SQUIDs And Low Noise Measurement Technique

⇒ How can SQUIDs measure Johnson Noise of a resistor at 100 mK?

Outline

Single Junction Physics

- Josephson Effect
- RSJ model
- I-V

SQUIDs

- Φ measuring device

Noise Theory + Amplifier Matching

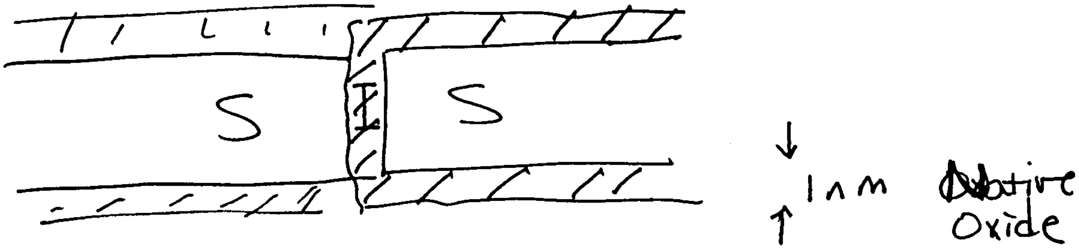
- Spectral density + Johnson noise
- Amplifier model (R_n + T_n)
- Noise Impedance Transformers

SQUID Noise + Matching

- Noise
- Matching
- Output Matching

TES Example

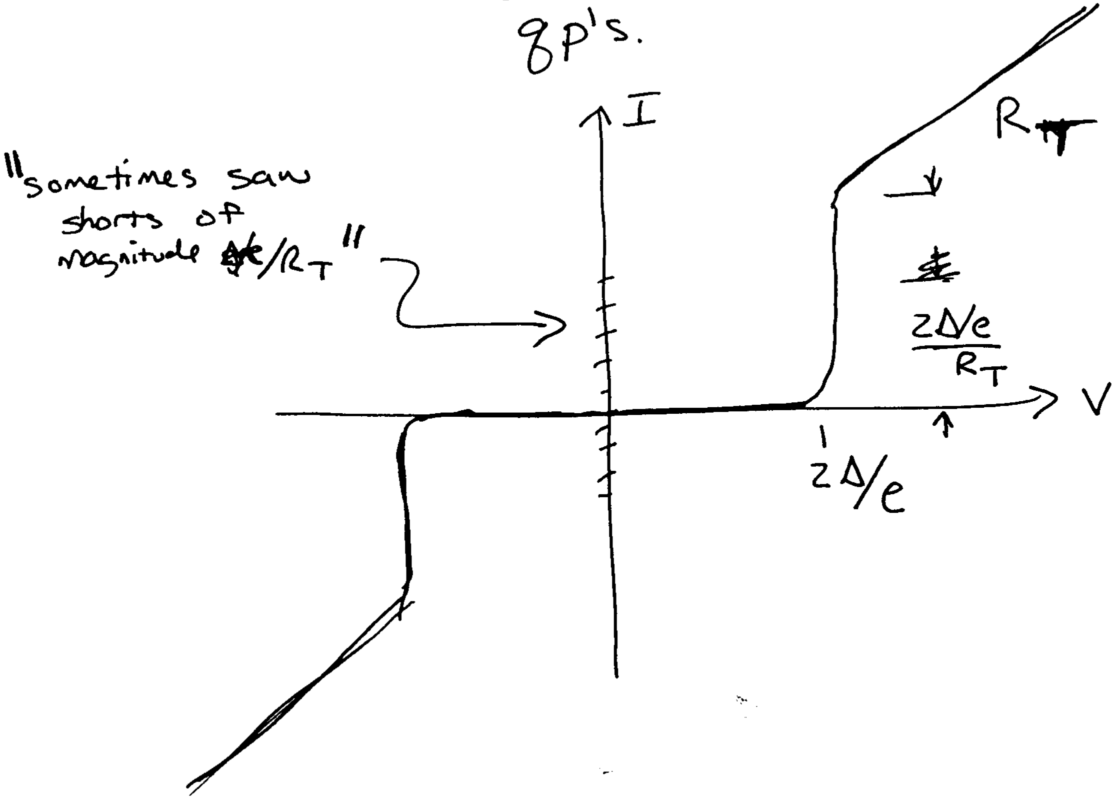
Tunnel Junctions



Tunneling of single electrons (quasiparticles)
I-V determined thr. D.O.S. (Golden Rule calc.)

$$I \sim \left| \langle f | \sum_{kk'} \underbrace{\left(T^* C_L^+ C_R + T C_L C_R^+ \right)}_{H_T} | 0 \rangle \right|^2$$

excited qp's.



Josephson Junction

How does tunneling affect gnd state? (supercurrent)

$$H_2 = -H_T^* \frac{1}{E_i} H_T$$

$$\delta E = -\langle 0| \sum_{k_L k_R} \left(T^* c_L^+ c_R^* + T c_L^* c_R^+\right) \frac{1}{E_i} \sum_{k_L' k_R'} \left(T^* c_L^+ c_R + T^* c_L c_R^+\right)|0$$

$T = 0$

need γ's to do properly ↓

$$\uparrow \quad \langle 0|c_L^+ c_L^+|0\rangle \neq 0\,! \quad \uparrow$$

(k_L matches $-k'$ ⇒ $|T|^2$

$$\simeq -\sum_{kk'} |T|^2 \langle 0| c_L^+ c_L^+ |0\rangle \frac{1}{E_i} \langle 0| c_R^* c_R |0\rangle + \text{c.c.}$$

$$\simeq \int_0^{\Delta} d\epsilon_1 \int_0^{\Delta} d\epsilon_2\, \rho_1 \rho_2 |T|^2 \frac{1}{2\Delta} e^{i\phi_L} e^{-i\phi_R} + \text{c.c.}$$

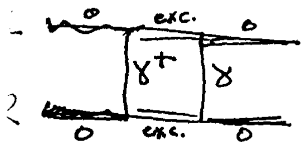
$$\simeq \frac{\left(R_K = \frac{h}{e^2}\right)}{R_T} \frac{\Delta^2}{2\Delta} e^{i(\phi_L - \phi_R)} + \text{c.c.}$$

$$\simeq \frac{h}{2e} \frac{\Delta/e}{R_T} \cos\Big(\phi_L - \phi_R + \underbrace{\overset{\text{from each } c^+ \text{ term}}{\frac{2e}{\hbar}} \int V dt}_{\hookrightarrow \text{ gauge transf.}}\Big)$$

$$I_{\text{Josephson}} = \frac{d(\delta E)/dt}{V} \leftarrow (\text{Power} = IV), \left(\text{can show from operators}\right)$$

$$= \frac{\pi}{2} \frac{\Delta/e}{R_T} \cos\left(\phi_L - \phi_R + \frac{2e}{\hbar}\int V dt\right)$$

↑ when work out correctly with γ^+, γ



1) Virtual tunneling of qp gives ~~current~~ charge transport.
2) Coherent nature of s.c. gnd state add's transport for macro current $I \sim I_{gap}$
3) Related to Real (single) Q.P. tunneling.

