Random walkers, $d=3$

Field theory: $H = m_\varphi^2 |\varphi|^2 + |\nabla\varphi|^2$

Probability of going from $\vec{x}$ to $\vec{y}$ in N steps:

$$P(x,y;N) \sim \frac{1}{N^{3/2}}\, e^{-\frac{(\vec{x}-\vec{y})^2}{2N}}$$

- $G(x,y) = \sum_N P(\vec{x},\vec{y};N) \sim \dfrac{1}{|\vec{x}-\vec{y}|^{d-2+\eta}}$
- $D(N) = \dfrac{1}{N} \sum_{\vec{x}} P(\vec{x},\vec{x};N) \sim \dfrac{1}{N^{\alpha}}$
- $\left(\langle |\vec{x}-\vec{y}|^2 \rangle\right)^{1/2} \sim N^{\Delta}$; Δ: Wandering exponent.

Gaussian case:

$$\boxed{\eta = 0, \quad \Delta = \frac{1}{2}, \quad \alpha = \frac{5}{2}}$$

Fractal dimension (Hausdorff-dim.) of random walks:

$$N_{max} \sim L^{D_H} \Leftrightarrow D_H = \frac{1}{\Delta}$$

$$\boxed{D_H^{r.w.} = 2}$$

Note: $\underline{\underline{\eta + D_H = 2}}$; $\underline{\underline{D_H = \dfrac{d}{\alpha - 1}}}$

J. Hove, S. Mo, + A.S., Phys. Rev. Lett. (in press)

Interacting vortex loops $T = T_c$

Field theory: $H = m_\varphi^2 |\varphi|^2 + |D\varphi|^2 + \frac{u}{2}|\varphi|^4 + \frac{1}{2}(\nabla \times A)^2$

$$P(\vec{x},\vec{y};N) \sim \frac{1}{N^{\varrho}}\, F\left(\frac{|\vec{x}-\vec{y}|}{N^{\Delta}}\right)$$

- Normalizability of P: $\underline{\underline{\varrho = d\Delta}}$
- $G(\vec{x},\vec{y}) = \sum_N P(\vec{x},\vec{y};N)$

$$= \frac{1}{|\vec{x}-\vec{y}|^{\frac{\varrho-1}{\Delta}}} = \frac{1}{|\vec{x}-\vec{y}|^{d-2+\eta}}$$

$$\underline{\underline{\frac{\varrho-1}{\Delta} = d-2+\eta}}$$

- $D(N) = \frac{1}{N}\sum_{\vec{x}} P(\vec{x},\vec{x};N) \Rightarrow \underline{\underline{\varrho = \alpha - 1}}$

Exact result, general case!!

$$\boxed{D_H = \frac{d}{\alpha-1} \qquad \eta + D_H = 2}$$

$\alpha > \frac{5}{2} \Rightarrow D_H < 2$

$\alpha < \frac{5}{2} \Rightarrow D_H > 2$

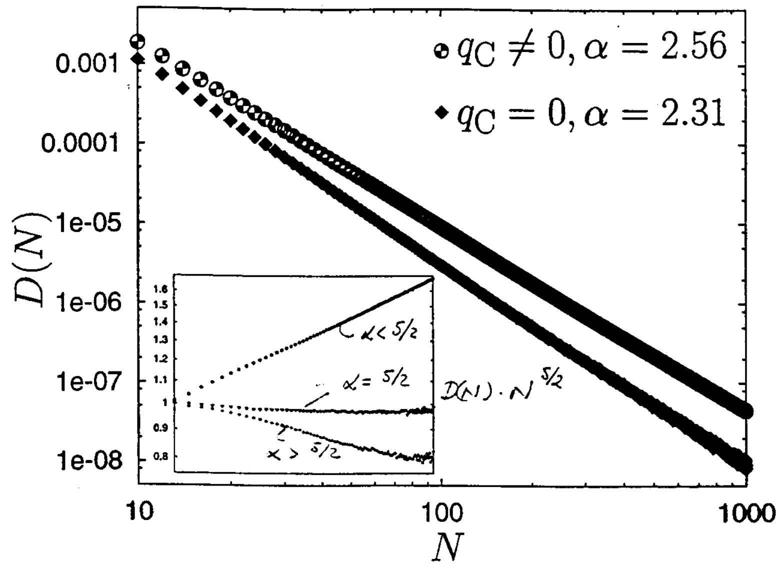
Original neutral: $\eta < 0 \Rightarrow D_H > 2$

Original charged: $\eta > 0 \Rightarrow D_H < 2$

Why does <u>neutral</u> system blow out more vortex-loops than <u>charged</u> system?

