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The Destructive Effect of a Magnetic Field

With decreasing size, pass through 4 regimes

1. Meissner effect
2. Orbital Pair Breaking
3. Spin Pair Breaking in Thin Films
4. Spin Pair Breaking in Nanoparticles

Will concentrate on type I superconductors ($\lambda < \xi$)

(reminder - in type II, magnetic vortices can penetrate

for interesting work on small type II samples, see A.K. Geim et al., PRL 79, 4653 (98)
Nature 390, 259 (98))

For samples smaller than ξ and λ, the distinction between type I and II is not meaningful.

References: M. Tinkham, Intro to Superconductivity

R Meservey and P.M. Tedrow, Physics Reports 238, [redacted] 173 (1994)

F. Braun et al, PRL 79, 921 (97)

Ralph
Lecture 3

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General Ideas

Superconductivity is based on the pairing of time-reversal-symmetric energy levels

A magnetic field disrupts time-reversal symmetry - interferes with superconductivity

We will see several trade-offs of energy scales.

Meissner Effect

1st ingredient – Condensation Energy

With no applied magnetic field, a superconducting state has a lower energy than the normal state, with a difference in free energy per unit volume

$$f_N(B=0) - f_S(B=0) \sim \tfrac{1}{2} g(\epsilon_F) \Delta^2$$

2nd ingredient – Add Magnetic Field – what changes?

(a) For a bulk type I superconductor (size >> λ) the field is expelled from the material (perfect diamagnetism) due to screening currents near the surface of the sample

$\frac{\text{Energy cost}}{\text{unit volume}}$ to maintain SC state $\frac{H^2}{8\pi}$

c.g.s. $(4 \times 10^{-2} H^2)$

(b) For a normal metal, the free energy is lowered slightly on account of Pauli paramagnetism

$\frac{\text{Energy decrease}}{\text{unit volume}}$ in N state $\mu_B^2 g(\epsilon_F) H^2$ $(1 \times 10^{-6} H^2$ for Al$)$ → small

Final free energy difference

$$f_N(H) - f_S(H) = \underbrace{\tfrac{1}{2} g(\epsilon_F) \Delta^2}_{\text{unchanged by B since no field inside SC.}} - \underbrace{\mu_B^2 g(\epsilon_F) H^2}_{\text{ignore for now (will come back.)}} - \frac{1}{8\pi} H^2$$

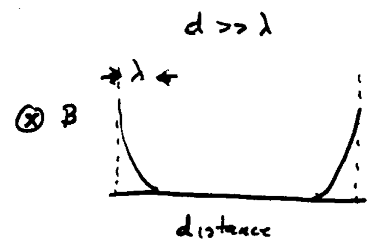
When $f_N(B) - f_S(B) < 0$, the SC state is unstable – 1st order transition to N state

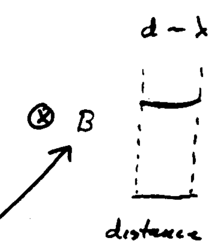
$$H_{c\,\text{Meissner}} \sim \sqrt{4\pi g(\epsilon_F)}\, \Delta \sim 100 \text{ Gauss for Al.}$$

Physical reason – Costs more energy to screen field than gained by remaining SC.

But things change for small samples, with at least one dimension comparable to λ – the magnetic field penetration depth. (≈ 100 nm)

Consider SC slab in a parallel magnetic field





field penetrates sample now – No screening cost to pay

Critical Field Can Go Much Higher.

Next Effect - Orbital Pair Breaking

A magnetic field alters the orbits of electrons inside the S.C. - disrupts superconductivity.

Time-reversed wave functions make up Cooper pairs. An applied field produces a relative energy shift between pairs.

$\mathcal{H} \propto \frac{e}{mc}\vec{p}_k \cdot \vec{A}$ for a state with wave vector $\vec{k}$

Time reversed orbital states - e.g. $\vec{k}$ and $-\vec{k}$ are split

$$\Delta E \sim \frac{2e}{mc}\vec{p}_k \cdot \vec{A} \sim \frac{2e}{c}\vec{v}_k \cdot \vec{A}$$

In a real (dirty) sample, lots of scattering. Plane wave states are not a good starting point.

Semiclassical picture - electrons do random walk in field.

It's easiest to characterize the strength of pair-breaking by adding up the phase difference $\varphi = \frac{\Delta E\, t}{\hbar}$ on each path segment.

