Lecture II, A. Sudbø

Geometric properties of critical fluctuations in abelian gauge-theories

$\underline{\underline{T = T_c}}$

- Anomalous scaling dimension of dual field, η
- Fractal dimension D_H of critical fluctuations
- Vortex-loop distribution $D(N) \sim N^{-\alpha}$
- $f(\alpha, D_H, \eta) = ?$ Scaling relations

Connect geometric properties of critical fluctuations to thermodynamic properties of dual field-theory

- NB!! Finite-field transitions

Fractal dimension D_H of critical fluctuations at zero field determines if finite-field transition can take place

Anomalous scaling dimensions, Ψ, ϕ

A.K.Nguyen and A. S., PRB **60**, 15307 (1999)

$$\lim_{q\to 0} < \Psi^*_q \Psi_{-q} > \; \sim \; \frac{1}{q^{2-\eta_\Psi}}; \quad T = T_c$$

$$\lim_{q\to 0} < \phi^*_q \phi_{-q} > \; \sim \; \frac{1}{q^{2-\eta_\phi}}; \quad T = T_c$$

$e = 0$: $\eta_\Psi = 0.038$ ($3DXY$ universality)

$e_d \neq 0$: $\eta_\phi = \; ?$ (Other universality?)

Vortex loop distribution function:

$$\begin{aligned} D(p) &= A\, p^{-\alpha} \; \exp[-\beta \;\; \varepsilon(T) \;\; p] \\ \varepsilon(T) &= \sigma_0 \, |T - T_c|^{\gamma_\phi}; \quad T \to T_c^- \\ \gamma_\phi &= 1.45 \pm 0.05 \end{aligned}$$

$$\varepsilon(T) \Leftrightarrow m_\phi^2$$

$$\lim_{q\to 0} \quad < \phi^*_q \; \phi_q > = \frac{1}{m_\phi^2} = \chi_\phi = |\tau|^{-\gamma_\phi}; T < T_c$$

$$\sim \quad |\tau|^{-\nu_\phi(2-\eta_\phi)} \Rightarrow \eta_\phi = -0.18 \pm 0.07 < 0!!$$

Geometric interpretation of $\eta_\phi < 0$

Neutral GLT:

$$H_\Psi = m_\psi^2|\Psi(r)|^2 + \frac{u_\psi}{2}|\Psi(r)|^4 + |\nabla\Psi(r)|^2$$

Charged DGLT:

$$\begin{aligned} H_{\phi,\mathbf{h}} &= m_\phi^2|\phi(r)|^2 + \frac{u_\phi}{2}|\phi(r)|^4 \\ +|(\frac{\nabla}{i} &- e_d\mathbf{h})\phi(r)|^2 + \frac{1}{2}(\nabla\times\mathbf{h})^2 \end{aligned}$$

ϕ-field $\Leftrightarrow$ Ψ-field at different point in H-space

$$<\phi^\dagger(x)\phi(0)> = \frac{1}{|x|^{1+\eta_\phi}} \quad \mathcal{G}_\pm(\frac{|x|}{\xi})$$

- $\eta_\Psi = 0.038$: Vortex-tangle less packed than free loops. $|\Psi|^4$-term: Steric repulsion.
- $\eta_\phi = -0.18 \pm 0.07$: Vortex-tangle more packed than random walkers, without collapsing! Biot-Savart.
- Only in $D = 3$!

$<\mathbf{A_q A_{-q}}>, <\mathbf{h_q h_{-q}}>, \quad e \neq 0$

J. Hove and A. S., condmat/0002197, to appear in PRL, 84, 3426 (2000)

Renormalized by vortex-loop fluctuations:

