Quantum Limits on Measurement

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Gurus: Michel Devoret, Steve Girvin, Aash Clerk

And many discussions with D. Prober, K. Lehnert, D. Esteve, L. Kouwenhoven, B. Yurke, L. Levitov, K. Likharev, …

Thanks for slides: L. Kouwenhoven, K. Schwab, K. Lehnert,…

Overview of Lectures

Lecture 1: Equilibrium and Non-equilibrium Quantum Noise in Circuits

Reference: "Quantum Fluctuations in Electrical Circuits," M. Devoret Les Houches notes

Lecture 2: Quantum Spectrometers of Electrical Noise

Reference: "Qubits as Spectrometers of Quantum Noise," R. Schoelkopf et al., cond-mat/0210247

Lecture 3: Quantum Limits on Measurement

References: "Amplifying Quantum Signals with the Single-Electron Transistor," M. Devoret and RS, Nature 2000.
"Quantum-limited Measurement and Information in Mesoscopic Detectors," A.Clerk, S. Girvin, D. Stone PRB 2003.

And see also upcoming RMP by Clerk, Girvin, Devoret, & RS

Outline of Lecture 3

- Quantum measurement basics:
  The Heisenberg microscope
  No noiseless amplification / No wasted information

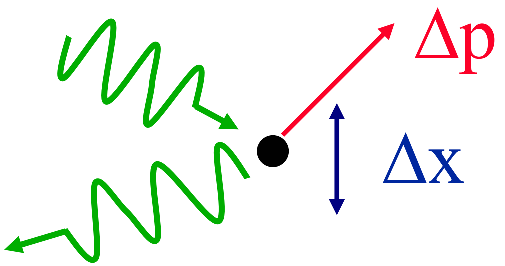
- General linear QND measurement of a qubit

- Circuit QED nondemolition measurement of a qubit
  Quantum limit?
  Experiments on dephasing and photon shot noise

- Voltage amplifiers:
  Classical treatment and effective circuit
  SET as a voltage amplifier
  MEMS experiments – Schwab, Lehnert

Heisenberg Microscope

Measure position of free particle:



Δx = imprecision of msmt.

wavelength of probe photon: $\lambda = hc / E_\gamma$

Δp = backaction due to msmt.

momentum "kick" due to photon: $\Delta p = E / c$

$$\Delta x \Delta p = \frac{hc}{E}\frac{E}{c} \sim h$$

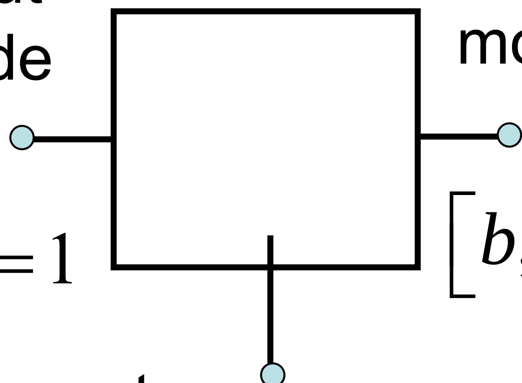
Only an issue if: 1) try to observe both x,p
or 2) try to repeat measurements of x

Uncertainty principle: $\Delta x \Delta p \geq \hbar / 2$

No Noiseless Amplification!

Clerk & Girvin,
after Haus & Mullen, 1962
and Caves, 1982

<u>Linear amplifier</u>



want: $b = \sqrt{G}\; a$

$b^\dagger = \sqrt{G}\; a^\dagger$

photon number gain, G

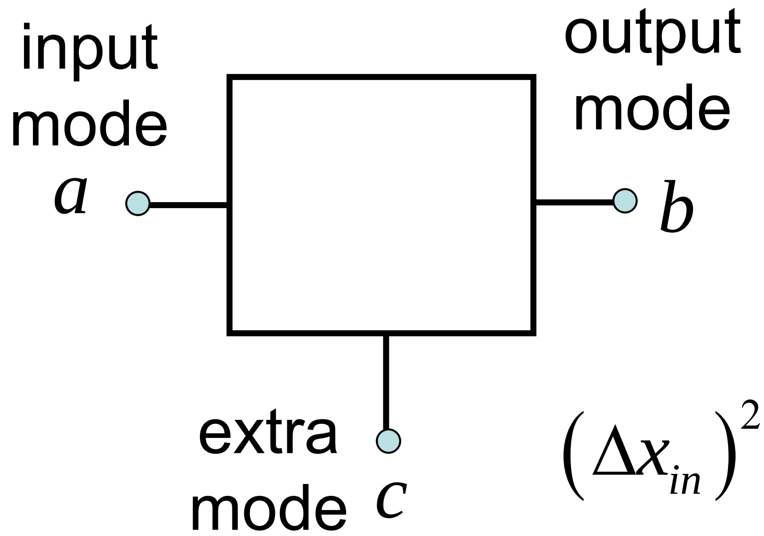
but then $\left[b, b^\dagger\right] = G\left[a, a^\dagger\right] \neq 1$

$$b = \sqrt{G}\; a + \sqrt{G-1}\; c^\dagger$$

$$b^\dagger = \sqrt{G}\; a^\dagger + \sqrt{G-1}\; c$$

$$\left[b, b^\dagger\right] = G\left[a, a^\dagger\right] + \left(G-1\right)\left[c^\dagger, c\right] = 1$$

No Noiseless Amplification! - II



$$b = \sqrt{G}\, a + \sqrt{G-1}\, c^{\dagger}$$

$$b^{\dagger} = \sqrt{G}\, a^{\dagger} + \sqrt{G-1}\, c$$

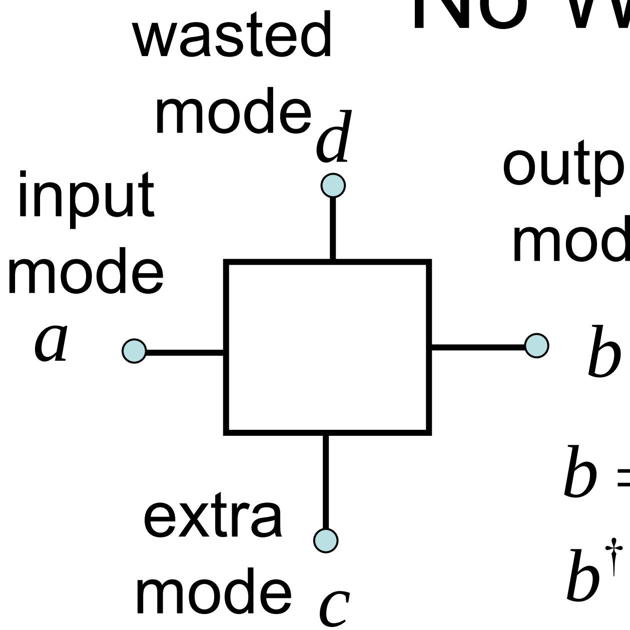
$$\left(\Delta x_{in}\right)^2 = \frac{1}{2}\left\langle aa^{\dagger} + a^{\dagger}a \right\rangle = n_a + \frac{1}{2}$$

$\boxed{G \gg 1}$

$$\left(\Delta x_{out}\right)^2 = \frac{1}{2}\left\langle bb^{\dagger} + b^{\dagger}b \right\rangle = \frac{G}{2}\left\langle \left\{ a + c^{\dagger}, a^{\dagger} + c \right\} \right\rangle$$

$$= G\left(n_a + \underbrace{\frac{1}{2}}_{\text{amplified input vacuum}} + \underbrace{n_c + \frac{1}{2}}_{\text{added noise}} \right)$$

No Wasted Information



(e.g. Clerk, 2003)

$$b = \sqrt{G}\, a + \sqrt{G-1}\left(c^{\dagger}\cosh\theta + d\sinh\theta\right)$$

$$b^{\dagger} = h.c.$$

$$\Rightarrow \left[b, b^{\dagger}\right] = 1$$

$$\boxed{G \gg 1}$$

$$\left(\Delta x_{out}\right)^2 = \frac{1}{2}\left\langle\left\{b, b^{\dagger}\right\}\right\rangle = \frac{G}{2}\left\langle\left\{a + c^{\dagger}\cosh\theta + d\sinh\theta, h.c.\right\}\right\rangle$$

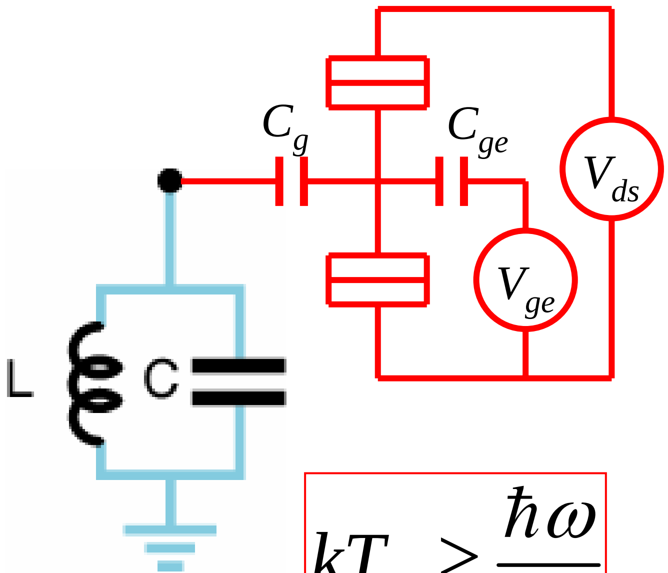
$$\left(\Delta x_{out}\right)^2 = G\left(n_a + \frac{1}{2} + \underbrace{\cosh^2\theta\left(n_c + \frac{1}{2}\right) + \sinh^2\theta\left(n_d + \frac{1}{2}\right)}_{\text{“Excess” noise above quantum limit}}\right)$$

Two Manifestations of Quantum Limit

<u>Position meas. of a beam</u>

Mech. HO with SET/APC detector

(Cleland et al.; Schwab et al.; Lehnert et al.)



