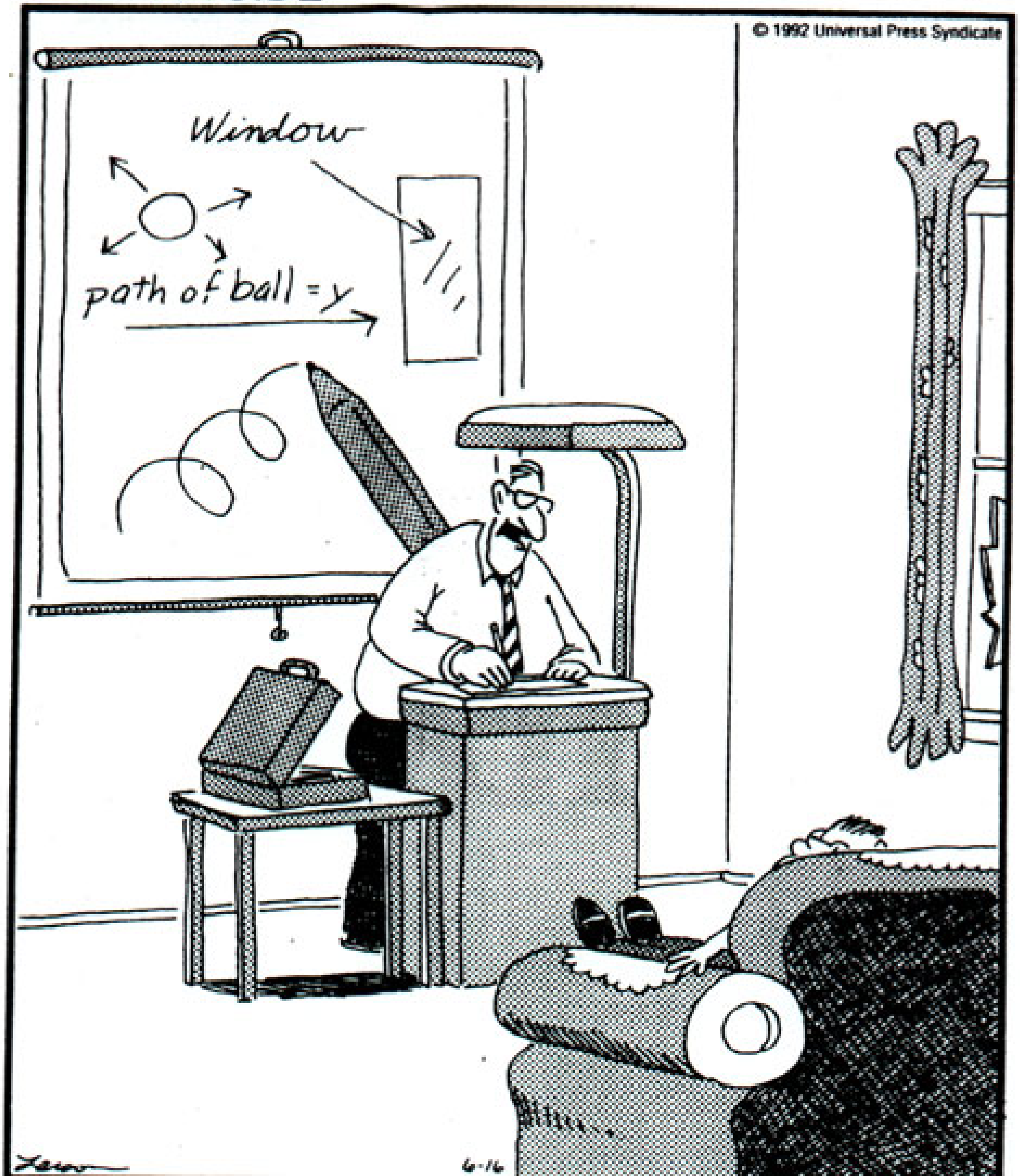
THE FAR SIDE



Eventually, Billy came to dread his father's lectures over all other forms of punishment.

Just What is Superconductivity Anyway?

Outline

1. What is condensed matter physics?

2. Phases, Phase Transitions, Ordering

3. Rigidity
   - in solids
   - in superconductors

4. Macroscopic Phenomenology

   Ginsburg - Landau Theory
   - Anderson-Higgs Mechanism / Meissner
   - Quantized Vortices
   - Monopole Confinement

5. Microscopic BCS Mechanism
   - Poor man's RNG

6. Broken Gauge Symmetry and All That

Rules of B. Matthias for discovering new superconductors

1. high symmetry is best
2. peaks in density of states are good
3. stay away from <u>oxygen</u>
4. stay away from <u>magnetism</u>
5. stay away from <u>insulators</u>
6. stay away from <u>theorists</u>

(courtesy MPA Fisher)

Force law is known: $F_{12} = \frac{Q_1 Q_2}{(r_{12})^2}$

but:

directly useful only on the scale of 1 Rydberg

log E

- ions
- 10 eV
- atoms
- 1 eV — bonds + bands
- 10^3 K — magnetism
- T_{room} — phonons
- LN_2 77K — high T_c superconductivity
- 10K — low T_c superconductivity, Integer QHE
- L^4He 4K — Fractional QHE
- 1K — Kondo
- Weird Fractional QHE
- 10^{-3} K — ^{3}He superfluidity

Thermodynamic limit:

- running coupling constants
- emergent phenomena
- different degrees of freedom at each scale
- surprises with each new material and scale

What is condensed matter physics?

a) assume all physics of <u>atoms</u>

b) gather up N atoms, $N \to \infty$

c) seek <u>collective</u> effects

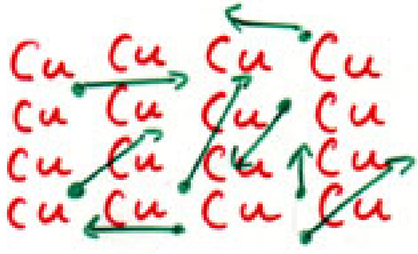
"The whole is greater than the sum of the parts."

"<u>Emergent</u> properties."

