Deterministic entanglement of trapped atomic ions II

NIST, Boulder, Ion Storage group:

M. Barrett (postdoc, Georgia Tech.)†
J. C. Bergquist (NIST)
B. Blakestad (student, CU)
J. J. Bollinger (NIST)
J. Britton (student, U. Colorado)
J. Chiaverini (postdoc, Stanford)
B. DeMarco (postdoc, U. Colorado)‡
W. Itano (NIST)
B. Jelenković (guest, Blegrade)¶
M. Jensen (U. Colorado)
J. Jost (student, U. Colorado)
E. Knill (NIST, computation Div.)
C. Langer (student, U. Colorado)
D. Leibfried (NIST)
W. Oskay (postdoc, U. Texas)
R. Ozeri (postdoc, Weizmann)
T. Rosenband (U. Colorado)
T. Schätz (postdoc, MPQ)
P. Schmidt (postdoc, Stuttgart)
D. J. Wineland (NIST)

† Present address: Otago University, NZ
‡ Present address: U. Illinois
¶ Present address: J.P.L.

Other ion groups pursuing entanglement:

Aarhus
Garching (MPQ)
Hamburg
Innsbruck
LANL
London (Imperial)
McMaster (Ontario)
Michigan
Oxford
Teddington (NPL)

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Deterministic entanglement of trapped atomic ions II

NIST, Boulder, Ion Storage group:

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M.

Other ion groups

ment:

entanglement-enhanced metrology:

- uncertainty relations
- improved interferometry

  e.g., Ramsey method of spectroscopy
- quantum information mapping
- improved detection efficiency

Oxford
Teddington (NPL)

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Uncertainty Relations:

Uncertainty relation for operators:

For operators $\tilde{O}_1$, $\tilde{O}_2$, Schwartz inequality $\Rightarrow$

$$\Delta\tilde{O}_1\Delta\tilde{O}_2 \geq \tfrac{1}{2}|\langle[\tilde{O}_1,\tilde{O}_2]\rangle| \quad (1)$$

e.g. for $\tilde{O}_1 = x$, $\tilde{O}_2 = p$ $\Rightarrow$ position/momentum uncertainty relation

Uncertainty relations for parameters and operators:

For operator $\tilde{O}(\lambda)$ ($\lambda \equiv$ parameter)

$$\Delta\tilde{O} \simeq |d\langle\tilde{O}\rangle/d\lambda|\,\Delta\lambda \quad (2a) \quad \Rightarrow \quad \boxed{\Delta\lambda \simeq \Delta\tilde{O}/|d\langle\tilde{O}\rangle/d\lambda|} \quad (2b)$$

Example: In (1), let $\tilde{O}_1 = \tilde{O}$, $\tilde{O}_2 = \mathbf{H}$ (Hamiltonian) $\Rightarrow$ $\Delta\tilde{O}\Delta\mathbf{H} \geq \tfrac{1}{2}|\langle[\tilde{O},\mathbf{H}]\rangle|$

But, $\hbar\, d\tilde{O}/dt = i[\mathbf{H},\tilde{O}] + \hbar\, \partial\tilde{O}/\partial t$ $\Rightarrow$ $\Delta\tilde{O}\Delta\mathbf{H} \geq \tfrac{1}{2}\hbar\, |d\langle\tilde{O}\rangle/dt|$ (for $\partial\tilde{O}/\partial t = 0$)

From (2a), $\Delta\tilde{O} = |d\langle\tilde{O}\rangle/dt|\,\Delta t$, $\Rightarrow$ $\Delta H\, \Delta t \geq \tfrac{1}{2}\hbar$ (time/energy uncertainty relation)

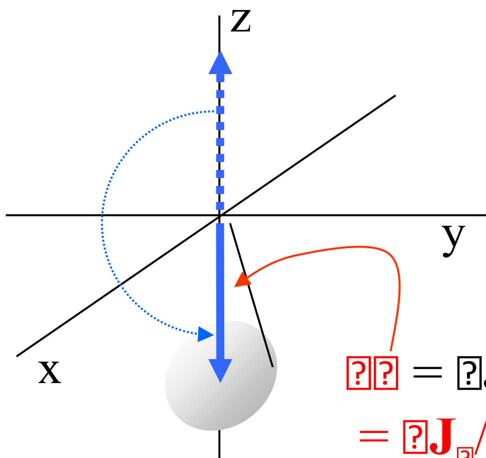
Quantum limits to (spin) rotation angle measurement

$\mathbf{J} = \sum_i \mathbf{S}_i$ (spin ½ particles, equivalent to ensemble of two-level systems (e.g., atoms))

(Feynman, *et al.* J. Appl. Phys. **28**, 49 (1957)

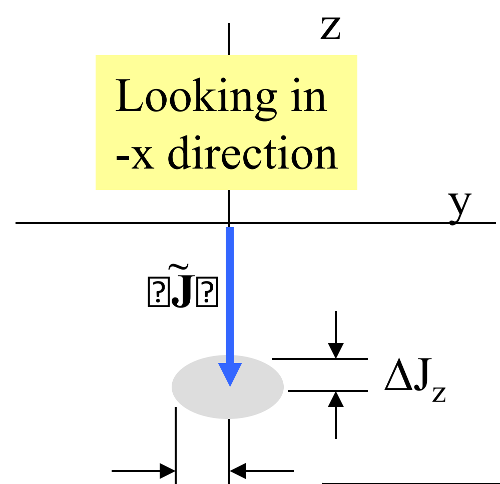
consider rotations about x axis:

$$\Delta\theta_x = \Delta \mathbf{J}_y / |d\langle \mathbf{J}_y \rangle / d\theta_x| = \Delta \mathbf{J}_y / |\langle \mathbf{J}_z \rangle|$$



$$\Delta\theta = \Delta \mathbf{J}_y / \langle \mathbf{J}_z \rangle$$
$$= \Delta \mathbf{J}_\perp / |\langle \mathbf{J} \rangle|$$

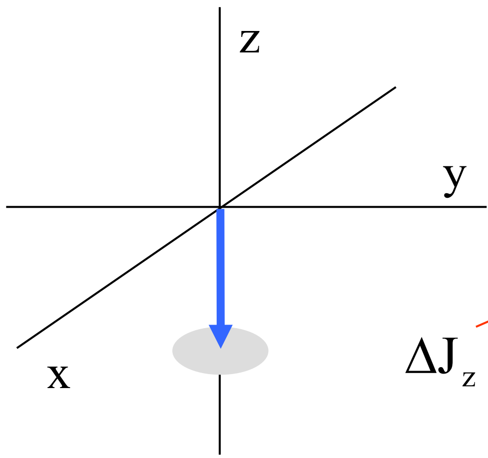
uncertainty "blob"

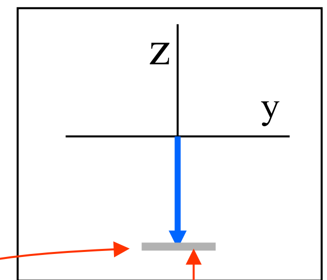


$$\Delta J_y = \sqrt{\langle (J_y - \langle J_y \rangle)^2 \rangle}$$

characterizes fluctuations of final measurements
on a series of identical measurements

Many spins (or atoms): Typical case (e.g., all atoms prepared in ground state):
$|\psi\rangle = |\downarrow\rangle|\downarrow\rangle\ldots|\downarrow\rangle_N = |J = N/2,\ m_J = -N/2\rangle$ ("coherent spin state")





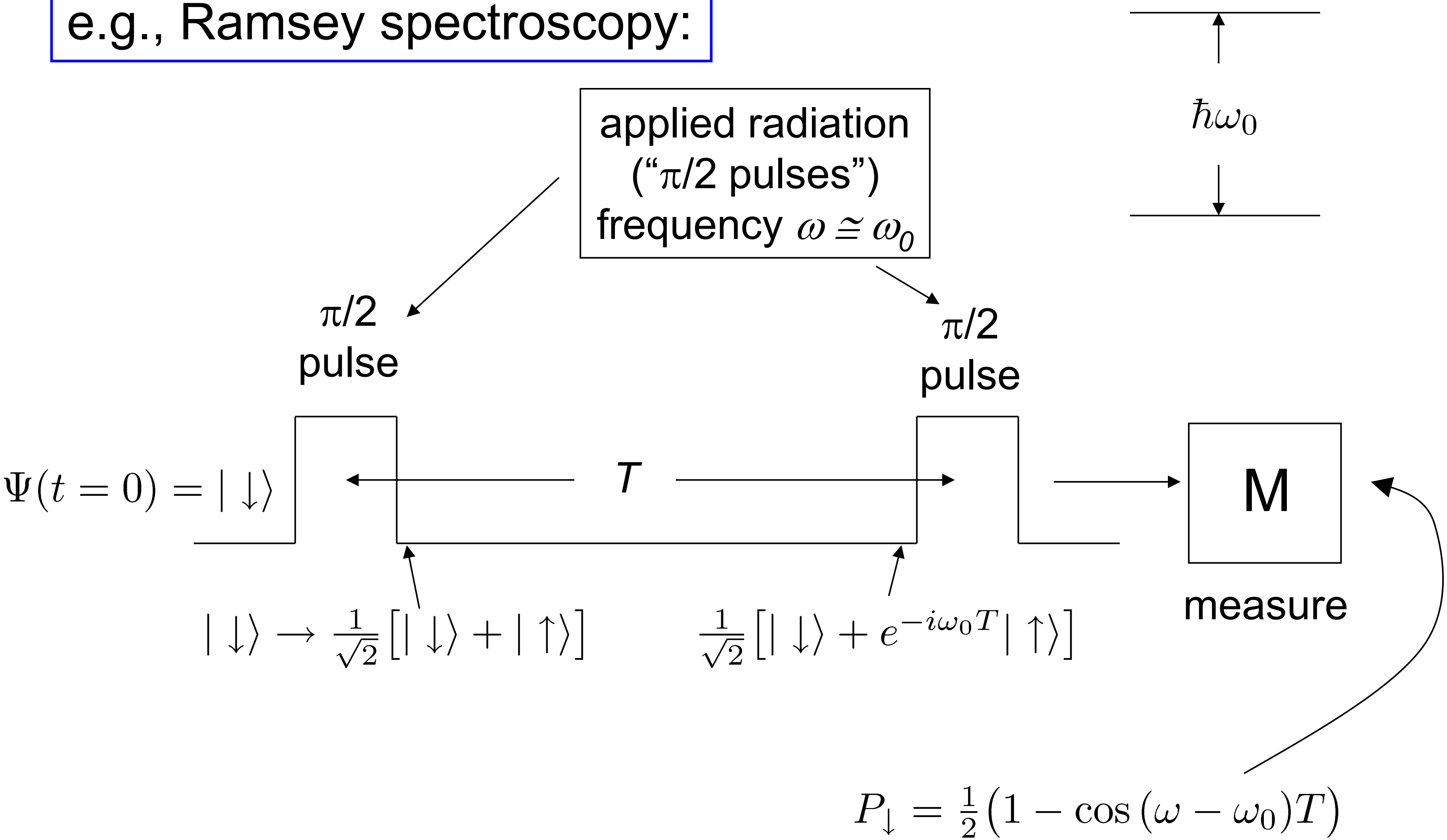
$$\Delta J_z = 0, \qquad \Delta J_x = \Delta J_y = \sqrt{J/2} = \tfrac{1}{2}\sqrt{N}$$

$$|\langle \mathbf{J_z} \rangle| = J = N/2$$

$$\boxed{\Delta\theta = \frac{1}{\sqrt{N}}}$$

"standard quantum limit"
or: "shot noise limit"