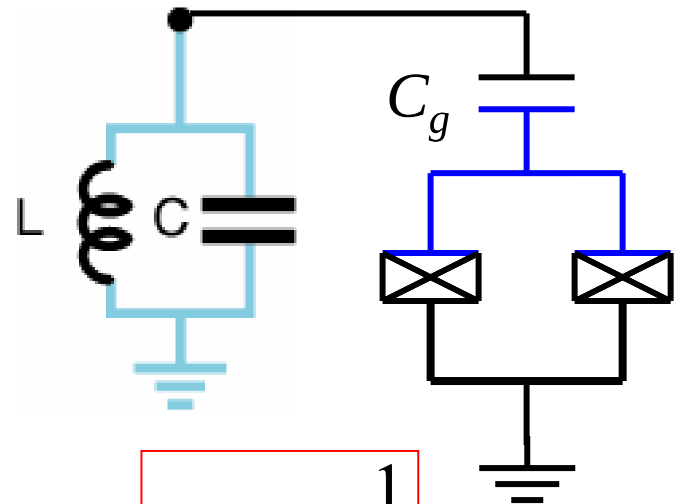
$$kT_N \geq \frac{\hbar\omega}{2}$$

min. noise energy of amplifier

<u>QND meas. of a qubit</u>

Circuit QED: Box + HO

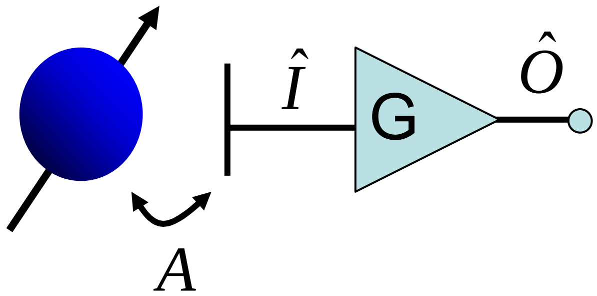
(Yale)



$$T_m \Gamma_\phi \geq \frac{1}{2}$$

meas. induces dephasing

Linear QND Measurement of Qubit



no transitions caused by measurement:

$$H_Q = -\frac{\hbar}{2}\omega_{01}\hat{\sigma}_z \qquad \hat{H}_1 = A\hat{\sigma}_z\hat{I} \qquad \left[\hat{H}_Q, \hat{H}_1\right] = 0$$

quantum nondemolition

in reality: $\left[\hat{H}_1, \hat{H}_{Universe}\right] \neq 0$

always some “demolition,” e.g. spontaneous emission

if $\left|\psi_q\right\rangle = \left|\sigma_z = \pm 1\right\rangle = \left|\uparrow\right\rangle \text{ or } \left|\downarrow\right\rangle$

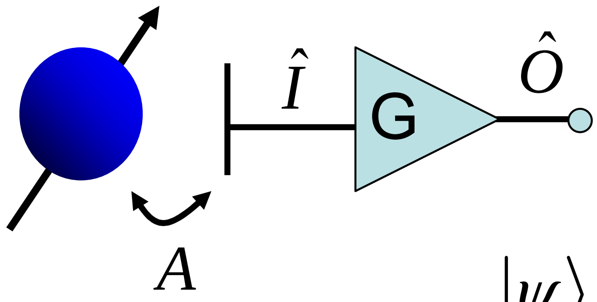
can measure repeatedly, no errors

but if $\left|\psi_q\right\rangle = \left|+,-\right\rangle = \left|\rightarrow\right\rangle \text{ or } \left|\leftarrow\right\rangle$

$\left\langle \hat{\sigma}_z \right\rangle = 0$

we get ± 1 at random

Linear QND Measurement - II



linear amplifier:

$$\left\langle \hat{O}(t) \right\rangle = A\int d\tau\, G\left(t-\tau\right)\left\langle \hat{\sigma}_z(\tau) \right\rangle$$

$$\left|\psi\right\rangle = \left|\psi\left(t=0\right)\right\rangle - \frac{i}{\hbar}\int_{-\infty}^{t} d\tau\, \hat{H}_1 \left|\psi(0)\right\rangle$$

$$\hat{H}_1 = A\hat{\sigma}_z \hat{I}$$

$$\left\langle \psi \right| = \left\langle \psi(0) \right| + \frac{i}{\hbar}\int_{-\infty}^{t} d\tau\, \left\langle \psi(0) \right| A\hat{\sigma}_z \hat{I}$$

$$\left\langle \psi \middle| \hat{O} \middle| \psi \right\rangle = -\frac{i}{\hbar}\int_{-\infty}^{\infty} d\tau\, \Theta(t-\tau) \left\langle \psi(0) \right| \left[\hat{O}, \hat{H}_1(\tau) \right] \left| \psi(0) \right\rangle$$

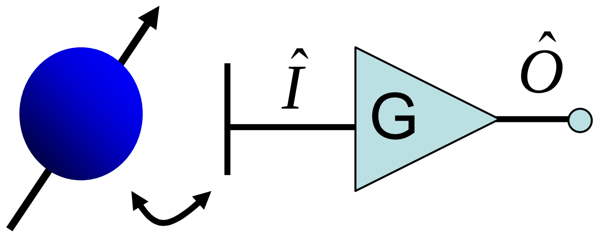
$$\left\langle \hat{O}(t) \right\rangle = -\frac{i}{\hbar}\int_{-\infty}^{\infty} d\tau\, A \left\langle \hat{\sigma}_z(\tau) \right\rangle \Theta(t-\tau) \left\langle \left[\hat{O}, \hat{I}(\tau) \right] \right\rangle$$

recognize $\boxed{\hbar G(t) = -i\Theta(t)\left\langle \left[\hat{O}(t), \hat{I}(0) \right] \right\rangle}$

but if $G \neq 0$ $\Longrightarrow$ $\left[\hat{O}(t), \hat{I}(0) \right] \neq 0$

input and output don't commute, and have noise!

Measurement Time



$$\hat{H}_1 = A\hat{\sigma}_z \hat{I}$$

Integrate output:

$$\hat{M}(t) = \int_0^t d\tau\, \hat{O}(\tau)$$

$$\langle \uparrow | \hat{M} | \uparrow \rangle = \int_0^t d\tau\, AG \langle \hat{\sigma}_z(\tau) \rangle = +AGt$$

$$\langle \downarrow | \hat{M} | \downarrow \rangle = -AGt$$

Distinguish when

$$\frac{\left(\langle \uparrow | \hat{M} | \uparrow \rangle - \langle \downarrow | \hat{M} | \downarrow \rangle \right)^2}{(\Delta M)^2} = \frac{(2AGt)^2}{S_O t} = \frac{4A^2}{S_O / G^2} t \sim 1$$

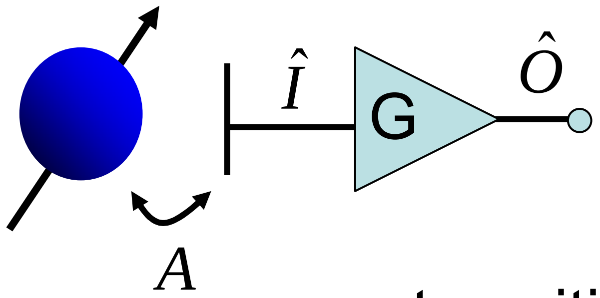
Measurement time

$$T_m = \boxed{\frac{S_O}{G^2}} \frac{1}{4A^2}$$

Stronger coupling, faster measurement

Spectral density of output noise, referred to input

Dephasing by QND Measurement



But $\hat{I}(t)$ also fluctuates!

$$\hat{H}(t) = -\hbar\omega_{01}\hat{\sigma}_z/2 + A\hat{\sigma}_z\hat{I}(t)$$
$$= -\hbar\left(\omega_{01} + \delta\omega(t)\right)\hat{\sigma}_z/2$$

so transition (Larmor) freq. fluctuates

$$|\psi(0)\rangle = |\hat{\sigma}_z = \pm 1\rangle \quad \Rightarrow \quad \text{unperturbed}$$

phase fluctuates!

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|+1\rangle + |-1\rangle\right) \quad \Rightarrow \quad |\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|+1\rangle + e^{i\phi(t)}|-1\rangle\right)$$

fluctuations Gaussian and rapid:

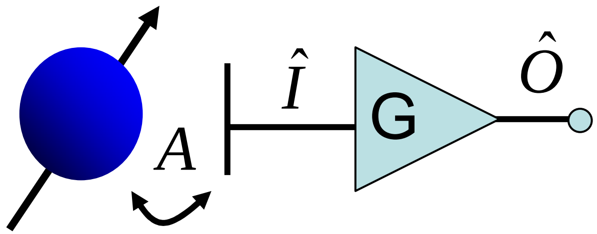
$$\phi(t) = \omega_{01}t + \int_0^t d\tau\,\delta\omega(\tau) = \omega_{01}t + \int_0^t d\tau\,\frac{A}{\hbar}\hat{I}(\tau)$$

$$\left(\Delta\phi\right)^2 = \frac{A^2}{\hbar^2}\left\langle\left(\Delta I\right)^2\right\rangle t = \frac{2A^2}{\hbar^2}S_I t = \Gamma_\phi t$$

spectral density of input dephasing rate

$\boxed{\text{Stronger coupling, faster dephasing!}}$

Quantum Limit for QND Measurement



Compare dephasing rate and measurement time:

Measurement time: $$T_m = \frac{S_O}{G^2}\frac{1}{4A^2}$$

Dephasing rate: $$\Gamma_\phi = \frac{2A^2}{\hbar^2} S_I$$

$$T_m \Gamma_\phi = \frac{S_O}{G^2}\frac{1}{4A^2}\frac{2A^2}{\hbar^2} S_I = \frac{S_O S_I}{2G^2\hbar^2}$$

independent of coupling!

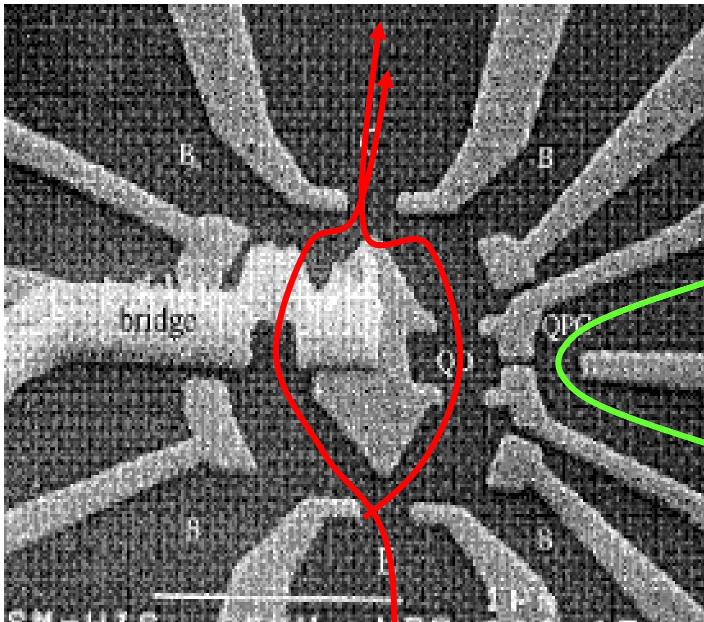
and since $\boxed{\hbar G \sim \left[\hat{O}(t), \hat{I}(0)\right]}$ $\Rightarrow$ $(\Delta O)^2 (\Delta I)^2 \geq (\hbar G)^2$

