(p.1)

Flux Quantization and Order Parameter Symmetry

KAM July 3, 2000

Outline

① fluxoid quantization in superconductors

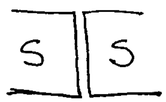
$$n\frac{hc}{e^*} = \frac{m^*c}{(e^*)^2 n_s}\oint \vec{J}_s \cdot d\vec{l} + \Phi$$

n — an integer

$\Phi_0 = \frac{hc}{2e}$ superconducting flux quantum

for $\vec{J}_s = 0$, $e^* = 2e$, $\boxed{\Phi = n\Phi_0}$

② possibility of π junctions



current $j = j_c \sin\Delta\phi$

energy $E = -\frac{\hbar j_c}{2e}\cos\Delta\phi$

conventional junction $j_c > 0$

"π-junction" $j_c < 0$

③ half-integral flux quantization with π junctions

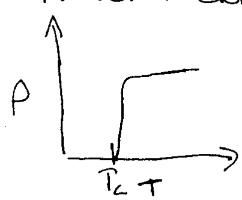
○ ×π $\Phi = (n+\frac{1}{2})\Phi_0$

Essential Background

1911 Discovery of perfect conductivity

viewgraph: photo of H. Kamerlingh Onnes

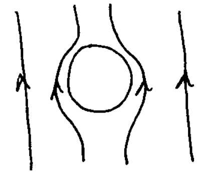
R vs. T data

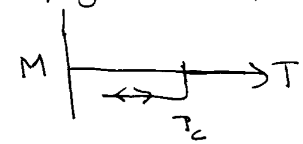


Moler
Lecture 1

(p.2)

1933 Discovery of perfect diamagnetism in some materials at cryogenic temperatures





both field-cooled (fc) and zero-field-cooled (zfc)

note that perfect zfc diamagnetism can be explained with perfect conductivity: the eddy currents aren't damped if R=0.

the fc diamagnetism is a completely different effect. Meissner effect

1935 London equations

1950 Ginzburg-Landau

phenomenological pseudo-wavefunction as superconducting order parameter

$\psi = |\psi| e^{i\phi}$

magnitude — $|\psi|$; phase — ϕ

superfluid density $n_s = |\psi|^2 = \frac{m^* c^2}{4\pi e^{*2}\lambda_L^2}$

λ_L — penetration depth

λ - penetration depth - length scale on which magnetic field can change

ξ - coherence length - length scale on which $|\psi|^2$ can change p.3

Cooper Pairs

2 electrons

interaction potential $V(\vec{r})$

($\downarrow\uparrow$)

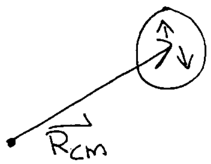
$$V_{\vec{k}\vec{k}'} = \frac{1}{\Omega}\int V(\vec{r})e^{i(\vec{k}'-\vec{k})}d\vec{r}$$

Ω (volume)

approximation:

$$V_{\vec{k}\vec{k}'} = \begin{cases} -V & \text{for } \varepsilon_{\vec{k}} < \hbar\omega_c \text{ and } \varepsilon_{\vec{k}'} < \hbar\omega_c \\ 0 & \text{for } \varepsilon_{\vec{k}} > \hbar\omega_c \end{cases}$$

$\varepsilon_{\vec{k}}$ single-particle energy relative to E_F



ξ coherence length
"size of a Cooper pair"
(1953 Pippard)

BCS Theory

pairing from attractive electron-electron interaction
(phonon-mediated)

gap self-consistency

$$\Delta_{\vec{k}} = -\frac{1}{2}\sum_{\ell}\frac{\Delta_{\vec{\ell}}\,V_{\vec{k}\vec{\ell}}}{(\Delta_{\vec{\ell}}^{2}+\varepsilon_{\vec{\ell}}^{2})^{1/2}}$$

$\Delta_{\vec{k}}$ energy gap – minimum energy required to create excitation
phenomenological order parameter w/ phase factor $e^{i\phi}$

w/ above simplifying approximation,

$$\Delta_{\vec{k}} = \begin{cases} \Delta & \text{for } |\varepsilon_{\vec{k}}| < \hbar\omega_c \\ 0 & \text{for } |\varepsilon_{\vec{k}}| > \hbar\omega_c \end{cases}$$

$\Rightarrow$ spherically symmetric, s-wave cooper pair

more generally,

$\Delta = \Delta(\vec{R})$

or $\psi = \psi(\vec{R})$

p.4

GL equations

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m^*}\left(\frac{\hbar}{i}\vec{\nabla} - \frac{e^*}{c}\vec{A}\right)^2\psi = 0$$

$$\vec{J}_s = \frac{c}{4\pi}\vec{\nabla}\times\vec{h} = \frac{e^*\hbar}{2m^* i}(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*) - \frac{e^{*2}}{m^*c}\psi^*\psi\vec{A}$$

$$\psi = |\psi|e^{i\phi}$$

$$\boxed{\vec{J}_s = \frac{e^*}{m^*}|\psi|^2\left(\hbar\vec{\nabla}\phi - \frac{e^*}{c}\vec{A}\right)}$$

Flux(oid) quantization

superconductor with inhomogeneous current and field distributions

path to integrate around

$$\oint \vec{J}_s\cdot d\vec{l} = \frac{e^* n_s \hbar}{m^*}\underbrace{\oint\vec{\nabla}\phi\cdot d\vec{l}}_{2\pi n} - \frac{(e^*)^2 n_s}{m^* c}\underbrace{\oint\vec{A}\cdot d\vec{l}}_{\Phi = \int\vec{h}\cdot d\vec{a}}$$

n = integer

$$\boxed{2\pi\frac{\hbar c}{e^*}\,n = \frac{m^* c}{(e^*)^2 n_s}\oint\vec{J}_s\cdot d\vec{l} + \Phi}$$

1961 what is $\frac{hc}{e^*}$? 4 back-to-back PRLs (2 experimental) demonstrated flux quantization

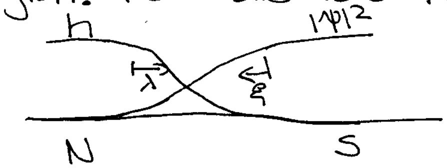
see viewgraphs

flux quantization applies inside a loop on which $\oint J_s \cdot d\vec{l} = 0$

$$\boxed{\Phi_0 = \frac{hc}{2e} = 20.7\,G\,\mu^2}$$ superconducting flux quantum

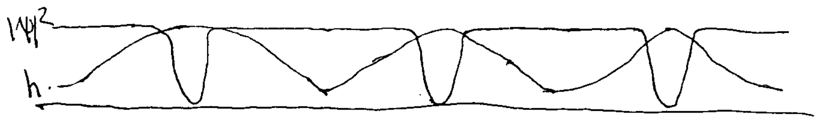
1962 fluxoid quantization demonstrated ($J_s \neq 0$) by measuring R vs. Φ for a tin cylinder close to T_c.
see viewgraph: Little-Parks experiment

1957 Abrikosov lattice (viewgraph) in Type-II superconductors
consider a domain wall between a superconducting and normal region. there are two types:



Type I: $\lambda < \xi/2$ normal inclusion doesn't save much magnetic energy, but it costs a lot of condensation energy
$\Rightarrow$ positive domain wall energy

Type II: $\lambda > \xi/2$ opposite situation
$\Rightarrow$ negative domain wall energy
$\Rightarrow$ depending on applied field (for $H > H_{c1}$) system likes to have many small domains
$\Rightarrow$ vortex (flux line) lattice forms



note: previous sketch is an oversimplified view of vortex structure!
see viewgraph Abrikosov's 57 paper

1967 First direct experimental observation of vortex lattice (by Bitter decoration)
U. Essman & H. Trauble
see viewgraphs:
- vortex lattice seen w/ Bitter decoration
- vortex lattice seen w/ STM (sees electronic properties, not magnetic)
- isolated trapped vortices (at field below H_{c1}) seen with scanning SQUID microscopy

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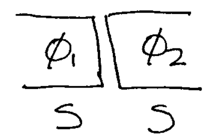
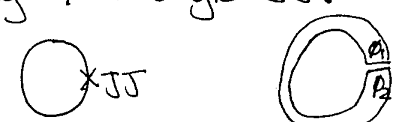
Josephson Junctions

j_c ≡ critical current density of junction



$\Delta\phi = \phi_1 - \phi_2$
$j = j_c \sin \Delta\phi$
$E = -\frac{\hbar j_c}{2e} \cos \Delta\phi$

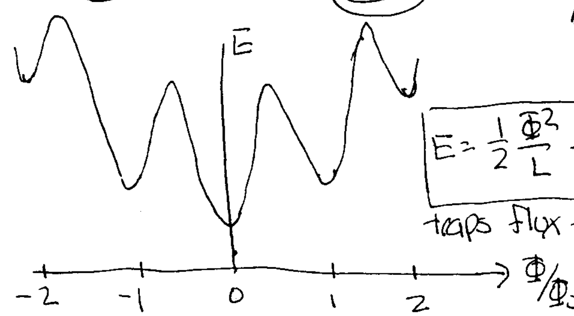
(superconducting) ring w/ a single JJ:



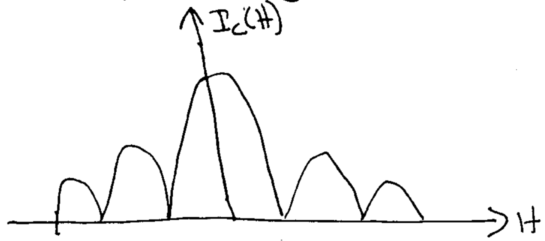
$E = \frac{1}{2} L I^2 - \frac{\hbar I_c}{2e} \cos\Delta\phi$
$\Delta\phi = 2\pi\Phi/\Phi_0$
$\Phi = IL$

$$\boxed{E = \frac{1}{2}\frac{\Phi^2}{L} - \frac{\hbar I_c}{2e} \cos\frac{2\pi\Phi}{\Phi_0}}$$

traps flux for large I_c



Fraunhofer diffraction pattern of single Josephson junction:



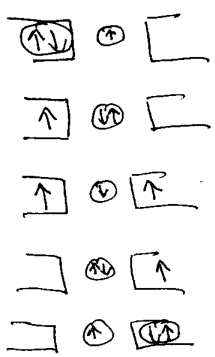
Josephson penetration depth λ_J: length scale on which field can change inside JJ. Typically $\lambda_J \gg \lambda_L$

"π Junction"

1977 L.N. Bulaevskii, V.V. Kuzii, and A.A. Sobyanin
1978 predicted $j_c < 0$! in a junction with magnetic impurities!

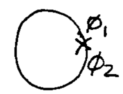
why? The Cooper pair has to hop through the junction one electron at a time:

see viewgraphs



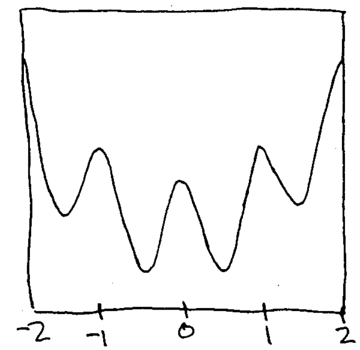
spin up & spin down are exchanged in this process
$\Rightarrow$ multiply wavefunction by -1
$\Rightarrow$ induce π phase shift
$(e^{-i\pi} = -1)$

consider the effect of $j_c < 0$ on a ring with a π-junction:



$$E = \frac{1}{2} L I^2 - \frac{\hbar I_c}{2e} \cos \Delta\phi$$

$$\boxed{E = \frac{1}{2}\frac{\Phi^2}{L} - \frac{\hbar I_c}{2e} \cos \frac{2\pi\Phi}{\Phi_0} \quad I_c < 0}$$



ground state has a spontaneously circulating current and carries flux $\Phi_0/2$ for sufficiently large I_c!

