Lecture IV

Inhomogeneous & Nonequilibrium States of Unconventional Superconductors

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I. Inhomogeneous SC : Andreev/Bogoliubov Equations

II. Vortex Core Structure : Bound states & Currents

III. Surface States of $D_{x^2-y^2}$ SC : { Broken Reflections / Broken T-symmetry

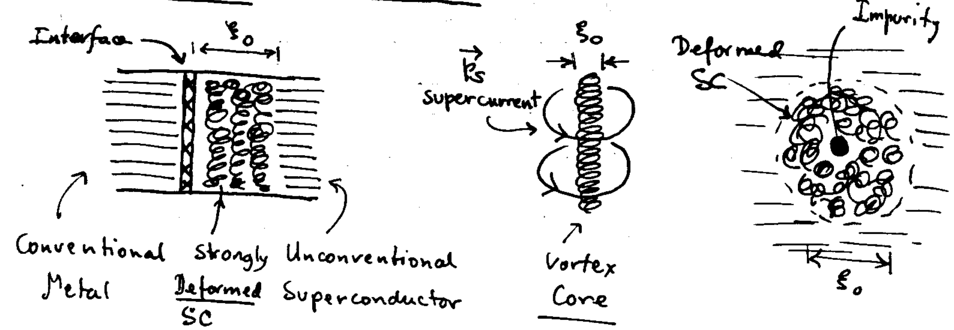
IV. Vortex Structure II: P-wave, S=1 w/ BTRS

V. Vortex Dynamics of Bound States

J. Sauls
Lecture 4

Inhomogeneous States of Superconductors

Many novel properties of superconductors involve electronic transport of charge, heat or magnetization across interfaces, or by local electronic states of vortices or impurities.



A proper theory of transport and non-equilibrium SC requires one to develop a theory for the electronic structure of inhomogeneous superconductors on length scales of order the coherence length and smaller.

Eilenberger's Transport Equation for the quasiclassical propagator, $\hat{g}(\vec{p}_f, \vec{R}; \epsilon_n)$, provides the most powerful formulation of inhomogeneous superconducting states. I first outline the connection between the Bogoliubov method and the quasiclassical Equations. This hopefully illustrates clearly the semi-classical aspects of superconductivity contained in Eilenberger's transport equation.

Bogoliubov's Equations

The key features of the BCS mean field Hamiltonian is that broken gauge symmetry (and ODLRO) are included directly by the mean-field of the pairs.

$$\begin{pmatrix} S=0 \\ s\text{-wave} \end{pmatrix} \quad \mathcal{H}_{BCS} = \sum_{\alpha} \int d\vec{r}\, \psi_{\alpha}^{\dagger}(\vec{r})\, \xi(-i\vec{\nabla})\, \psi_{\alpha}(\vec{r}) + \int d\vec{r} \left\{ \Delta(\vec{r})\, \psi_{\uparrow}^{\dagger}(\vec{r})\, \psi_{\downarrow}^{\dagger}(\vec{r}) + h.c. \right\}$$

$$\text{w/} \quad \Delta(\vec{r}) = -V_0 \langle \psi_{\uparrow}(\vec{r}) \psi_{\downarrow}(\vec{r}) \rangle$$

The Hamiltonian can be diagonalized by a canonical transformation,

$$\psi_{\uparrow}(\vec{r}) = \sum_{n}{}' \left\{ U_n(\vec{r})\, \gamma_{n\uparrow} - V_n^{*}(\vec{r})\, \gamma_{n\downarrow}^{\dagger} \right\}$$

$$\psi_{\downarrow}(\vec{r}) = \sum_{n}{}' \left\{ U_n(\vec{r})\, \gamma_{n\downarrow} + V_n^{*}(\vec{r})\, \gamma_{n\uparrow}^{\dagger} \right\}$$

where the new operators $\gamma_{n\sigma}, \gamma_{n\sigma}^{\dagger}$ obey Fermion anti-commutation relations, and destroy or create quasiparticle states in the superconducting phase.

$$\{ \gamma_{n\alpha}, \gamma_{n'\beta}^{\dagger} \} = \delta_{nn'} \delta_{\alpha\beta}$$

$$\{ \gamma_{n\alpha}, \gamma_{n'\beta} \} = 0$$

Particle-Hole Coherence

The central feature of the Bogoliubov quasiparticles is that they are coherent superpositions of normal-state quasiparticle & quasi-hole excitations,

$$\gamma_{n\uparrow}^{\dagger} = \int d\vec{r} \left\{ U_n(\vec{r})\, \underset{\sim}{\psi_{\uparrow}^{\dagger}(\vec{r})} + V_n(\vec{r})\, \underset{\sim}{\psi_{\downarrow}(\vec{r})} \right\}$$

The U_n and V_n amplitudes which diagonalize $\mathcal{H}_{BCS}$ obey Bogoliubov's Equations, $\left(\mathcal{H}_{BCS} = \sum_{n\alpha}{}' \underset{\sim}{\epsilon_n}\, \gamma_{n\alpha}^{\dagger} \gamma \right.$

$$\begin{pmatrix} \frac{1}{2m^*}\left(-i\vec{\nabla} - \frac{e}{c}\vec{A}\right)^2 - \mu & \Delta(\vec{r}) \\ \Delta^{*}(\vec{r}) & -\frac{1}{2m^*}\left(+i\vec{\nabla} - \frac{e}{c}\vec{A}\right)^2 + \mu \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix} = \underset{\sim}{\epsilon} \begin{pmatrix} U \\ V \end{pmatrix}$$

$\vec{\varphi} \equiv \begin{pmatrix} U \\ V \end{pmatrix}$ is the two-component particle-hole spinor

$\Delta(\vec{r})$ determines the relative amplitude & phase of the quasiparticle and quasihole components of the Bogoliubov quasiparticles.

Self Consistency $\updownarrow$

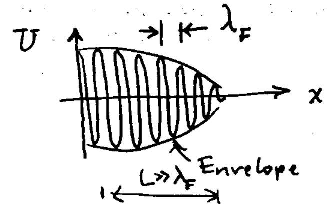
$$\Delta(\vec{r}) = V_0 \sum_{n}{}' U_n(\vec{r})\, V_n(\vec{r}) \tanh\left(\frac{\epsilon_n}{2T}\right)$$

Andreev's Equations (1964)

Andreev introduced a semi-classical approximation to the Bogoliubov equations. Andreev's equations are closely related to Eilenberger's equation (for clean SC).

Envelope Expansion: $\xi_0 = \dfrac{\hbar v_f}{2\pi k_B T_c} \gg \lambda_f = \dfrac{\hbar}{p_f}$

$$\vec\varphi(\vec r) = \underbrace{e^{i\vec p_f\cdot\vec r}}_{\text{"fast"}}\ \underbrace{\vec\Psi_{\vec p_f}(\vec r)}_{\text{"slow"}}$$



Andreev's Approximation: $\left|\dfrac{\hbar^2}{2m^*}\nabla^2\vec\Psi_{\vec p_f}\right| \ll \left|\vec v_f\cdot\dfrac{\hbar}{i}\vec\nabla\vec\Psi_{\vec p_f}\right|$ leads to

$$\boxed{\left[\epsilon\,\overset{\begin{pmatrix}1&0\\0&-1\end{pmatrix}}{\hat\tau_3} - \hat\Delta(\vec p_f,\vec r)\right]\vec\Psi_{\vec p_f} + i\,\vec v_f\cdot\vec{\mathcal{D}}\,\vec\Psi_{\vec p_f} = 0}$$

w/
$$\hat\Delta(\vec p_f,\vec r) = \begin{pmatrix} 0 & \Delta(\vec p_f,\vec r) \\ -\Delta^*(\vec p_f,\vec r) & 0\end{pmatrix}$$
(Simple to generalize to anisotropic pairing)

$$\vec{\mathcal{D}} = \vec\nabla_{\vec r} - i\frac{e}{\hbar c}\vec A(\vec r)\,, \qquad \vec\Psi_{\vec p_f} = \begin{pmatrix} u_{\vec p_f}(\vec r) \\ v_{\vec p_f}(\vec r)\end{pmatrix}$$

Some Solutions to Andreev's Equations

1. Ballistic Wavepackets of quasiparticles (Normal state)

Set $\Delta = 0$ and $\vec A = 0$. Also $\epsilon \to i\frac{\partial}{\partial t}$

The equations for $u_{\vec p_f}$ and $v_{\vec p_f}$ decouple at each point on the Fermi surface, $\vec p_f$;

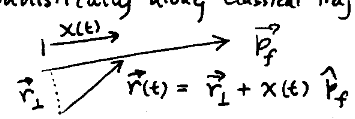
$$\left(\frac{\partial}{\partial t} + \vec v_f\cdot\vec\nabla_{\vec r}\right)u_{\vec p_f} = 0 \qquad \left(\frac{\partial}{\partial t} - \vec v_f\cdot\vec\nabla_{\vec r}\right)v_{\vec p_f} = 0$$

and correspond to the two linearly independent particle and hole excitations for each $\vec p_f$ and excitation energy ϵ.

$$\vec\Psi_{\vec p_f +} \sim \begin{pmatrix}1\\0\end{pmatrix} e^{-i\epsilon t}\, e^{i\frac{\epsilon}{v_f}\hat v_f\cdot\vec r}\ ;$$
• particle excit
$\vec v_+ = +\vec v_f$ (group velocit

$$\vec\Psi_{\vec p_f -} \sim \begin{pmatrix}0\\1\end{pmatrix} e^{-i\epsilon t}\, e^{-i\frac{\epsilon}{v_f}\hat v_f\cdot\vec r}\ ;$$
• hole excitation
$\vec v_- = -\vec v_f$

The excitations move ballistically along classical trajectories defined by $\vec p_f$.



