Fluctuations, Dissipation, and Phase Transitions in Superconductors

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1. Mean-Field Phenomenology: conventional wisdom
2. What's Missing?
   - (a) Fluctuations
   - (b) Disorder
3. Experimental Consequences
   - (a) Novel phases of "superconducting" matter
   - (b) New phase transitions
   - (c) Dissipation in a "superconducting" state
4. Conclusions
   - (a) Summary
   - (b) Connection to other problems
   - (c) Future directions

- **MOTIVATION:**
  - Basic exciting physics (superconductivity, glassy elastic systems)
  - New vortex states of matter (liquid and glass of elastic lines)
  - Applications (MRI, train levitation, particle accelerators)

(conventional, low-T_c superconductors)

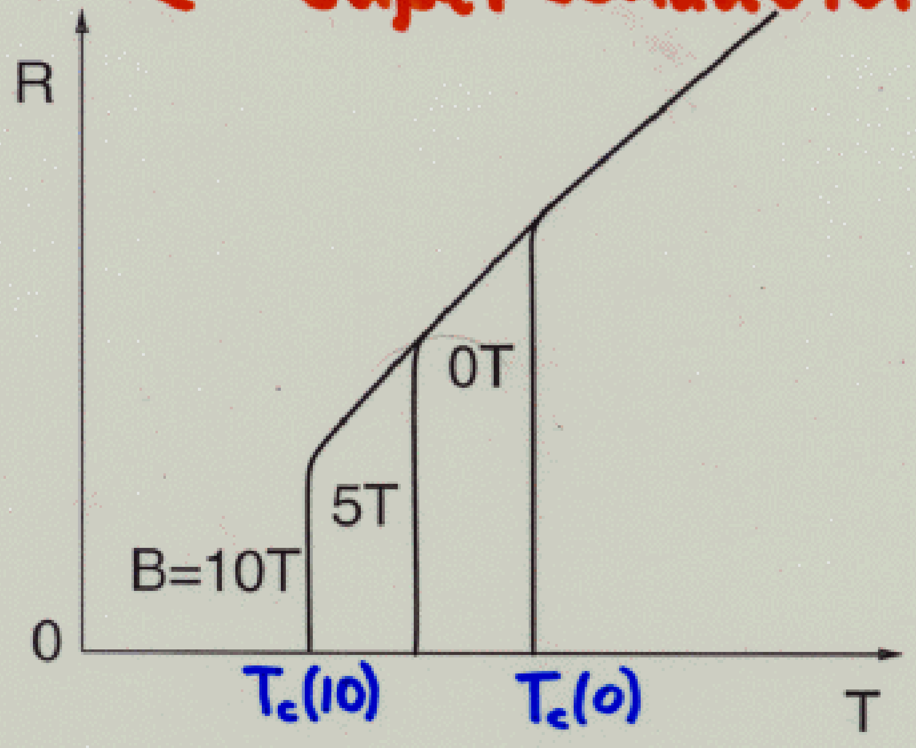


FIG. 1. Schematic of the behavior of resistance, as function of temperature, for increasing values of B-field, in low T_c conventional superconductors. Note the sharpness of the transition even at finite B.

- microscopic mechanism
- low energy phenomenology ⟸ focus of this talk

- **GINZBURG-LANDAU THEORY:** derivable from BCS (Gorkov)

kinetic

$$G_{GL} = \left[\frac{\hbar^2}{2m_z} \left| \left(\frac{\partial}{\partial z} - \frac{i2e}{\hbar c} A_z(\mathbf{r}) \right) \Psi(\mathbf{r}) \right|^2 + \frac{\hbar^2}{2m} \left| \left(\vec{\nabla}_\perp - \frac{i2e}{\hbar c} A_\perp(\mathbf{r}) \right) \Psi(\mathbf{r}) \right|^2 \right.$$

$$\left. + \underbrace{\alpha |\Psi(\mathbf{r})|^2 + \frac{1}{2}\beta |\Psi(\mathbf{r})|^4}_{\text{condensation}} + \frac{1}{8\pi} \left(\vec{H} - \vec{\nabla} \times \vec{A}(\mathbf{r}) \right)^2 \right] + G_{normal}$$

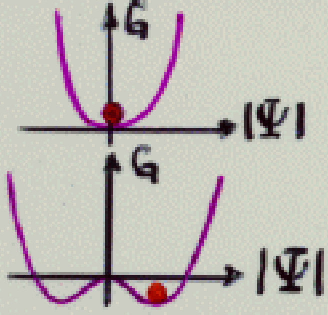
magnetic

$\Psi(\mathbf{r}) = |\Psi| e^{i\Theta}$ – Superconducting order parameter,
bosonic BCS pair wavefunction of electric charge $2e$.

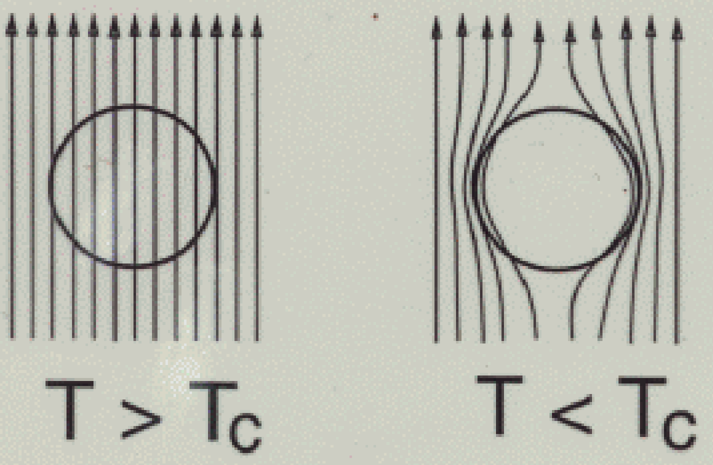
$\alpha \sim T - T_c^{MF}$,

$\gamma^{-1} \equiv \sqrt{m/m_z}$, high T_c s.c.'s have strong anisotropy, $\gamma^{-1} << 1$

MEAN-FIELD THEORY: find $\Psi(\mathbf{r})$, $\vec{A}(\mathbf{r})$ that *minimize* G_{GL}.



- **Normal Phase**: $\alpha > 0$, $\Psi = 0$, $\vec{B} = \vec{H}$
- **Superconducting Meissner Phase:**
  - $-\alpha < 0$
  - $-\Psi \neq 0 = |\Psi| e^{i\Theta}$, "broken" gauge invariance, ODLRO
  - $-\vec{B} \sim e^{-r/\lambda} \to 0$ in the bulk of the superconductor



Flux expulsion
(Higg's mechanism)

– Vortex line is a fundamental defect, with $\Psi(\mathbf{r}) \to 0$ in small core, and phase $\Theta(\mathbf{r})$ winds 2π around the vortex line, flux $\phi_0 = hc/2e$

– $\xi \sim \hbar/\sqrt{m\alpha}$ coherence length on which Ψ varies

– $\lambda \sim 1/|\Psi|$, London penetration length on which $B(r)$ varies

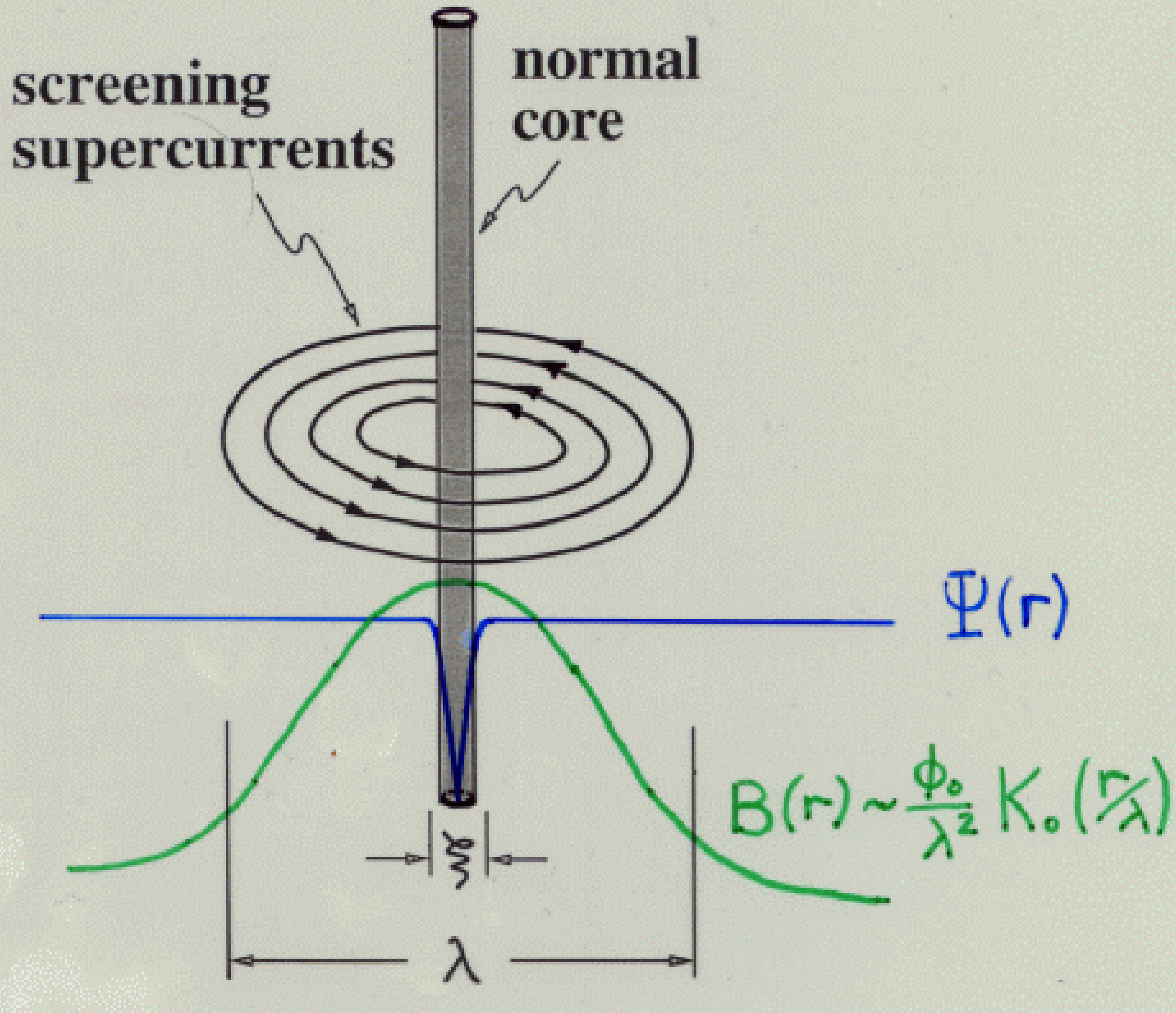


FIG. 2. Schematic of the an isolated flux line vortex, with relevant length scales

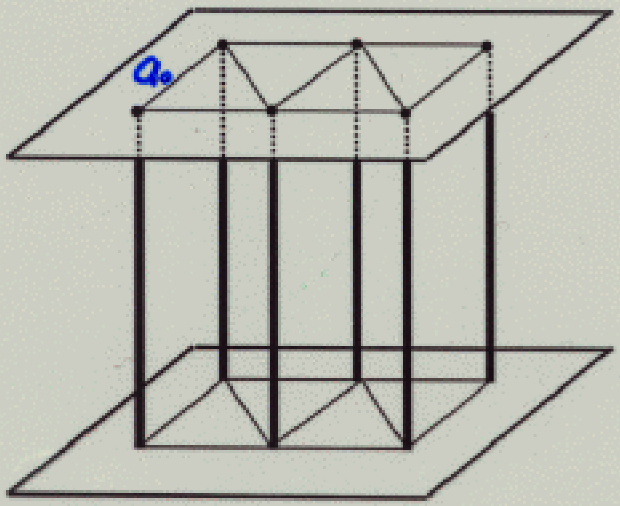
- **"Superconducting" Abrikosov Phase:**
  - Intermediate phase of type II superconductors ($\kappa = \lambda/\xi > 1$), Gibb's energy:

$$g \approx N\left[\underbrace{(\phi_0/4\pi\lambda)^2\log(\kappa)}_{\text{vortex kinetic energy}} - \underbrace{H\phi_0/4\pi}_{\text{magnetic pressure}}\right] \equiv N\left[H_{c1} - H\right]$$

  - For $H_{c1} < H < H_{c2}$, vortices enter the sample
  - Vortex-vortex interaction

$$\left(\frac{\phi_0}{4\pi\lambda}\right)^2 K_0(r/\lambda)\begin{cases} V(r) \sim e^{-r/\lambda}/r^{1/2}\ , \text{ for } r >> \lambda \\ \quad\sim \log(\lambda/r)\ , \text{ for } \xi << r << \lambda \end{cases}$$

  - 3d *crystal* of infinitely long parallel vortex lines, which breaks
    - translational and rotational symmetry
    - U(1) symmetry of $\Psi \to \Psi e^{i\phi}$



VORTEX LATTICE

$$B = \phi_0 n_v$$

$$a_0 \simeq \sqrt{\frac{\phi_0}{B}}$$

Mean-Field Picture

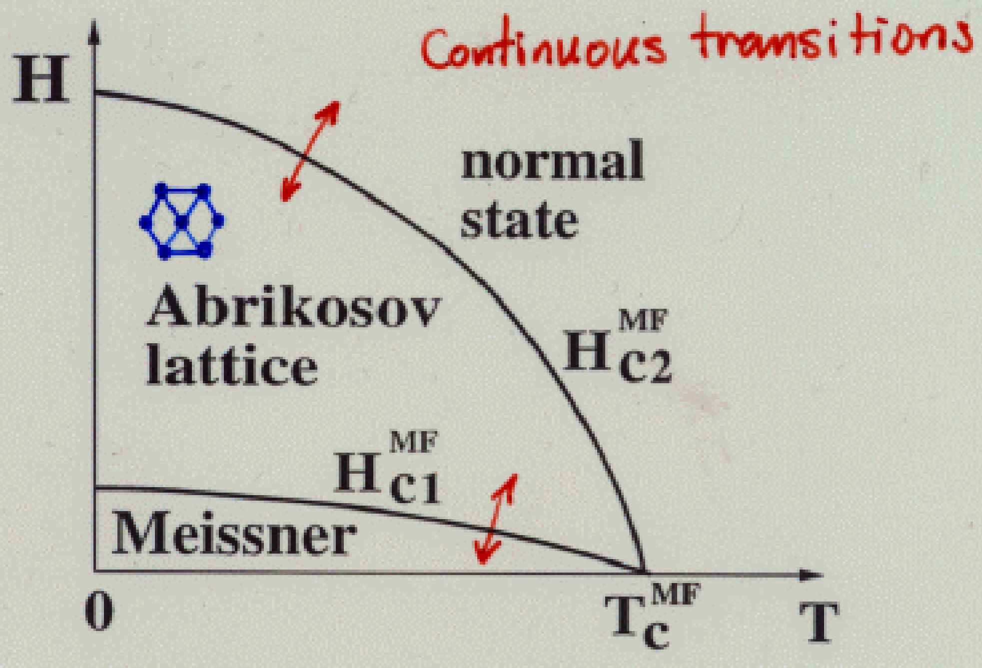


FIG. 3. Mean-field phase diagram appropriate for conventional low T_c superconductors.

grain boundary

each twin orients flux lattice, but they are incompatible, so flux lattice has a grain boundary

P. Gammel, et al. (YBCO)

YBCO twins

20 Gauss
4.2 K

Neutron Scattering

S(k)