(even if I_c is not large enough compared to L for the energetics to favor spontaneous flux generation, the $j_c < 0$ will show up in the Fraunhofer diffraction pattern as an $I_c(H)$ that has a local minimum at $H=0$ instead of the usual local maximum)

note: a spontaneously generated half-integral flux quantum caused by magnetic impurities has never been observed, as far as I know.

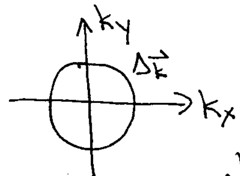
unconventional order parameters

in BCS, the simplifying approximation was made that the electron-electron interaction potential $V(\vec{r})$ is spherically symmetric. this led to a spherically symmetric Cooper pair, with a gap function:

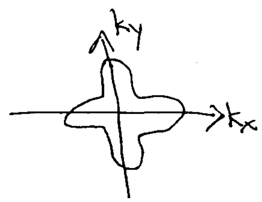
$$\Delta_{\vec{k}} = \begin{cases} \Delta_0 \text{ for } |\varepsilon_{\vec{k}}| < \hbar\omega_c \\ 0 \text{ for } |\varepsilon_{\vec{k}}| > \hbar\omega_c \end{cases}$$

more generally $\Delta_{\vec{k}}$ can depend on the direction of $\vec{k}$ (and this dependence is directly related to the internal structure of a Cooper Pair)

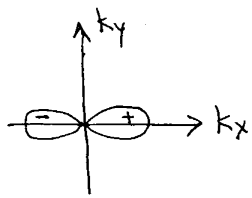
$\Delta_{\vec{k}}$ can be s-wave:



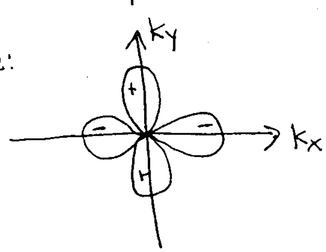
anisotropic s-wave:



p-wave:



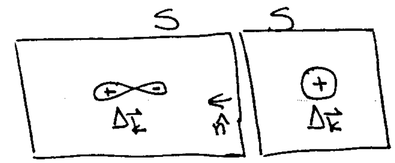
d-wave:



both the amplitude and phase of $\Delta_{\vec{k}}$ **can depend** on $\vec{k}$. (A phase change of π is a sign change)

1986 Geshkenbein, Larkin, and Barone
1987 pointed out that unconventional order parameters can lead to pi-junction like effects.

see viewgraphs

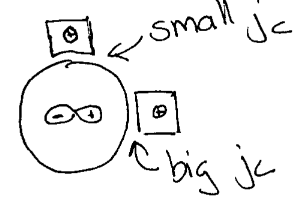


$$F = \mathrm{Re} \int \Delta_1 \Delta_2^* G_1 G_1 |T|^2 G_2 G_2 \, d\vec{k}_1 d\vec{k}_2$$

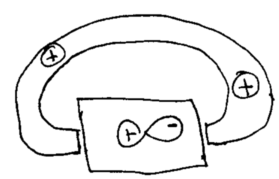
for normal incidence tunneling, expressions for tunneling depend on the gap $\Delta_{\vec{k}}$ with $\vec{k} \parallel \hat{n}$, so the tunneling probability is a measure of the angular dependence of the gap.

Three experimental implications:

① $I_c(\Phi)$



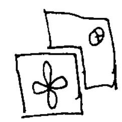
②



half-integral flux quantum can form

or

③



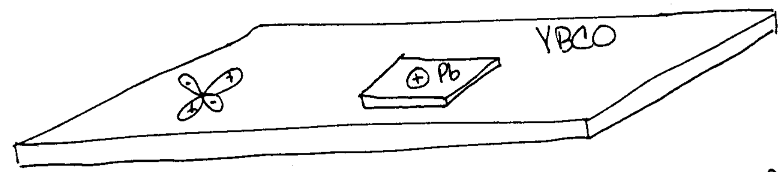
$I_c(H)$ has minimum at $H=0$

<u>1990's</u>

<u>see</u> viewgraphs and references:
the existence of π phase shifts established in several high-T_c materials.

A famous red herring:
c-axis tunneling

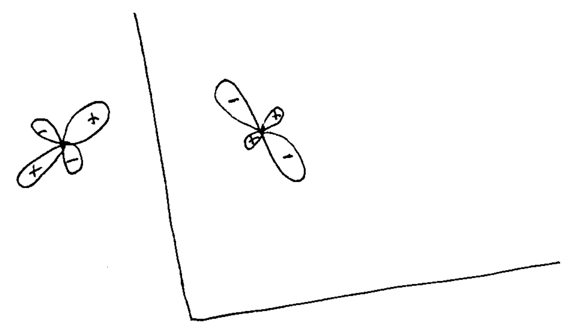
c-axis s-wave - d-wave junctions shouldn't show any tunneling



but: they do! Sun et al 1994.

this coupling is expected for an <u>untwinned</u> orthorhombic "d+s" order parameter

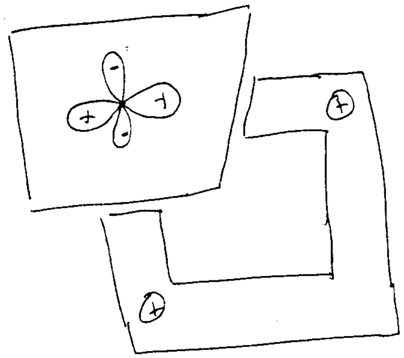
but the experiments were ~~also~~ done on <u>heavily twinned</u> samples, where no coupling is expected for a <u>random</u> distribution of twins



one possibility: correlate w/ twin density

Kouznetsov et al 1997: junction across a single twin

Fractional Vortices (other than $\frac{1}{2}\Phi_0$)



$$\frac{\Phi}{\Phi_0} = n + \frac{\delta}{2\pi}$$

0 conventionally

π d-wave in some geometries

any arbitrary value for complex order parameters

Beasley, Lew, and Laughlin 1995

Theoretically, a complex order parameter can develop locally at interfaces
Sigrist, Bailey, and Laughlin 1995
Sigrist, Kuboki, Lee, Millis, and Rice 1995

Lots of hints of fractional vortices, but no convincing evidence so far

Complex order parameters at interfaces:
upcoming talks by Sauls
Greene

Fluxoid Quantization References

Ginzburg-Landau: Order Parameter or Phenomenological Pseudowave-function:
V.L. Ginzburg and L.D. Landau, *Zh. Eksperim I Teor. Fiz.* **20**, 1064 (1950).

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Half-Integral Flux Quantization: early theory
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Paramagnetic Meissner Effect, aka the Wohlleben Effect
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J.R. Kirtley, A.C. Mota, M. Sigrist, T.M Rice, "Magnetic imaging of the paramagnetic Meissner effect in the granular high-T_c superconductor $Bi_2Sr_2CaCu_2O_x$," *Journal of Physics: Condensed Matter* **10**, L97 (1998).

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Half-Integral Flux Quantization: experiments

D.A. Wollman, D.J. Van Harlingen, W.C. Lee, D.M. Ginsberg, A.J. Leggett, "Experimental determination of the superconducting pairing state in YBCO from the phase coherence of YBCO-Pb DC SQUIDs," *Physical Review Letters* 71, 2134 (1993).

D.A. Brawner, H.R. Ott, "Evidence for an unconventional superconducting order parameter in $YBa_2Cu_3O_{6.9}$," *Physical Review B* **50**, 6530 1994.

C.C. Tsuei, J.R. Kirtley, C.C. Chi, L.S.Yu-Jahnes, A. Gupta, T. Shaw, J.Z. Sun, M.B. Ketchen, "Pairing symmetry and flux quantization in a tricrystal superconducting ring of $YBa_2Cu_3O_{7-\delta}$," *Physical Review Letters* **73**, 593 (1994).

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What About c-axis Tunneling?

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K.A. Kouznetsov, A.G. Sun, B. Chen, A.S. Katz, S.R. Bahcall, J. Clarke, R.C. Dynes, D.A. Gajewski, S.H. Han, M.B. Maple, J. Giapintzakis, J.-T. Kim, D.M. Ginsberg, "c-axis Josephson tunneling between $YBa_2Cu_3O_{7-\delta}$ and Pb: direct evidence for mixed order parameter symmetry in a high-T_c superconductor," *Physical Review Letters* **79**, 3050 (1997).

What about twins? M. Walker

This is definitely not a complete list of references, but if you read all these papers you will have a good understanding of this issue.

