Lecture III

- Suppression of Δ by disorder:
  - Collective mode formalism
  - Plasma + Carlson-Goldman modes
  - Why fluctuating modes reduce Δ
  - Why Coulomb singularity is cancelled

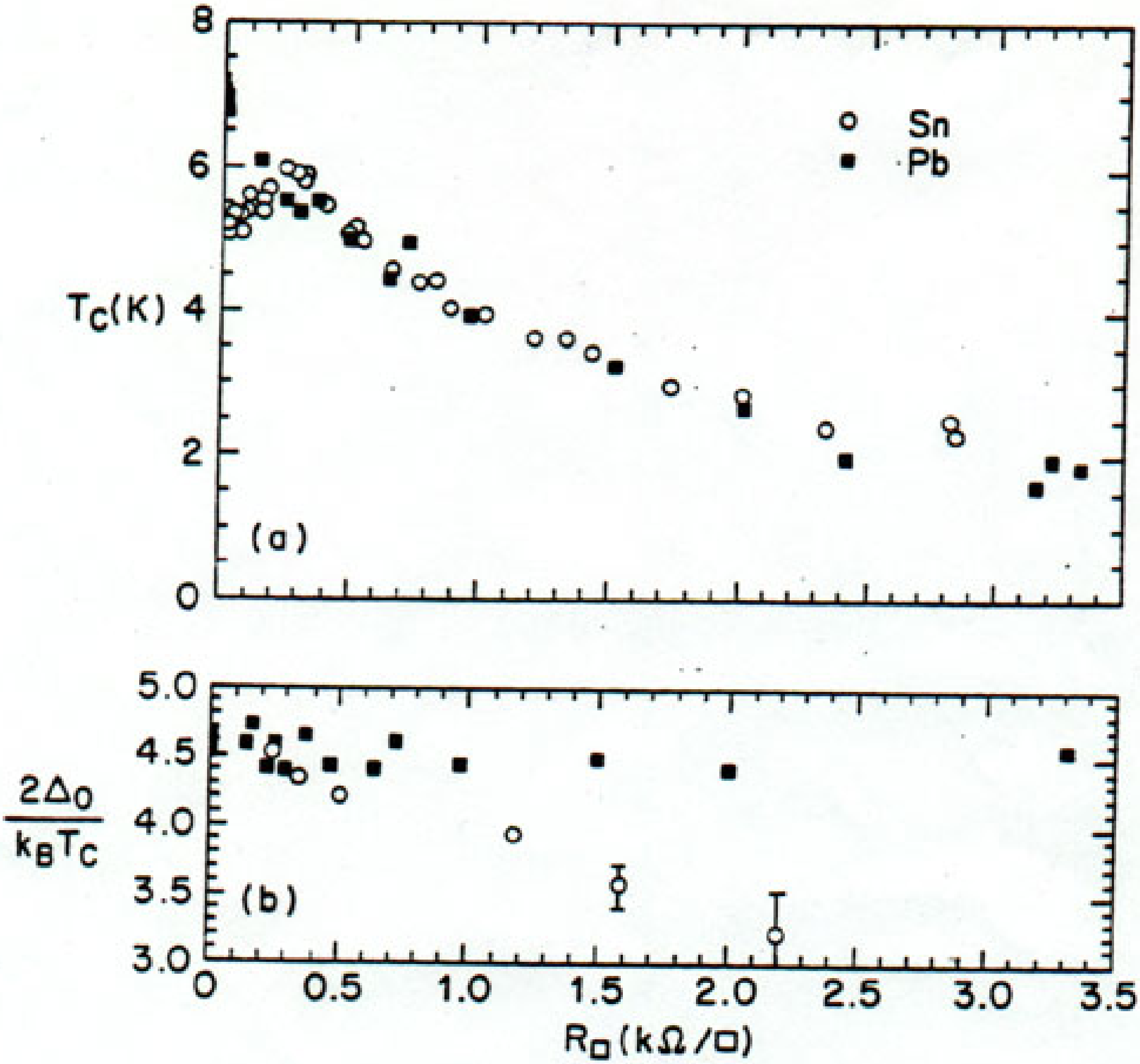
$$F[\Delta, T] \to \Delta(T) \to T_c$$

- Strong coupling + disorder:
  - Why include strong coupling: Pb, PbBi...
  - Eqns for T_c, $\Delta(\omega)$, $Z(\omega)$

Conclusion:
- Previous nonperturbative approach <u>IS</u> strong coupling
- Fully consistent theory of phonons + Coulomb + disorder!

Suppression of Superconducting Gap in Pb + Sn Films

[Valles et al PRB 40 6680 (1989)]



Expt + theory show $\frac{2\Delta}{k_B T_c}$ roughly constant as $R_\square$ increased

The Collective Mode Approach

- The idea is that fluctuations of collective modes lead to reduction of the average order parameter in superconductor.