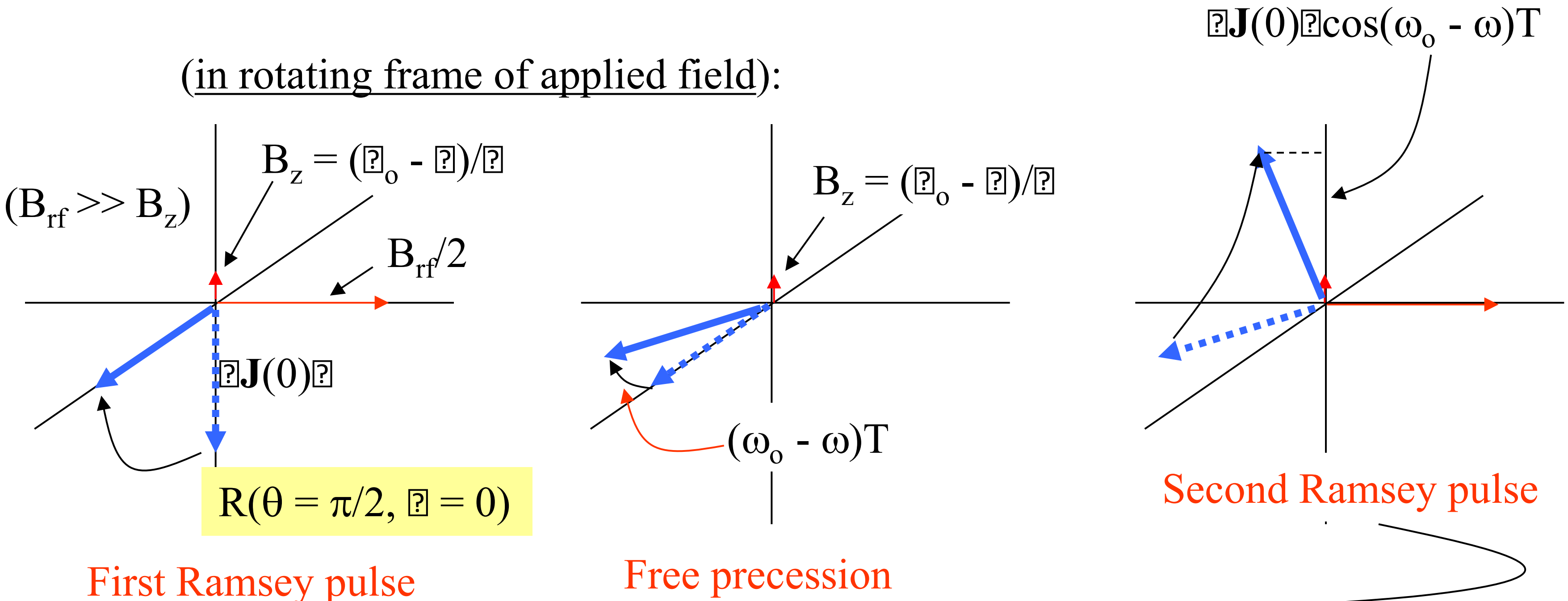
Interferometry
e.g., Ramsey spectroscopy:



Ramsey spectroscopy (equivalent spin-½ picture):

H_i = -**□**$_{\mathbf{i}}$•**B** = ħ□$_o$$S_{zi}$ (S_i = ½), □$_o$ = □B **B** = **z** B

J = □$_i$ **S**$_i$ □$_{intial}$ = □↓□□□↓□…□↓□$_N$ = □J = N/2, m_J = -N/2□ (“coherent spin state”)



Measure number of spins in |↑⟩ state:

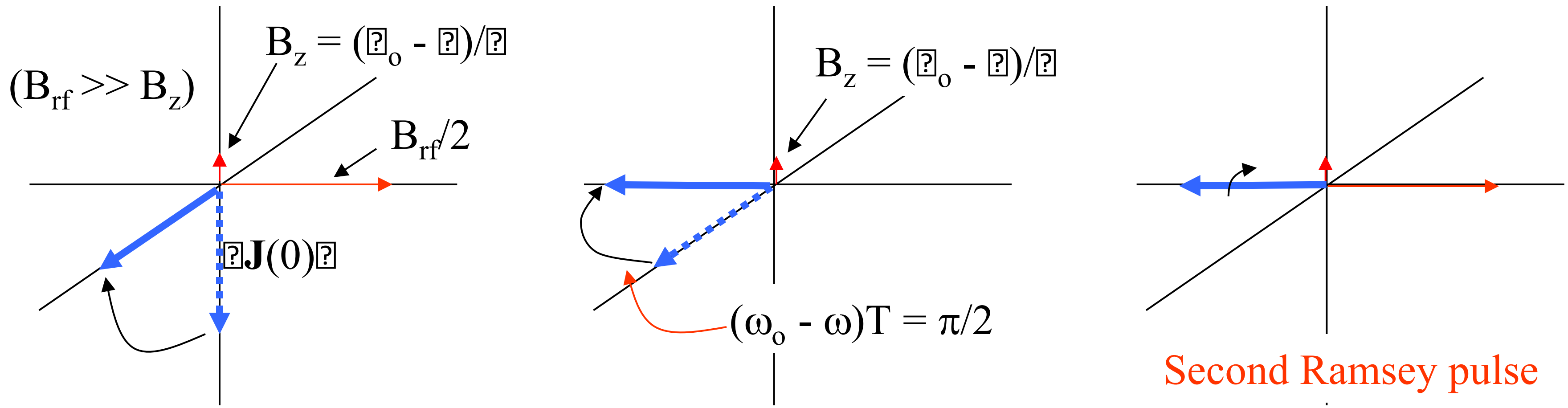
Operator Õ = $\tilde{N}_{\uparrow}(t_f)$ = $\hat{J}_z$ + J$\hat{I}$ $\langle \tilde{N}_{\uparrow}(t_f) \rangle$ = N/2(1 + cos(ω_o - ω)T)

in laser experiments, laser fields lead to effective B_{rf}

J = □$_i$ **S**$_i$ (S_i = ½), □$_{intial}$ = □↓□□↓□…□↓□$_N$ = □J = N/2, m_J = -N/2□ (“coherent spin state”)

$$\boxed{(\omega_0 - \omega)T = +\pi/2}$$

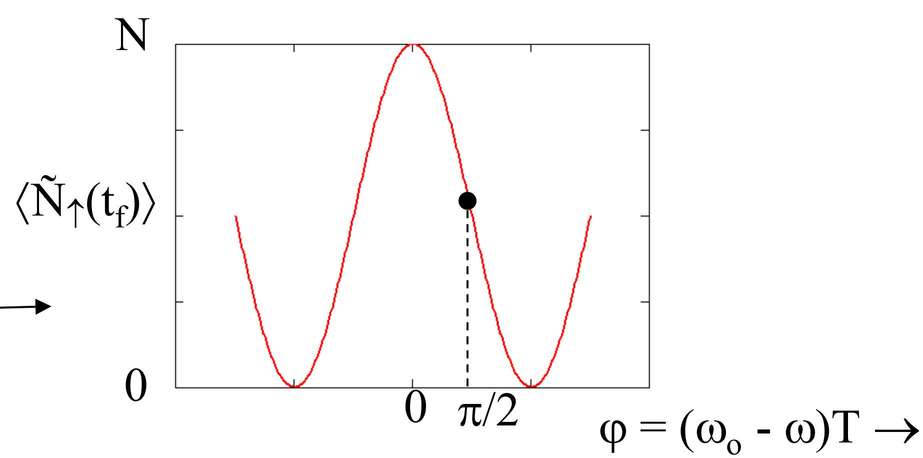
(in rotating frame of applied field):



First Ramsey pulse

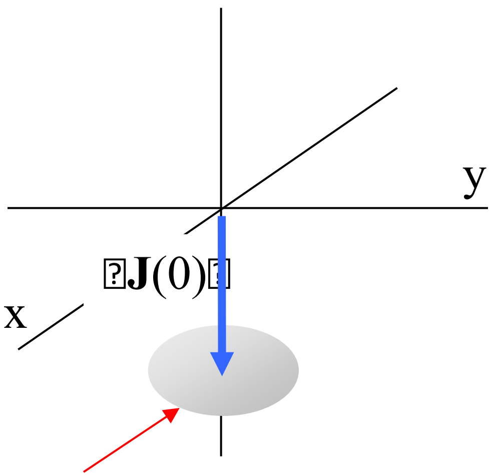
Free precession

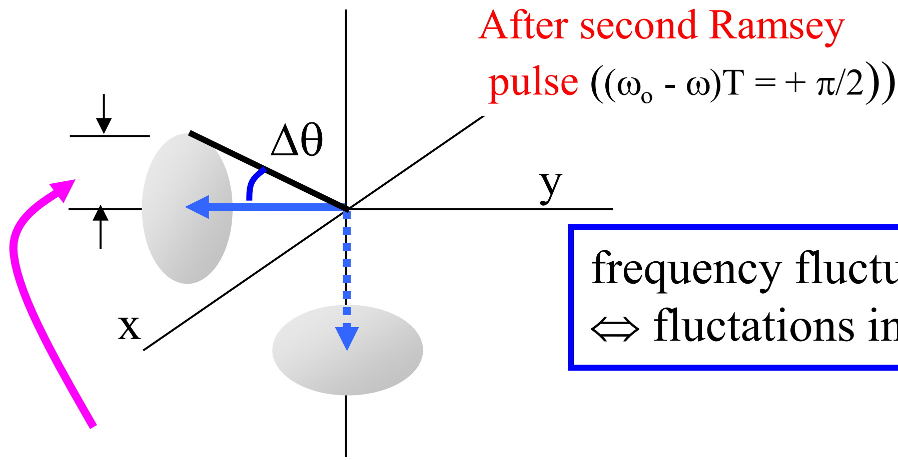
$\langle\tilde{N}_\uparrow(t_f)\rangle = N/2(1 + \cos(\omega_o - \omega)T)$



quantum noise:

For $\psi(0) = |J, -J\rangle$ ("coherent" spin state)



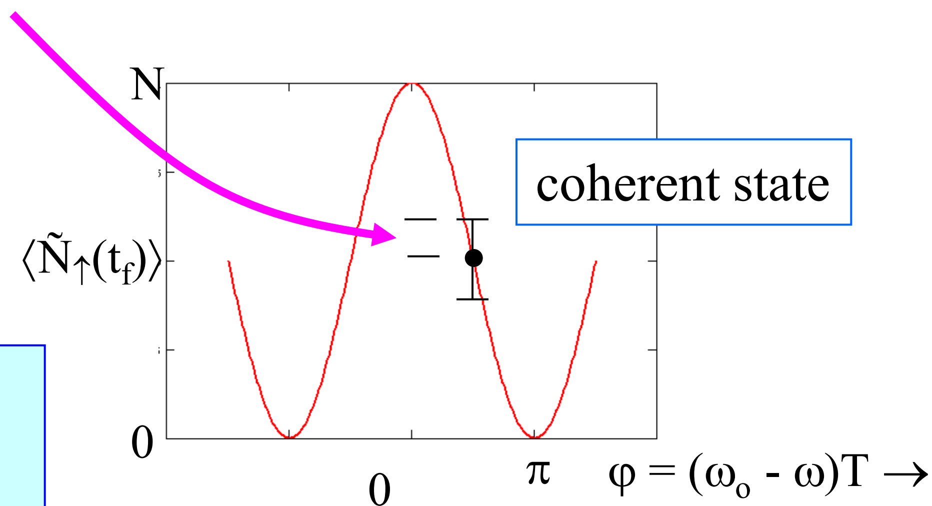


frequency fluctuations
$\Leftrightarrow$ fluctations in θ

$$\Delta\tilde{N}_{\uparrow}(t_f) = \Delta J_z(t_f) = \Delta J_y(0)$$

$$\Delta J_z(0) = 0,$$

$$\Delta J_x(0) = \Delta J_y(0) = \sqrt{J/2} = \tfrac{1}{2}\sqrt{N}$$



$\Delta\phi = \Delta\{(\omega_o - \omega)T\} = N^{-1/2}$

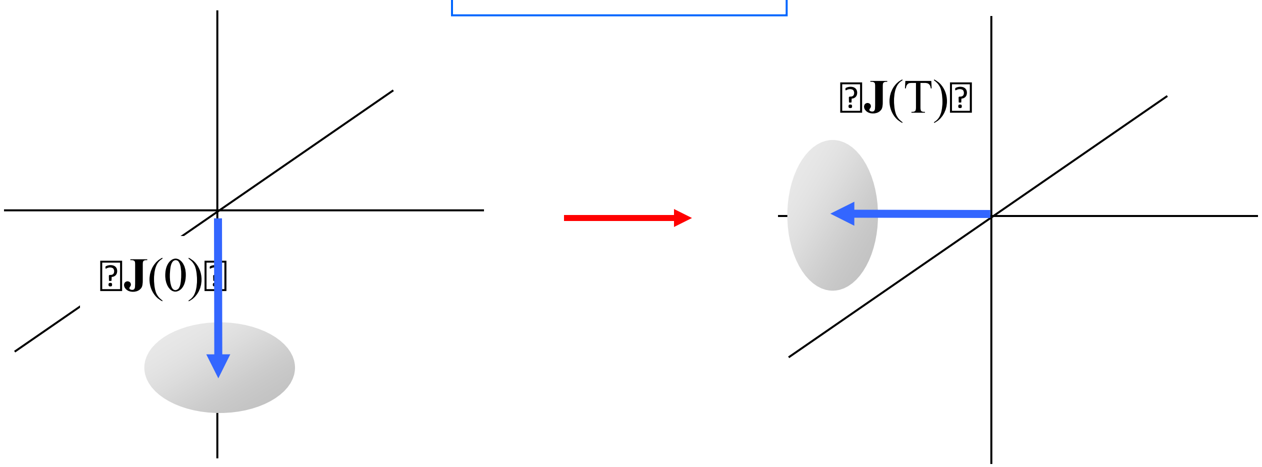
observed for coherent spin states:

- Itano *et al.*, Phys. Rev. A47, 3554 (1993).
- Santarelli *et al.*, Phys. Rev. Lett. **82**, 4619 (1999).