Easiest to compute: $\underline{\underline{\alpha}}$; $D(N) \sim N^{-\alpha}\big|_{T=T_c}$

J. Hove, S. Mo, + A.S., PRL (in press)



$$H = -\sum_{ij} \cos(\theta_i - \theta_j - q_c A_{ij}) + \frac{1}{2}(\nabla \times \vec{A})^2$$

Figure 1.

J. Hove, S. Mo and A. Sudbø : Hausdorff dimension of critical fluctuations in abelian gauge theories.

$q_c \neq 0 : \alpha > \frac{5}{2} \Rightarrow D_H < 2$

$q_c = 0 : \alpha > \frac{5}{2} \Rightarrow D_H > 2$

Infinitely much more screening (large) vortex-loop in neutral system compared to charged system.

$D_H > 2$: Loop-transition survives finite field (Extreme type-II superconductor)

NB!!

Vortex loops, entropy, and compressibility

- Original neutral system has L.R. X-ions between vortex segments. Incompressible vortex-system
- Original charged: L.R. X-ions are screened. Compressible vortex system
- Gain in configurational entropy easier for compressible vortex system

Finite compressibility: Good
Incompressibility: Bad, because entropy is "frustrated"!

Infinitely large amount of screening (large) vortex loops in neutral case acts as substitute for gauge-field fluctuations to make vortex tangle of neutral system compressible

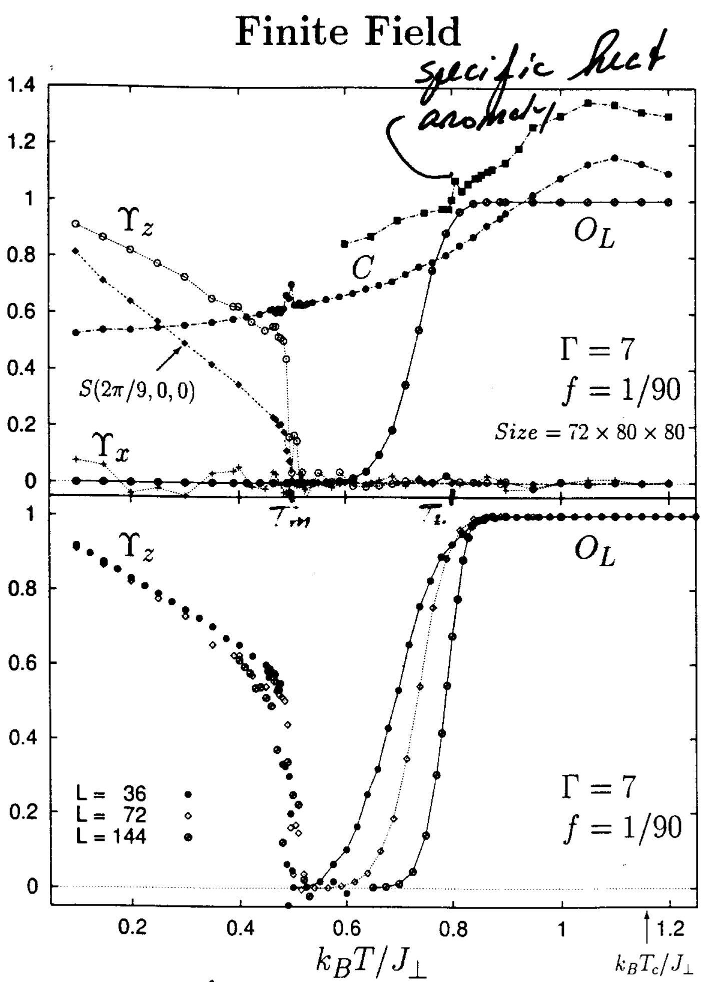
$D_H = 3$: Vortex-tangle collapses on itself $\Rightarrow \eta = -1$

Scaling relation; Critical exponents $\beta = \frac{\nu}{2}(1+\eta)$; $\eta \to -1 \Rightarrow \beta \to 0$

1. order P.T.