- We identify three bosonic variables whose variation leads to propagating modes:
  - order parameter amplitude $O_\Delta = \psi_\uparrow^+ \psi_\downarrow^+ + \psi_\downarrow \psi_\uparrow$
  - order parameter phase $O_\phi = i[\psi_\uparrow^+ \psi_\downarrow^+ - \psi_\downarrow \psi_\uparrow]$
  - electronic density $O_\rho = \psi_\uparrow^+ \psi_\uparrow + \psi_\downarrow^+ \psi_\downarrow$

- All three can be written in terms of Nambu operators and Pauli matrices

$$O_i = \Psi^+ \tau_i \Psi$$

$$= (\psi_\uparrow^+, \psi_\downarrow) \tau_i \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow^+ \end{pmatrix}$$

$\tau_1 \Rightarrow O_\Delta$

$\tau_2 \Rightarrow O_\phi$

$\tau_3 \Rightarrow O_\rho$

- The total electron-electron interaction (BCS + Coulomb) is then:

$$\sum_q \left\{ -\tfrac{1}{2}\lambda O_\Delta(q) O_\Delta(-q) - \tfrac{1}{2}\lambda O_\phi(q) O_\phi(-q) + V_c(q) O_\rho(q) O_\rho(-q) \right\}$$

Screened Effective Potentials

- Any Pauli matrix τ_1, τ_2, τ_3 can occur at a vertex corresponding to emission of the appropriate collective mode

- To find collective modes we evaluate screened effective potentials:

$$V_{ij} = V^0_{ij} + V^0_{ik}\,\Pi_{kl}\,V_{lj}$$

$$V^0 = \begin{bmatrix} -\lambda & & \\ & -\lambda & \\ & & 2V_c(q) \end{bmatrix} \qquad \Pi = \begin{bmatrix} \Pi_{\Delta\Delta} & & \\ & \Pi_{\phi\phi} & \Pi_{\phi\rho} \\ & \Pi_{\rho\phi} & \Pi_{\rho\rho} \end{bmatrix}$$

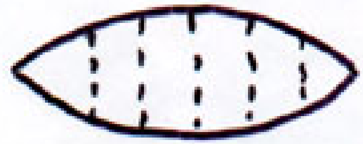
- Coupling of order parameter phase with electronic density is due to gauge invariance.

- Collective modes given by:

$$\underline{-\lambda^{-1} + \Pi_{\Delta\Delta}(q,\omega) = 0}$$

$$\underline{\left[-\lambda^{-1} + \Pi_{\phi\phi}(q,\omega)\right]\left[(2V_c(q))^{-1} + \Pi_{\rho\rho}(q,\omega)\right] + \Pi_{\phi\rho}^2(q,\omega) = 0}$$

- Lowest order approximation for dirty superconductors is usual impurity ladder:

$\mathrm{II} \equiv$ 

- At $T=0$ this gives collective mode eqns:

$$\underline{1 + \frac{Dq^2}{8\Delta} - \frac{\omega^2}{12\Delta^2} = 0}$$

$$\underline{\left[\frac{\pi D q^2}{4\Delta} - \frac{\omega^2}{4\Delta^2}\right]\left[1 + \frac{1}{2N(0)V_c(q)}\right] + \frac{\omega^2}{4\Delta^2} = 0}$$

- The order parameter amplitude mode is always gapped at $\omega \sim \Delta$.

- If we could ignore coupling to Coulomb, get fourth sound mode

$$\omega = c_4 q \qquad c_4^2 = \pi D \Delta = \frac{n_s}{n}\frac{v_F^2}{3}$$

where n_s is superfluid density.

- In 3D Coulomb interaction raises this mode to plasma frequency

$$\omega^2 = 8\pi^2 N(0) e^2 D \Delta = \frac{4\pi n_s e^2}{m}$$

- In 2D Coulomb interaction gives a dispersion $\omega \sim q^{1/2}$

$$\underline{\omega^2 = 4\pi^2 N(0) e^2 D \Delta q = \frac{d}{2\lambda_{TF}^2} \cdot \frac{n_s}{n} \cdot \frac{v_F^2}{3} q}$$

where λ_{TF} is 3D Thomas-Fermi screening length and d is film thickness.

- In 1D wire dispersion will be $\omega \sim q$.

- There is a second mode with linear dispersion corresponding to counterflow of normal + superfluid densities: Carlson-Goldman mode. It is overdamped except very close to T_c:

$$\underline{\omega_{CG} = \sqrt{a^2 q^2 - \Gamma^2/4} - \frac{1}{2} i \Gamma}$$

$$\underline{a^2 = \frac{n_s}{n} \frac{4T}{\pi \Delta} \frac{v_F^2}{3}} \qquad \underline{\Gamma = \frac{n_s}{n} \frac{1}{\tau}}$$

Observation of 2D Plasma Mode in a Superconducting Al Film

[Buisson et al PRL 73 3153 (1994)]