2. Continuum Excitations in the SC state ($\epsilon > |\Delta|$)

For a spatially uniform gap Δ there are two propagating solutions corresponding to continuum excitations above the gap.

i) Particle-like branch

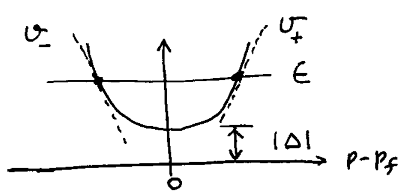
$$\vec{\Psi}_{\vec{p}_f +} = A_+ \begin{pmatrix} \epsilon + \sqrt{\epsilon^2 - |\Delta(\vec{p}_f)|^2} \\ \Delta(\vec{p}_f) \end{pmatrix} e^{+i \frac{\sqrt{\epsilon^2-|\Delta|^2}}{v_F} \hat{p}_f \cdot \vec{r}}$$

$$\xrightarrow[\Delta \to 0]{} \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad ; \quad \vec{v}_+(\epsilon) = + v_F \left(\frac{\sqrt{\epsilon^2-|\Delta|^2}}{\epsilon} \right)$$

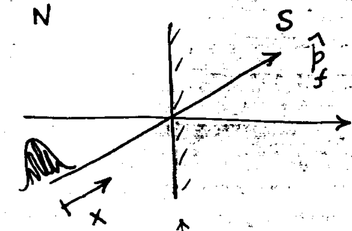
ii) Hole-like branch

$$\vec{\Psi}_{\vec{p}_f -} = A_- \begin{pmatrix} \Delta(\vec{p}_f) \\ \epsilon + \sqrt{\epsilon^2 - |\Delta(\vec{p}_f)|^2} \end{pmatrix} e^{-i \frac{\sqrt{\epsilon^2-|\Delta|^2}}{v_F} \hat{p}_f \cdot \vec{r}}$$

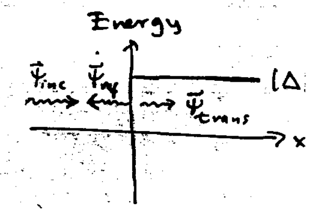
$$\xrightarrow[\Delta \to 0]{} \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad ; \quad \vec{v}_-(\epsilon) = - v_f \left(\frac{\sqrt{\epsilon^2-|\Delta|^2}}{\epsilon} \right)$$



3. Andreev Reflection at an NS interface



High Transmission NS interface.



Andreev Reflection Amplitude

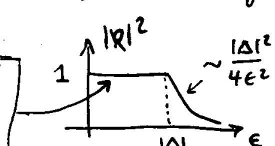
$$\underline{x<0} : \quad \vec{\Psi} = \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{i \frac{\epsilon}{v_F} x}}_{\vec{\Psi}_{inc}} + \underbrace{R(\epsilon) \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-i \frac{\epsilon}{v_F} x}}_{\vec{\Psi}_{ref}}$$

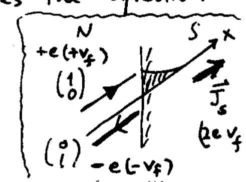
$\underline{x>0}$: For energies below $|\Delta|$ only 'evanescent waves' are allowed. The solution that decays for $x>0$ is

$$\vec{\Psi}_{trans} = A \begin{pmatrix} \Delta \\ \epsilon - i\sqrt{|\Delta|^2 - \epsilon^2} \end{pmatrix} e^{-\sqrt{|\Delta|^2-\epsilon^2}\, x/v_F}$$

* Matching the solutions at $x=0$ gives the reflection amplitude

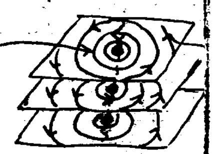
$$\boxed{R(\epsilon) = \frac{\epsilon - i\sqrt{\Delta^2 - \epsilon^2}}{\epsilon}}$$

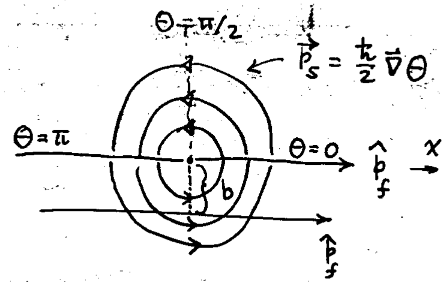




4. Bound States in Vortex Cores

Consider a 2D "pancake" vortex



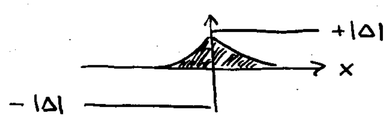


$$\oint_C \vec{\nabla}\Theta \cdot d\vec{\ell} = 2\pi$$

Solve Andreev's Equation for $|\epsilon| < |\Delta|$ along a trajectory that passes through the vortex center.

$$\left(\epsilon\hat{\tau}_3 - \hat{\Delta}(\vec{p}_f, \vec{R})\right)\vec{\Psi}_{\vec{p}_f}(\vec{R}) + i\,\vec{v}_f\cdot\vec{\nabla}_R\,\vec{\Psi}_{\vec{p}_f} = 0$$

- We can neglect the vector potential for $|\vec{R}| \ll \lambda$.
- "Zero-Radius" Vortex



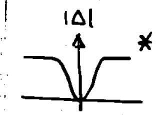
- For each $\vec{p}_f$ passing thru the center there is a bound-state in the core with $\epsilon = 0$.

$$\boxed{\vec{\Psi}_{\vec{p}_f} = \sqrt{\frac{|\Delta(\vec{p}_f)|}{2v_F}}\; e^{-\frac{|\Delta(\vec{p}_f)|}{v_F}|x|}\begin{pmatrix}1\\ i\end{pmatrix}}$$

Decay Length $\sim \frac{v_F}{|\Delta|} \sim \xi_0$.

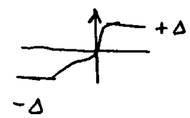
Zero Modes of the Vortex Core

1. The zero energy bound state arises from the π-phase change of Δ along the trajectory thru the vortex center.

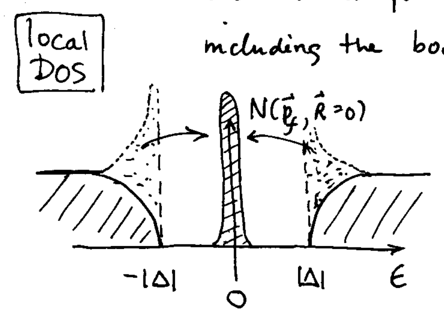


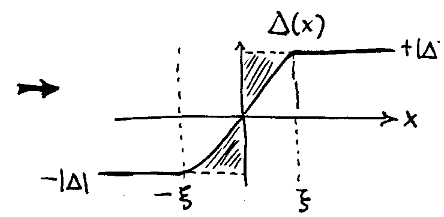
* (The Zero Modes are not due to an effective potential provided by $|\Delta(\vec{R})|$.)

2. Zero modes are generic to Andreev's Equation (Dirac-like Hamiltonian's) for fermions coupled to $\Delta(\vec{R})$ that is real and changes sign.



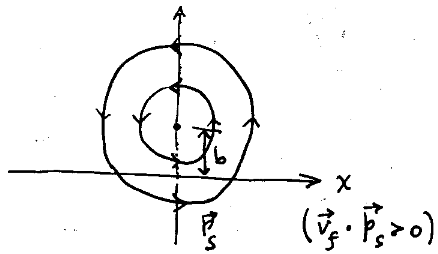
3. Pair-breaking, i.e. $|\Delta(\vec{R})| \underset{R\to 0}{\rightarrow} 0$, in the vortex core results for self-consistency of $\Delta(\vec{R})$ including the bound-state solutions for $\vec{\Psi}$.

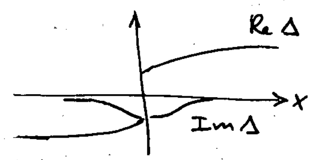




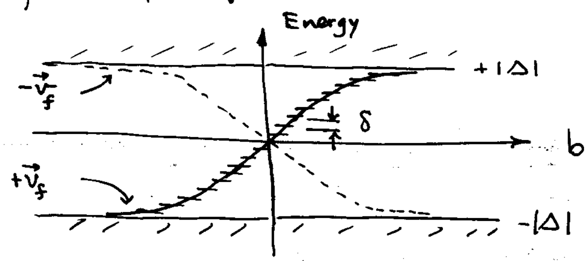
Bound States with Finite "Impact parameter"

Consider trajectories that pass thru the Vortex core with non-zero impact parameter, b.





There is a bound-state solution with $|\epsilon| < |\Delta|$, for all finite impact parameters.



For $|b| \ll \xi_0$ the spectrum is linear in b,

$$\boxed{\epsilon_{bound} \simeq 2(b/\xi_0)|\Delta|}$$

Caroli, deGennes, Matricon → Cylindrical Symmetry (s-wave):

→ Quantize $L_z = p_f \cdot b = \mu \cdot \hbar \Rightarrow \boxed{\epsilon_b = \mu * \overbrace{\left(\frac{2}{\pi}\frac{\Delta^2}{E_F}\right)}^{\delta}}$

Connection between Andreev ⟺ Eilenberger

For clean superconductors, we can construct the quasiclassical propagator $\hat{g}(\vec{p}_f, \vec{R}, \epsilon_n)$ directly from solutions of Andreev's Equation.

For Matsubara Energies: $\epsilon\hat{\tau}_3 \rightarrow i\epsilon_n\hat{\tau}_3$

$\hat{M} = i\epsilon_n\hat{\tau}_3 - \hat{\Delta}(\vec{p}_f, \vec{R})$; $|\psi\rangle = \begin{pmatrix} u \\ v \end{pmatrix}$, $\langle\psi| \equiv (u^* \; v^*)$

1). $i\vec{v}_f \cdot \vec{\nabla} |\psi_\pm\rangle + \hat{M} |\psi_\pm\rangle = 0$

2). $-i\vec{v}_f \cdot \vec{\nabla} |\tilde{\psi}_\pm\rangle + \hat{M} |\tilde{\psi}_\pm\rangle = 0$

The latter equation is useful for constructing $\hat{g}$. Eq. (1) has exploding (+) and decaying (−) solutions for each trajectory ($\vec{v}_f$). Likewise for Eq. (2). However, if $|\psi_+\rangle$ is exploding then $|\tilde{\psi}_+\rangle$ is decaying. This tells us how to construct physical solutions for $\hat{g}(\vec{p}_f, \vec{R}, \epsilon_n)$.

$$\hat{g}(\vec{p}_f, \vec{R}; \epsilon_n) = \sum_{\mu=\pm} G_\mu |\psi_\mu\rangle\langle\tilde{\psi}_\mu|$$

Obeys $\boxed{\left[i\epsilon_n\hat{\tau}_3 - \hat{\Delta}(\vec{p}_f, \vec{R}), \hat{g}(\vec{p}_f, \vec{R}; \epsilon_n)\right] + i\vec{v}_f \cdot \vec{\nabla}\hat{g} = 0}$

Fix G_μ so that $\hat{g}^2 = -\pi^2 \hat{1}$ (Normalization).