Quantum Limit! $$\boxed{T_m \Gamma_\phi \geq \frac{1}{2}}$$

Measurement is dephasing

Measurement Dephasing – Quantum Dots

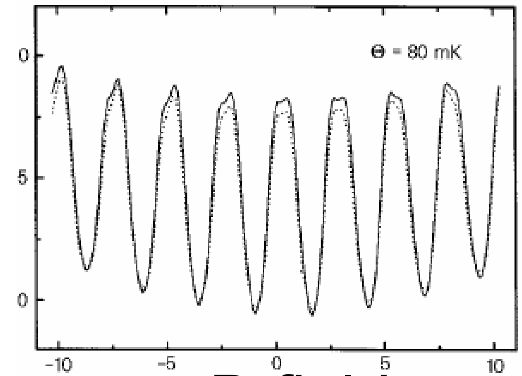
A “which path” experiment in mesoscopics - Heiblum group, Weizmann 1998



QPC “detector”

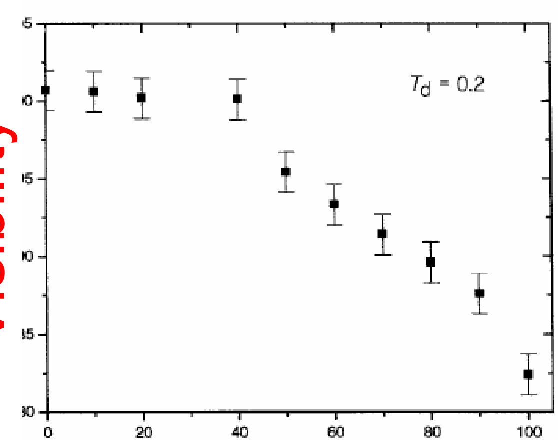
Quantum dot in a ring

A-B oscillations of ring tests coherence



G_{ring}

B-field



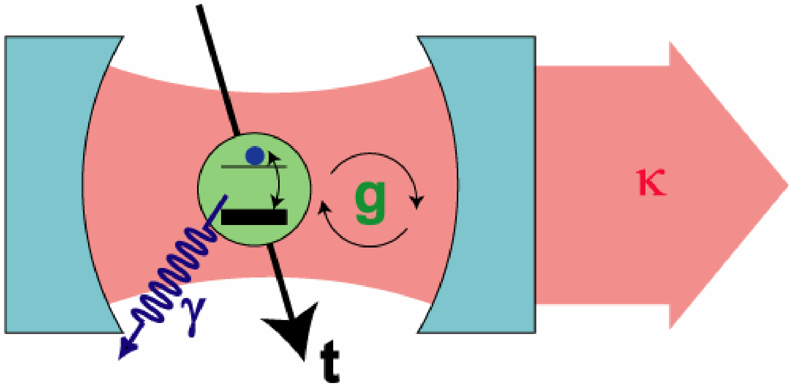
Visibility

QPC current

QPC current senses which way electrons go around ring, destroys fringes.

E. Buks et al., Nature **391**, 871 (1998)

“Circuit QED” – Box + Transmission Line Cavity

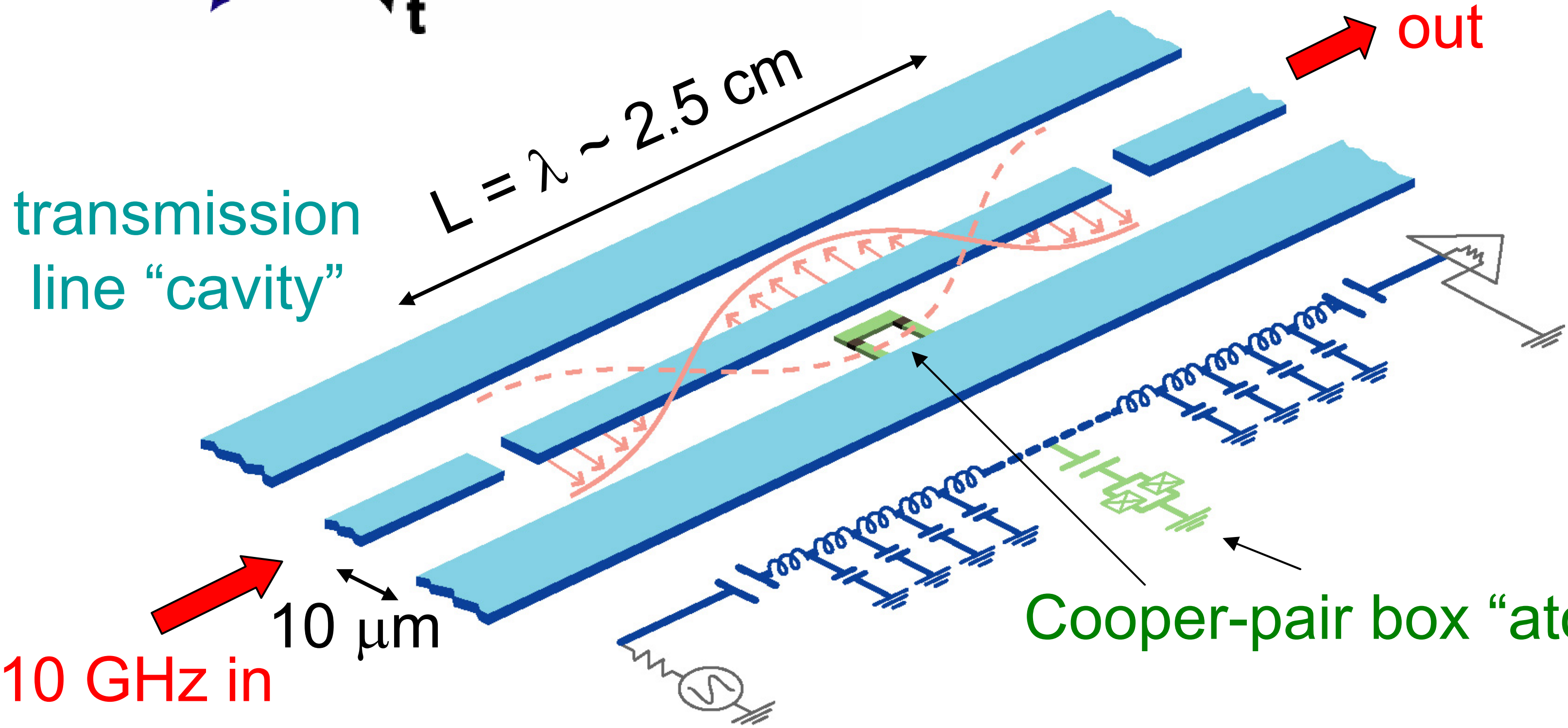


$2g$ = vacuum Rabi freq.

κ = cavity decay rate

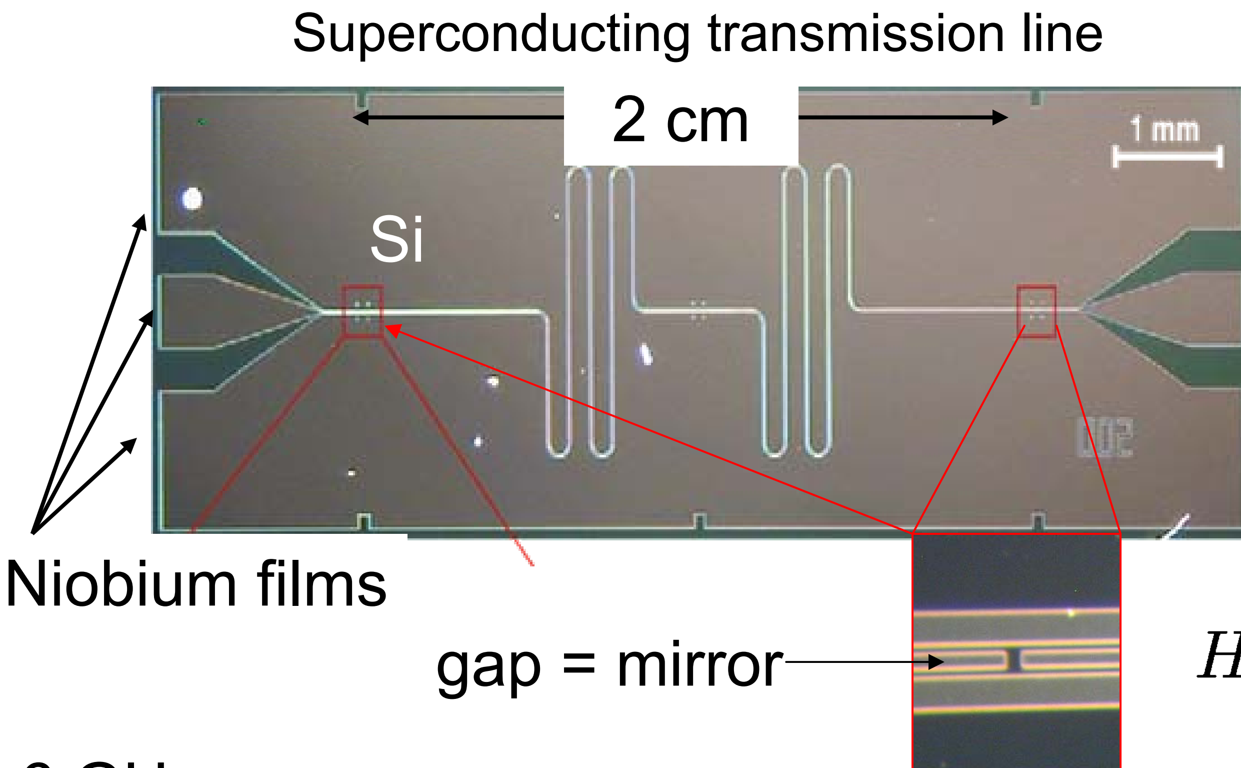
γ = “transverse” decay rate

Strong Coupling = **g** $> \kappa , \gamma$



Theory: Blais et al., *Phys. Rev. A* **69**, 062320 (2004)

Implementation of Oscillator on a Chip



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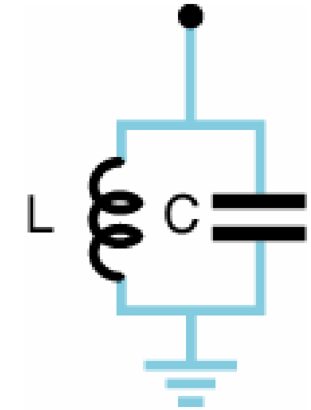