Example:

1 copper atom does <u>not</u> conduct electricity

but: Copper wire <u>does</u>



Matter occurs in different phases

- insulator/conductor/superconductor
- solid/liquid/vapor
- magnetic/non-magnetic

Phases have order

Phase transitions

- solid → liquid : melting
- insulator → conductor ('melting' of electrons)

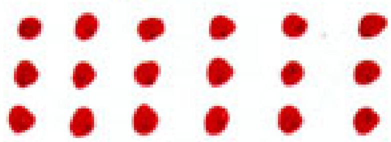
UNIVERSALITY : deep similarities among many transitions

type A order → type B order

What is the order in a phase?

NO essential difference between gas and liquid.

solid is highly ordered:

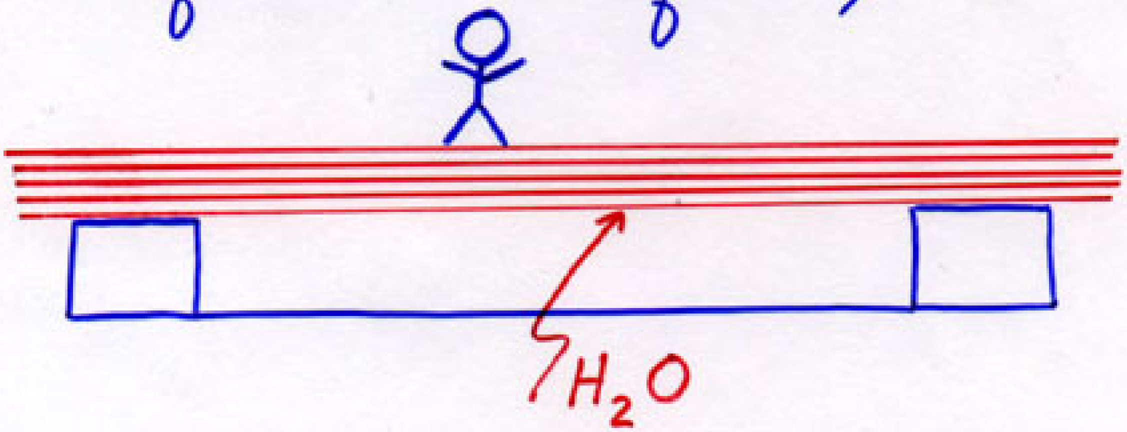


"Broken translation symmetry"

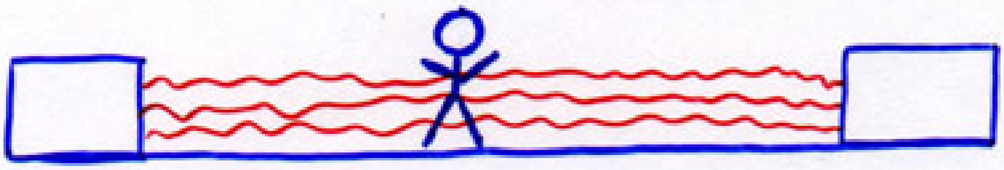
Children know better:

"solids are <u>rigid</u>"

"liquids are squishy"

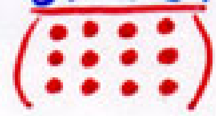


$T < 0°C$



$T > 0°C$

1. what is a superconductor?
2. what is the microscopic <u>order</u>?



3. what is the macroscopic <u>rigidity</u>?

I. INTRODUCTION

It gives me great pleasure to have the opportunity to join my colleagues John Bardeen and Leon Cooper in discussing with you the theory of superconductivity. Since the discovery of superconductivity by H. Kamerlingh Onnes in 1911, an enormous effort has been devoted by a spectrum of outstanding scientists to understanding this phenomenon. As in most developments in our branch of science, the accomplishments honored by this Nobel prize were made possible by a large number of developments preceding them. A general understanding of these developments is important as a backdrop for our own contribution.

On December 11, 1913, Kamerlingh Onnes discussed in his Nobel lecture (1) his striking discovery that on cooling mercury to near the absolute zero of temperature, the electrical resistance became vanishingly small, but this disappearance "did not take place gradually but *abruptly*." His Fig. 17 is reproduced as Fig. 1. He said, "Thus, mercury at 4.2 K has entered a new state

"friction"