Advantages to Eilenberger's Approach

1. $\hat{g}(\vec{p}_f, \vec{R}; \epsilon_n) = \begin{pmatrix} g & f \\ \underline{f} & \underline{g} \end{pmatrix}$

gives directly the standard Green's Function, g, and the off-diagonal propagator, f, for pairs.

2. Local DOS:

$$N(\vec{p}_f, \vec{R}; \epsilon) = -\frac{1}{\pi} \mathrm{Im}\left[g(\vec{p}_f, \vec{R}; i\epsilon_n \to \epsilon + i0) \right]$$

3. Order Parameter (locally)

$$\ln(T/T_c)\, \Delta(\vec{p}_f, \vec{R}) = T \sum_{\epsilon_n} \int d^2\vec{p}_f\, \mathcal{Y}^*(\vec{p}_f) \left[f(\vec{p}_f, \vec{R}; \epsilon_n) - \frac{\pi\Delta}{|\epsilon_n|} \right]$$

* 4. Eilenberger's Transport Equation is more general:

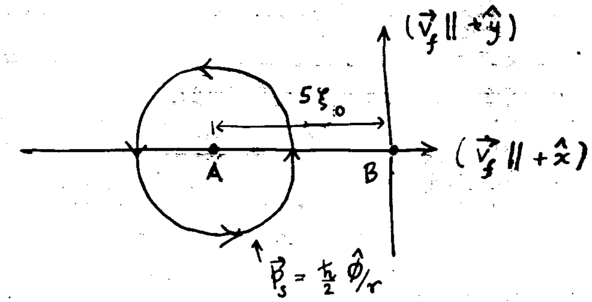
i) Impurity Scattering

ii). Inelastic e-e & e-ph scatt.

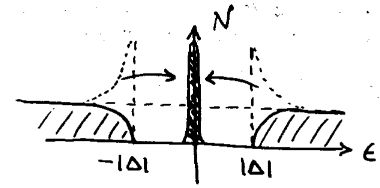
iii) Strong-coupling

* 5). Extends to Non-Equilibrium Dynamics of coupled quasiparticles & pairs.

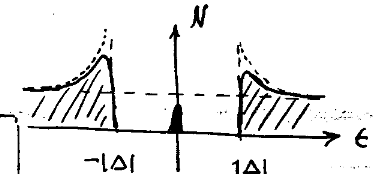
Local DOS in a (clean: $\ell \to \infty$) Vortex Core (s-wave)



1. $N(\vec{p}_f \parallel +\hat{x}, A; \epsilon)$



2. $N(\vec{p}_f \parallel +\hat{x}, B; \epsilon)$, $B = 5\xi_0$

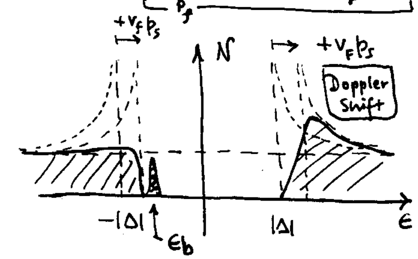


$$\boxed{|\psi_{\vec{p}_f}(x)|^2 \propto \int_{-\gamma}^{+\gamma} d\epsilon\, N(\vec{p}_f \parallel +\hat{x}, x; \epsilon)} \rightarrow \boxed{|\psi_{\vec{p}_f}(x)|^2 \sim e^{-\frac{2|\Delta(\vec{p}_f)|}{v_F} x}}$$

3. $N(\vec{p}_f \parallel +\hat{y}, B; \epsilon)$

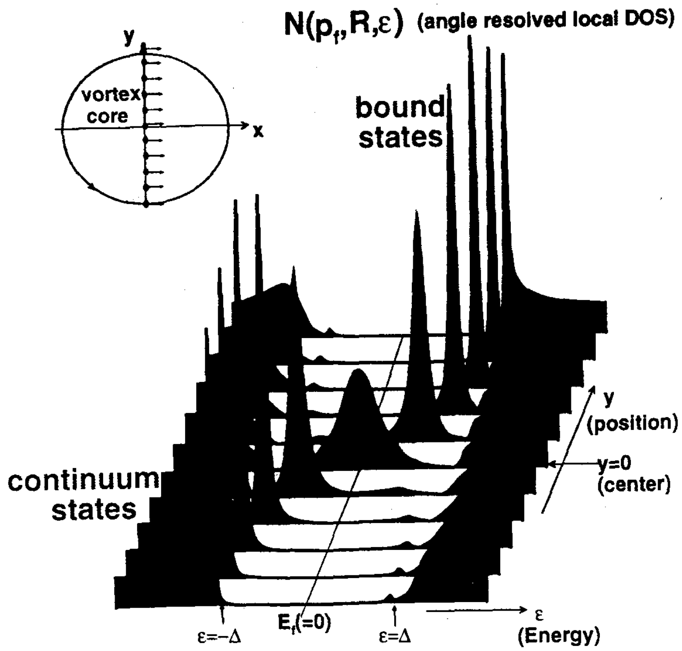
$\epsilon_b \simeq -0.9|\Delta|$

↑ Carries the Current!

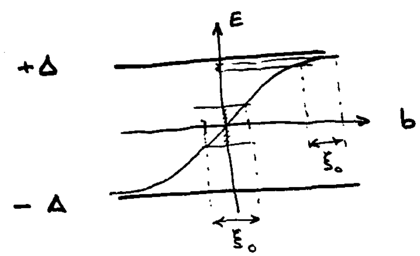


Spectrum of bound states in the vortex core in equilibrium

$l=10\xi_0,\ \kappa^{-1}=0,\ T=0.3T_c$



broadening of zero energy bound state: $1/\tau > \Delta^2/E_f$

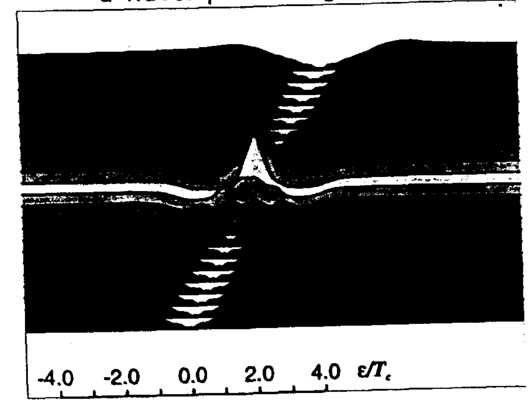


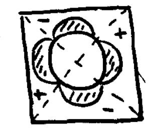
M. Eschrig, et al. (PRB 1999)

Equilibrium DOS spectra of d-wave vortex

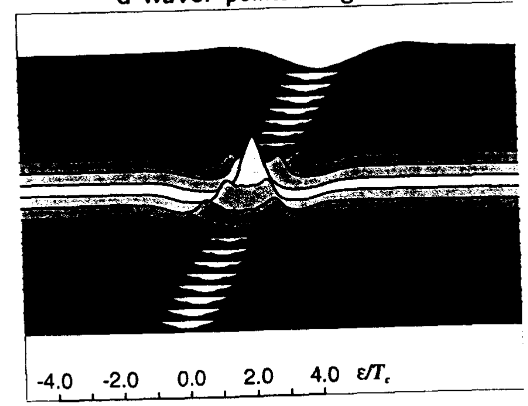
$l = 10\,\xi_0$

d-wave: points along antinode:





d-wave: points along node:



M. Eschrig, et al.

c.f. N. Schopohl + K. Maki (Phys. C 1995)

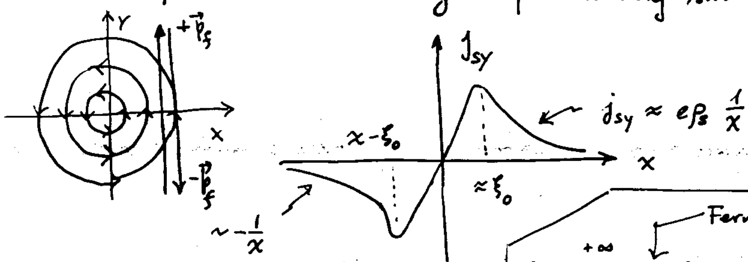
(17)

Vortex Current Density

We can calculate the supercurrent of a vortex in equilibrium directly from the quasiclassical propagator,

$$\vec{j}_s(\vec{R}) = N_f \int d^2\vec{p}_f \, [e\vec{v}_f] \, T \sum_{\epsilon_n} g(\vec{p}_f, \vec{R}; \epsilon_n),$$

once we have solved the Eilenberger Equation and gap equation self consistently. The structure of the current density is qualitatively shown here,



Fermi Distribution

$$\vec{j}_s(\vec{R}) = \int_{-\infty}^{+\infty} d\epsilon \, f(\epsilon) \int d^2\vec{p}_f \; \vec{J}(\vec{p}_f, \vec{R}; \epsilon)$$

The current density in the vicinity of the core ($r \sim$ few ξ_0) is generally large. The spectral resolution of the current shows that these currents are due to the Andreev Bound States

