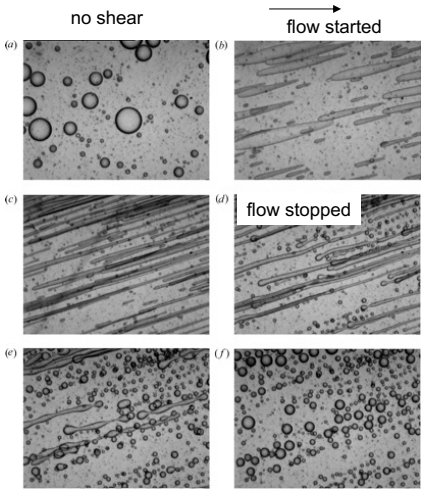
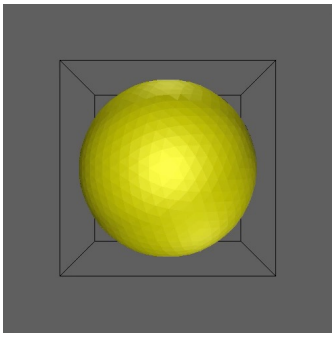
Complex fluids: Interfaces

Petia M. Vlahovska
Northwestern University





Iza&Bousmina (2000)

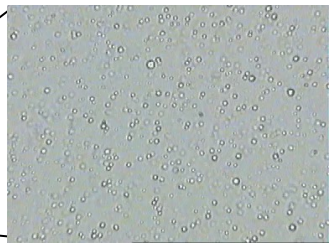
1

Complex fluids/soft materials are structured

Zoom x 1000

drop size ~ microns

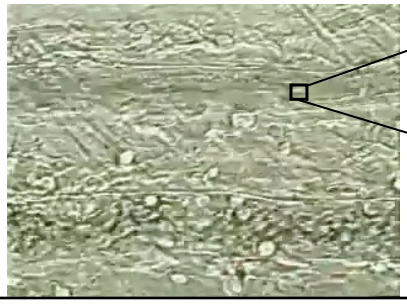
milk

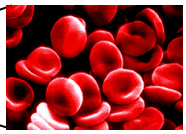


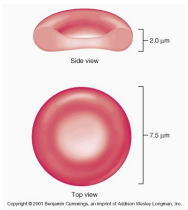
Drops (spherical)

RBCs (biconcave disks)

blood







Blood dense suspension:
RBC volume fraction ~ 40-45%

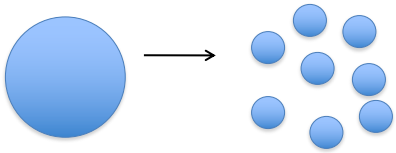
2

large surface to volume ratio

⇕

unstable

example: Take a sphere, chop it in N spheres, same volume but area increases ?? fold



thermal noise is important

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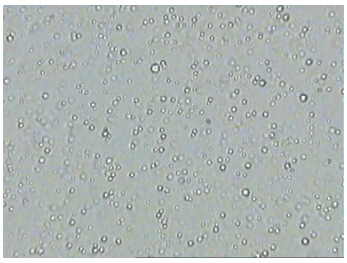
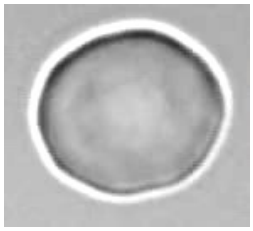
Brownian motion, fluctuations





https://www.youtube.com/watch?v=VwhNLaRCD-4

video Timo Betz

Turlier et al. Nature Physics (2016)

deformable microstructure

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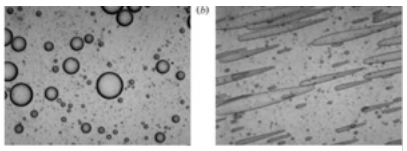
nonlinear response
(viscoelasticity,
non-Newtonian fluid mechanics,
nonlinear elasticity)

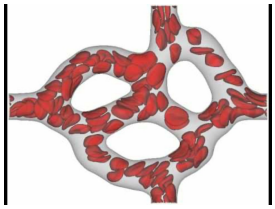
no shear → shear



Iza&Bousmina (2000)

(mayonnaise spreading)

blood flow in the microcirculation



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Interfaces: simple and complex, statics and dynamics

