Sample Problems: Geometrically-Frustrated Assembly

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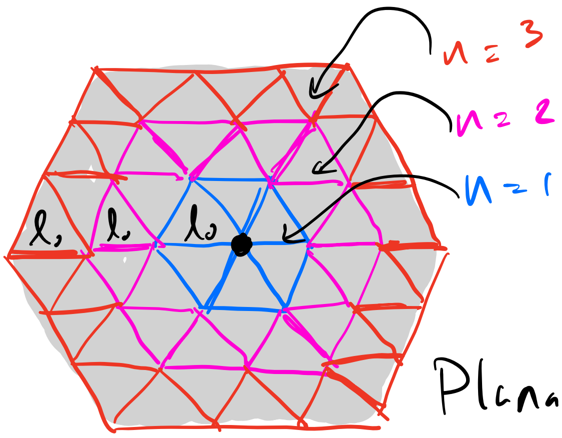
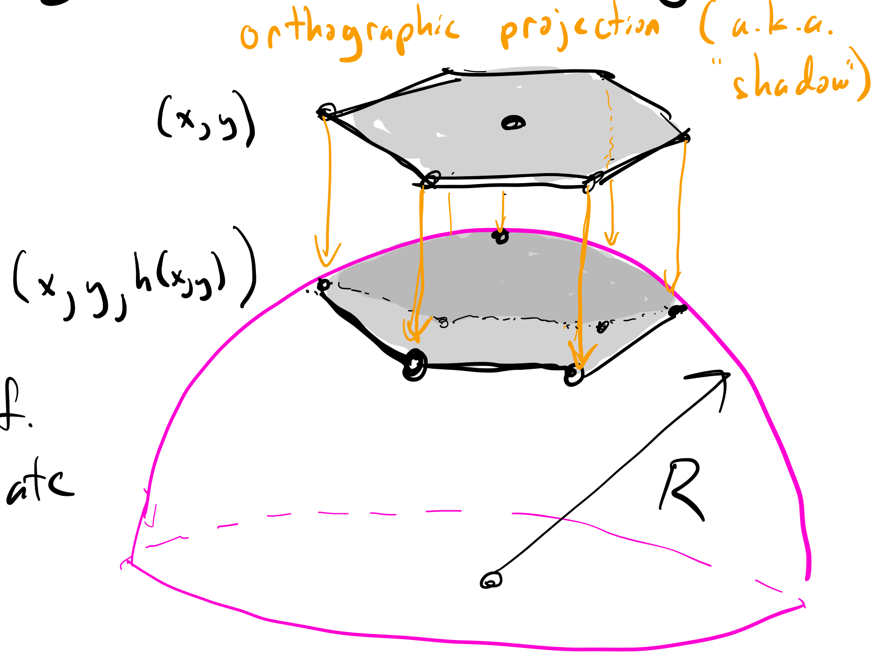
1. Spherical Projections of Triangular Lattice

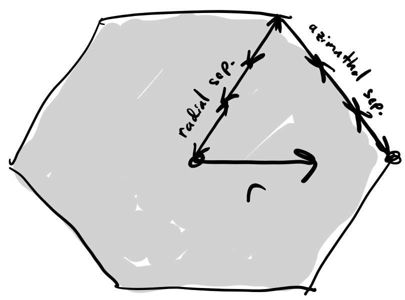
Consider n layer triangular lattice of spacing ℓ_0



Planar ref. state



- Assuming $n\ell_0 \ll R$, estimate $\ell(n)$ for orthographic projection along radially oriented bonds (i.e radial distance between layers).



- Now consider "azimuthal-equidistant projection", i.e. preserve distance r along radial geodesics from center. Estimate $\ell(n)$ for azimuthally seperated points (i.e. within layer).

2. Frustration in 2D bend-nematic domains

Consider simple model of banana (bent core) mesogens on the plane

$$F = \frac{1}{2}\int dA \left[K_1 (\nabla_\perp \cdot \hat{n})^2 + K_3 (\nabla_\perp \times \hat{n} - b_0)^2 \right]$$

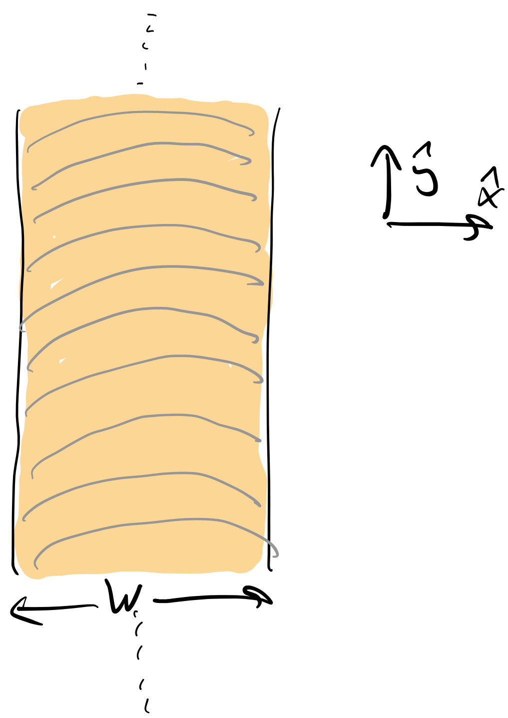
elastic free energy

preferred ber

planar orientation:

$\hat{n} = \cos\theta(x)\,\hat{x} + \sin\theta(x)\,\hat{y}$

Assume an infinite "ribbon" domain of width W. Assuming $\partial_y \theta = 0$, derive and solve equilibrium equations for ground state as function of $b_0 W$. (Assume $b W_0 \ll 1$ if needed).