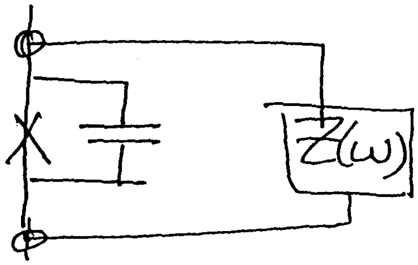
Josephson Equations

Classical Eq. of Motion.

$$I_J = I_0 \cos\phi$$

$$\dot{\phi} = \frac{2eV}{\hbar}$$

classical variable!



Solve via standard circuit element techniques.

ϕ can be q.m!

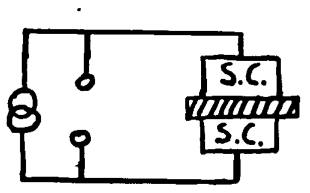
small flucts: Macros quantum tunneling

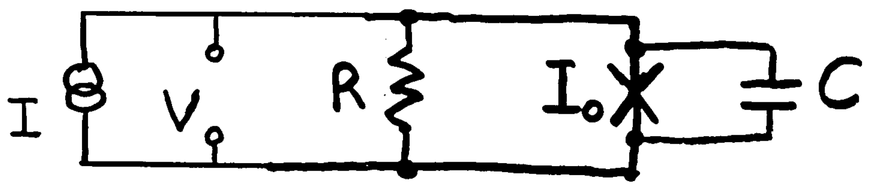
large q-flucts: $\frac{(2e)^2}{2C} \gg I_0 \Phi_0/2\pi$

$$|Z(\omega)| \gg h/e^2$$

CURRENT-VOLTAGE CHARACTERISTIC

(Classical Description of δ)

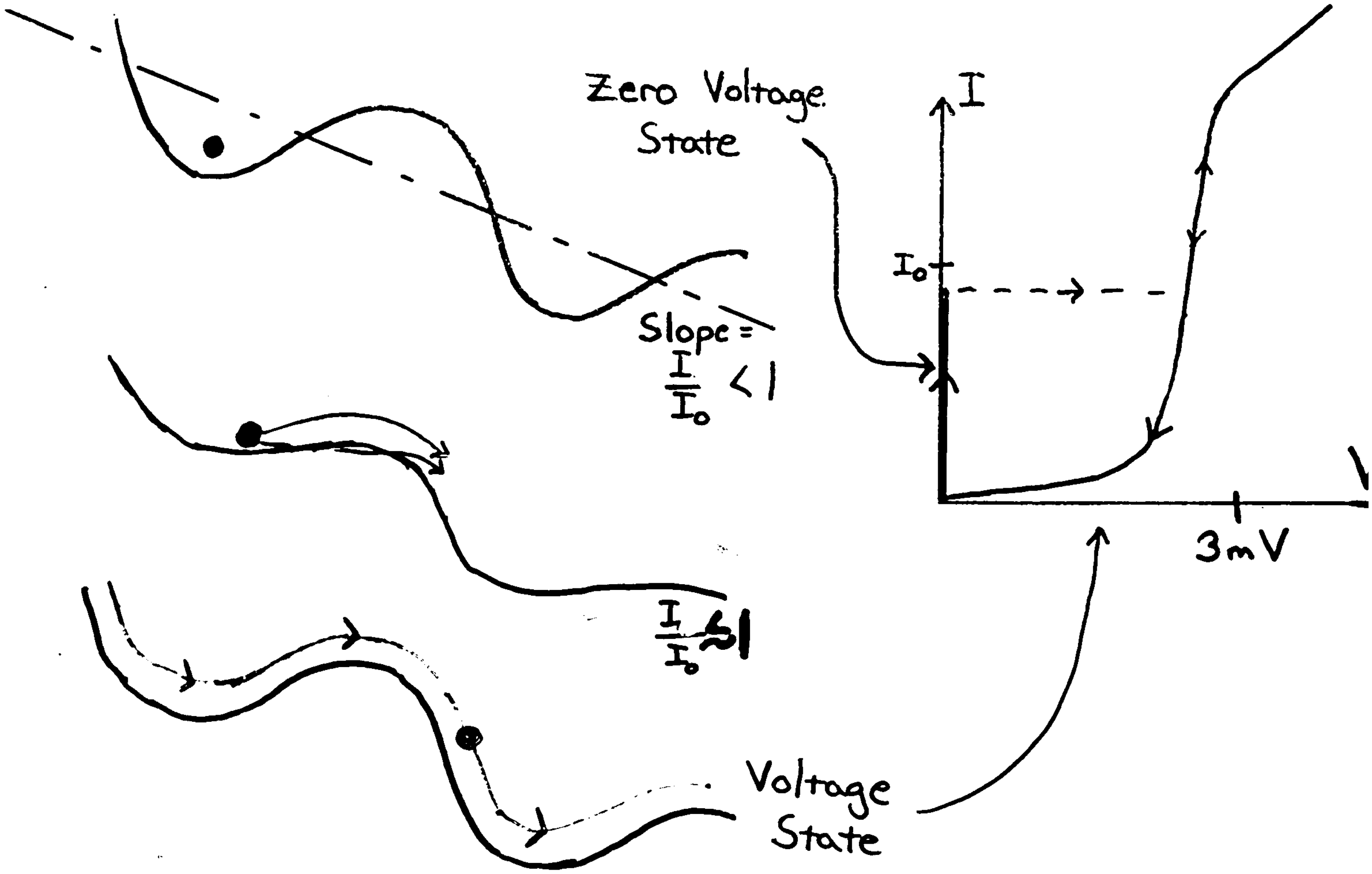




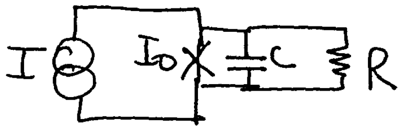
Equations of motion:

$$\begin{cases} I = \dfrac{V}{R} + I_0 \sin\delta + C\dot{V} \\ V = \dfrac{\Phi_0}{2\pi}\dot{\delta} \end{cases}$$

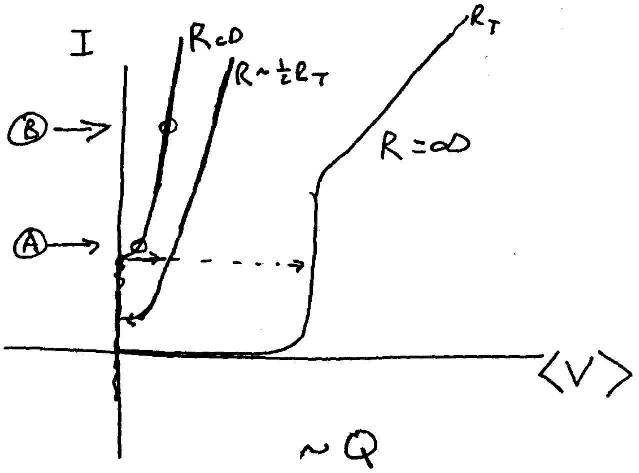
$$\underbrace{\left[C\left(\frac{\Phi_0}{2\pi}\right)^2\right]}_{\text{mass}}\ddot{\delta} + \underbrace{\left[\frac{1}{R}\left(\frac{\Phi_0}{2\pi}\right)^2\right]}_{\text{damping}}\dot{\delta} + \frac{\partial}{\partial\delta}\underbrace{\left[-I_0\frac{\Phi_0}{2\pi}\cos\delta - I\frac{\Phi_0}{2\pi}\delta\right]}_{\text{potential } U(\delta)} = 0$$



