Superconductivity in Small Samples

- What does "superconductivity" mean if the sample contains a fixed number of electrons?
- μm - sized samples
- nm - sized samples

references:

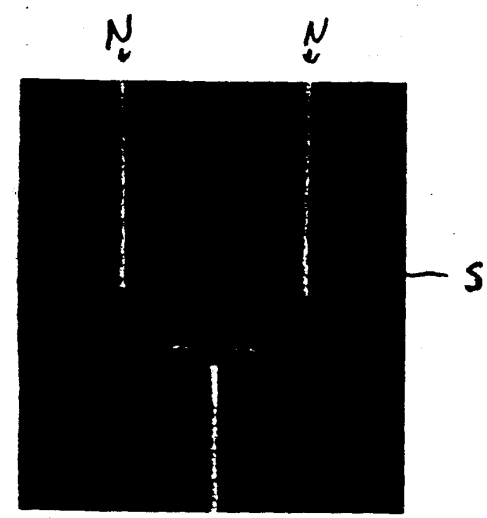
M. Tinkham, Intro to Superconductivity 2nd Ed
Ch. 7

J. von Delft, to appear in Physics Reports
http://www-tfp.physik.uni-karlsruhe.de/~vondelft/Habil/physicsreports.ps

J. M Hergenrother PhD Thesis 1995

Conventional Electron-Beam Lithography:

70 x 20 x 2000 nm^3 island



NSN tunneling transistor

10^9 conduction electrons

- J. Hergenrother.

Ralph
Lecture 2

Small Samples and Charging

Coulomb Charging Energy $\sim \frac{e^2}{2C}$

For 100nm x 100nm junction $C \sim 10^{-15} F$, $\frac{e^2}{k_B 2C} \sim 1\,K$.

3nm x 3nm $\quad C \sim 10^{-18} F \quad \frac{e^2}{k_B 2C} \sim 1000\,K$.

Thermal Charge Fluctuations

For $k_B T \ll \frac{e^2}{2C}$, thermal fluctuations are suppressed.

Quantum Charge Fluctuations

are suppressed if the lifetime broadening of energy levels $\frac{\hbar}{\Delta t} \ll \frac{e^2}{2C}$

$\Delta t \sim RC$ lifetime of charge on capacitor

$\Rightarrow$ need $\frac{\hbar}{RC} \ll \frac{e^2}{2C}$ or $R \gg \frac{2\hbar}{e^2}$

Conclusion – The charge on a metal grain is controllable to unit accuracy if

$$kT \ll \frac{e^2}{2C} \qquad R \gg \frac{h}{e^2}$$

even if the total number of electrons $> 10^9$!

Tuning the Charge – Effects of Gate Voltage.

Let n be the number of excess charges on a metal island.

$$E(n) = \frac{(ne)^2}{2C_\Sigma} - ne\frac{C_g}{C_\Sigma}V_g$$

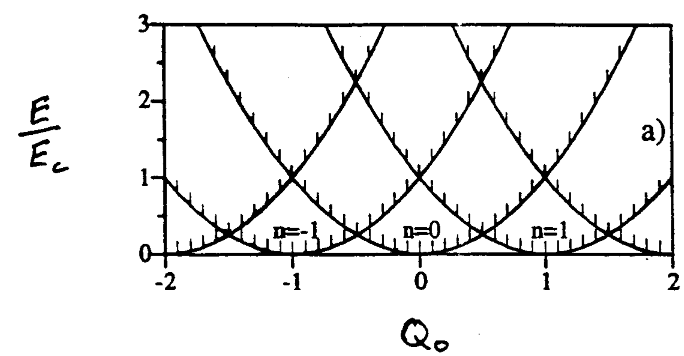
$$= \frac{1}{2C_\Sigma}(ne - C_gV_g)^2 - \frac{(C_gV_g)^2}{2C_\Sigma}$$

ignore – doesn't depend on n

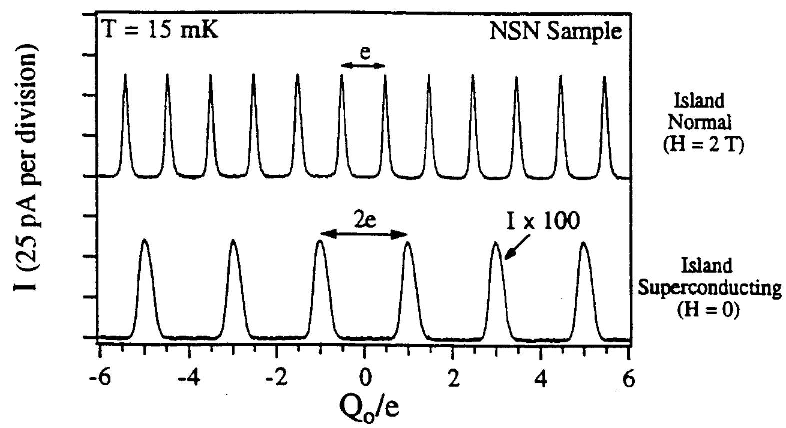
units of charge – call this $Q_0 = C_g V_g$

$$= \frac{1}{2C_\Sigma}(ne - Q_0)^2$$

For a given charge, energy depends quadratically on Q_0 (gate voltage)