"projection noise"

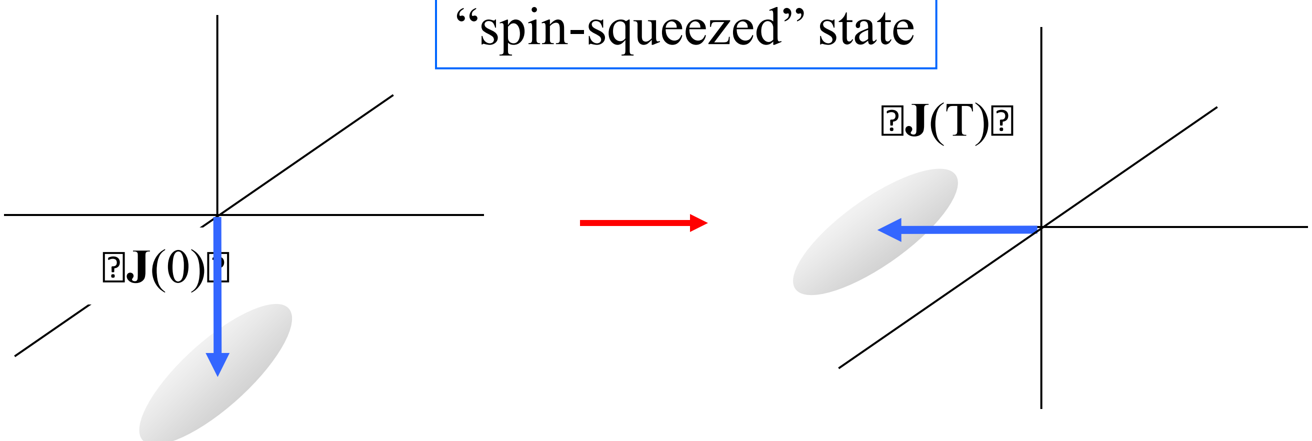
$\Delta\phi = N^{-1/2}$ independent of $\omega_o - \omega$

coherent state



WANT:

“spin-squeezed” state

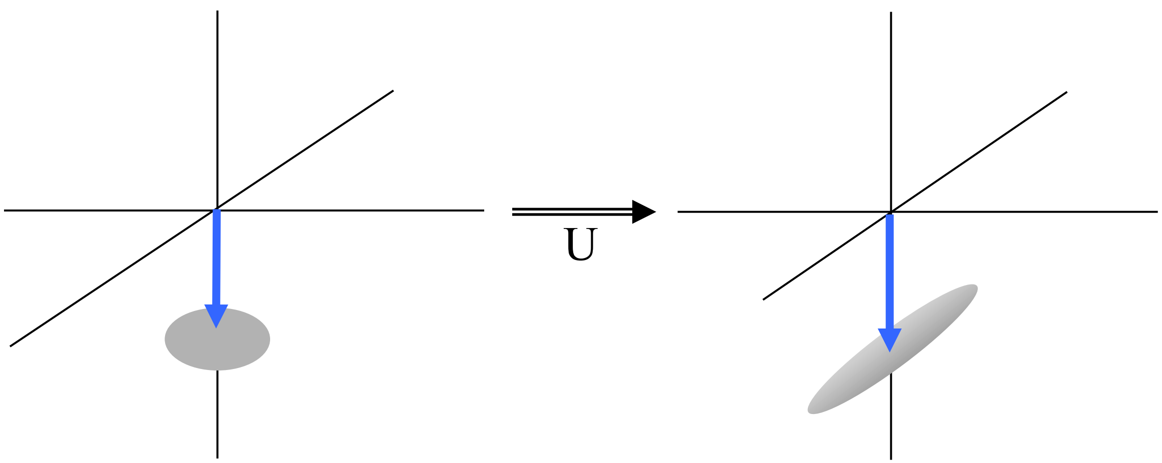


Yurke *et al.*, PRA**33**, 4033 (1986)

Generate spin squeezing with $H_I = \hbar\chi J_x^2, \Rightarrow U = \exp(-i\chi t J_x^2)$

- Sanders, Phys. Rev. A**40**, 2417 (1989)
  (nonlinear beam splitter for photons)
- Kitagawa and Ueda, Phys. Rev. A**47**, 5138 (1993)
  (potentially realized by Coulomb interaction in electron interferometers)
- Milburn, Schneider and James, Fortschr. Physik **48,** 801 (2000)
  (trapped ions)
- Sørensen & Mølmer, Phys. Rev. A**62**, 022311 (2000)
  (trapped ions)

$J_x^2 = J_z^2$ (in rotated basis). Can implement J_z^2 with geometric phase gate (previous lecture).



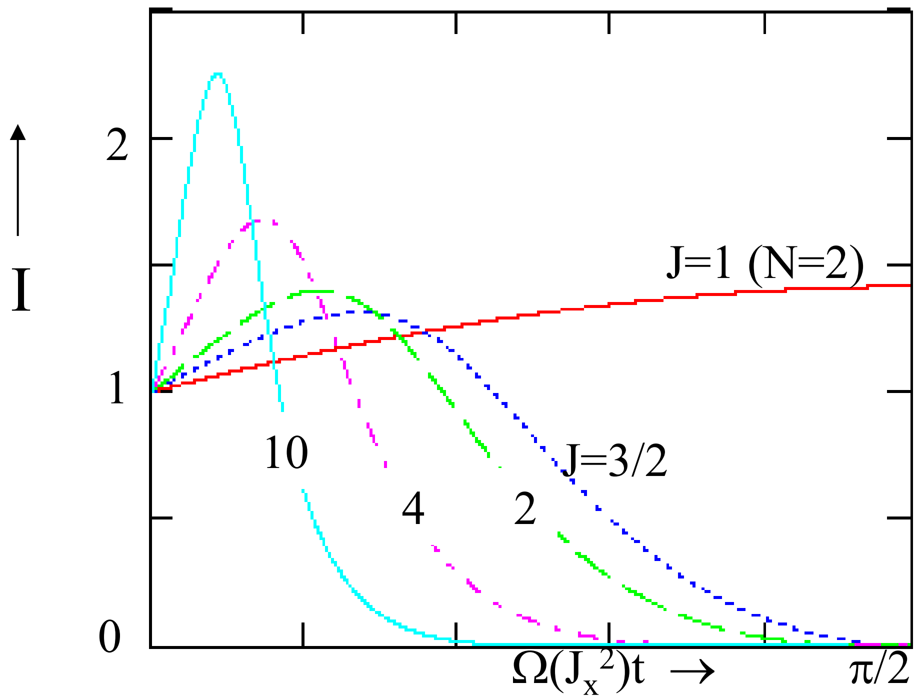
Improvement?

$$I = \frac{\Delta\theta(\text{coheren t})}{\Delta\theta(\text{squeezed})} = \frac{1/\sqrt{2J}}{\Delta J_\perp / \left|\langle \vec{J} \rangle\right|}$$

$H_I = \hbar ? J_x^2$ (or $\hbar ? J_z^2$)

Improvement in S/N: $I = \dfrac{\Delta\theta(\text{coherent})}{\Delta\theta(\text{squeezed})} = \dfrac{1/\sqrt{2J}}{\Delta J_\perp / \left|\langle \vec{J} \rangle\right|}$

Kitagawa and Ueda (Phys. Rev. A**47**, 5138 (1993))



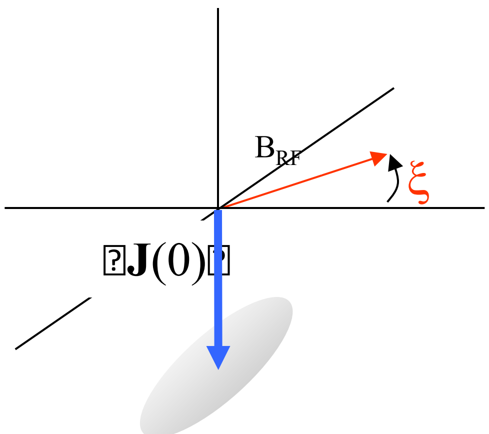
For large N,
$I_{max} \approx 0.78 N^{0.35}$
integration time $\tau_{avg.}$
reduced by ~ $(0.78)^2 N^{0.7}$

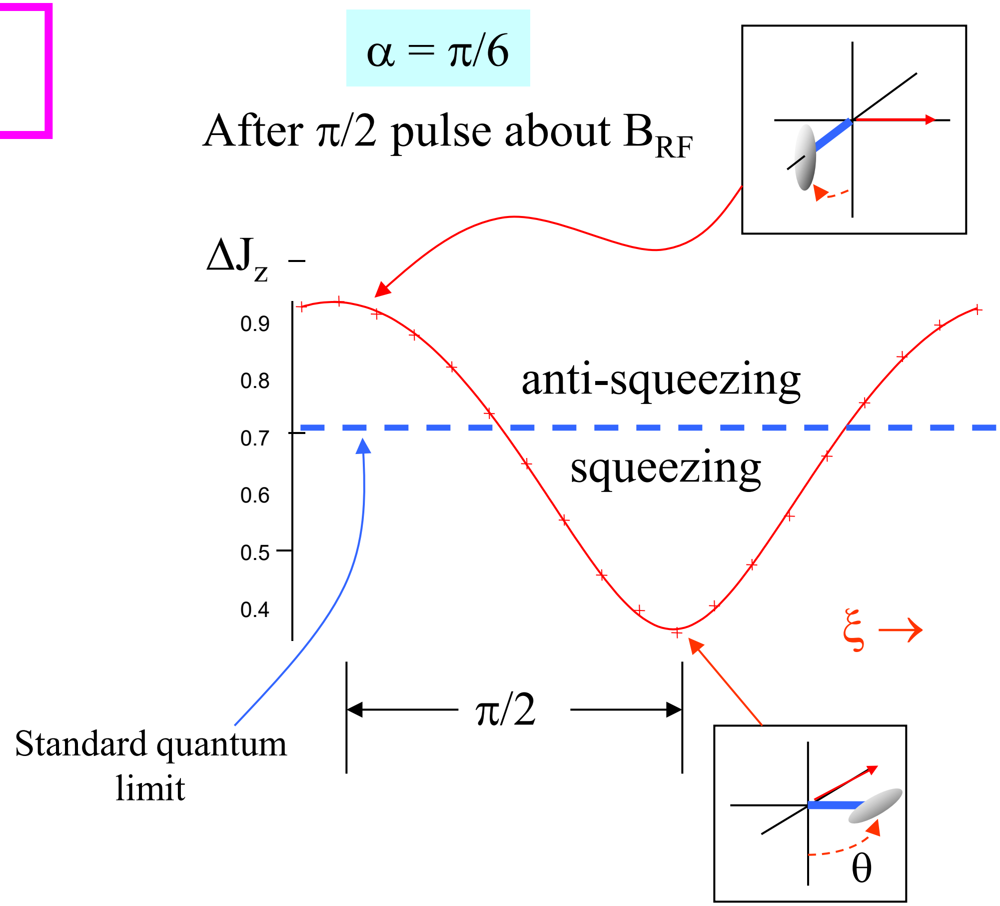
signal-to-noise ratio: $\propto \dfrac{1}{\Delta\theta} \times \sqrt{N_{\text{experiments}}} \propto \dfrac{1}{\Delta\theta} \times \sqrt{\tau_{\text{avg.}}}$

Simple experiment, N = 2 ($^9Be^+$): Meyer *et al.*, PRL **86**, 5870 (2001)