RSJ (+C) Model



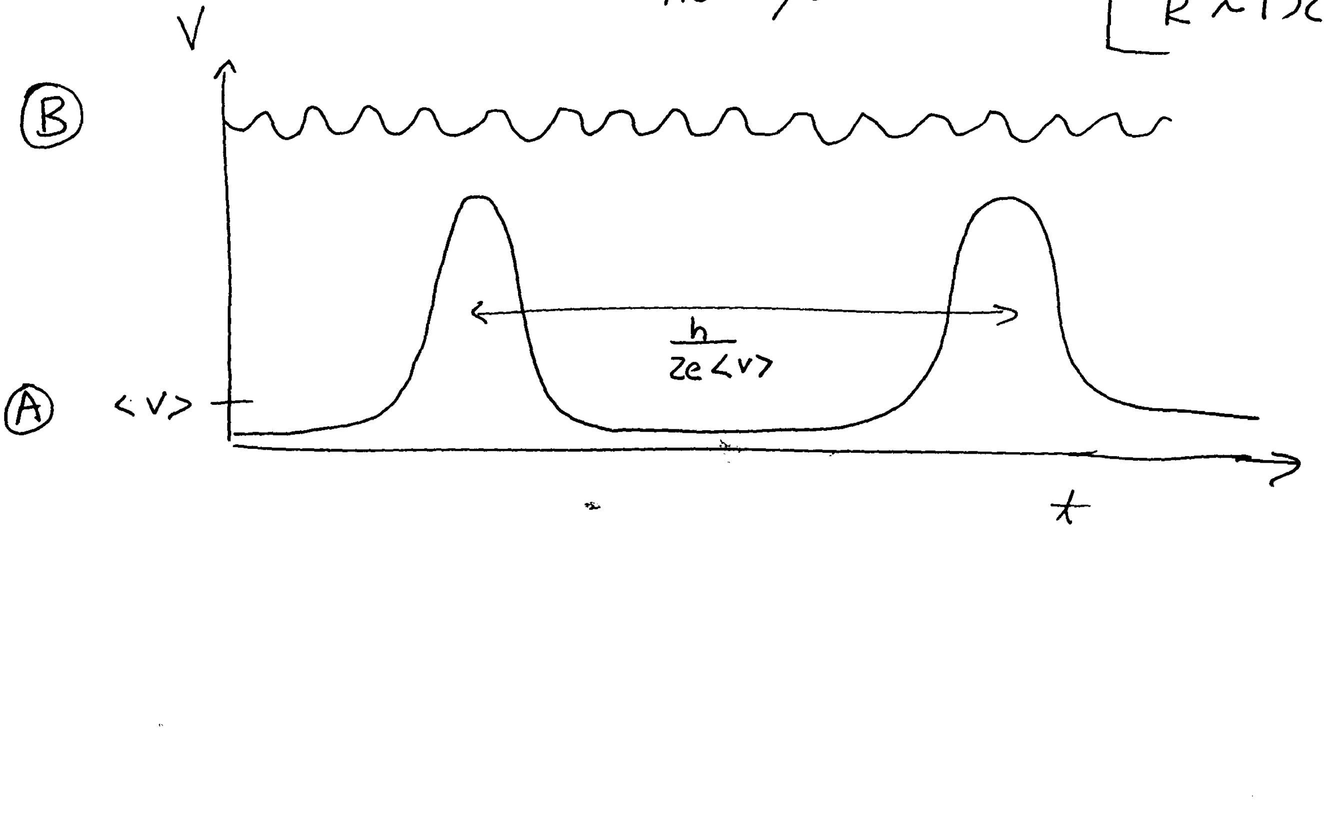
→ Dynamics from "particle in Washboard" (see ⑥)



$$\beta_c = \overset{\sim Q}{I_0 R^2 C \frac{2\pi}{\Phi_0}} = 1$$

critical damping,
no hysterisis

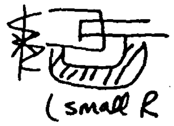
3μm x 3μm
$I_0 \sim 100\mu A$
$C \sim 2pF$
$R \sim 1\Omega$



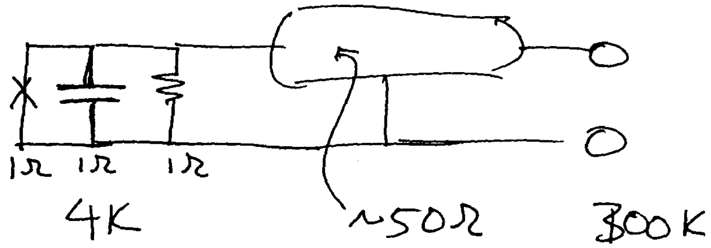
Limitations of RSJ

- Special Case of Real Model $Z(\omega) \to R$
  Can get <u>qual.</u> difference for other models!

- RSJ works well when μfab. resistor



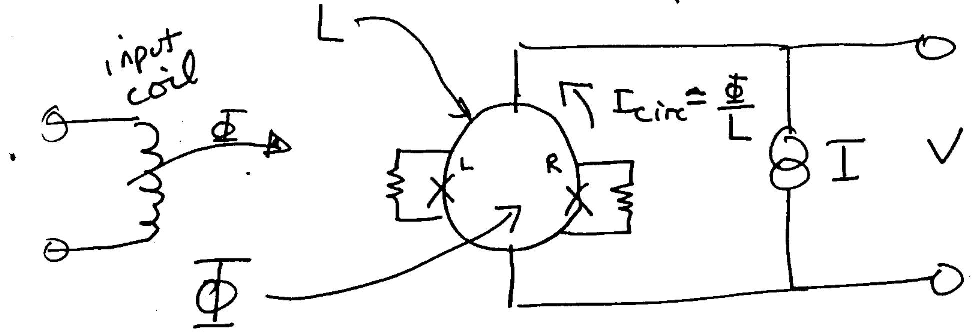
- Ignores wiring impedance
  OK for "normal" "SQUID + JJ" circuits



$$\left|Z_{\substack{\cancel{\text{wire}}\\ \text{junct}}}\right| \;\cancel{\ll}\; \left|Z_{\substack{\cancel{\text{junct}}\\ \text{wire}}}\right|$$

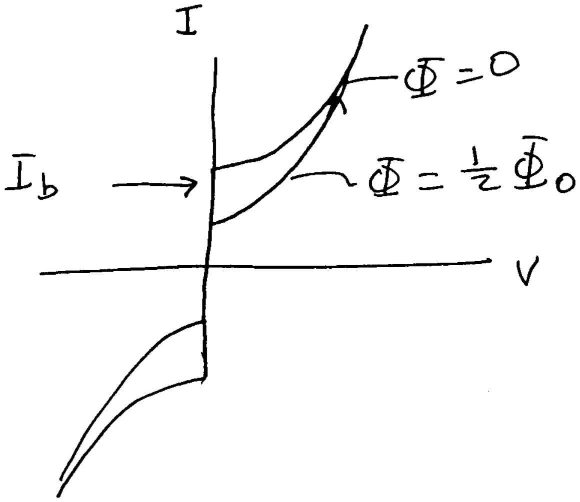
[Q.M. of phase when $Z_{\text{junct}} \gtrsim h/e^2$]