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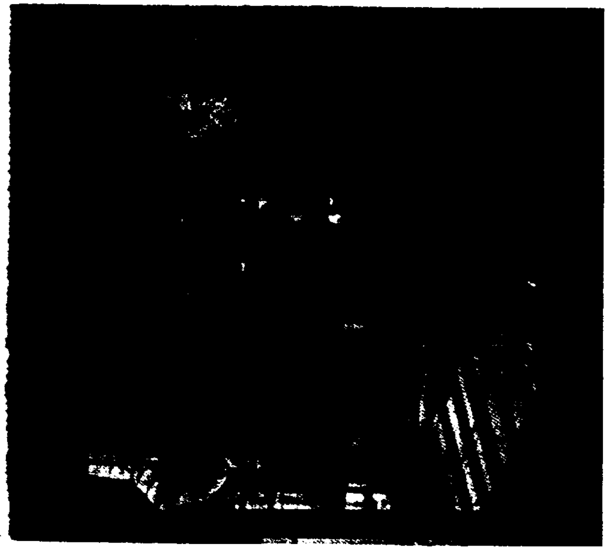
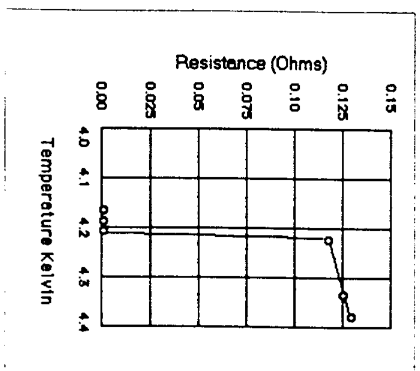


Discovery of superconductivity 1911

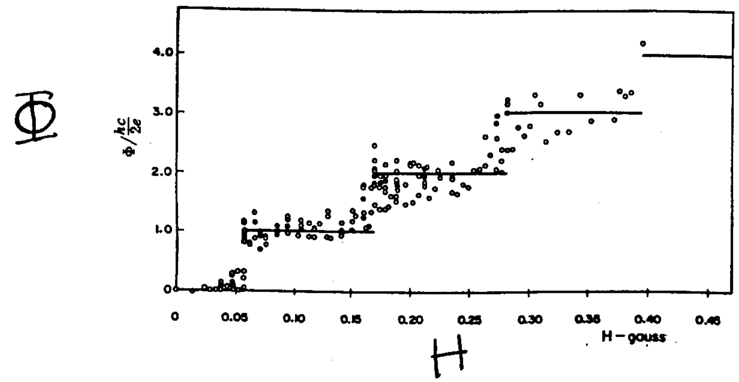


EXPERIMENTAL EVIDENCE FOR QUANTIZED FLUX IN SUPERCONDUCTING CYLINDERS*

Bascom S. Deaver, Jr., and William M. Fairbank
Department of Physics, Stanford University, Stanford, California

(Received June 16, 1961)

We have observed experimentally quantized values of magnetic flux trapped in hollow superconducting cylinders. That such an effect might occur was originally suggested by London[1] and Onsager,[2] the predicted unit being hc/e. The quantized unit we find experimentally is not hc/e, but $hc/2e$ within experimental error.[3]



of flux, the extra unit being shoved out the side of the cylinder. This is especially probable for sample No. 1 since the x-ray photograph showed a break in the tin coating near the middle of the cylinder. Also, it is known that flux can create a normal region in a superconductor by shrinking in size until the critical field is exceeded. In this experiment we were unable to measure independently the signals from the two coils. However, in future experiments this will be done to remove this ambiguity. It is interesting to note that no intermediate points are found outside the expected scatter of the data near the first step. One point for which no flux was trapped was found near the center of the first step with sample No. 1.

In conclusion, we find:

1. The flux trapped in a superconducting cylinder both in the presence and absence of an applied magnetic field is not continuous but exhibits a step behavior, the first step occurring for $\Phi = hc/2e$, within experimental error in the data. Considering all sources of error, we estimate that the value of the trapped flux at the first step is $hc/2e \pm 20\%$. If the correction to the size of the cylinder due to the presence of the copper should prove invalid, an additional 11% error could arise for the large cylinder and 17% for the small cylinder.

2. The data seem to indicate additional steps at hc/e, $3hc/2e$, and $2hc/e$. The points appearing between these levels will be investigated further.

3. The ratio of the fields at which the steps occur are approximately 1, 3, 5, and 7. In the first cylinder (for which the effective cross-sectional area of the cylinder is 2.33 times the area of the hole), the first jump occurs when the flux passing through the total effective cross section of the cylinder in the normal state is approximately $hc/2e$.

For cylinder No. 2 (in which the effective cross-sectional area of the cylinder is 1.1 times the area of the hole), the first jump occurs when the flux passing through the total effective cross section of the cylinder in the normal state is approxip.51

(19)

'AL PROOF OF MAGNETIC FLUX QUANTIZATION IN A SUPERCONDUCTING RING*

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(Received June 19, 1961)

ical considerations, based on wave
ıdon[1] concluded that the magnetic
ı a twofold-connected superconduct-
or tube) should not have any arbi-
t only such values which are inte-
f a basic unit ϕ_0,

$$= 4.12\times10^{-7}\ \text{gauss cm}^2. \qquad (1)$$

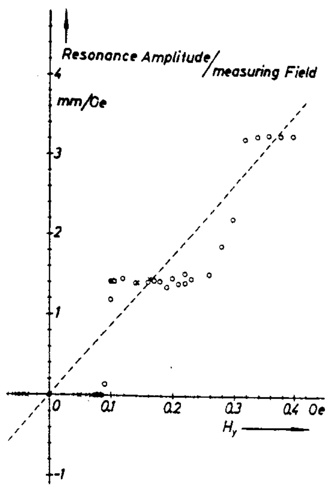
magnetic flux should be quantized.
:hrieffer[2] also agreed with this con-

the lead tube, Eq. (2) predicts for the interval of the magnetic field strength corresponding to one flux unit a value of $H_y = 0.5$ oe. The experimentally observed interval, however, reaches only 0.2 oe, that is about 40% of the calculated value. So far the reason for this discrepancy is not clear. For example, an error of 60% in the determination of the lead tube's diameter would explain the difference, but such an error is improbable.

The experiments are being continued with higher fields H_y and other superconductors of various diameters.

Mercereau and Vant-Hull[5] also tried to verify London's postulate of the quantization of magnetic flux in a superconducting ring. The result of their experiments was negative.

The authors are indebted to Professor W. Meissner who made possible and promoted this work. The authors would further like to thank Professor F. X. Eder for encouragement and helpful discussions.



:esonance amplitude divided by measuring
ı function of the applied field H_y. The
proportional to the frozen-in flux. ×– First
nd run.

*Presented at the Conference on Fundamental Research in Superconductivity, IBM Research Center, Yorktown Heights, New York, June 15-17 (1961).

[1]F. London, Superfluids (John Wiley & Sons, New York, 1950), Vol. I, p. 152.

[2]J. Bardeen and I. R. Schrieffer, Progress in Low-Temperature Physics (North-Holland Publishing Company, Amsterdam, 1961), Vol. III, p. 182.

[3]A. Einstein and W. J. de Haas, Verhandl. deut. physik. Ges. 17, 152 (1915).

[4]R. Doll, Z. Physik 153, 207 (1958).

[5]J. E. Mercereau and L. L. Vant-Hull, Bull. Am. Phys. Soc. 6, 121 (1961).

(20)

0.46

mately $0.6hc/2e$. In a following Letter, Byers and Yang[6] conclude that in a thin ring the first jump should occur at $0.5hc/2e$.

4. Since the time constant of our measuring circuit is 25 seconds, this experiment gives only a large upper limit for the time involved in reaching these quantized flux values. Mercereau and Vant-Hull[7] have reported a negative experiment designed to observe quantized flux in a 1-mm ring cooled 6000 times per second through the superconducting transition in a small magnetic field. It is possible that the difference in their results and the results of our experiment are due to a minimum time necessary to establish equilibrium. We are planning to investigate this relaxation time.

We have had the pleasure of discussing the results of this experiment with N. Byers, C. N. Yang, and L. Onsager, whose interpretation of these results appear in the following Letters.[6,8] One of us (WMF) also wishes to acknowledge his indebtedness to F. London and M. J. Buckingham who greatly influenced his concept of the superfluid state. We also wish to thank F. Bloch, L. I. Schiff, and J. D. Bjorken for many stimulating discussions of the experiment. We wish to acknowledge the invaluable assistance of M. B. Goodwin.

*Work supported in part by grants from the National Science Foundation, the Office of Ordnance Research (U. S. Army), and the Linde Company.

[1]F. London, Superfluids (John Wiley & Sons, New York, 1950), p. 152.

[2]L. Onsager, Proceedings of the International Conference on Theoretical Physics, Kyoto and Tokyo, September, 1953 (Science Council of Japan, Tokyo, 1954), pp. 935-6.

[3]Such a possibility was mentioned by Lars Onsager to one of us (WMF) at the conference on superconductivity in Cambridge, England, 1959 (unpublished).

[4]S. Foner, Rev. Sci. Instr. 30, 548 (1959).

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[7]J. E. Mercereau and L. L. Vant-Hull, Bull. Am. Phys. Soc. 6, 121 (1961).

[8]L. Onsager, this issue [Phys. Rev. Letters 7, 50 (1961)].

THEORETICAL CONSIDERATIONS CONCERNING QUANTIZED MAGNETIC FLUX IN SUPERCONDUCTING CYLINDERS*

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(Received June 16, 1961)

In a recent experiment,[1] the magnetic flux through a superconducting ring has been found to be quantized in units of $ch/2e$. Quantization in twice this unit has been briefly discussed by London[2] and by Onsager.[3] Onsager[4] has also considered the possibility of quantization in units $ch/2e$ due to pairs of electrons forming quasi-bosons.

The previous discussions[3] leave unresolved the question whether quantization of the flux is a new physical principle or not. Furthermore, sometimes the discussions seem[2] to be based on the assumption that the wave function of the superconductor in the presence of the flux is proportional to that in its absence, an assumption which is not correct. We shall show in this Letter that (i) no new physical principle is involved in the requirement of the quantization of magnetic flux through a superconducting ring, (ii) the Meissner effect is closely related to the requirement that the flux through any area with a boundary lying entirely in superconductors is quantized, and (iii) the quantization of flux is an indication of the pairing of the electrons in the superconductor.

Macroscopic discussion. Consider a multiply connected superconducting body P with a tunnel O (Fig. 1). We shall only discuss macroscopic

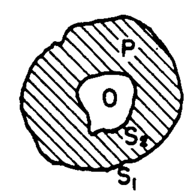

FIG. 1. Multiply connected superconductor.

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MAGNETIC FLUX THROUGH A SUPERCONDUCTING RING

p. 50

Lars Onsager*
University of California, La Jolla, California
(Received June 15, 1961)

London[1] recognized that the magnetic flux embraced by a superconducting ring ought to be quantized. He argued as follows: The current density is proportional to the average of

$$\vec{p} - (Q/c)\vec{A},$$

where $\vec{A}$ denotes the vector potential and Q the charge of the current-carrying particle. In the interior of the superconductor the current density vanishes, so that the condition

$$\oint \vec{p} \cdot d\vec{x} = -nh$$

for a single-valued wave function implies

$$\Phi = \oint \vec{A} \cdot d\vec{x} = -nhc/Q.$$

Substituting $Q = -e$ for the charge of the electron, he arrived at the conclusion

$$\Phi = nhc/e.$$

It is possible to recast London's discussion in a form which is completely Lorentz and gauge invariant; the details need not concern us here.

London's result inspired the suggestion[2] that the quantization of flux might be an intrinsic property of the electromagnetic field.

Not much later, Schafroth[3] pointed out that electron pairs held together by attractive interactions would obey Bose statistics and be capable of superfluid (Einstein) condensation. A likely source of the requisite attractive interaction—by way of the phonon field—had been suggested by Fröhlich[4] and by Bardeen.[5] These ideas form the basis of more detailed theories, which explain the various observed properties of superconductors so well that they have been generally accepted.