- "simple" interfaces: surface tension, capillarity
- "complex interfaces": surfactants, biological membranes, charged interfaces

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“SOFT”

Typical Length Scales, Energies, Forces:

Length:
1 nm = 10^{-9} m, red blood cells ~10 µm = 3-5*10^{-6} m
average cell in our body: approx. 50 µm

Energy:
Thermal energy
1 k_BT =4.16 × 10^{-21} J
≈ 0.6 kcal/mol ≈ 0.6 * 4.1868 kJ/mol
= 2.5 kJ/mol (298K)
= 4.1 pN nm
Boltzmann constant k_B=1.38*$10^{-23}$$JK^{-1}$

Force:
1 pN = 10^{-12} N

C-C bond: approx. 100 k_BT stable at room temperature
H-bond: approx. 10 k_BT

Surface tension:
water/oil 10^{-3} N/m~ 25 k_BT/nm^2

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What is the shape of a soap bubble?

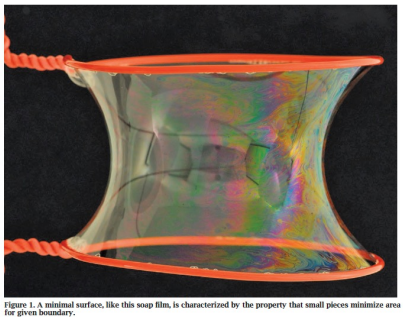

Figure 1. A minimal surface, like this soap film, is characterized by the property that small pieces minimize area for given boundary.

surface tension keeps drops and bubbles spherical
What is surface tension?
How do describe and determine shapes?

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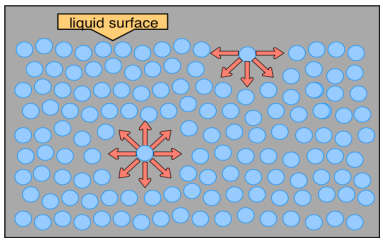
Surface tension

Surface tension can be interpreted both as surface energy per unit area (thermodynamic definition) and as a force per unit length (mechanical definition)

Surface tension as unbalanced intermolecular force



in the bulk: no net force on a molecule
at the surface: imbalance

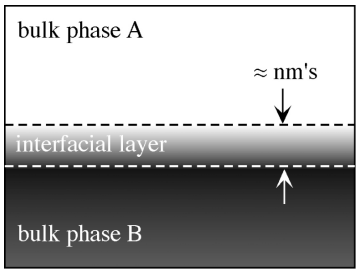
=>stretched-membrane effect
water/air =70mN/m

$$\sigma \sim \frac{A}{b^2}$$

A: Hamaker constant

b: molecule size

Surface tension as pressure deficit in the interfacial layer



Interfaces are not sharp

(discuss Gibbs dividing surface)

For a flat surface

$$\sigma = \int_{-\infty}^{\infty} (p_N - p_T)dz$$ (Bakker equation)