P. Gammel, etal. (1994)

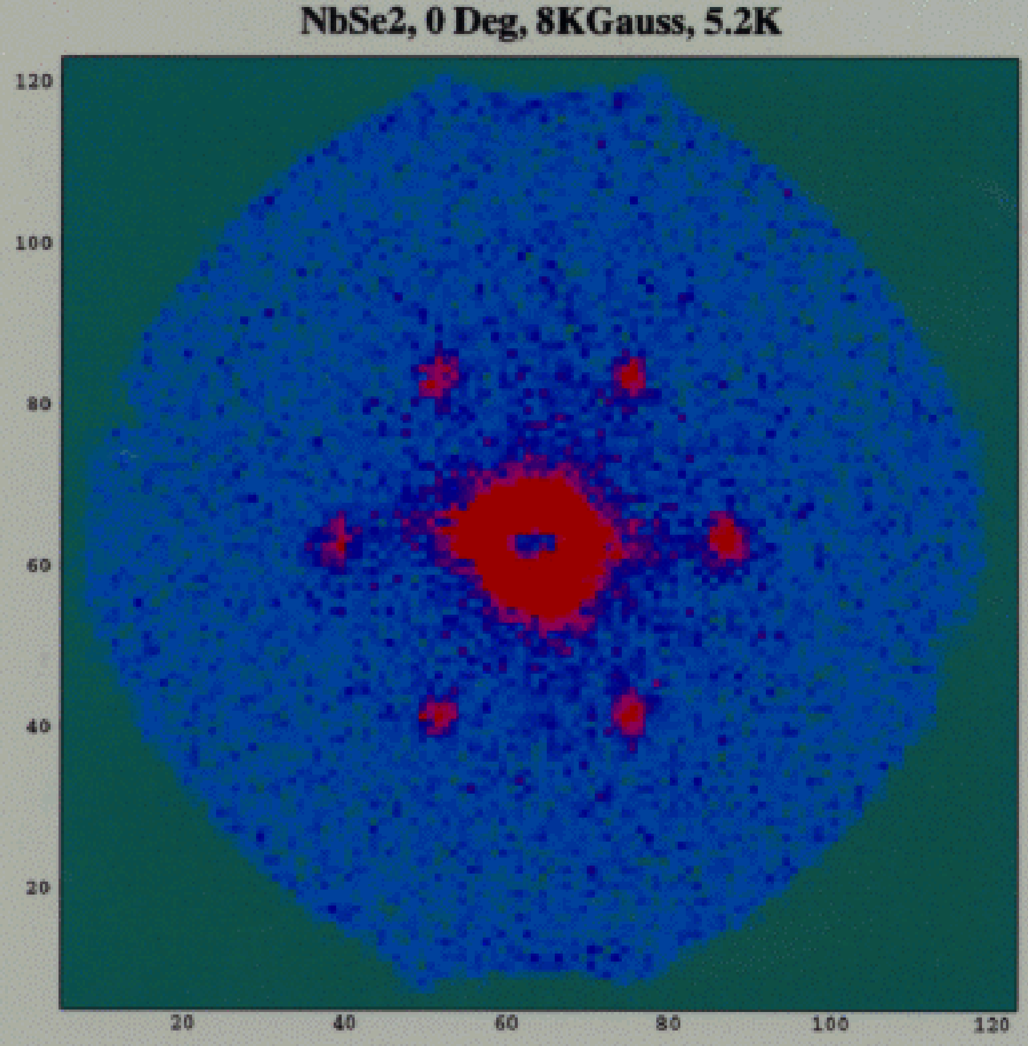


Figure 4. Reciprocal space image of the Abrikosov flux lattice, obtained using neutron small angle ($\sim 1^o$) scattering from $NbSe_2$, a strongly type II low-T_c superconductor ($T_c \approx 7$K). The image was obtained with 8 KGauss magnetic field applied along the c-axis, at $T = 5.2$K. The six hexagonally placed small spots are the magnetic Bragg scattering from the flux lattice. The large spot at the center is a nonmagnetic background that includes small angle scattering from imperfections in the sample. The scattering at the higher-order Bragg spots is too weak to see under these conditions.

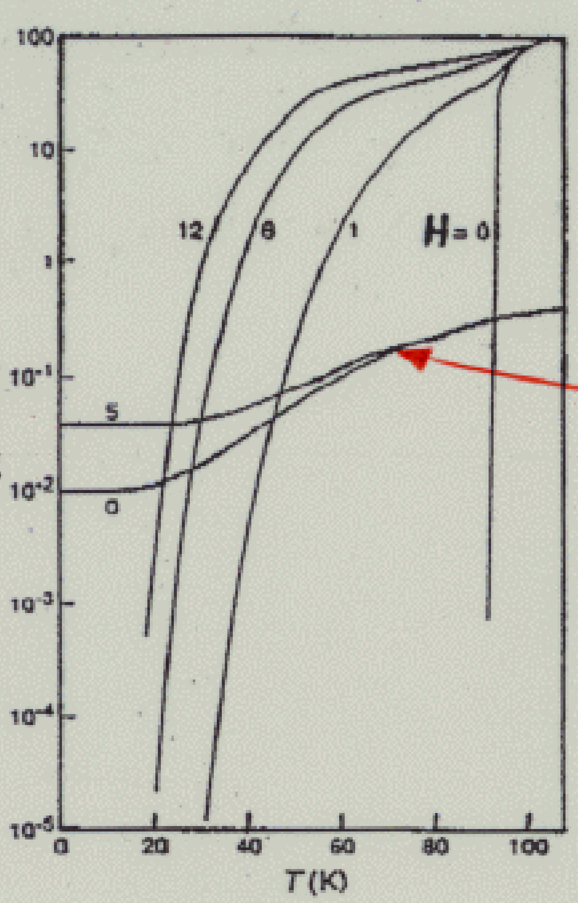
High-T_c Superconductors

- **Linear Resistivity of BSCCO:**

from:
Huse, Fisher, Fisher; Nature (1992)

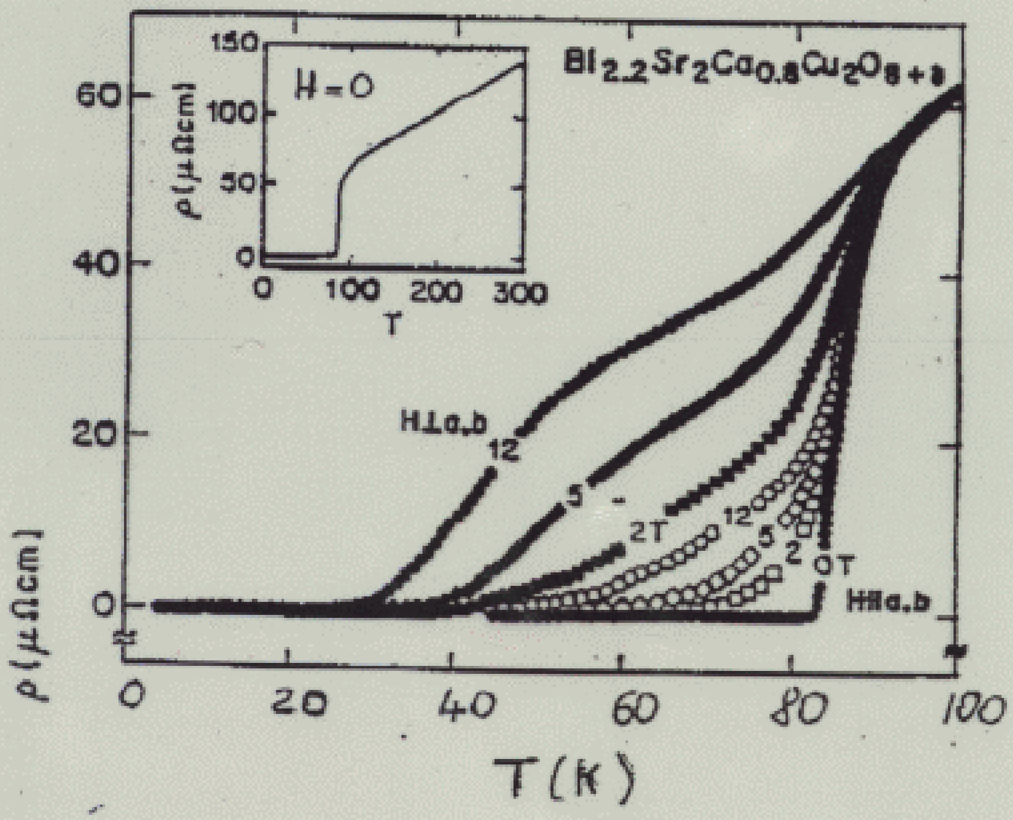
data from AT&T Bell Labs

$Bi_2Sr_2CaCu_8O_{8+\delta}$



Copper

from:
Palstra et al. AT&T (1988)



- **High-T_c Superconductors:**

Well described by Ginzburg-Landau theory, with unusual parameters: $\kappa \equiv \lambda/\xi \approx 100 >> 1$, high T, large anisotropy $\gamma \equiv m_z/m_\perp >> 1$

IMPORTANT CONSEQUENCES:

– Strong thermal fluctuations → melting, dissipation

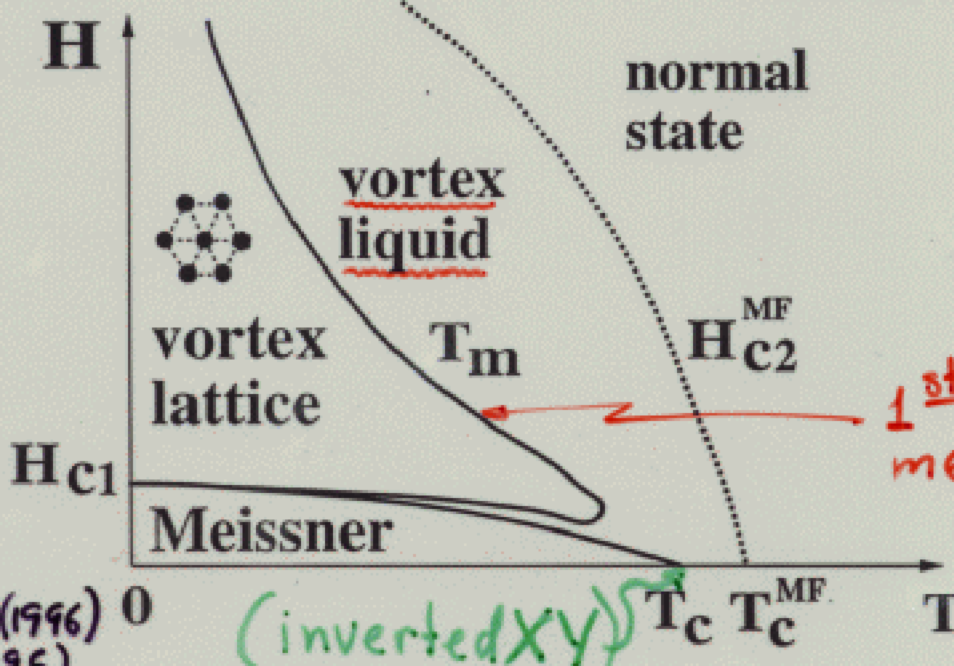
– Importance of Disorder → pinning, irreversability

Nelson, Seung, Marchetti, Brandt, Huse, L. R., Tesanovic, Moore, ...

Eilenberger ('67)!

"CLEAN" LIMIT

Is continuous melting possible in 3D ? (L. R. 1995)



Zeldov et.al. (1995)

Pastoria et.al. (1996)

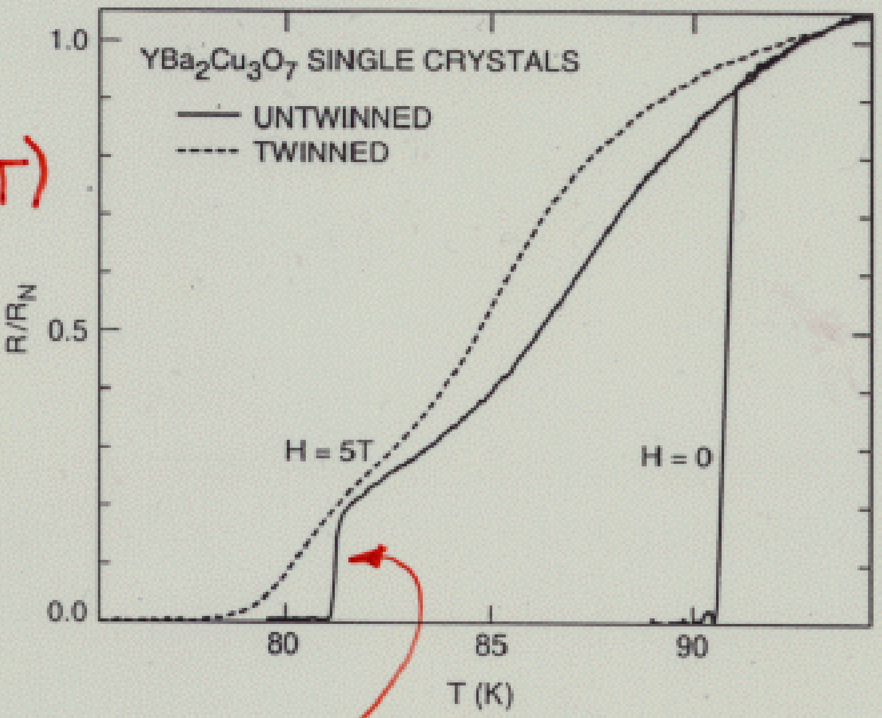
Liang et.al (1996)

- Jump discontinuity ΔM in BSCCO

  $\Delta M > 0$ ($B_{solid} < B_{Liquid}$, c.f. ice, quantum melting)

- $\Delta S \sim 6 \times 10^{-2}\ k_B$/vortex/layer (latent heat)

R(T)

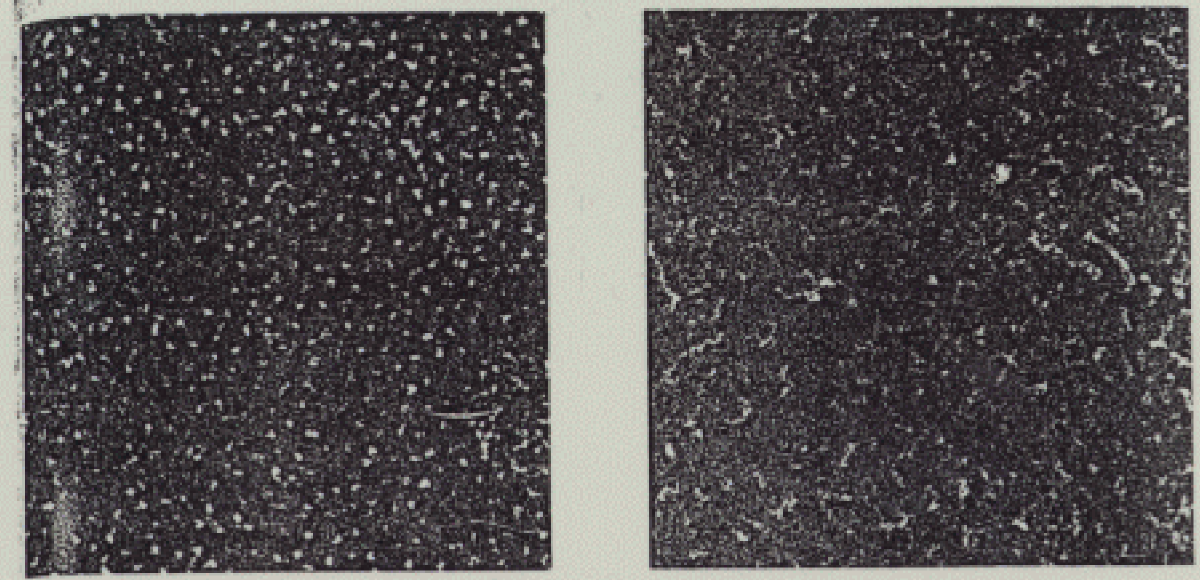


AT&T
Bell Labs
Gammel, et al.

jump discontinuity upon freezing at $T_m(B_m)$

Physics Today, March 1989 (P. Gammel, et al. AT&T Bell Labs)