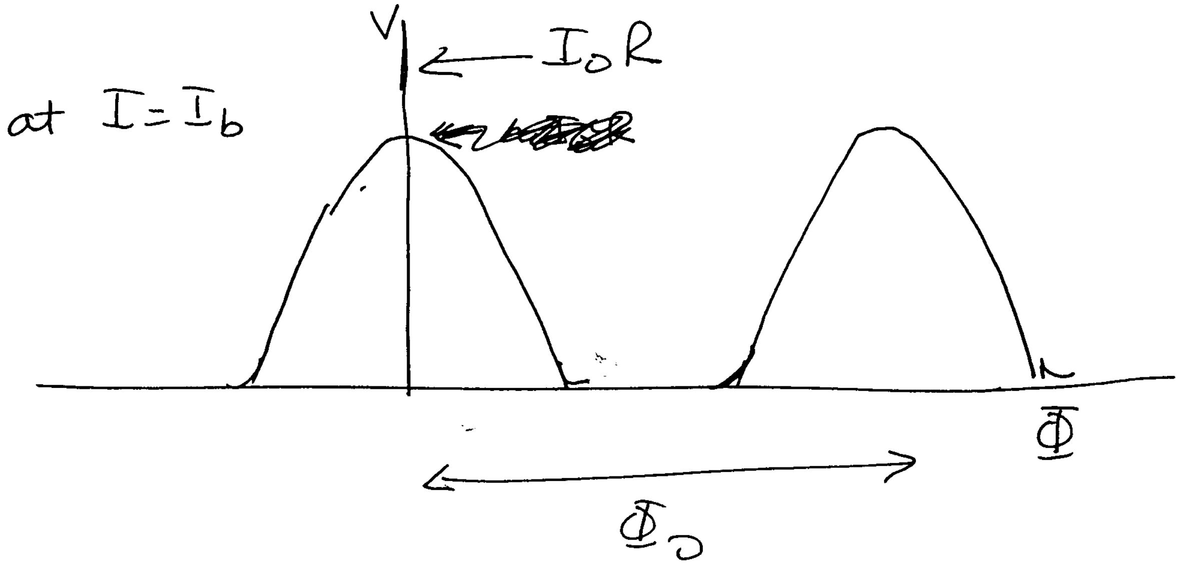
SQUIDS



$$I_{J-L} \sim I + \Phi/L$$
$$I_{J-R} \sim I - \Phi/L$$

← Modulates effective I_0 of SQUID

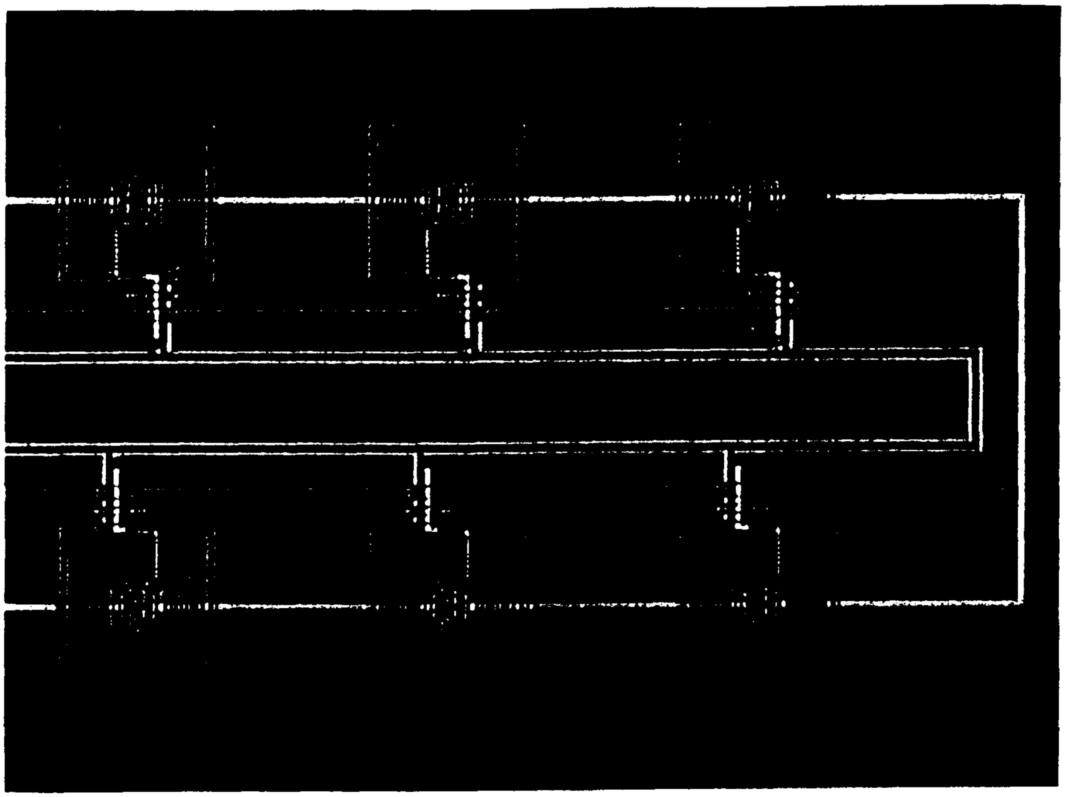


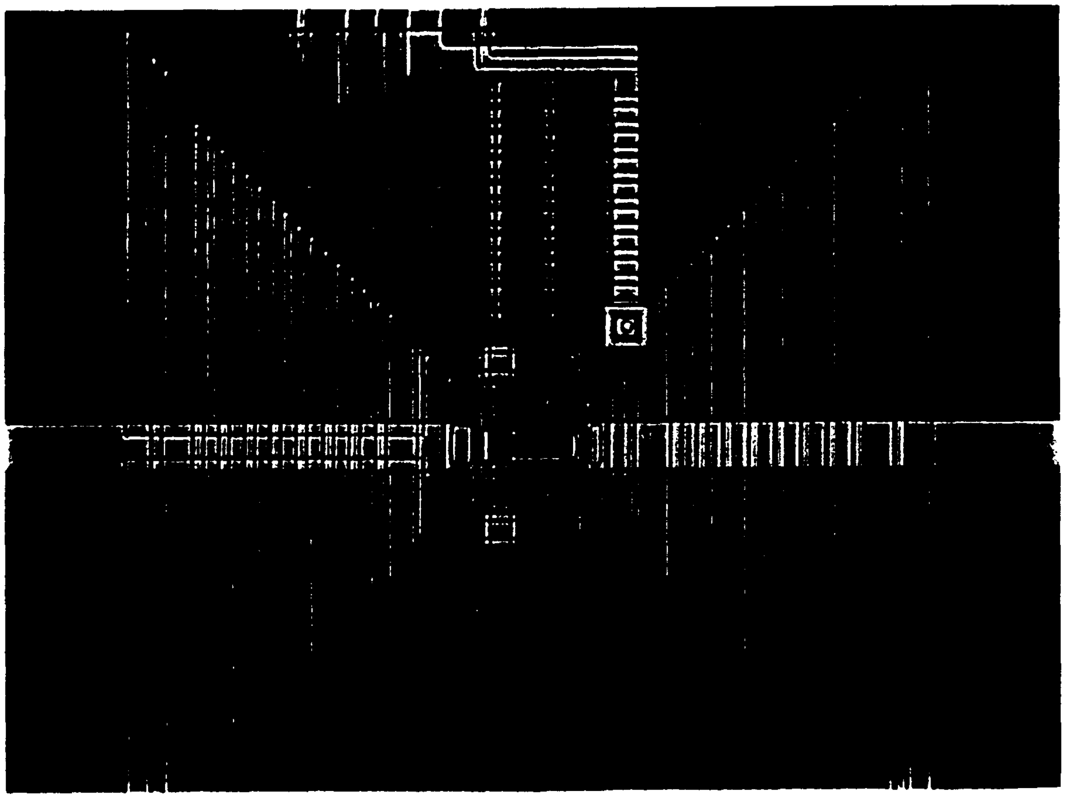


Best SQUID Noise Performance at

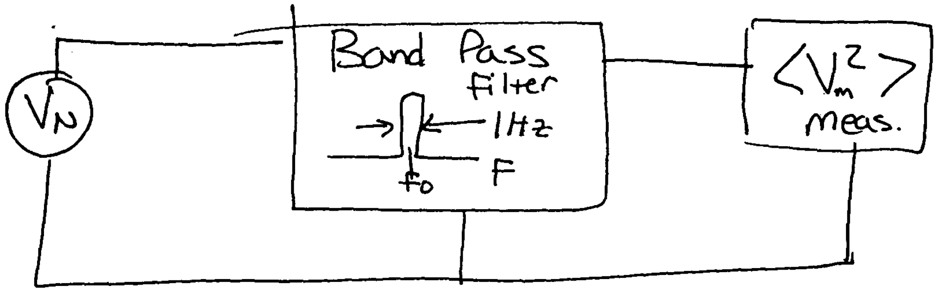
$$\beta = \frac{2\pi L I_0}{\Phi_0} = 1$$

(large I_0 mod, but L not too small)





Measurement of Noise – Spectral Density

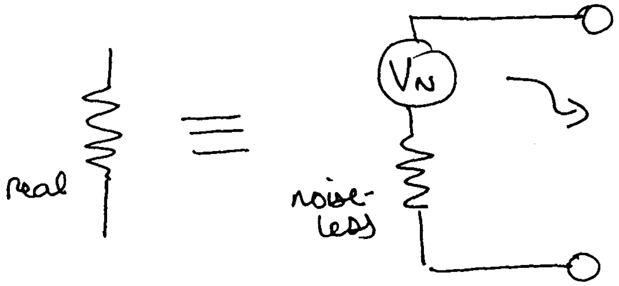