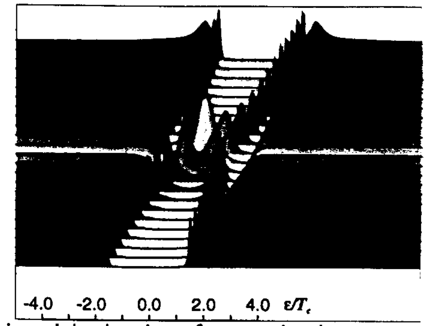
Waxman, etal. (1996)

$$\vec{J}(\vec{p}_f, \vec{R}; \epsilon) \equiv e\vec{v}_f \left[N(+\vec{p}_f, \vec{R}; \epsilon) - N(-\vec{p}_f, \vec{R}; \epsilon) \right]$$

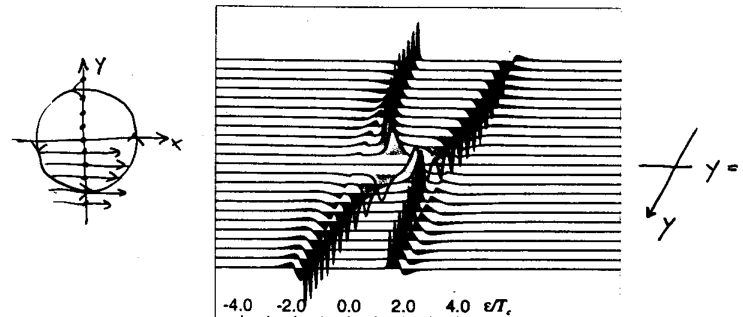
(18)

Equilibrium DOS spectra of s-wave vortex

$\ell = 10\xi_0$, Born limit scattering, $T = 0.3T_c$

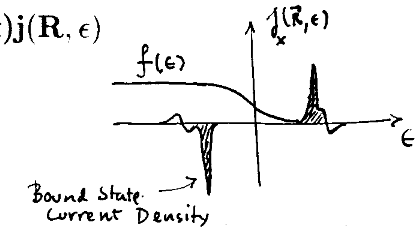


local quasiparticle density of states in the vortex core region



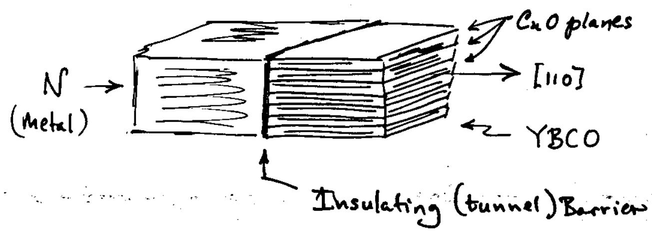
local spectral current density, $j(\mathbf{R}, \epsilon)$, in the vortex core region

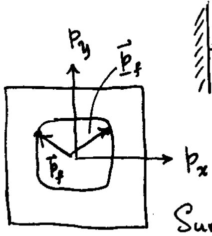
$$\mathbf{j}(\mathbf{R}) = \int d\epsilon \, f(\epsilon) \, \mathbf{j}(\mathbf{R}, \epsilon)$$

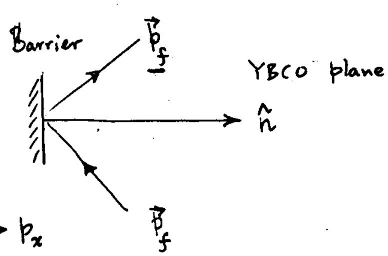


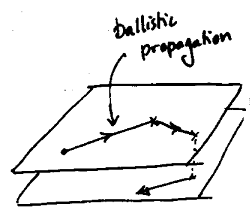
Surface-induced Andreev Bound States in D-wave SC

Because of the Broken Reflection Symmetry of the $d_{x^2-y^2}$ order parameter, Andreev Bound States analogous to the Vortex core states in a S-wave SC are formed at interfaces. These states carry current and are essential for understanding transport across ab-interfaces in high T_c (and other Unconv. SC).





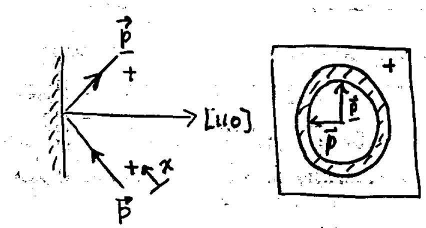


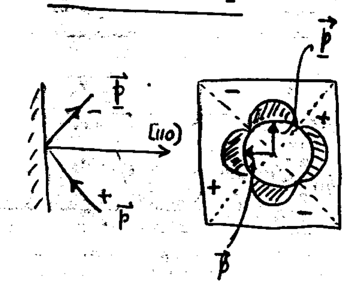


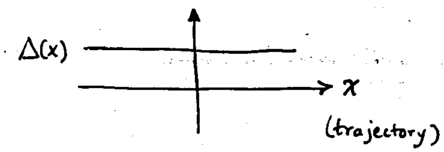
Surface Scattering $\Rightarrow$ $\Delta\vec{p} \cong 2(\hat{n}\cdot\vec{p}_f)\hat{n} \approx \vec{p}_f$.

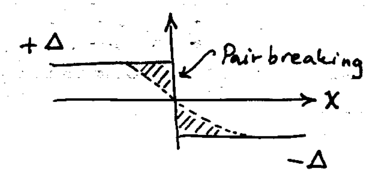
↳ Pairing Symmetry Matters!

S-wave | **D-wave [110]**









- $\Delta(x) = \Delta_0$
- No Andreev Reflections

→ "Anderson Theorem" for non-magnetic surface

- $\Delta(x)$ changes sign.

⇓

- Formation of an $\epsilon = 0$ Andreev Bound State

⇓

- Maximal Pair Breaking

$$\vec{\psi}_{\vec{p}_f} \sim e^{-|\Delta(\vec{p}_f)||x|/v_F} \begin{pmatrix} 1 \\ i \end{pmatrix} \Rightarrow \text{Same as the Vortex Core state.}$$

Local DOS:

$$N(\vec{p}_f, x; \epsilon) = \mathrm{Im}\left\{ \frac{(\epsilon + i\gamma)}{\sqrt{|\Delta(\vec{p}_f)|^2 - (\epsilon+i\gamma)^2}} - \frac{|\Delta(\vec{p}_f)|^2}{\epsilon + i\gamma} * \frac{e^{-\lambda(\vec{p}_f,\epsilon)\cdot|x|}}{\sqrt{|\Delta(\vec{p}_f)|^2 - (\epsilon+i\gamma)^2}} \right\}$$

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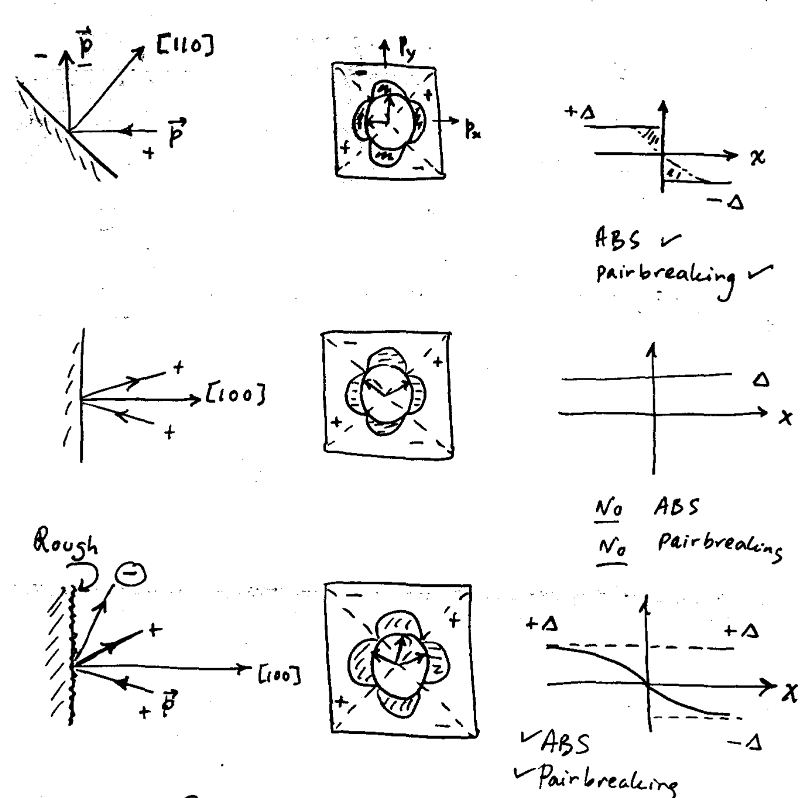
Orientation Dependence of the Surface ABS



→ Surface Roughness Matters

→ ABS ↔ Pairbreaking

— C.-R. Hu (94) — Buchholtz, etal (95)

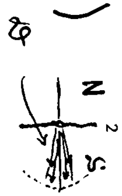
— Tanaka + Kashiwaya (95)

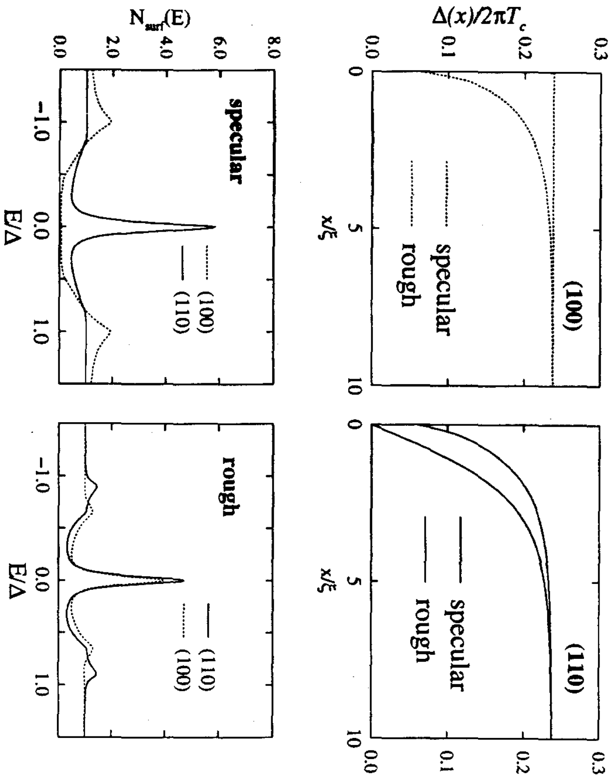
22

d-wave Order Parameter Near Surfaces

• Fogelström, Rainer + JAS (PRL 1997)

$$G(V) = \frac{dI}{dV} = \frac{1}{R_N}\int d\varepsilon \int d\vec{p}_f\, \mathcal{D}(\vec{p}_f)\, N_{surf}(\varepsilon,\vec{p}_f)\left(-\frac{\partial f}{\partial \varepsilon}\right)$$





