Beyond "conventional" spin liquids.

Def^{n} of QSL (in terms of excitations)

- ~~8pin~~ Lattice spin models, finite-range ints, may impose some symmetries, T=0 equilibrium

QSL is a stable phase with ~~exc~~:

(1) fractional q.p. excitations (not locally createable)

or

(2) gapless exc. not from SSB

<u>often</u> also no SSB of microscopic symms. (but we don't require this, e.g. chiral spin liquid)

excludes: conventional SSB, SPT order.

includes: · topologically ordered QSLs (gap, fractional exc (in 2d ~ anyons)) } "conventional"

· ~~many~~ interesting gapless states, some w/o q.p. description, conventional effective QFT description

· fracton state

(1 1/2)

- Theory of fractons is challenging & still in a primitive state
- But things are happening (we are far from being stuck)

Things to do when do you discover (theoretically) new class of phases...

	T-invariant free-fermion top. phases	Fractons	"Conventional" QSLs
Find examples	✓	✓	✓
Phenomena	✓	Some	✓
Characterize	✓	Very little (+ recent (current) progress)	✓
Classify	✓	—	~~A lot~~ Major progress
+ Find realizations	✓	—	Ongoing
	2004 - 2009	2004 — present	

✓ ~~gapped~~ in some cases (but not gapless QSLs)

Not exclusive list but covers many big questions.

"Pros & cons of living in interesting times."

- CM theory (+ theoretical Q-physics) rapid developments over past 20+ years. Many major new theoretical discoveries. Exciting!
- The community moves fast. Easy to get the impression that if most people move on from a topic, it's "done". This is wrong & fractons is an example.
  ↳ as far as qualitatively new things vs. details
- Some problems are challenging/subtle, take more time (not same as being stuck)

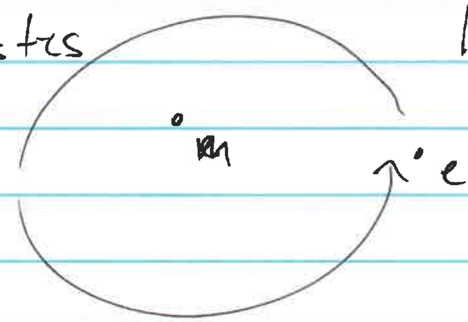
Bottom line: you have to decide for yourself what's interesting

Example: "$\mathbb{Z}_2$ QSL" ($d=2$ spatial dim)

- Gap to all excitations.
- Three non-trivial types of exc: e, m, $f = \underbrace{e+m}_{\text{fuse } e \,\&\, m \text{ together.}}$

Fractionalized means: not locally creatable $\overset{e}{\cdot}\!\sim\!\underset{S_e}{\sim}\!\overset{e}{\cdot}$

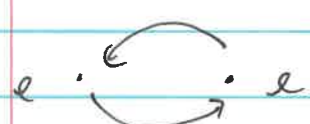
Braiding statistics



$|\psi\rangle \to e^{i\Theta_{em}}|\psi\rangle$.

$\Theta_{em} = \pi$

Also $\Theta_{ef} = \Theta_{mf} = \pi$.



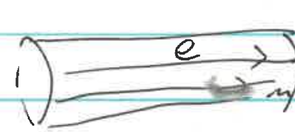
$\theta_e = \theta_m = 0 \quad \theta_f = \pi$.

Realized in e.g. $\mathbb{Z}_2$ toric code, by certain effective theories of spin models ($\mathbb{Z}_2$ gauge theories)
Kitaev honeycomb model (~~...~~ if suff. anisotropic)

⋮

Often in many examples, In $SO(3)$-inv't systems, ~~...~~ e and f carry half-odd integer spin (or integer spin). Example of ~~...~~ symmetry ~~...~~ fractionalization.

GSD $= 4$ on torus



$S_e = \pm 1$
$S_m = \pm 1$.

[Depends on b.c! If $e \leftrightarrow m$ when passing thru bdy, GSD $= 2$. In infinite Euclidean plane, GSD $= 1$ (unique!)
→ GSD not a bulk property!!!