$$S_V(f_0) = \langle V_m^2 \rangle$$ RMS voltage per 1Hz BW at $f = f_0$ $\left[\frac{V^2}{Hz}\right]$

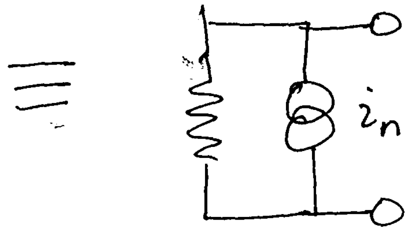
$$\langle V^2 \rangle = \int_{f_1}^{f_2} S_V(f)\,df$$ for arbitrary filter

$$\propto (f_2 - f_1)$$ for white noise

Resistor

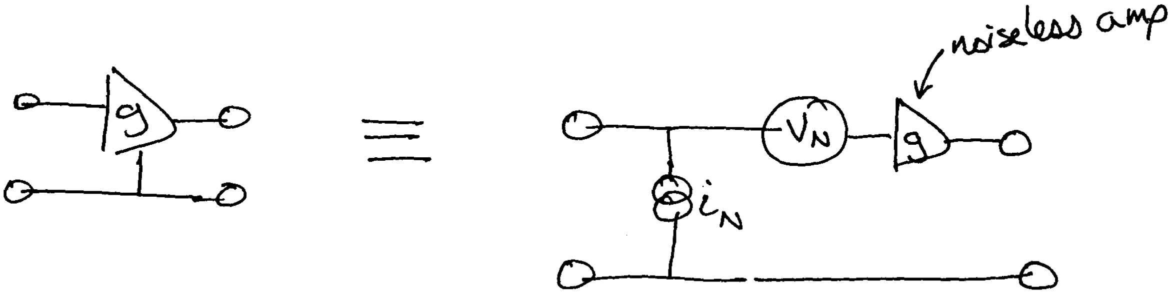


$\rightarrow S_V(f) = 4kTR$



$$S_I(f) = \frac{4kT}{R}$$

Voltage Amplifier Noise



2 ports (in + out)
⇒ noise associated with each port

S_{V_N}

S_{I_N}

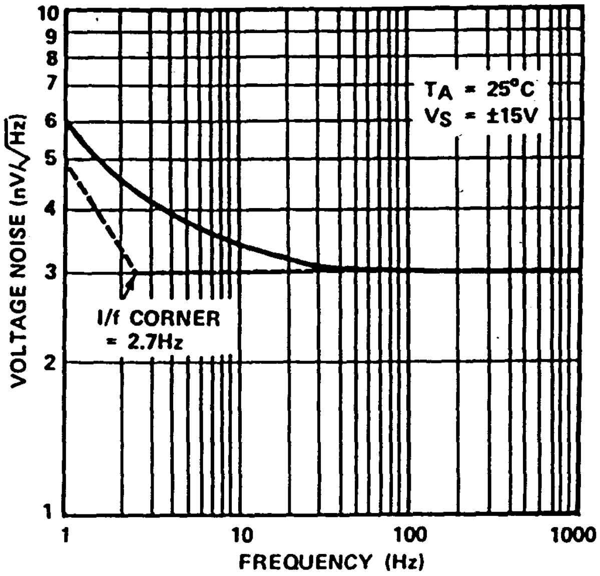
$\left[S_{I_N V_N} \sim 0 \right]$

For Op-Amp.