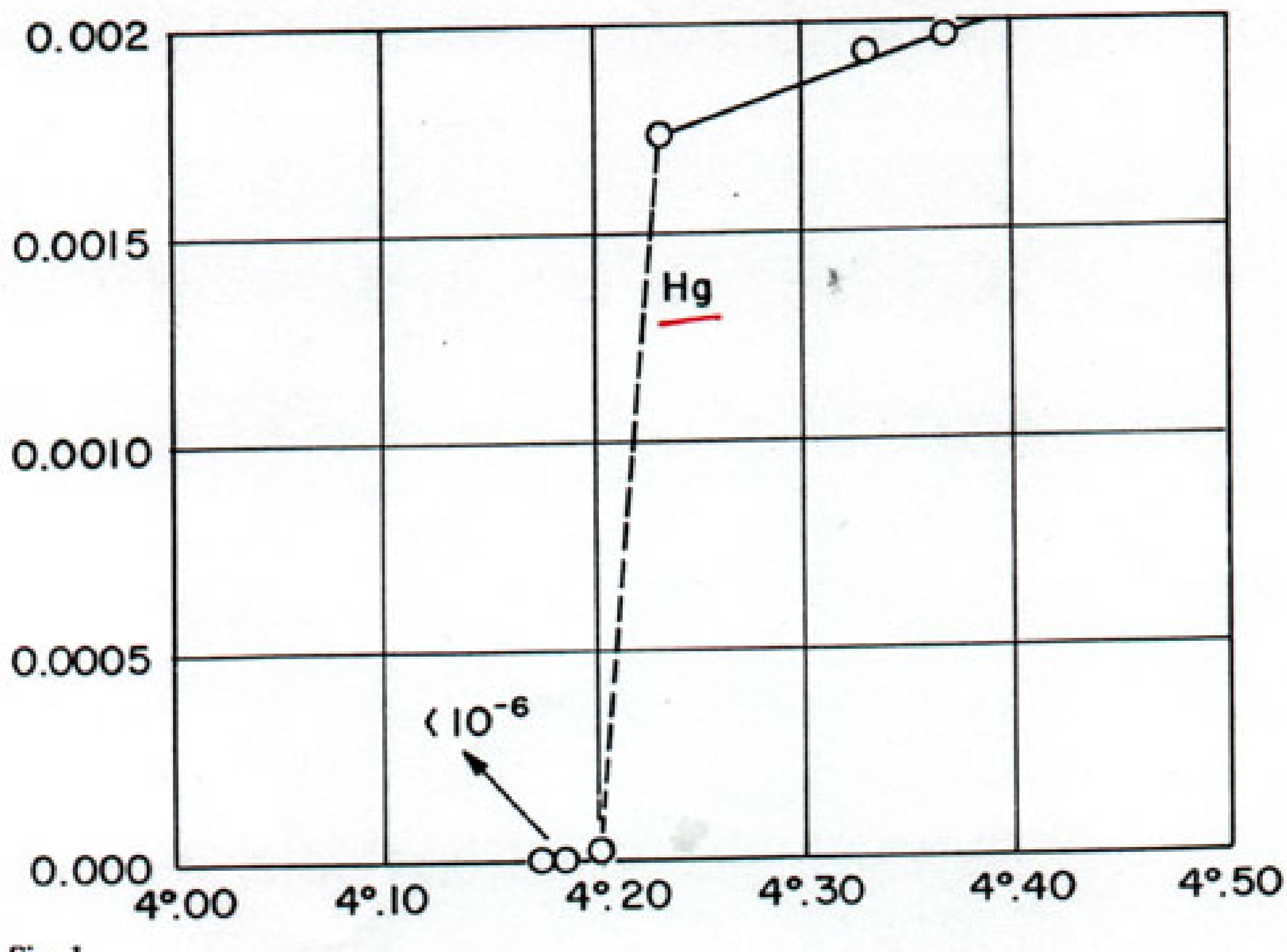


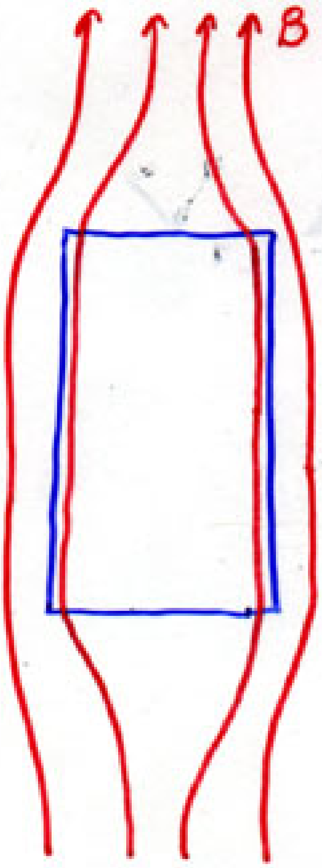
Fig. 1

Kamerlingh Onnes Fig. 17
J.R. Schrieffer Fig. 1

"Rigidity" in a superconductor

a) infinite conductivity (persistent currents)

b) Meissner effect
- expulsion of B fields



magnetic pressure

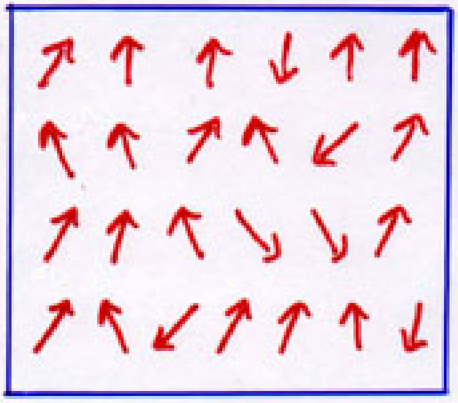
$$\frac{B^2}{8\pi}$$

1 Tesla = 5 atmospheres

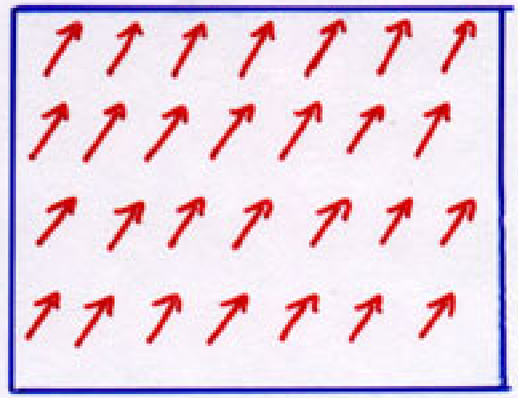
perfectly resisted by superconductor

rigidity in a solid (ice)

microscopic <u>order</u> that produces <u>rigidity</u>



$T > T_c$



$T < T_c$

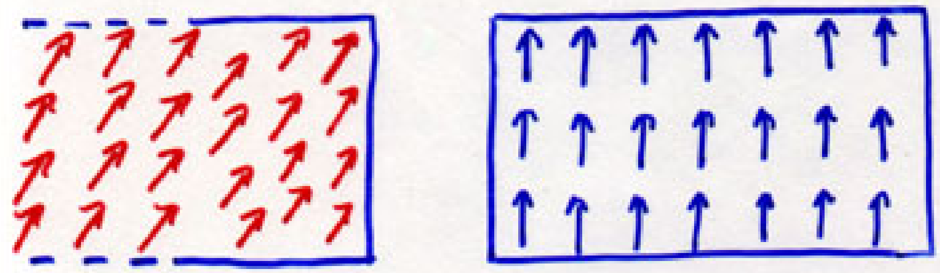
analogous to xy ferromagnet

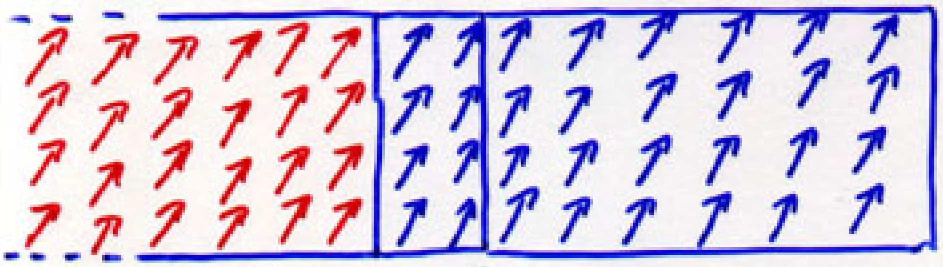
$$+J\,|\vec{\nabla}\vec{S}\,|^2 \quad \text{or} \quad \sum_{\langle ij\rangle} -\vec{S}_i\cdot\vec{S}_j$$