d ~ 100 A°

$SrTiO_3$ substrate

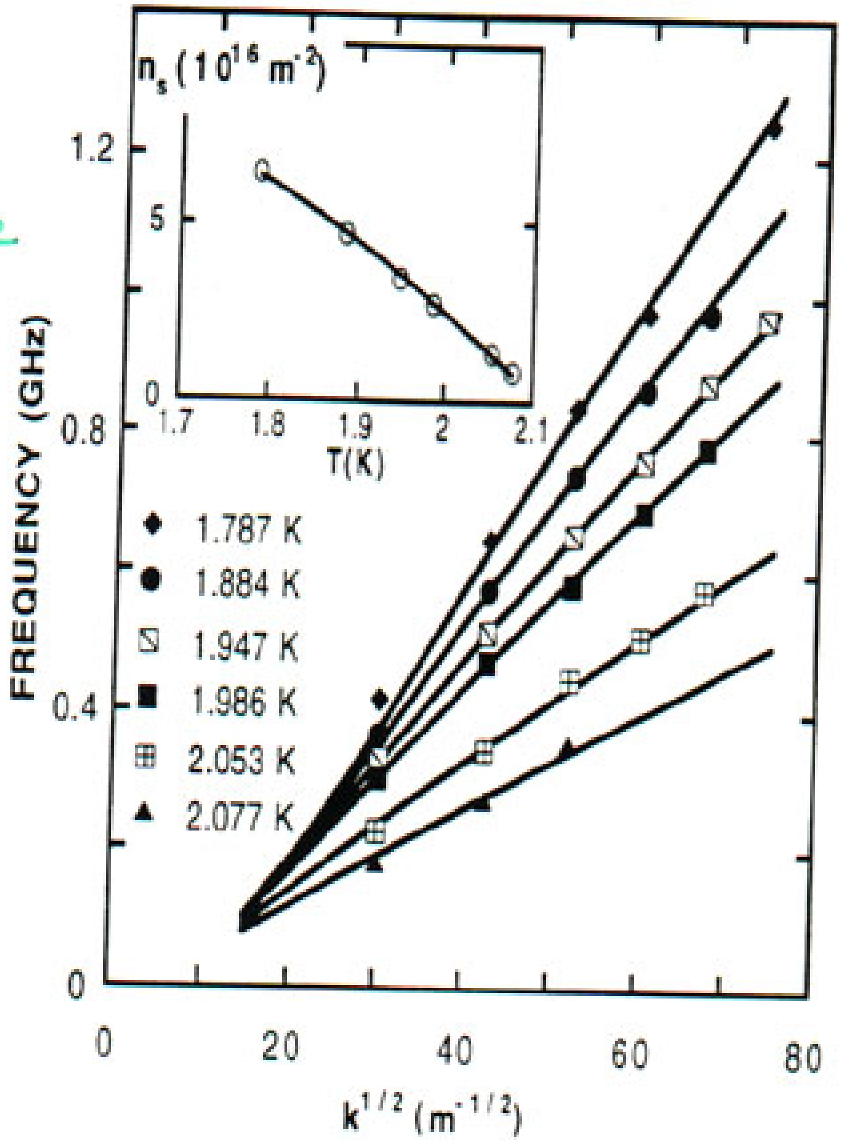
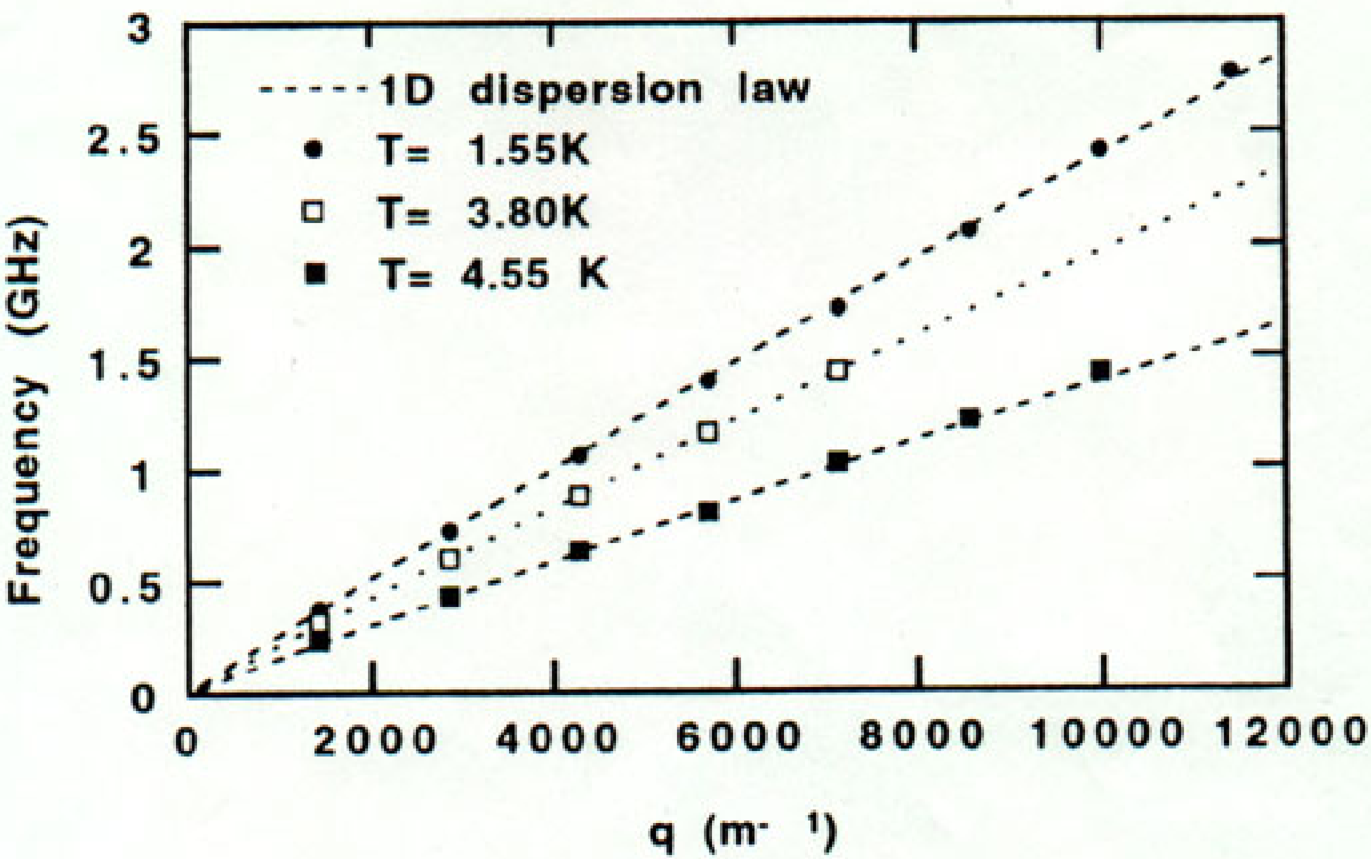