(Analogy - Aharanov-Bohm)

$$\frac{d\varphi}{dt} = \frac{\Delta E}{\hbar} = \frac{2e}{\hbar c} v_k \cdot A$$

so on one segment, in mean free time τ

$$\Delta\varphi \sim \tau \frac{2e}{\hbar c} v_k \cdot A$$

In the course of the random walk, the phase difference grows $\propto \sqrt{N}$, where $N = \#$ of segments $= \frac{t}{\tau}$

Total phase accumulated $\sim (\tau t)^{1/2} \frac{2e}{\hbar c} \langle v_k \cdot A \rangle_{k}$ rms, some average

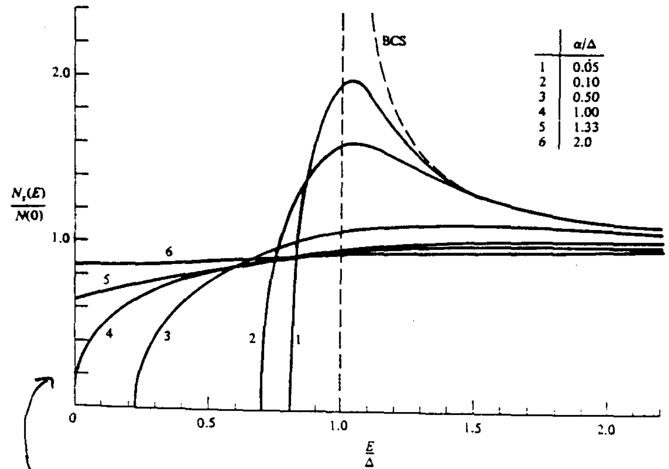
$$\sim (\tau t)^{1/2} \frac{2e}{\hbar c} v_F \sqrt{\langle A^2 \rangle}$$

It is conventional to characterize the strength of pair breaking by defining a parameter $\frac{1}{\tau_K}$, where τ_K is the time needed to accumulate a phase difference ≈ 1

$$\frac{1}{\tau_K} = \tau \left(\frac{2e}{\hbar c} v_F\right)^2 \langle A \rangle^2 \sim \frac{v_F^2 \tau e^2}{\hbar^2 c^2} H^2 d^2$$

(for a thin film of thickness d in a parallel field)

The magnetic-field-induced shifts in energy levels can be seen directly in the quasiparticle density of states



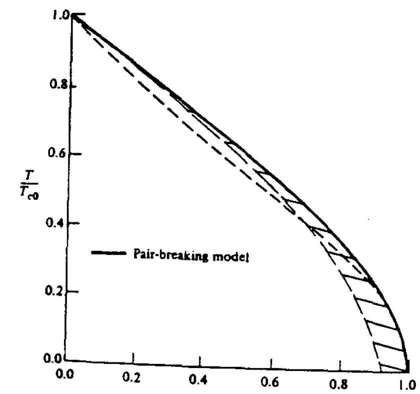
tunneling gap fills in before SC is destroyed

– gapless superconductivity –

As long as the paths are sufficiently random (ergodic), the same density of states also applies for phase breaking in other geometries. due to magnetic impurities, type II, etc

A detailed calculation shows that the critical field is determined by the condition $\frac{\hbar}{\tau_K} = \Delta_0$

$\frac{T}{T_c}$



$= \frac{\hbar}{\tau_K \Delta_0}$

$$\frac{v_F^2 \tau e^2 H_c^2 d^2}{\hbar c^2} \sim \Delta$$

$$\Rightarrow \quad H_c^2 \sim \frac{\hbar c^2 \Delta}{v_F^2 \tau e^2 d^2} \sim \frac{\hbar c^2 \Delta}{v_F e^2 \ell d^2} \qquad \text{where } \ell = v_F \tau$$

normally $\ell \sim d$ in very thin films

Intuitive, qualitative interpretation

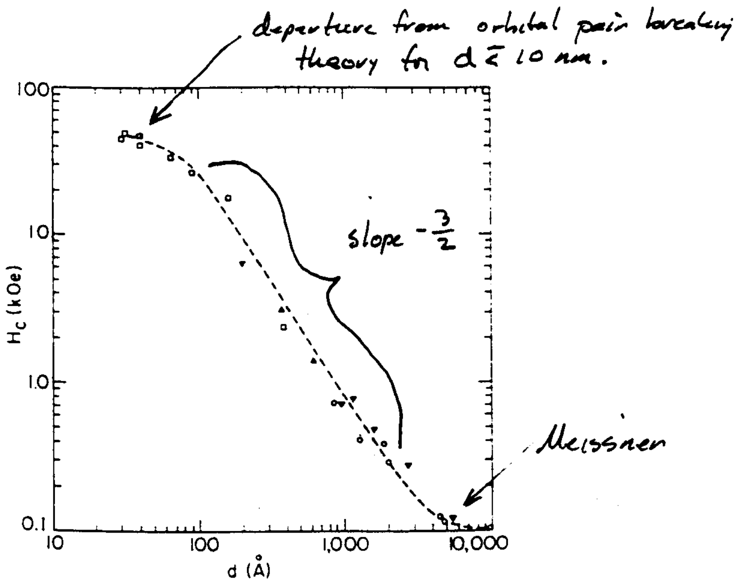
At the critical field, one flux quantum $\Phi_0 = \frac{hc}{2e}$ penetrates through an area ξd ← thickness of film