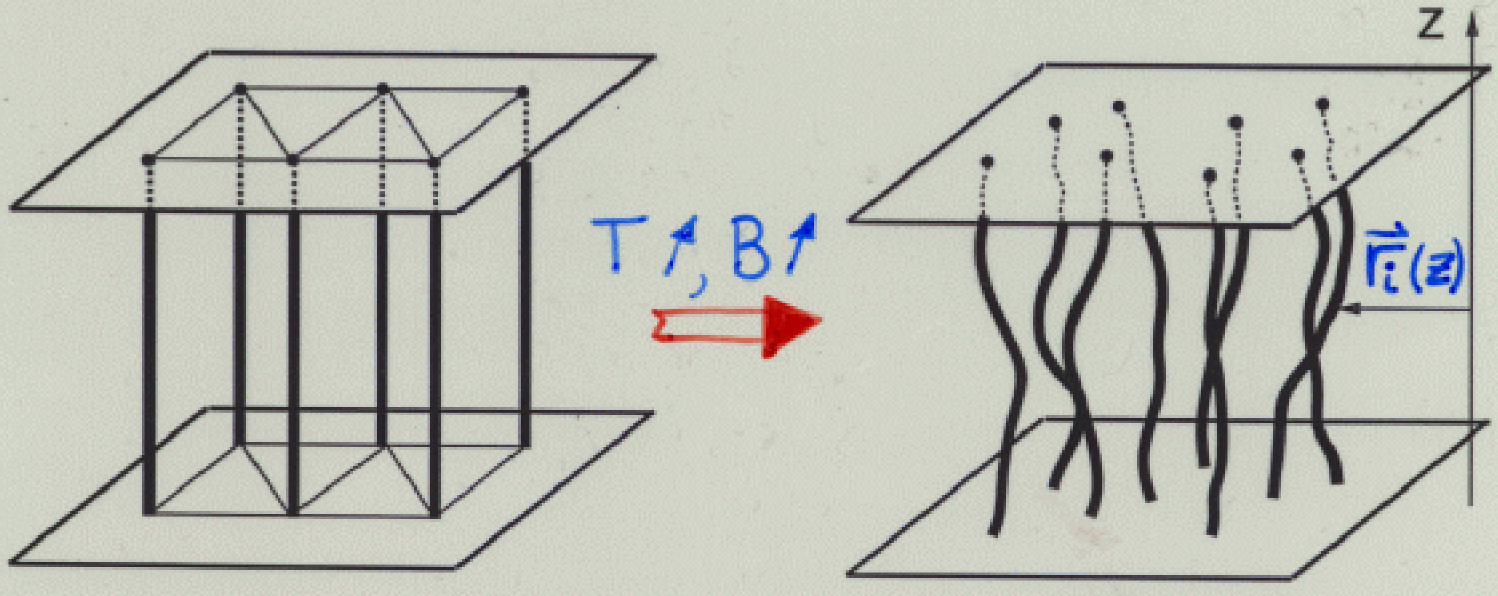
YBCO
crystal



BSCO
liquid

Flux configurations averaged over 1 sec, the time scale of the "decoration" experiment, at 15 K for a sample of Y-Ba-Cu-O (left) and Bi-Sr-Cu-O (right). These scanning electron micrographs of the surfaces of superconductors placed in an atmosphere of magnetic particles show the positions of flux lines because magnetic particles (white spots) settle preferentially on regions of nonzero magnetic flux. At the field values used (20 gauss), the yttrium superconductor is far below its flux-lattice melting temperature, so the flux lines move little over the 1-sec decoration time and give sharp points. The bismuth superconductor is close to flux-lattice melting, and the white spots are blurred because of the motion of flux lines. The average distance between the spots is 1 μm. (Courtesy of AT&T Bell Labs.)

T = 15 K, B = 20 Gauss



VORTEX LATTICE

VORTEX LIQUID

(Nelson+Seung, Marchetti, Huse, L.R.+Frey)

- Effective vortex line Hamiltonian (London limit):

2D Quantum Boson Analogy (duality)

$$F[\vec{r}_i(z)] = \frac{\tilde{\epsilon}_l}{2} \sum_{i=1}^{N} \int_0^L dz (d\vec{r}_i/dz)^2 + $$
$$+ \sum_{i>j=1}^{N} \int_0^L dz V[\vec{r}_i(z) - \vec{r}_j(z)] + \sum_{i=1}^{N} \int_0^L dz U[\vec{r}_i(z)]$$

- Lindemann criterion for melting:

$$\langle |\vec{u}(r)|^2 \rangle \sim \frac{k_B T}{a_\bullet \sqrt{\mu K}}$$

Lattice melts when:

$$\langle |\vec{u}(r)|^2 \rangle^{1/2} \approx a_\bullet ,$$ → melting curve $T_m(B_m)$

"DIRTY" LIMIT

- **Point Disorder (O_2 vacancies):**

VORTEX GLASS (Fisher, Fisher and Huse)

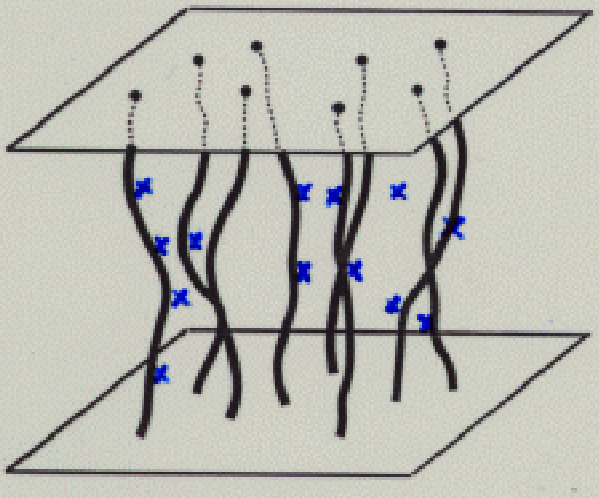
vortex crystal is unstable to even weak disorder (Larkin '70)

– $\rho_L = 0$

– Off-Diagonal Long-Range Order

– Frozen liquid

– Continuous transition into vortex glass

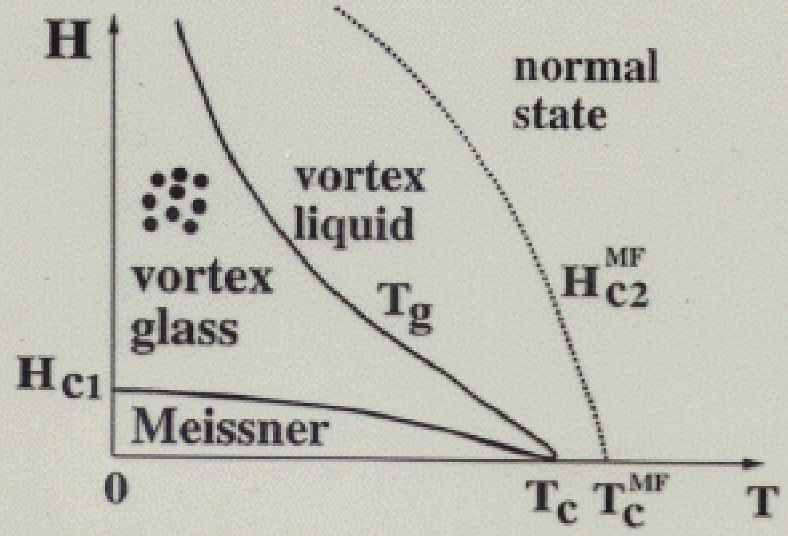


point disorder is not very effective

Thermal renormaliz.:

$U_0 \to U_0 e^{-T^3} \ll U_0$

probably only exists in 3d



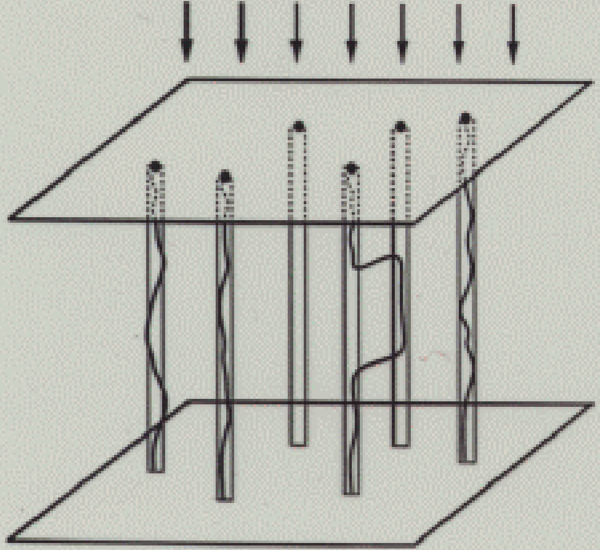
How to suppress vortex liquid regime?

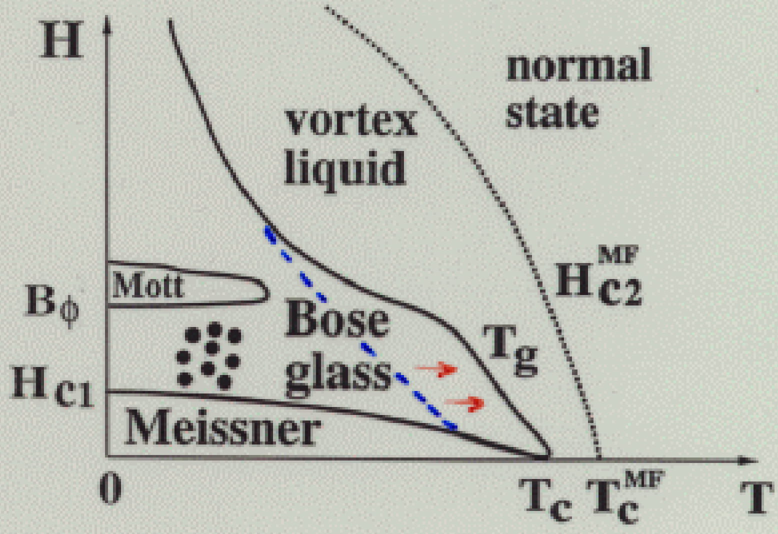
- **Correlated Disorder (columnar defects, twin planes):** (Civale, et al)

BOSE GLASS (Nelson and Vinokur) (M.P.A. Fisher + D.H. Lee)

- $\rho_L = 0$ and Off-Diagonal Long-Range Order
- Continuous transition into Bose glass (Fisher, Weichmann, et al.) analogy with quantum Bosons (2+1)
- Upward shift in $T_m(B) \rightarrow T_{BG}(B)$, for $B < B_\phi$
- Two-fluid model (L.R.)
  * Strongly and weakly pinned Bose glass, and interstitial liquid
  * Mott insulator - Meissner phase for interstitial vortices
  * Analogies with He^4 films on random substrates

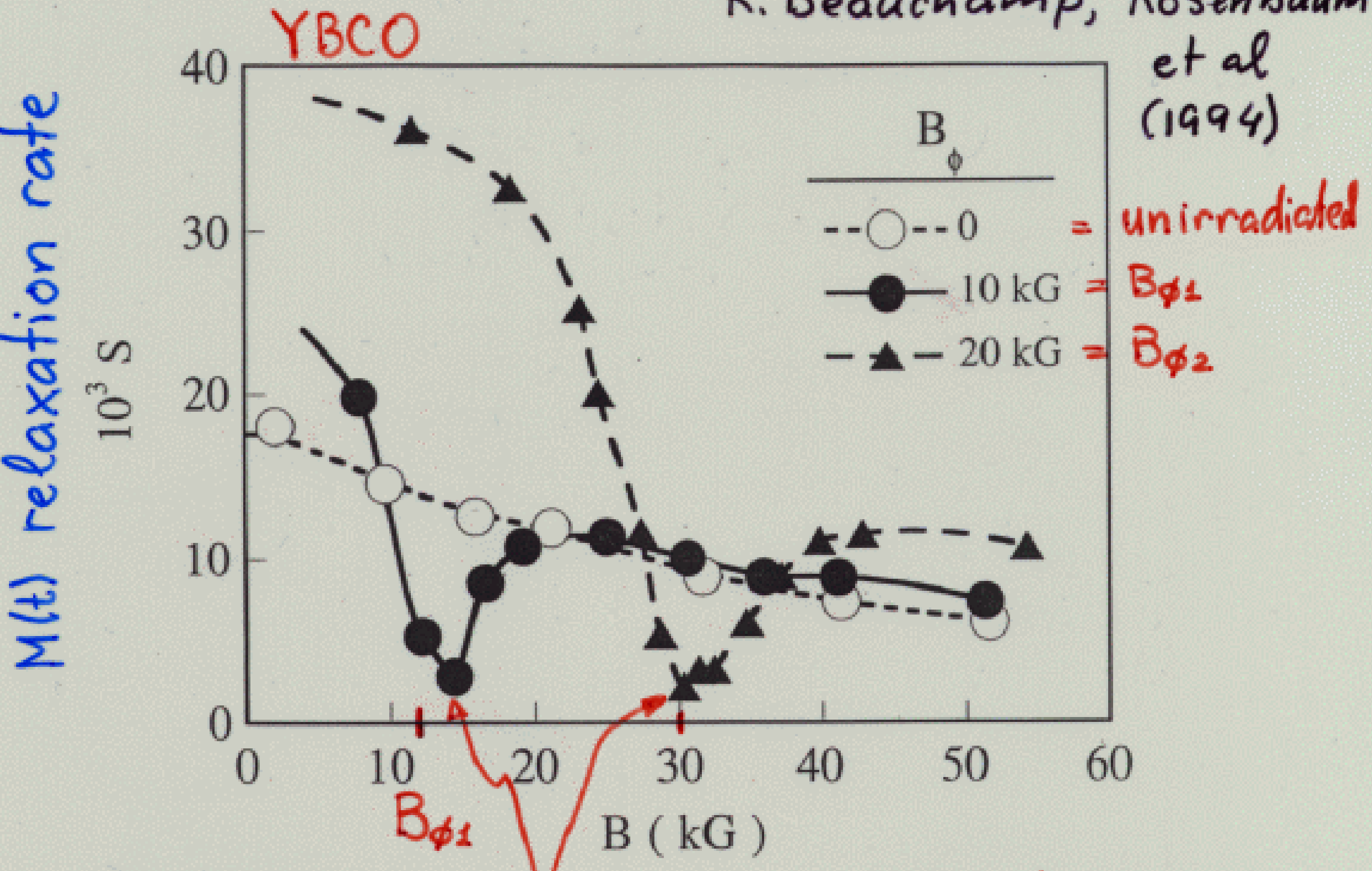
Ion Irradiation (Civale, Budhani)





Magnetization Relaxation Rate: Evidence of Mott insulator

K. Beauchamp, Rosenbaum et al (1994)



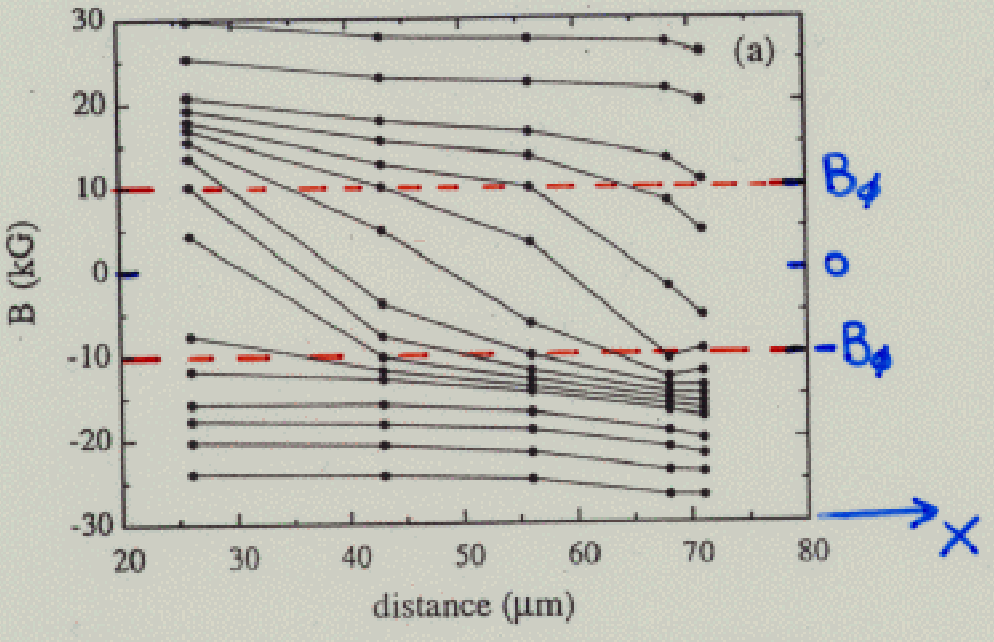
Suppression of $\frac{dM(t)}{dt}$ for $B \simeq B_\phi$

Evidence for Mott insulator?

