Lecture II

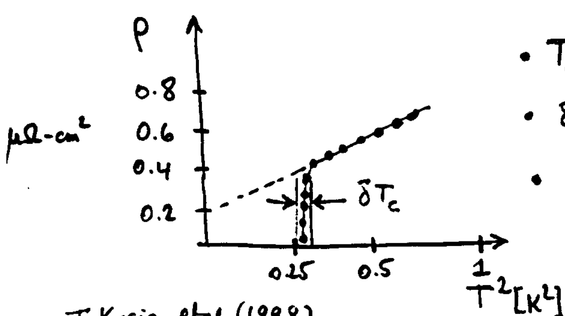
A { • Multiple SC phases of UPt_3
  • Ginzburg-Landau Theory

B { • Microscopic Free Energy Functional
  • GL Theory derived from Fermi Liquid / BCS

$\boxed{UPt_3}$ Heavy Fermion Metal, $T < T^* \sim 10K$

- $\rho = \rho_0 + A T^2$ w/ $A \simeq$, $\rho_0 \simeq 0.1 \mu\Omega \, cm^2$
- $C_V \simeq \gamma T$ w/ $\gamma/\gamma_0 = \frac{m^*}{m} \simeq 500$!
- $\chi/\chi_0 \simeq$ constant

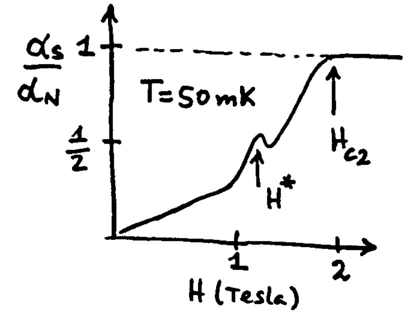
► Superconductivity (G. Stewart, etal. 1984)



J. Kycia, etal (1998)

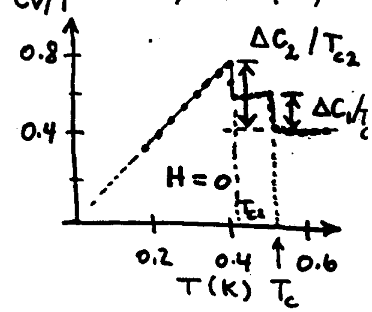
- $T_c \simeq 526$ mK
- $\delta T_c \simeq 1$ mK
- $RRR = \frac{\rho(300)}{\rho_0} \simeq 1{,}200$
- $\Delta C / \gamma T_c \simeq 1$

Multiple SC phases in UPt_3



Qian, etal (1987)
– Müller, etal (1987)

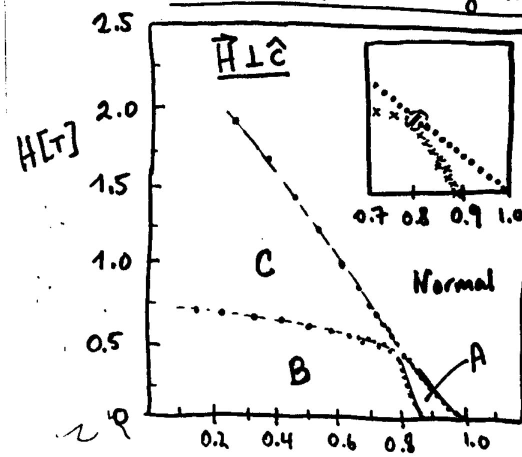
Fisher, etal (1989)



splitting $\frac{\Delta T_c}{T_c} \simeq 10\%$

$\frac{\Delta C_2 / T_{c2}}{\Delta C_1 / T_{c1}} \simeq 1.25$

► H vs. T phase diagram (S. Adenwalla, etal. 1990)



- 3 Flux Phases
- 2 Meissner Phases
- Tetracritical point for $\vec{H} \perp \hat{c}$ & $\vec{H} \not\perp \hat{c}$