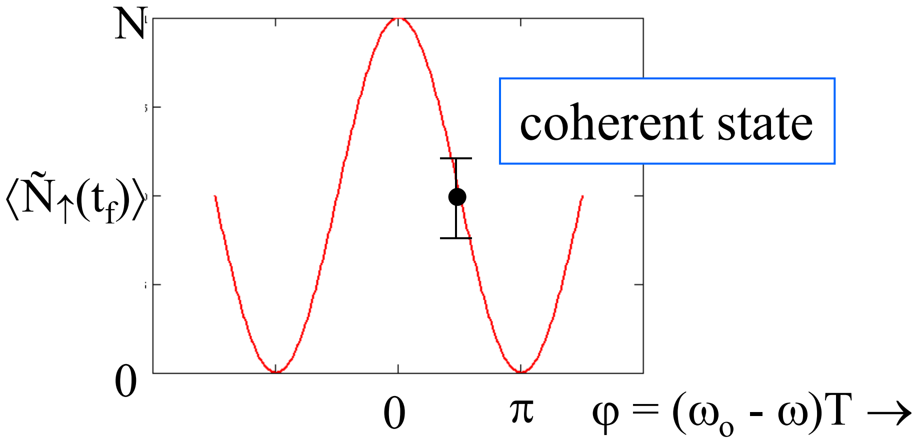
apply $H_I \propto J_x^{\,2}$. For N = 2,

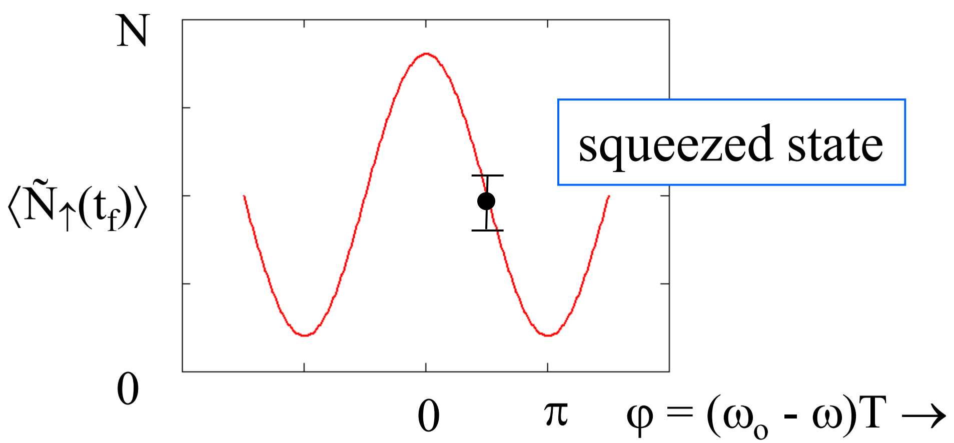
□□□□ □ cos□ □□□□ + sinα □□□□





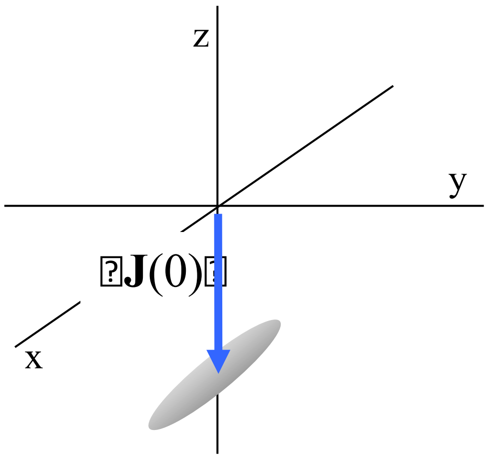
<u>But</u>, in general, □**J**(0)□ shrinks with squeezing





in Meyer *et al.*, PRL **86**, 5870 (2001), I = 1.09

“Spin-squeezed” states: Maximum sensitivity?



$$\Delta J_x \cdot \Delta J_y \ge \tfrac{1}{2}\left|\left\langle \left[J_x , J_y\right]\right\rangle\right| = \tfrac{1}{2}\left|\left\langle J_z \right\rangle\right|$$

$$\Delta\theta = \frac{\Delta J_y}{\left|\left\langle J_z \right\rangle\right|} \ge \frac{1}{2\Delta J_x} \ge \frac{1}{2\Delta J_{x,max}} = \frac{1}{N}$$

$\Delta\theta = 1/N$ “Heisenberg” limit; holds for other operators used to determine rotation angle

Kitagawa and Ueda (Phys. Rev. A**47**, 5138 (1993)): $H_I = i\hbar\Omega(J_+^{\,2} - J_-^{\,2})$

(not known how to efficiently implement this operator)

Other possibilities for generating spin squeezing:

1. Transfer squeezing from harmonic oscillator to spins (e.g., via $J_+a + J_-a^\dagger$ coupling)
   - Wineland *et al*., PRA **46**, R6797 (1992); Wineland *et al*., PRA**50**, 67 (1994)
   - Kuzmich *et al*., PRL **79**, 4782 (1997); Hald *et al*., PRL **83**, 1319 (1999)

     (experiment: squeezing transferred from laser beam to atoms)

2. QND measurements:
   - Kuzmich *et al*., PRL **85**, 1594 (2000)
   - Geremia *et al*., *Science* **304**, 270 (2004)

     (experiment: deterministic spin squeezing with QND measurement + feedback)

3. Collisions in cold atoms
   - Sørensen *et al*., *Nature* **409**, 63 (2001)

     (proposal: collision operators for cold (BEC) atoms looks like $H_I \propto J_z^2$)

Different strategy? recall: ⍰⍰ ≃ ⍰Õ/|d⍰Õ⍰/d⍰|

spin-squeezing relies on measuring operator Õ = **J**. Use other operators?

Bollinger *et al.*, Phys. Rev. A**54**, R4649 (1996)

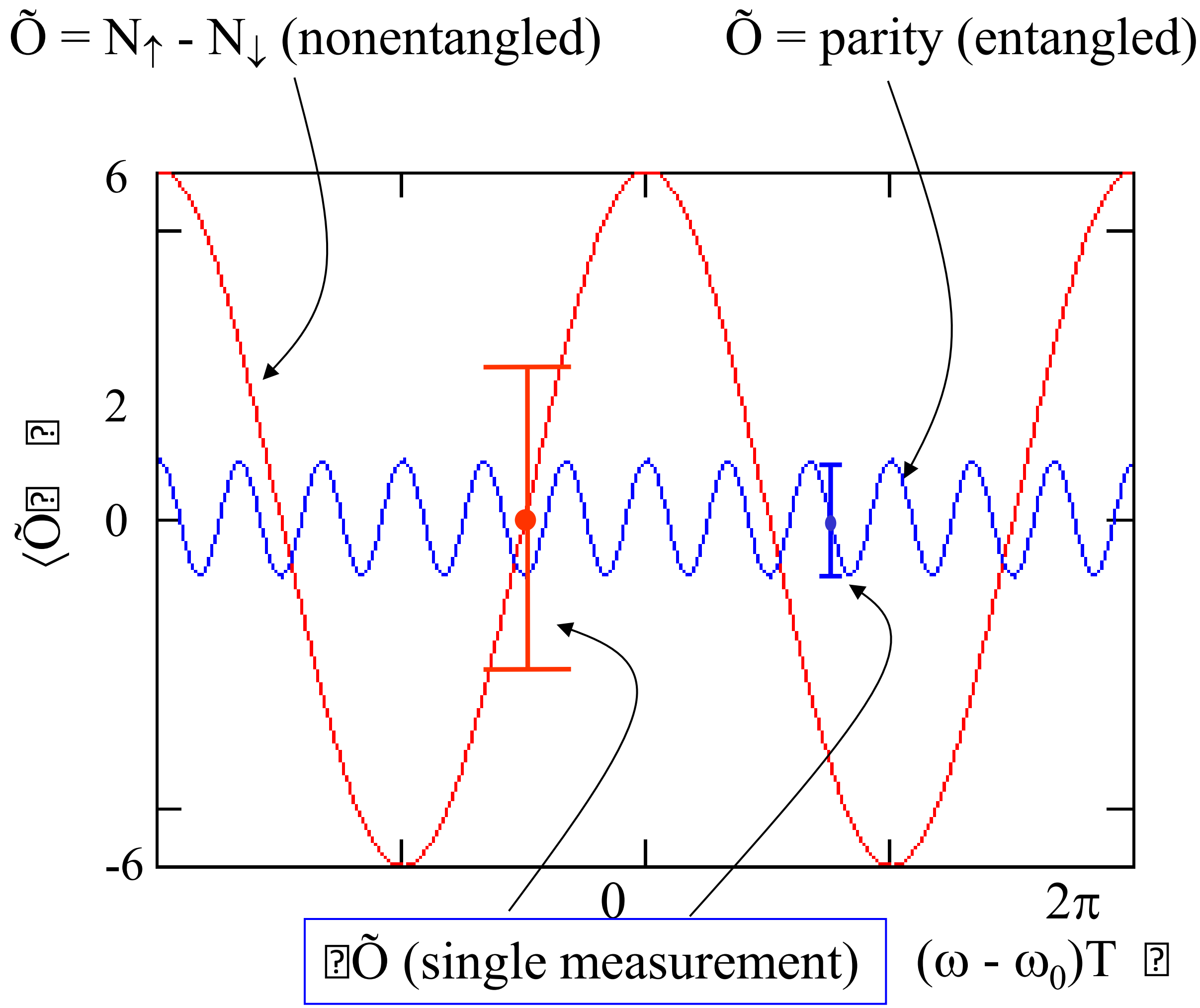
After first "(entangling) Ramsey pulse":

$$\Psi = \frac{1}{\sqrt{2}}\left[|\downarrow\rangle_1|\downarrow\rangle_2\cdots|\downarrow\rangle_N + e^{-i\mathbf{N}\omega_0 t}|\downarrow\rangle_1|\downarrow\rangle_2\cdots|\downarrow\rangle_N\right]$$ (note: $\langle\mathbf{J}\rangle = 0$)

After second (normal) Ramsey pulse, measure parity operator:

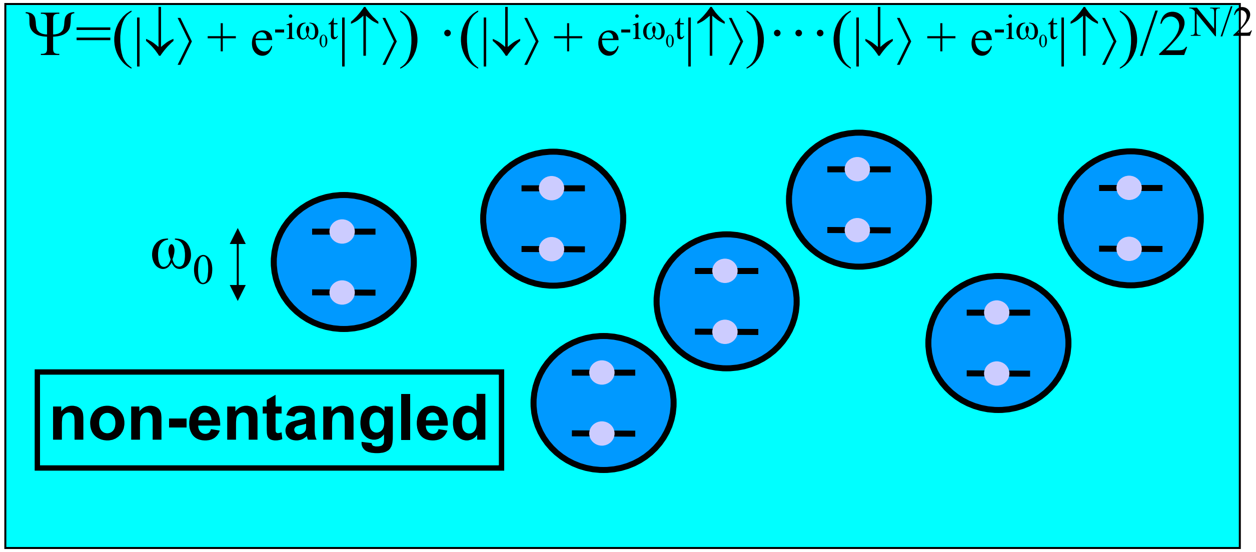
$$\tilde{O} = \tilde{P} = \prod_{i=1}^{N}\sigma_{zi}$$ (two values possible: 1, -1)