Deaver and Fairbank[6] have found that the flux embraced by a superconducting annulus varies in steps of half the size proposed by London. This is readily explained by Schafroth's theory, which requires

$$Q = -2e,$$
$$\Phi = nhc/2e.$$

The discovery of steps just this size provides a convincing direct proof for the pairing of electrons.

The notion that the electromagnetic field itself might be subject to a similar condition seems untenable now, for singly charged bosons exist (deuterons) and a condition imposed on the electromagnetic field ought to be equally compatible with all charged particles.

Instead, we arrive at the remarkable result that we can measure the magnetic flux (except for an additive undetermined multiple of $hc/2e$) embraced by a given closed path without examining the enclosed field; a superconductor placed along the path will respond with a supercurrent which compensates the fractional excess of flux. Complete Meissner effect in a multiply-connected superconductor requires coherent ring closure. We may infer that such closure will not take place unless the free energy liberated by the matching of phases exceeds the kinetic energy of the necessary supercurrent plus the added magnetic field energy. The detailed kinetic mechanism is not yet known.

I am indebted to F. Bloch for a discussion which clarified the closure problem, and to B. Deaver and W. M. Fairbank for the communication of their unpublished results.

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[1] F. London, Superfluids (John Wiley & Sons, Inc., New York, 1950), Vol. 1, p. 152.
[2] L. Onsager, Proceedings of the International Conference on Theoretical Physics, Kyoto and Tokyo, September, 1953 (Science Council of Japan, Tokyo, 1954), p. 935.
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[6] B. Deaver and W. M. Fairbank, this issue [Phys. Rev. Letters 7, 43 (1961)].

22

OBSERVATION OF QUANTUM PERIODICITY IN THE TRANSITION TEMPERATURE OF A SUPERCONDUCTING CYLINDER*

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(Received May 10, 1962; revised manuscript received June 15, 1962)

Deaver and Fairbank[1] and Doll and Näbauer[2] have shown experimentally that the flux which is trapped in a superconducting cylinder is an integral multiple of the unit $hc/2e$. It has been pointed out[3,4] that this result follows because the free energy of the superconducting state is periodic in this unit of the flux if the electrons are paired in the manner described by the Bardeen-Cooper-Schrieffer (BCS) theory.[5] The free energy of the normal state, on the other hand, is virtually independent of the flux. Consequently, the transition temperature T_c, which is the temperature at which the free energy of the normal and superconducting states are equal, must also be a periodic function of the enclosed flux ϕ. The magnitude of the change in T_c was calculated for a thin cylindrical sample using the BCS model in which the possible pairing of particles with net momentum was included. This calculation showed that the binding energy of each pair was reduced by the amount of energy required to provide the center-of-mass motion necessary to maintain the fluxoid,

$$\frac{1}{h}\oint\left(m\vec{v}_s + \frac{2e}{c}\vec{A}\right)\cdot d\vec{s}, = \text{an integer}$$

an integer. Each integer n corresponds to a different superconducting state characterized by a particular pairing arrangement and a different transition temperature. The transition temperature is found to vary as

$$\Delta T_c = \frac{\hbar^2}{16m^*R_0^2}\left(\frac{2e}{hc}\phi + n\right)^2.$$

The choice of n which gives the tightest binding and the highest transition temperature switches from 0 to -1, -1 to -2, etc., when ϕ is given by $\frac{1}{2}(hc/2e)$; $\frac{3}{2}(hc/2e)$, etc. We note also that the binding energy of the pair is a minimum at these points and varies periodically with the flux. At the transition temperature the penetration depth becomes infinite and consequently the flux ϕ, enclosed by the cylinder, is determined entirely by the external field. T_c is given by a periodic array of parabolas, each of which is centered on a flux unit (see Fig. 1). One can estimate the expected magnitude of ΔT_c by taking $m^* = m_e$ and a reasonable diameter of say 1 micron for the cylinder. ΔT_c is then approximately 5×10^{-5} K° which is of measurable magnitude in the liquid helium temperature range.

We have observed such an effect with a thin

FIG. 2. Lower trace: variation of resistance of tin cylinder at its superconducting transition temperature as a function of magnetic field. Upper trace: magnetic field sweep.

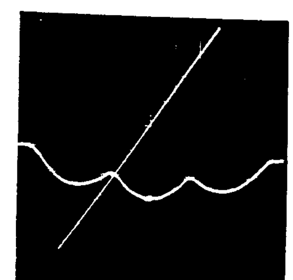

FIG. 3. Enlarged view of parabolic variation of the resistance of tin cylinder for pairs in quantum states -1, 0, and +1. Straight line is magnetic field variation with zero field at the center of the picture.

SOVIET PHYSICS JETP VOLUME 5, NUMBER 6 DECEMBER 15, 1957

On the Magnetic Properties of Superconductors of the Second Group

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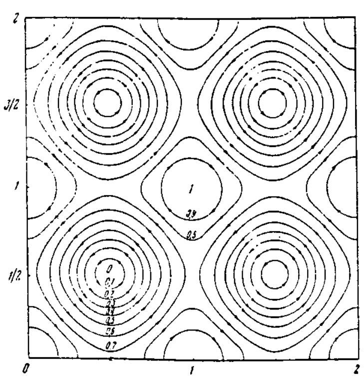
(Submitted to JETP editor November 15, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1442-1452 (June, 1957)

A study is made of the magnetic properties of bulk superconductors for which the parameter $\varkappa$ of the Ginzburg-Landau theory is greater than $1/\sqrt{2}$ (superconductors of the second group). The results explain some of the experimental data on the behavior of superconductive alloys in a magnetic field.



Techniques for "seeing" individual vortices

Bitter decoration
U. Essman and H. Trauble, 1967

* **SSM**: scanning SQUID microscopy
F. P. Rogers, 1983

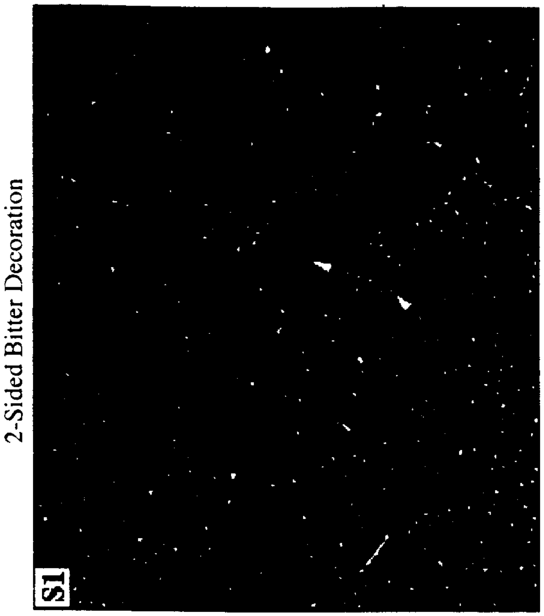
STM: scanning tunneling microscopy
Harald Hess, Bell Labs 1991

* **SHM**: scanning Hall probe microscopy
H. Hess, A. Chang, and coworkers, 1992

Electron holography
A. Tonomura and coworkers, 1993

MFM: magnetic force microscopy
H.-J. Guntherodt and coworkers, 1994

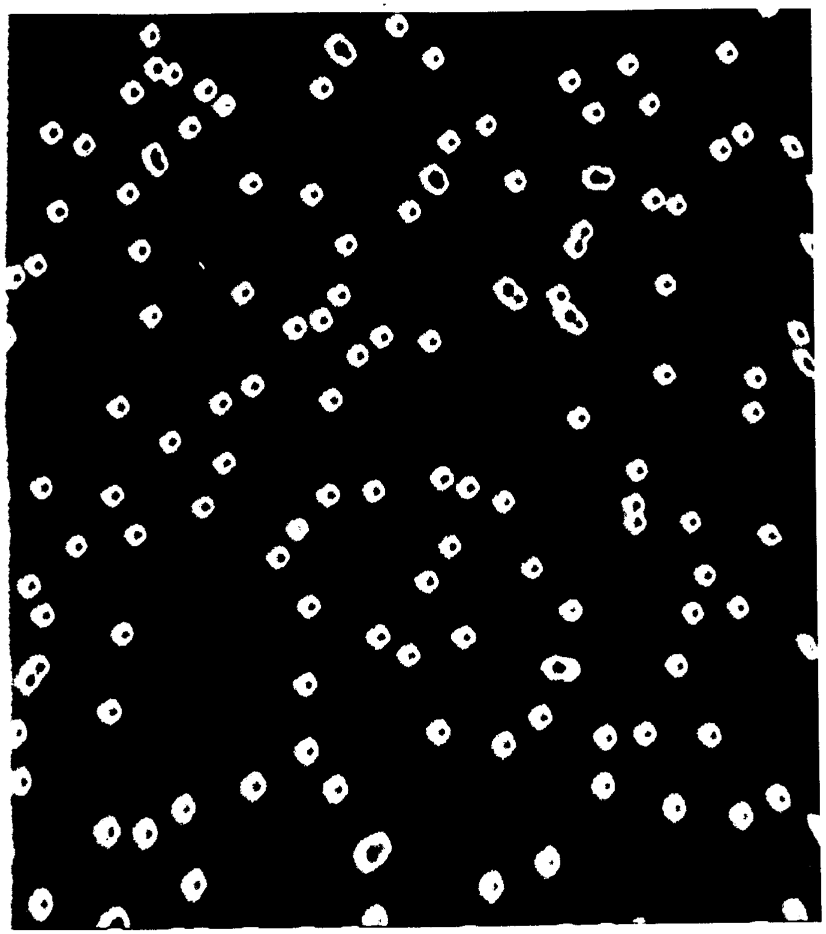
2-Sided Bitter Decoration



Yao, Yoon, Dai, Fan, & Lieber 1994

Scanning SQUID Microscopy
· quantitative measure of flux
· high sensitivity to flux
· relatively poor spatial resolution
· limited field & temperature range

25



512µm x 512µm

26

Solid State Communications, Vol. 25, pp. 1053-1057, 1978. Pergamon Press. Printed in Great Britain

ON POSSIBILITY OF THE SPONTANEOUS MAGNETIC FLUX IN A JOSEPHSON JUNCTION CONTAINING MAGNETIC IMPURITIES

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(Received 15 January 1978 by V.M.Agranovich)

The Josephson junction containing localized magnetic moments in a dielectric layer between two superconductors is considered. Conditions are studied under which the phase difference between superconductors in the state with energy minimum is equal to π (such a junction we call π-junction). In addition we consider "one-dimensional" Josephson junction one part (2) of which is π-junction, the other (1) being the usual Josephson junction (0-junction). Conditions are found under which in such a system there is a spontaneous vortex with the centre at the boundary between the parts 1 and 2 and magnetic flux associated with this vortex. The vortex appears by second order phase transition as temperature decreases from T_c.