$\boxed{\text{Sauls Lecture 2}}$

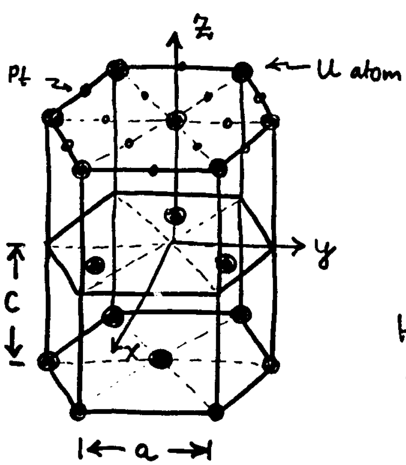
Multiple Superconducting Phases

⇓

Multi-Component Order Parameter
$F_{\alpha\beta}(\vec{p})$

1). Representation with Dim > 1

2). Two (Nearly) Accidentally Degenerate Representations



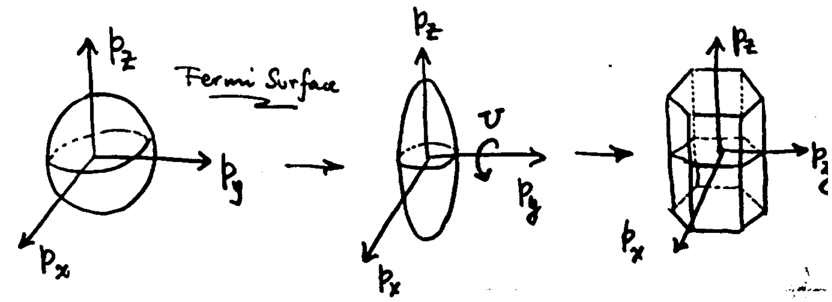
$$G = \Big(\overbrace{SO(3)_{spin}}^{\text{for weak spin-orbit coupling}}\Big) \times D_{6h} \times T \times U(1)$$

Hexagonal point group w/ Inversion

Time-reversal

Gauge

Representations of D_{6h} (derived from SO(3))



$$SO(3) \longrightarrow SO(2)\times U \longrightarrow \underbrace{D_{6h}}_{}\quad \overbrace{C_6,\ ?,\ U,\ \sigma_h}$$

$\boxed{l=1}$

'2D vector'

$$\Gamma_1 = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} \begin{matrix} \nearrow \\ \searrow \end{matrix} \begin{matrix} \begin{pmatrix} p_x \\ p_y \end{pmatrix} \longrightarrow \begin{pmatrix} p_x \\ p_y \end{pmatrix} \Rightarrow E_{1u} \\ (p_z) \longrightarrow (p_z) \Rightarrow A_{1u} \end{matrix}$$

$\boxed{l=2}$

$$\Gamma_2 = \begin{pmatrix} (p_x+ip_y)^2 \\ (p_x-ip_y)^2 \\ p_z(p_x+ip_y) \\ p_z(p_x-ip_y) \\ p_z^2-1/3 \end{pmatrix}$$

$$\xrightarrow{e^{\pm i2\phi}} \begin{pmatrix} (p_x+ip_y)^2 \\ (p_x-ip_y)^2 \end{pmatrix} \xrightarrow{\sim} \begin{pmatrix} (p_x^2-p_y^2) \\ 2p_xp_y \end{pmatrix} \Rightarrow E_{2g}$$

$$\xrightarrow{e^{\pm i\phi}} \begin{pmatrix} p_z(p_x+ip_y) \\ p_z(p_x-ip_y) \end{pmatrix} \xrightarrow{\sim} \begin{pmatrix} p_z p_x \\ p_z p_y \end{pmatrix} \Rightarrow E_{1g}$$

$$\xrightarrow{1} (p_z^2-1/3) \longrightarrow (p_z^2-1/3) \Rightarrow A_{1g}$$

E_{1u} Model $\longrightarrow$

- Similar Analysis for E_{1g}, E_{2g}, E_{2u}
- Model for Sr_2RuO_4 (Agterberg + Sigrist)