Also exp. by:
E. Nowak, et al. (1996)
in Thallium

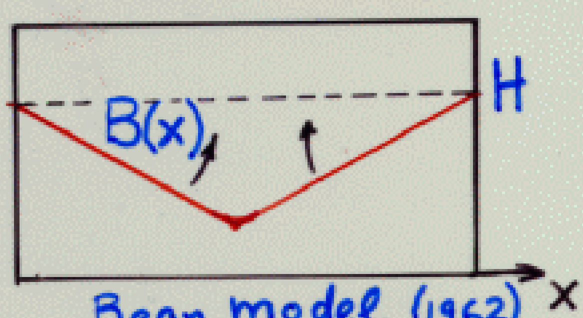
Modified Bean model for Bose glass superconductors

$B_\phi = 10\,kG,\ T = 5K$



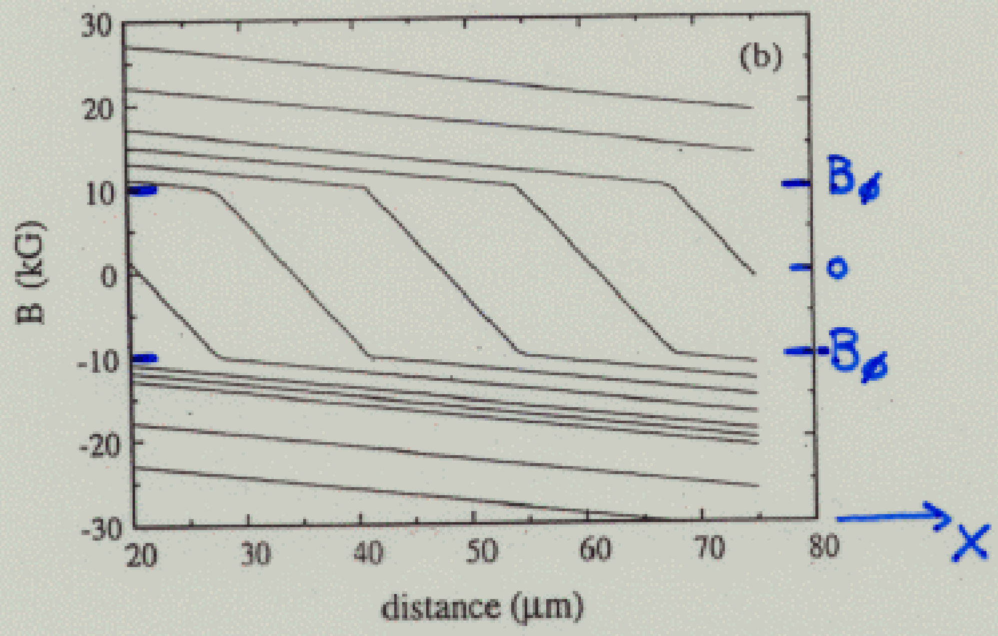
Experiment by:

K. Beauchamp et.al. (1995)



Bean model (1962)

$\vec{\nabla} \times \vec{B} = \vec{J}_c$



Theory:

L. R. (1995)

$$\frac{dB}{dx} = J_c(B) \begin{cases} J_c(B < B_\phi) \simeq J_{c1} \\ J_c(B > B_\phi) \simeq J_{c2} \end{cases}$$

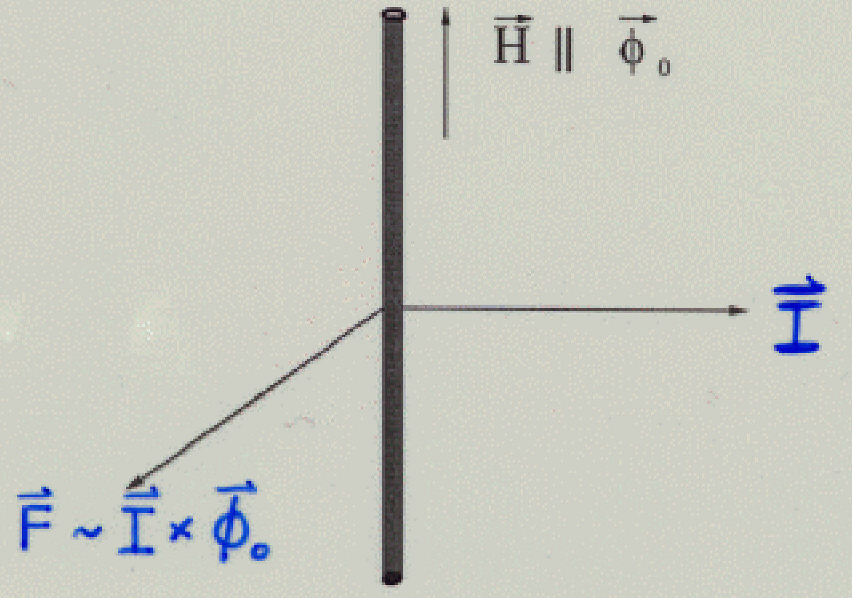
two-fluid model

$J_{c1} \gg J_{c2}$

Electrical Transport Properties:

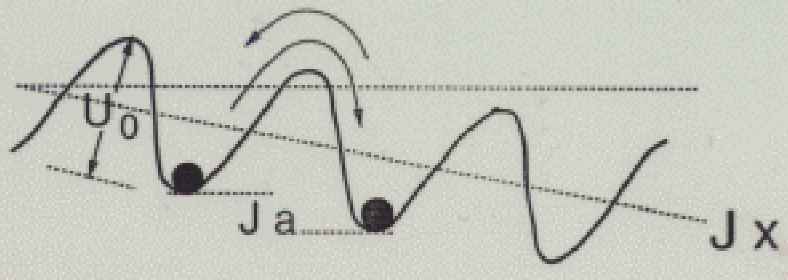
CURRENT ($\vec{J}$) $\rightarrow$ FLUX MOTION ($\vec{v}_v$) $\rightarrow$ DISSIPATION ($\vec{E}$)

- Current $I \sim$ driving force $\vec{F} = \vec{J} \times \vec{\phi}_0$ Lorentz & Magnus
  (screening)
- Voltage $V \sim$ vortex velocity $\vec{v}_v$ (Josephson Effect)
- $\rho = (B/H_{c2})\rho_n$ (Bardeen-Stephen) $\frac{d\phi}{dt} = \frac{e}{\hbar} V$
- Need pinning disorder to stop vortex motion and dissipation that it causes



- **Dissipation in Vortex State:**

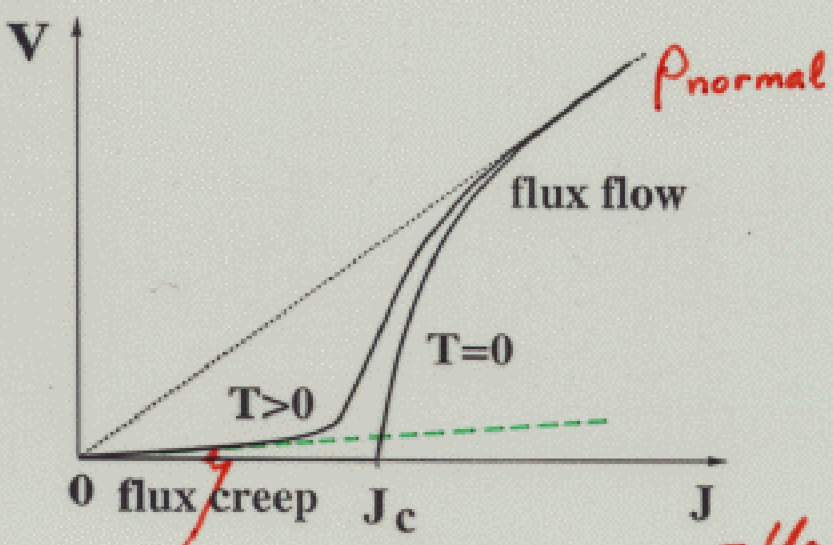
INDEPENDENT PARTICLE MODEL (Anderson and Kim)



$$E \propto e^{-(U_0 - Ja)/T} - e^{-(U_0 + Ja)/T}$$

$$E \propto \sinh(Ja/T) e^{-U_0/T}$$

$$\lim_{J \to 0} E \propto J \left(\underbrace{e^{-U_0/T}}_{\rho_\ell \neq 0} \right)$$

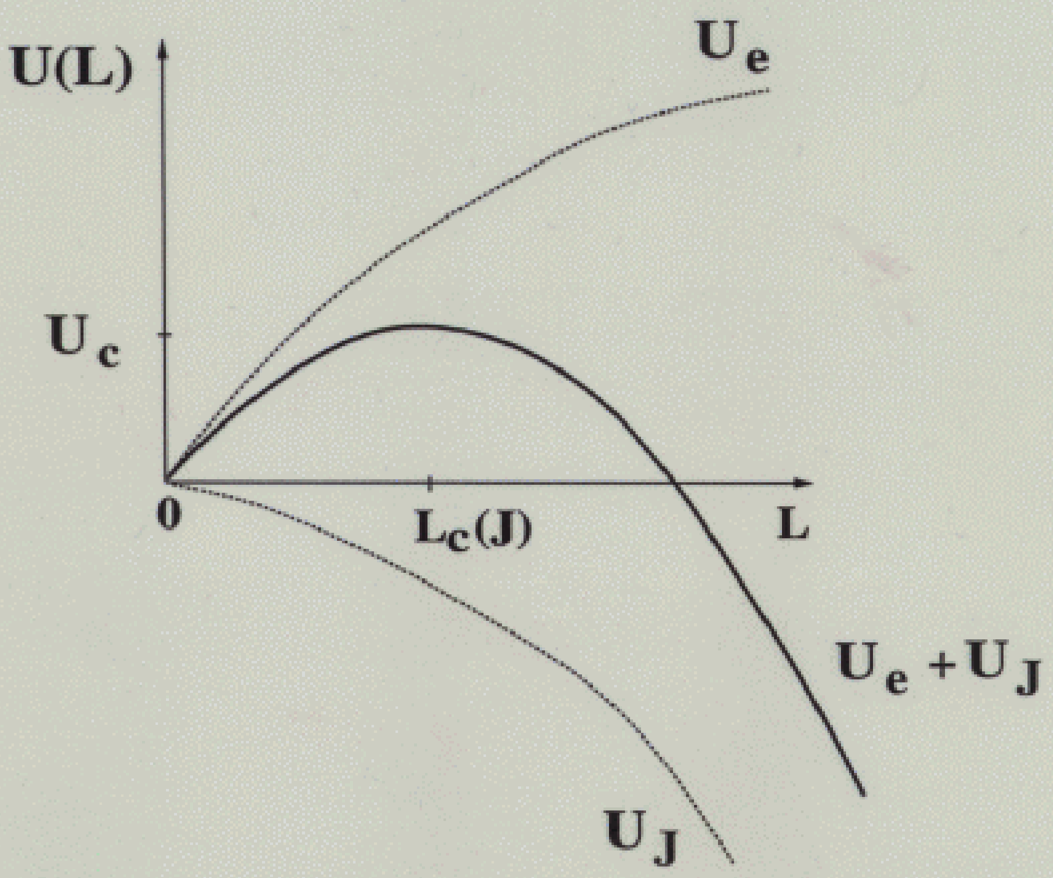


$J_c \sim \dfrac{U_0}{a}$

(limit of metastability)

finite slope $= \rho_\ell \sim e^{-U_0/k_B T}$

NUCLEATION DRIVEN DISSIPATION IN DIRTY SUPERCONDUCTORS



– Nonequilibrium state (vs globally well-defined free energy)

– Vortex *line* defects $U_B(L) \sim L^1$ (vs domain walls $U_B(L) \sim L^{d-1}$)

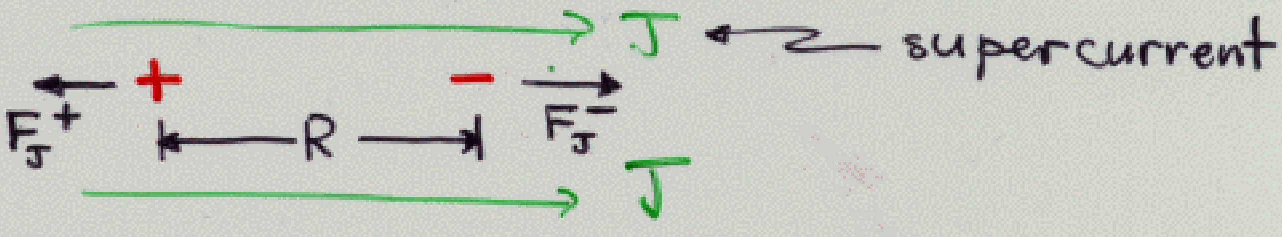
– Current J is the driving mechanism (vs H in spin systems)

Dissipation in S.C. Films

(2D warm up)

Kosterlitz-Thouless phase $T \le T_{KT}$:

$$H = \frac{1}{2}\rho_s \int d^2r \,(\vec{\nabla}\theta)^2$$



$$H[R] = \pi \rho_s \ln R/a - \phi_0 J d R$$

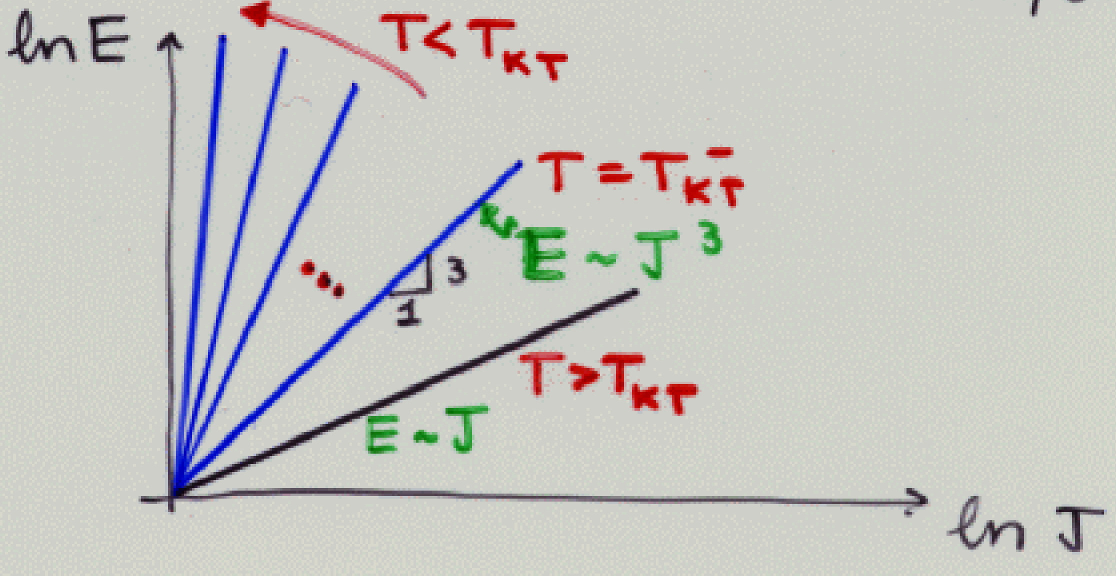
minimize: $\left.\frac{\delta H}{\delta R}\right|_{R_c} = 0 \Rightarrow R_c = a\,\frac{J_0}{J}$

$$\left(J_0 = \frac{\pi \rho_s}{\phi_0 d a}\right)$$

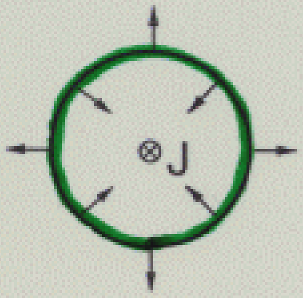
$$\Rightarrow H[R_c(J)] = \pi \rho_s \ln J_0/J + \text{const.}$$