↳ effective coherence length $= (\xi_0 d)^{1/2}$ where $\xi_0 = \frac{\hbar v_F}{\pi\Delta}$

(true if the mean free path $\ell = d$)

Check: $H_c (\xi_0 d)^{1/2} d = \Phi_0$

$$H_c \sim \left(\frac{h\, c^2 \Delta}{v_F\, e^2}\right)^{1/2} \frac{1}{d^{3/2}}$$



$\boxed{\text{Spin Pair Breaking}}$ (Clogston, Chandrasekhar)

For very thin films - the electron orbits are very compressed - do not intersect much field ⇒ little shifting by Aharonov-Bohm effect.

But we have not yet considered the effects of H on the electron spin.

Recall from discussion of Meissner Effect:

$$f_N(H) - f_S(H) = \underbrace{\tfrac{1}{2} g(\epsilon_F)\Delta^2}_{\text{condensation energy}} - \underbrace{\tfrac{1}{8\pi} H^2}_{\text{Meissner screening cost in bulk} \to 0 \text{ for thin samples}} - \underbrace{\mu_B^2 g(\epsilon_F) H^2}_{\text{Spin paramagnetism in N state (Pauli)}}$$

Previously we ignored Pauli term. But if there are no orbital effects, at high fields this term by itself may make the N state more energetically favorable than SC state:

$f_N(H) - f_S(H) < 0$ when $\frac{1}{2} g(\epsilon_F)\Delta^2 < \mu_B\, g(\epsilon_F) H^2$

$$\Rightarrow \boxed{H_{c\,\text{spin}} = \frac{\Delta}{\sqrt{2}\,\mu_B}}$$

Note the density of states drops out.

1st order transition

Physical Interpretation:

At the critical field, it becomes energetically beneficial to break ~~lots~~ of Cooper pairs (by flipping spins), and gain the Zeeman energy in the applied magnetic field

\# of broken spins

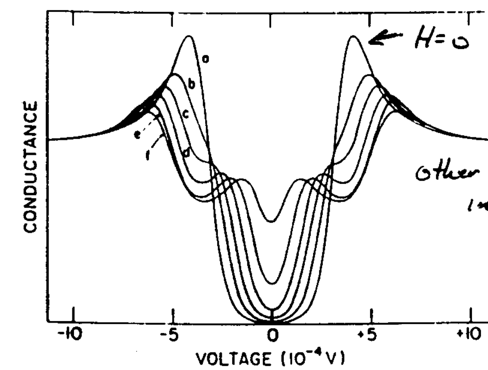
$$\sim \frac{\mu_B H_c}{(\delta E)} \overset{4\ \text{Tesla}}{=} \frac{0.2\ \text{meV}}{\delta E} \sim 10^5$$

level spacing in sample

for 20x20x2000 nm sample

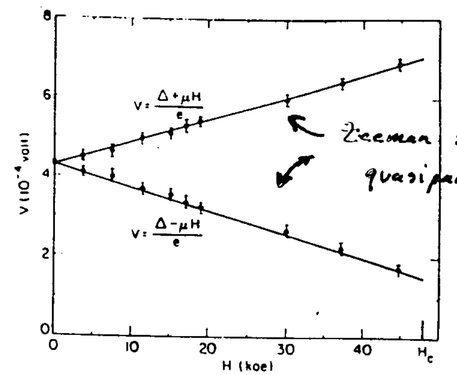
Beautiful Experimental Confirmation - Meservey & Tedrow

Tunneling into a thin Al sample in parallel magnetic field



← H=0

other curves for increasing H



Zeeman splitting of quasiparticle states (still S.C.)

suddenly, no more S.C. at predicted critical field.