- Odd parity $\to S=1$
- Equal Spin pairing
- $F_{\alpha\beta}(\vec{p}) = \hat{d}\cdot(i\vec{\sigma}\sigma_y)_{\alpha\beta}\ \psi(\vec{p})$
- Strong Spin-Orbit Coupling

$$\Delta E_{S\text{-}O} \gg k_B T_c \Rightarrow \hat{d} \text{ locked to lattice}$$

choose $\hat{d} \parallel \hat{z}$

$$\hat{F}(\vec{p}) = \begin{pmatrix} 0 & \psi(\vec{p}) \\ \psi(\vec{p}) & 0 \end{pmatrix} \quad ; \ S_z = 0$$

2-Dim Orbital State

$$\psi(\vec{p}) = (\eta_x \hat{p}_x + \eta_y \hat{p}_y)$$

$$\boxed{\vec{\eta} = \begin{pmatrix} \eta_x \\ \eta_y \end{pmatrix} = \text{Order Parameter}}$$

2D vector (complex)

Ginzburg - Landau Free Energy Functional

$$\Delta\Omega[\vec{\eta},\vec{A}] = \int d^3R \Big\{ \underbrace{\alpha(T)\,\vec{\eta}\cdot\vec{\eta}^*}_{\text{Instability}} + \underbrace{\beta_1(\vec{\eta}\cdot\vec{\eta}^*)^2 + \beta_2|\vec{\eta}\cdot\vec{\eta}|^2}_{\text{Condensation Energy}}$$

$$+\left(K_1+\frac{K_{23}}{2}\right)\left[|\vec{D}_\perp \eta_x|^2 + |\vec{D}_\perp \eta_y|^2\right] \leftarrow \text{Kinetic Energy of Supercurrents + Gradient Energies}$$

$$+ K_4 \left[|D_z \eta_x|^2 + |D_z \eta_y|^2\right] \leftarrow \text{Uniaxial Anisotropy}$$

$$+ \frac{K_{23}}{2}\left[(D_x\eta_x)(D_y\eta_y)^* + \text{c.c.}\right] \leftarrow \text{Intrinsic (Broken Symmetry) ab-plane Anisotropy}$$

$$+ K_a\left(\frac{2e}{\hbar c}\right) i\,(\vec{\eta}\times\vec{\eta}^*)\cdot(\vec{\nabla}\times\vec{A}) \leftarrow \text{Spontaneous Orbital Magnetism}$$

$$+ \frac{|\vec{\nabla}\times\vec{A}|^2}{8\pi} \Big\}$$

- 7 Material parameters
- $\vec{D} \equiv \vec{\nabla} - i\frac{2e}{\hbar c}\vec{A}$; $\vec{D} = (\vec{D}_\perp, D_z)$

GL Equations

$$\frac{\delta \Delta\Omega[\vec{\eta},\vec{A}]}{\delta \eta_i^*} = 0 \qquad \frac{\delta \Delta\Omega[\vec{\eta},\vec{A}]}{\delta A_i} = 0$$

$$\alpha \eta_x = 2\beta_1 |\vec{\eta}|^2 \eta_x + 2\beta_2 (\vec{\eta}\cdot\vec{\eta})\eta_x^* + K_{123} D_x^2 \eta_x + K_1 D_y^2 \eta_x + (K_2 D_x D_y + K_3 D_y D_x)\eta_y$$

$$\alpha \eta_y = 2\beta_1 |\vec{\eta}|^2 \eta_y + 2\beta_2 (\vec{\eta}\cdot\vec{\eta})\eta_y^* + K_{123} D_y^2 \eta_y + K_1 D_x^2 \eta_y + (K_2 D_y D_x + K_3 D_x D_y)\eta_x$$

$$\left[\vec{\nabla}\times(\vec{\nabla}\times\vec{A})\right]_i = -\frac{16\pi e}{\hbar c}\,\mathrm{Im}\Big[K_4\, \delta_{iz}\, \eta_j (D_z \eta_j)^* + K_1 \eta_j (D_{\perp i}\eta_j)^* + K_2 \eta_i (D_{\perp j}\eta_j)^* + K_3 \eta_j (D_{\perp j}\eta_i)^* \Big]$$

Zero-Field Solutions (Homogeneous Phases)