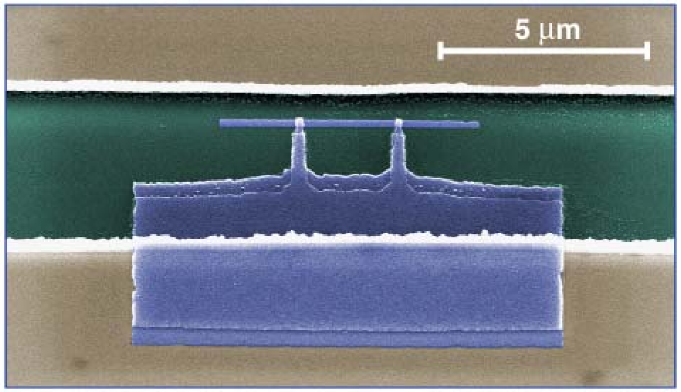
$$H_r = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right)$$

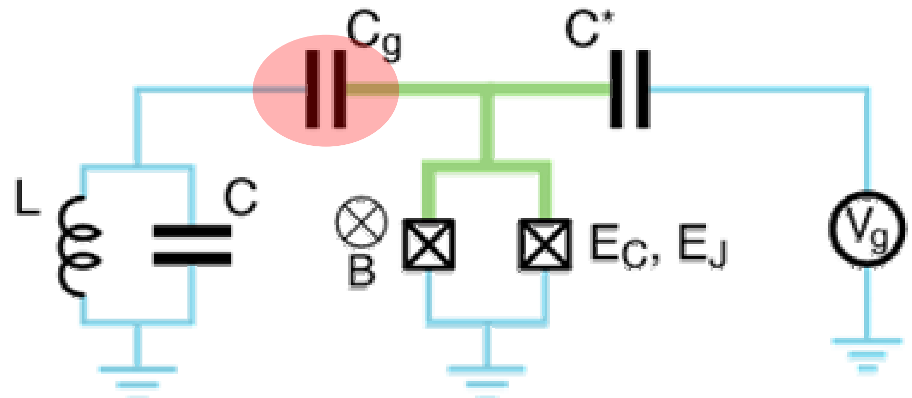
<u>6 GHz:</u>

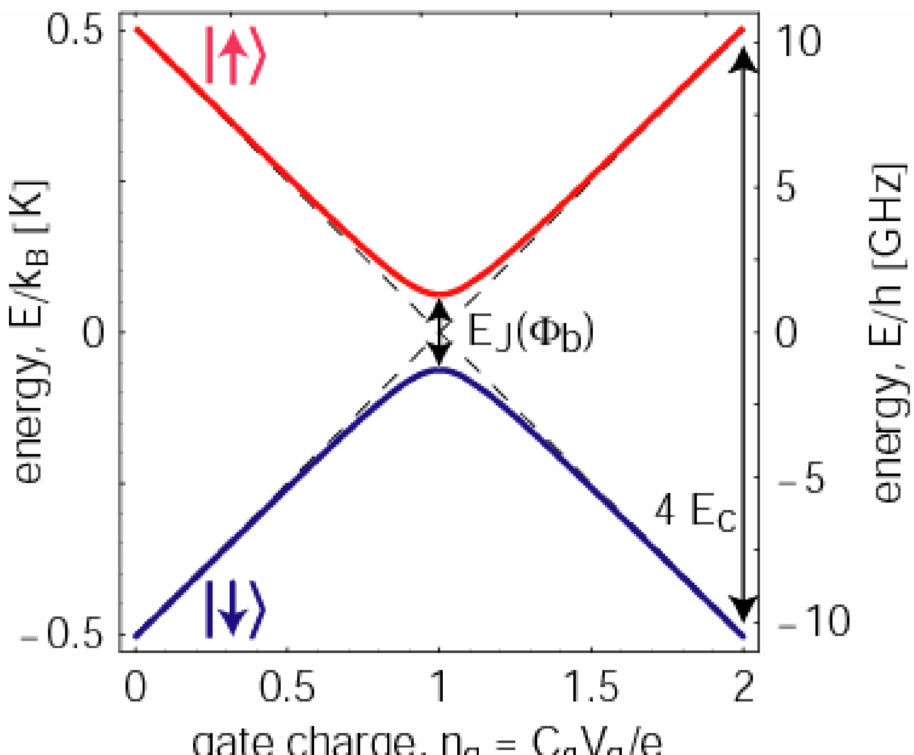
$$\hbar\omega = 300\,mK \quad \Rightarrow \quad \langle n_\gamma \rangle \ll 1 \quad @\ 20\ \mathrm{mK}$$

<u>RMS voltage:</u> $V_0 = \sqrt{\dfrac{\hbar\omega_R}{2C_R}} \sim 1\ \mu\mathrm{V}$ even when $\langle n_\gamma \rangle = 0$

Energy Levels of Cooper Pair Box







$$H = \frac{E_{\text{Coulomb}}}{2}\sigma^{x} - \frac{E_{\text{Josephson}}}{2}\sigma^{z}$$

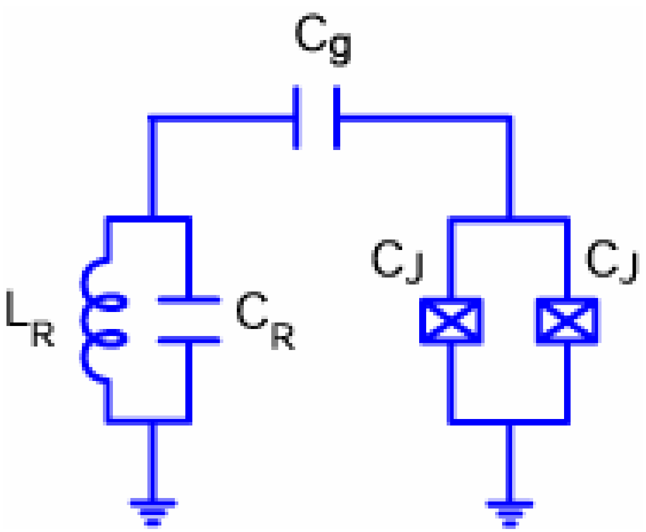
Tune σ^x with voltage: (Stark)

$$E_{\text{Coulomb}} = 4E_C\left(n_g - 1\right)$$

Tune σ^z with Φ: (Zeeman)

$$E_{\text{Josephson}} = E_{J_{\max}} \cos\left[\pi\Phi_b / \Phi_0\right]$$

Box Coupled to Oscillator



L_R ~ ½ nH; C_R ~ ½ pF

$$\frac{1}{2} C_R \langle V_0^2 \rangle = \frac{1}{4} \hbar\omega_R$$

$$V_0 = \sqrt{\frac{\hbar\omega_R}{2C_R}} \sim 1\ \mu\mathrm{V}$$

$$\hat{H}_{box} = -\frac{E_J}{2}\hat{\sigma}_z$$

$$\hat{H}_{HO} = \hbar\omega_R(a^\dagger a + 1/2)$$

$$\hat{H}_{\text{int}} = -e\left(\frac{C_g}{C_\Sigma}\right)\hat{V}\hat{\sigma}_x$$

$$= -\hbar g(\sigma^- a^\dagger + \sigma^+ a)$$

Jaynes-Cummings

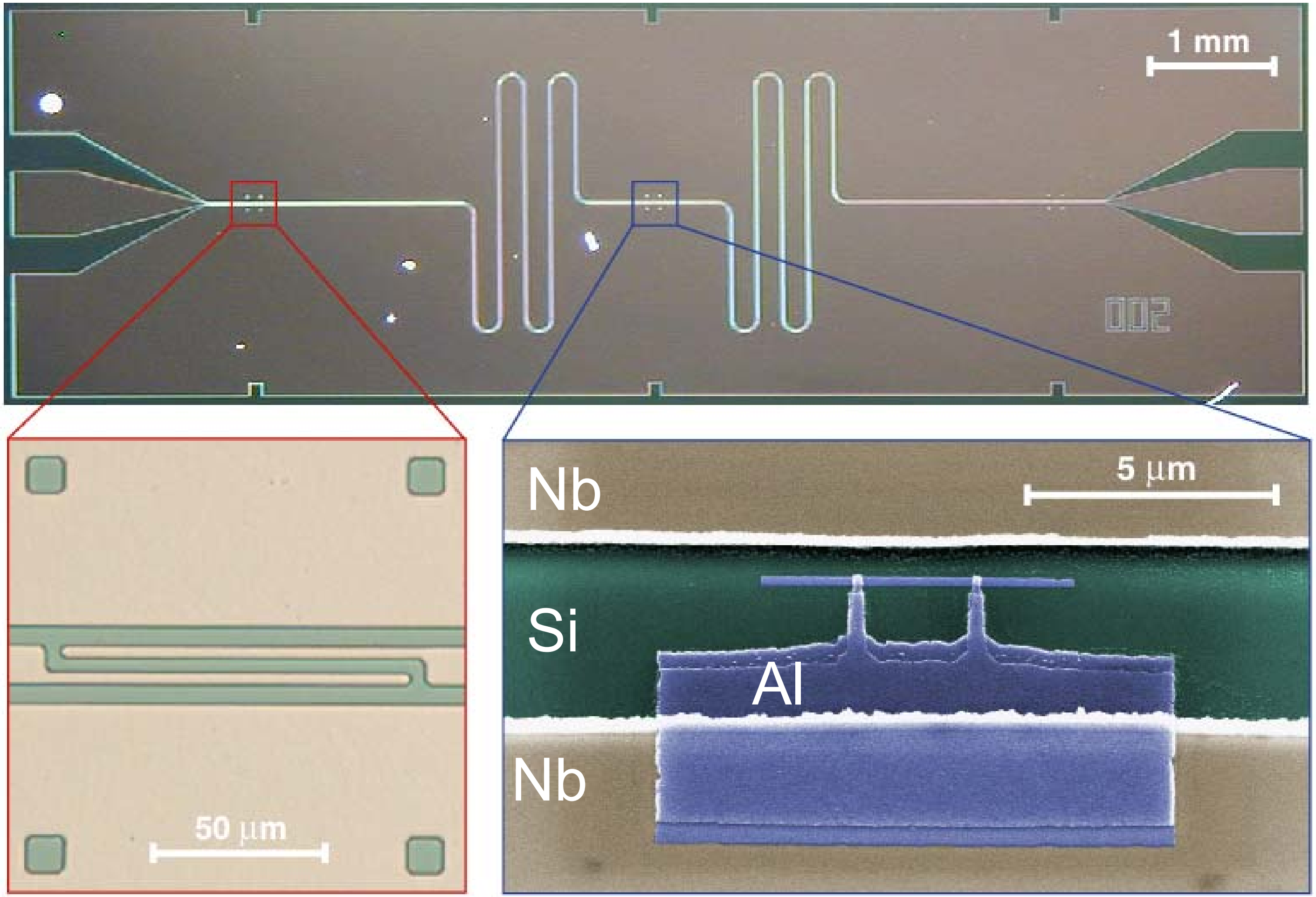
$$g = \frac{1}{2\hbar}\frac{eC_g}{C_\Sigma}\sqrt{\frac{\hbar\omega_R}{2C_R}}$$

So for: $C_g / C_\Sigma = 0.1$

$\Rightarrow$ $g \sim 10 - 100\ \mathrm{MHz}$

The Chip for Circuit QED

Wallraff et al., Nature **431**, 162 (2004).