1. Conditions for the π-junction Realization.

Let us consider first a Josephson junction with magnetic impurities within the dielectric layer between two superconductors A and B [1]. The tunnel Hamiltonian for such a junction can be written as follows [2-4]:

$$H = \sum_{k,k',s,s'} \Big[t_{kk'} \delta_{ss'} + \sum_n \big(v_{kk'n} \vec{S}_n \vec{\sigma}_{ss'} + t_{kk'n} S \delta_{ss'} \big) \Big] a^+_{ks} b_{k's'} + h.c., \tag{1}$$

where a^+_{ks} (b^+_{ks}) is a creation operator for the conductivity electron with the wave vector k and spin s of the superconductor A (B), $\vec{S}_n$ is the operator of the localized spin in the dielectric layer in site $\vec{n}$ and $\vec{\sigma}$ are the Pauli matrices.

According to [3,4] the matrix elements $v_{kk'n}$ and $t_{kk'n}$ take into account the electron tunneling from superconductor B to superconductor A via magnetic impurities:

$$v_{kk'n} = \frac{U}{\varepsilon_d (U - \varepsilon_d)} t_{Akn} t^*_{Bk'n},$$

$$t_{kk'n} = \frac{U - 2\varepsilon_d}{\varepsilon_d (U - \varepsilon_d)} t_{Akn} t^*_{Bk'n}, \tag{2}$$

where t_{Akn} (t_{Bkn}) is the matrix element for electron tunneling from the localized state in site $\vec{n}$ to the superconductor A (B), ε_d is the energy increase of the system when one electron has tunneled from a localized state to a superconductor and $U - \varepsilon_d$ is the energy increase when one electron has tunneled from superconductor to a localized state (the energy U is dependent on the Coulomb and exchange interaction of electrons in localized state, the exchange interaction being absent if $S = 1/2$). The matrix elements $t_{kk'}$, include all other types of tunneling from superconductor B to A except for tunneling via magnetic impurities.

With the tunnel Hamiltonian (1) we obtain expressions for the stationary Josephson current density j and Josephson energy density E of the junction [2]:

$$\boxed{j = j_c \sin\varphi, \quad E = -\frac{\hbar}{2e} j_c \cos\varphi,}$$

$$j_c = -\frac{8e}{\hbar^2} \operatorname{Im} \int_0^\infty dt \Big[\sum_{k,k'} |t_{kk'}|^2 + \sum_{m,n} \big(S^2 t_{kk'm} t^*_{kk'n} - v_{kk'm} v^*_{kk'n} \langle \vec{S}_m(0) \vec{S}_n(t) \rangle \big) \Big] F(k,t) F(k',t), \tag{3}$$

where φ is the phase jump in the Josephson junction, $F(k,t)$ is the anomalous Green's function for superconductors A and B, the angular brackets mean quantum and thermodynamic average over the spin system.

For calculating j_c we assume firstly that direct tunneling from one superconductor to the other one as well as the tunneling via magnetic impurities are almost diffusive. Hence in the sum over magnetic sites m, n we omit all the terms with $m \neq n$. Owing to diffusive nature of tunneling the dependence of the matrix elements $t_{kk'}$, $v_{kk'n}$ and

$t_{kk'n}$ on the wave vector difference $\vec{k}-\vec{k}'$ is very week and we substitute quantities $|t_{kk'}|^2$, $|V_{kk'n}|^2$ and $|t_{kk'n}|^2$ by their mean values at the Fermi surface t^2, v^2 and t'^2 respectively.

Secondly, all the localized spins $\vec{S}_n$ are considered as free. In order to this condition be fulfilled the concentration c of magnetic impurities in the dielectric layer between superconductors should be small enough, $c \ll 1$ (here $c = N_{imp}/N_{at}$, where N_{imp} is the total number of magnetic impurities in dielectric layer between superconductors, N_{at} is the total number of metal atoms at the surface of the superconductor inside the junction). More precisely we assume that c is so small that spin ordering temperature is lower than the temperature T under consideration.

The spin correlation function $\langle \vec{S}_n(0)\vec{S}_n(t)\rangle$ vanishes at $t \to \infty$ due to spin-lattice and spin-spin interaction. We assume at third that the corresponding relaxation time is much greater than the time of Cooper pairs tunneling through the junction of order $\hbar/\Delta(0)$, where $2\Delta(0)$ is the energy gap of the superconductors at $T=0$. This condition may be easily fulfilled if $c \ll 1$. So we take $\langle \vec{S}_n(0)\vec{S}_n(t)\rangle = \vec{S}^2(0) = S(S+1)$ and j_c in (3) may be written as

$$j_c = \frac{2\pi^2 e}{\hbar}\Big[t^2 + ct'^2 - c\,S(S+1)\,v^2 \Big] N^2(0)\Delta(T)\,\mathrm{th}\frac{\Delta(T)}{2T}, \qquad (4)$$

where $N(0)$ is the density of states near Fermi level.

Now we note that the value j_c can be made negative by taking the energy ε_d sufficiently small. The quantities $|t_{kk'}|$ and $|t_{kkn}\, t^{0}_{kk'n}|$ decrease exponentially as functions of junction thickness d: $\sim \exp(-\varkappa_1 d)$ and $\exp(-\varkappa_2 d)$ respectively, the exponents $\varkappa_1$ and $\varkappa_2$ being approximately equal each other. So we estimate $|t_{kkn}\, t^{0}_{kk'n}| \approx \varepsilon_0 |t_{kk'}|$, where ε_0 is the order of atomic energy (several eV). Thus for $\varepsilon_d \ll U$ we obtain $v^2 \approx t'^2$ and hence $t^2/cv^2 \approx \varepsilon_d^2/c\,\varepsilon_0^2$. So in order to obtain $j_c<0$ the condition $\varepsilon_d^2/c\,\varepsilon_0^2 \ll 1$ must be performed. In addition, as mentioned above, one must have the concentration $c \ll 1$. It seems to us that these two conditions can be fulfilled experimentally. For example, the choice $\varepsilon_0 \approx 1$ eV, $\varepsilon_d \approx 0.1$ eV, $c \approx 0.05$ may be appropriate.

Neglecting the term t^2 in (4) we obtain finally

$$j_c = -\frac{2\pi^2 e}{\hbar}\, c\,S\,v^2 N^2(0)\Delta(T)\,\mathrm{th}\frac{\Delta(T)}{2T}. \qquad (5)$$

We emphasize once more that the negative sign of j_c results from conditions $\varepsilon_d^2/c\,\varepsilon_0^2 \ll 1$, $\varepsilon_d \ll U$, $c \ll 1$ and nonzero difference between $\vec{S}^2 = S(S+1)$ and S^2.

In the case $j_c<0$ the Josephson energy as well as current have opposite signs compared to the ones for the usual junction (see (4) with $c=0$). The ground state of the junction with $j_c<0$ corresponds to the phase jump $\varphi=\pi$ and we call it π-junction, the usual junction being 0-junction.

2. Spontaneous Vortex in the Infinite Josephson Junction.

Previously [1] we have considered the bulk superconductor completed in a ring by the Josephson π-junction. We have shown that if the ring is large enough the system can possess spontaneous electric current and magnetic flux.

In the present work we deal with the Josephson junction one part (1) of which is 0-junction the other part (2) being π-junction (see Fig. 1). Let us show at first that if the width L_1 of the part 1 as well as the width L_2 of the part 2 satisfy the conditions $L_1 \gg \lambda_1$, $L_2 \gg \lambda_2$ (λ_1 and λ_2 are the Josephson lengths of the parts 1 and 2 respectively) the spontaneous vortex appears at the boundary between parts 1 and 2 and possesses the magnetic flux equal to $\Phi_0/2$, where $\Phi_0 = \pi c\hbar/e$ is the magnetic flux quantum.

Indeed, in the absence of the external magnetic field all the currents in such a system vanish far from the boundary. We draw round the boundary the closed contour taken within superconductors forming the junction and so to cross the parts 1 and 2 far enough from the boundary (in distances much greater than the lengths λ_1 and λ_2). The variations of phase φ along the contour under consideration can be written as

$$\hbar\nabla\varphi - \frac{2e}{c}\vec{A} = j/\beta = 0$$

in bulk superconductors,

$$\Delta\varphi_1 - \frac{2e}{c}\int_1 \vec{A}\,d\vec{l} = \arcsin(j_1/\beta_1) = 0$$

in junction part 1,

$$\Delta\varphi_2 - \frac{2e}{c}\int_2 \vec{A}\,d\vec{l} = \arcsin(j_2/\beta_2) = 0$$

in junction part 2, where $\nabla\varphi$ is the phase gradient, $\Delta\varphi_{1,2}$ are the phase jumps in junction, j, $j_{1,2}$ are the current densities, $\vec{A}$ is the vector potential of magnetic field, the coefficients β, $\beta_{1,2}$ are proportional to the density of superconducting electrons. Summing the phase changes along the contour and equating this sum to $2\pi n$ (n is an integer) we find the magnetic flux through the contour $\Phi = (2n+1)\Phi_0/2$, the state

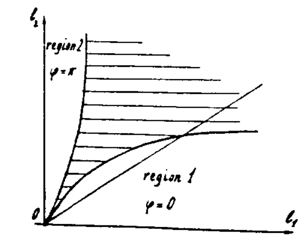

Fig.2. Phase diagram of finite Josephson junction. The shaded region is that for vortex states. The regions 0 and π correspond to homogeneous states without vortex. The straight lines represent the ways of variation of parameters ℓ_1, ℓ_2 as the temperature decrease from T_c for different fixed values of the junction widths L_1 and L_2.

and 2 the homogeneous states of the junction are realized.

For the magnetic flux in the vortex states near the lower phase transition line we obtain at fixed ℓ_1 and small difference $\ell_2 - \ell_{2c}$ where $\ell_{2c} = \mathrm{arctg}(A^{-1}\,\mathrm{th}\,\ell_1)$:

$$\Phi(\ell_1,\ell_2) = \frac{\varphi_2-\varphi_1}{2\pi}\Phi_0 = \pm\frac{2\Phi_0}{\pi D}(1-\alpha)\sqrt{\ell_2-\ell_{2c}}, \qquad (12)$$

$$D^2 = \ell_{2c} - A^{-1}\alpha^4\ell_1 + (\sin 2\ell_{2c})(1-\alpha^2)/2,$$

$$\alpha = \cos\ell_{2c}/\mathrm{ch}\,\ell_1.$$

Near the apper phase transition curve we find the expression for $\Phi(\ell_1,\ell_2)$ at fixed ℓ_2 and small difference $\ell_1 - \ell_{1c} = \ell_1 - \mathrm{arctg}(A\,\mathrm{th}\,\ell_2)$ which may be obtained from (12) by exchanging $\ell_2 \to \ell_1$, $\ell_1 \to \ell_2$ and $A \to A^{-1}$.