(3)

Effective field theory: TQFT (can be described as a CS-theory).

→ Brief on abstract TQFT

~~[illegible]~~

Another example "Gapped U(1) QSL", $d=3$

Effective theory is a U(1) gauge theory

$$H_{eff} = \underbrace{E^2 + B^2}_{\text{gapless photon}} + \underbrace{H_{charge}}_{\text{gapped e-charge}} + \underbrace{H_{monopoles}}_{\text{gapped magnetic charge}}.$$

charges ρ requires closed e only... cannot be done by local process inside V.

Both charges & monopoles are fractional. → argument $\int dV \rho = \int dV\, \nabla\cdot e = \int d\vec{a}\, \vec{e}\cdot\vec{da}$

"Quantum spin ice" realization; ~~[illegible]~~

$S^z = \pm\frac{1}{2}$ interpret as vector field on diamond lattice

$\vec{e}$

Exact mapping to U(1) gauge theory w/ matter, $\nabla\cdot e = \rho$

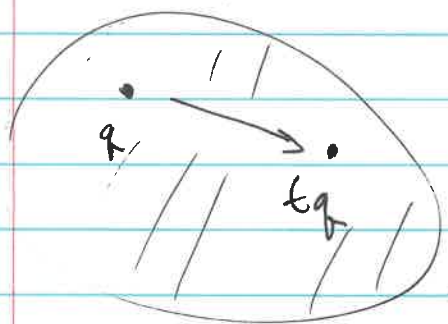
$$-\sum_{\langle ij \rangle} S_i^z S_j^z \leadsto \sum_i (\nabla\cdot e)_i^2$$

matter is gapped,

If this term dominates, low-energy dynamics is pure gauge theory, can be in deconfined phase ⇝ U(1) QSL.

Actually happens in realistic spin models, maybe in some $A_2B_2O_7$ pyrochlores

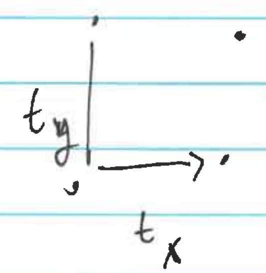
Both examples: ~~charges~~ excitations are <u>mobile</u>. Assume translation symmetry T.



$t \in T$ is a translation

local operator U destroys q & creates tq.

Not quite always possible: e.g. $\mathbb{Z}_2$ QSL

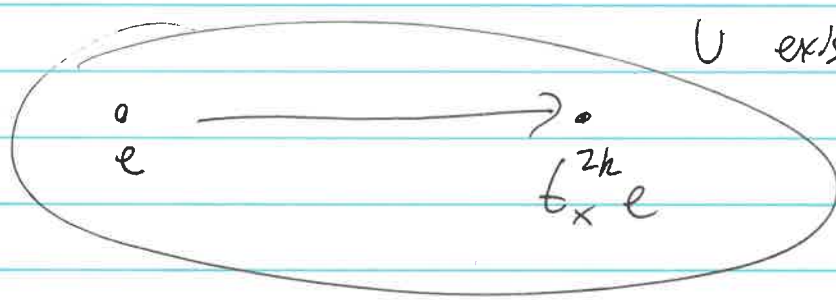


$t_x e \simeq m \quad t_x m \simeq e$

$t_y e \simeq m \quad t_y m \simeq e$

$\Rightarrow$ No local op exists to transform $e \to t_x e$ (or $t_y e$)

But $t_x^2 e \simeq e$, so e is still mobile along x



U exists.

~~Mobility is a long distance property~~

"mobility" $\Leftrightarrow$ Excitations are mobile over arbitrarily long distances.