Roughness

J. Geerk, et al Z. Phys. B 1988

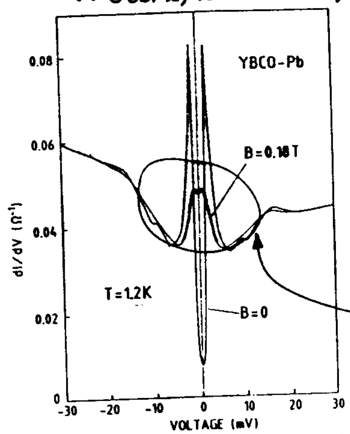
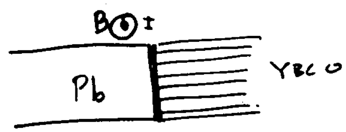


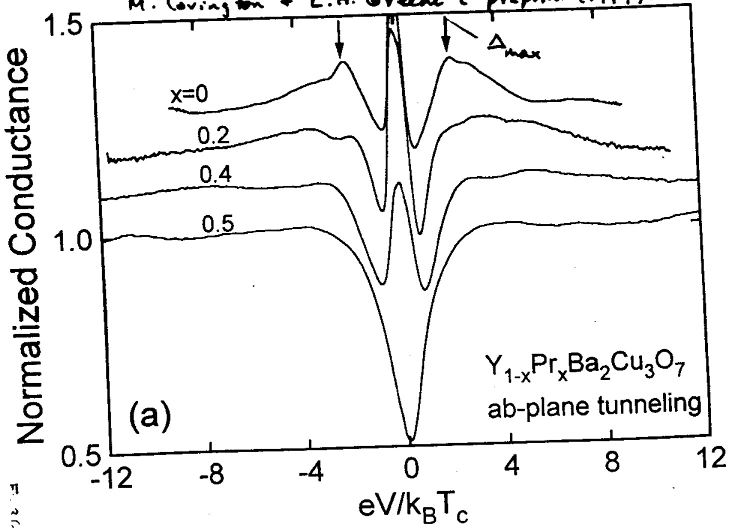
Fig. 2. dI/dV vers. V of a YBCO-Pb junction with the Pb counter electrode in the normal and superconducting state



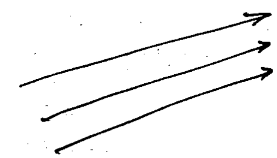
ZBA splits in B-field.

⇒ J. Lesueur, et al. (1990) (Bell Core)

M. Covington + L.H. Greene { LT21 Proc. (1996). preprint (1997).



Superflow & Doppler Shifts



$\vec{p}_s = \frac{\hbar}{2}\vec{\nabla}\theta$

$$\hat{\Delta}(\vec{p}_f,\vec{R}) = \begin{pmatrix} 0 & \Delta_o e^{i\theta} \\ -\Delta_o e^{-i\theta} & 0 \end{pmatrix}$$

Δ_o = Reference O.P. $= \Delta_o^*$

Andreev's Equation in the presence of $\vec{\nabla}\theta$ is:

$$\left[\epsilon\hat{\tau}_3 - \hat{\Delta}(\vec{p}_f,\vec{R})\right]\vec{\Psi}_{\vec{p}_f} + i\vec{v}_f\cdot\vec{\nabla}_R\vec{\Psi}_{\vec{p}_f} = 0$$

Introduce a local gauge transformation,

$$\boxed{\hat{U}(-\theta) \equiv e^{i\theta\hat{\tau}_3/2}} \quad ; \quad \hat{U}(-\theta)\hat{U}^{\dagger}(-\theta) = \hat{1}$$

$$\hat{U}(-\theta)\hat{\Delta}\hat{U}^{\dagger}(-\theta) = \hat{\Delta}_o = \begin{pmatrix} 0 & \Delta_o \\ -\Delta_o & 0 \end{pmatrix}$$

$$\vec{\Psi}'_{\vec{p}_f} \equiv \hat{U}(-\theta)\vec{\Psi}_{\vec{p}_f} \quad \& \quad \boxed{\hat{U}(-\theta)\vec{v}_f\cdot\vec{\nabla}\hat{U}^{\dagger}(-\theta) = -i/2(\vec{v}_f\cdot\vec{\nabla}\theta)\hat{\tau}_3}$$

$$\boxed{\left[(\epsilon + \vec{v}_f\cdot\vec{p}_s)\hat{\tau}_3 - \hat{\Delta}_o\right]\vec{\Psi}'_{\vec{p}_f} + i\vec{v}_f\cdot\vec{\nabla}\vec{\Psi}'_{\vec{p}_f} = 0}$$

Doppler Shift of states

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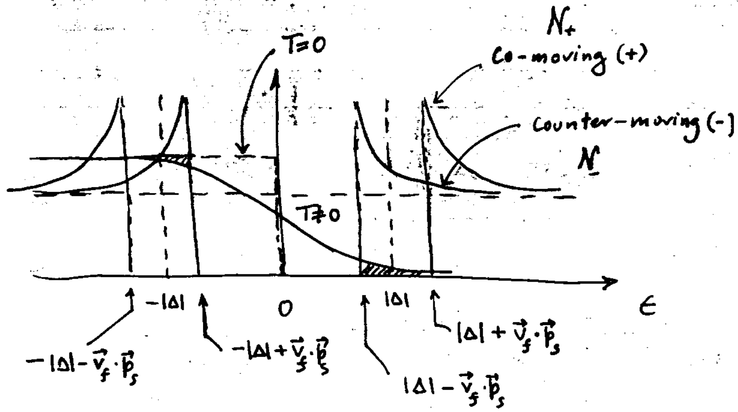
Superfluid Density (Uniform Superflow)

D. Xu, et al (1994)



$$\vec{J}_s = N_f \int_{-\infty}^{+\infty} d\epsilon\, f(\epsilon) \int_{(>0)} d^2\vec{p}_f \,[e\vec{v}_f]\left(N_+(\vec{p}_f,\epsilon) - N_-(\vec{p}_f,\epsilon)\right)$$

$\vec{v}_f \rightrightarrows \vec{p}_s$ $\qquad$ $-\vec{v}_f \leftarrow$, $\vec{p}_s \rightarrow$

► Zero-Temperature, Clean limit → Condensate ($\epsilon < 0$).

$$\vec{J}_s = e N_f \int_{(>0)} d^2 p_f \; \vec{v}_f \left(2\vec{v}_f \cdot \vec{p}_s\right) = \overbrace{\left(\frac{e}{4} N_f v_f^2\right)}^{\rho_s(T=0)} \left(\vec{\nabla}\Theta - \frac{2e}{\hbar c}\vec{A}\right)$$

Bulk Critical Current: $\underline{J_c = \frac{e}{2} N_f v_f |\Delta|}$ $\leftarrow (v_f p_c = |\Delta|)$.

► $\underline{T > 0}$: Thermal excitation to the counter-moving band w/ $\epsilon > 0 \rightarrow \rho_s(T) < \rho_s(T=0)$.

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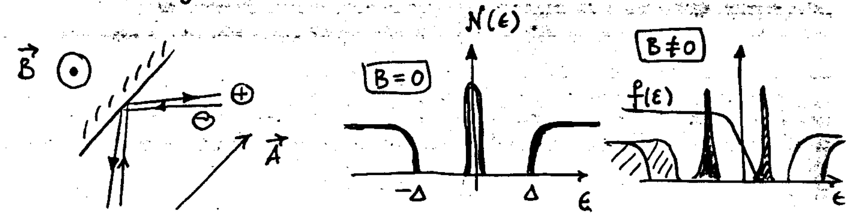
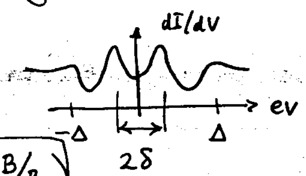
Doppler Shifts of the Surface Andreev States

For an ideal [110] interface the surface states can be grouped into time-reversed pairs, which are degenerate in $\vec{B} = 0$.