The square-root dependence of Φ on variable $\ell_2 - \ell_{2c}$ is typical for the second order phase transition. The magnitude of the spontaneous magnetic flux can be considered in this case as the order parameter of the phase transition.

In the case which seems most plausible experimentally $\ell_1 \gg 1$, $A \ll 1$ (the limit of such a case is the π-junction shorted by superconductor) we obtain $\Phi = \pm 2\pi^{-1}\Phi_0\sqrt{(\ell-\ell_{2c})/\ell_{2c}}$ where $\ell_{2c} = \pi/2 - A$.

If the junction widths L_1 and L_2 are fixed the parameters ℓ_1 and ℓ_2 will vary with temperature and near the superconductor transition temperature T_c one has $\ell_1, \ell_2 \sim (T_c - T)^{1/4}$. As temperature decreases from T_c the variation of values ℓ_1, ℓ_2 can be presented by movement of point (ℓ_1, ℓ_2) along the straight line from the origin of the coordinates as it is shown in Fig.2 (the temperature dependences of λ_1 and λ_2 are the same). We see from Fig. 2 that as the temperature decreases the junction initially will be at the homogeneous state, then at some temperature $T_0 < T_c$ the magnetic flux spontaneously will appear if the value of L_1 or L_2 is large enough. For $T_0 - T \ll T_0$ the flux $\Phi \sim \sqrt{(T_0-T)/T_0}$.

The difference in the free energy between the vortex state near the lower curve in Fig.2 and the homogeneous state $\varphi(x)=0$ is equal to

$$\Delta F = -\frac{\hbar j_{c1}\lambda_1 A}{e}\left(\frac{\ell_2-\ell_{2c}}{D}\right)^2 \qquad (13)$$

From (13) and the expressions for ℓ_1, ℓ_2 near T_0

$$\frac{\ell_1-\ell_{10}}{\ell_{10}} = \frac{\ell_2-\ell_{20}}{\ell_{20}} = \left(\frac{d\ln\lambda_1}{dT}\right)_{T_0}(T_0 - T), \qquad (14)$$

$$\ell_{10} = \ell_1(T_0); \quad \ell_{20} = \ell_2(T_0)$$

we can find the specific heat jump at the phase transition point T_0 for lower curve of Fig.2

$$\Delta C = \frac{2\hbar j_{c1}\lambda_1}{e}\,\frac{A}{D^2}\left(\frac{d\ln\lambda_1}{dT}\right)^2_{T_0}\left(\ell_{20} - \alpha^2 A^{-1}\ell_{10}\right)^2. \qquad (15)$$

In the particular case when $\ell_1 = A\ell_2$ (in Fig.2) this line concerns both phase transition curves at the origin of coordinates) the temperature of the phase transition to the vortex state T_0 coincide with the superconducting critical temperature T_c. Here the magnetic flux $\Phi \sim (T_c - T)$ near T_c.

So we have considered here the possibility of appearance of the spontaneous current and magnetic flux in the 0-π Josephson junction. It was shown that the vortex appear by the second order phase transition. It is an interesting example of the phase transition on the bondary between two parts of complex system.

It should be remarked in conclusion that all these interesting phenomena could be observed if the π-junction could be constructed. Whether it is possible or not remains the main question to be solved experimentally.

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The Josephson effect in superconductors with heavy fermions

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Pis'ma Zh. Eksp. Teor. Fiz. 43, No. 6, 306–309 (25 March 1986)

The dependence of the Josephson current that flows between an ordinary and extraordinary superconductor on the temperature and on the angle between the surface of the contact and the crystal axes is determined.

rotation around which results in the multiplication of the order parameter by the phase factor. The symmetry classes of the order parameter are listed in Ref. 2. A highly simplified angular dependence of the Josephson current which satisfies these requirements is given by

$$\boxed{I(\varphi) = A\,\mathrm{Im} f(\mathbf{n}) e^{i\varphi},}$$

where $f(\mathbf{n})$ for different representations is:

cubic group (UBe_{13})

$$A_1 : n_x n_y n_z (n_x^2 - n_y^2)(n_y^2 - n_z^2)(n_z^2 - n_x^2)\,;\; A_2 : n_x n_y n_z\,;$$

$$E : n_x n_y n_z (\eta_1 (n_x^2 + e^{-2\pi i/3} n_y^2 + e^{2\pi i/3} n_z^2) - \eta_2 (n_x^2 + e^{2\pi i/3} n_y^2 + e^{-2\pi i/3} n_z^2))\,;$$

$$F_2 : \eta_1 n_x (n_y^2 - n_z^2) + \eta_2 n_y (n_z^2 - n_x^2) + \eta_3 n_z (n_x^2 - n_y^2)\,;\; F_1 : \eta_1 n_x + \eta_2 n_y + \eta_3 n_z\,;$$

tetragonal group ($CeCu_2Si_2$)

$$A_1 : n_x n_y n_z (n_x^2 - n_y^2)\,;\; A_2 : n_z\,;$$

$$B_1 : n_x n_y n_z\,;\; B_2 : n_z (n_x^2 - n_y^2)\,;$$

$$E : \eta_1 n_y - \eta_2 n_x\,;$$

hexagonal group (UPt_3)

$$A_1 : n_z (n_x^3 - 3 n_x n_y^2)(n_y^3 - 3 n_y n_x^2)\,;\; A_2 : n_z\,;$$

$$B_1 : n_x^3 - 3 n_x n_y^2\,;\; B_2 : n_y^3 - 3 n_y n_x^2\,;$$

$$E_1 : \eta_1 n_y - \eta_2 n_x\,;$$

$$E_2 : \eta_1 n_z (n_x - i n_y)^2 - \eta_2 n_z (n_x + i n_y)^2.$$

For other than the one-dimensional representations, the possible values of the parameters η_i are given in Ref. 2. For some phases these values are ambiguous. In this case, domain walls can exist in the bulk of the crystal. As to whether these domain walls interact with the surface requires further study. If we are dealing with a magnetic phase and a complex order parameter, we will have $I(\varphi) = I_1 \cos\varphi + I_2 \sin\varphi$, where $I_c^2 = I_1^2 + I_2^2$.

The minimum energy of the contact does not always correspond to the point $\varphi = 0$. If, for example, a plate with an odd phase between two contacts is inserted into a superconducting circuit, the vector $\mathbf{n}$ and hence the energy of these contacts will have opposite signs. If, for example, a plate with an odd phase between two contacts is inserted into a superconducting circuit, the vector $\mathbf{n}$ and hence the energy of these contacts will have opposite signs. If the minimum energy of one contact corresponds to the phase difference $\varphi = 0$, the minimum energy of the other contact will correspond to the phase difference $\varphi = \pi$. This circuit will therefore have a half-integer number of fluxoids. $\Phi_0/2$

The absence of this effect in a UPt_3 single crystal may conceivably stem from such positioning of the contact plane relative to the crystal axes at which the Josephson current vanishes. The presence of a current in the Josephson junction of an ordinary superconductor with $CeCuSi_2$ does not mean that there is an ordinary pairing in this contact

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Vortices with half magnetic flux quanta in "heavy-fermion" superconductors

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It is shown that in "heavy-fermion" superconductors a new vortex state can occur characterized by the existence of half magnetic flux quanta. Vortices in polycrystals should exist even in the absence of an externally applied magnetic field. The internal structure of the vortices is also investigated.

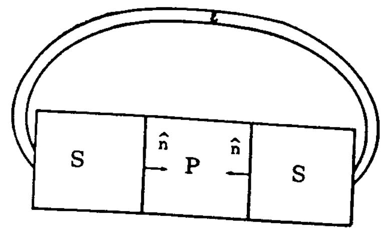


FIG. 1. Sketch of a sandwhich structure *S-P-S* (see text) closed by a superconducting loop.

proposed geometry to observe $\Phi_0/2$:

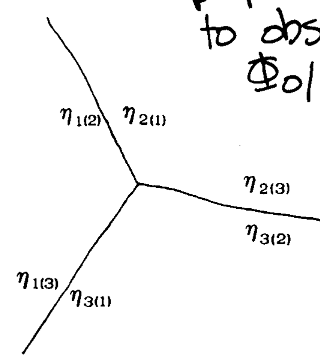
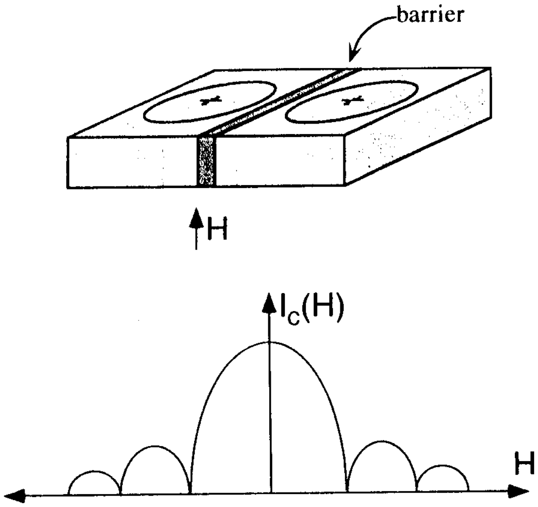
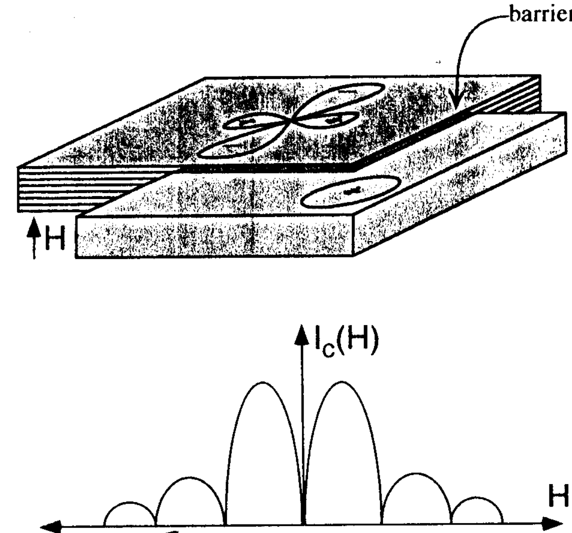


FIG. 2. Values of the order parameters at the grain boundaries.