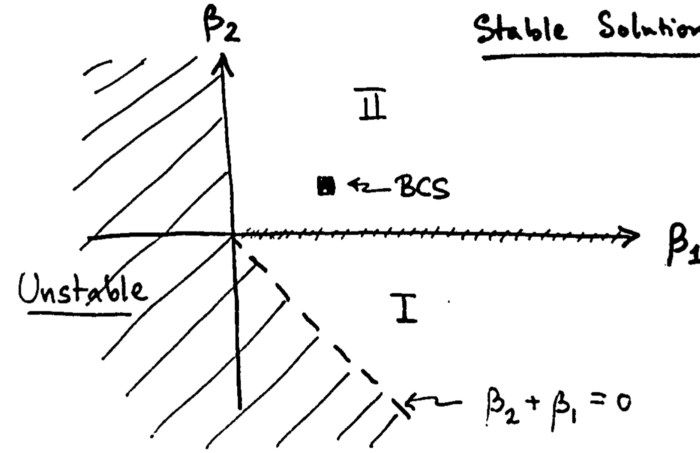
$$\mathrm{Min}\ \Delta\Omega[\vec{\eta}] = \int dR \left\{ \alpha |\vec{\eta}|^2 + \beta_1 |\vec{\eta}|^4 + \underbrace{\beta_2 |\vec{\eta}\cdot\vec{\eta}|^2}_{} \right\}$$

$$\vec{\eta} = \begin{pmatrix} \eta_x \\ \eta_y \end{pmatrix} = e^{i\theta} \begin{pmatrix} |\eta_x|\, e^{i\zeta/2} \\ |\eta_y|\, e^{-i\zeta/2} \end{pmatrix} \qquad \boxed{-\beta_2 |\eta_x|^2 |\eta_y|^2 \times \sin^2(\zeta)}$$

$\Theta(\vec{R})$ = Global phase (Broken $U(1)$) $\rightarrow \vec{p}_s \sim \vec{\nabla}\theta$

$\zeta(\vec{R})$ = Internal, relative phase (Broken $\mathcal{T}$-symm.)
↳ Orbital Magnetism

Stable Solutions ($T<T_c$)



I. $\beta_1 > 0$ & $0 > \beta_2 > -\beta_1$

$$\vec{\eta} = \eta_0 \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} e^{i\theta}$$

Degenerate w/

$C_6 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$G = D_{6h} \times T \times U(1)_\theta$$

$$G' = \underbrace{[D_{2h}]} \times [T]$$

Broken Rotational Symm.

$$\vec{j}_s = 2e \left[\overleftrightarrow{\rho}_s\right] \cdot \left(\vec{\nabla}\theta - \frac{2e}{\hbar c}\vec{A}\right)$$

$$\overleftrightarrow{\rho}_s = \eta_0^2 \begin{array}{c} \begin{pmatrix} \boxed{K_{123}} & 0 & 0 \\ 0 & \boxed{K_1} & 0 \\ 0 & 0 & K_4 \end{pmatrix} \end{array} \begin{array}{c} x \\ y \\ z \end{array}$$

$$j_{sx} \sim \frac{1}{\lambda_x^2}\left(\nabla_x\theta - \frac{2e}{\hbar c}A_x\right)$$

$$j_{sy} \sim \frac{1}{\lambda_y^2}\left(\nabla_y\theta - \frac{2e}{\hbar c}A_y\right)$$

$$\left.\right\} \rightarrow \boxed{\frac{\lambda_y^2}{\lambda_x^2} = \frac{K_{123}}{K_1}}$$

► Consequences:

Anisotropic Current Flow in the ab-plane $\}\rightarrow$ Deformed Vortex Lattice

$\underline{\vec{H} \parallel \hat{z}}$

II. $\beta_1 > 0$ & $\boxed{\beta_2 > 0}$

Minimize

$\beta_2 |\vec{\eta}\cdot\vec{\eta}|^2$

$=$

$$\vec{\eta} = \eta_0 \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$G = D_{6h} \times T \times U(1)_\theta$$

$$G' = [D_{6h}]_{R-\theta}$$

ab-plane isotropy

$$\overleftrightarrow{\rho}_s = \eta_0^2 \begin{pmatrix} \boxed{K_1 + \frac{K_{23}}{2}} & 0 & 0 \\ 0 & \boxed{K_1 + \frac{K_{23}}{2}} & 0 \\ 0 & 0 & K_4 \end{pmatrix}$$