I. Tunneling DOS → (Doppler Splitting) of the Zero-Bias Peak.

$dI/dV \propto N_T(\epsilon = eV)$



$$\boxed{\delta(H) = \frac{e}{c}\vec{v}_F \cdot \vec{A} = \Delta_0 \left(B/B_D\right)}$$

$B_D = \frac{c}{e}\frac{\Delta_0}{v_f \lambda_{ab}} / \sin\phi_c$

$B_D \simeq \underline{10\ \text{Tesla}}$

II. Paramagnetic Anomaly in the Screening Current → $\left\{ \begin{array}{l} \delta\lambda(T) \text{ vs } T \\ T \ll T_c \end{array}\right.$

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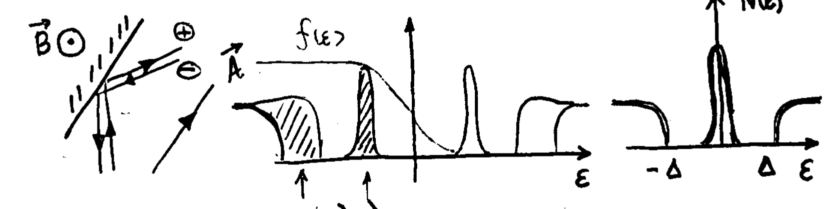
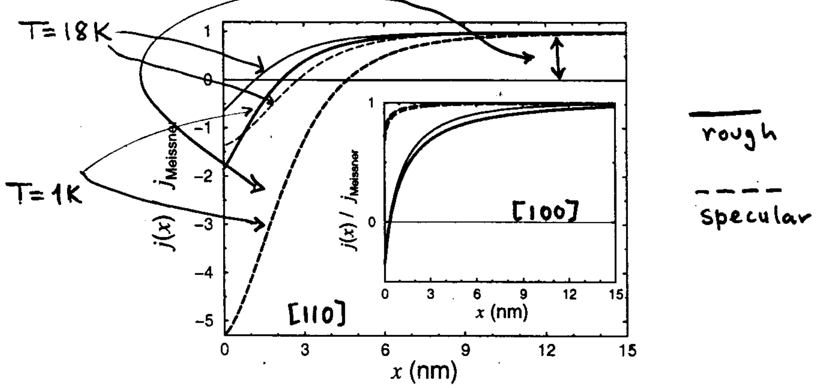
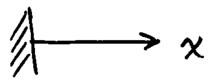
Anomalous Surface Currents

Doppler Shift: $\boxed{\Delta \mathcal{E} = \frac{e}{c}\, \vec{v}_f \cdot \vec{A}}$

Time-reversed States







- Fogelström, Rainer + JAS (1997 PRL)
- Burkhardt, Rainer + JAS (1998, PRL)

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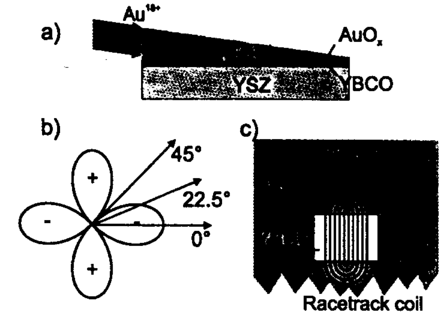
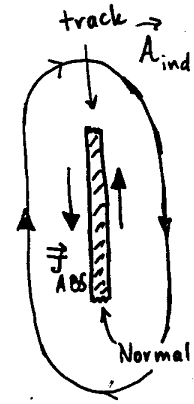
Anomaly in the Penetration Depth

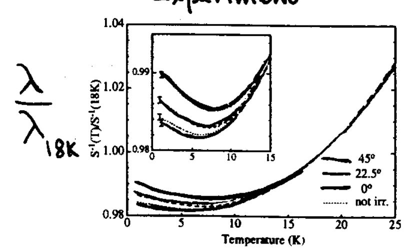
H. Walter, et al., PRL 80, 3598 (1998).

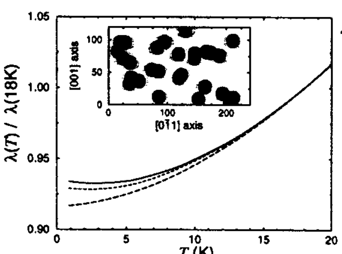




Theory: Anomaly is due to ABS current

Experiment



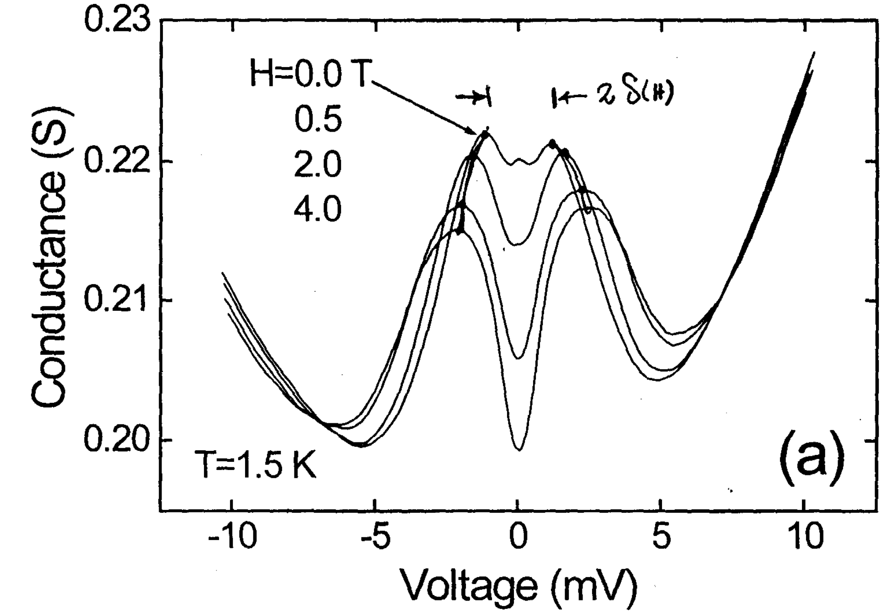


Burkhardt, Rainer, JAS (PRL, '88)

- $\frac{\delta\lambda}{\lambda} \sim 1\%$
- Depends on orientation

- $\delta\lambda/\lambda \sim \xi_0/\lambda_0 \sim 1\%$
- Orientation ⇒ $\boxed{d_{x^2-y^2}}$

Field Evolution of dI/dV.



Covington, et al., Fig. 2a
PRL 1997

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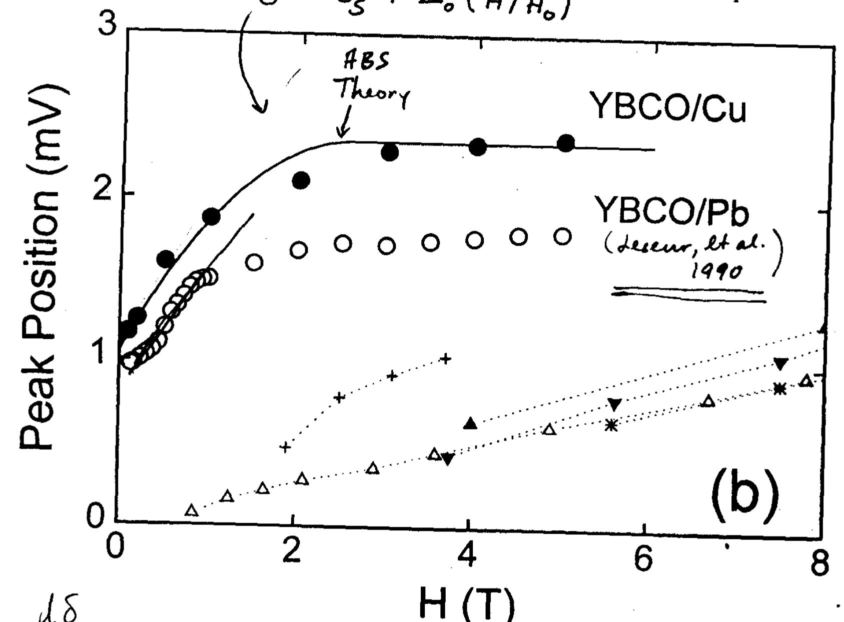
$\delta = \overset{\text{spont.}}{\delta_S} + \overset{\text{Doppler}}{\Delta_0 (H/H_0)}$



* $\frac{d\delta}{dH} \simeq$ ~~[illegible]~~ $1\ mV/T$

Covington, et al., Fig 2b
PRL (1997)

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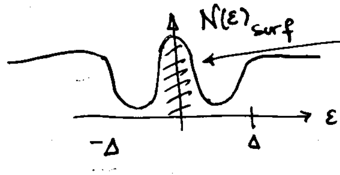
Surface Phase Transitions ?

{ Matsumoto & Shiba (1996)
Fogelström, etal (1997)

Surface scattering in a $d_{x^2-y^2}$ SC produces a large DOS in the vicinity of the surface.



Can these quasiparticles form a new condensate ?

Possible sub-dominant pairing channels

1. electron-phonon-electron → s-wave (A_{1g})
2. AFM spin fluctuations → g-wave (A_{2g})

BCS-Ginzburg-Landau for $d_{x^2-y^2} \oplus s \rightarrow$ $d \pm is$
(Broken $\mathcal{T}$-Symmetry)

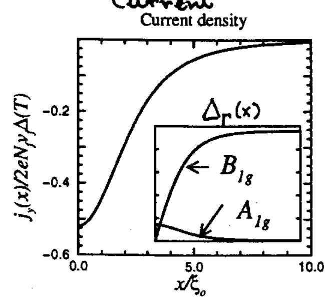
↓

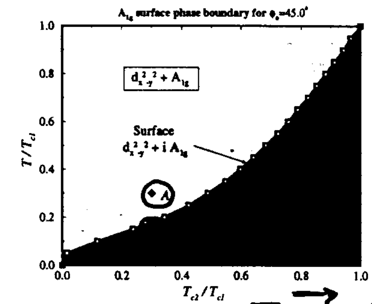
Signatures of BTRS from sub-dominant pairing

1. Spontaneous Splitting of the Zero-Bias Conductance for $T < T_{surf} \ll T_c$
2. Onset of Surface Currents & Induced Local $\vec{B}(\vec{r})$

Surface-induced broken $\mathcal{T}$-symmetry

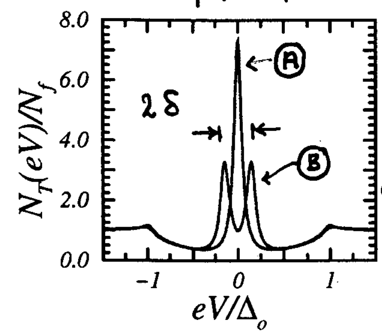
Spontaneous Surface Current

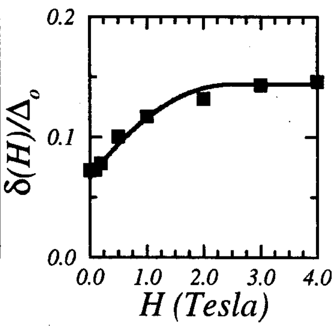




$T_{c\text{-}s\text{wave}} / T_c$

Spontaneous Splitting of the Spectrum





Fogelström, etal. (1997).