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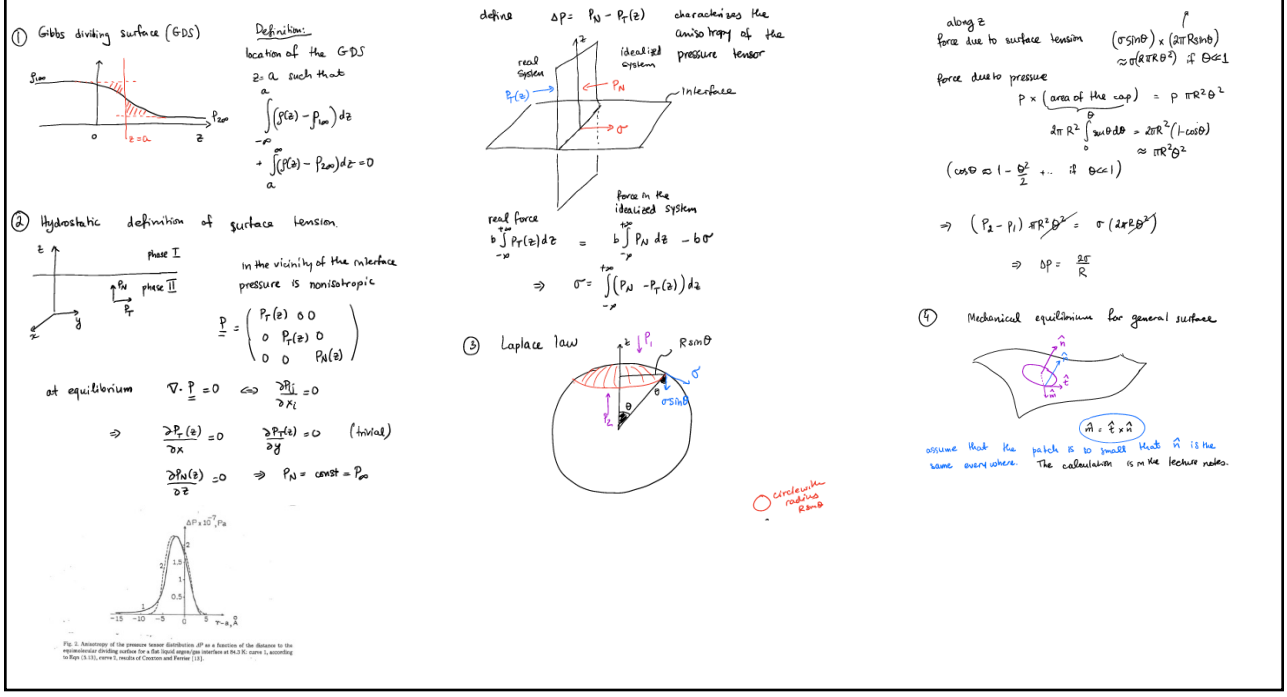

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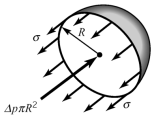
curved interface:

$$2\pi R\sigma = \pi R^2 \left(p_{in} - p_{out}\right)$$



Laplace' s law

$$\Delta p = \frac{2\sigma}{R}$$

What if the interface is not a sphere?????

fluid 2

fluid 1

$$\frac{d\mathbf{t}}{ds} = -H\mathbf{n}$$

$$\frac{d\mathbf{n}}{ds} = H\mathbf{t} \leftrightarrow H = \nabla_s \cdot \mathbf{n}$$

$$\mathbf{f}ds = \sigma(s+ds)\mathbf{t}(s+ds) - \sigma(s)\mathbf{t}(s)$$

$$\mathbf{f} = \frac{d}{ds}(\sigma\mathbf{t})$$

This local force is balanced by the jump in hydrodynamic traction,

$$\mathbf{f} + (\mathbf{T}_2 - \mathbf{T}_1)\cdot\mathbf{n} = 0$$

note: normal points into 2

Mechanics of interfaces: surface tension

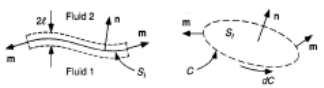


Figure 1: Figure from Dean "Analysis of Transport Phenomena"

Consider a surface area with normal **n**. Conservation of momentum at the interface shows that inertia is negligible (due to its thinness, the interface mass is negligible). The mechanical equilibrium at the interface then reads

$$\int_S (\mathbf{T}_2 - \mathbf{T}_1)\cdot\mathbf{n}dS = \int_C \mathbf{f}_s dC \qquad (4)$$

$(\mathbf{T}_2 - \mathbf{T}_1)\cdot\mathbf{n}$ is the jump in the bulk hydrodynamic stress. $\mathbf{f}_s$ are interfacial forces (surface tension, viscoelastic stresses, bending, etc...)

For an interface with surface tension $\mathbf{f}_s = \sigma\hat{\mathbf{m}}$, $\hat{\mathbf{m}} = \hat{\mathbf{t}} \times \mathbf{n}$

$$\int_C \mathbf{f}_s dC = \int_S \left[(\mathbf{n}\times\nabla)\times(\sigma\mathbf{n})\right] dS \qquad (5)$$

$$(\mathbf{n}\times\nabla)\times(\sigma\mathbf{n}) = (\sigma\nabla\cdot\mathbf{n})\,\mathbf{n} - \nabla_s\sigma, \quad \nabla_s = (\mathbf{I} - \mathbf{nn})\cdot\nabla$$