Fracton example I: scalar charge theory (Pretko 2016)

- $d=3$
- Tensor E-field $E_{\mu\nu} = E_{\nu\mu}$ $\quad \mu,\nu = x,y,z$.
- Gauss Law: $\partial_\mu \partial_\nu E_{\mu\nu} = \rho$
- Effective Hamiltonian for deconfined phase

$$H_{eff} = E^2 + \underbrace{B^2}_{} + \underbrace{H_e + H_m}_{\text{dynamics for gapped e \& m charges.}}$$

"B" constructed from tensor potential $A_{\mu\nu}$

$$A_{\mu\nu} \to A_{\mu\nu} + \partial_\mu \partial_\nu \phi$$

$$B_{\mu\sigma} = \varepsilon_{\mu\nu\lambda} \partial_\nu A_{\lambda\sigma}$$

- Gapped electric & magnetic charges, gapless "photon".
- Can be realized on cubic lattice, $E_{\mu\mu}$ on sites
  E_{xy}, E_{yz}, E_{xz} on faces

$$H = \sum (\partial\partial E)^2 + \cdots$$

- Not a ~~realistic~~ microscopically realistic model, but conceptually similar to quantum spin ice, results in stable phase.

Like in U(1) vector gauge theory, consider

$$Q_V = \int_V \rho \, dV = \int_V \partial_\mu \partial_\nu E_{\mu\nu} \, dV = \int_A \partial_\nu E_{\mu\nu} \cdot dn_\mu \quad (V)_A$$

$\Rightarrow$ Q_V is "conserved" (cannot be changed by local ops)

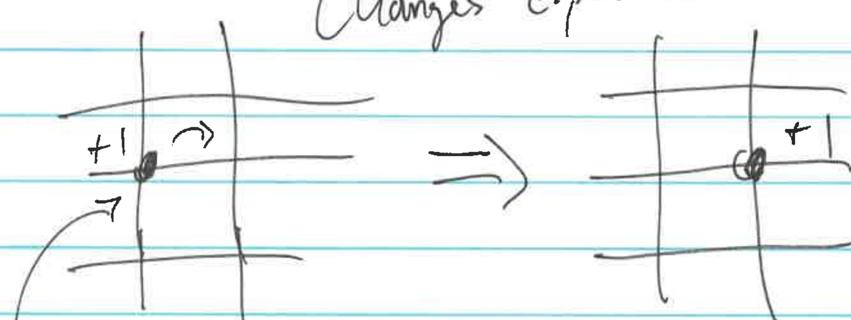
But now consider dipole moment:

$$D^V_\mu = \int_V x_\mu \, \rho \, dV = \int_V x_\mu \, \partial_\nu \partial_\lambda E_{\nu\lambda} \, dV \overset{\text{int. by parts twice}}{=} \int_A f_\mu(E) \cdot dn_\nu$$

$\Rightarrow$ D^V_μ also conserved.

Consequence:

Changes dipole moment (by $a\hat{x}$)



$Q = +1$

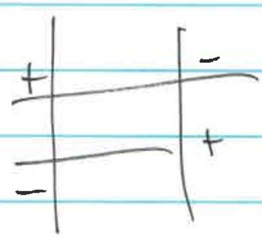
No local process can move a charge

... by any distance

⇒ Gauge charges in scalar charge theory are "fractons": can't move in isolation by any local process.

"Cooperative mobility"

- Quadrupole moment not conserved, most local processes moving charges are:



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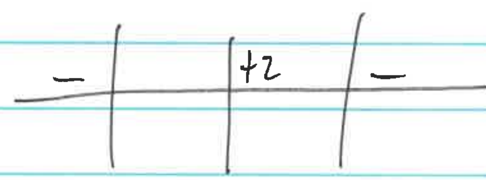
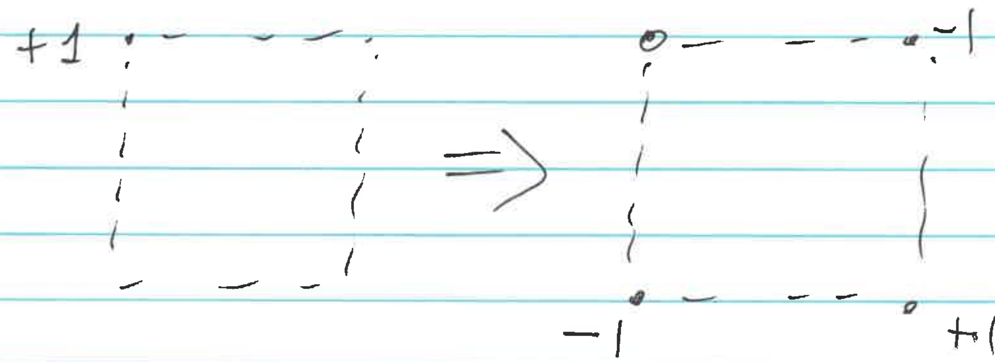


- (Can be viewed as dipole hopping processes)

- Single charges can move by creating more charges, and/or groups of charges can move.

e.g.