e.g., N = 6



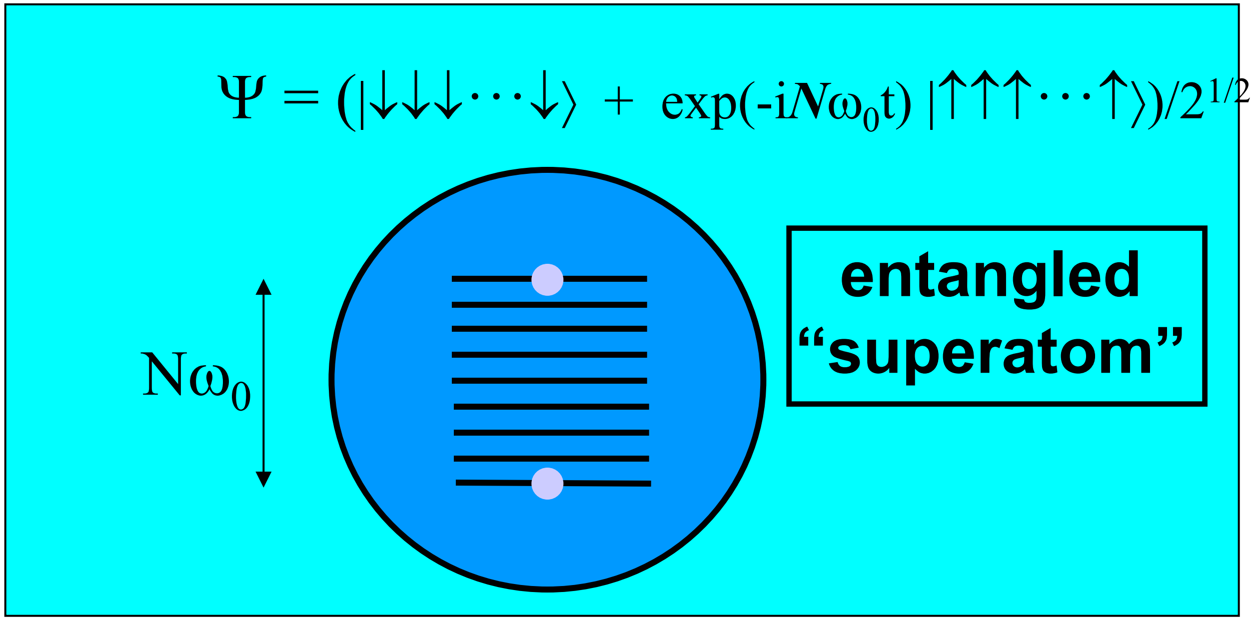
$\Delta\varphi = 1/N$, independent of $(\omega - \omega_0)T$

demonstrated in Meyer *et al.*, PRL **86**, 5870 (2001) (N = 2)



"standard quantum limit:"

$$\Delta\omega = \frac{1}{T\sqrt{N}}$$



Heisenberg limited:

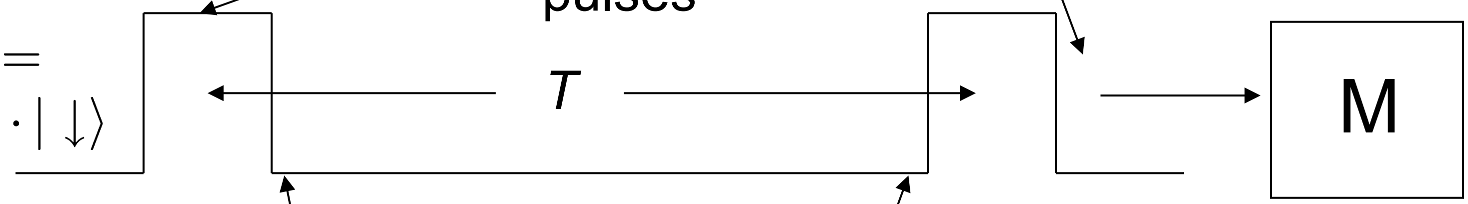
$$\Delta\omega = \frac{1}{TN}$$

The Ramsey method (with 2 entangling pulses):

$$\sin(N\phi/2)|\downarrow\rangle|\downarrow\rangle\cdots|\downarrow\rangle + \cos(N\phi/2)|\uparrow\rangle|\uparrow\rangle\cdots|\uparrow\rangle]$$

entangling "π/2" pulses

$$\Psi(t=0) = |\downarrow\rangle|\downarrow\rangle\cdots|\downarrow\rangle$$



measure

$$\frac{1}{\sqrt{2}}[|\downarrow\rangle|\downarrow\rangle\cdots|\downarrow\rangle + |\uparrow\rangle|\uparrow\rangle\cdots|\uparrow\rangle]$$

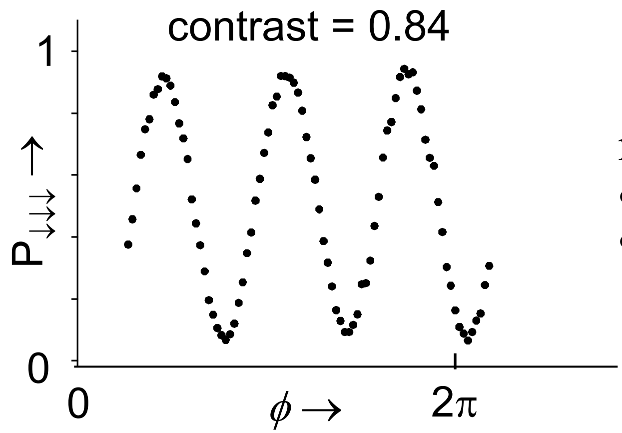
$$\frac{1}{\sqrt{2}}[|\downarrow\rangle|\downarrow\rangle\cdots|\downarrow\rangle + e^{-N\omega_0 T}|\uparrow\rangle|\uparrow\rangle\cdots|\uparrow\rangle]$$

$$P_{\downarrow\downarrow\cdots\downarrow} = \frac{1}{2}\left(1 - \cos\boxed{N}(\omega - \omega_0)T\right)$$

three-ion demonstration

$$I = 0.84 \times \sqrt{3} = 1.45$$

(Didi Leibfried *et al.* *Science* **304**, 1476 (2004))

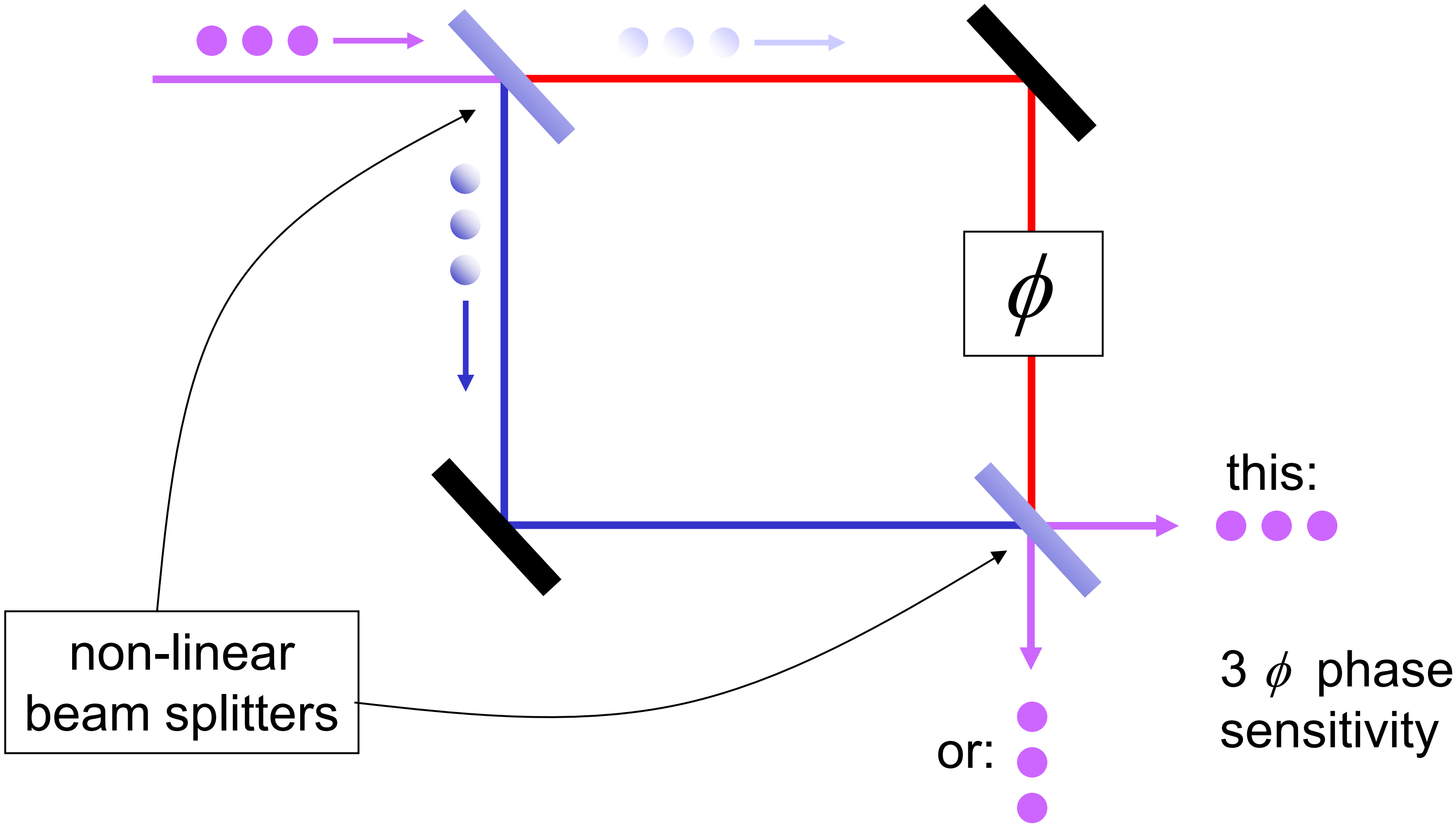


related experiments with photons:

- Walther *et al.*, *Nature* **429**, 158 (2004)
- Mitchell *et al.*, *Nature* **429**, 161 (2004)

Particle (e.g., photons, atoms) interferometers?

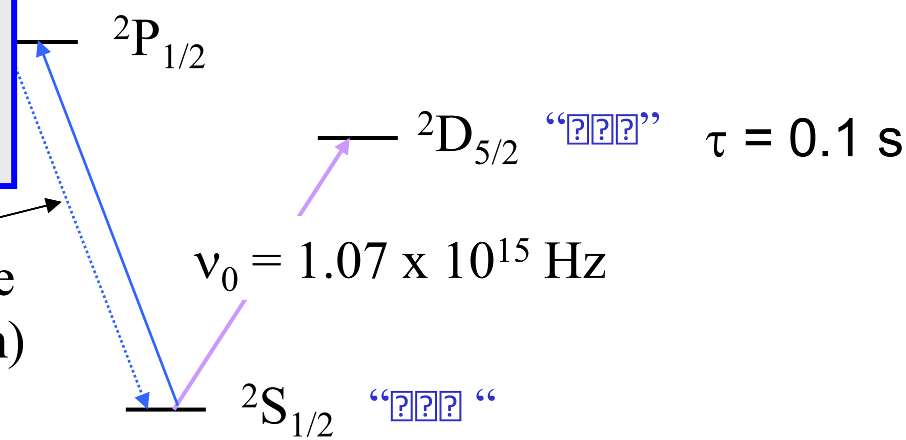
analog to the Ramsey experiment with 2 entangling “π-pulses:”