complex 'wave function' order parameter field

$$\Psi(\vec{r}) = |\Psi|\, e^{i\varphi(\vec{r})}$$

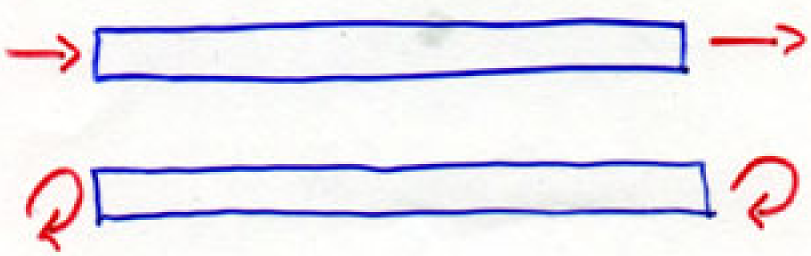
Analog of rigidity



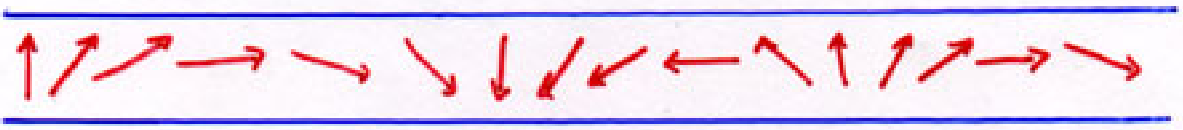


- phase arrows rigidly connected
- twist of phase at one end is transmitted to the other

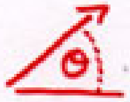
just as rigid rods transmit "push" and "torsion"



twist strain energy represents energy of current flow



$\Psi_{(x)} = e^{i\,\theta(x)}$

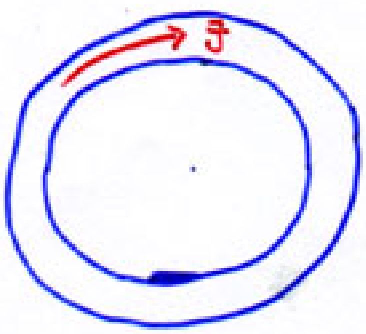


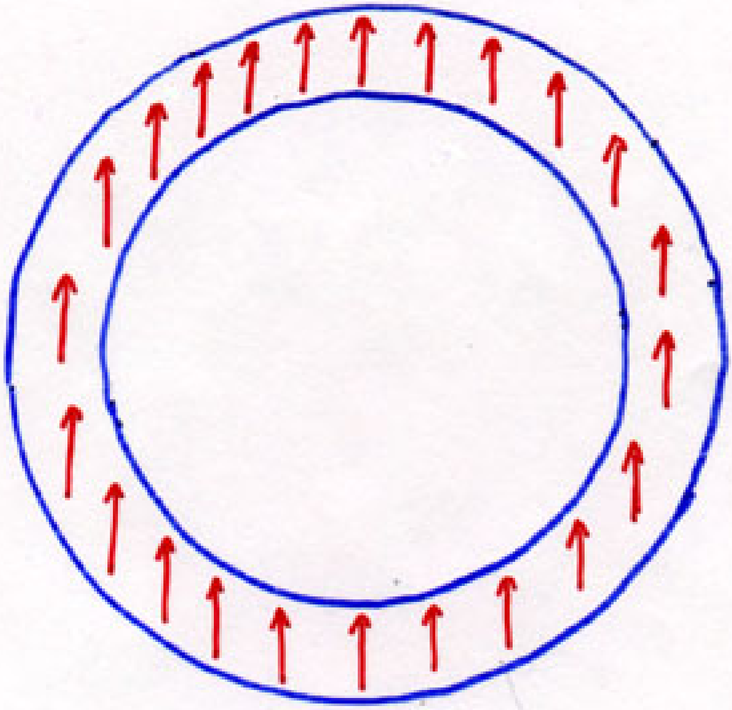
$$F = \frac{1}{2}\,\rho_s\,(\vec{\nabla}\theta)^2$$

"stiffness"