Effects of Coulomb Blockade on Charge Flow



Is Superconductivity Still Possible With a Fixed Number of Electrons?

- BCS theory assumes a grand canonical ensemble. The ground state is written as a superposition of states with different numbers of electrons

$$|\Psi_G\rangle = \prod_i (u_i + v_i c_{i\uparrow}^{+} c_{i\downarrow}^{+}) |vac\rangle$$

- The superconducting order parameter is normally defined as

$$\Delta_{BCS} = \lambda(\delta E) \sum_i \langle c_{i\downarrow} c_{i\uparrow} \rangle$$

$$= 0 \quad \text{for a fixed number of electrons.}$$

- The superconducting phase is canonically conjugate to the electron number. If the electron # is fixed, then the phase is completely uncertain (No breaking of gauge invariance.)

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In Reality – No Problem

The BCS pairing Hamiltonian can be solved exactly using a canonical ensemble (fixed # of electrons.)
- R W Richardson (1960's - 70's)

Attractive e-e interactions still lead to a pair-correlated ground state, with a gap to excitations. (Just like ordinary BCS theory.)

(Important for many years in nuclear physics)

Can define a good order parameter

$$\Delta_{can}^2 = (\lambda\,\delta E)^2 \sum_{i,j} \left(\langle c_{i\uparrow}^+ c_{i\downarrow}^+ c_{j\downarrow} c_{j\uparrow} \rangle - \langle c_{i\uparrow}^+ c_{j\uparrow} \rangle \langle c_{i\downarrow}^+ c_{j\downarrow} \rangle \right)$$

The experiments I describe will demonstrate these pairing correlations.

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Effects of Pairing — Even/Odd Effects

For Even # of Electrons:

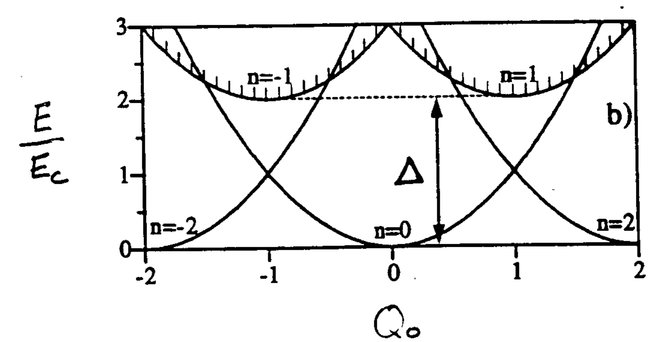
Ground State – Fully paired

1st Excited State – 1 broken pair (higher energy by 2Δ.)

For Odd # of Electrons

Ground State – 1 unpaired electron (quasiparticle)

Relative to even-electron ground state, higher in energy by Δ (plus charging energy).



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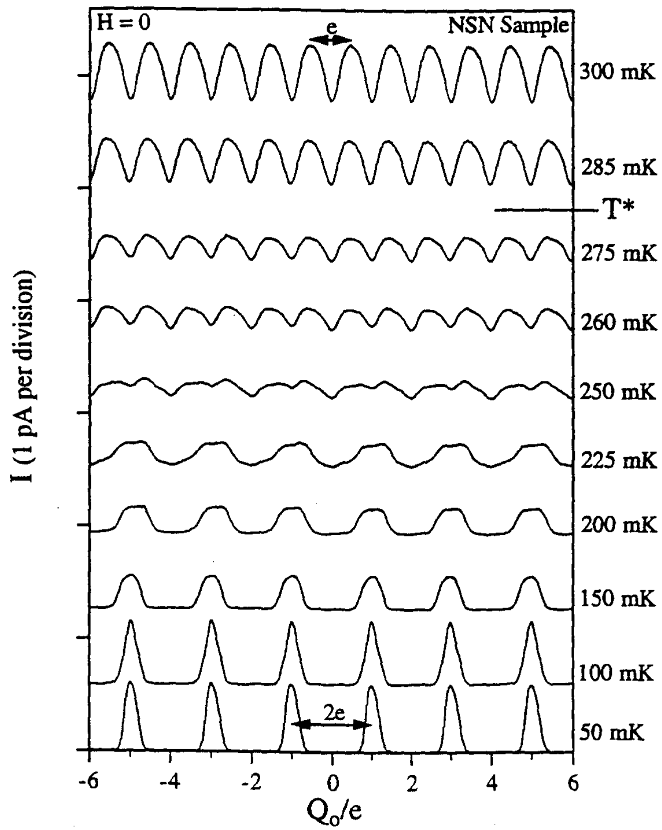
J.M. Hergenrother