Misconception: "Tensor gauge theories ~~were~~ advance fracton theory because they give continuum QFT realizations of fractons." People said this kind of thing soon after Pretko. ~~Th~~ What actually happened is that this motivated work on weird continuum theories.

~~All~~ Misleading at best. [illegible] "is" a continuum theory, but its relationship w/ lattice tensor gauge theory is not so clear.

- Photon-only continuum theory $H = E^2 + B^2$, no issues.
- Continuum theory with electric charges has continuous ∞ of excitation types (put charge anywhere in space). Only countable ∞ on lattice.
- On lattice, # $Q=1$ charge excitations within fixed volume V below fixed energy is finite, ... is ∞ in continuum theory.

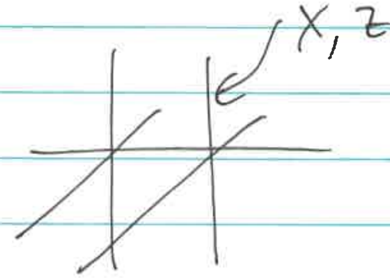
Continuum vector gauge theories $\xrightarrow{\text{effective QFTs of}}$ Lattice vector gauge theories $\xrightarrow{\text{effective models of}}$ QSLs

Continuum tensor gauge theories $\rightsquigarrow$?? Lattice tensor gauge theories $\longrightarrow$ Some fracton phases

Why tricky? Continuum limit, get continuous ∞ of different types of excitation, ~~[illegible]~~ on ∞ lattice only countable ∞

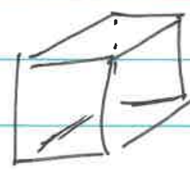
Fracton example II : X-cube model (Vijay, Haah & Fu 2016)

- $d=3$, commuting Pauli Hamiltonian on simple cubic lattice

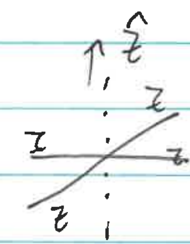


$$H = -\sum_c B_c - \sum_v \sum_{\mu = x,y,z} A_v^\mu .$$

$$B_c = \prod_{\ell \in \text{cube } c} X_\ell$$



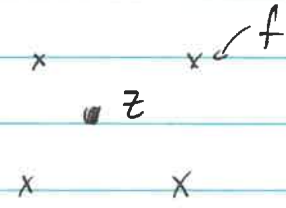
A_v^z



$[A_v^\mu, B_c] = 0$ g.s.: $B_c = A_v^\mu = 1$

- Unique g.s. infinite space
- On $L\times L\times L$ 3-torus, $\log_2 \text{GSD} = 6L-3$, top. order

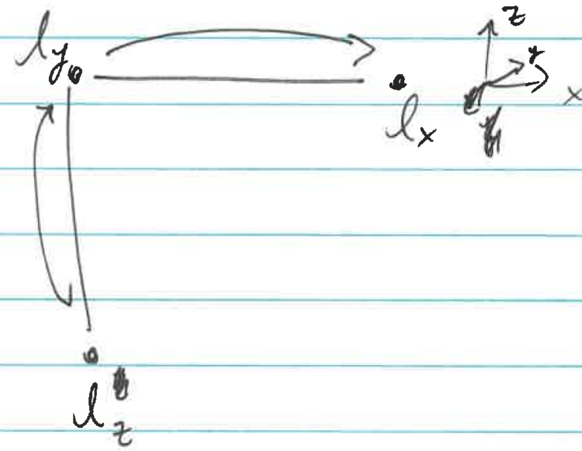
Excitations: $B_c = -1 \rightsquigarrow$ fractons f

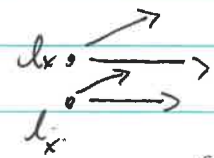


"fracton dipole" $\rightsquigarrow$ "planon" moves in plane $\perp z$.