No wires attached to qubit!

realization of superconducting cavity QED circuit

Dispersive QND Qubit Measurement

approximate diagonalization for $|\Delta| = |\omega_a - \omega_r| \gg g$

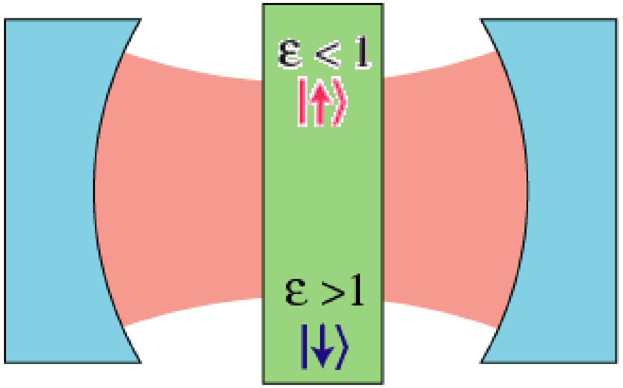
$$H \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right)a^\dagger a + \frac{1}{2}\hbar\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

//

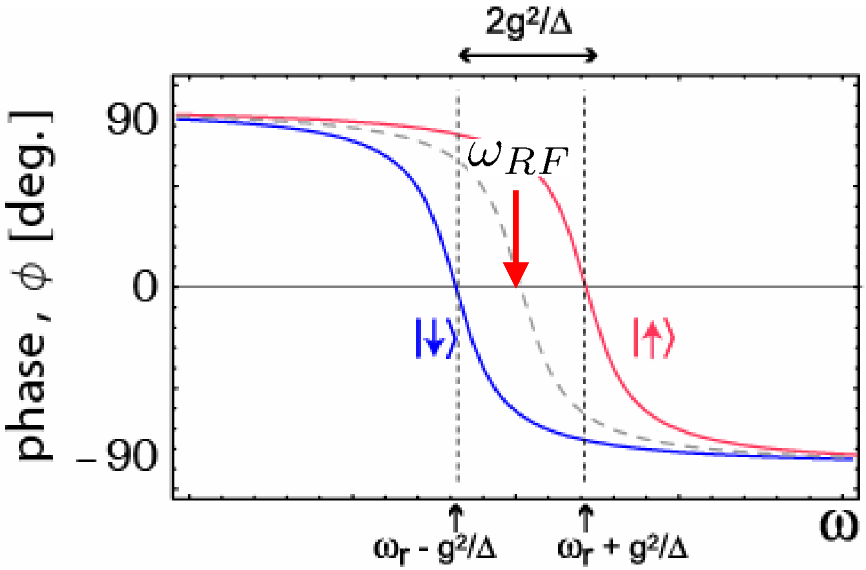
cavity frequency shift
and qubit ac-Stark shift

//

Lamb shift



reverse of Nogues et al., 1999
(Ecole Normale)
QND of single photon
using Rydberg atoms!



A. Blais, R.-S. Huang, A. Wallraff, S. M. Girvin, and RS, *PRA* **69,** 062320 (2004)

Alternate View of the QND Measurement

$$H_{\text{eff}} \approx \hbar\left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right)a^\dagger a + \frac{\hbar}{2}\left(\omega_a + \frac{g^2}{\Delta}\right)\sigma_z$$

cavity freq. shift

atom ac Stark shift

$= 2\bar{n} \times$ cavity pull

Lamb shift

vacuum ac Stark shift

$$H_{\text{eff}} \approx \hbar\omega_r a^\dagger a + \frac{\hbar}{2}\left(\omega_a + \boxed{2\frac{g^2}{\Delta}}\left[\boxed{a^\dagger a} + \frac{1}{2}\right]\right)\sigma_z$$

A

$\hat{n} \sim \hat{I}$

cQED Measurement and Backaction - Predictions

Input = photon number in cavity   Output = voltage outside cavity

phase shift on transmission: $$\theta_0 = \frac{2g^2}{\kappa\Delta}$$

measurement rate:

$$\Gamma_m = \frac{1}{T_m} = 2\theta_0^2\left(\frac{P}{\hbar\omega_r}\right) = 2\theta_0^2\bar{n}\kappa$$

(expt. still ~ 40 times worse)

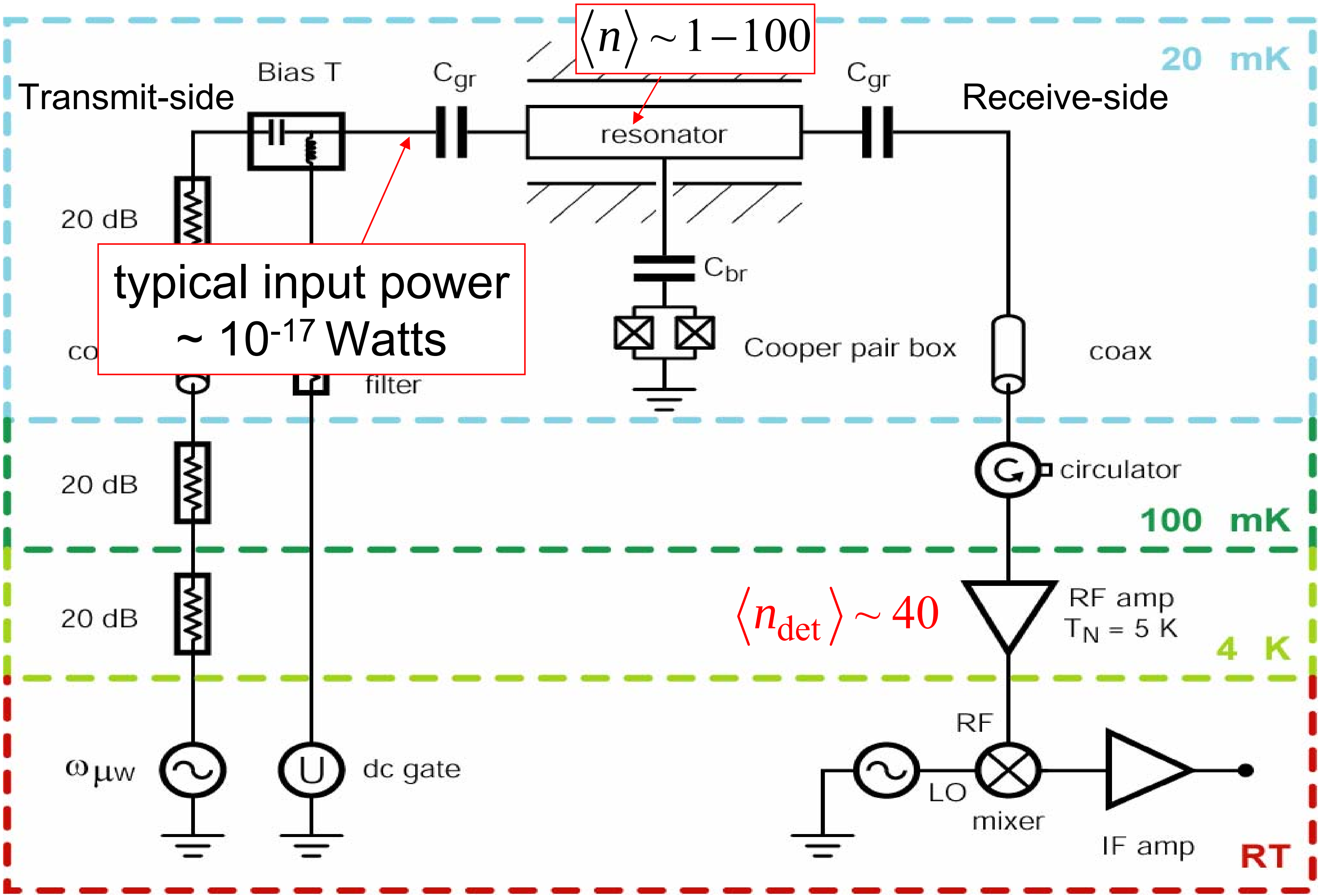
dephasing rate:

$$\Gamma_\phi = 2\theta_0^2\left(\frac{P}{\hbar\omega_r}\right) = 2\theta_0^2\bar{n}\kappa$$