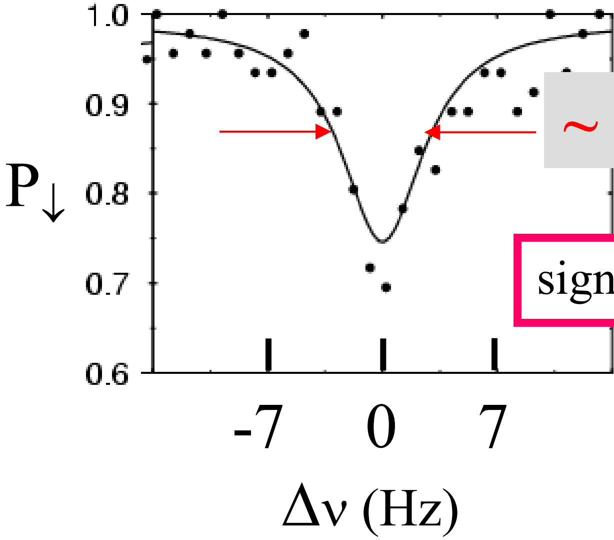
simulate non-linear beam splitters for N = 1,2,3 with motional modes of trapped ion (Leibfried *et al.*, PRL **89**, 247901 (2002))

single $^{199}Hg^{+}$-ion optical frequency standard (Jim Bergquist *et al.*)

entry level:
inaccuracy < 1 part in 10^{15}

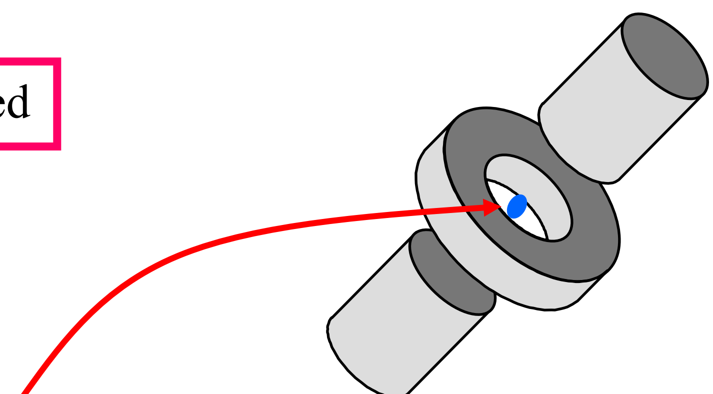


Observe fluorescence
(? = 194 nm)



~ 6.5 Hz, Q = 1.6x10^{14}

signal is lifetime limited



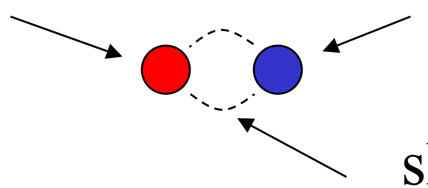
$^{2}D_{5/2}$ state has quadrupole moment.
Measure ν_0 along three B-field directions (Wayne Itano)

quadrupole shift

Quantum information mapping:

e.g. : cooling and detection of clock ions
with quantum information processing methods

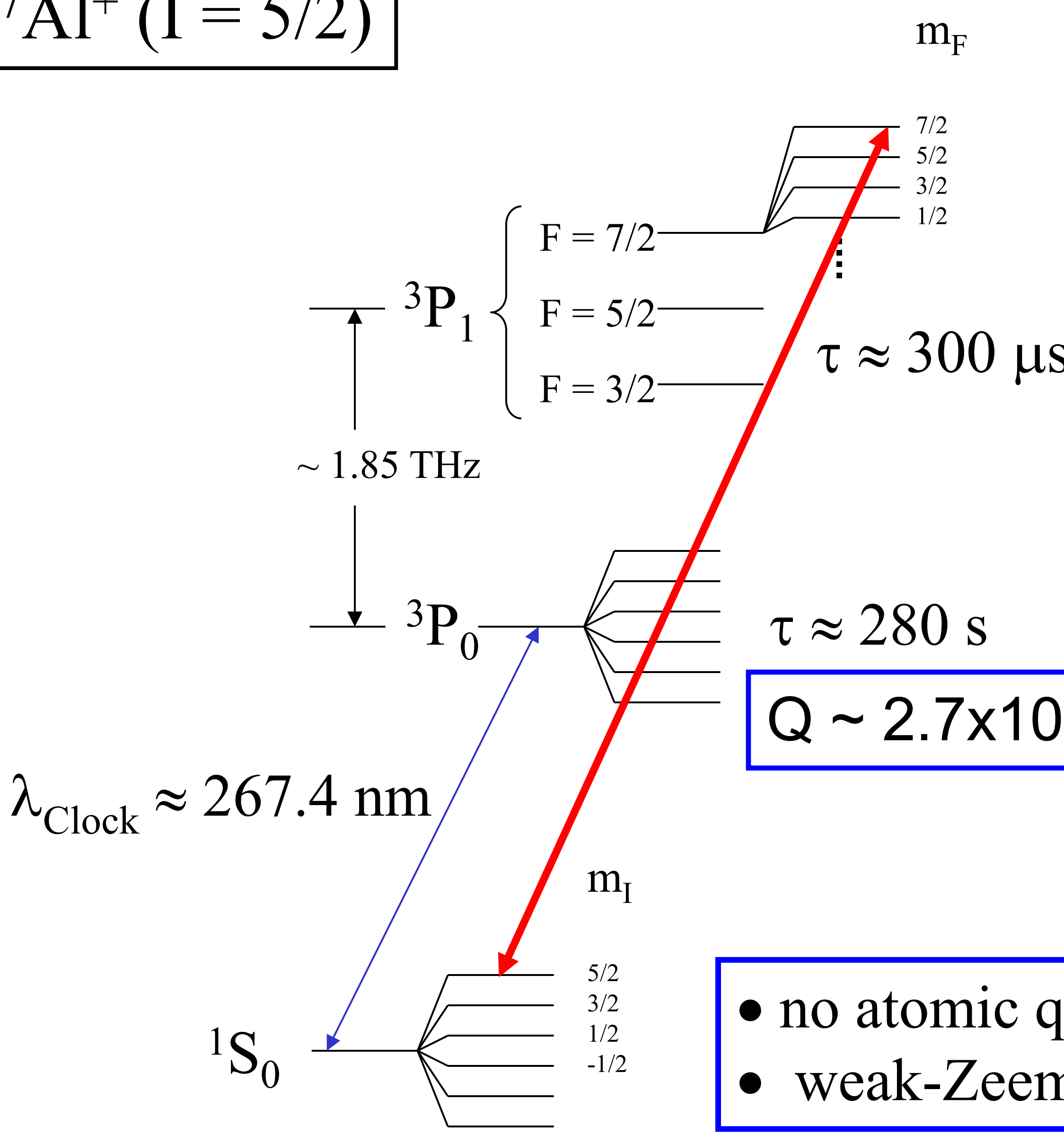
Basic idea (2 trapped ions):

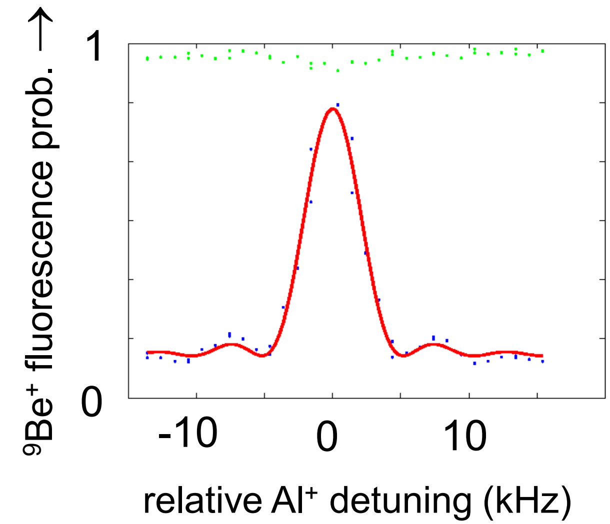


- Cool Clock ion with Logic ion
- Detect Clock ion by mapping state to Logic ion through motion
  - increases clock ion possibilities
  - sympathetic (laser) cooling <u>during</u> clock transition

$^{27}Al^{+}$ (I = 5/2)

Preliminary experiment
Al^{+} state mapped to Be^{+}
(Piet Schmidt, Till Rosenband)

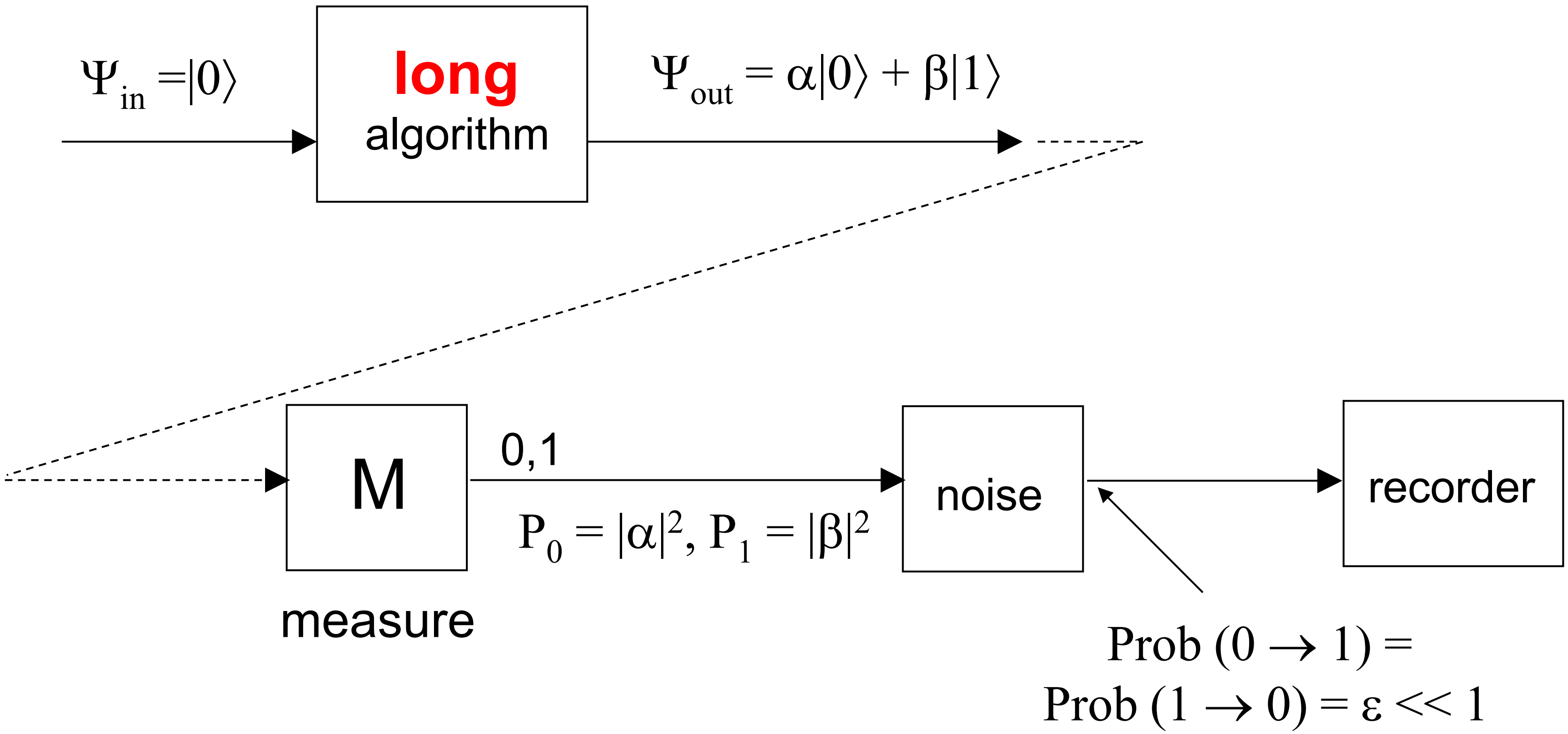




$Q \sim 2.7 \times 10^{18}$

- no atomic quadrupole shift
- weak-Zeeman effect

Enhanced quantum state detection with Quantum Information Processing



Make copies of Ψ_{out} and measure all copies ?