Energy Levels in Superconducting Nanoparticles

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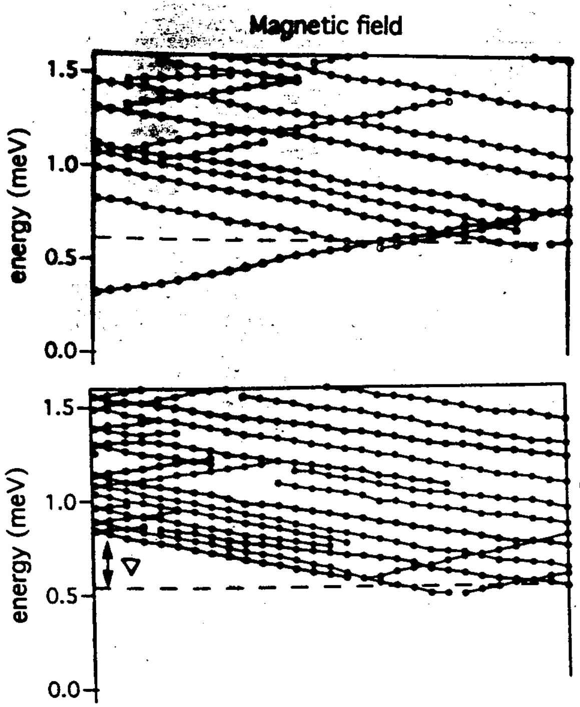
Level Crossings at Large Magnetic Fields



odd n_0 → even n_0+1 (probes n_0+1 states above chemical potential)

even n_0+1 → odd n_0 (probes filled n_0 states)

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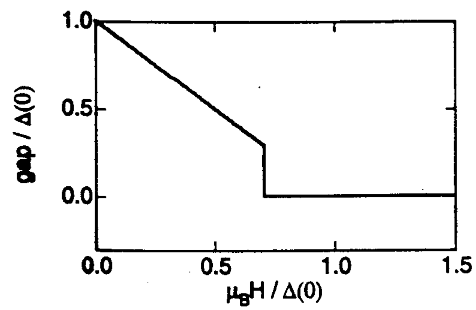
Why does the tunneling threshold evolve continuously as a function of H?

<u>Clogston and Chandrasekhar (1966)</u>
Predicted a discontinuous transition under the influence of spin pair-breaking, for a continuum density of states.

Competition between the superconducting condensation energy and the Pauli paramagnetism of the normal state:

$$\frac{\Delta^2}{2(\delta E)} \text{ vs. } \frac{(\mu_B H)^2}{\delta E}$$

Discontinuous transition when $H_c = \dfrac{\Delta}{\mu_B \sqrt{2}}$.



A discontinuous transition is expected whenever $\delta S > 1$ at the transition.

Why break one Cooper pair at a time in a nanoparticle?

- Need a more careful consideration of what is the ground state in a magnetic field.

1. Superconducting state with some broken pairs?
   - Can we optimize the energy by having a small (non-zero) spin, while still maintaining the superconducting condensation energy?

or

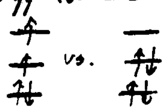
2. Is breaking one Cooper pair enough to destroy the pairing correlations in a nanoparticle?

Energies to take into account:

- Zeeman energy for states with unpaired spins

  $-g\mu_B S H$

- Kinetic energy for exciting states to higher orbital levels.



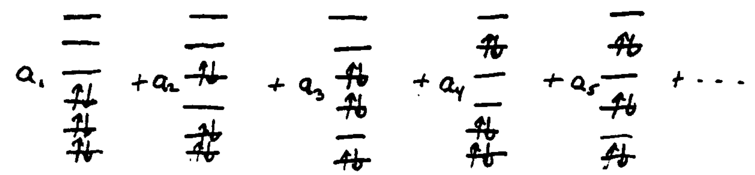
- Condensation energy including gap suppression

  New ingredient

  An unpaired electron suppresses the condensation energy.

Idea of Gap Suppression

Recall: with no unpaired electrons, the S.C. ground state is a linear superposition of Slater determinant states with different arrangements of pairs.

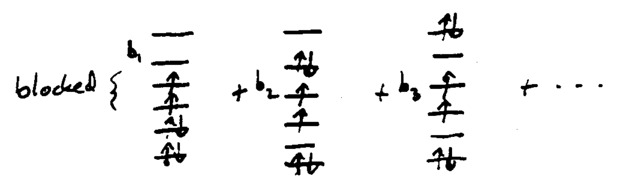


Allows maximum energy benefit from pair scattering

$$\langle \Psi_G | V_{ee} | \Psi_G \rangle$$

$$V_{ee} = K \sum_{i,j} c_{i\uparrow}^{\dagger} c_{i\downarrow}^{\dagger} c_{j\downarrow} c_{j\uparrow}$$

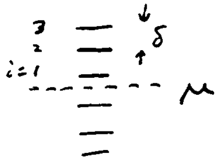
If pairs are broken, some states are blocked from participating in the linear superposition.