quantum limit?: $$T_m\Gamma_\phi = 1$$

2x limit, since half of information wasted in reflected beam

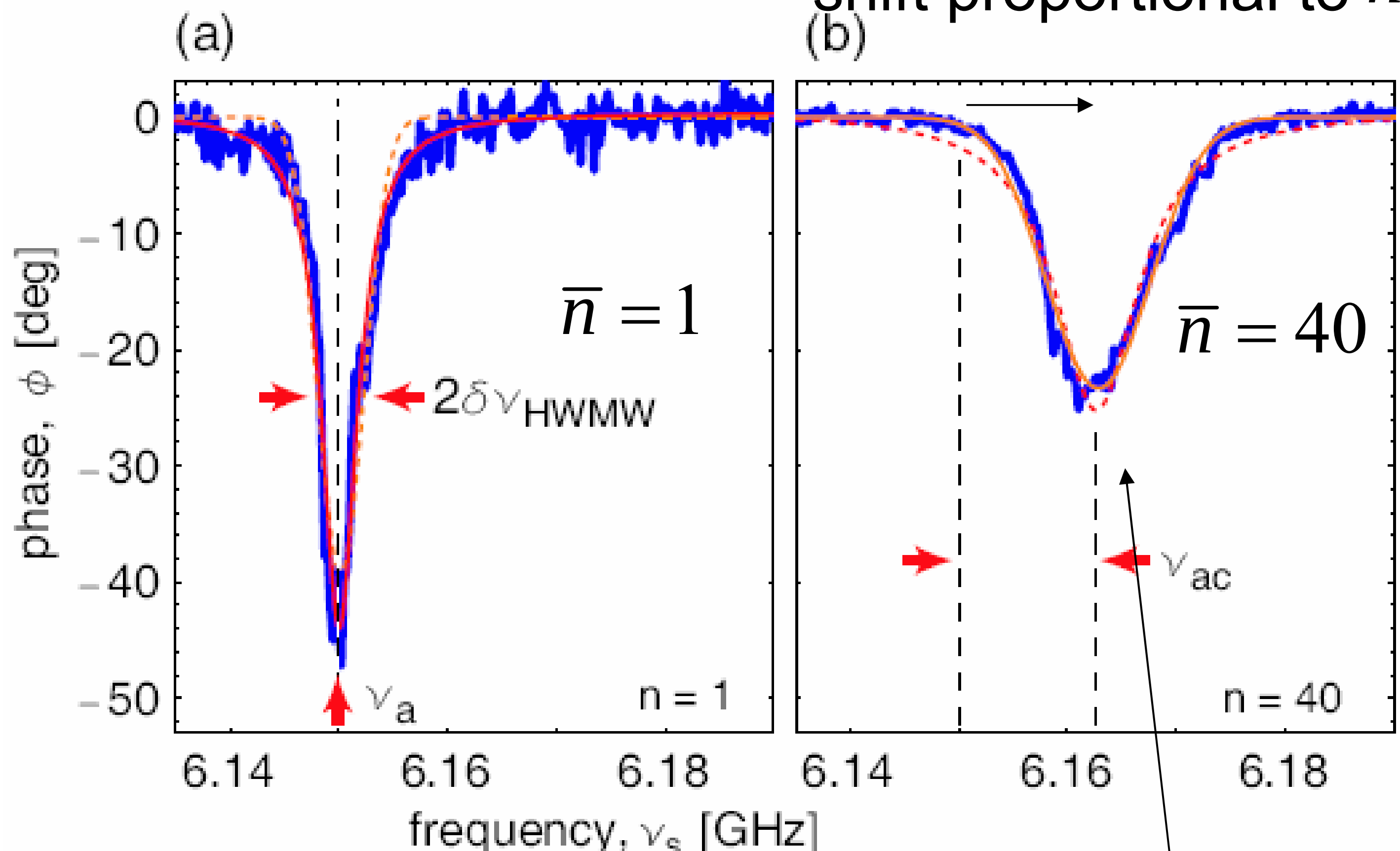
Microwave Setup for cQED Experiment



Observing ac Stark Shift

Measure absorption spectrum of CPB w/ continuous msmt.

shift proportional to $\overline{n}$

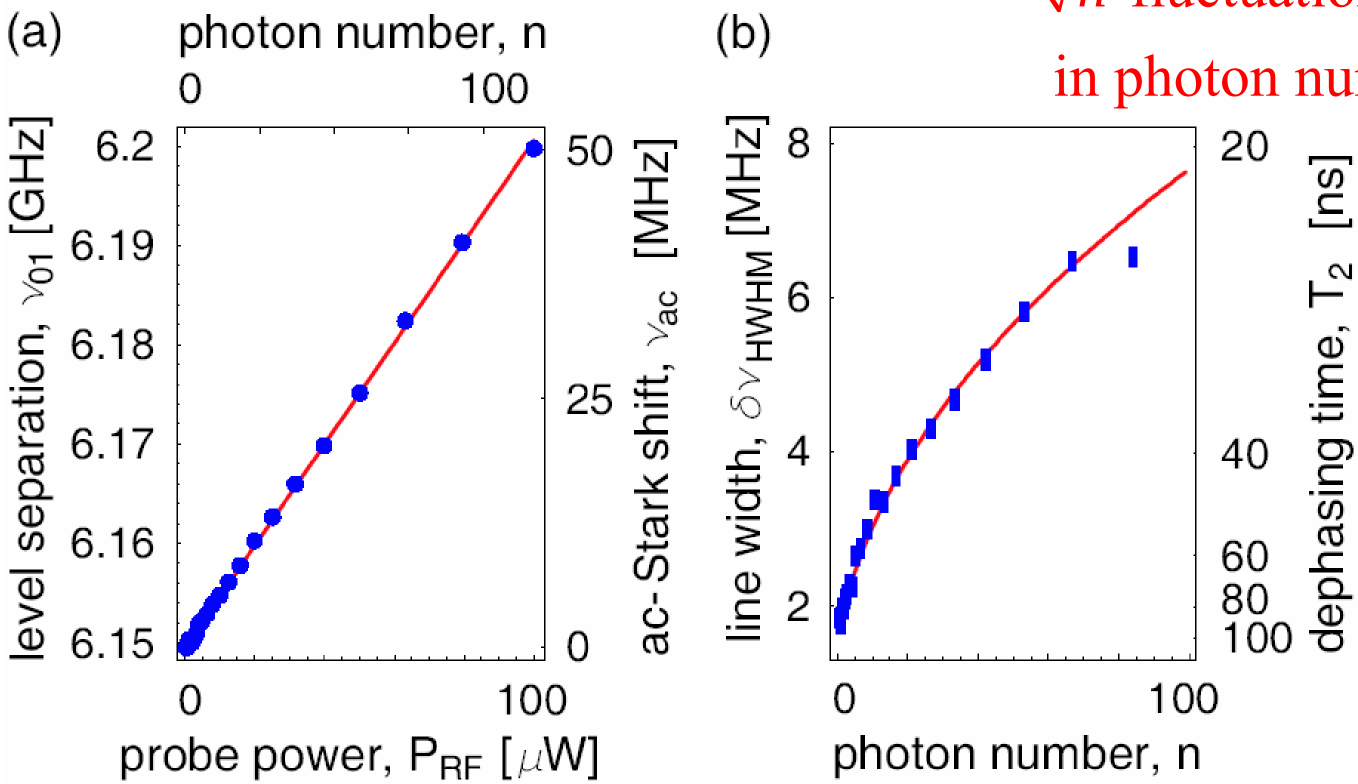


Line broadened as qubit
is dephased by photon shot noise

Observing Backaction of Measurement

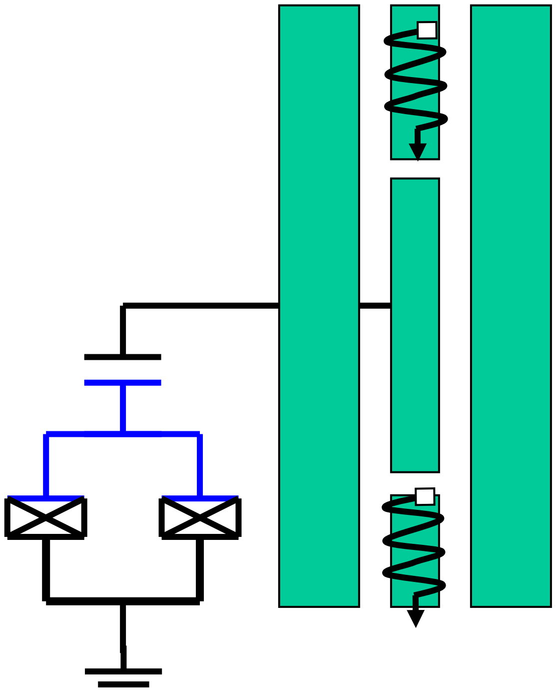
$$H_{\mathrm{eff}} \approx \omega_{\mathrm{r}} a^{\dagger} a - \frac{1}{2}\left(\omega_a + 2\frac{g^2}{\Delta}\left[a^{\dagger}a + \frac{1}{2}\right]\right)\sigma_z$$

$\sqrt{n}$ fluctuations in photon number

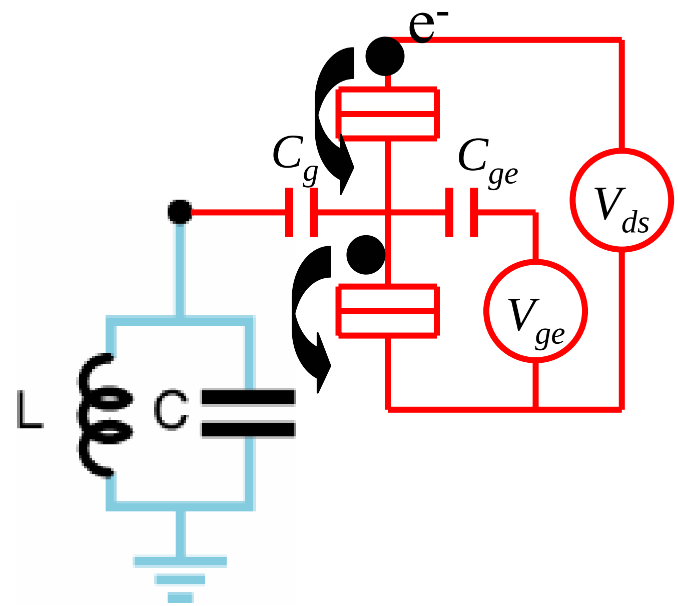


expt: Schuster et al., PRL **94**, 123602 (2005).

Cavity QED - SET Analogy



photon shot noise
induces qubit dephasing



shot noise of SET
current causes
backaction

Summary of Lecture 3

- Quantum limit on measurement comes from $\left[a, a^{\dagger}\right] = 1$

- Two equivalent manifestations of quantum limit:

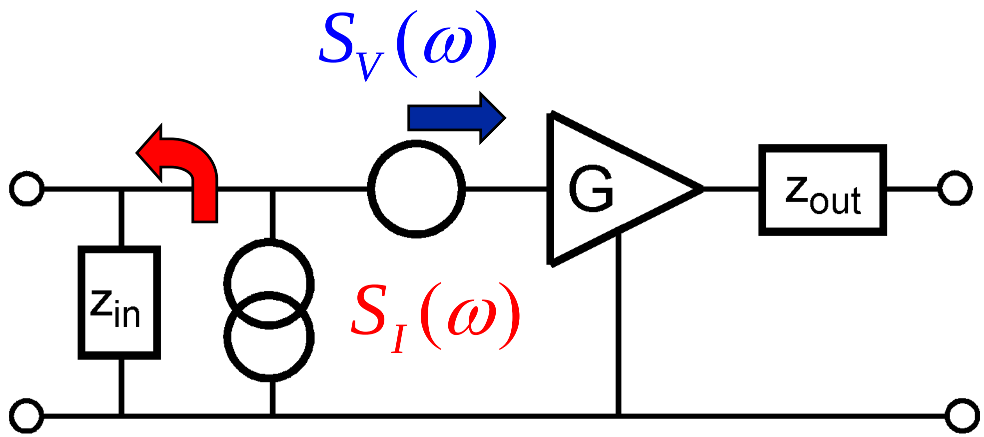
Min. noise temperature $\boxed{T_N \geq \dfrac{\hbar\omega}{2k}}$ Meas. induced dephasing $\boxed{T_m \Gamma_\phi \geq \dfrac{1}{2}}$

- Mesoscopic expts. can approach these limits:
  Sensitivity ~ 10-100 times limit obtained
  Dephasing due to measurement observed

- But true quantum limit not yet observed/tested!