Proof:

$$\left[(\mathbf{n}\times\nabla)_j \times (\sigma\mathbf{n})_k\right]_i = \varepsilon_{ijk}(\mathbf{n}\times\nabla)_j(\sigma n_k) = \varepsilon_{ijk}\varepsilon_{jpq}n_p\frac{\partial}{\partial x_q}(\sigma n_k) = \varepsilon_{ijk}\varepsilon_{jpq}n_p n_k\frac{\partial\sigma}{\partial x_q} + \varepsilon_{ijk}\varepsilon_{jpq}n_p\sigma\frac{\partial n_k}{\partial x_q} =$$

using that $\varepsilon_{ijk} = \varepsilon_{jki}$ and

$$\varepsilon_{jki}\varepsilon_{jpq} = \delta_{kp}\delta_{iq} - \delta_{kq}\delta_{ip}$$

leads to

$$= n_k n_k\frac{\partial\sigma}{\partial x_i} + \sigma n_k\frac{\partial n_k}{\partial x_i} - n_i n_k\frac{\partial\sigma}{\partial x_k} - \sigma n_i\frac{\partial n_k}{\partial x_k} = \left[(\mathbf{I} - \mathbf{nn})\cdot\nabla\sigma + \sigma\mathbf{n}\cdot\nabla\mathbf{n} - \sigma\mathbf{n}\nabla\cdot\mathbf{n}\right]_i$$

note that $\mathbf{n}\cdot\nabla\mathbf{n} = 0$ because $\nabla(\mathbf{n}\cdot\mathbf{n}) = 0$

Thus

$$(\mathbf{T}_2 - \mathbf{T}_1)\cdot\mathbf{n} = (\sigma\nabla\cdot\mathbf{n})\,\mathbf{n} - \nabla_s\sigma \qquad (6)$$

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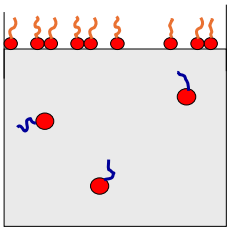
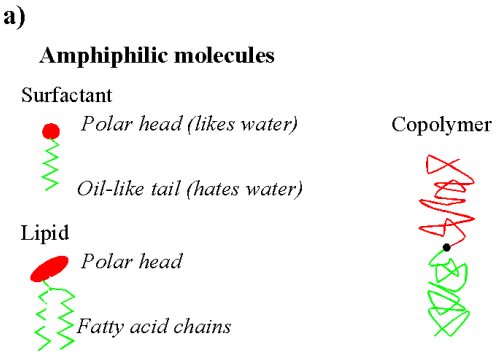
Note: at equilibrium surface tension is uniform, and there are no gradients. If surface tension is nonuniform (e.g. due to temperature or surfactant nonuniformities) this drives flow

Surfactants: adsorb at interfaces and lower surface tension





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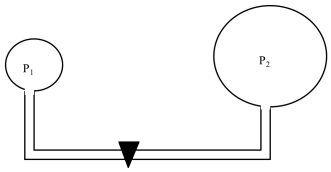
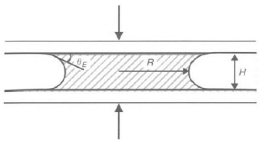
Questions:



Two bubbles are connected by a tube.
What happens when you open the valve?



Capillary adhesion: Two wetted surfaces can adhere strongly if the liquid wets both surfaces. Estimate the force

Self -propelling drop. (see paper by Lorenceau and Quere “Drops on a conical wire" , J. Fluid Mech. 2003)
An asymmetric capillary device may prompt spontaneous drop propulsion. An example of such
motion is illustrated in the Figure. A drop of silicone oil is deposited on a conical copper wire (mean radius of 250 um), whose radius increases from the right to the left with a gradient dr/dz of the order of 1 %. Note that silicone oil totally wets copper. Four successive photographs are taken. The time interval between two snapshots is about 1 second.

In which direction does the drop move?

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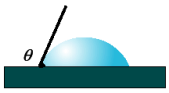
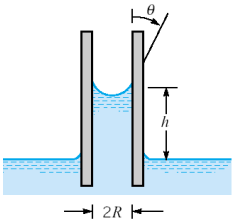
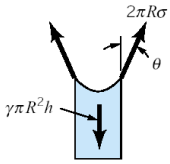
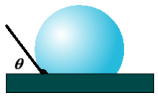






$$h = \frac{2\sigma \cos\theta}{\rho g R}$$

What if the liquid is nonwetting?



but why is the interface curved?

