d \neq 0)\,!}$$

THIS ASSURES THAT DUAL THEORY HAS THE SAME THERMODYNAMICS AS THE ORIGINAL THEORY!

REMARKABLY, THERE IS SUCH A SOLUTION:

$$\nu_{dual} = \nu_{SC} = \nu_{3DXY} \sim 0.67$$

WITH $\mathbb{C}$ THUS FIXED $\Rightarrow$ $\boxed{\eta_{dual} \sim -0.2\,!}$

ALSO, $\boxed{\eta_{A_d} = 1 \text{ – EXACT !}}$

COMPARE TO SIMULATIONS BY SUDBØ ET AL.