- Future: back-action evasion, squeezing,
  quantum feedback, …

Equivalent Circuit of an Amplifier



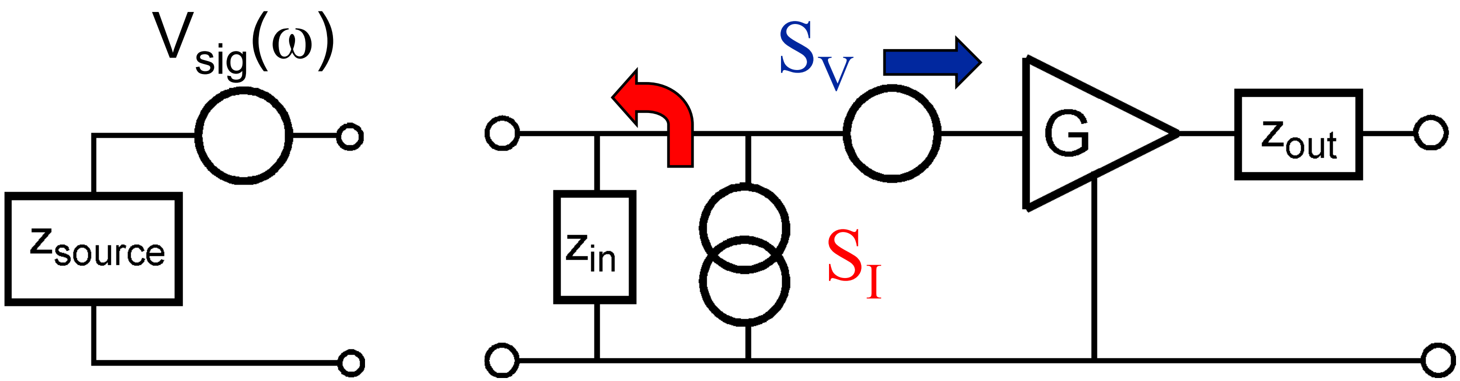
$S_V(\omega)$ "ficticious noise source" (V^2/Hz)
= output noise referred to input

$S_I(\omega)$ a real noise (A^2/Hz) driven thru input terminals

here assume uncorrelated, though typically not!

Noise Temperature of an Amplifier

Def'n (IEEE) : temperature of a load @ input which doubles the system's output noise (assumes Rayleigh-Jeans)



total noise at input: $S_V^{tot} = S_V + S_I \left| \frac{Z_{in} Z_S}{Z_{in} + Z_S} \right|^2$

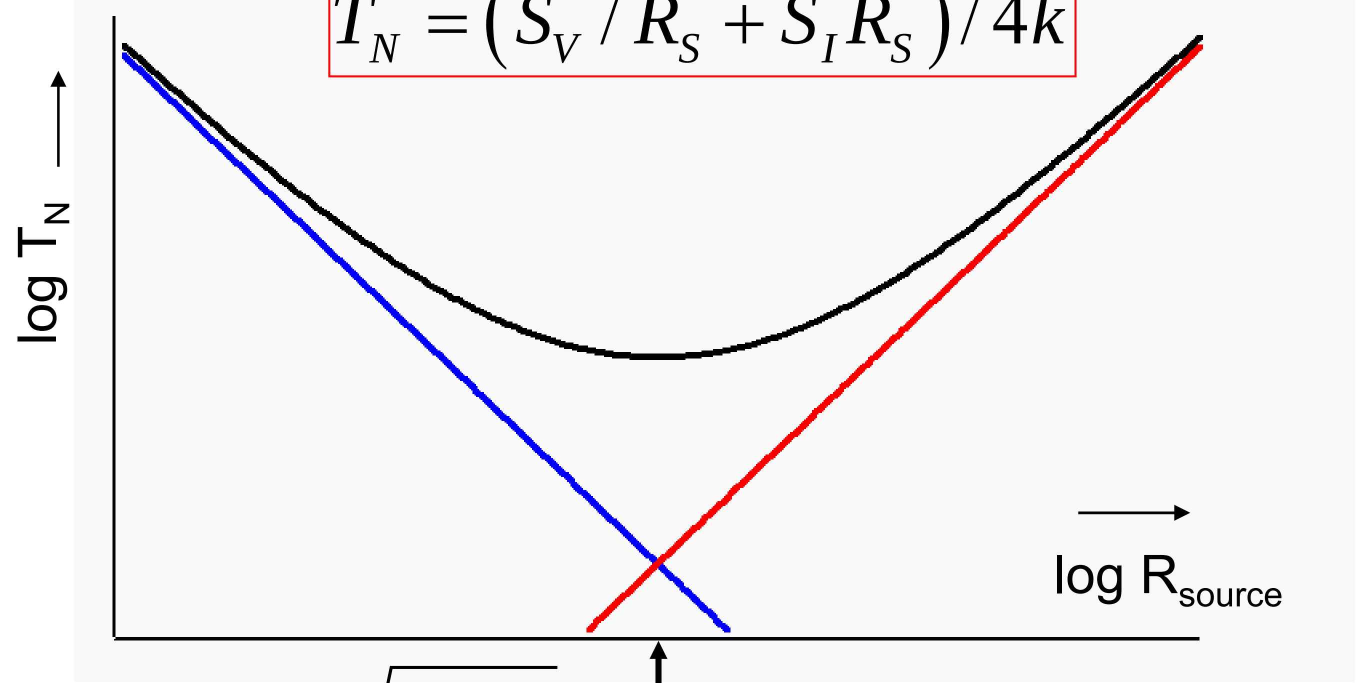
equate to Johnson noise of source: $S_V^{tot} = 4kT_N \, \mathrm{Re}[Z_s]$

for $Z_{in} = R_{in} \gg R_s = Z_s$ $\quad \boxed{T_N = \left(S_V / R_S + S_I R_S \right) / 4k}$

T_N depends on source impedance

Optimum Noise Temperature of Amplifier

$$\boxed{T_N = \left(S_V / R_S + S_I R_S\right)/4k}$$



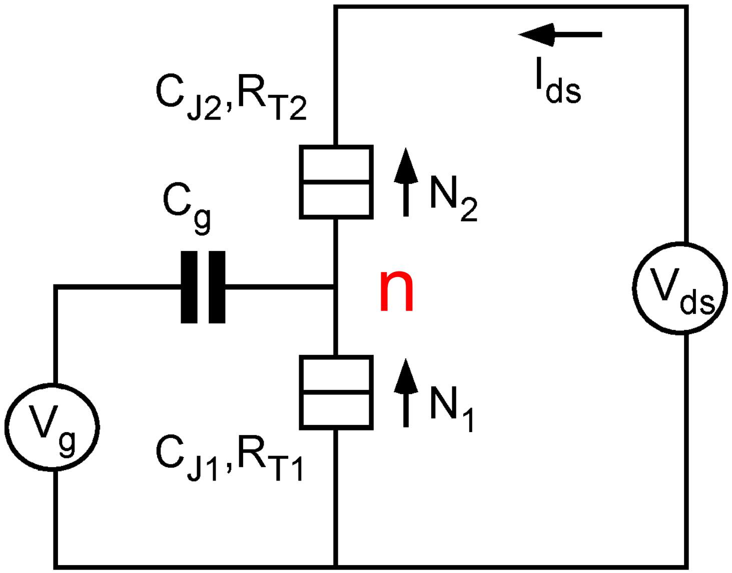
$$R_{opt} = \sqrt{S_V / S_I}$$

$$T_{N_{opt}} = \sqrt{S_V S_I}/2k \qquad E_{N_{opt}} = kT_{N_{opt}} = \sqrt{S_V S_I}/2$$

E_N is energy of signal that can be detected with SNR = 1

QM imposes minimum: $\boxed{E_N \geq \hbar\omega/2}$

Noise of a Single Electron Transistor



charge advance, k

$$k = (N_1 + N_2)/2$$

$$I_{ds} = e\left\langle \frac{dk}{dt} \right\rangle$$

island charge, n

$$n = N_1 - N_2$$

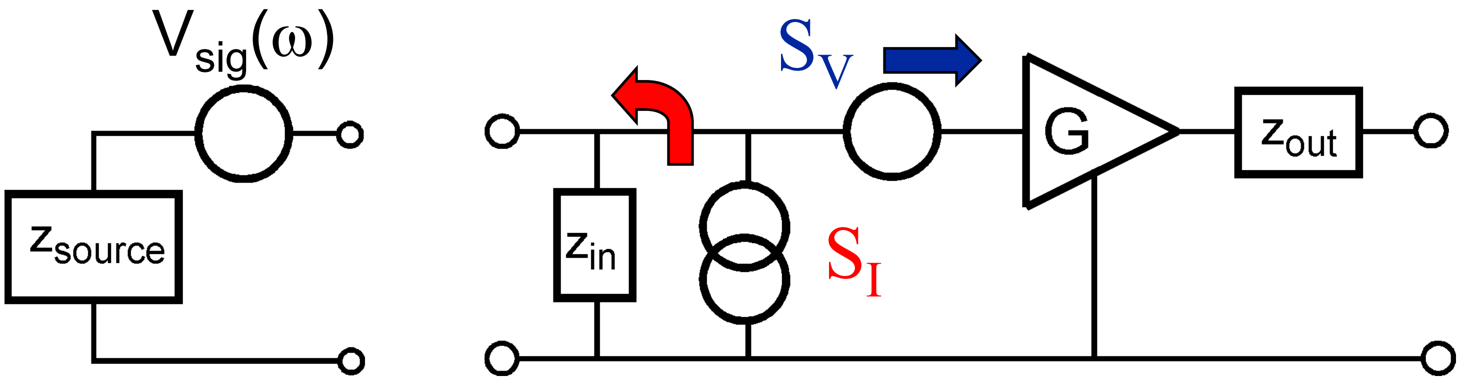
Ideally, SET has only **shot noise** (T=0, ω<V/eR)

Fluctuations of k limit msmt. of response (I_{ds})

Fluctuations of n cause island potential to change

➡ current flows thru gate @ $\omega \neq 0$

Properties of an SET Amplifier



In limit of:
normal state,
T=0,
no cotunneling,
ω << V/eR

$$S_V(\omega) = \frac{(1-\alpha^2)(1+\alpha^2)}{8\alpha^2} e V_{ds} R_\Sigma \left(\frac{C_\Sigma}{C_g}\right)^2$$

indep. of ω

$$S_I(\omega) = \frac{(1-\alpha^2)}{4} \frac{e V_{ds}}{R_\Sigma} \left(\frac{C_g}{C_\Sigma}\right)^2 \left(\frac{e\omega R_\Sigma}{V_{ds}}\right)^2$$

~ ω^2

$$\alpha = (2C_g V_g - e)/C_\Sigma V_{ds}$$

M. Devoret and RS (2000),
similar results by Schon et al,
Averin, Korotkov

Noise Energy of SET

Sequential Tunneling: (e.g. Devoret & RS, 2000)

$$E_N(\omega) = \sqrt{S_V S_I} = \frac{\pi\left(1-\alpha^4\right)}{2\alpha}\sqrt{\frac{\left(1+\alpha^2\right)}{2}}\frac{R_\Sigma}{R_K}\hbar\omega$$

$$E_N < 2\hbar\omega \qquad R_{opt} \approx \frac{1}{\omega C_g} \sim 10^8\ \Omega \quad \text{at 16 MHz}$$