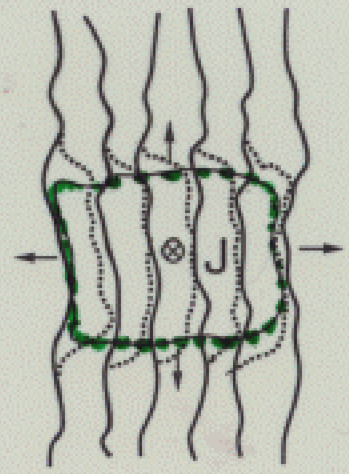
$$\Rightarrow E(J) = \rho(J)\, J \simeq e^{-H[J]/T}\, J$$

$$\Rightarrow E(J) = J\left(\frac{J}{J_0}\right)^{\frac{1}{2\eta}}, \quad 0 < \left(\eta \equiv \frac{k_B T}{2\pi \rho_s}\right) < 1/4$$



DISSIPATION IN MEISSNER ($H < H_{c1}$) and GLASS PHASES





– Excitation energy:

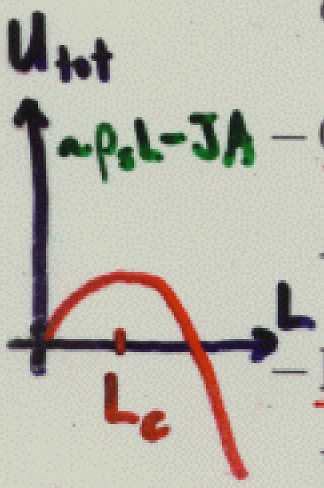
$U_e \sim \rho_s L$ $\qquad$ $U_e \sim \rho_s L^\theta$ $(\theta < 1)$

– Area of excitation "bubble":

$A \sim L^2$ $\qquad$ $A \sim L^\tau$ [redacted]

– Energy from current J:

$U_J \sim f_J A \sim J L^2$ $\qquad$ $U_J \sim J L^\tau$



– Critical "bubble" size, balance $E_e \approx E_J$:

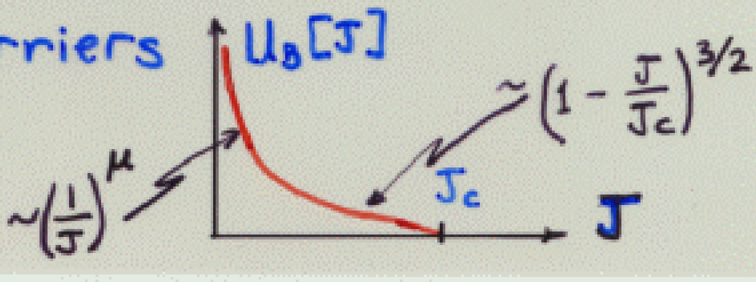
$L_c(J) \sim 1/J$ $\qquad$ $L_c(J) \sim 1/J^{(\tau-\theta)}$

– Barrier size $E_B[L] \sim L^\psi$ to activate over via $e^{-U_B[L]/T}$:

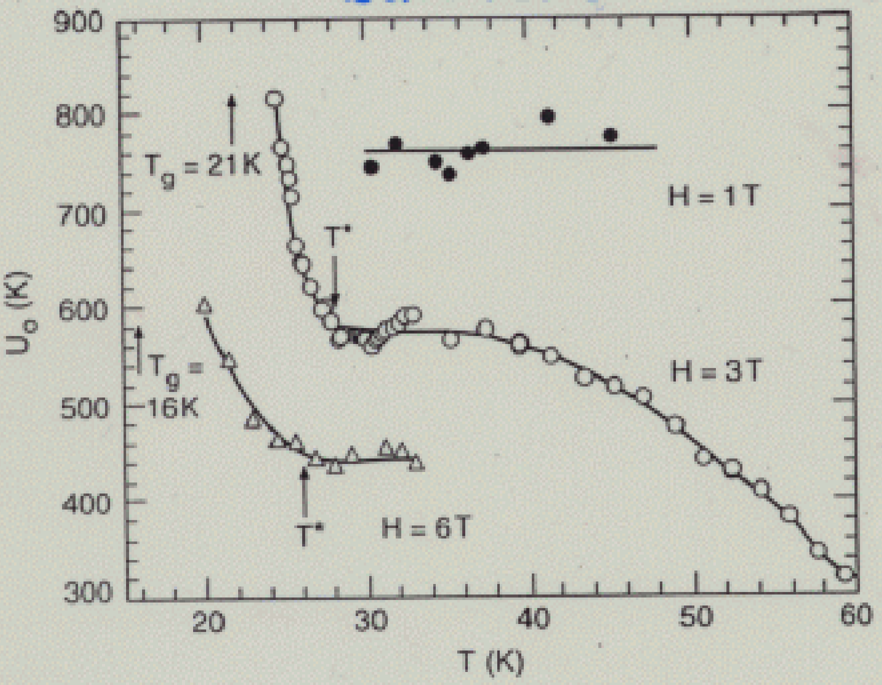
$V(J) \sim e^{-1/J}$ $\qquad$ $V(J) \sim e^{-1/J^\mu}$

Arrhenius rate of production: $e^{-\frac{U_B[J]}{T}}$ $\qquad$ $(\mu = \psi/(\tau - \theta))$

but with divergent barriers



Diverging activation barriers



AT&T Bell Labs
BISCCO
(2212)

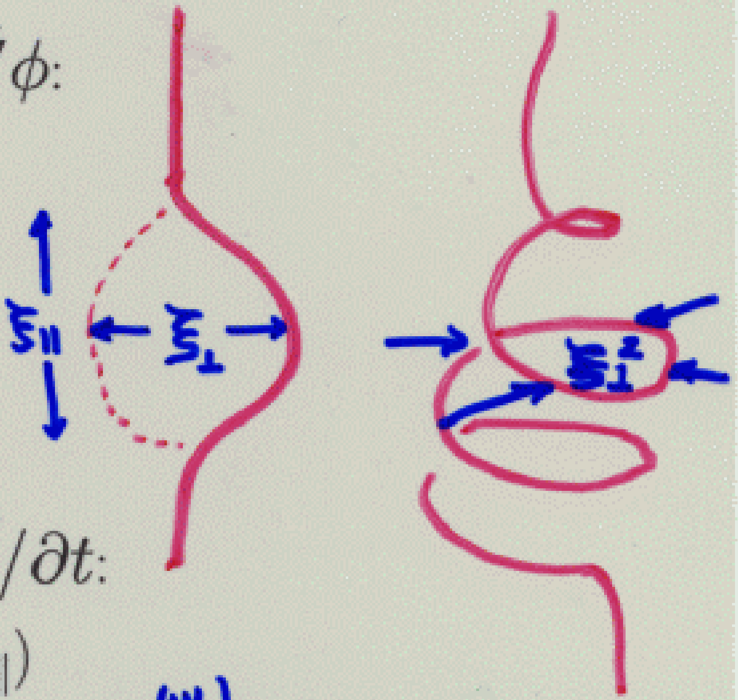
Expect: $U_B(T) \sim \dfrac{1}{|T-T_g|^{\mu\nu}}$

Dissipation near glass transition:

- Bose glass scaling theory (F+F+H, N+V, L.R.)

  M. Wallin + S. Girvin

  - $\xi_\perp \sim |T - T_{BG}|^{-\nu}$, $\xi_\parallel \sim \xi_\perp^2$, $\tau \sim \xi_\perp^z \to \infty$
  - Free energy density:
    $f \sim 1/(\xi_\perp^2 \xi_\parallel)$
  - Vector potential $\vec{A} \sim \vec{\nabla}\phi$:
    $A_\perp \sim 1/\xi_\perp$, $A_\parallel \sim 1/\xi_\parallel$
  - Current $\vec{J} = \partial f / \partial \vec{A}$:
    $J_\perp \sim 1/(\xi_\perp \xi_\parallel)$, $J_\parallel \sim 1/\xi_\perp^2$
  - Electric field $\vec{E} = -\partial \vec{A}/\partial t$:
    $E_\perp \sim 1/\xi_\perp^{1+z}$, $E_\parallel \sim 1/(\xi_\perp^z \xi_\parallel)$
  - IV relation:



$J_\perp^{(NL)} \to 0$, as $T \to T_g^+$

$J > J^{NL} \to$ non-linear

$$E_\perp \sim \xi_\parallel \xi_\perp^{-z} J_\perp G_\pm^\perp(\underbrace{\xi_\perp \xi_\parallel}_{} J_\perp \phi_o / cT) \sim \xi_\perp^{2-z}(\xi_\perp^{3+\delta} J_\perp)$$

$$E_\parallel \sim \xi_\parallel^{-1} \xi_\perp^{2-z} J_\parallel G_\pm^\parallel(\underbrace{\xi_\perp^2}_{} J_\parallel \phi_o / cT)$$

$\delta = \frac{z-2}{3}$

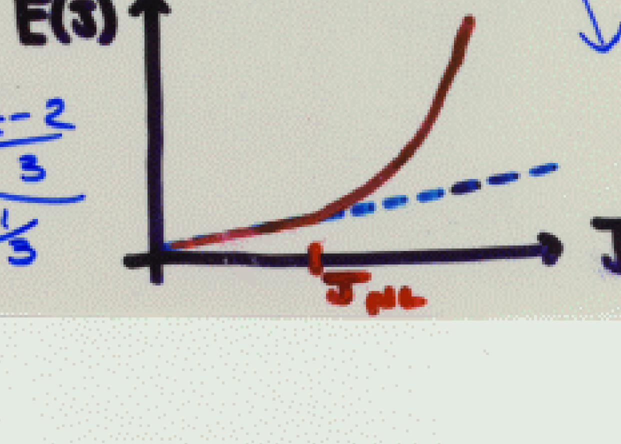
$J < J^{NL} \to$ linear

$$E(J) = \rho J G_\pm(J/J_{nl})$$

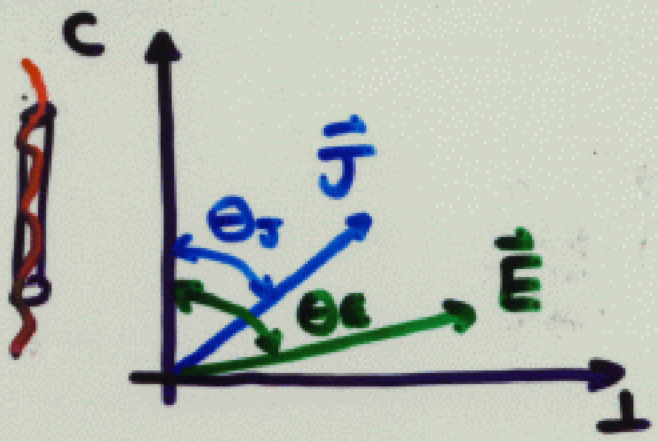
$J_\parallel^{(NL)} \to 0$, as $T \to T_g^+$

i.e. $G_+(x \to 0) \to 1$

$$E_\perp(J_\perp) \sim J_\perp^{1 + \frac{\frac{z-2}{3}}{\frac{z+1}{3}}}$$



$-T > T_{BG},\ E_i = \rho_i J_i$:



$* \rho_\perp \sim |T - T_{BG}|^{\nu(z-2)}$

$* \rho_\parallel \sim |T - T_{BG}|^{\nu z}$

$* \tan(\theta_E) \propto \tan(\theta_J)/(T - T_{BG})^{2\nu}$

$\theta_E(T \to T_{BG}^+) \approx \pi/2 - (T - T_{BG})^{2\nu} \cot(\theta_J) \to \pi/2$

$-T = T_{BG},\ \rho = 0$:

$* E_\perp \sim J_\perp^{(1+z)/3}$

$* E_\parallel \sim J_\parallel^{(2+z)/2}$

$-T < T_{BG},\ \rho = 0$:

$E \sim e^{-1/J^\mu}$

- **Bose glass scaling theory** (F+F+H, N+V, L.R.)

$-\xi_\perp \sim \dfrac{1}{|T-T_g|^\nu},\ \xi_\parallel \sim \xi_\perp^2,\ \tau \sim \xi_\perp^z$

Exp. by:
P. Gammel, etal.
AT&T (1991)

$-E(J) = J\rho F_\pm(J/J_{nl})$

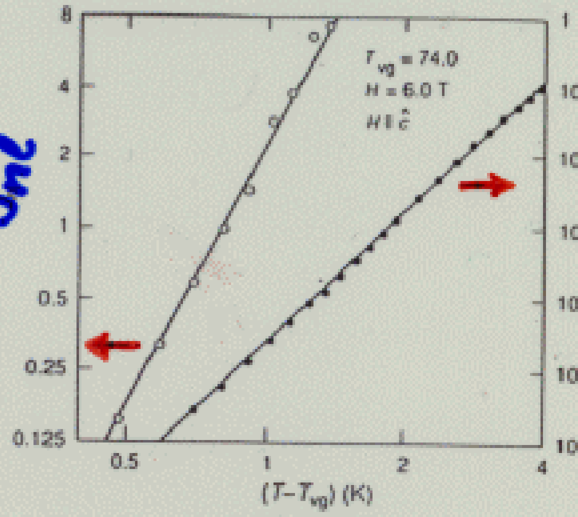
$-T > T_{BG}\ \ E = \rho J$:

Nelson+L.R.
(1996)

$* \rho_\perp \sim |T - T_{BG}|^{\nu(z-2)}$

$* \rho_\parallel \sim |T - T_{BG}|^{\nu z}$

J_{nl}



ρ/ρ_n

$T-T_{vg}$

$-T = T_{BG}\ \ \rho = 0$:

$* E_\perp \sim J_\perp^{(1+z)/3}$

$* E_\parallel \sim J_\parallel^{(2+z)/2}$

Reverse of intrinsic material anisotropy ($\rho_\perp \ll \rho_\parallel$)

$-T < T_{BG}\ \ \rho = 0$:

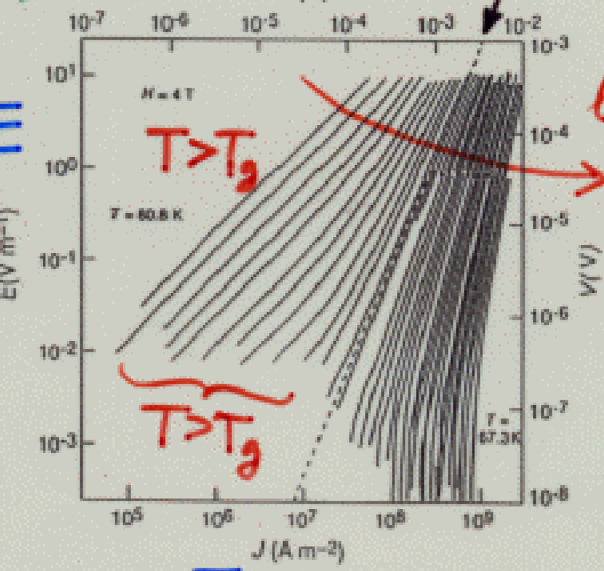
$E \sim e^{-1/J^\mu}$

Exp. by:
R. Koch, etal. (1989)
IBM

Point Disorder

$T = T_g$

E



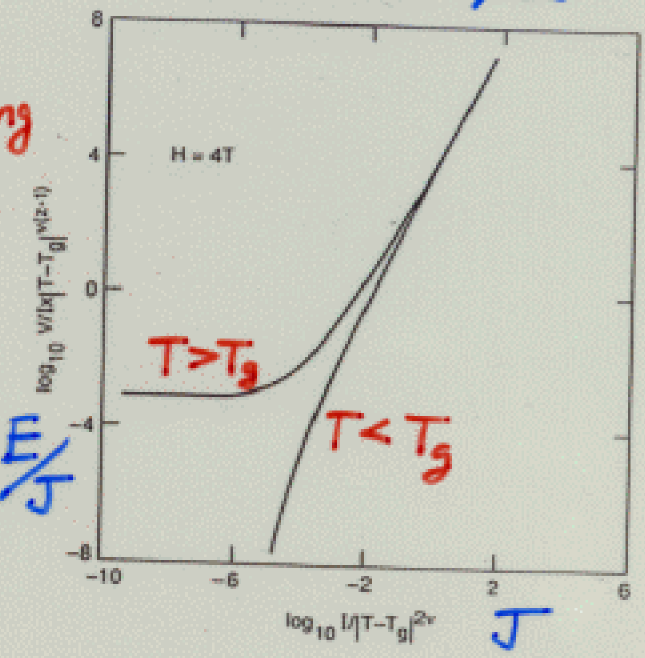
lowering T

J

T<T_g

(Point disorder)

Data collapse

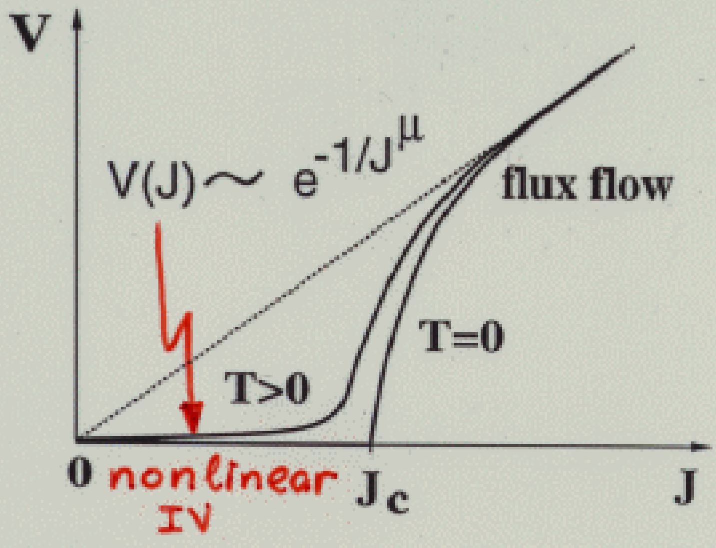


E/J

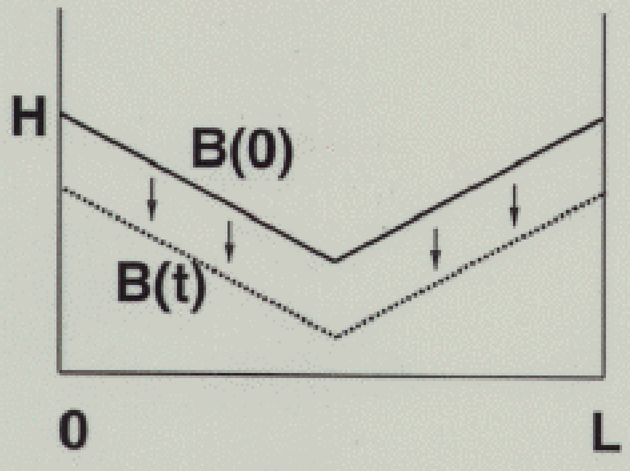
J

– Meissner and Glass phases are true superconductors:
$\rho_l = 0$ (in contrast to vortex liquid)

c.f. Anderson-Kim
$\rho_e \sim e^{-U_0/T} \neq 0$



– Slow (logarithmic) magnetization relaxation:



Bean model (1962)

$$\frac{dB}{dx} = J_c$$

smooth fnc of B, T

$$M(t) \sim J_s(t) = ?$$

$$dE_s/dt \sim \tfrac{d}{dt}(\rho_s I^2) \sim I dI/dt = -IV(I)$$

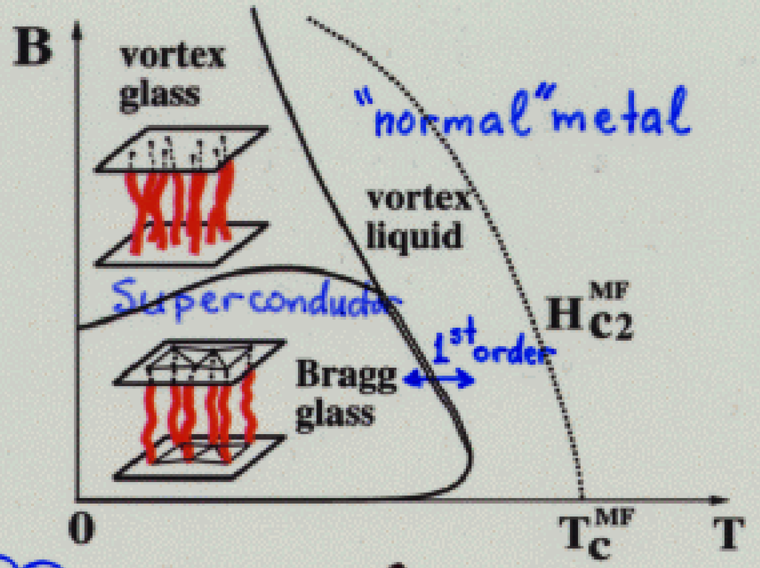
$$dI/dt \sim dM/dt \sim V(I) \sim e^{-1/I^{\mu}}$$

$$M(t) \sim M_0 \left[1 + \mu \tfrac{T}{U_0} \log(t/t_0)\right]^{-1/\mu}$$

Experiments:
Beauchamp, et al.
Nowak, et al.

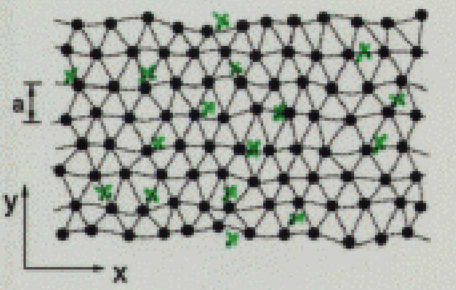
• *EQUILIBRIUM* **VORTEX STATES IN DIRTY S.C.'S**:

(lattice is unstable for $d < 4$ (Larkin))



Bragg glass: $H = \int_r \left[K(\nabla \vec{u})^2 + V_p(\mathbf{r}, \vec{u})\right]$

– Elastically disordered but topologically ordered



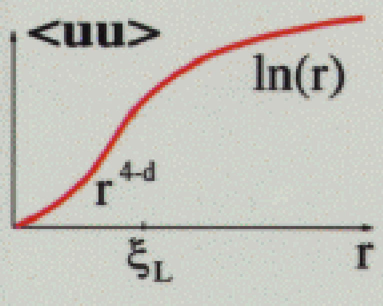
bound dislocations

(*assumed* by Giamarchi+LeDoussal '94)

(*simulated* by Gingras+Huse '96)

(*proved for XY model* by D.S. Fisher '97)

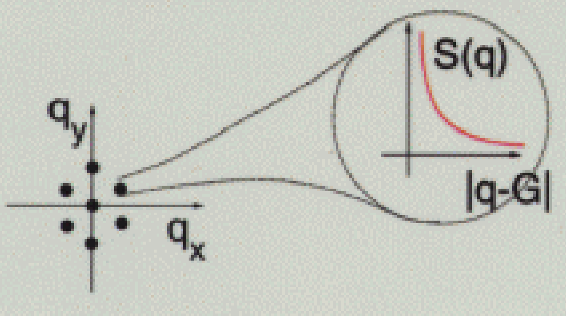
– $\overline{\langle u(r)u(0)\rangle} \sim \underbrace{(4-d)}_{\text{universal}} \ln(r/\xi_L)$, for $2 < d < 4$



(Giamarchi+LeDoussal '94)

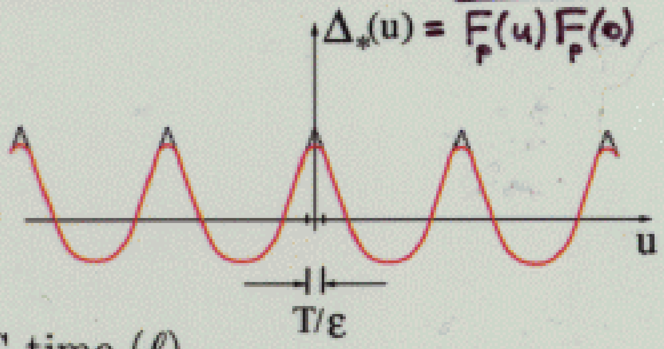
(Villain + Fernandez '86)

– Algebraic "Bragg" peaks $S(\mathbf{q}) \sim |\mathbf{q} - \mathbf{G}|^{-\eta}$ in $2<d<4$



– Pinned glassy state, described by $T = 0$ f.p.

$*$ Static FRG (D.S. Fisher '85):



$$\partial_\ell \Delta(u,\ell) = (4 - d - T\partial_u^2)\Delta - \ldots$$
$$\partial_\ell T(\ell) = -(d-2)T$$

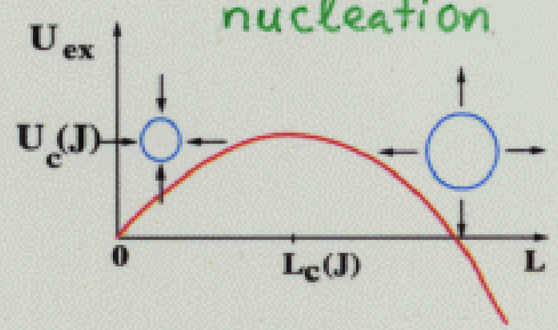
cusps in $\Delta(u)$ develop in finite RG time (ℓ)

$*$ Vanishing linear response $\Rightarrow$ slow $\log(t)$ relaxation of $M(t)$

· *Scaling theory* (D.S.Fisher+Huse '85, Nattermann '90)

$$U_{ex} \sim KL^{d-2} - JL^d \sim L^\theta$$
$$\sim (1/J)^{(d-2)/2} \to \infty \text{ as } J \to 0$$

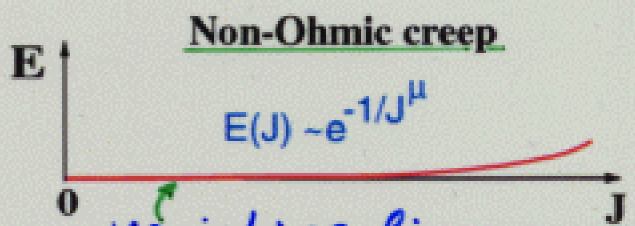
excitation energy



· *Dynamic FRG* (Radzihovsky '97, Balents+D.S.Fisher '97)

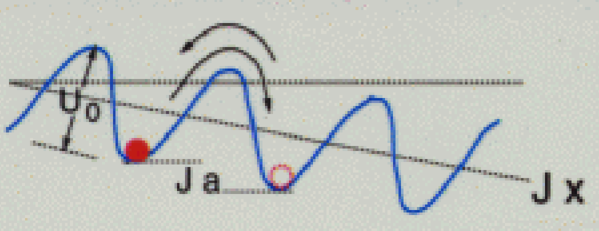
dynamic exponent
$$z(\ell) = 2 + |\Delta''(0,\ell)| \to \infty$$
$$\tau \sim e^{L^{d-2}} \to \infty$$
$$\mu = (d-2)/2$$

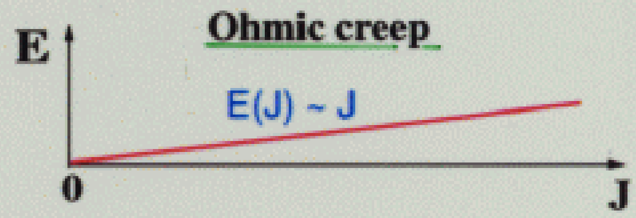
time scale for excitation of size L



vanishing linear response ($\rho_\ell = 0$)

$*$ Strongly contrasts with Anderson-Kim *independent* particle picture

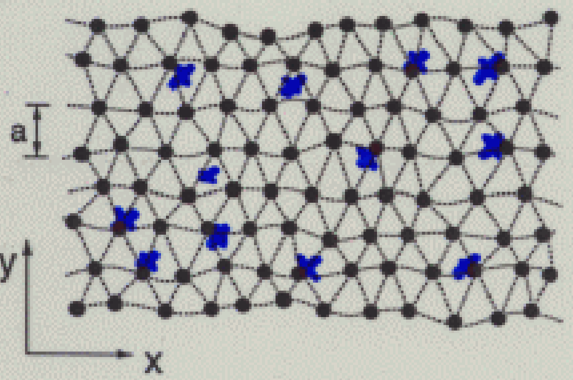




$$E \sim \underbrace{e^{-(U_0 - Ja)/T}}_{J_+} - \underbrace{e^{-(U_0 + Ja)/T}}_{J_-} \sim_{\lim_{J\to 0}} \big(\underbrace{e^{-U_0/T}}_{\rho_\ell}\big) J$$

- **Dynamic Functional Renormalization Group of Vortex Glass** [redacted] (Balents, Fisher, L.R) '97
  - Equilibrium equation of motion:

$$\gamma \partial_t u(\mathbf{r},t) = K\nabla^2 u + F_p[u,\mathbf{r}] + \zeta(\mathbf{r},t) \; ,$$



  - Quenched Gaussian random pinning force (zero mean):

$$\overline{F_p[u(\mathbf{r},t),\mathbf{r}]F_p[u(\mathbf{r}',t'),\mathbf{r}']} = \Delta(u(\mathbf{r},t) - u(\mathbf{r},t'))\ \delta^d(\mathbf{r}-\mathbf{r}') \; .$$

    * $\Delta(u-u') = \Sigma_Q \Delta_Q \cos[Q(u-u')]$ is a periodic function due to descreteness of vortex lines
    * For $d > 2$, $T = 0$ fixed point $\rightarrow$ all harmonics are equally relevant

  - Thermal Gaussian noise (zero mean):

$$\langle \zeta(\mathbf{r},t)\zeta(\mathbf{r}',t')\rangle = 2T\gamma\delta^d(\mathbf{r}-\mathbf{r}')\delta(t-t') \; .$$

MARTIN-SIGGIA-ROSE (MSR) FORMALISM:

– Map EOM onto field theory:

$$Z = \overline{\langle \delta(\gamma \partial_t u - K\nabla^2 u - F_p[u, \mathbf{r}] - \zeta) \rangle} \, ,$$

$$Z = \int [d\tilde{u} \, du] e^{-S_0[\tilde{u},u] - S_p[\tilde{u},u]} \, ,$$

$$S_0 = \int_{\mathbf{r},t} \left[i\tilde{u}(\mathbf{r},t) \left(\gamma \partial_t - K\nabla^2 \right) u(\mathbf{r},t) + \gamma T \tilde{u}(\mathbf{r},t)^2 \right] ,$$

$$S_p = \frac{1}{2} \int_{\mathbf{r},t,t'} \tilde{u}(\mathbf{r},t) \tilde{u}(\mathbf{r},t') \Delta(u(\mathbf{r},t) - u(\mathbf{r},t')) \, .$$

* $\tilde{u}(\mathbf{r},t)$ is auxiliary response field
* treat $S_p[\tilde{u},u]$ in Wilson's perturbative RG
* need dynamic *functional* RG for $2 < d < 4$
* controlled with $\epsilon = 4 - d$–expansion

