Lecture I, A. Sudbø

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Topological excitation in

2D, 3D superfluids/superconductors

- Phase fluctuations of s.c. OP $\Psi = |\Psi| e^{i\theta}$
  - i) Vortex-antivortex pairs (2D) (Kosterlitz-Thouless)
  - ii) Closed vortex loops (3D) ($\neq$ vortex rings)

  Vortex loop of perimeter p in volume $V = L^3$

  $p_{max} \sim L^{\omega}$ $\omega = 1$: Vortex ring
  $\omega = 2$: Random walkers

  In genl. $\omega \notin Z \Rightarrow$ vortex loops are fractal objects on long length scales

- Local field theory for (critical) vortex loop tangle: Dual theory

φ: Dual matter field φ } Physical
$\vec{h}$: Dual gauge field $\vec{h}$ } interpretation

- φ provides "hidden" OP for novel PT in extreme type-II s.c's and superfluids

- Experimental verification?

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NTNU

Ginzburg-Landau theory

OK for s-wave, d-wave, ... close to T_c

$$\boxed{\begin{aligned} H_{\Psi,\mathbf{A}} &= m_\psi^2 |\Psi(r)|^2 + \frac{u_\psi}{2} |\Psi(r)|^4 \\ &+ |(\frac{\nabla}{i} - 2e\mathbf{A})\Psi(r)|^2 + \frac{1}{2}(\nabla \times \mathbf{A})^2 \end{aligned}}$$

$$\Psi(\vec{r}) = |\Psi(\vec{r})| \exp[i\,\theta(\vec{r})]$$

$|\Psi|^2$: Local Cooper-pair density
$\nabla\theta - e\vec{A}$: Superflow

Complex matter field Ψ coupled to massless gauge-field $\mathbf{A}$ with charge $2e$.

$He^4 : e = 0$

$$\boxed{H_\Psi = m_\psi^2 |\Psi(r)|^2 + \frac{u_\psi}{2} |\Psi(r)|^4 + |\nabla \Psi(r)|^2}$$

$$\Psi(r) = |\Psi(r)| \exp[i\,\theta(r)]$$

- $e \neq 0 \Rightarrow H_{\Psi,\mathbf{A}}$ exhibits local gauge-symmetry $\theta(\vec{r}) \to \theta(\vec{r}) + \int^{\vec{r}} d\vec{l} \cdot \vec{A}(l)$, $\vec{A} \to \vec{A} + \nabla\theta$
- $e = 0 \Rightarrow H_\Psi$ exhibits global $U(1)$-symmetry $\theta(\vec{r}) \to \theta(\vec{r}) + \theta_0$

Extreme type-II (doped Mott-Hubbard insulators): $\vec{A}$-fluctuations strongly suppressed.

$M^2S - HTSC - VI$. Houston. February 20-25 2000

$M_\psi^2 \sim 1 - \frac{T}{T_c^{MF}}$

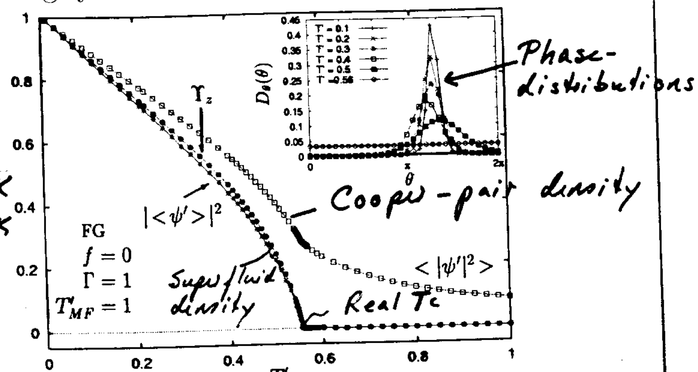
T_c^{MF}: „Mean-field T_c" (input-parameter in GLT)

Amplitude and phase fluctuations

A. K. Nguyen and A. S. PRB **60**, 15307 (1999)

Mean-field T_c" ~ 150K

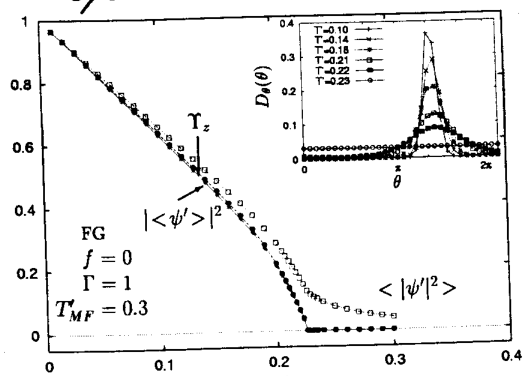
Real T_c ~ 100K



System sizes: $140 \times 140 \times 140$

Mean-field T_c" ~ 50K

Real T_c: ~30K



Υ: Superfluid density

Gauge-fluctuations not included!