FIG. 3. Experimental resonance frequencies versus the permitted $\sqrt{k}$ for different temperatures. The solid lines correspond to the theoretical dispersion law obtained from Eq. (1). Inset: the surface density of superconducting electrons $n_s(T)$ (open circles) is fitted by the Gorter-Casimir law (line) using $T_c = 2.110$ K and $n_s(0) = 13 \times 10^{16}$ m^{-2}.

Observation of 1D Plasma Mode in a Superconducting Nb Wire

[Camarota et al JLTP 118 589(2000)]

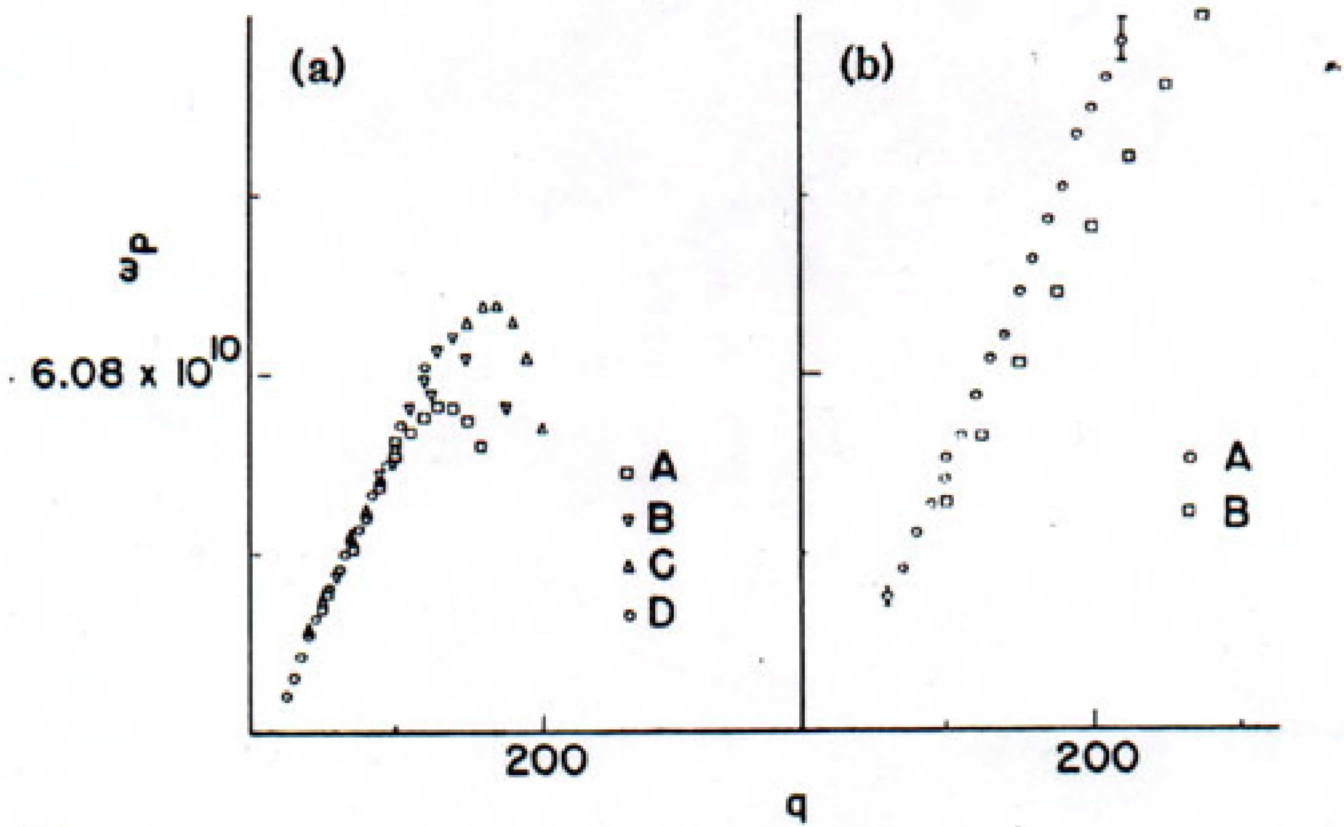


$W = 3\,\mu m \quad d \sim 110\,A^{\circ}$

$SrTiO_3$ substrate

Carlson-Goldman Mode in an Al Film

[Carlson + Goldman PRL 34 ~~$$~~ (1975)]



Points found from peak in pair structure function measured by tunneling. q is altered by in-plane B-field.