$\vec{J} = \rho_s\,\vec{\nabla}\theta$ current density

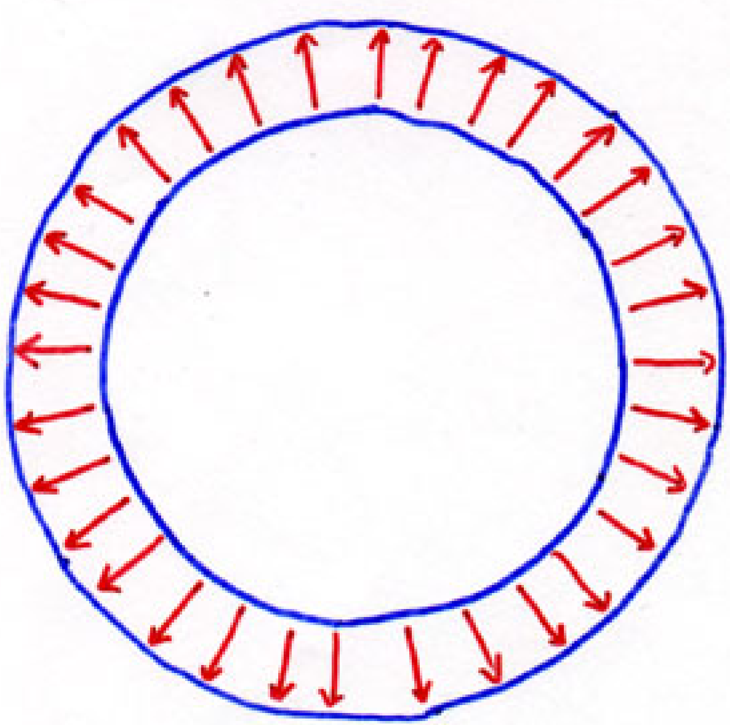
'rigidity' explains persistent currents





$I = 0$

large barrier to decay of current

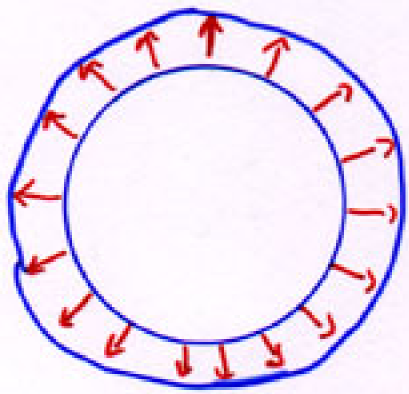


$I = +1$

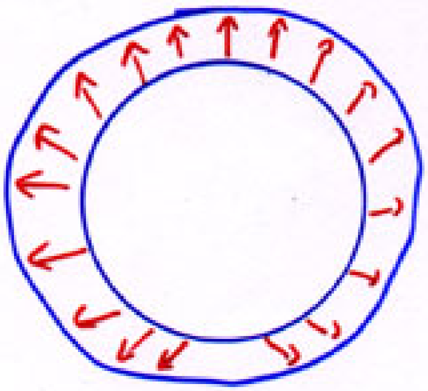
$$I \propto \oint d\vec{r} \cdot \vec{\nabla}\varphi = 2\pi\, n_{winding}$$

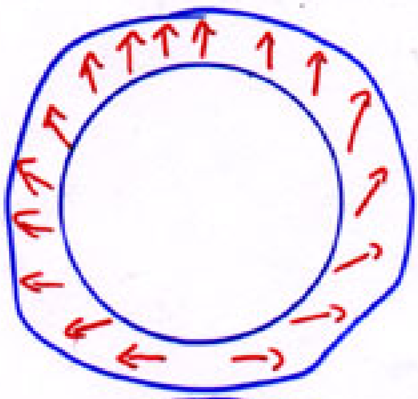
"topological stability"

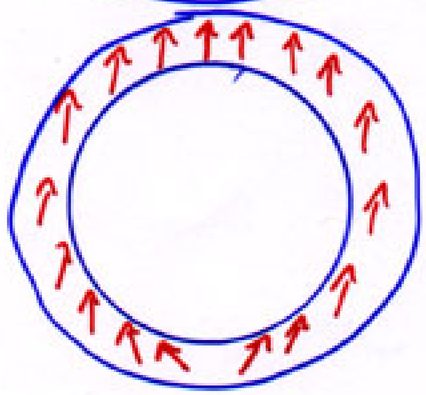
unwinding the phase twist



$I = +1$

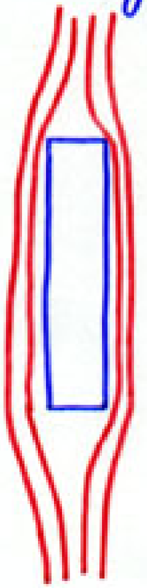






$I = 0$

"rigidity" explains Meissner effect



$$f = \frac{1}{2}\rho_s \left|\vec{\nabla}\theta - \vec{a}\right|^2 \neq 0$$

transverse

$$\vec{\nabla}\times\vec{a} = \vec{B}$$

longitudinal

gauge choice $\vec{\nabla}\theta = \vec{0}$

$$F = \frac{1}{2}\rho_s \int d^3r\, |\vec{a}|^2 = \frac{1}{2}\rho_s \sum_{\vec{q}} \frac{|\vec{q}\times\vec{a}_q|^2}{q^2}$$

$$F = \frac{1}{2}\rho_s \sum_{\vec{q}} \frac{b_q^2}{q^2}$$

Arrows Story

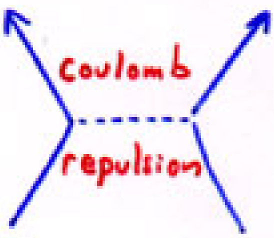
"B field wants each arrow to turn 10° more than its neighbor"

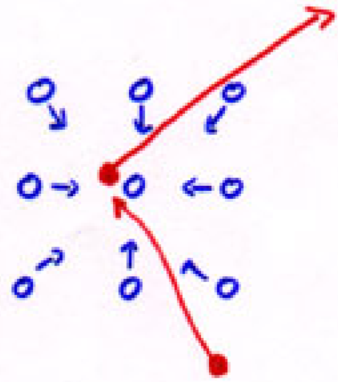
lattice:

$$\sum_{\langle ij\rangle} \cos(\theta_i - \theta_j - a_{ij})$$

$$a_{ij} = \int_{R_i}^{R_j} d\vec{r}\cdot\vec{A}\left(\frac{2e}{\hbar c}\right)$$

Microscopic Physics: BCS theory
(low T_c)