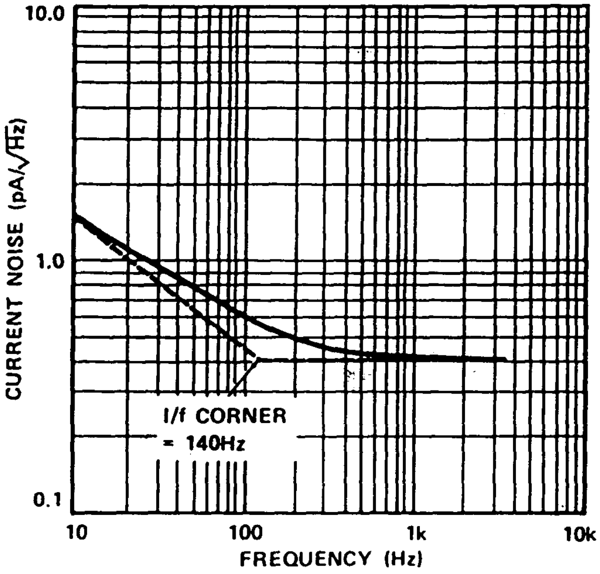
V_N i_N white,
except 1/f at low freq. due to param. flucts caused by μscopic motion of atoms.

OP-37

VOLTAGE NOISE DENSITY vs FREQUENCY



CURRENT NOISE DENSITY vs FREQUENCY

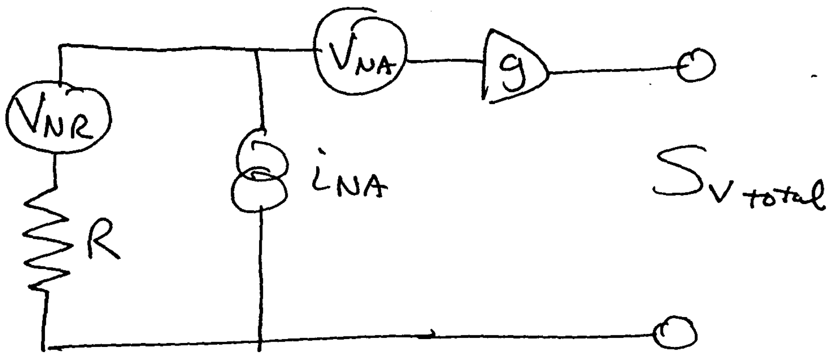


Noise Measurement of a Resistor – + Optimal Matching.

⇒ Measure Johnson Noise of Resistor
[Generic, as typically limitation to a measurement]

[What R, amplifier to use?]

$$\underbrace{\frac{S_V}{R} = 4kT}_{\text{free choice of } R}$$



$$\frac{S_{V_{total}}}{g^2} = \underbrace{4kTR}_{sig.} + \underbrace{S_{V_{NA}} + S_{I_{NA}} R^2}_{noise}$$

$$\frac{\text{Signal}}{\text{Noise}} = \frac{4kT}{\frac{S_{V_{NA}}}{R} + S_{I_{NA}} R}$$

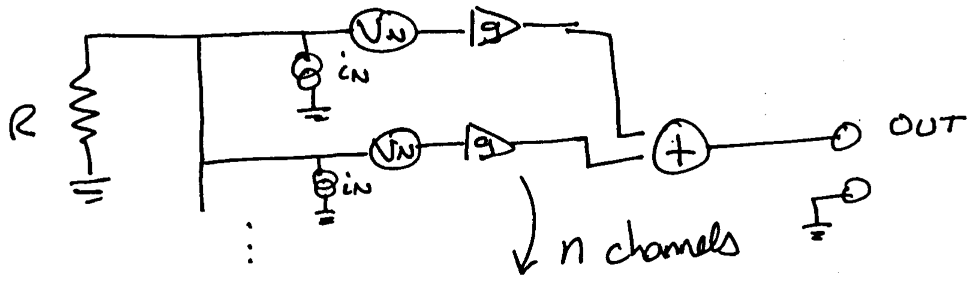
$$\frac{S}{N}\Big|_{max} = \frac{4kT}{2\underbrace{\sqrt{S_{V_{NA}} S_{I_{NA}}}}_{\hookrightarrow\ \boxed{\equiv kT_N}}} \quad \text{maximum at } \boxed{R = R_N = \sqrt{\frac{S_{V_{NA}}}{S_{I_{NA}}}}}$$

$$= \frac{2T}{T_N}$$

- Choose $R = R_N$ for optimal S/N
- T_N is a measure of how cold a resistor that can be measured (when matched)

Changing R_N of an Amplifier

⇒ What if R is fixed? → Use "transformer"



$$S_V = \overset{\text{incoher.}\ \downarrow}{n}\, g^2 S_{VNA} + \underset{\uparrow\ \text{incoh.}}{(n S_{iA} R^2)}\ \underset{\uparrow\ \text{coherent}}{(ng)^2}$$

$$\frac{S_V}{(ng)^2} = \frac{S_{VNA}}{n} + n\, S_{IA} R^2$$

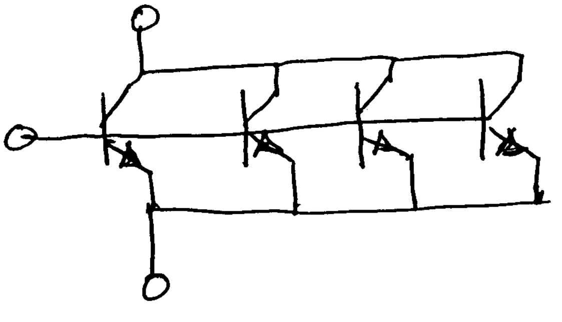
$T_N = \sqrt{S_{VNA} S_{INA}}$ same

$R_N = \sqrt{\frac{S_{VNA}}{S_{INA}}}\ \frac{1}{n^2}$ changed!

Thus can match $R = R_N$ by choosing n!

OP-07
$V_N = 10\ \text{nV}/\sqrt{\text{Hz}}$ $R_N = 71\text{K}\Omega$
$i_N = 0.14\ \text{pA}/\sqrt{\text{Hz}}$ $T_N = 100\text{K}$

AD797
$V_N = 0.9\ \text{nV}/\sqrt{\text{Hz}}$ $R_N = 450\Omega$
$i_N = 2\ \text{pA}/\sqrt{\text{Hz}}$ $T_N = 130\text{K}$



Parallel Transistors

ABSOLUTE MAXIMUM RATINGS[1]

Supply Voltage	±18 V
Internal Power Dissipation @ +25°C[2]	
Input Voltage	±V_S
Differential Input Voltage[3]	±0.7 V
Output Short Circuit Duration	Indefinite Within max Internal Power Dissipation
Storage Temperature Range (Cerdip)	-65°C to +150°C
Storage Temperature Range (N, R Suffix)	-65°C to +125°C
Operating Temperature Range	
AD797A/B	-40°C to +85°C
AD797S	-55°C to +125°C
Lead Temperature Range (Soldering 60 sec)	+300°C

NOTES

[1] Stresses above those listed under "Absolute Maximum Ratings" may cause permanent damage to the device. This is a stress rating only, and functional operation of the device at these or any other conditions above those indicated in the operational section of this specification is not implied. Exposure to absolute maximum rating conditions for extended periods may affect device reliability.

[2] Internal Power Dissipation:
8-Pin SOIC = 0.9 Watts $(T_A-25°C)/\theta_{JA}$
8-Pin Plastic DIP and Cerdip = 1.3 Watts - $(T_A-25°C)/\theta_{JA}$
Thermal Characteristics
8-Pin Plastic DIP Package: θ_{JA} = 95°C/W
8-Pin Cerdip Package: θ_{JA} = 110°C/W
8-Pin Small Outline Package: θ_{JA} = 155°C/W

[3] The AD797's inputs are protected by back-to-back diodes. To achieve low noise, internal current limiting resistors are not incorporated into the design of this amplifier. If the differential input voltage exceeds ±0.7 V, the input current should be limited to less than 25 mA by series protection resistors. Note, however, that this will degrade the low noise performance of the device.