In-plane phase-sensitive Josephson experiments

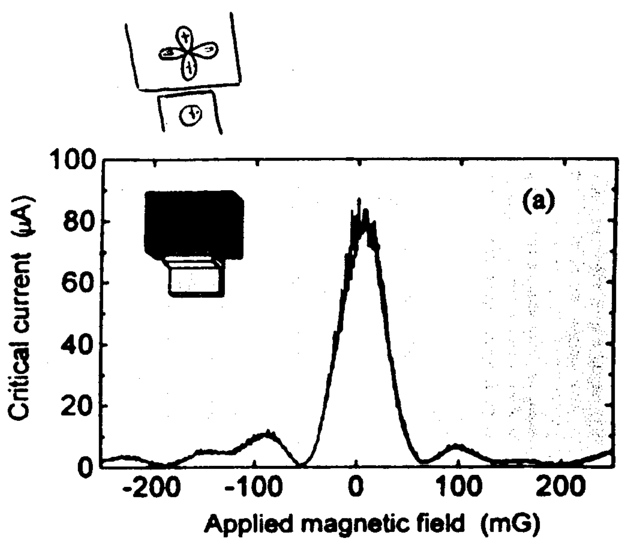


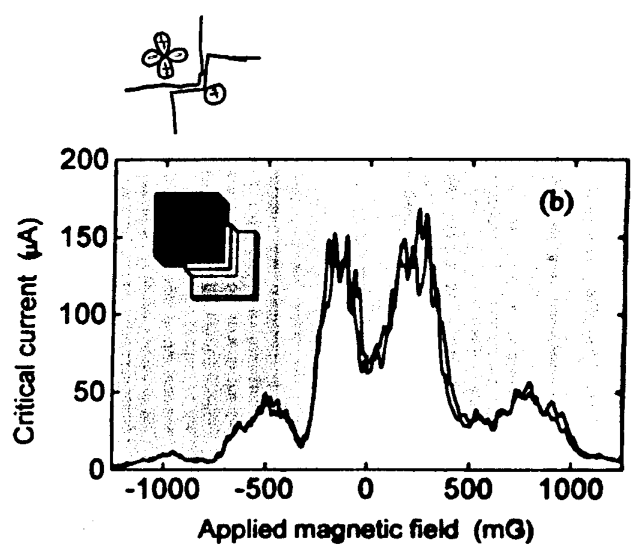


short-junction limit

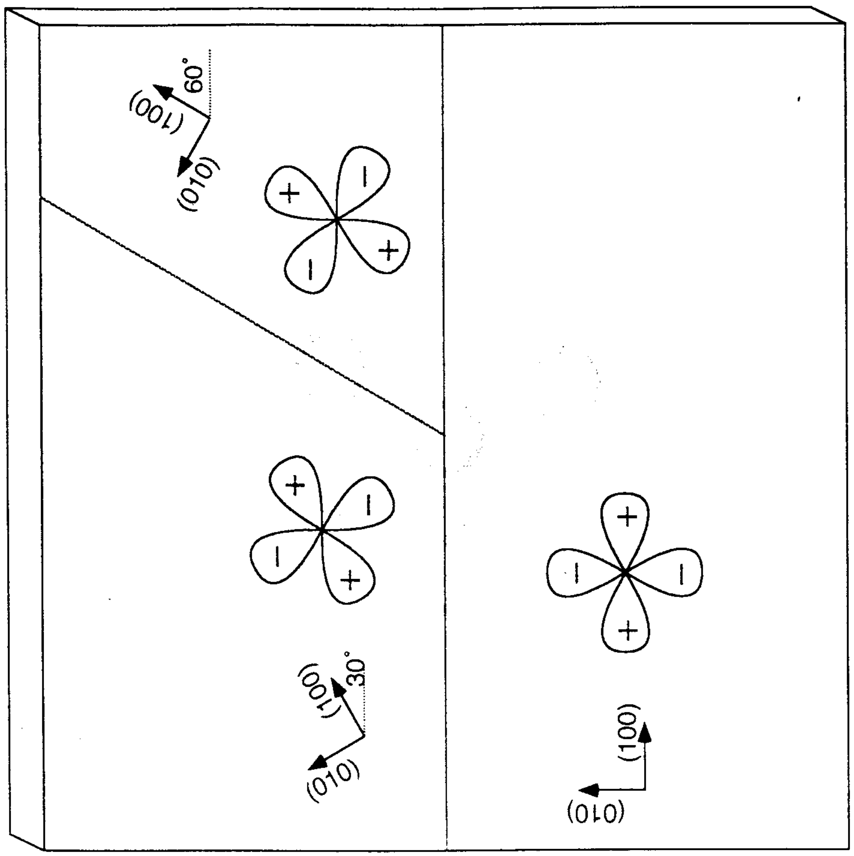
- $I_c(H)$ dramatic for $L \ll \lambda_J$
- spontaneous flux appears for $L \gg \lambda_J$