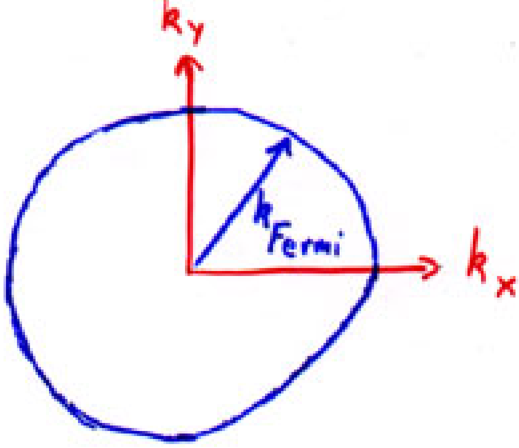


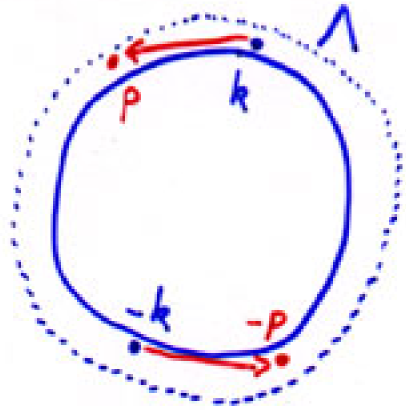


retardation beats Coulomb

attractive 'phonon exchange' for low energies $\omega < \Lambda$

Fermi liquids are <u>unstable</u> to weak attractions

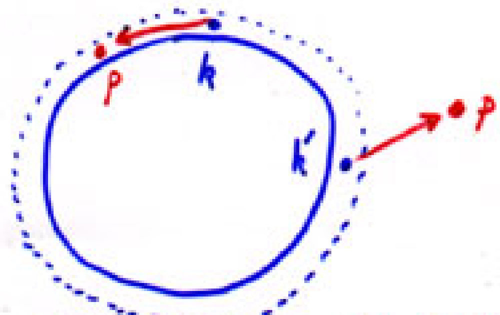




good

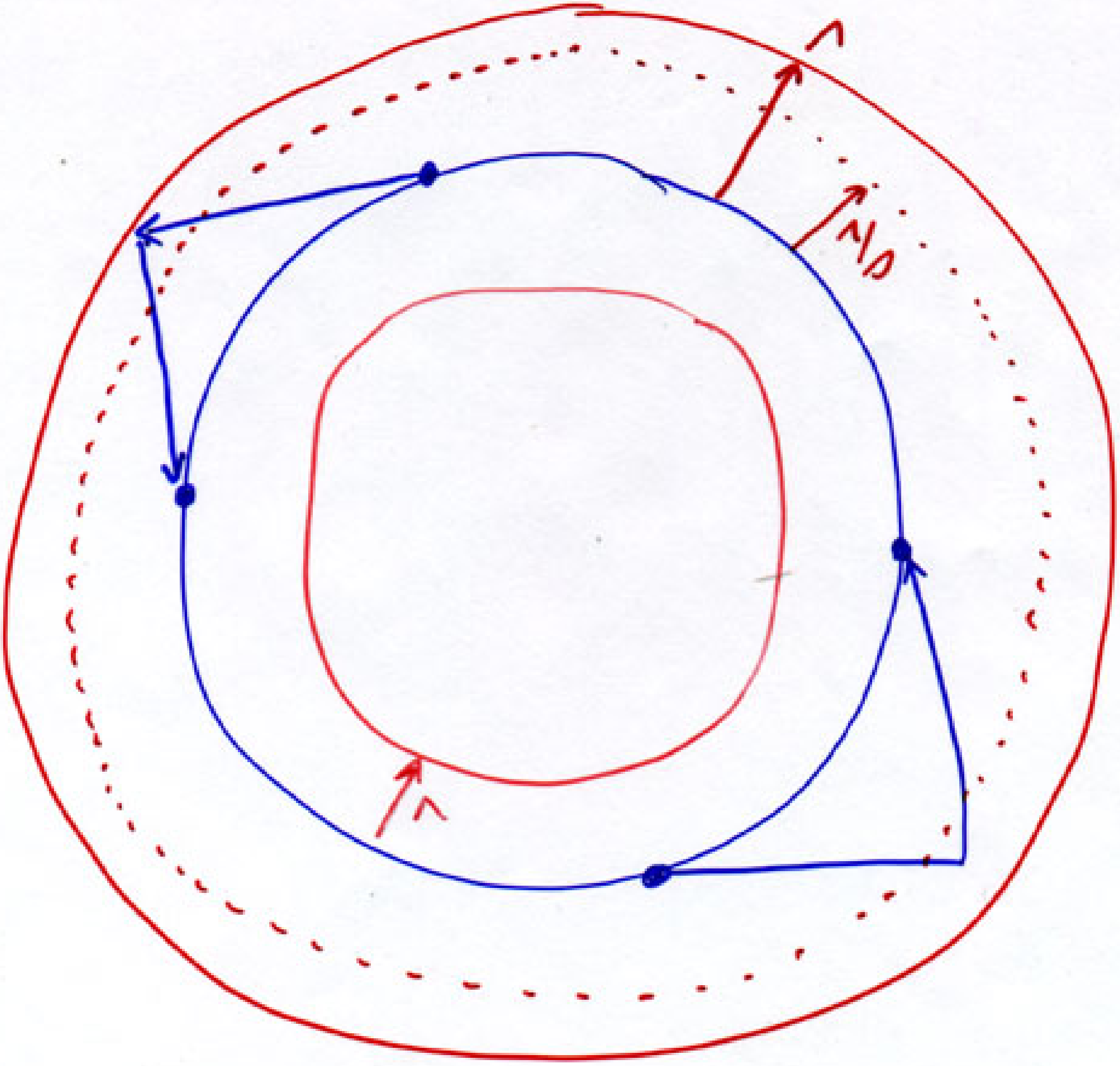
pairs of time-reversed particles crucial

$$c^\dagger_{\vec{k}\uparrow} \, c^\dagger_{-\vec{k}\downarrow}$$



bad

— DROP FROM H FOR NOW —



2nd order perturbation theory

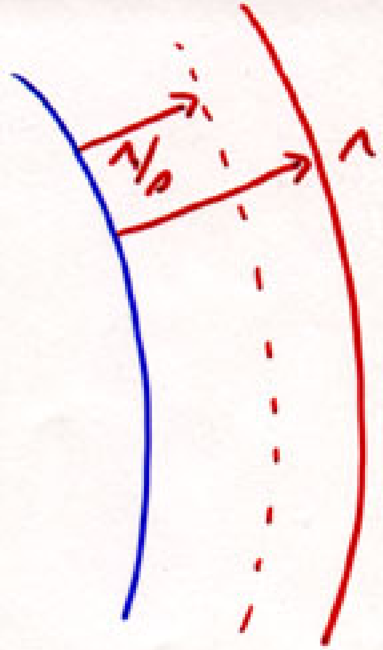
$$v_{\Lambda/s} = v_\Lambda + 2\int_{\Lambda/s}^{\Lambda} d\epsilon\, g(\epsilon)\; v_\Lambda \frac{1}{-2\epsilon}\, v_\Lambda$$

$$v_{\Lambda/s} = v_\Lambda - g v_\Lambda^2 \ln s$$

dimensionless coupling $vg \equiv u$

$$\boxed{\frac{du(s)}{d\ln s} = -u^2(s)}$$ "flow of coupling"