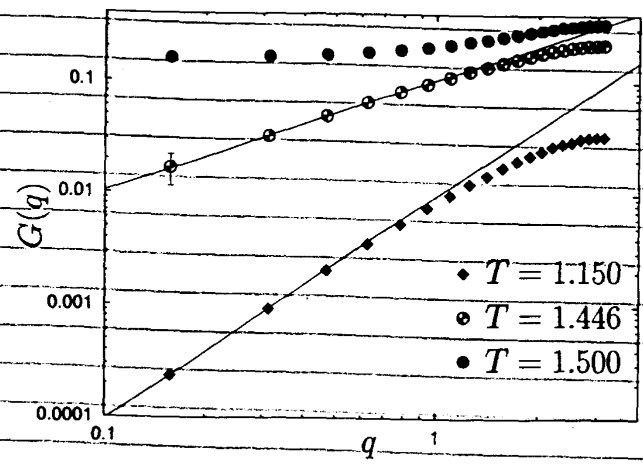
$$\begin{aligned} <\mathbf{A_q A_{-q}}> &= \frac{1}{q^2+e^2}\left(1 + \frac{4\pi^2\beta e^2\ G(\mathbf{q})}{q^2\ (q^2+e^2)}\right) \\ <\mathbf{h_q h_{-q}}> &= \frac{2\beta}{q^2+e^2}\left(1 - \frac{2\pi^2\beta\ G(\mathbf{q})}{q^2+e^2}\right) \\ G(\mathbf{q}) &= <\mathbf{m_q m_{-q}}> \end{aligned}$$

$G(\mathbf{q})$ probes a vortex-loop blowout at T_c:

$$\begin{aligned} \lim_{q\to 0} G(\mathbf{q}) &= q^2, \quad T < T_c \\ \lim_{q\to 0} G(\mathbf{q}) &= q^\eta, \quad T = T_c \\ \lim_{q\to 0} G(\mathbf{q}) &= C, \quad T > T_c \end{aligned}$$

$$\begin{aligned} \lim_{q\to 0} <\mathbf{A_q A_{-q}}> &\sim \frac{1}{q^{2-\eta}}; \quad T = T_c \\ \lim_{q\to 0} <\mathbf{h_q h_{-q}}> &= \frac{2\beta}{q^2}; \quad T = T_c \end{aligned}$$

Vortex-correlator, $\langle \mathbf{m_q m_{-q}} \rangle$, $e \neq 0$



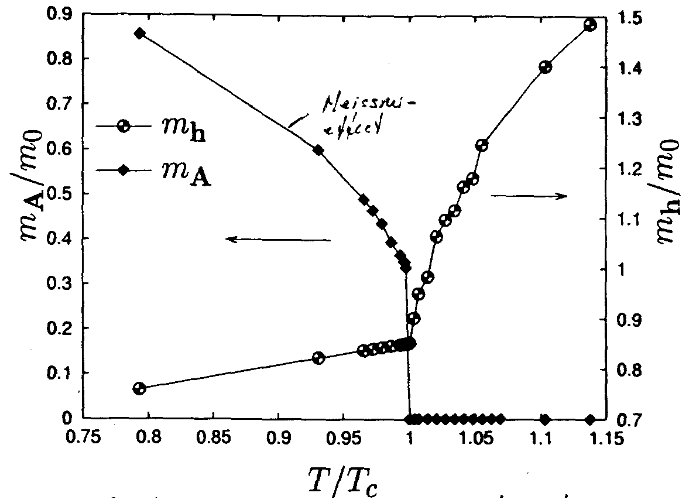
$$\frac{\partial e'^2}{\partial \ln l} = (D-4+\eta_A)\, e'^2$$

FP: $\eta_A = 4-D$; $e'^2 \neq 0$

Figure 1.

J. Hove and A. Sudbø : Anomalous scaling dimensions and stable charged fixed-point of type-II superconductors.

Gauge-field masses $m_{\mathbf{A}}$ and $m_{\mathbf{h}}$ $e \neq 0$



$\vec{h}$-field mediates Biot-Savart interactions between vortex-segments

Non-analytic change in screening at T_c

Figure 2.

J. Hove and A. Sudbø : Anomalous scaling dimensions and stable charged fixed-point of type-II superconductors.

$< \mathbf{A_q A_{-q}} >, < \mathbf{h_q h_{-q}} >, \quad e = 0$

Renormalized by vortex-loop fluctuations:

$$< \mathbf{A_q A_{-q}} > \; = \; \frac{1}{q^2}\left(1 + \frac{4\,\pi^2\,\beta\,e^2\,G(\mathbf{q})}{q^4}\right) \simeq \frac{1}{q^{2-\eta_A}}$$