- How does orientational strain vary with position?
- How does free energy depend on domain size?
- What plays role of K_G, to measure frustration?

3. Self-limiting width of double-twist cylinders

Consider an infinite cylindrical LC fiber described by Frank–Oseen elastic energy for chiral nematic:

chirality

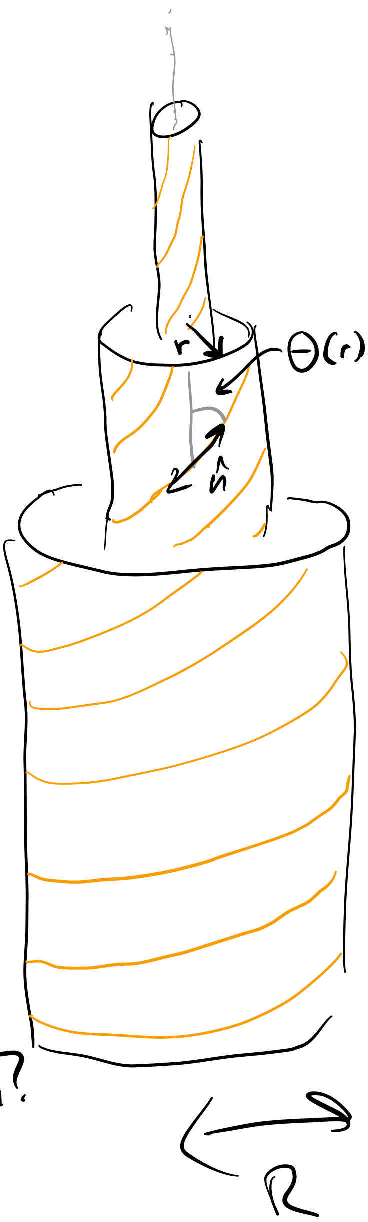
$$F = \frac{1}{2}\int dV \Big\{ K_1(\nabla\cdot\hat{n})^2 + K_2\left(\hat{n}\cdot(\nabla\times\hat{n}) + q_0\right)^2 + K_3\left[(\hat{n}\cdot\nabla)\hat{n}\right]^2$$

$$- K_{24}\nabla\cdot\left[\hat{n}(\nabla\cdot\hat{n}) - (\hat{n}\cdot\nabla)\hat{n}\right]\Big\}$$

saddle-splay

Assuming screw-symmetric texture $\hat{n} = \sin\theta(r)\,\hat{\phi} + \cos\theta(r)\,\hat{z}$, show

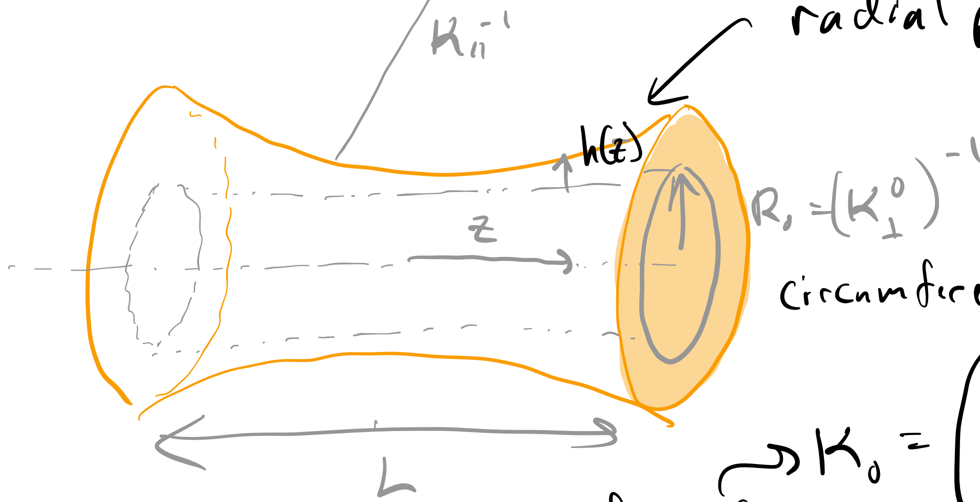
$$F = \frac{1}{2}\int dr \Big\{ K_2\left[\theta' + \frac{\theta}{r} + q_0\right]^2 + K_3\frac{\sin^4\theta}{r^2} - \frac{K_{24}}{r}\frac{d}{dr}\left(\sin^2\theta\right)\Big\}$$



- Assuming $\theta = \Omega r \ll 1$, find Ω for a given R.
  const.
- Find $F(R)$. What "measures" frustration?
- Include surface energy of fiber and find self-limiting radius R_*.

4. Strain accumulation & flattening in trumpets

Consider a 2D solid membrane with a preferred "trumpet" shape



radial position $= R_0 + h(z)$

deflection from cylinder

circumferential

$$\text{preferred curv.} \to K_0 = \begin{pmatrix} K_\perp^0 & 0 \\ 0 & -K_\parallel^0 \end{pmatrix}$$

along tube

By assuming axisymmetry, can show:

$$E_{elas} = \frac{1}{2} \int dA \left[Y \left(\frac{h}{R_0} \right)^2 + B \left(\partial_z^2 h - K_\parallel^0 \right)^2 \right] \qquad \partial_z^2 h \simeq K_\parallel$$

- Find groundstate shape $h(z)$ for trumpet of length L.
(* hint; free BCs @ ends: $\partial_z^2 h = K_\parallel^0$ & $\partial_z^3 h = 0$)

- Show that there is a transition from superextensive energy @ small L to extensive growth for large L. What parameters control the length scale of the transition?