– High k mode elimination and rescaling:

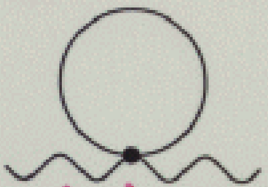
$u(\mathbf{r},t) = u^<(\mathbf{r},t) + u^>(\mathbf{r},t)$, integrate out shell $\Lambda e^{-\ell} < q < \Lambda$

$$\begin{aligned} r &= r' e^{\ell} \, , \\ t &= t' e^{z\ell} \, , \\ u^<(\mathbf{r},t) &= e^{\chi \ell} u(\mathbf{r}',t') \, , \quad \chi = 0 \\ \tilde{u}^<(\mathbf{r},t) &= e^{\tilde{\chi}\ell} \tilde{u}(\mathbf{r}',t') \, . \end{aligned}$$

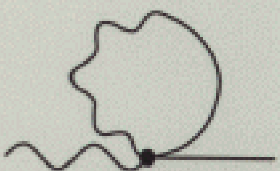
— Correlation and response functions:

$$C(\mathbf{q},\omega) \equiv \langle u(\mathbf{q},\omega)u(-\mathbf{q},-\omega)\rangle ,$$
$$= \frac{2T\gamma}{(\gamma\omega)^2 + (Kq^2)^2} ,$$
$$G(\mathbf{q},\omega) \equiv \langle u(\mathbf{q},\omega)\tilde{u}(-\mathbf{q},-\omega)\rangle ,$$
$$= \frac{1}{-\gamma\omega + iKq^2} ,$$

— Diagrammatic corrections to γ, T and Δ:



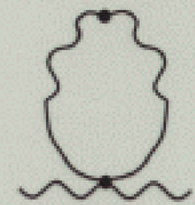
$\delta(\gamma T)$, $\delta\Delta^{(1)}$



$\delta\gamma$

$\delta S_2^{(a)}$

$\delta\Delta^{(2)}$



$\delta S_2^{(b)}$

$\delta\Delta^{(2)}$

— Renormalization group flow equations:

dynamics (new) →
gives dynamic interpretation to cusps

friction $\sim \mu^{-1}$

$$\frac{d\gamma}{d\ell} = \left(d + \tilde{\chi} - \tilde{\Delta}''(0)\right)\gamma\,,$$

$$\frac{d\tilde{T}}{d\ell} = (2-d)\tilde{T}\,,$$

exact (FDT + $u(r,t) \to$ $\to u(r,t) + \chi(d$

disorder

recover statics (D.S. Fisher) →

$$\frac{\partial \tilde{\Delta}(u)}{\partial \ell} = \left(4 - d + \tilde{T}\frac{\partial^2}{\partial u^2}\right)\tilde{\Delta}(u) - \tilde{\Delta}'(u)^2 - \tilde{\Delta}''(u)\left(\tilde{\Delta}(u) - \tilde{\Delta}(0)\right)\,.$$

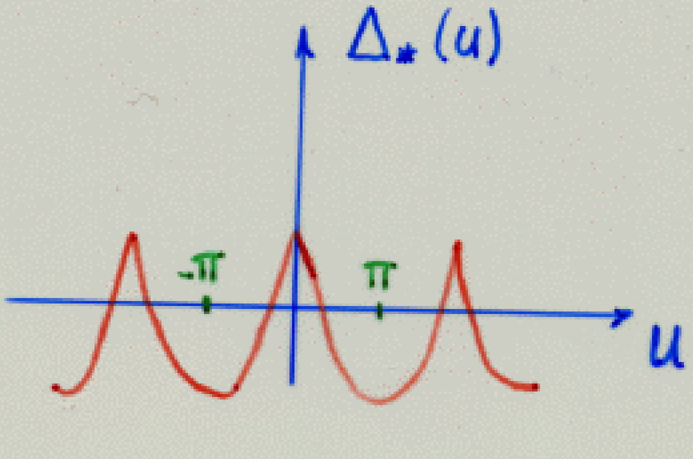
* $\tilde{T}(\ell) \approx \tilde{T}e^{-(d-2)\ell}$, i.e., $T(\ell)$ flows to 0, for $d > 2$
* $\Delta_*(u, \ell \to \infty) = \frac{\epsilon}{6}\left[(u-\pi)^2 - \frac{\pi^2}{3}\right]$ $\sim \epsilon|u|$ around $u=0$
* $\Delta_*''(0, \ell \to \infty) \to -\infty$
* $\gamma \sim$ time scale diverges, *freezing*
  $z = 2 - \tilde{\Delta}''(0)^* \to \infty$

$\langle (u(r,t) - u(r',t))^2 \rangle \sim$

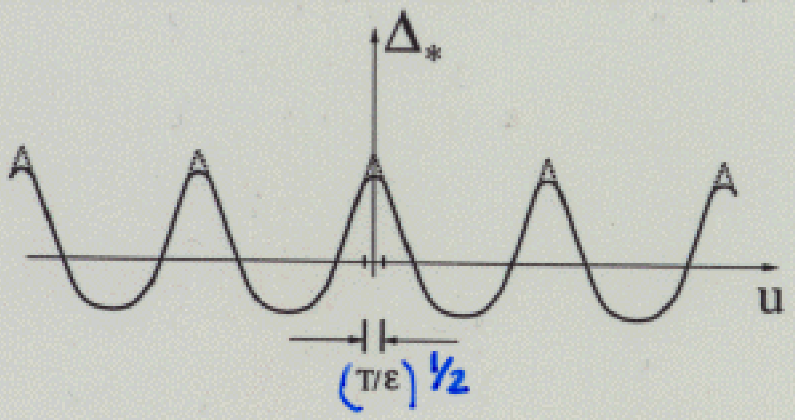
$\sim A \log|r - r'|$

universal const.

(D.S. Fisher; Giamarchi + LeDoussal)

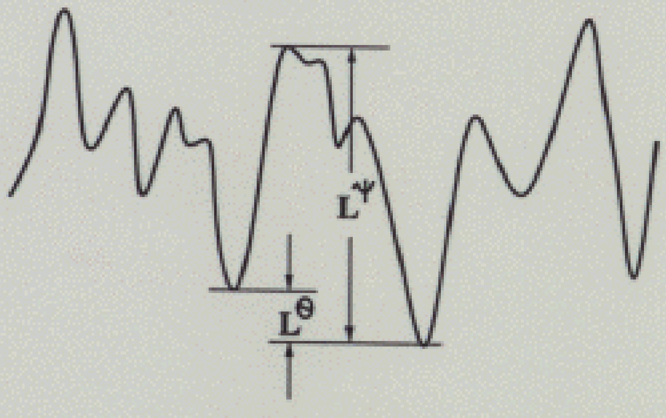


* Cut off the cusp divergence by $T(\ell)$



$$\Delta''(0,\ell) \approx -\frac{\epsilon^2\pi^2}{9}\frac{1}{\tilde{T}(\ell)} \to -\infty\,, \text{ as } \ell \to \infty$$

* Barriers that diverge with length scale L



$$\gamma_R(\ell) \approx \gamma e^{a\epsilon^2 e^{(d-2)\ell}/\tilde{T}}\,,$$

$$\gamma_R(L) \approx \gamma e^{U[L]/T}$$

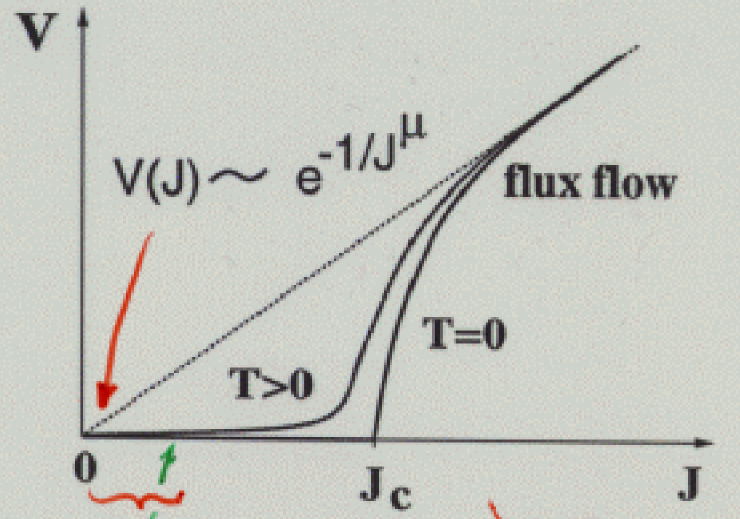
$$U[L] \sim L^\theta\,, \quad \psi = \theta = d-2\,.$$

$U[L] \sim \ln L$ in 2d

"Dirty" S.C. ARE really superconducting

$*$ Vanishing linear response $\rho = \gamma^{-1}$

$$V(J) \sim e^{-(T^*/T)(J^*/J)^{\mu}}$$



near equilibrium

vanishing slope

⇒ superconduct.

(L.R. '97)

et.al.

nonequil. dynamics

depinning "transition"

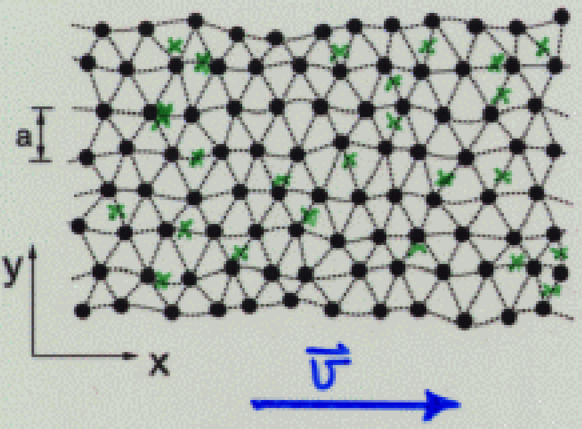
(D.S. Fisher + O. Narayan)

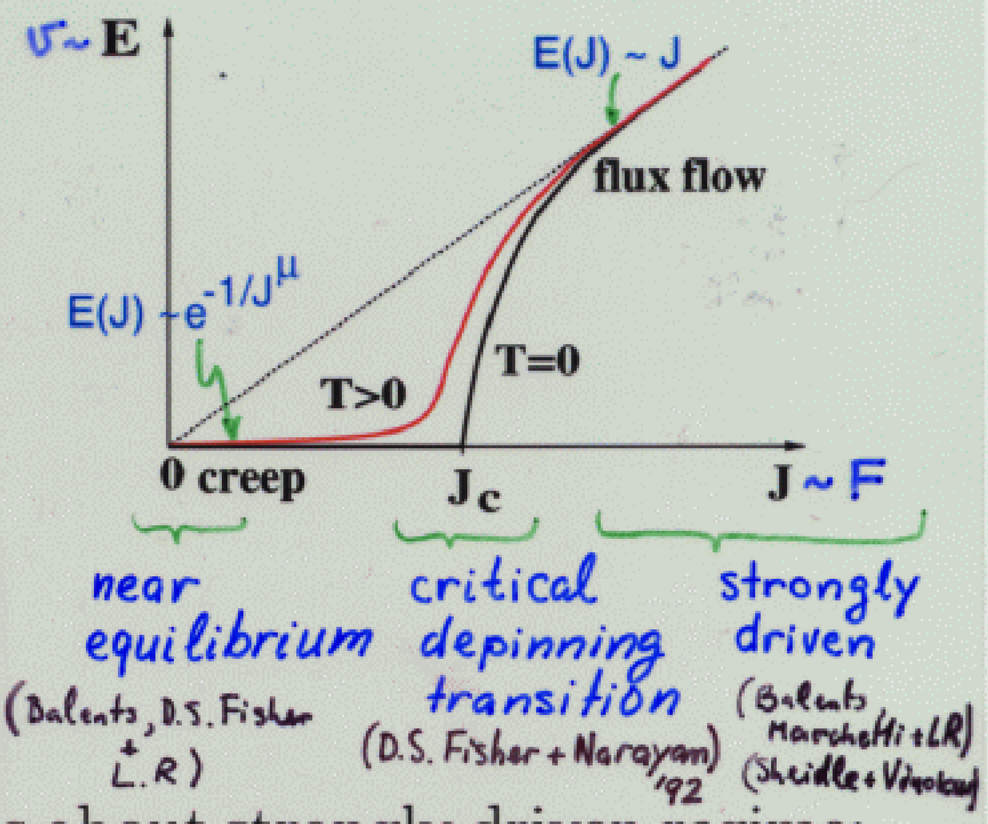
A. Middleton

nonequilibrium dynamics

(L. Balents, C. Marchetti 2 L.R. '97)

• **DRIVEN DYNAMICS:**





– Important questions about strongly driven regime:

- ∗ hydrodynamic modes and equation of motion
- ∗ E(J) (unique or hysteretic?)
- ∗ phases (classifications, correlation function,...)
- ∗ phase transitions (universality classes and order parameters)

– Earlier work

- ∗ *Experiments*:
  Neutron scattering show increase in ξ_T (Yaron, et al. '94)
  Transport show ordering above dV/dI peak (Bhattacharya, et.al. '93)
- ∗ *Simulations*:
  Decrease in dislocation/disclination defect density with increasing drive (Shi+Berlinsky '91, Koshelev+Vinokur '94)
- ∗ *Theory*:
  Current-driven crystallization at $T_{eff} = T + T_{sh}(J)$, with $T_{sh} \sim \Delta/J^2$ (Koshelev+Vinokur '94)
  Moving Bragg glass with $u_x \approx 0$ (ordered), u_y rough and pinned → *finite transverse critical current* (Giamarchi+LeDoussal '96)

– Recent theoretical developments

(Balents, Marchetti, Radzihovsky '97)

* **Nonequilibrium *lattice* equation of motion**:

"convection" elasticity nonequil. pinning force

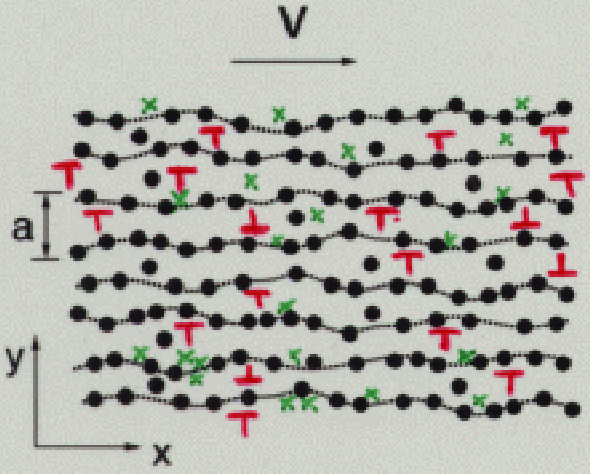
$$\gamma_{ij}\partial_t u_j = A_{i\alpha j}\partial_\alpha u_i + B_{i\alpha\beta j}\partial_\alpha\partial_\beta u_j + C_{i\alpha j\beta k}\partial_\alpha u_j \partial_\beta u_k + \tilde{F}_i[\mathbf{r},\mathbf{u},t] + \eta_i(\mathbf{r},t)$$

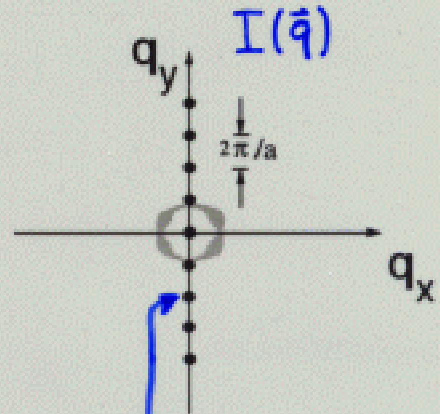
friction "KPZ" nonlinearity "thermal" noise

- stability of translational order?
- stability of temporal order?