$\eta_A = 0$

$$< \mathbf{h_q h_{-q}} > \; = \; \frac{2\beta}{q^2}\left(1 - \frac{2\,\pi^2\,\beta\,G(\mathbf{q})}{q^2}\right) \simeq \frac{1}{q^{2-\eta_h}}$$

$$G(\mathbf{q}) \; = \; < \mathbf{m_q m_{-q}} >$$

$G(\mathbf{q})$ probes a vortex-loop blowout at T_c:

$$\lim_{q\to 0} 2\beta\pi^2 G(\mathbf{q}) \overset{T\leq T_c}{=} (1 - C_2)\; q^2 + \ldots$$

$$\lim_{q\to 0} 2\beta\pi^2 G(\mathbf{q}) \overset{T=T_c}{=} q^2 - C_3\; q^{2+\eta_h} + \ldots$$

$$\lim_{q\to 0} 2\beta\pi^2 G(\mathbf{q}) \overset{T\geq T_c}{=} q^2 - C_4\; q^4 + \ldots$$

$$\lim_{q\to 0} < \mathbf{h_q h_{-q}} > \; = \; \frac{2\beta C_2}{q^2}; \quad T < T_c$$

$$\lim_{q\to 0} < \mathbf{h_q h_{-q}} > \; = \; \frac{2\beta C_3}{q^{2-\eta_h}}; \quad T = T_c$$

$$\lim_{q\to 0} < \mathbf{h_q h_{-q}} > \; = \; 2\beta C_4; \quad T > T_c$$

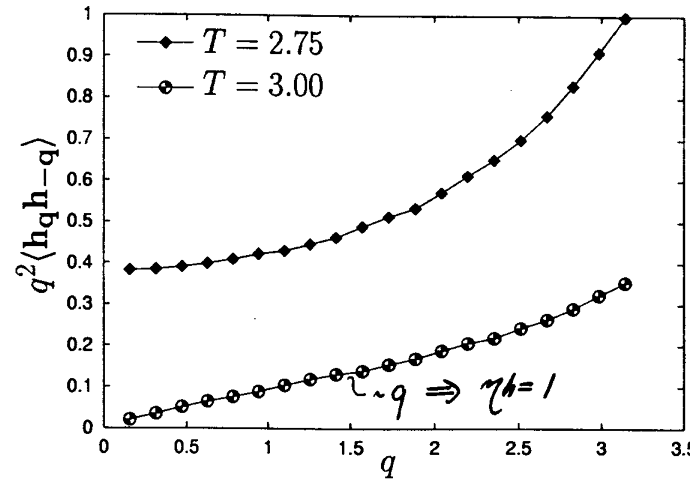
„Dual Meissner-effect" above T_c!

$q^2 < \mathbf{m_q m_{-q}} >, \quad e = 0$

Dual charged case, $e_d = 2\pi < |\Psi|^2 >^{1/2}$

$$q^2 \; < \mathbf{h_q h_{-q}} > \; \sim \; q^{\eta}$$

$$\eta = 1: \quad T \; = \; T_c$$



$$\frac{\partial e_d'^2}{\partial \ln l} = (D - 4 + \eta_h)\, e_d'^2 \qquad \text{(Gauge-invariance)}$$

FP, $e_d^{*} \neq 0 \Rightarrow \eta_h = 4 - D$

Figure 3.

J. Hove and A. Sudbø: Anomalous scaling dimensions and stable charged fixed-point of type-II superconductors.
to be published, PRL, condmat/0002197

Experimental consequences

Superfluid density: $\rho_s \sim \xi^{2-D}$
Free energy density: $f \sim l^{-D}$
Gauge-field: $A^2 \sim l^{-(D-2+\eta_A)}$

$$\rho_s \sim \frac{\partial^2 f}{\partial A^2}$$
$$\rho_s \sim l^{-2+\eta_A} \sim \lambda^{-2+\eta_A}$$

$$\lambda \sim \xi^{\frac{D-2}{2-\eta_A}}$$

$$e = 0, D = 3 : \eta_A = 0 \Rightarrow \lambda \sim \sqrt{\xi}$$
$$e \neq 0, D = 3 : \eta_A = 1 \Rightarrow \lambda \sim \xi$$

SUMMARY

- Two separate fixed points in type-II superconductors:
  i) 3DXY (unstable, $e \neq 0$),
  $$\lambda \sim \sqrt{\xi}$$
  ii) Charged fixed point (stable), $e \neq 0$,
  $$\lambda \sim \xi$$
- The dual field ϕ provides a local OP for a $U(1)$-symmetry breaking phase-transition inside the vortex liquid phase, which is not provided by Ψ in GLT.

e^2	η_A, η_Ψ	FPGLT	η_h, η_ϕ	FPDGLT
$=0$	0, 0.038	3DXY	1, -0.18	Charged
>0	1, -0.18	Charged	0, 0.038	3DXY