Less "phase space" for pair scattering, smaller condensation energy.

Amusing Exercise

Consider BCS gap equation with equally-spaced discrete levels



$$1 = \frac{V}{2} \sum_{i=1}^{\hbar\omega_c/\delta} \frac{1}{[\Delta^2 + (E_i-\mu)^2]^{1/2}}$$

$$E_i - \mu = \frac{\delta}{2} + (i-1)\,\delta$$

In the continuum limit ($\delta \to 0$) in weak coupling ($\frac{V}{\delta} \ll 1$), solving the gap equation gives usual answer

$$\Delta \sim 2\hbar\omega_c\, e^{-\frac{\delta}{V}} \qquad \left(g(\epsilon_F)(\mathrm{Vol}) = \tfrac{1}{\delta}\right)$$

To do: a) Solve gap equation with non-zero δ ($< \Delta$)

b) Solve equation for 1 state blocked = begin sum at $i = 2$.

Can see the effects of gap suppression directly (in the BCS mean-field approximation)

Braun, von Delft

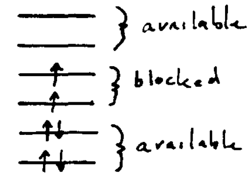
Variational Technique for Finding Correlated Eigenstates

For each spin s, find variational eigenstates of the form

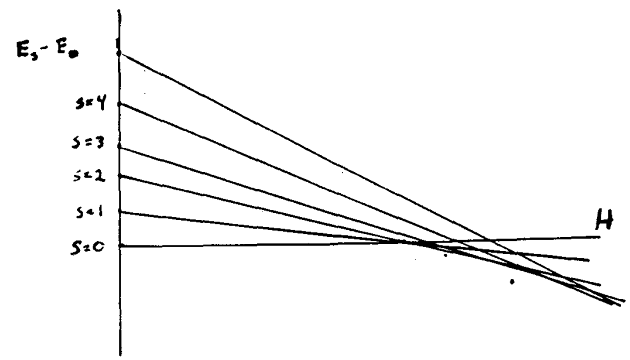
$$|s\rangle = \prod_{j\ \text{available}} \left(u_j^{(s)} + v_j^{(s)} c_{j,\uparrow}^{\dagger} c_{j,\downarrow}^{\dagger}\right) |\mathrm{Vac}\rangle$$

in the BCS framework.

For s=1



As H is increased, what is s for the eigenstate whose energy crosses first below the SC ground state energy?

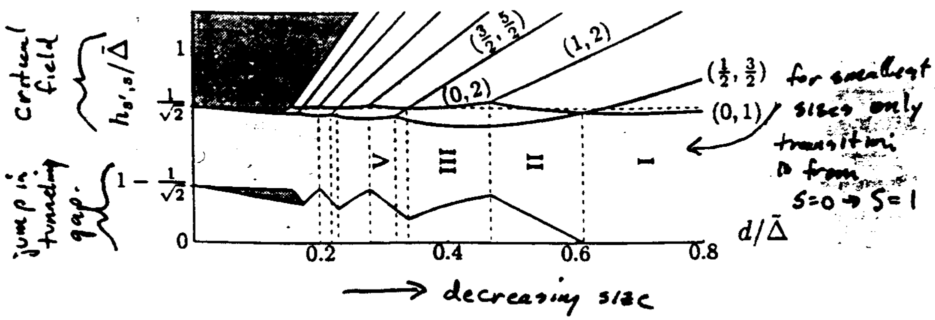


Whether s=1 crosses s=0 before any s > 1 state does determines the nature of the transition.

For $\delta E \to 0$, (C&C limit) $s_{crit} \approx \Delta / (\sqrt{2}\,\delta E)$.

(Exact Solution · Richardson)

The Order of Level Crossings varies with Level Spacing



It turns out, in general, that the state existing after the transition has the gap suppressed completely to zero. — At no magnetic field is there a ground state with non-minimal S and S.C. correlations.

Breaking one Cooper pair is sufficient to kill superconductivity in a nanoparticle.

(All true in this simple 1 band, s-wave, variational model.)

Conclusions:

A magnetic field is not good for superconductivity

For Aluminum, critical field can range from

100 Gauss – bulk samples

to

>40,000 Gauss – thin films and small particles.

Orbital effects dominant at larger sizes,

Spin pair breaking below ~ 10 nm.

Further complications possible:

- Spin-orbit scattering -

True eigenstates become superpositions of ↑ and ↓

Decreases Zeeman spin splitting

⇒ Even larger magnetic fields can be applied before Zeeman energy overwhelms the condensation energy.