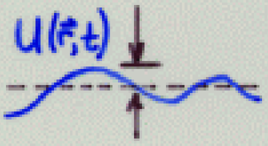
* **Perturbative analysis in weak disorder**:
  - $(u_x^{\rm rms})^2 \sim L_x^{(3-d)/2} \sim L_y^{3-d} \to \infty$ for $d \leq 3$ (c.f. G+L $u_x \approx 0$)
  - dislocations with $\mathbf{b}||\mathbf{v}$ unbind $\top \; \perp \; \perp \; \top \; \top$
  - shear melting into a moving *Transverse Smectic*

* **Nonequilibrium *smectic* equation of motion**:





- *periodic* stack of *liquid* sheets
- ρ and u are *independent* hydrodynamic modes
- neutron scattering probes r-order: $I(q) \sim |q_x^2 + (\delta \mathbf{q}_\perp)^4|^{-(3-\eta)/4}$
- narrow band noise probes t-order: $S(\omega) \sim \langle E(\omega)E(-\omega)\rangle$



$$\gamma(\partial_t + v\partial_x)u = K\nabla^2 u + \kappa\partial_y\rho + F_p(u,\mathbf{r}) + \eta(\mathbf{r},t)$$ (KPZ irrelevant)

layer displacement

$$(\partial_t + v_1\partial_x)\rho = -v_2\partial_x\partial_y u + D\nabla^2\rho$$ (continuity eqn.)

vortex density ⇒ permeation mode

* Nonequilibrium FRG analysis of transverse smectic:

friction coeff.
⇔ time scale increase
⇒ freezing

$$\frac{d\gamma}{d\ell} = \frac{2|\tilde{\Delta}''(0)|}{2+\tilde{v}^2}\,\gamma$$

disorder "heats" the smectic

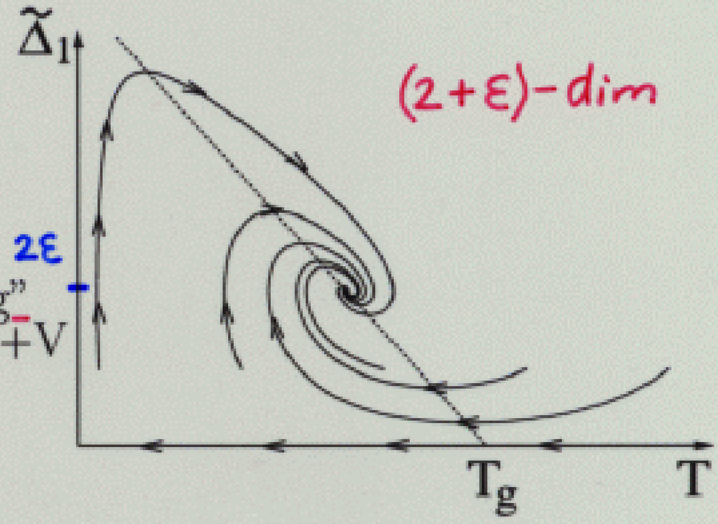
$$\frac{d\tilde{T}}{d\ell} = \left[2 - d + \overbrace{\left(\frac{1}{2} - \frac{1}{2+\tilde{v}^2}\right)}^{\neq 0}|\tilde{\Delta}''(0)|\right]\tilde{T}$$

$$\frac{\partial\tilde{\Delta}(u)}{\partial\ell} = \left(3 - d + \tilde{T}\frac{\partial^2}{\partial u^2}\right)\tilde{\Delta}(u) - \tilde{\Delta}''(u)\left(\tilde{\Delta}(u) - \tilde{\Delta}(0)\right)$$

=> $d_{uc}^{nonequil} = 3$ (cf. $d_{uc}^{equil} = 4$)

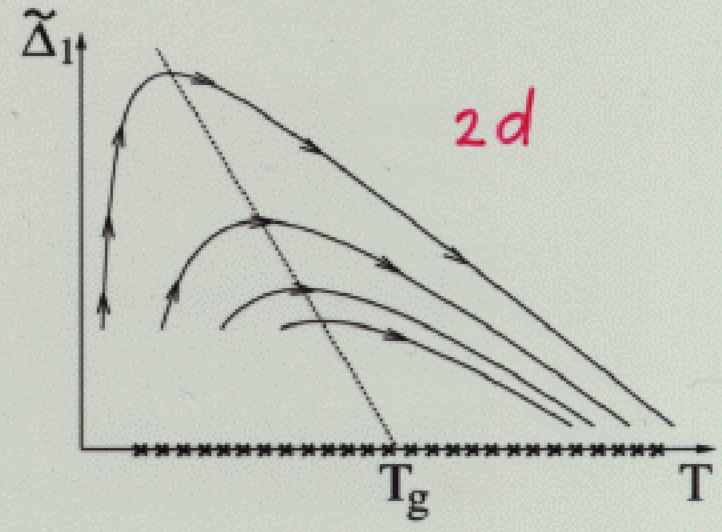
=> QLRO in 3d

=> Lack of FDT − > disorder "heating" *multiplicative* not additive as in K+V



① => Finite T fixed point

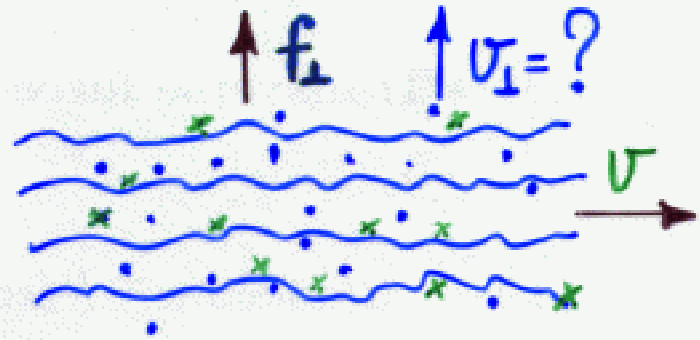
=> Effectively disorder-free 2d smectic at $\tilde{T}_{\rm eff} = 1 + \sqrt{1 + \tilde{T}(\tilde{\Delta}_1 + \tilde{T} - 2)}$



② => Permeation mode $\rho(\mathbf{r},t)$

①+② => *Analytic* transverse force response

∗ Transverse force response:



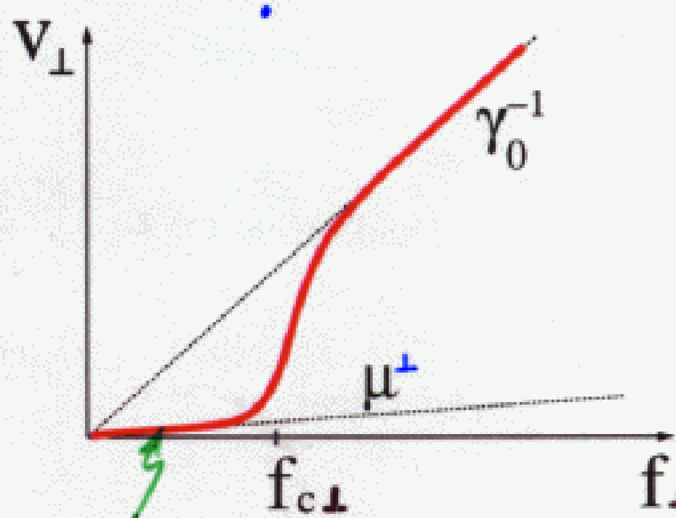
$$v_\perp \sim \underbrace{(\mu^\perp_{\text{collective}} + \mu^\perp_{\text{permeation}})}_{\mu^\perp_{\text{smectic}} \neq 0} f_\perp$$

vanishes as $v \to 0$

$$\mu^\perp_{\text{collective}} \sim e^{-(K/\gamma v)^2}$$

$$\mu^\perp_{\text{permeation}} \sim e^{-U_d/T}$$

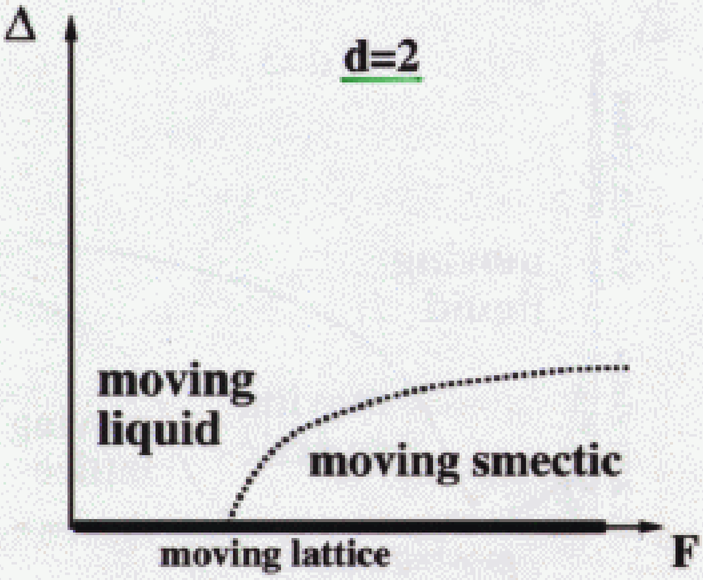
permeation mode mobility

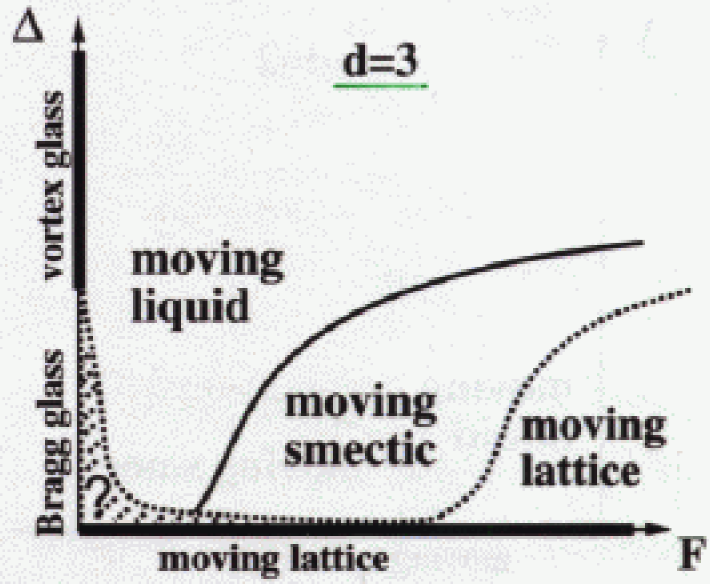


finite linear mobility

transverse

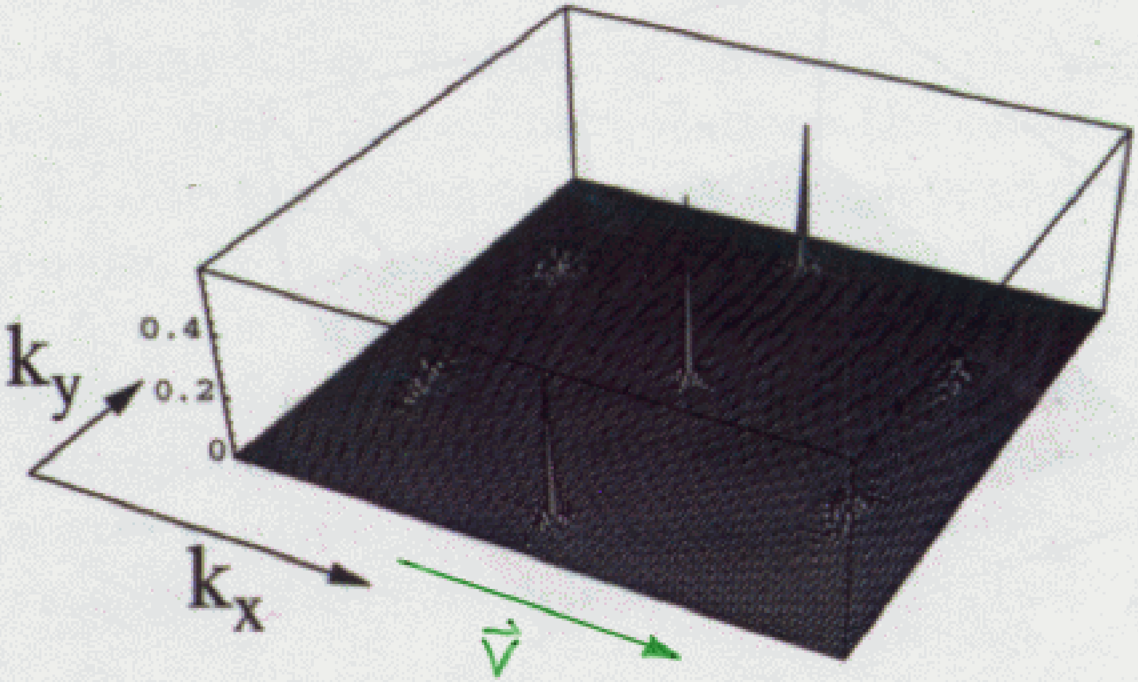
∗ Phase diagram for a driven solid:





– Simulations (Moon, et. al.'96, also see Ryu, et al.'96)

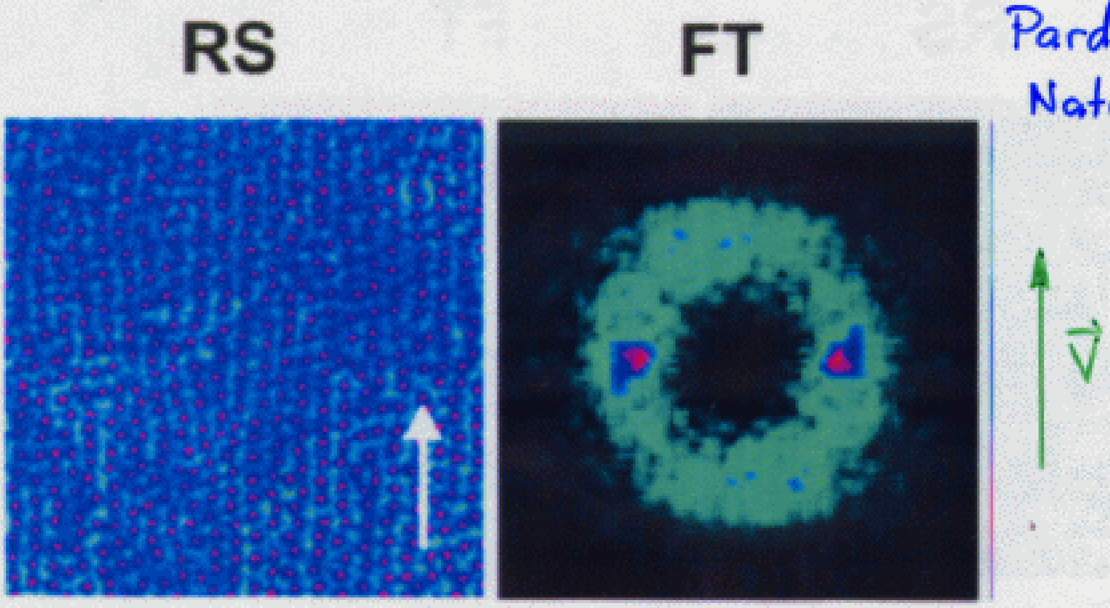
- Algebraic Bragg peaks $\perp \vec{V}$
- Channel flow with phase slips



– Decoration experiments on $NbSe_2$ (Pardo, et al.'97, also see Marchevsky, et al.'97)

4K, 5G

Pardo, etal, Nature, v. 396, pg 348 (1998)



Moving transverse smectic

- Summary and Conclusions:
  - Rich phenomenology of high T_c superconductors
  - Strong thermal fluctuations and pinning disorder
  - New vortex phases: Vortex liquid, Vortex glass, and Bose glass (few others)
  - Dissipation
  - Phase transitions
  - Equilibrium and non-equilibrium dynamics of disordered periodic media
- Remaining Questions (many!)
  - Topological defects (dislocations)
  - Nonequilibrium phase transitions