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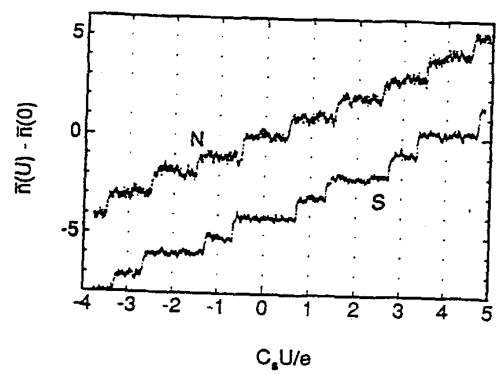
Direct Measurements of average charge on the island.



P. Lafarge et al.

PRL 70, 994 (93).

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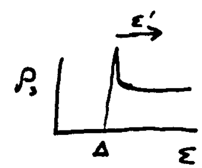
When raising T, why does the even-odd effect go away when $T \ll T_c$?

<u>Thermally-excited quasiparticles</u> - If a sample contains enough quasiparticles, no significant difference between even & odd.

Easy calculation - At what T does a sample contain on average 1 quasiparticle. (Think about easiest case - no parity restrictions.)



$$\langle N_{qp}(T)\rangle = 2(\mathrm{Vol})\int_0^\infty \rho_s(\varepsilon)\, f(\varepsilon,T)\, d\varepsilon$$

$$\approx 2(\mathrm{Vol})\int_\Delta^\infty \rho_s(\varepsilon)\, e^{-\beta\varepsilon}\, d\varepsilon \quad \text{for } kT \ll \Delta$$

$$\overset{\varepsilon=\Delta+\varepsilon'}{=} e^{-\beta\Delta}\, \underbrace{2(\mathrm{Vol})\int_0^\infty \rho_s(\varepsilon')\, e^{-\beta\varepsilon'}\, d\varepsilon'}$$

$N_{eff}(T)$ = # of available states within kT of Δ

$\approx 10^4$, weakly T dependent

Crossover temperature T_o^*

$$1 \sim N_{eff}\, e^{-\Delta/k_B T_o^*}$$

$$\Rightarrow k_B T_o^* \sim \Delta/\ln(N_{eff}) \sim \frac{1.8\, kT_c}{\ln(10^4)} \sim \frac{kT_c}{5}.$$

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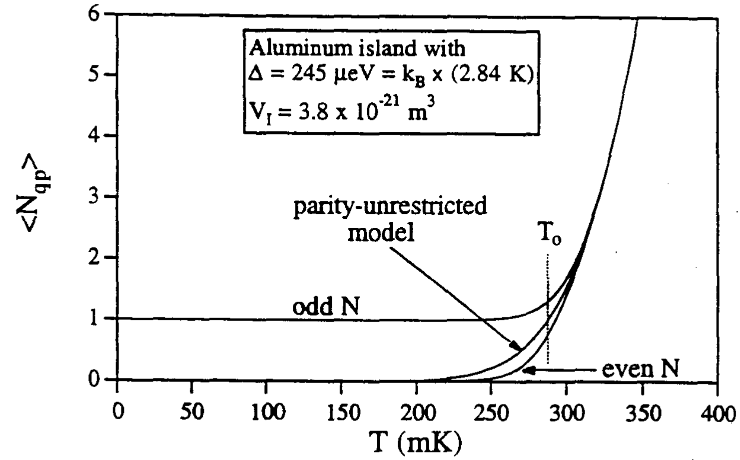
Results of a proper calculation.



How small a metal particle is required in order to resolve individual electronic states?

Need $\delta E > k_B T$.