- Broken T-symmetry
- Relative Gauge-Rot. Symm. Broken

$$\rightarrow \lambda_x^2 = \lambda_y^2 \neq \lambda_z^2$$

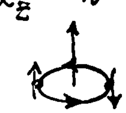
Broken Time-Reversal Symmetry

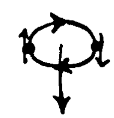
$$\vec{\eta}_+ = \frac{\eta_0}{\sqrt{2}}\begin{pmatrix} 1 \\ +i \end{pmatrix} \qquad \vec{\eta}_- = \frac{\eta_0}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

2-fold degenerate

$$\Psi_+(\vec{p}) \sim (p_x + i p_y) \qquad \Psi_-(\vec{p}) \sim (p_x - i p_y)$$

$d_z = +\hbar$





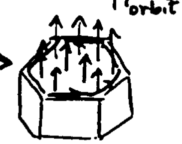
$d_z = -\hbar$

Orbital Magnetism of Pairs

$$\vec{M}_{orbit} = -K_a\left(\frac{2e}{\hbar c}\right) i(\vec{\eta}\times\vec{\eta}^*) = \left(\frac{2e}{\hbar c}\right) K_a \eta_0^2\ \hat{z}$$

- What is the Source of $\vec{M}_{orbit}$?

$\longrightarrow$ (Edge Currents, $\Delta r \sim \xi_0$) $\longrightarrow$



$$\vec{\eta} = \eta_0(\vec{R})\begin{pmatrix}1\\ i\end{pmatrix} \Rightarrow \vec{J}_{edge} = \vec{\nabla}\times\vec{M}_{orbit}$$

* Common to SC w/ Broken $\mathbb{T}$ and $\mathbb{P}$ symmetries

- Screening of $\vec{M}_{orbital}$ (Gorkov)

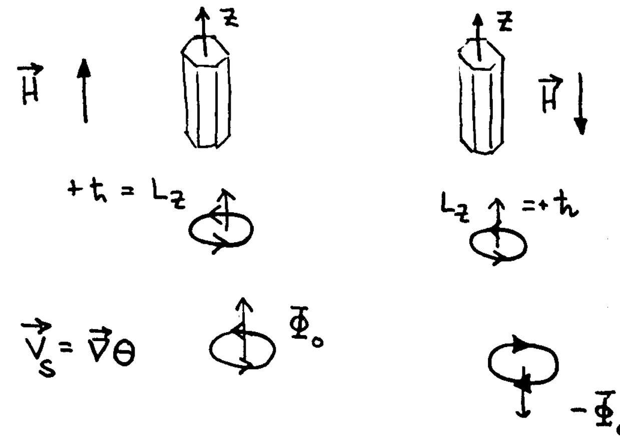
$$4\pi M_{orbit} < H_{c1} \rightarrow 4\pi(\vec{M}_{orbit} + \vec{M}_{Meissner}) \rightarrow 0$$

BCS/clean: $M_{orbit} = 0$ $(\Delta r \gg \lambda)$

BCS + : $M_{orbit} \sim N(E_f)\left(\frac{\Delta^2}{E_F}\right)\left(\frac{e\hbar}{m^*c}\right) \sim H_{c1}\left(\Delta/E_F\right)^2 \ll H_{c1}$.

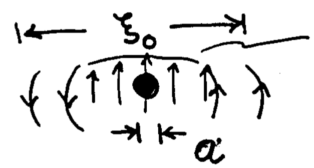
Signatures of Broken $\mathbb{T}$-symmetry

1. Asymmetry in $H_{c1} = \frac{4\pi}{\Phi_0}\left\{\pi\rho_s \ln\frac{\lambda}{\xi} + \varepsilon_{core}\right\}$



Tokuyasu, et al. $\quad H_{c1}^{\uparrow} \neq H_{c1}^{\downarrow} \leftarrow \boxed{\Delta\mathcal{E}_{core}}$