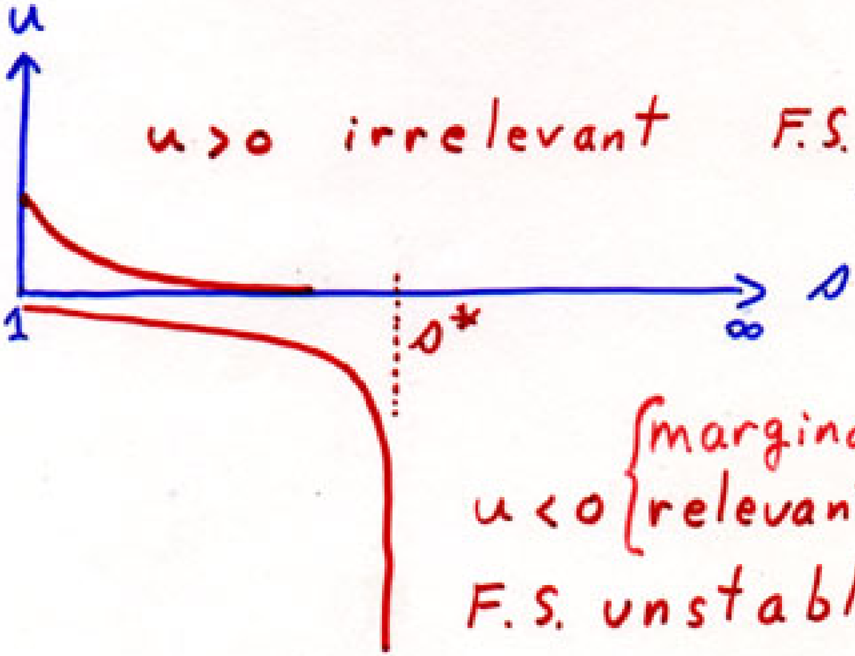
$$\Lambda \to \Lambda/s$$



$$u(s) \equiv g\nu$$

$$\frac{du}{d\ln s} = -u^2(s)$$

$$u(s) = \frac{u(1)}{1 + u(1)\ln s}$$



$u>0$ irrelevant F.S. stable

$u<0$ $\left\{\begin{array}{l}\text{marginally} \\ \text{relevant}\end{array}\right.$

F.S. unstable if $T < \frac{\Lambda}{s^*}$

$$\ln s^* = -\frac{1}{u(1)}$$

$\frac{\Lambda}{s^*}$ is energy scale defining T_c

$$\boxed{T_c = \Lambda\, e^{-\frac{1}{|u(1)|}}}$$

large $\Lambda \leftrightarrow \omega_{Debye}$ retardation

large $u \leftrightarrow$ strong coupling to phonons

structural instabilities ?

Non-perturbative in bare u.

BCS pairing model

$$H = \sum_k \frac{\hbar^2 k^2}{2m} c_k^+ c_k + v_{eff} \sum_{k,p} c_{k\uparrow}^+ c_{-k\downarrow}^+ c_{-p\downarrow} c_{+p\uparrow}$$

$v_{eff} \rightarrow -\infty$

# of excited particles becomes macroscopic

$$c^+c^+cc \rightarrow \langle c^+c^+\rangle cc + c^+c^+\langle cc\rangle$$

Mean field theory of broken gauge symmetry

c^+c^+ acts like boson b^+

field condenses

$$\psi(\vec{r}) = \langle cc\rangle = \langle b\rangle$$

macroscopic wave function
coherent state of bose field

attraction flows to strong coupling

$$V \Rightarrow -|u| \sum_{\vec{k}\vec{p}} c^+_{\vec{k}\uparrow} c^+_{-\vec{k}\downarrow} c_{\vec{p}\uparrow} c_{-\vec{p}\downarrow}$$

Peculiar mean field theory ('broken gauge symmetry')

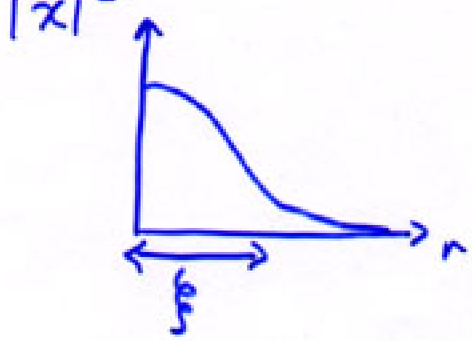
$$V = -|u| \left\{ \langle c^+ c^+ \rangle c\,c + c^+ c^+ \langle c c \rangle \right\}$$

$$\langle c(\vec{r}_1)\, c(\vec{r}_2) \rangle = \Psi\left(\vec{R} = \frac{\vec{r}_1 + \vec{r}_2}{2}\right) \chi(\vec{r}_1 - \vec{r}_2)$$

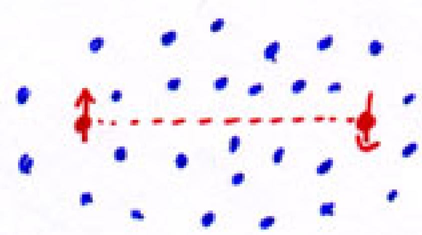
macroscopic order parameter

microscopic Cooper pair internal wave function

low T_c : mean field theory extremely good



$\xi \sim 10^3\,\mathring{A}$



$N = \rho \xi^3 \sim 10^7$ electrons in Cooper pair volume

very different from <u>BEC</u>

fluctuation corrections $\sim \frac{1}{\sqrt{N}}$

$$\boxed{\text{High } T_c \quad N \sim 10^1 \quad \text{danger!}}$$

What is broken symmetry?

- Ising model example

What is broken gauge symmetry?