Figure 1

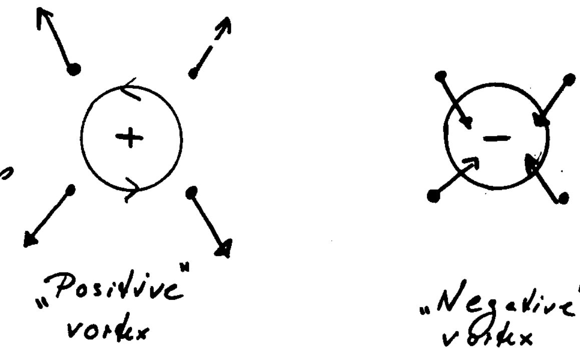
Amplitude-fluctuations unimportant at Tc

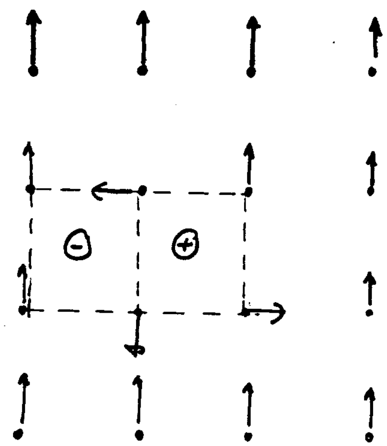
T>Tc: Incoherent Cooper-pair liquid. Pseudogap?

2D superfluids

$\psi = |\psi| e^{i\theta}$

Phase-fluctuations





Thermal fluct. of $\theta \Rightarrow$ Vortex-antiv. pairs

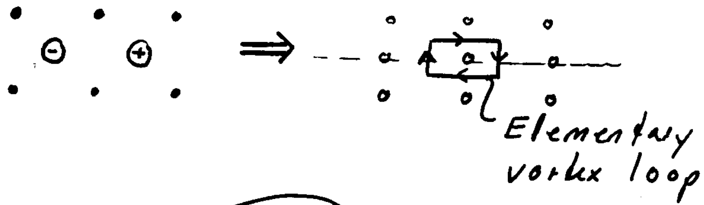
$T = T_c$: 2D Coulomb system = critical sector of the 2D GLT

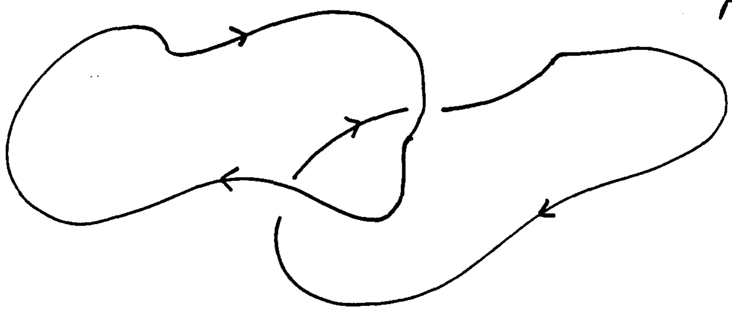
Unbinding of „dipoles" = Kosterlitz Thouless transition. Vanishing superfluid density. No local OP.

3D: Longitudinal phase-flucts. do not destroy Long Range Order

Transverse phase-fluctuations do! $\Leftrightarrow$ closed vortex-loops

3D superfluids: Vortex segments cannot start or end inside supercond.