NTNU

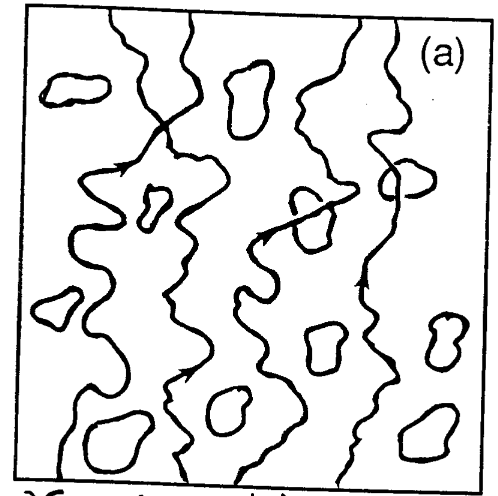
A. K. Nguyen + A. S., Europhys. Lett., 46, 780 (1999)



See also A. K. Nguyen + A.S. PRB 60, 15307 (1999)

Vortex-loop „blowout", finite magn. field

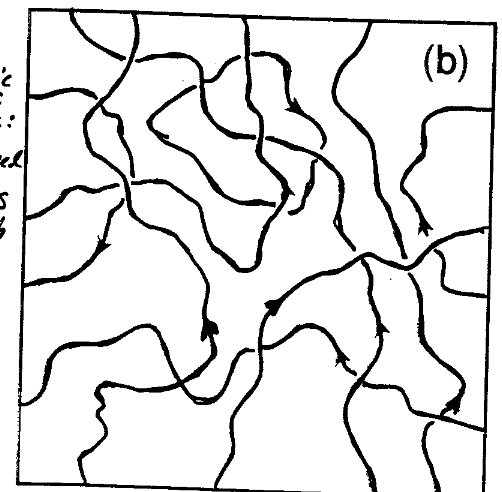
U(1)-symmetric vortex state: Conserved # vortex lines going through the system



$T > T_m$
$T < T_L$: Vortex-line liquid
Directed vortex lines with nonzero linetension

$\Upsilon_z \sim \langle|\psi| e^{i\theta}\rangle = 0$
$\langle\Phi\rangle = 0$

U(1)-nonsymmetric vortex state: Non-conserved # vortex lines going through the system



$T > T_L$: Vortex-line tension is zero. No longer a vertex liquid of directed vortex lines

$\Upsilon_z \sim \langle|\psi| e^{i\theta}\rangle = 0$; $\langle\Phi\rangle \neq 0$

Phase-transition at $T_L(B)$: No signature in local O.P. $\langle|\psi| e^{i\theta}\rangle$. Signature: Change in characteristics of ...

Field-temperature phase-diagram

A. K. Nguyen and A. S. Europhys. Lett., **46**, 780 (1999)

A. K. Nguyen and A. S. Phys. Rev. **B 60**, 15307, (1999).

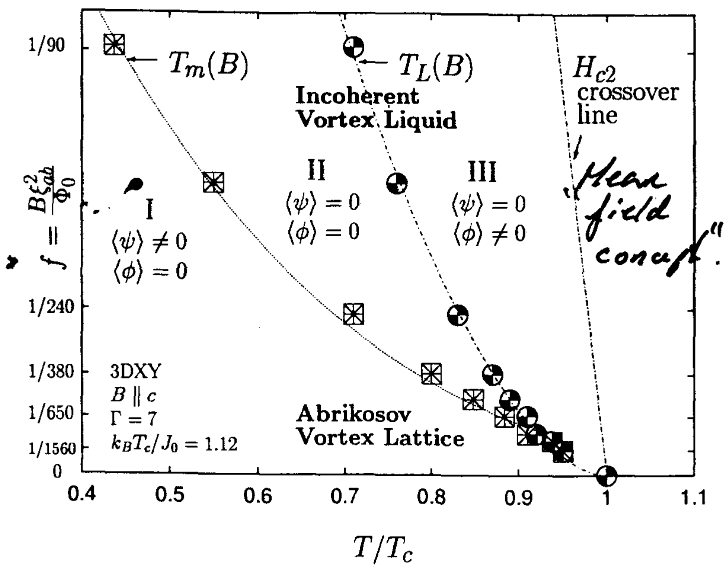


Figure 12

I: Vortex lattice
II: Vortex-LINE liquid
III: "Vortex-blown out" state.

Relevant papers

- Z. Tesanovic, Phys. Rev. **B 59**, 6449 (1999)
- S. K. Chin, A. K. Nguyen, and A. Sudbø, Phys. Rev. **B 59**, 14017 (1999)
- A. K. Nguyen and A. Sudbø, Europhys. Lett., **46**, 780 (1999)
- A. K. Nguyen and A. Sudbø, Phys. Rev. **B 60**, 15307, (1999).
- A. Sudbø, A. K. Nguyen, and J. Hove, cond-mat/9907386
- J. Hove and A. Sudbø, cond-mat/0002197, to be published in PRL 84, 3426, 2000
- I. F. Herbut and Z. Tesanovic, Phys. Rev. Lett. **76**, 4588 (1996)
- I. F. Herbut, J. Phys. **A 30**, 423 (1997)
- J. Hove, S. Ho, + A.S. Phys. Rev. Lett. (in press)