Why Fluctuations Reduce Order

- We basically want to calculate the free energy which is related to partition function by:

$$F = -k_B T \ln Z \qquad Z = \mathrm{Tr}\, e^{-\beta H}$$

$$= \int D\phi(x)\, e^{-S[\phi]}$$

functional integral

- Imagine integrating a function $e^{-f(x)}$ where $f(x)$ has minimum at x_0:

$$Z = \int_{-\infty}^{\infty} e^{-f(x)}\,dx = \int_{-\infty}^{\infty} e^{-f(x_0)}\, e^{-\frac{1}{2}f''(x_0)(x-x_0)^2}\,dx$$

$$= \left(\frac{2\pi}{f''(x_0)}\right)^{1/2} e^{-f(x_0)}$$

$$\Rightarrow \quad F = -\ln Z = f(x_0) + \frac{1}{2}\ln f''(x_0)$$

minimum value of energy

free energy is increased by fluctuations around mimimum

- Only difference in functional integral is that there is an independent variable for each $(q, i\Omega)$:

$$F = F_0 - k_B T \sum_{q,\Omega} \{ \ln[-\lambda^{-1} + \Pi_{DD}(q,\Omega)]$$

$$+ \ln[\{-\lambda^{-1} + \Pi_{\phi\phi}(q,\Omega)\}\{(2V_c(q))^{-1} + \Pi_{\rho\rho}(q,\Omega)\} + \Pi^2_{\phi\rho}(q,\Omega)]\}$$

- Knowledge of screened effective potentials is sufficient to find correction to free energy.

- Minimising F with respect to Δ will yield order parameter self-consistency eqn.

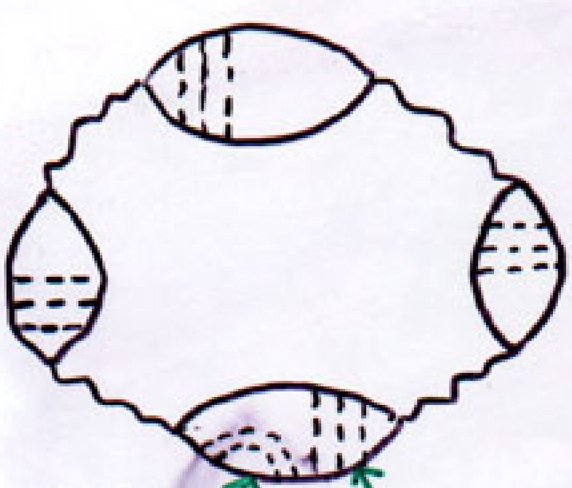
- At $T = T_c$, $\Delta \to 0$ and we obtain T_c eqn by linearising with respect to Δ.

- All the T_c suppression diagrams are thus hidden in the above expression!

$$F(\Delta) \Rightarrow \frac{\partial F}{\partial \Delta} = 0 \qquad \Rightarrow \left.\frac{\partial^2 F}{\partial \Delta^2}\right|_{\Delta=0} = 0$$

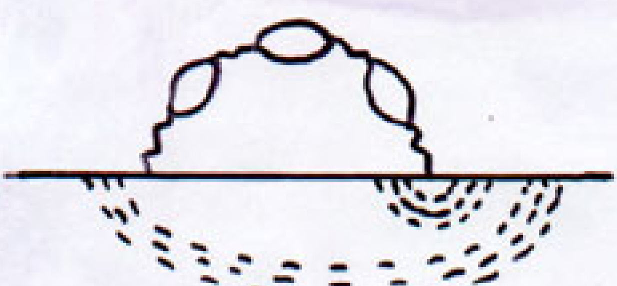
Δ eqn $\qquad$ T_c eqn

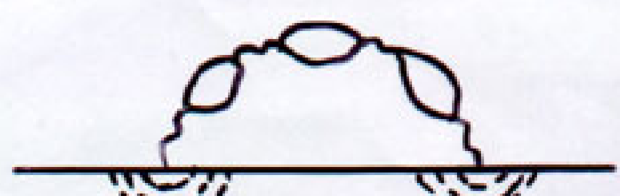
$F(\Delta) \rightarrow \Delta(T) \rightarrow T_c$ in Diagrams



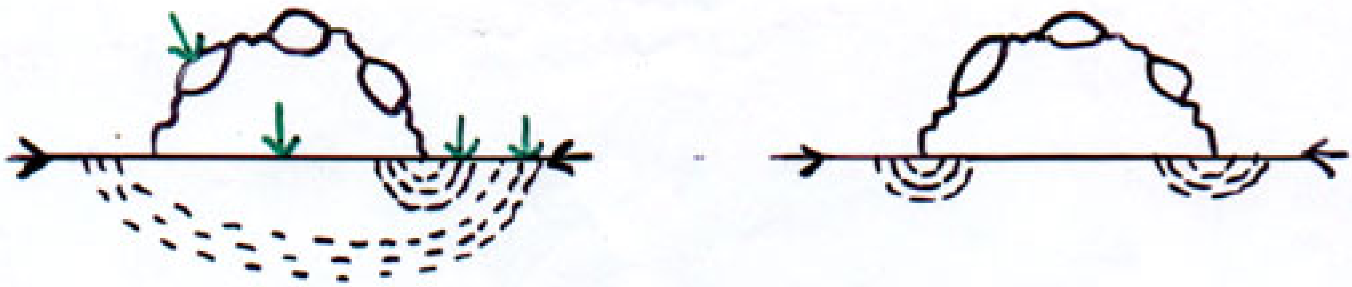
$F(\Delta)$ = "string of bubbles"

$\frac{\partial}{\partial \Delta}$ breaks Green function line in all possible ways