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Wetting and contact angle

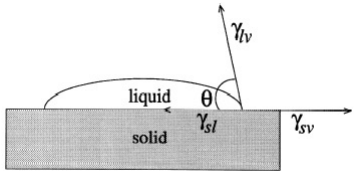
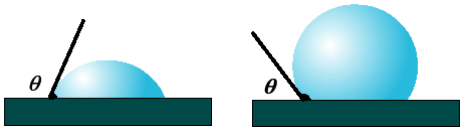


Fig. 1. Schematic of a sessile-drop contact angle system.

Kwok and Neumann, 1999

$$\gamma_{lv}\cos\theta_Y = \gamma_{sv} - \gamma_{sl}$$

Young's relation



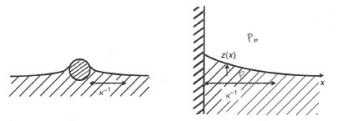


FIGURE 2.2. A small floating object (a) or a wall (b) perturbs the surface of the liquid over a distance κ^{-1} called the "capillary length."

(de Gennes book)

Show that

$$z(x) \sim \exp(-z\kappa)$$

(in axisym geometry – Bessel functions!)

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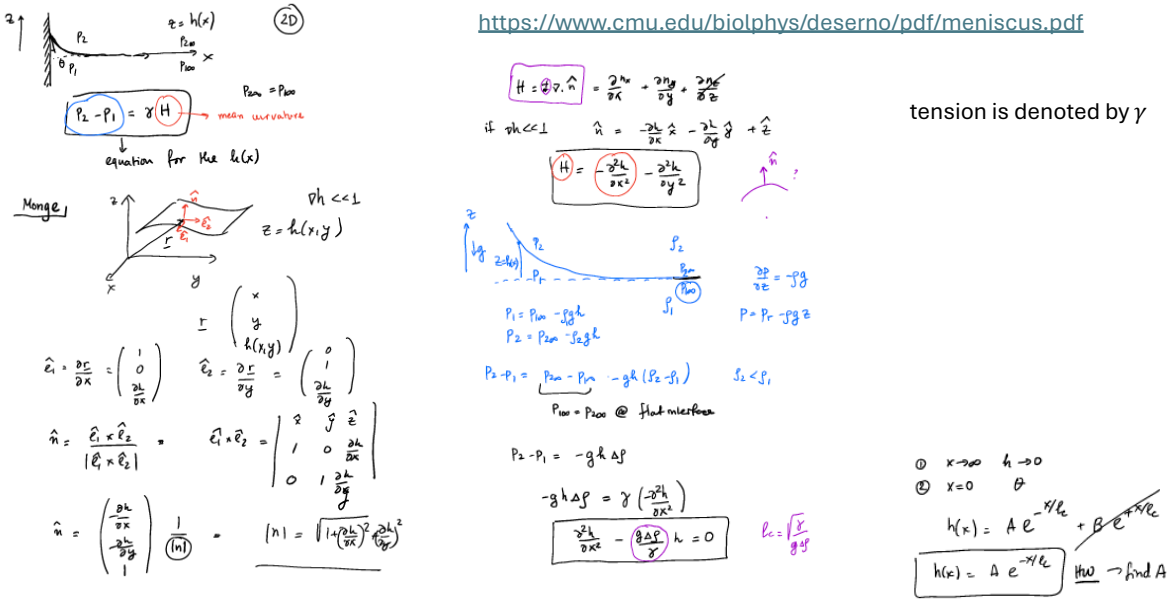

https://www.cmu.edu/biolphys/deserno/pdf/meniscus.pdf

tension is denoted by γ

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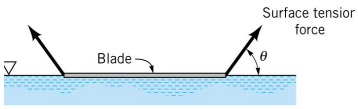

Q: The razor blade is heavier than the water. Why does it float?