Edge & corner YBCO/Au/Pb Josephson junctions

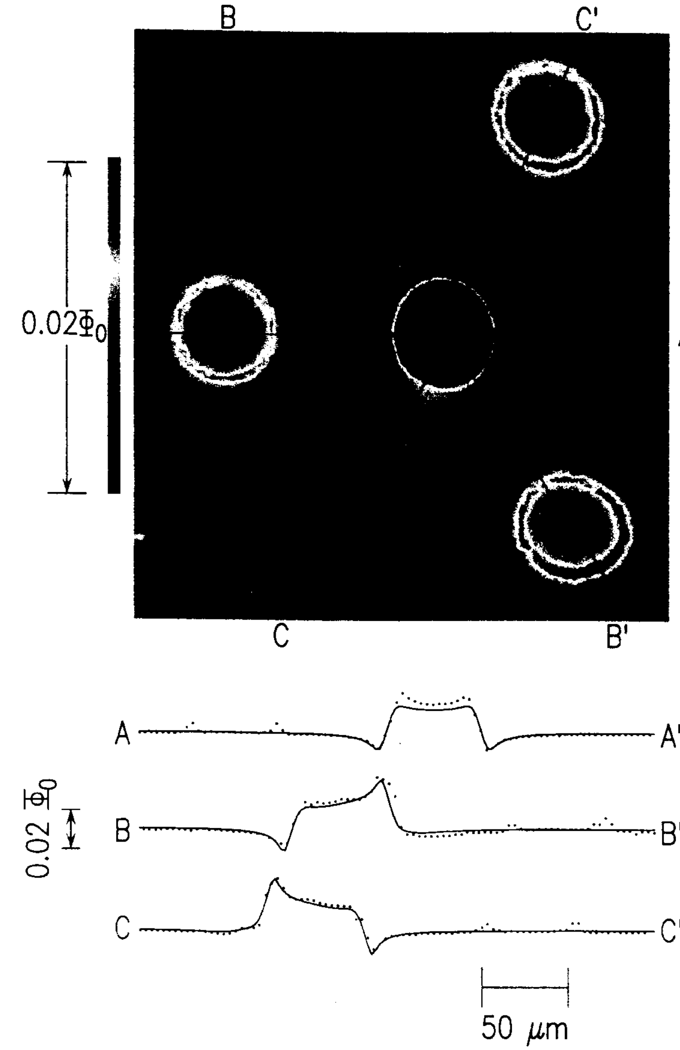




IBM, 1994



$YBa_2Cu_3O_{7-\delta}$ film on $SrTiO_3$ substrate

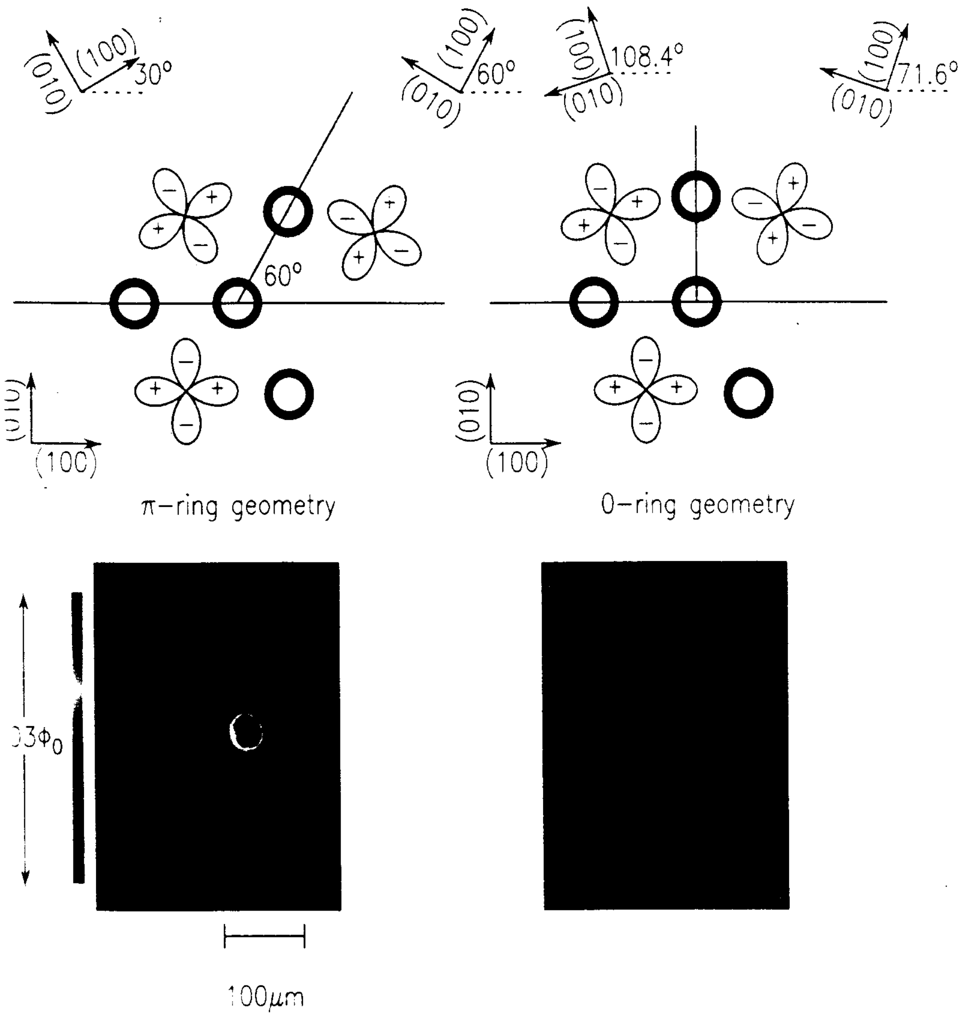


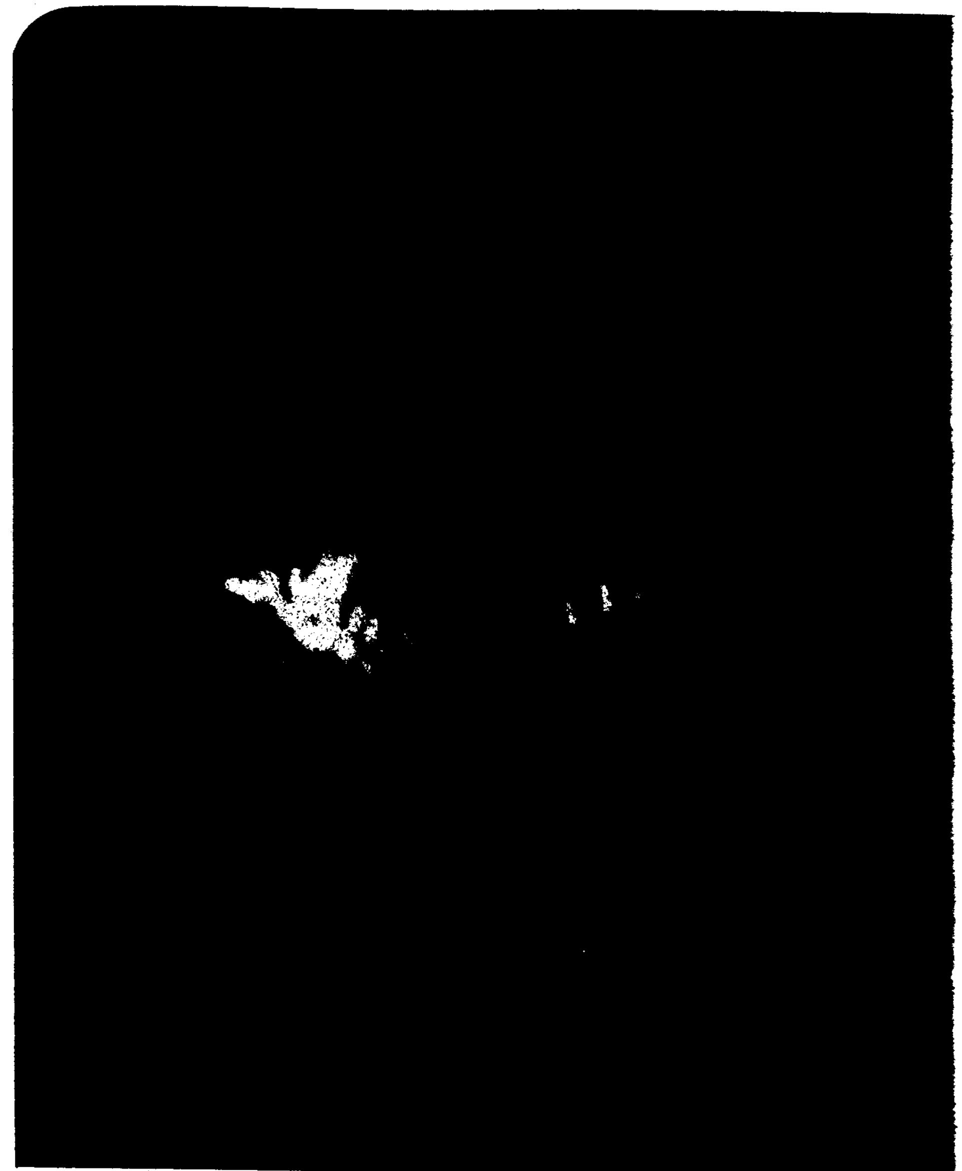
$\frac{\Phi_0}{2}$!

(YBCO)

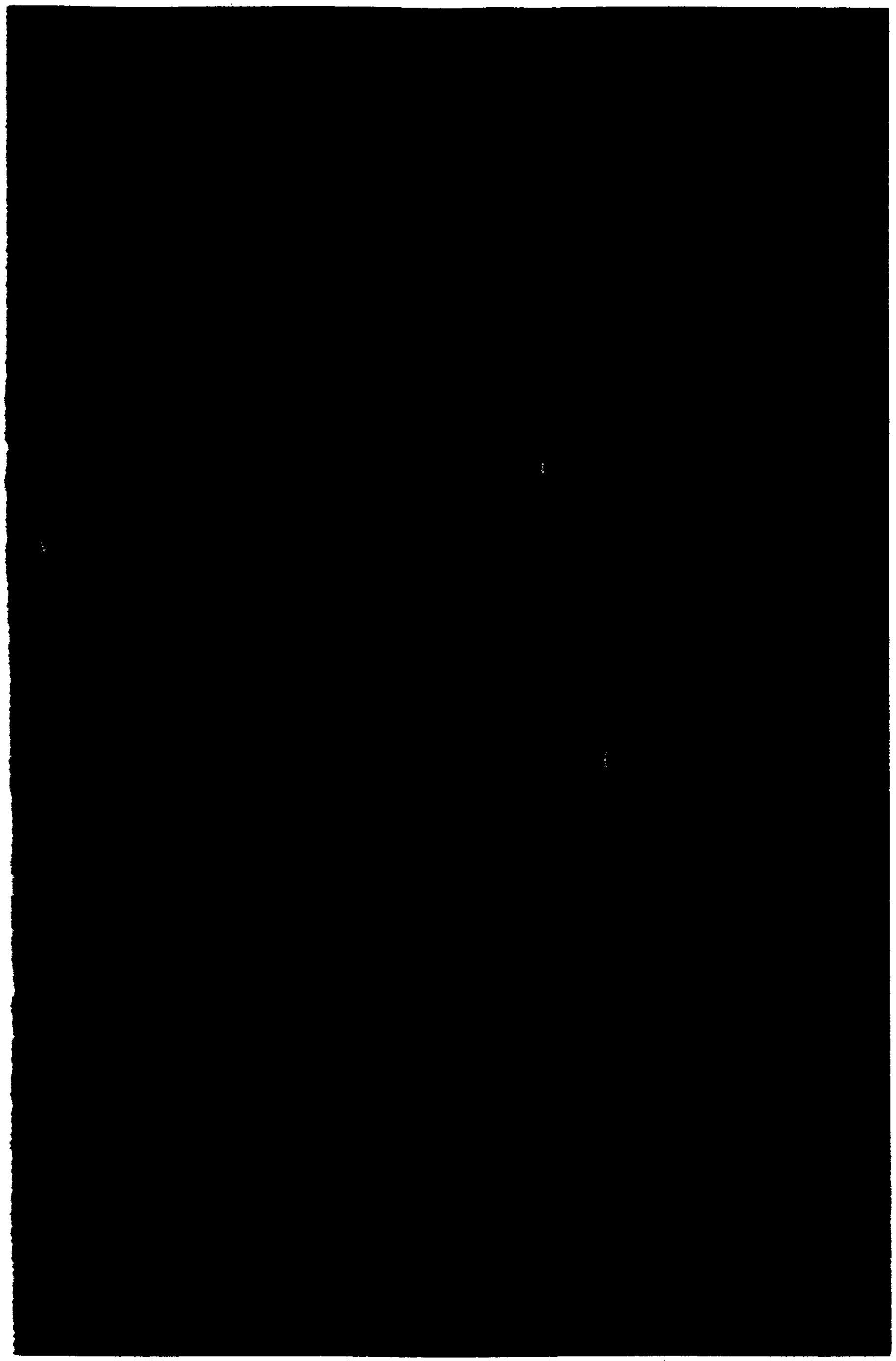
Tsuei, Kirtley, et al 1994

many samples
other materials
several geometries

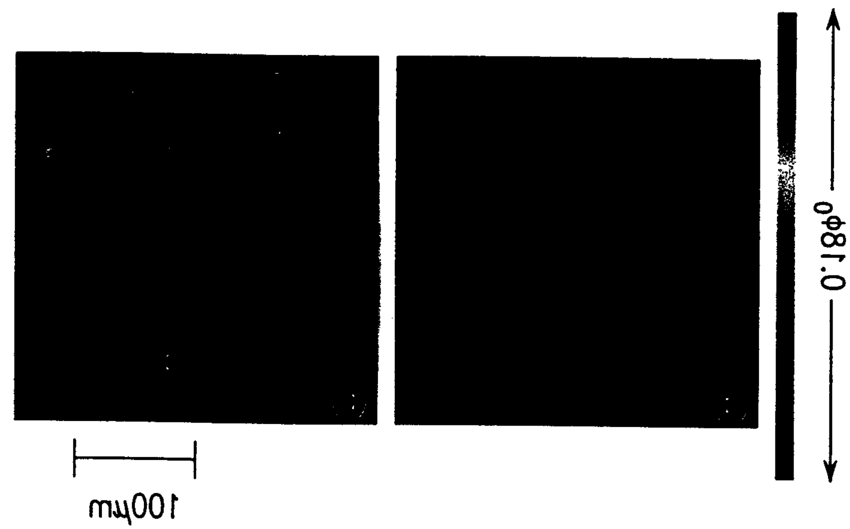




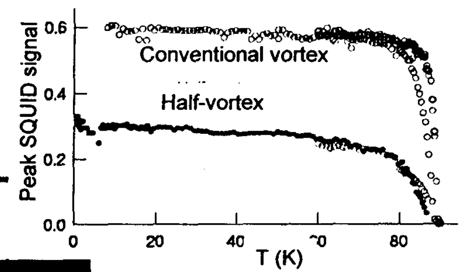
$Tl_2Ba_2CuO_{6+\delta}$
C.C. Tsuei et al. F

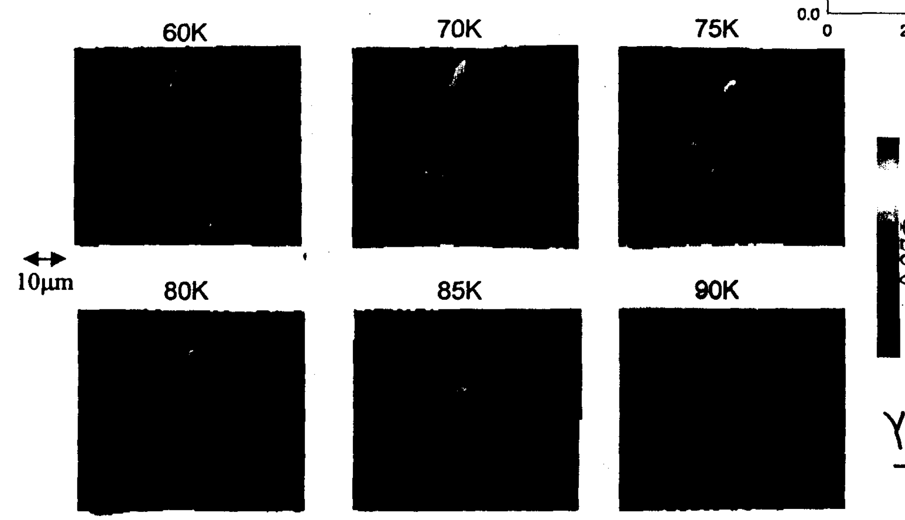


des to keep



Observed T dependence of a half-integral flux quantum





YBCO tricrystal
$T_c = 90K$
$\Delta T_c = 2K$

- Flux spreads out near Tc without any abrupt changes
- $\Phi_0/2$ integrated flux for $0.45K < T < 88K$ within experimental error
- In zero field, the order parameter of optimally doped YBCO is predominantly d-wave over the temperature range from 0.45 K to 88 K in two samples with Tc = 90 K.