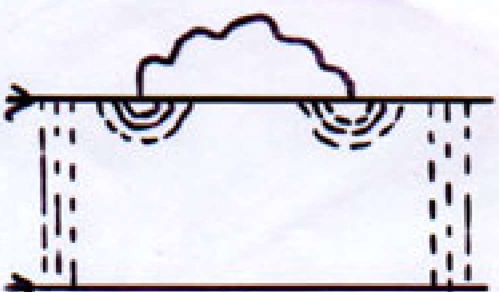




$\Delta(T)$: correction to superconducting self-energy due to collective modes



$\frac{\partial}{\partial \Delta}$ breaks $\rightarrow\!\!\!\leftarrow$ to $\rightarrow \quad \leftarrow$ in all possible ways yielding corrections to pair propagator



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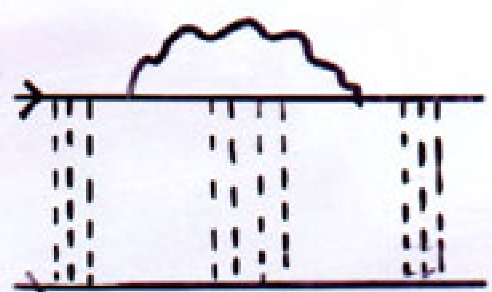

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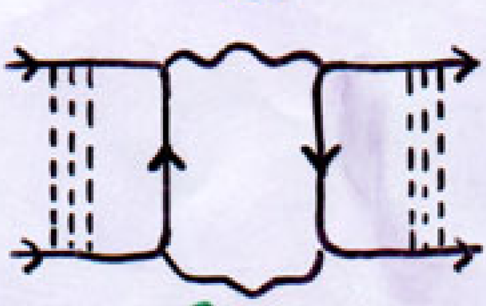

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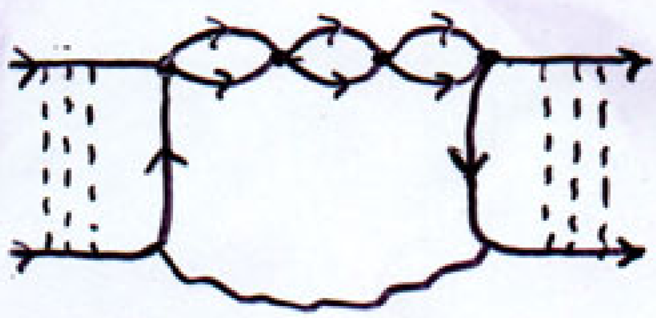


5

arrows mean one
~ is Coulomb,
one is pair propagator

Expressions for Screened Potentials

$$(-\lambda^{-1} + \Pi_{\Delta\Delta}) = \pi N(0) T \sum_{\omega} \left\{ \left[1 + \frac{\omega\omega' - \Delta^2}{WW'} \right] \frac{1}{Dq^2 + W + W'} - \frac{1}{W} \right\}$$

$$(-\lambda^{-1} + \Pi_{\phi\phi}) = \pi N(0) T \sum_{\omega} \left\{ \left[1 + \frac{\omega\omega' + \Delta^2}{WW'} \right] \frac{1}{Dq^2 + W + W'} - \frac{1}{W} \right\}$$

$$\Pi_{\rho\rho} = N(0) - \pi N(0) T \sum_{\omega} \left[1 - \frac{\omega\omega' + \Delta^2}{WW'} \right] \frac{1}{Dq^2 + W + W'}$$

$$\Pi_{\phi\rho} = \pi N(0) T \sum_{\omega} \underbrace{\frac{\Delta\Omega}{WW'}}_{\text{coherence factor}} \underbrace{\frac{1}{Dq^2 + W + W'}}_{\text{diffusion propagator}}$$

$$W = \sqrt{\omega^2 + \Delta^2} \qquad W' = \sqrt{\omega'^2 + \Delta^2} \qquad \omega' = \omega + \Omega$$

The "Magical" Coulomb Cancellation

- In Ambegaokar II it was proved that charge conservation + self-consistency give:

$$\Omega\, \Pi_{\rho\rho}(0,\ell) = 2\Delta\, \Pi_{\phi\rho}(0,\ell)$$

$$\Omega\, \Pi_{\rho\phi}(0,\Omega) = 2\Delta\left[-\lambda^{-1} + \Pi_{\phi\phi}(0,\ell)\right]$$

$$\Rightarrow \underline{\Pi_{\rho\rho}(0,\ell)\left[-\lambda^{-1} + \Pi_{\phi\phi}(0,\ell)\right] + \Pi_{\phi\rho}^{2}(0,\ell) = 0}$$

$$\Rightarrow \underline{\left[(2V_c(q))^{-1} + \Pi_{\rho\rho}(q,\ell)\right]\left[-\lambda^{-1} + \Pi_{\phi\phi}(q,\ell)\right] + \Pi_{\phi\rho}^{2}(q,\ell) = O(q^{d-1})}$$

- All effective potentials $V_{\rho\rho}$, $V_{\phi\phi}$, $V_{\phi\rho} = -V_{\rho\phi}$ have long-range $1/q^{d-1}$ behaviour below T_c.

- However q^{d-1} singularity occurs under the $\ln[\;]$ in correction to free energy: upon differentiating w.r.t. Δ to get self-consistency eqn it vanishes.

- Hence ultimately it is charge conservation that ensures cancellation of long range Coulomb behaviour in T_c eqn.