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The cheerios effect

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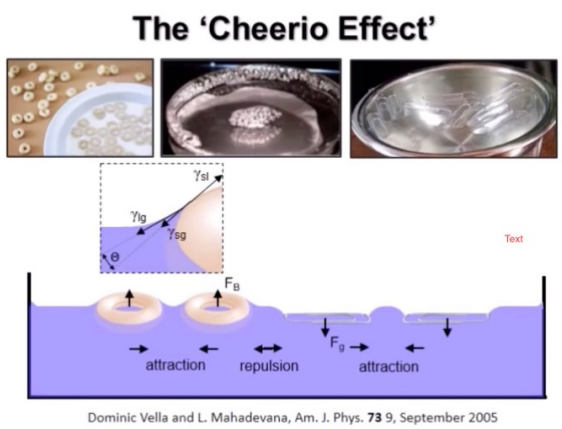
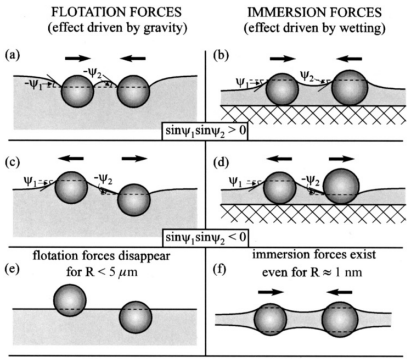


Dominic Vella and L. Mahadevana, Am. J. Phys. **73** 9, September 2005

https://www.youtube.com/watch?v=NIcNrsTlbVU



P.A. Kralchevsky, K. Nagayama "Advances in Colloid and Interface Science" 85 (2000) 145-192

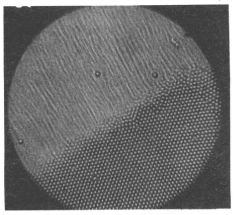


Figure 1. Photograph of the 2D array growth: the tracks of the particles rushing towar ordered phase are seen.

Denkov, N.; Velev, O.; Kralchevsky, P.; Ivanov, I.; Yoshimura, H.; Nagayama K. (1992). "Mechanism of formation of two-dimensional crystals from latex particles on substrates". Langmuir 8: 3183

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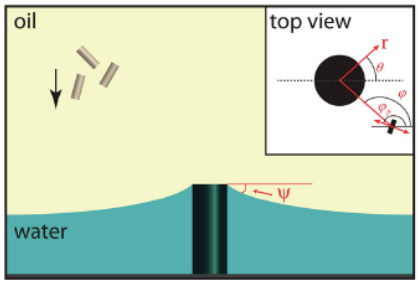
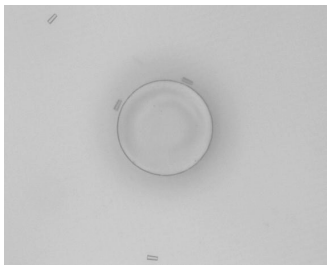
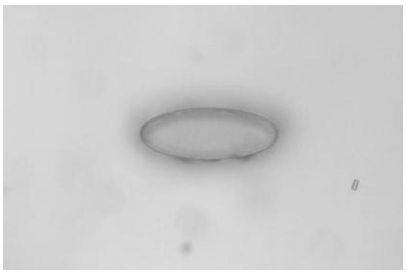
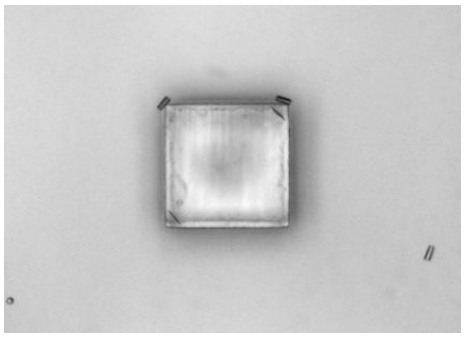


Fig. 1. Schematic of a curved oil–water interface with slope $-\tan\psi$ formed around a circular micropost. Cylindrical microparticles sediment through the oil phase, attach to the interface, and create deformations that interact with the interface curvature field. (*Inset*) Top view of the micropost with polar coordinates (r,θ) and of the particle-induced deformation, approximated as a quadrupole. The angle φ_p indicates the orientation of the quadrupolar rise axis, represented as a two-headed arrow. The angle φ indicates the orientation of the principal axis of curvature.