ESD SUSCEPTIBILITY

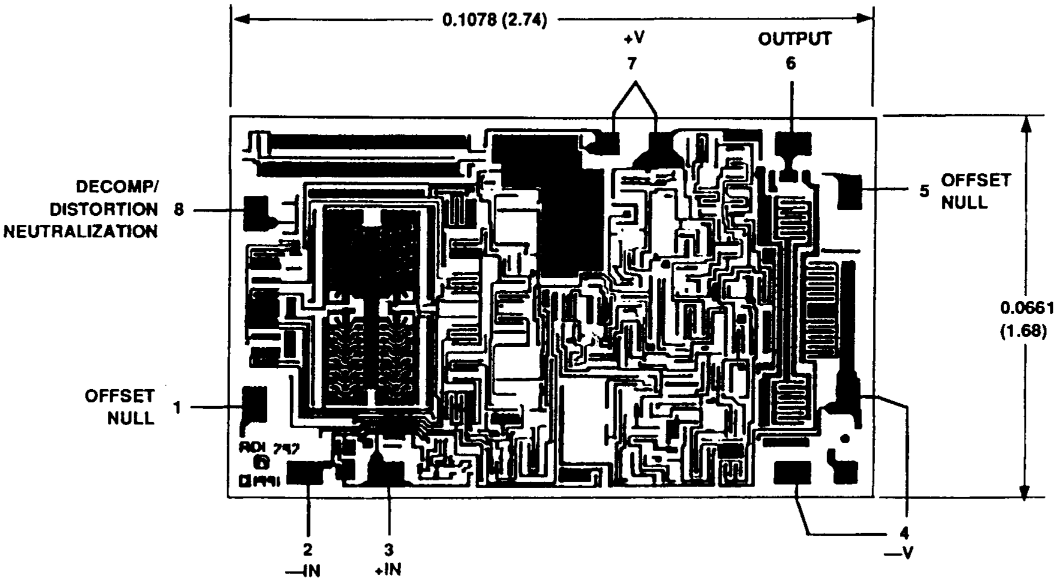
ESD (electrostatic discharge) sensitive device. Electrostatic charges as high as 4000 volts, which readily accumulate on the human body and on test equipment, can discharge without detection. Although the AD797 features proprietary ESD protection circuitry, permanent damage may still occur on these devices if they are subjected to high energy electrostatic discharges. Therefore, proper ESD precautions are recommended to avoid any performance degradation or loss of functionality.

ORDERING GUIDE

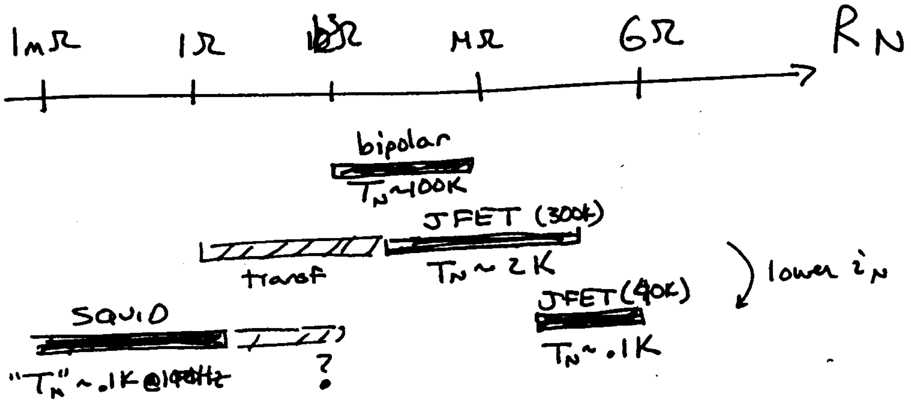
Model	Temperature Range	Package Description	Package Option
AD797AN	-40°C to +85°C	8-Pin Plastic DIP	N-8
AD797BN	-40°C to +85°C	8-Pin Plastic DIP	N-8
AD797BR	-40°C to +85°C	8-Pin Plastic SOIC	SO-8
AD797BR-REEL	-40°C to +85°C	8-Pin Plastic SOIC	SO-8
AD797BR-REEL7	-40°C to +85°C	8-Pin Plastic SOIC	SO-8
AD797AR	-40°C to +85°C	8-Pin Plastic SOIC	SO-8
AD797AR-REEL	-40°C to +85°C	8-Pin Plastic SOIC	SO-8
AD797AR-REEL7	-40°C to +85°C	8-Pin Plastic SOIC	SO-8
5962-9313301MPA	-55°C to +125°C	8-Pin Cerdip	Q-8

METALIZATION PHOTO

Contact factory for latest dimensions.
Dimensions shown in inches and (mm).

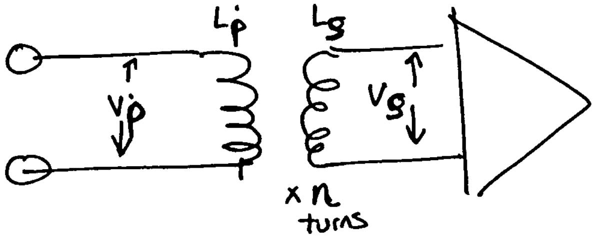


NOTE
The AD797 has double layer metal. Only one layer is shown here for clarity.


1mΩ
1Ω
10³Ω
MΩ
GΩ
R_N
bipolar
T_N~100K
JFET (300K)
transf
T_N ~ 2K
lower i_N
JFET(40K)
T_N~.1K
SQUID
"T_N"~.1K@100Hz
?

Transformer Matching

[Physics Exp's]



$V_s \simeq n V_p$

$L_s \simeq n^2 L_p$

$R \sim \omega L_p$

$R_n \sim \omega L_s$

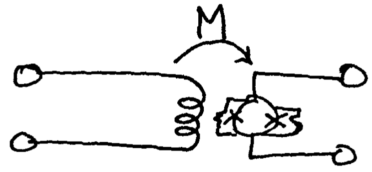
↳ Narrow Bandwidth

$$R_n|_{eff} \simeq \frac{R_n|_{amp}}{n^2}$$

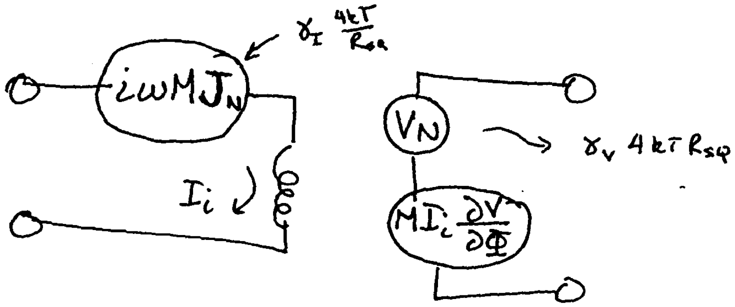
$R_n|_{eff}$ can be $\sim 1\Omega$!