Universal (unitary) cloning machine:

$$U_{clone}|\Psi_{out}\rangle|0\rangle = |\Psi_{out}\rangle|\Psi_{out}\rangle$$

$$U_{clone}|\Phi_{out}\rangle|0\rangle = |\Phi_{out}\rangle|\Phi_{out}\rangle$$

inner products:

$$\langle 0|\langle\Phi_{out}|\overbrace{(U_{clone})^{\dagger}\, U_{clone}}^{\boxed{?}}|\Psi_{out}\rangle|0\rangle$$

$$= \langle 0|\langle\Phi_{out}|\Psi_{out}\rangle|0\rangle = \langle\Phi_{out}|\Psi_{out}\rangle$$

$$\langle\Phi_{out}|\langle\Phi_{out}|\Psi_{out}\rangle|\Psi_{out}\rangle = \langle\Phi_{out}|\Psi_{out}\rangle^2$$

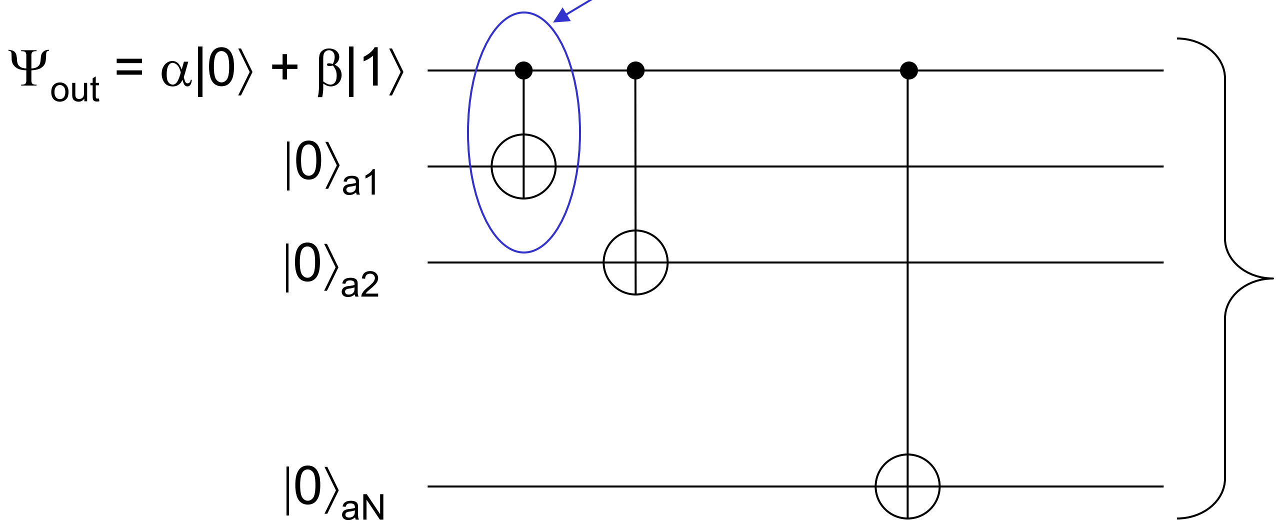
$$\neq \langle\Phi_{out}|\Psi_{out}\rangle$$

“no cloning” theorem:

Wootters and Zurek, *Nature*, **299**, 802 (1982)

Use controlled-nots: (DiVincenzo, in *Scalable Quantum Computers* (Wiley-VCH, 2001))

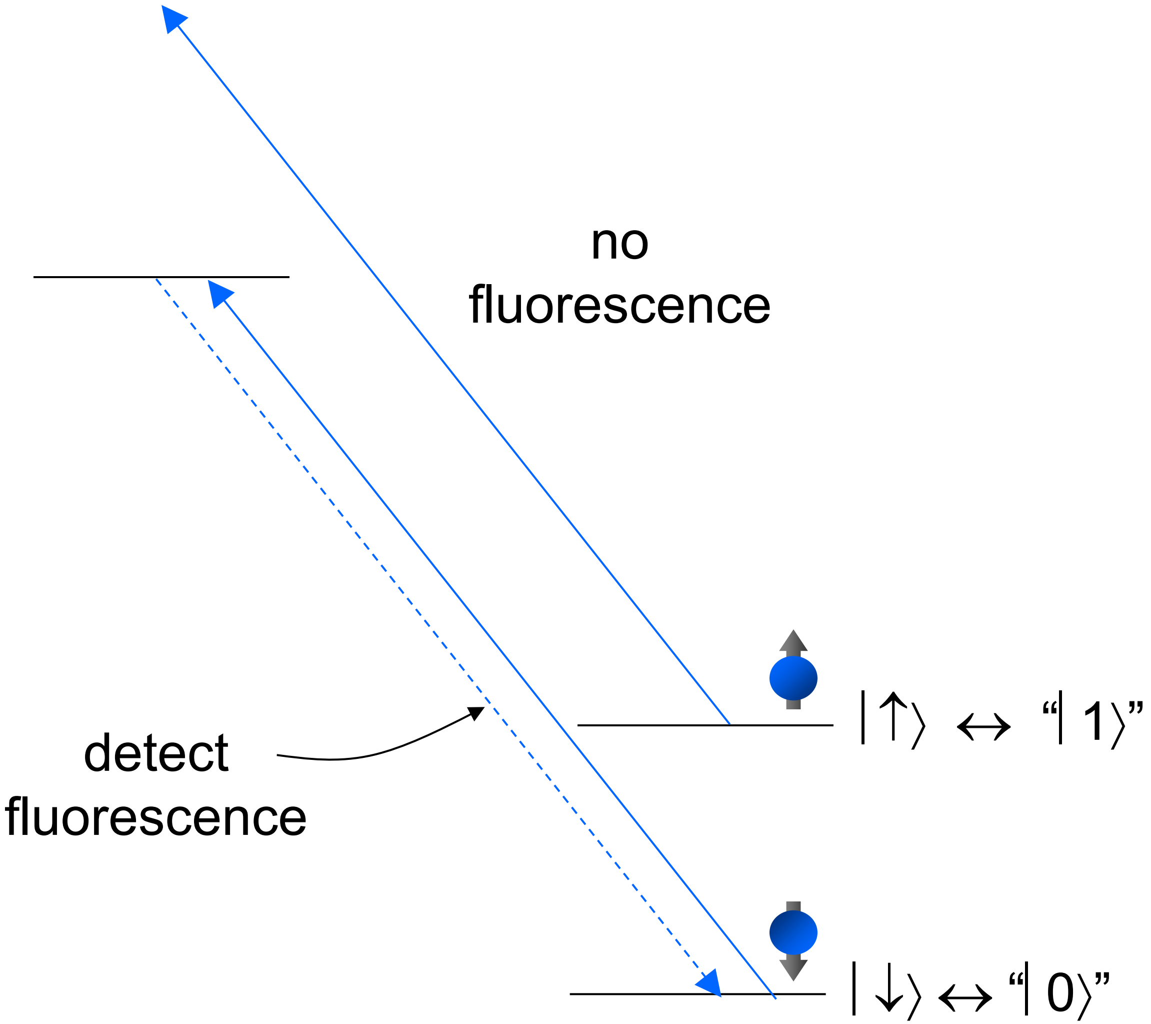
CNOT $(\alpha|0\rangle + \beta|1\rangle)\otimes|0\rangle_{a1} \rightarrow \alpha|0\rangle|0\rangle_{a1} + \beta|1\rangle|1\rangle_{a1}$



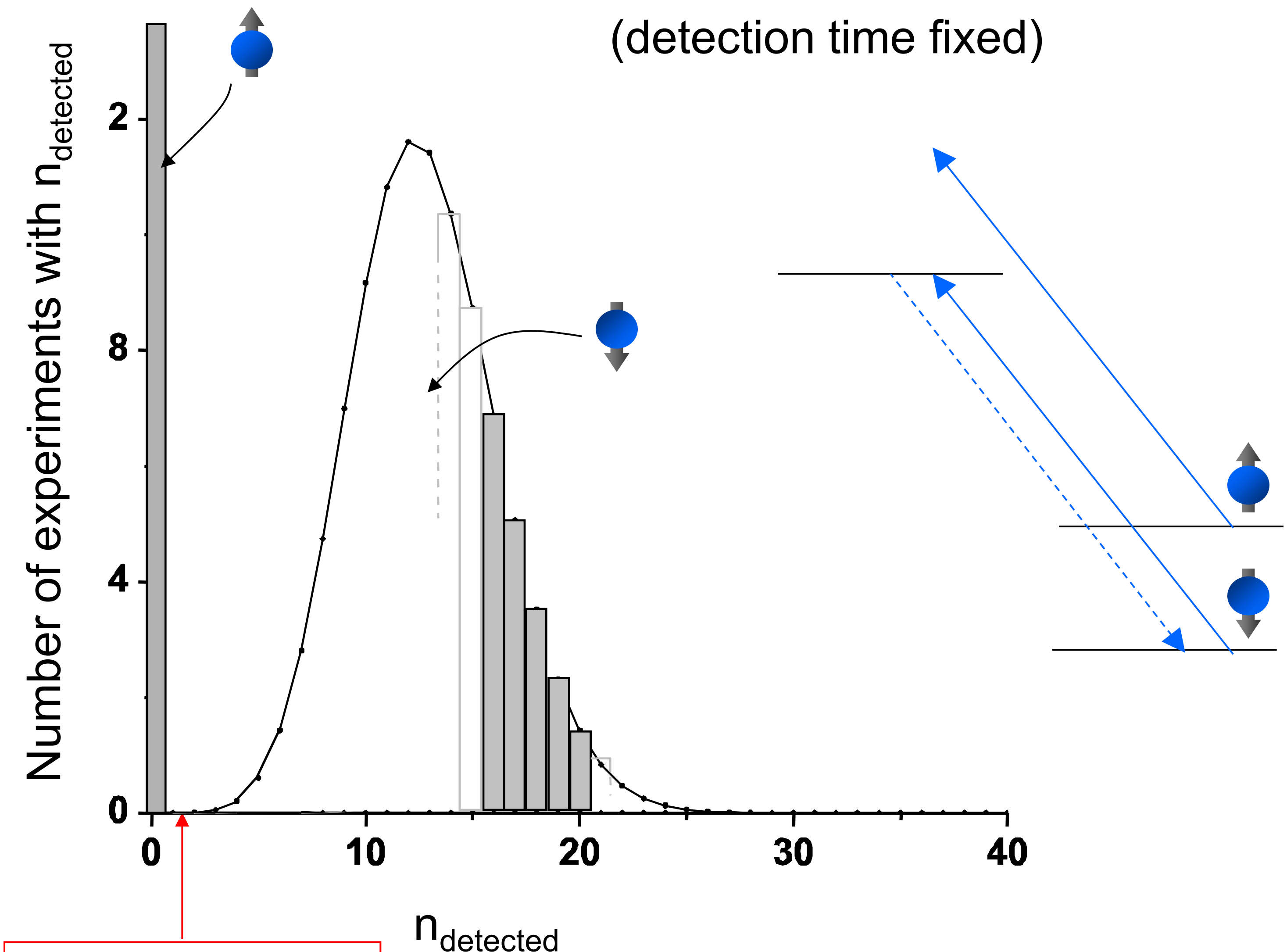
$$\Psi_{out} = (\alpha|0\rangle + \beta|1\rangle)\otimes|0\rangle_{a1}\otimes|0\rangle_{a2} \cdots \otimes|0\rangle_{aN}$$

$$\rightarrow \alpha|0\rangle|0\rangle_{a1}|0\rangle_{a2} \cdots |0\rangle_{aN} + \beta|1\rangle|1\rangle_{a1}|1\rangle_{a2} \cdots |1\rangle_{aN}$$

e.g., measure all bits, take a majority vote example: state detection with atomic qubits:



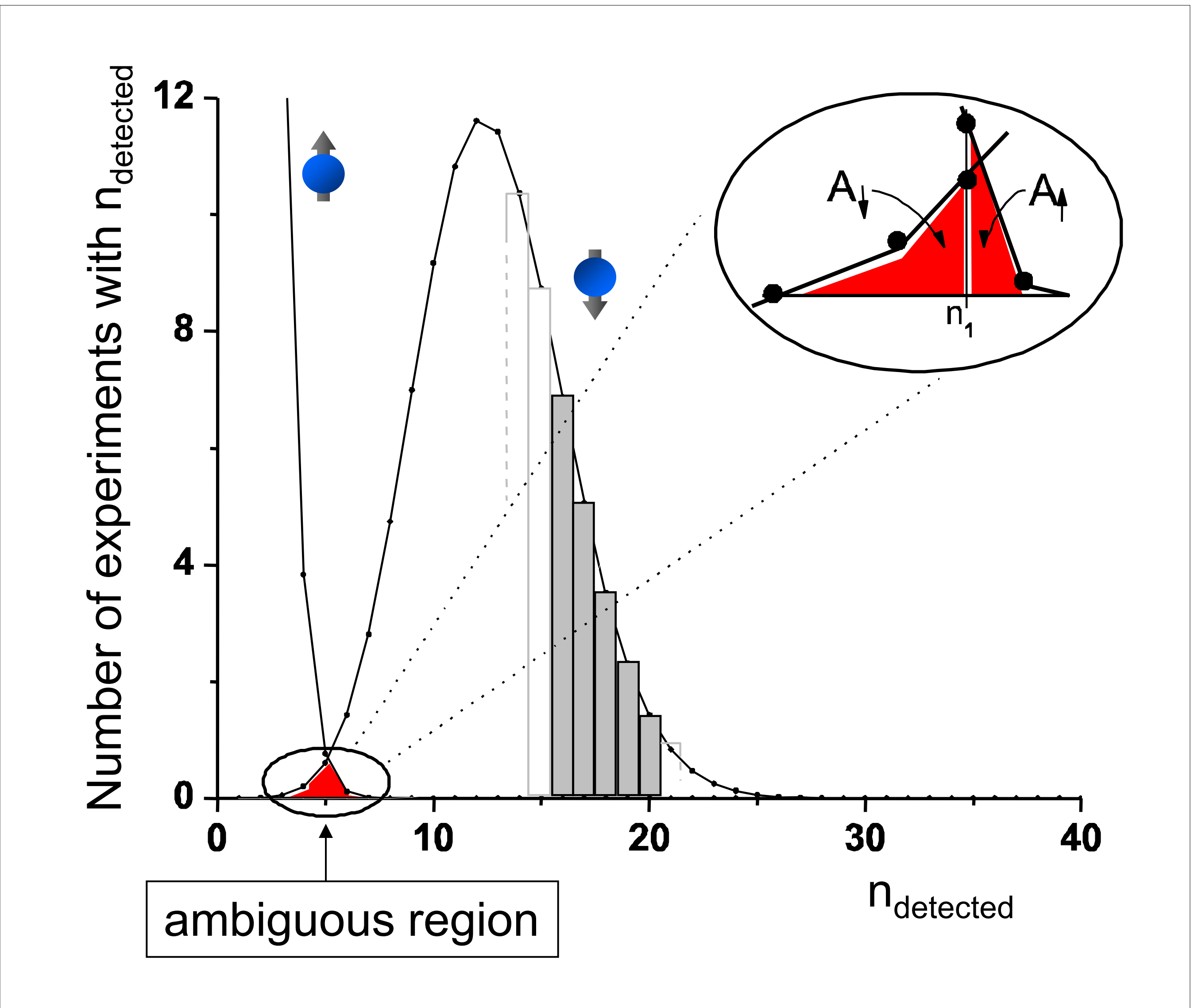
Ideal case,
(detection time fixed)



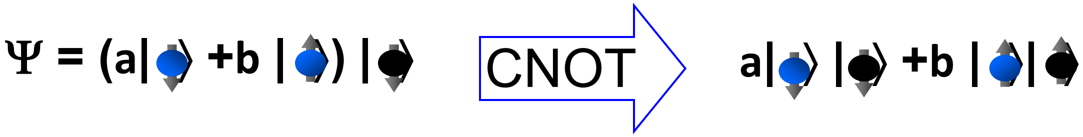
set discriminator here

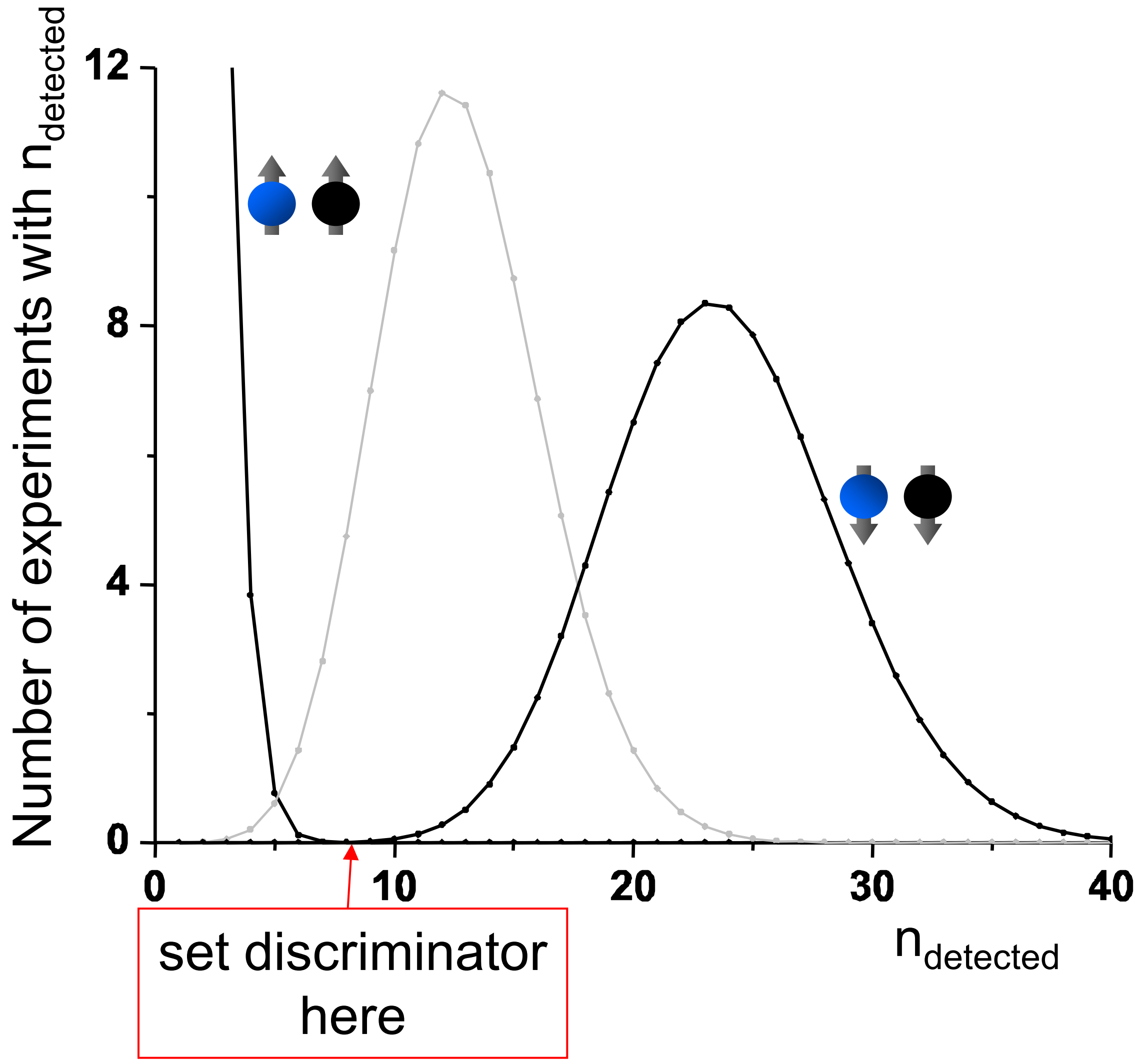
non-ideal case (added background noise)

$$\langle \dot{n}_{background} \rangle = 0.125 \langle \dot{n}_{\downarrow} \rangle$$



Amplification with one ancilla bit





detect both bits simultaneously (i.e., not majority vote)

(experimental demonstration: Tobias Schaetz *et al*. ‘04)

summary:

- improved interferometry
  - e.g., spectroscopy and atomic clocks
  - ideas applicable to photon & atom interferometers
- quantum information mapping
- improved detection efficiency

future:

- more and better
- new kinds of entanglement (and operators) for metrology?

Appendix: Ramsey spectroscopy – connection with (particle) interferometers?

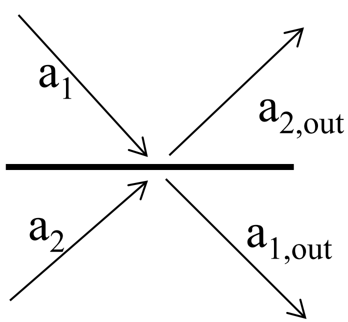
$J_x = ½(a_1^\dagger a_2 + a_2^\dagger a_1)$
$J_y = -½i(a_1^\dagger a_2 - a_2^\dagger a_1)$ (Schwinger operators)
$J_z = ½(a_1^\dagger a_1 - a_2^\dagger a_2) = ½(\tilde{n}_1 - \tilde{n}_2)$

$[J_x,J_y] = iJ_z$, etc.

Eigenstates:

$$\left|J, m_J\right\rangle = \left|\frac{n_1+n_2}{2}, \frac{n_1-n_2}{2}\right\rangle = \left|n_1\right\rangle\left|n_2\right\rangle, \text{(two mode Fock states)}$$

Application to (linear) beam splitters (B. Yurke, *et al.*, Phys. Rev. A **33**, 4033 (1986).)



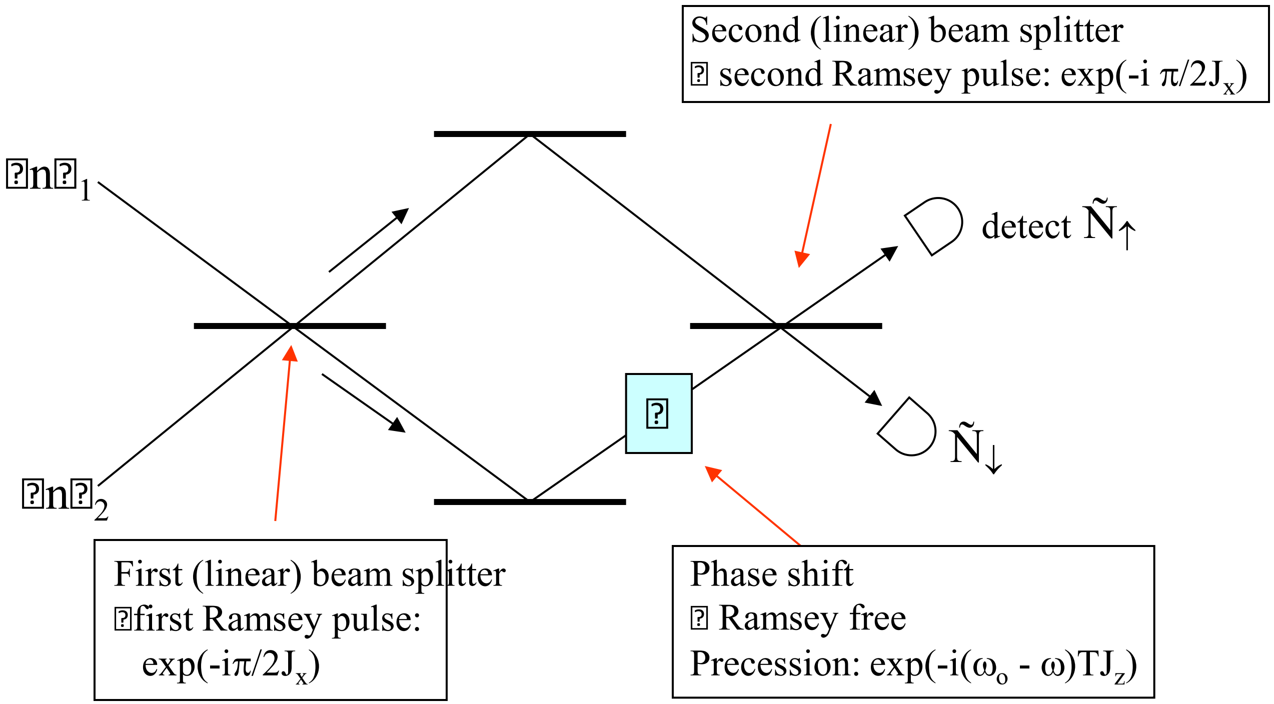
$$\left|\psi\right\rangle_{in} = \sum_{n_1,n_2=0}^{\infty} C(n_1,n_2)\left|n_1\right\rangle\left|n_2\right\rangle$$

$$\left|\psi\right\rangle_{out} = \exp(-i\alpha\, J_x)\left|\psi\right\rangle_{in}$$

Beam splitter with transmission $\cos^2(\alpha/2)$

Heisenberg limited interferometry with "dual Fock state input" and linear beam splitters:

- Holland and Burnett, Phys. Rev. Lett. **71**, 1355 (1993).
- Bouyer and Kasevich, Phys. Rev. A**56**, R1083 (1997).
- Pfister, Holland, Noh, and Hall, Phys. Rev. A**57**, 4004 (1998).



Dual Fock state: $|n\rangle_1|n\rangle_2 = |J = n, m_J = 0\rangle$

Measure variance: $\tilde{O} = \tilde{N}_\uparrow^2 - \langle\tilde{N}_\uparrow\rangle^2$ or J_z^2

$\delta\phi \approx 1/N$, for $\phi \approx 0$