Cavallaro et al Curvature-driven capillary migration and assembly of rod-like particles PNAS 2011

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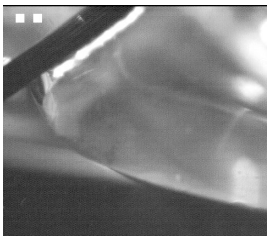

http://www.nature.com/nature/journal/v424/n6949/abs/nature01793.html

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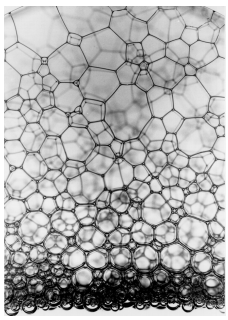
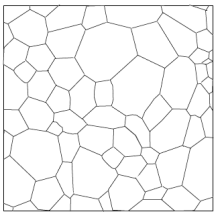
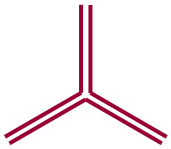
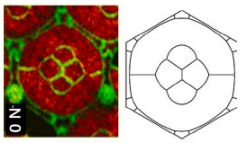
Liquid interfaces are under tension ➡ ”Shape” of foam







For the intersection of soap film lamellae in a foam all tensions are the same, hence the angle is 120



Fruit fly eye

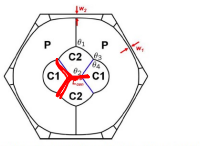


Fig. 2. Nomenclature and geometry of the modeled ommatidium. Indicated are the C1 (anterior and posterior cone), C2 (equatorial and polar cone), and P (primary pigment) cells, and the angles θ_i for the upper-right quadrant. The edge between the two C2 cells has length l_{cell}, and the widths of the secondary pigment cells attached to one (w_2) or two (w_1) P cells are shown. All edges carry *E*-cadherin, whereas the edges between C cells (blue) carry both *E*- and *N*-cadherin.

Physical modeling of cell geometric order in an epithelial tissu
Hilgenfeldt et al, PNAS 2008

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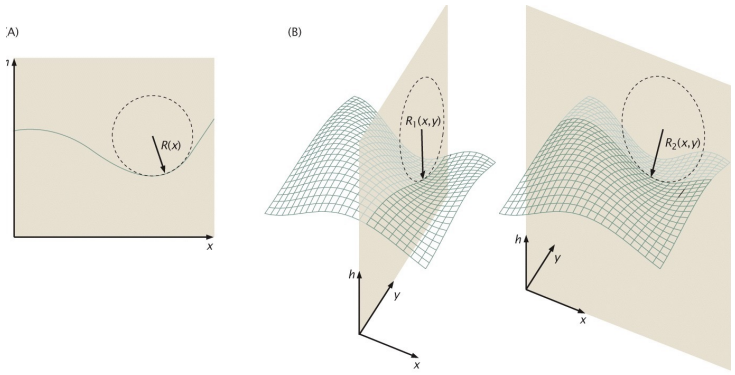
Note on geometry:



igure 11.15 Physical Biology of the Cell, 2ed. (© Garland Science 2013)

Giomi, 2013 review

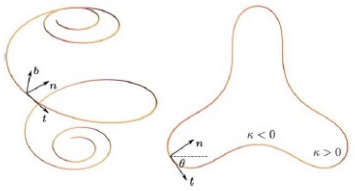


Fig. 1 Example of the Frenet frame in three (left) and two (right) dimensions. In three dimensions the curvature κ is conventionally taken with the positive sign and the normal vector $\boldsymbol{n}$ is directed toward the center of curvature. If a closed plane curve is oriented counterclockwise, on the other hand, $\boldsymbol{n}$ is directed toward the interior of the curve and $\kappa > 0$ ($\kappa < 0$) corresponds to points where the curve is convex (concave).

Frenet frame of the tangent $\boldsymbol{t} = \mathrm{d}\boldsymbol{r}/\mathrm{d}s$, normal $\boldsymbol{n}$ and binormal vector $\boldsymbol{b}$ is described by the Frenet–Serret formulae:

$$\boldsymbol{t}' = \kappa \boldsymbol{n}, \tag{1a}$$

$$\boldsymbol{n}' = -\kappa \boldsymbol{t} + \tau \boldsymbol{b}, \tag{1b}$$

$$\boldsymbol{b}' = -\tau \boldsymbol{b}, \tag{1c}$$

as well as the following cross-product relationship:

$$\boldsymbol{t} = \boldsymbol{n} \times \boldsymbol{b}, \quad \boldsymbol{n} = \boldsymbol{b} \times \boldsymbol{t}, \quad \boldsymbol{b} = \boldsymbol{t} \times \boldsymbol{n}. \tag{2}$$

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