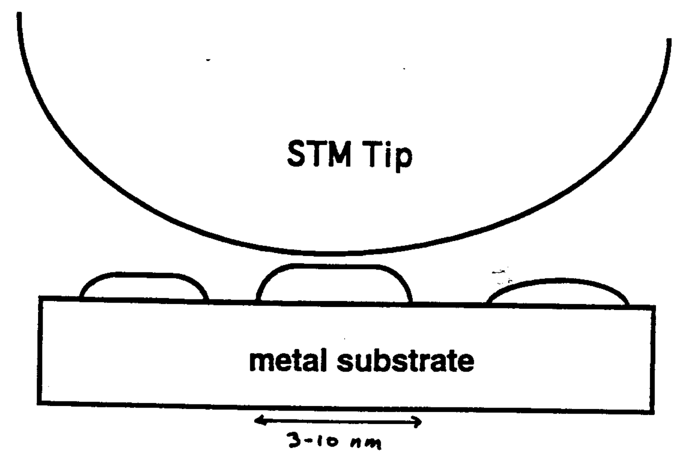
$$\delta E \approx \frac{4\varepsilon_F}{3N} = \frac{2\pi^2\hbar^2}{mk_F\mathcal{V}}$$

$$\mathcal{V} \approx \frac{90\ \text{nm}^3}{\delta E / \text{meV}}$$

Suppose we want $\delta E > k_B$ (1 K) ~0.1 meV.

$\Rightarrow \mathcal{V} < (10\ \text{nm})^3$.

Scanning-Tunneling Microscope:



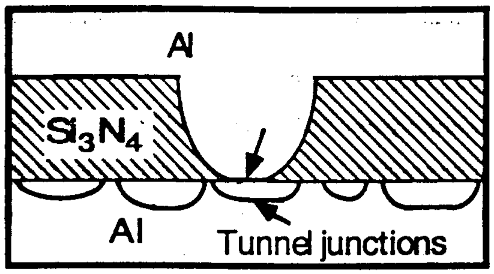
STM studies have been attempted, but discrete electronic states on the particles have never been resolved.
(some recent progress)

Problems:
Mechanical instability
Difficult to cool to mK temperatures
Often, electrical leads not filtered effectively

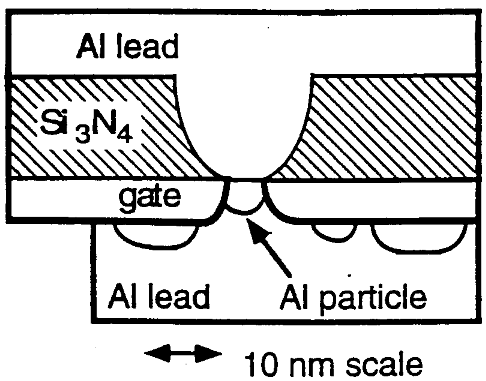
(15)

DEVICE CONCEPT

Without gate:



With gate:



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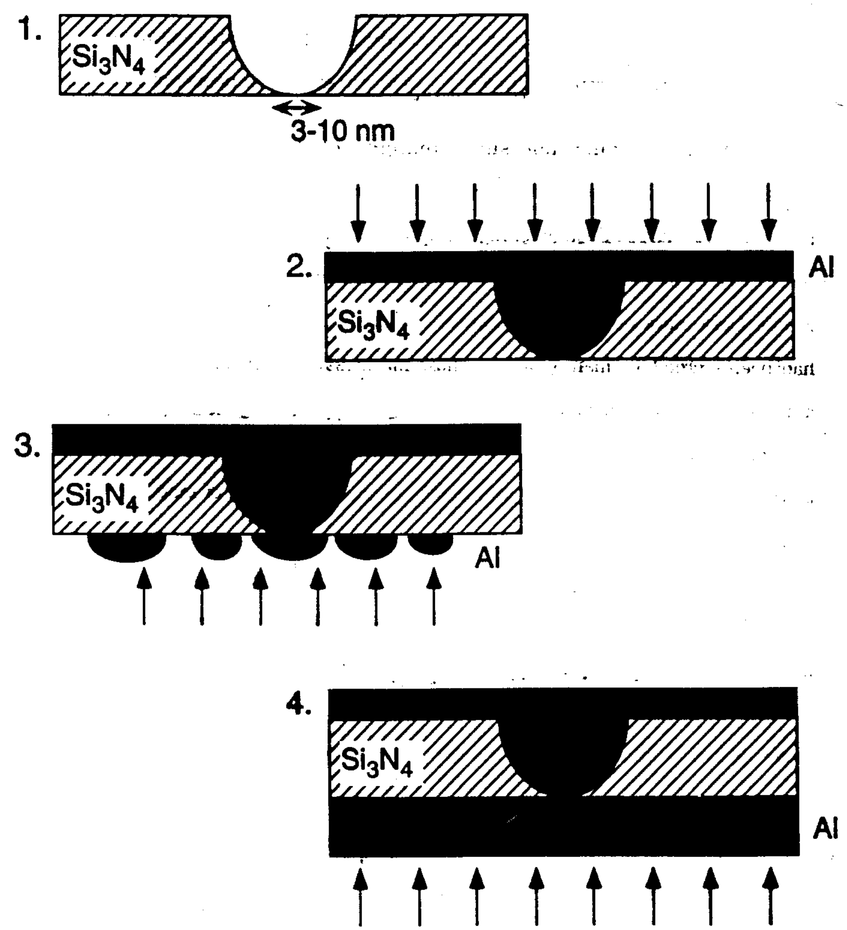
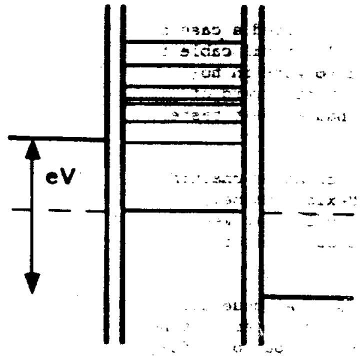
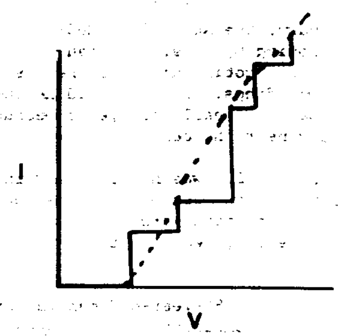
Device Fabrication