$T = T_c$: Vortex-loop tangle = critical sector of Ginzburg-Landau th. in 3D. Field theory of this sector: Dual theory

Contributions to the energy of the interacting vortex-system:

- Vortex-line tension (3D)
- Steric repulsion between vortex segments (short distances)
- Biot-Savart interaction between vortex-segments (also at long distances)
- Bending energy (3D)

3D vortex-loop theory:

Dual theory

~ vortex-line tension

$$H_{\phi,\mathbf{h}} = m_\phi^2|\phi(r)|^2 + \frac{u_\phi}{2}|\phi(r)|^4 + |(\frac{\nabla}{i} - e_d\mathbf{h})\phi(r)|^2 + \frac{1}{2}(\nabla\times\mathbf{h})^2 + \frac{e^2}{2}\mathbf{h}^2; \quad \boxed{\frac{\partial e^2}{\partial \ln l} = e^2}$$

Exact. Gauge-invariance

$$\boxed{\begin{array}{c}\phi(r) = |\phi(r)|\ \exp[i\ \omega(r)] \\ e_d^2 = 2\pi < |\Psi|^2 > \end{array}}$$

"Dual" charge

$\underline{e=0}$: $$H_{\phi,\mathbf{h}} = m_\phi^2|\phi(r)|^2 + \frac{u_\phi}{2}|\phi(r)|^4 + |(\frac{\nabla}{i} - e_d\mathbf{h})\phi(r)|^2 + \frac{1}{2}(\nabla\times\mathbf{h})^2$$ Like the original charged case

Dual of neutral superfluid ⇔ superconductor

$\underline{e \neq 0}$: $e^2 \to \infty, l \to \infty \Rightarrow \mathbf{h}$ supressed

$$H_\phi = m_\phi^2|\phi(r)|^2 + \frac{u_\phi}{2}|\phi(r)|^4 + |\nabla\phi(r)|^2$$ Like the original neutral case

Dual of superconductor ⇔ neutral superfluid

GLT	DGLT
Complex matter field ψ coupled to massless gauge-field $\vec{A}$	• Complex matter field $\varphi = \lvert\varphi\rvert e^{i\omega}$ coupled to gauge-field $\vec{h}$
$\lvert\psi\rvert^2$: Local Cooper-pair density	• $\lvert\varphi\rvert^2$: Local vortex density
$\nabla\theta$: Supercurrent	• $\nabla\omega$: Vortex-current
$\nabla\times\nabla\theta$: Vorticity of supercurrent $\Leftrightarrow$ local magnetic field	• $\nabla\times\nabla\omega$: Vorticity of vortex-current $\Leftrightarrow$ local supercurrent
$\vec{A}$: Gauge-field that mediates Coulomb-interactions between electrons $\vec{\nabla}\times\vec{A}$ = magnetic field	• $\vec{h}$: Gauge-field that mediates interactions (Biot-Savart) between vortex-segments $\vec{\nabla}\times\vec{h}$ = electric field
$\langle\psi\rangle \neq 0$: Broken symmetry $\Leftrightarrow$ onset of superconducting order	• $\langle\varphi\rangle \neq 0$: Broken symmetry $\Leftrightarrow$ onset of disorder $\Leftrightarrow$ OP for normal state
$\lvert\psi\rvert^4$-term: „Coulomb" repulsion (short-range) repulsion between Cooper-pairs.	• $\lvert\varphi\rvert^4$-term: Steric (short range) repulsion between vortex segments

Vortex path probability and dual symmetry breaking

O_L

Probability $G(\vec{x}, \vec{y})$ of finding connected vortex-path between $\vec{x}$ and $\vec{y}$

$$G(\vec{x},\vec{y}) = \langle \phi^*(\vec{x})\, \phi(\vec{y}) \rangle$$

$$\lim_{|\vec{x}-\vec{y}|\to\infty} G(\vec{x},\vec{y}) = \langle \phi^* \rangle \ \langle \phi \rangle$$

O_L: Probability of finding at least one connected vortex path connecting opposite sides of system.