Strong Coupling + Disorder

- Electron-phonon interaction so strong that quasiparticles no longer long-lived at phonon frequency above F.S.

$$\Rightarrow \quad \omega_D \lesssim \frac{1}{\tau_{qp}(\omega_D)}$$

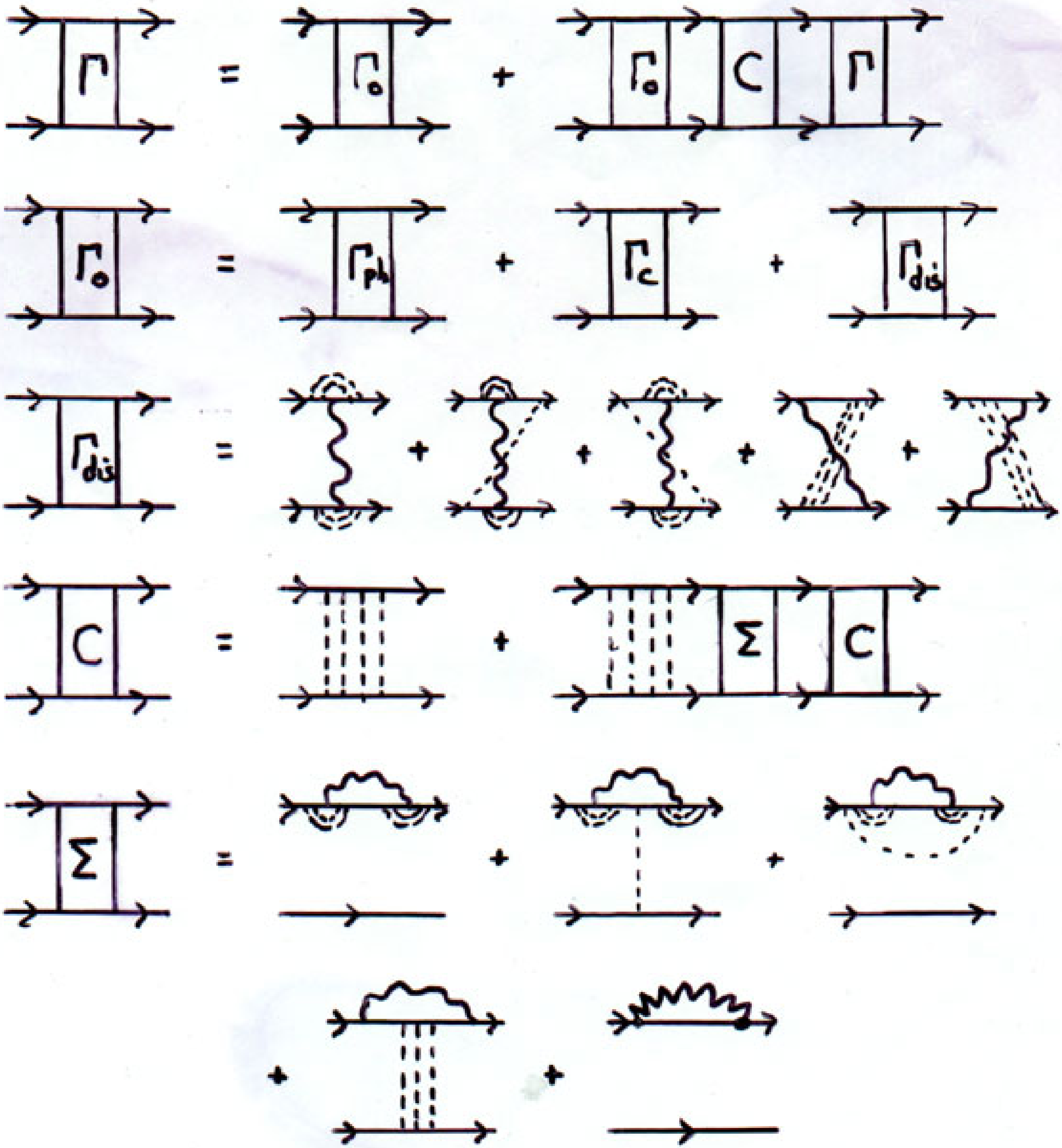
- Details of phonon spectra then become important + Green function methods unavoidable. Average over F.S. so that only phonon frequency structure needed.

- Disordered Coulomb effects are similar as a frequency dependence of potential comes from quantum interference.

- Strong coupling effects should make theory harder, but needed for systems used : Pb, PbBi, MoGe.

- Actually theory SIMPLIFIES - WE'VE BEEN DOING STRONG COUPLING ALL ALONG!!!

Strong Coupling Equations for T_c



- Three types of interaction:
  - Phonons : $\Gamma_{ph}(\omega,\omega') = \lambda(\omega-\omega')$
  - Featureless Coulomb $(k-k' \sim k_F \,;\, \omega-\omega' \sim \varepsilon_F)$

    $$\Gamma_c(\omega,\omega') = -\mu$$
  - Disordered Coulomb $(k-k' \lesssim 1/l \,;\, \omega-\omega' \lesssim 1/\tau)$

    $$\Gamma_{dis}(\omega,\omega') = -t\ln\left[\frac{1/\tau}{|\omega|+|\omega'|}\right]$$

- Strong coupling eqns are then:

$$Z(\omega)\Delta(\omega) = \pi T \sum_{|\omega'|<\omega_c} \left[\lambda(\omega-\omega') - \mu^* - t\ln\left(\frac{1/\tau}{|\omega|+|\omega'|}\right)\right]\frac{\Delta(\omega')}{|\omega'|}$$