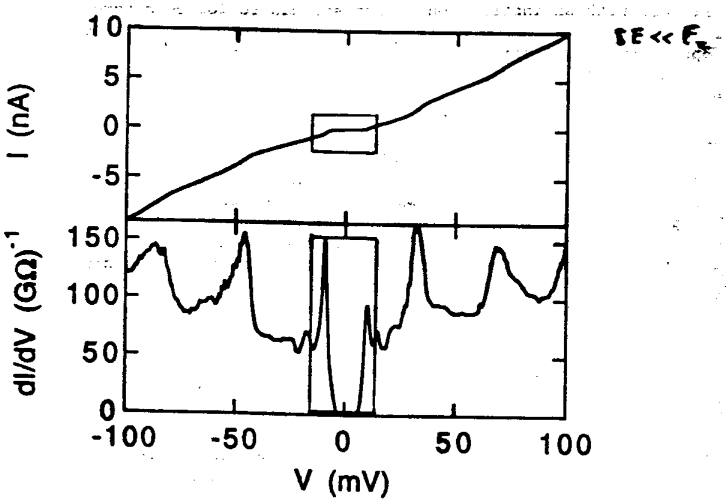


TUNNELING VIA DISCRETE STATES

NORMAL METAL LEADS

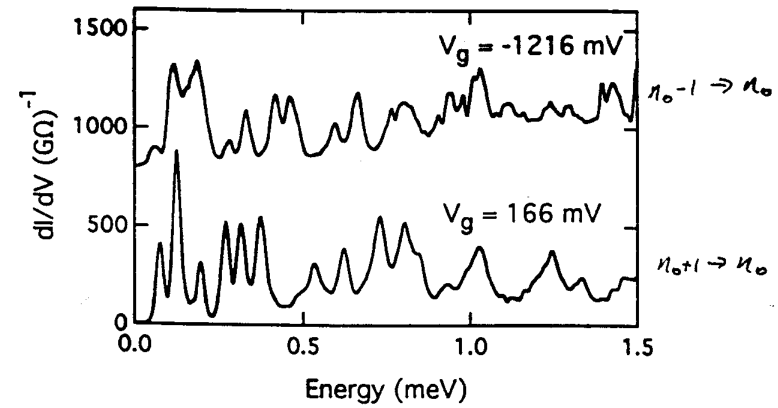




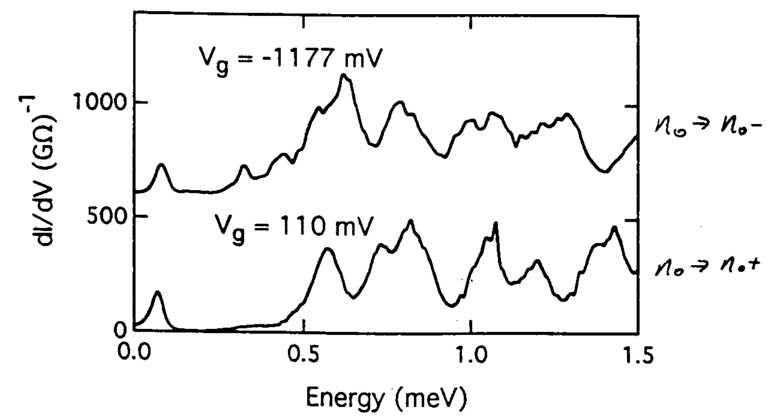


Superconducting Particle

even to odd transitions (50 mK, 0.05 Tesla)