Scaling Ansatz: $G(x,y) = \dfrac{1}{|x-y|^{d-2+\eta}}\, \mathcal{G}\!\left(\dfrac{|x-y|}{\xi}\right)$

$$\boxed{O_L \neq 0 \ \Leftrightarrow \ \langle o^* \rangle \ \langle o \rangle \neq 0 \Leftrightarrow \langle o \rangle \neq 0}$$

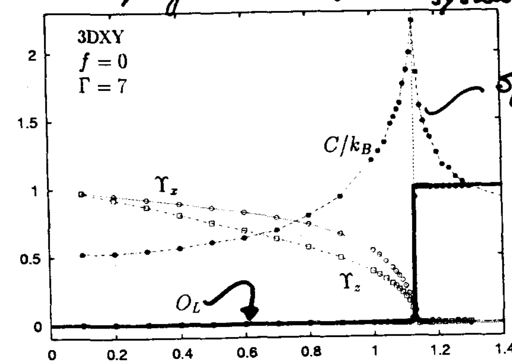
O_L	$\neq 0$	:	U(1) − symmetry broken
O_L	$= 0$	:	U(1) − symmetry conserved
$\langle o \rangle$	$= 0$	:	Vortex − loop confinement
$\langle o \rangle$	$\neq 0$	:	Vortex − loop blowout

$o(\vec{r})$: Local order parameter for vortex-loop blowout. Probed by computing O_L.

Critical point: Vortex-loop blowout

A. K. Nguyen and A. S. Europhys. Lett., **46**, 780 (1999); Phys. Rev. **B60**, 15307 (1999)

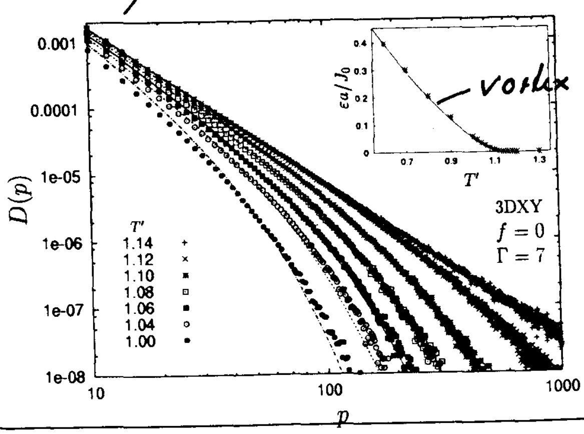
O_L: Probability of connecting opposite sides of system via connected vortex path



Specific heat.

$O_L : 0 \to 1 \Leftrightarrow$ "geometric" transition in vortex-tangle: Percolation

System size 360 × 360 × 360



vortex line tension

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Figure 4

$$D(p) = A p^{-\alpha} \exp(-\beta p \,\epsilon(T)) ; \; L_0 = \frac{k_B T}{\epsilon}$$

Vortex-loop "blowout"
zero magnetic field

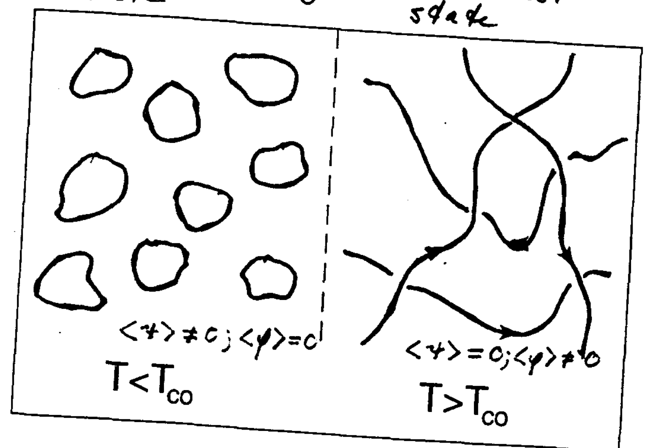
$$D(p) \sim p^{-5/2} \cdot e^{-\frac{\epsilon p}{k_B T}}$$

$T < T_{c_0}$ vortex : Typical length of loops: $L = k_B T/\epsilon$

Superconducting state

Normal state

$\epsilon > 0$



$\epsilon = 0$

$|\psi| \neq 0$
$\langle|\psi|\rangle \neq 0$
$\langle|\psi| e^{i\theta}\rangle \neq 0$; $\langle\phi\rangle = 0$

$|\psi| \neq 0$
$\langle|\psi|\rangle \neq 0$
$\langle|\psi| e^{i\theta}\rangle = 0$; $\langle\phi\rangle \neq$

due to large phase-fluctuations $\Leftrightarrow$ explosion of vortex-loops

Signature of transition: Change in local o.p. $\langle|\psi| e^{i\theta}\rangle$, equivalently a change in characteristics of vortex paths in system