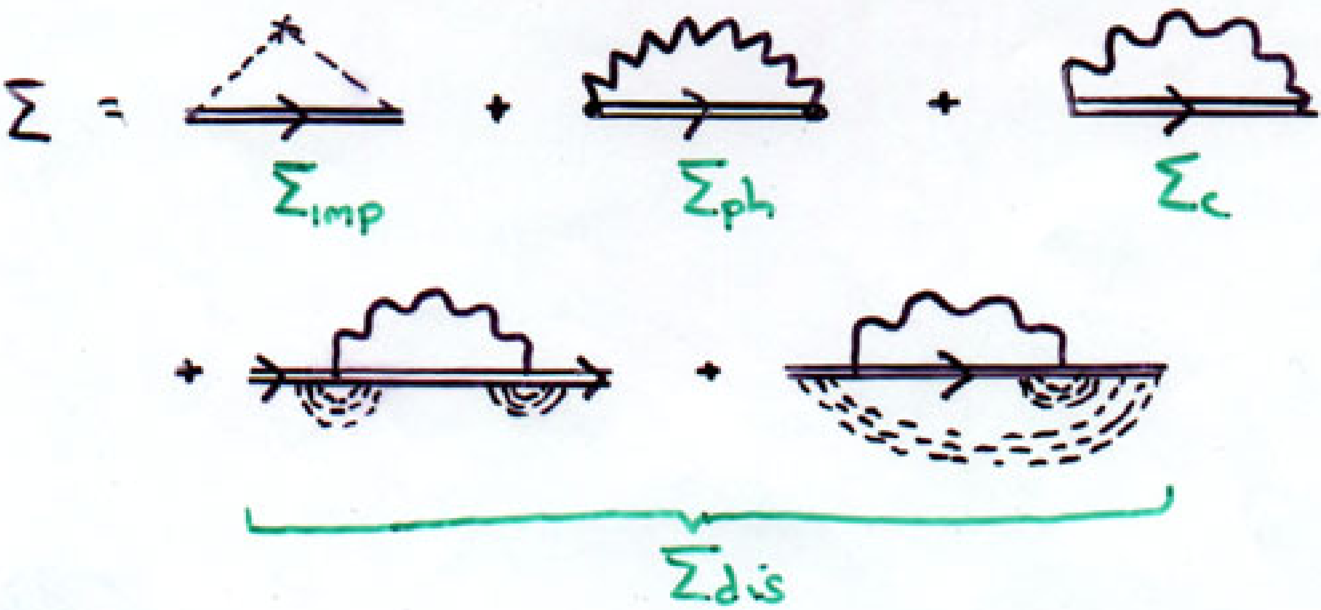
$$[Z(\omega)-1]\,\omega = \pi T \sum_{|\omega'|<\omega_c} \left[\lambda(\omega-\omega')\,\mathrm{sgn}(\omega') - t\omega\frac{1}{|\omega|+|\omega'|}\right]$$

- In the limit $t = R_\square/R_0 \to 0$ this reduces to usual strong coupling theory.

- In the limit $\lambda(\omega,\omega') \to \lambda\,\Theta(\omega_D-|\omega|)\,\Theta(\omega_D-|\omega'|)$ recover nonperturbative theory discussed in Smith II.

Strong Coupling Equations for $\Delta(\omega)$

- Work below T_c using Nambu-Gorkov diagrams including self-energy diagrams for disordered Coulomb deduced from $F(\Delta)$



- Eliashberg eqns below T_c are then

$$\omega\left[Z(\omega)-1\right] = \pi T \sum_{\omega'} \lambda(\omega-\omega')\,\frac{\omega'}{W'}$$

$$-\,t\pi T \sum_{\omega'} \frac{\omega}{W+W'}\left[1 - \frac{\omega\omega' + \Delta(\omega)\Delta(\omega')}{WW'}\right]$$

$$+\,t\pi T \sum_{\omega'} \frac{\Delta(\omega)}{W}\left[\frac{\omega'\Delta(\omega) - \omega\Delta(\omega')}{WW'}\right] \ln\left[\frac{1/\tau}{W+W'}\right]$$

$$Z(\omega)\Delta(\omega) = \pi T \sum_{\omega'} \left[\lambda(\omega-\omega') - \mu^*\right] \frac{\Delta(\omega')}{W'}$$

$$+ t\pi T \sum_{\omega'} \frac{\Delta(\omega)}{W+W'} \left[1 - \frac{\omega\omega' + \Delta(\omega)\Delta(\omega')}{WW'}\right]$$

$$+ t\pi T \sum_{\omega'} \left[\frac{\omega'\Delta(\omega) - \omega\Delta(\omega')}{WW'}\right] \frac{\omega}{W} \ln\left[\frac{1/\tau}{W+W'}\right]$$

- This reproduces previous T_c eqn when we set $\Delta \to 0$ and linearise.

- We therefore have a complete consistent theory to analyse effects of phonon spectrum, Coulomb repulsion and disorder (both magnetic + nonmagnetic) on superconductivity.

- Given $\alpha^2 F(\omega)$, μ^*, $t = R_0/R_0$ we can calculate T_c, $Z(\omega)$, $\Delta(\omega)$. The latter quantities can be accessed via tunneling experiments.

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