2. Impurity-Induced Local Fields (Choi, Muzikar) (non-magnetic)



$$\vec{B}(0) \approx n\left(\frac{\Delta}{E_f}\right)^2\left(\frac{e\hbar}{m^*c}\right)(ak_f)^2$$

$$\approx 10^{-2} - 10^{-1}\ \text{Gauss}$$

Symmetry Breaking Perturbations

► Strain Coupling to the SC Order Parameter (ab-plane)

- Strain Tensor: $\varepsilon_{ij} = \varepsilon_{ji} = \epsilon_{ij} + \underbrace{\tfrac{1}{3}\varepsilon\,\delta_{ij}}_{\text{Dilitation}}$
- ε_{ij} is a 2nd rank Tensor.

► New 2nd Order Invariant

$$\Delta\Omega_\varepsilon = \int d^3R\ \tau\left(\eta_i\ \varepsilon_{ij}\ \eta_j^*\right)$$

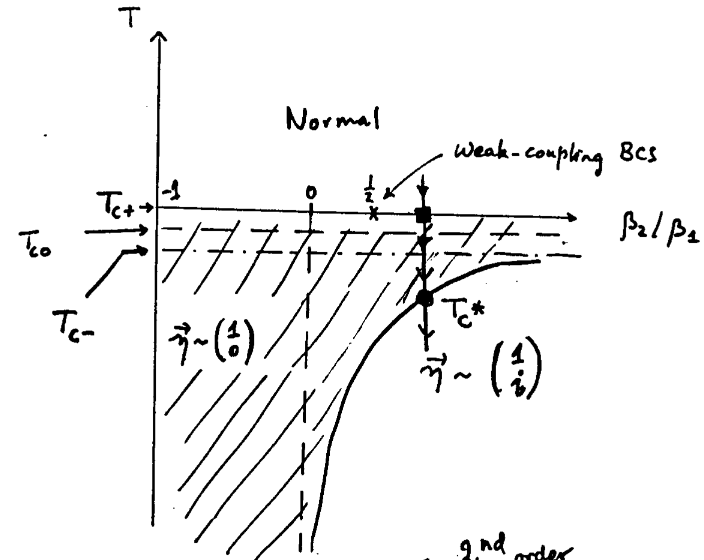
- $\varepsilon \rightarrow T_c(\varepsilon)$
- ε_{ij} → Orients degenerate $\binom{1}{0}$ states $(\beta_2<0)$
  → Splitting of the $H=0$ transition $(\beta_2>0)$

► Orthorhombic Strain

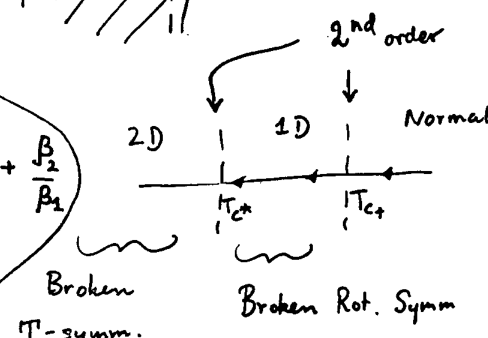
$$\Delta\Omega_\varepsilon = \int d^3R\ \tau\,\varepsilon\left(|\eta_x|^2-|\eta_y|^2\right) \rightarrow T_c^{\pm} = T_{co} \mp \frac{\tau\varepsilon}{\alpha_0}$$

Zero-field phase diagram

E_1 (or E_2) SC with SBF



$$\frac{\Delta C^*/T_{c^*}}{\Delta C^+/T_{c+}} = 1+\frac{\beta_2}{\beta_1} > 1$$



Beyond the GL theory

- What is the ground state?

  β_2 / β_1

  Coupling to SBF $<$ Strain / Magnetism

- What determines the Anisotropy of the Currents?

  K_4 , $K_{1,2,3}$

- What is the magnitude, sign, origin of the Spontaneous Orbital Magnetism?

  K_a , Boundary Currents

- How does (weak) disorder effect SC?

  $T_c(\tau_{elastic})$, β_i

- What is the structure of vortices & vortex lattices in multi-component SC?

  Internal states $\rightarrow$ $H_{c1}^{+} - H_{c1}^{-}$