Recall the Ising model

$$H = -J \sum_{\langle ij \rangle} S_i S_j - h \sum_k S_k \quad ; \quad S = \pm 1$$

If $h = 0$, H invariant under $\{S_j\} \to \{-S_j\}$

(time reversal)

$$\Rightarrow \quad \langle S_k \rangle = 0$$

$$= \frac{1}{Z} \sum_{\{S\}} e^{-\beta H} S_k$$

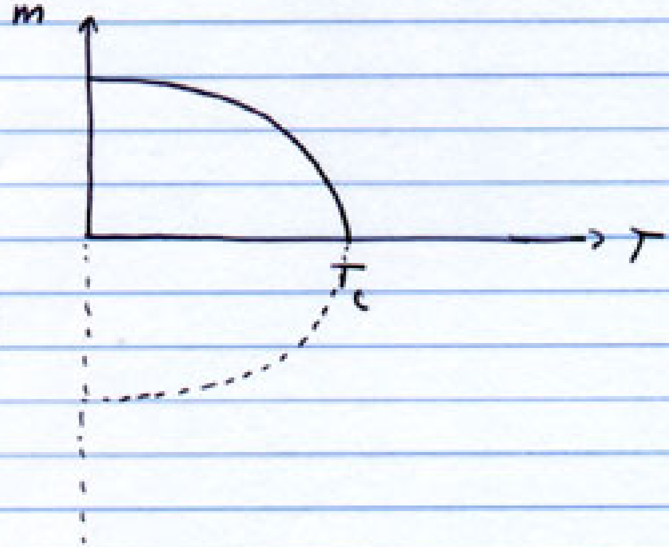
$\langle S_k \rangle \neq 0$ for $h = 0 \Rightarrow$ state has less symmetry than H

"broken symmetry"

Order of limits

$$\lim_{N\to\infty} \lim_{h\to 0^+} \langle S_k \rangle = \lim_{N\to\infty} \lim_{h\to 0^-} \langle S_k \rangle = 0$$

$$\lim_{h\to 0^+} \lim_{N\to\infty} \langle S_k \rangle = - \lim_{h\to 0^-} \lim_{N\to\infty} \langle S_k \rangle = m \neq 0$$



$F(T,h) = -k_B T \ln Z$ is analytic in T, h for any finite N

Mean Field Theory

$$H = -J \sum_{\langle ij \rangle} S_i S_j \cancel{- h \sum_k S_k}$$

$$H = -J \sum_{\langle ij \rangle} S_i \langle S_j \rangle$$

$$H = -J \sum_i h_{eff} S_i$$

Mean field theory appears to not have the correct symmetry — until we recognize that heff changes sign when $\langle s \rangle$ changes sign.

Mean Field Theory breaks symmetry 'by hand' but has 2 solutions

$$\langle S_i \rangle = +|m| \quad -|m|$$

Gauge symmetry

1st quantization

$$\psi(\vec{r}) \to U\,\psi(\vec{r})$$

$$U = e^{i\theta(\vec{r})}$$

$$U\left(\vec{p} + \frac{e}{c}\vec{A}\right)U^{\dagger} = \vec{p} + \frac{e}{c}\vec{A} - \hbar\vec{\nabla}\theta$$

$$\vec{A} \to \vec{A} - \frac{\hbar c}{e}\vec{\nabla}\theta \qquad \text{gauge change}$$

2nd quantization

$$U = e^{i\int d^3r\, \theta(\vec{r})\, \hat{n}(\vec{r})}$$

$$\hat{n} \equiv \psi^{\dagger}(\vec{r})\,\psi(\vec{r})$$

$$U\psi^{\dagger}U^{\dagger} = e^{i\theta(\vec{x})}\psi^{\dagger}$$

$\vec{A}$ and therefore H is <u>invariant</u> only if $\vec{\nabla}\theta = \vec{0}$; $\theta =$ constant **AND**

H conserves charge

$$\left[H, \int d^2r\, \hat{n}(\vec{x})\right] = 0$$

$\psi^{\dagger}, \psi$ transform oppositely

But the SC order parameter

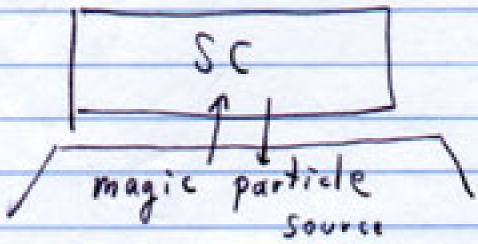
$$\langle \psi^{\dagger}_{\uparrow}\psi^{\dagger}_{\downarrow} \rangle \to \langle U\psi^{\dagger}_{\uparrow}\psi^{\dagger}_{\downarrow}U^{\dagger} \rangle = e^{2i\theta}\langle \psi^{\dagger}_{\uparrow}\psi^{\dagger}_{\downarrow} \rangle$$

"charge 2 order parameter"

Not gauge invariant

add formal symmetry breaking term to H

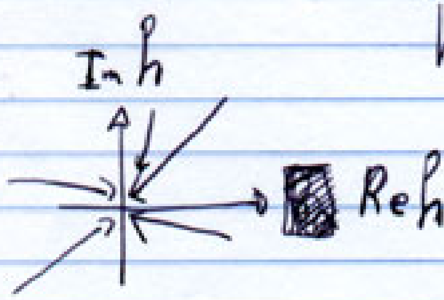
$$-h\,\psi_\uparrow^\dagger\psi_\downarrow^\dagger - h^*\psi_\uparrow\psi_\downarrow$$



$$\lim_{|h|\to 0}\ \lim_{V\to\infty}\ \langle\psi_\uparrow^\dagger\psi_\downarrow^\dagger\rangle = |\Psi|\,e^{+i\varphi}$$

$h=|h|e^{-i\varphi}$

depends on the way $h\to 0$ limit taken



Mean Field Theory breaks symmetry by hand

$$H_0+\left\{-v\,\psi_\uparrow^\dagger\psi_\downarrow^\dagger\langle\psi_\downarrow\psi_\uparrow\rangle + h.c.\right\}$$

- analogies
- pictures
- a little math
- lots of fun

~~numbers~~

(at least not until you understand the picture)