Local constraint $A_v^x A_v^y A_v^z = 1$

$A_v^x = A_v^y = -1$ $A_v^z = 1$ lineon ℓ_z





"linear dipole" again a planon.

- Can prove: f is not composite of lineons + planons } "intrinsic" fractons + lineons.
  l " " " " planons }

<u>Summarize</u>: • fully gapped state in $d=3$, only point-like excitations (no loops like 3d toric code)

- Not described by the (conventional) TQFT
  - GSD depends on L
  - Need foliation structure to define on spatial mfld (foliated TQFT?)

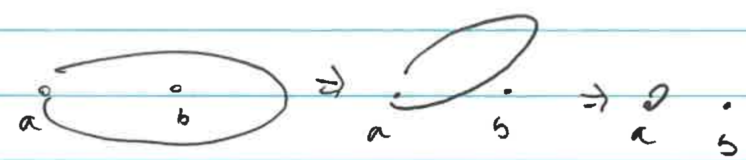
Important point: Gapped phase $\neq$ $\underbrace{\text{topological phase}}_{\text{means } \exists \text{ TQFT description}}$

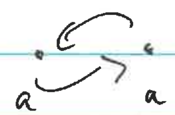
(used to be conventional wisdom)

11

Statistics

Mobile particles in $d=3$:
No mutual statistics

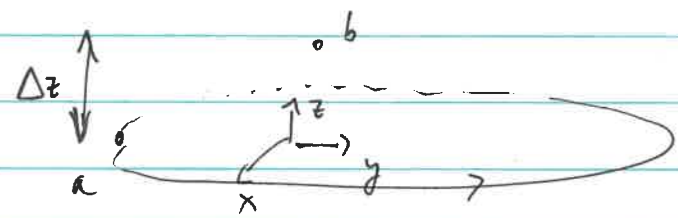




$\theta_a = \pm 1$ only (two exchanges can be deformed to no exchange)

Changes ~~fda~~ with restricted mobility.

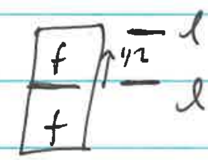
Simple: planon - planon statistics (planons of same orientation)



$\Theta_{a,b}$ can be nontrivial (but should $\to 1$ as $|\Delta z| \to \infty$).

X-cube: a = fracton dipole, b = lineon dipole

$$\Theta_{a,b} = \begin{cases} -1 & \text{if } |\Delta z| = \frac{1}{2} \\ 1 & \text{otherwise} \end{cases}$$



f-l, l-l, ~~fd~~ f-f : use slides

~~X-1 statistics can be used to prove X-cube & semionic X-cube models in different phases~~

~~Subtle - Need to consider whole excitation spectrum (see paper)~~

~~- Depends on how phase is defined.~~

~~"Standard" def^{n} of gapped phase (rough)~~

~~∘ Deform H without closing gap (or 1st order phase transition)~~
~~∘ Allow for stacking with trivial product states~~
~~↳ [Why? "Continuum approximation principle"~~
~~[Lattice models are idealizations of continuum systems]~~

<u>Characterizing gapped fracton phases</u> (with Evan Wickenden + others)

Wanted: <u>universal props of phases</u>, easily comparable

→ tricky: multiple def^{n}s in literature.
Claim: solved.

also tricky.

Focus on exc, generalize (A, θ).

$\rightsquigarrow (S, ?)$

• S is a big object, need simpler "invariants"
• How to include statistics?

- QSS

To make more progress, need simpler classes of fracton orders

"planar-only"  Not just decoupled layers.

$$\tilde{e}_i^z \, \tilde{m}_{i+1}^z = e_i^A \, e_{i+1}^B$$