odd to even transitions

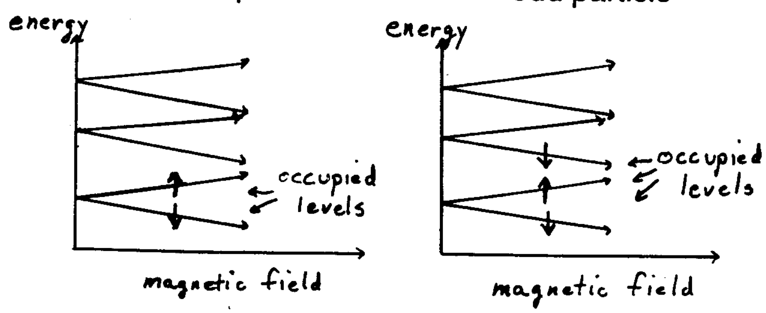


EVEN/ODD ELECTRON EFFECT IN MAGNETIC FIELD

For **even** number of electrons, the first tunneling state exhibits Zeeman spin splitting.
For **odd** number of electrons, the first state does **not** exhibit Zeeman splitting.

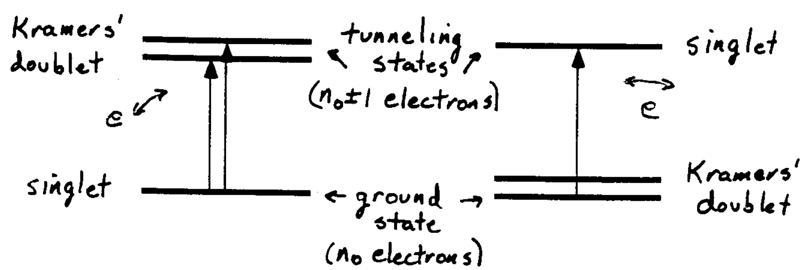
Independent Electron Picture:

even particle | odd particle

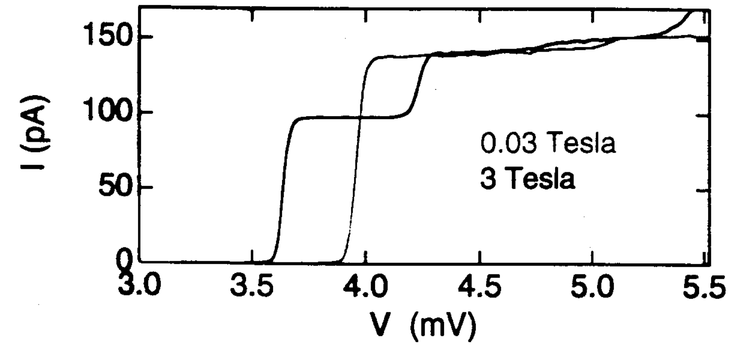


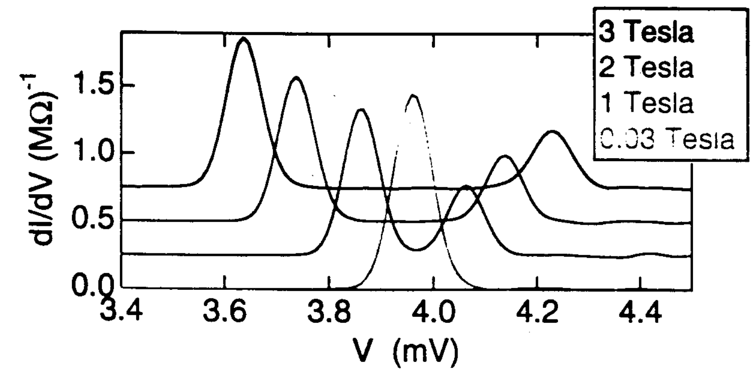
Many-body Picture: *(Magnetic field fixed, nonzero)*