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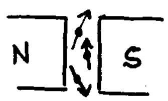
Field Scales for $\delta(H)$

Experiment: $\delta = \overbrace{\Delta_0}^{\approx 1.5\,mV/T} * (H/\underbrace{H_{exp}}_{\cong\ \underline{10T}})$

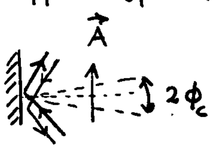
Zeeman Splitting of Resonant Impurities:
(Appelbaum, Anderson)



$$\delta = g\mu_B H = \Delta_0 (H/H_p)$$

$$H_p = \frac{\Delta_0}{g\mu_B} \simeq 125\,T \quad \longrightarrow \text{or } g \simeq 10\text{-}15$$

Doppler Splitting of ABS:



$$\delta = \frac{e}{c}\vec{v}_f \cdot \vec{A} = \frac{e}{c} v_f H \lambda \sin\phi_c$$

$$= \Delta_0 (H/H_D)$$

▶ $H_D = \frac{c}{e}\frac{\Delta_0}{v_f \lambda} / \sin\phi_c = \frac{H_c}{\sin\phi_c} \sim \boxed{1\text{-}10T}$ ($\phi_c = 10°$)

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Summary of Surface Bound States

- ▷ Pair breaking ↔ Zero-energy (ABS) surface states are intrinsic to $d_{x^2-y^2}$ (most UnSC).

- ▷ ABS are observed in ab-plane tunneling
  (Equivalent signature as π-shift in SQUID) → (Early evidence for $d_{x^2-y^2}$ symmetry) { J. Geerk (1988), J. Lesever, etal (1992) }

- ▷ ABS are observed in transverse EM response as a paramagnetic anomaly in $\delta\lambda_{ab}(T)$

- ▷ Field Evolution of ABS ⟹ Doppler Shifts

- ▷ ABS + Sub-dominant pairing ⟹ Spontaneous BTRS at surfaces.

- Observable: ① Spontaneous Splitting of the Zero-Bias Peak
  ② Spontaneous Surface Currents / Fields.

- Evidence: Splitting $\delta_s \simeq 1.5\,meV$ for $T \lesssim 7K°$. (YBCO)

Vortex Structure for Odd-Parity (p-wave), S=1 w/ Broken T-symmetry

✓ Superfluid ^3He-A

✓ UPt_3 : Triplet, E_{2u} (f-wave) (hexagonal)

? Sr_2RuO_4 : Triplet, E_u (p-wave) (tetragonal)

$\boxed{Sr_2RuO_4}$

1). Strong suppression of T_c w/ non-magnetic Impurities (Mackenzie, etal. 1998) → Unconventional pairing

2). Knight shift un suppressed below T_c (Ishida, etal. 1998) → Triplet w/ $\vec{d} \parallel \hat{z}$

3). μSR linewidths → Broken T-symmetry onsets at T_c (G. Luke, etal (1998))

• Theoretical Model (Agterberg, etal): E_u w/ $\vec{d} \parallel \hat{z}$ ESP

$$\vec{\Delta}(\vec{p}_f, \vec{R}) = \vec{d}\,\{\Delta_+(\vec{R})(p_x + ip_y) + \Delta_-(\vec{R})(p_x - ip_y)\}$$

(Same as E_u model for UPt_3)

Ground State ($\vec{B}=0$): $\Delta_+ = \Delta_0 \neq 0$; $\Delta_- = 0$

Broken Time-Reversal for the Ground State (BTRS)

There are two degenerate orbital ground states of the E_{1u} model corresponding to time-reversed orbital pair states.

$$\vec{\Delta}_+ = \vec{d}\,(p_x + ip_y) = \vec{d}\, e^{+i\theta} \Rightarrow l_z^{int} = +\hbar$$

$$\vec{\Delta}_- = \vec{d}\,(p_x - ip_y) = \vec{d}\, e^{-i\theta} \rightarrow l_z^{int} = -\hbar$$

Vortex Core Order Parameter ($\vec{\Delta}_+$ is the Ground State)

Consider an m-quantized vortex in the BTRS state with $L_z = +\hbar$. The most general O.P. is

$$\Delta(\vec{p}_f, \vec{R}) = \left(\Delta_+(\vec{R})\, e^{i\theta}\, e^{im\phi} + \Delta_-(\vec{R})\, e^{-i\theta}\, e^{ip\phi}\right)$$

i) The global phase winding at $(R\to\infty)$ is $2\pi \cdot m$

ii) $\Delta_+(\vec{R}) \xrightarrow[R\to\infty]{} \Delta_0$ & $\Delta_-(\vec{R}) \xrightarrow[R\to\infty]{} 0$

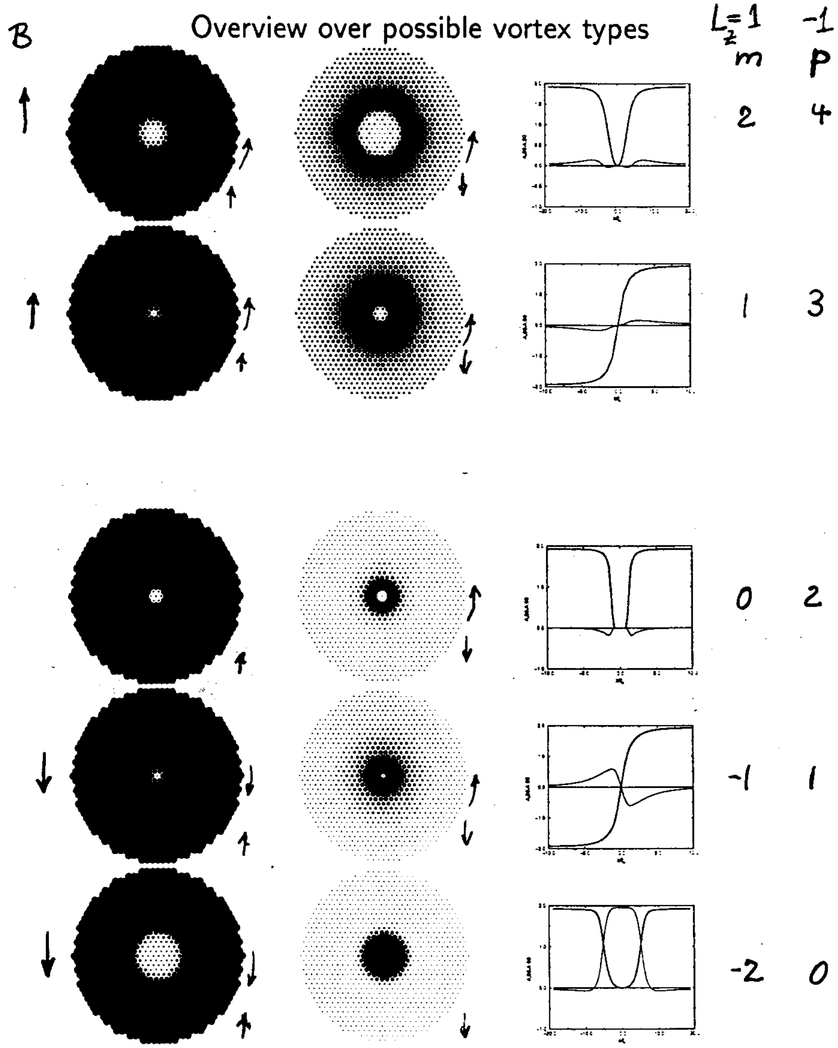
roximate

→ iii). "Cylindrical" Symmetry at $R \gg \xi_0$: $Q_z = l_z^{int} + l_z^{CM}$

$Q_z\, \Delta(\vec{p}_f, \vec{R}) = q\, \Delta(\vec{p}_f, \vec{R})$ $\boxed{m+1 = p-1}$

Code: $\begin{cases} \blacksquare \rightarrow \cos(m\phi) > 0 \\ \blacksquare \rightarrow \cos(m\phi) < 0 \end{cases}$

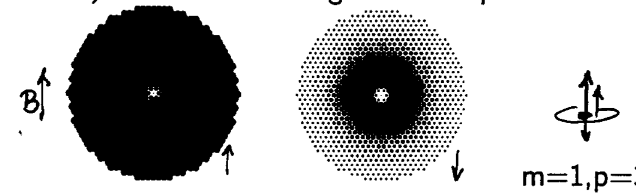
Overview over possible vortex types



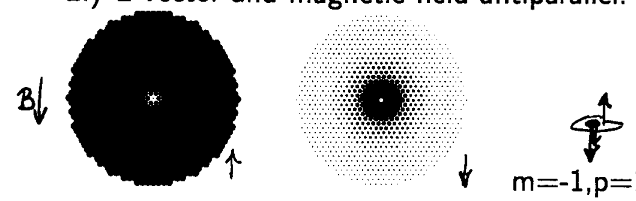
↳ Energetically favored!

Two types of singly quantized vortex

1.) L-vector and magnetic field parallel:



2.) L-vector and magnetic field antiparallel:

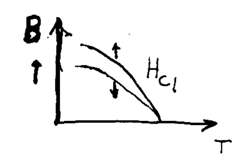


Coming from the Meissner state the spontaneously chosen L-vector defines the type of singly quantized vortex in the vortex phase.