SQUID AMPLIFIER

Current measmt → low impedance, <u>Dual</u> of J-FET



Resistors: output volt noise
circulating current
noise in L

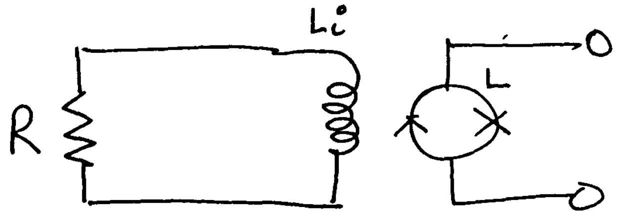


1) Hard to see input noise → Neglect
(T$_N$ low, need C to resonate/remove $i\omega L_i$)

2) $\Phi_N = \dfrac{V_N}{\frac{\partial V}{\partial \Phi}}$; refered to "input"

$\sim 1\,\mu\Phi_0/\sqrt{Hz}$ for typical SQUIDs

SQUID Impedance Matching



$$M \simeq n L$$
$$L_i \simeq n^2 L$$

Can always increase (current sensitivity) of amplifier by increasing n!

→ But at cost of $BW = \frac{R}{2\pi L_i}$

for $L = 4pH$

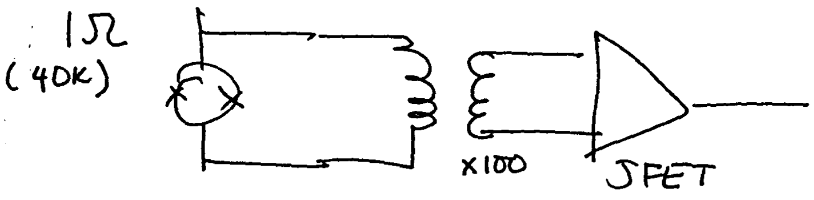
R	$4kT/R$ (0.1K)	n	Φ_N	L_i	$\frac{R}{2\pi L_i}$
1Ω	2.3 pA/$\sqrt{Hz}$	200	$0.8\mu\Phi_0/\sqrt{Hz}$	160nH	1MHz
0.01Ω	23 pA/$\sqrt{Hz}$	20	$0.8\mu\Phi_0/\sqrt{Hz}$	1.6nH	1MHz

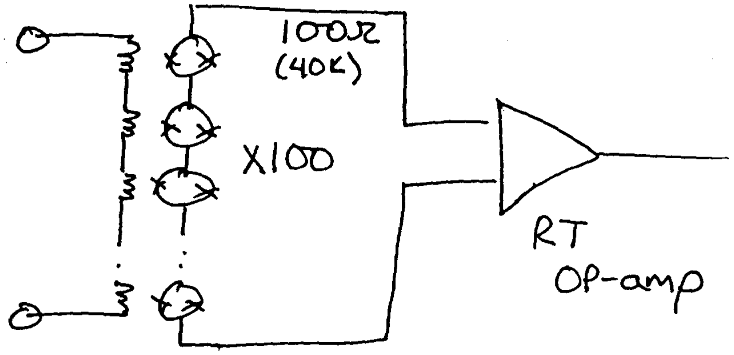
$R \Rightarrow n$; "matching" condition

$BW \Rightarrow$ "T_n"

[Show R_N slide]

SQUID OUTPUT COUPLING



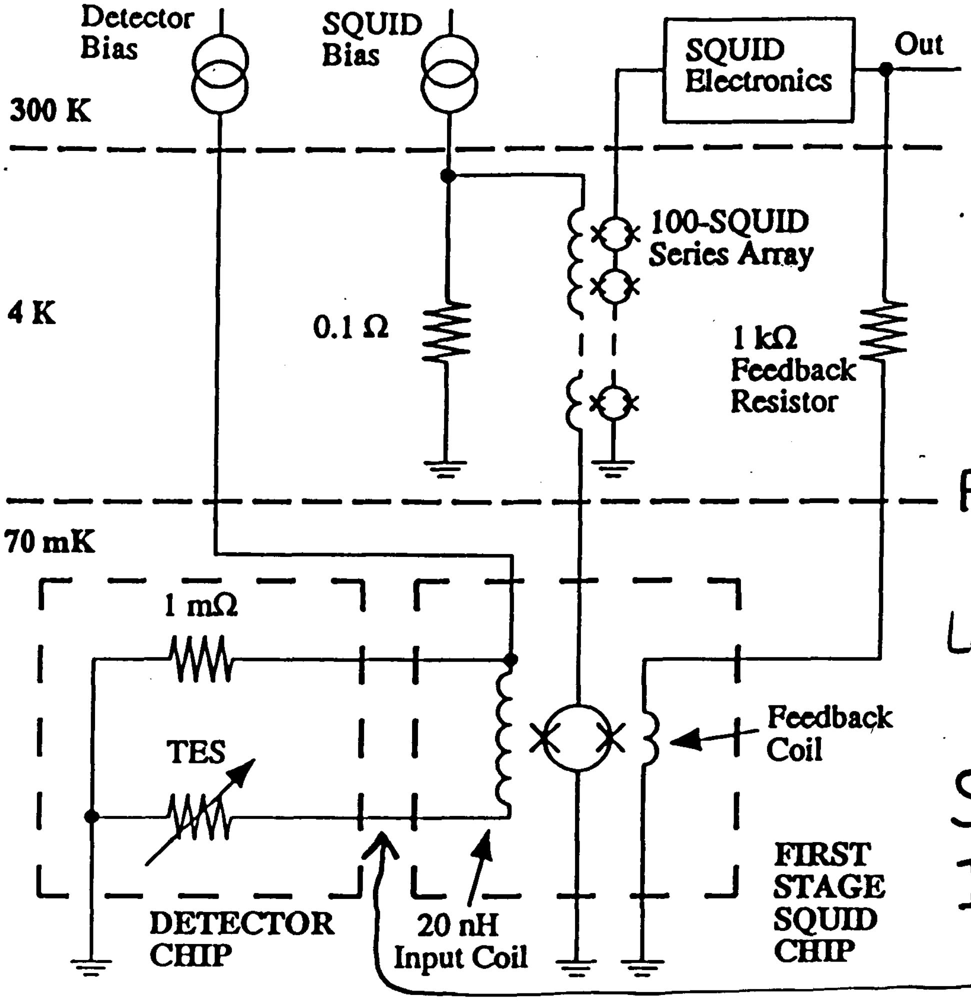


[Use slightly more M
to elevate noise
above RT amp.]

Advantages

- Large BW output
- Simple operation
- 10 mV output make diagnosis easier
- No harder to make 100 SQUIDS

PRESENT DETECTOR CIRCUIT



Fast Detector

⇓

Low TES resist

⇓

Small Str

Inductanc

"Testbed" for mux.

Summary

- Evalute R_{eff} + T_{eff} of an experiment
- Then choose amplifiers + transf to match, $T_N < T$
- $10^{-3}\Omega$ to $10\Omega \rightarrow$ SQUID
  1Ω to $10^{8}\Omega \rightarrow$ JFET