even particle | odd particle



Zeeman splitting with even-electron island





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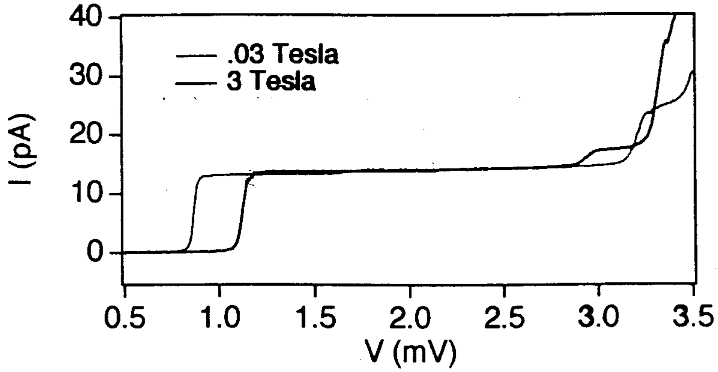
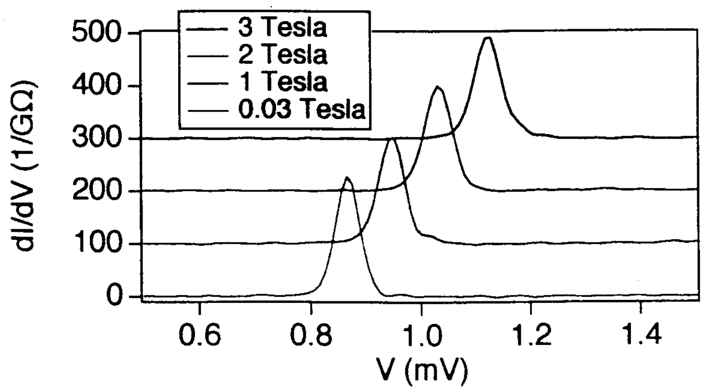
Magnetic field dependence for odd-electron particle





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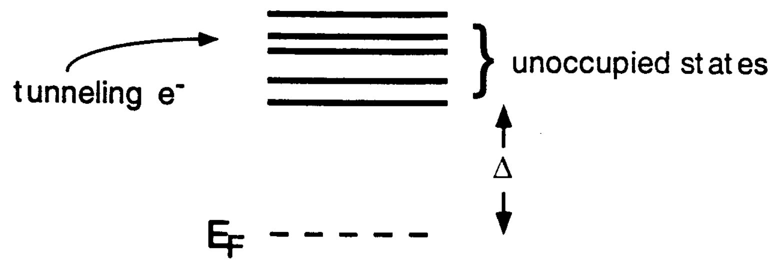
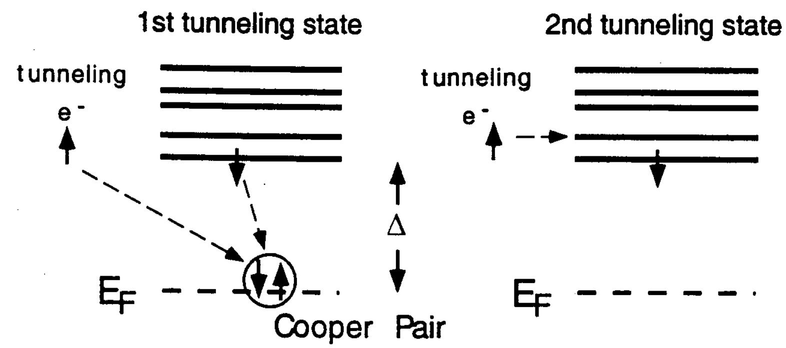
Discrete States in a Superconducting Particle

Even-to-odd tunneling:

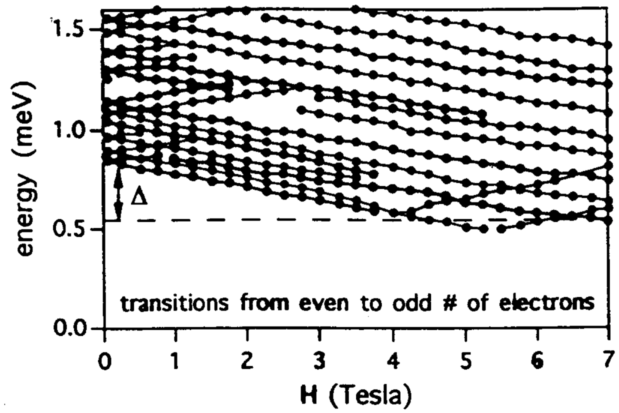


Odd-to-even tunneling:

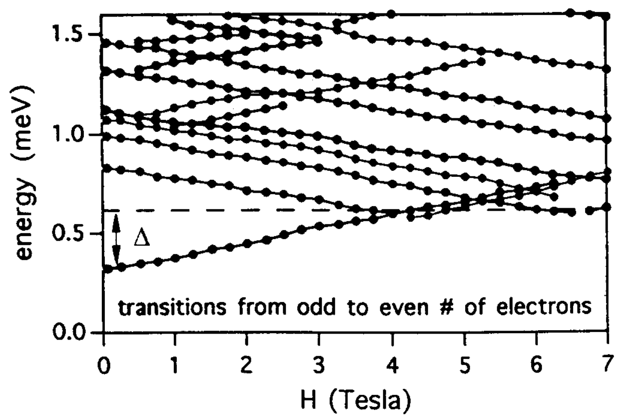


First two tunneling states are separated in energy by ~ 2Δ

SUPERCONDUCTIVITY in nm-SCALE PARTICLES



$n_0+1 \to n_0$



$n_0 \to n_0+1$

$\Delta = 0.30$ mev

Superconductivity destroyed by spin pair breaking

Transition is continuous, not first order.