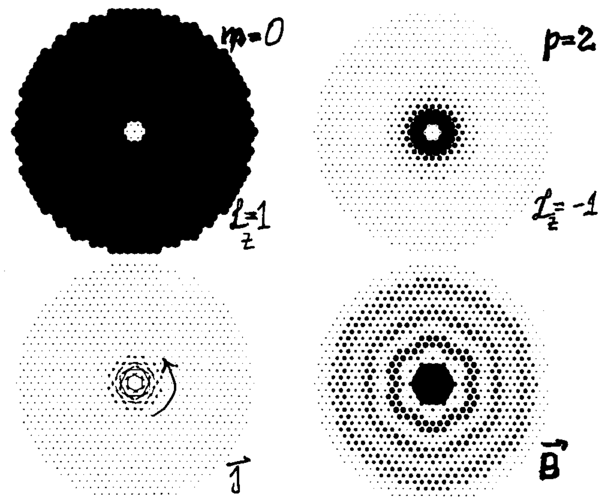
Different free energy for vortices corresponding to different field directions →

Splitting of H_{c1} for fields parallel or antiparallel to the internal angular momentum vector L

Tokuyasu, et al. (1990)



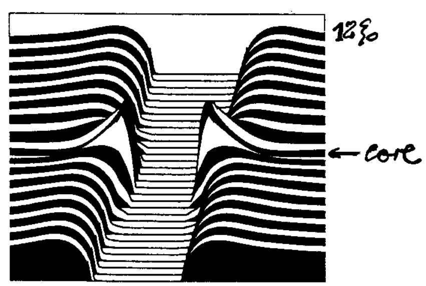
Structure of inhomogeneities in the Meissner phase



spontaneous supercurrents and local magnetic fields

Choi, Muzikar ('89)

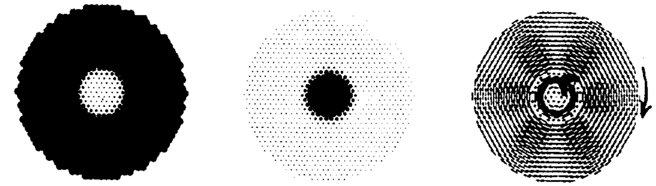
Impurity induced $\vec{B}$

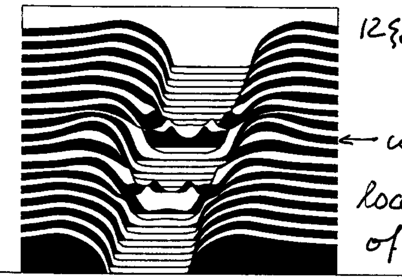


Local density of states

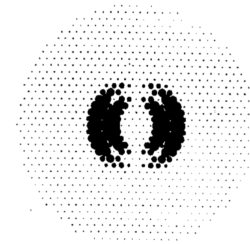
$l = 10\xi_0$

The doubly quantized vortex with homogeneous core

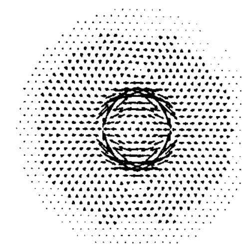




local density of states



Out of phase charge response for $\omega = 0.2\Delta$

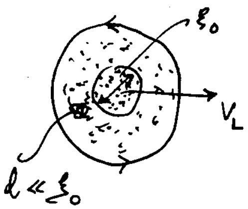


Absorptive current response for $\omega = 0.2\Delta$

$l = 10\xi_0$

Vortex Core Dynamics

Conventional Type II SC with $\xi_0 > \ell = v_f \tau_{el}$ are described by the Bardeen-Stephen Model of a "normal core".



$\sigma_{core} \cong \sigma_{Normal}$

$\rho_{ff} \approx \underline{(B/H_{c_2})}\, \rho_N$

(Flux-Flow Resistance)

However, in the clean limit,

$\frac{\hbar}{p_f} \sim a_0 < \xi_0 < \ell$, but not ultra-clean

$\ell < \frac{E_F}{\Delta_0} \cdot \xi_0$

The vortex core is described by a spectrum of Andreev Bound States w/ very different dynamics than Normal e^-.

Non-Equilibrium Theory of Superconductivity

For Equilibrium studies of Inhomogeneous SC we focussed on the spectral properties and the self-consistent order parameter.

Non-equilibrium Dynamics introduces new degrees of freedom and complexity to the theoretical description and framework.

Dynamics → Non-equilibrium Distribution Fnct. for excitations & pairs.

Andreev processes → Quantum transitions between Branches of quasiparticles, and excited pairs (collective modes)

→ Spectral Dynamics of the excitations e.g. $\vec{v}_f \cdot \vec{A}(\vec{R}, t)$.

→ Coupling of excitations & collective modes.

(self-consistency is essential for conservation laws.)

Keldysh Green's Function

$$\check{G}(\vec{p}_f \vec{R}; \epsilon t) = \begin{pmatrix} \hat{g}^R & \boxed{\hat{g}^K} \\ 0 & \hat{g}^A \end{pmatrix}$$

$\hat{g}^K$ = "generalized" Distribution Function

$$\hat{g}^K_{equil.} = \underbrace{\left(f(\epsilon)-\tfrac{1}{2}\right)}_{\text{equilib. distrib. fnct}} \underbrace{\left[\hat{g}^R_{eq} - \hat{g}^A_{eq}\right]}_{\text{Spectral function.}}$$

Normalization: $\hat{g}^R \otimes \hat{g}^R = -\pi^2 \hat{1}$

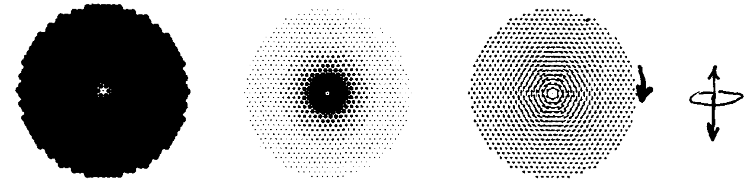
$$\hat{g}^R \otimes \hat{g}^K - \hat{g}^K \otimes \hat{g}^A = 0$$

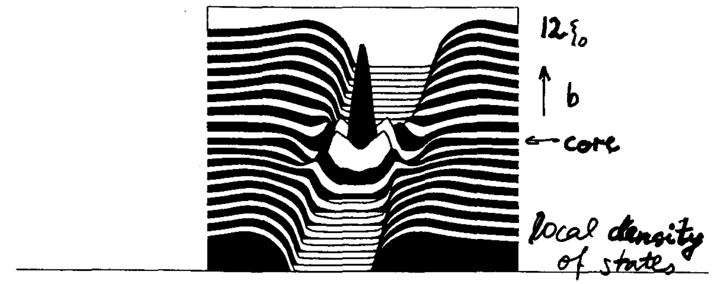
Transport Equations:

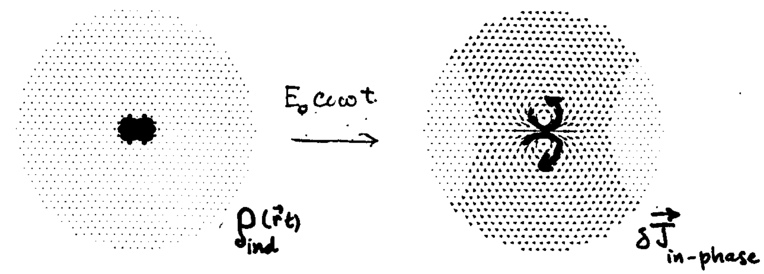
1) $[\epsilon\hat{\tau}_3 - \hat{V}_{ext} - \hat{\sigma}^{R,A}, \hat{g}^{R,A}]_\otimes + i\vec{v}_f\cdot\vec{\nabla}\hat{g}^{R,A} = 0$

2) $(\epsilon\hat{\tau}_3 - \hat{V}_{ext} - \hat{\sigma}^R)\otimes\hat{g}^K - \hat{g}^K\otimes(\epsilon\hat{\tau}_3 - \hat{V}_{ext} - \hat{\sigma}^A) - \hat{\sigma}^K\otimes\hat{g}^A + \hat{g}^R\otimes\hat{\sigma}^K + i\vec{v}_f\cdot\vec{\nabla}\hat{g}^K = 0$

Singly quantized vortex

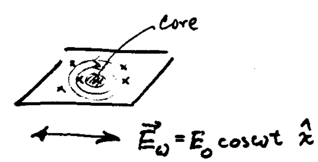






Out of phase charge response for $\omega = 0.2\Delta$

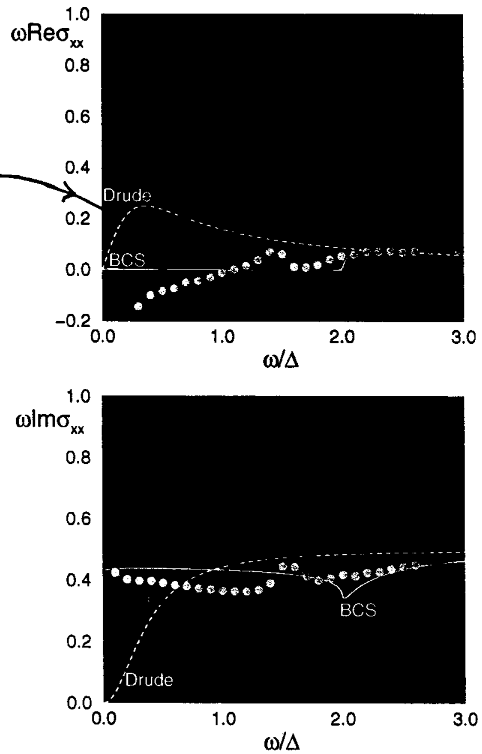
Absorptive current response for $\omega = 0.2\Delta$



$\ell = 10\xi_0$

Conductivity in the center of the vortex core
for an isolated s-wave vortex
(or p-wave (2D))

Bardeen-Stephen



$\chi = 0$

$x = \frac{\pi}{4}\xi_0$

$x = \frac{\pi}{2}\xi_0$

▸ $Re\,\sigma_{xx} < 0$ → "Hot spots"

- Energy Transport from $x = 0 \rightarrow x \sim \xi_0$

▸ $\overline{Re\,\sigma_{xx}} = \frac{1}{Area}\int dx \int dy \; Re\,\sigma_{xx} > 0$.

Conclusions

▸ (Dynamically) Fully Self-Consistent Calculations of Vortex Core Structure & EM Response for pancake Vortices w/ s- and p-wave symm.

▸ Broken T-symmetry of p-wave states:

- Spontaneous local currents/fields
- Doubly quantized Vortices w/ novel cores
- Splitting of H_{c2} for $\vec{B} \parallel \hat{l}$ vs $\vec{B} \parallel -\hat{l}$

▸ A.C. Response of Vortex Cores → Bound States

- s-wave & p=1 (p-wave) → Zero Energy Core States
- p=2 (p-wave) → finite energy Bound states
- − Surface dipole outside core. charge distribution at core interface