What is the Lower Size Limit for Superconductivity?

<u>Anderson (1959)</u>

$\delta E_{crit} \sim \Delta_0$

<u>von Delft, Zaikin, Golubev, Tichy (PRL 77, 3189 (96))</u>

Calculate Δ_{even}, Δ_{odd} using MFT with uniform level spacings

$\delta E_{even,crit} = 3.56\ \Delta_0$

$\delta E_{odd,crit} = 0.89\ \Delta_0$

<u>Smith and Ambegaokar (PRL 77, 4962 (96))</u>

MFT with random level spacings

$\langle \delta E_{even,crit} \rangle = 13.8\ \Delta_0$

$\langle \delta E_{odd,crit} \rangle = 1.80\ \Delta_0$

<u>Matveev and Larkin (PRL 78, 3749 (97))</u>

Include quantum fluctuations of the order parameter

Parity effects in ground state persist for $\delta E > \Delta_0$.

<u>Exact and DMRG solutions:</u>

Richardson (J. Math. Phys. 18, 1802 (77))

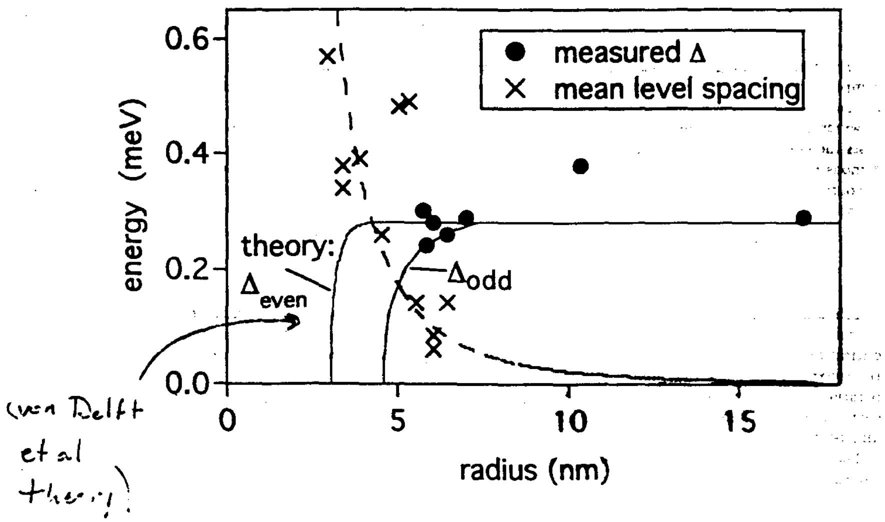
Mastellone, Falci, Fazio (PRL 80, 4542 (98))

Berger and Halperin (PRB 58, 5213 (98))

Dukelsky and Sierra (PRL 83, 172 (99))

Braun and von Delft (cond-mat/9907402)

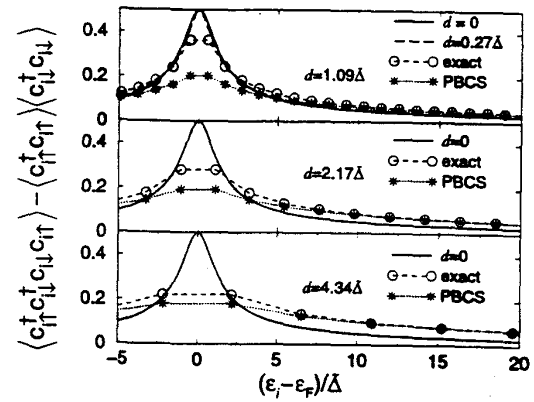
SIZE-DEPENDENCE OF SUPERCONDUCTIVITY?



Note $\xi_0 \equiv \frac{\hbar v_F}{\pi \Delta} \sim 2000\ nm$.

Cross-Over to a Fluctuation-Dominated Regime for Very Small Nanoparticles

No phase transition.
Qualitative change in behavior.



Exact solution: Richardson (1960's)

Review: Jan von Delft, Habilitation thesis

Conclusions

- Superconducting pairing is allowed even in samples with a fixed number of electrons. (The use of a grand canonical ensemble in BCS theory is only a calculational convenence.)

- In micron-sized samples (10^9 electrons), superconducting pairing causes even-electron ground states to be energetically favorable

- In nm-sized samples ($\sim 10^4$ electrons), pairing is visible directly in the "electron-in-a-box" level spectrum

A paired ground state persists as long as

$\Delta \gtrsim \delta E$

the level spacing