Cotunneling limit: (e.g. Averin, Korotkov)

$$E_N \to \hbar\omega/2$$

Resonant Cooper-pair tunneling (DJQP): (e.g. Clerk)

$$E_N \to \hbar\omega/2$$

Experimentally:

still factor of 10-100 from intrinsic shot noise limit

Other Amplifiers Near Quantum Limit

Josephson parametric amplifier at 19 GHz

$T_N = 0.45\text{K} \sim h\nu/2k$

Yurke et al.;
Movshovich et al., PRL **65**, 1419 (1990)

SIS mixer at 95 GHz
(heterodyne detection using quasiparticle nonlinearity)

Noise added = 0.6 photons

Mears et al., APL **57**, 2487 (1990)

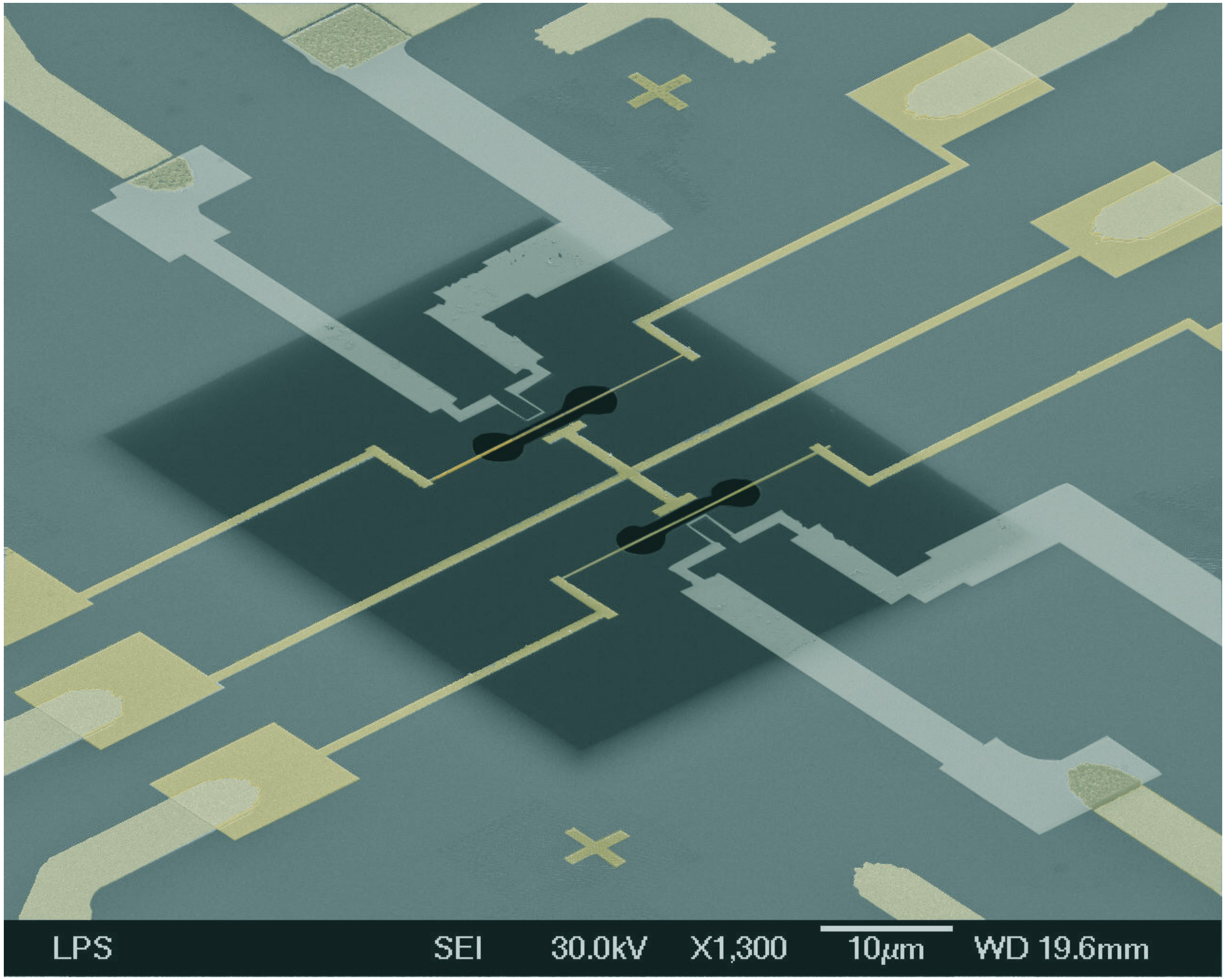
Microwave SQUID amplifier at 500 MHz

$T_N = 50\text{ mK} \sim 2h\nu/k$

Muck, Kycia, and Clarke, APL **78**, 967 (2001)

No measurement of crossover, or backaction yet.

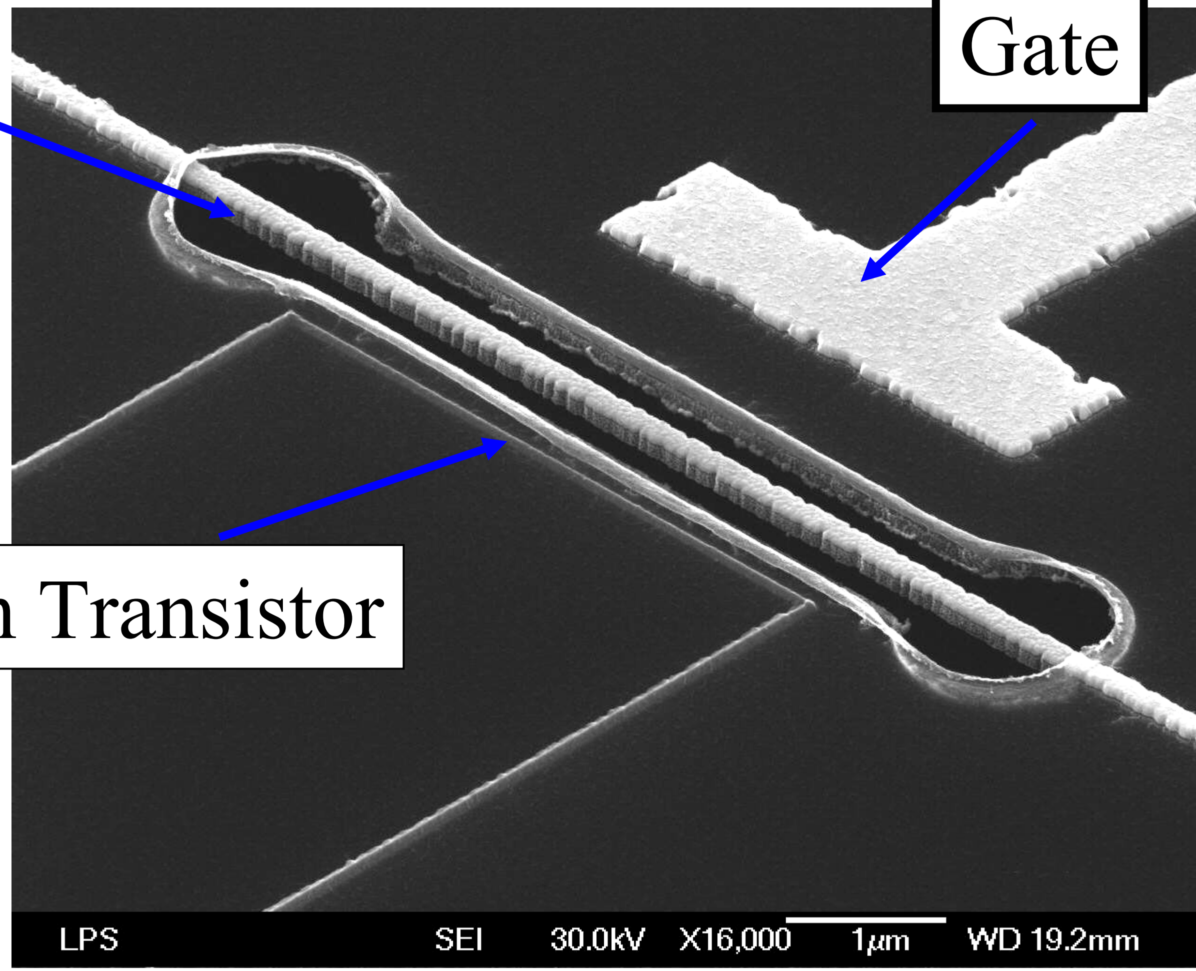
NEMS Oscillator Measured by SET – Schwab group



Sample

Beam

Silicon Nitride
8μm X 200nm X 100nm
f_O = 19.7MHz
Q ~ 30-45000



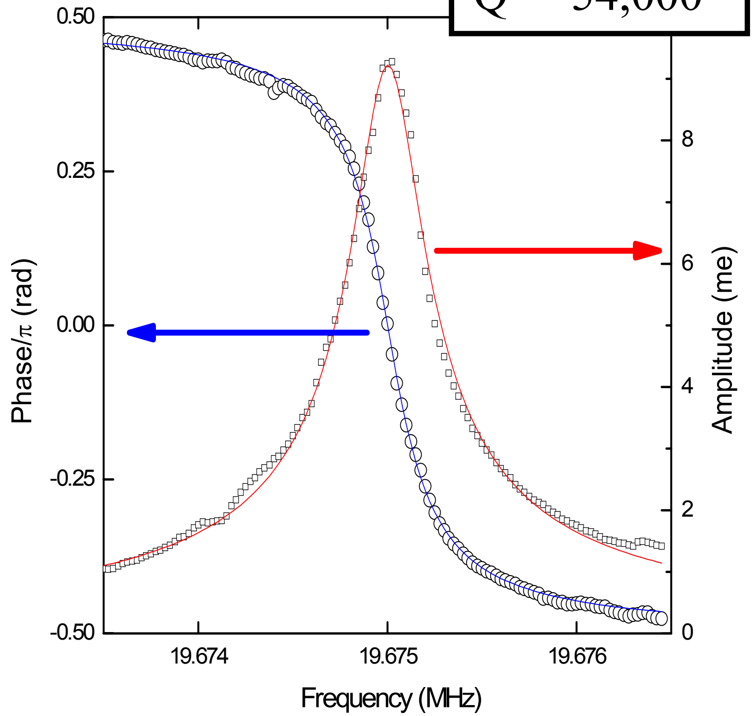
Single Electron Transistor

$Al/Al_xO_y/Al$ Junctions
2K Charging Energy
70kΩ Resistance
70 MHz Bandwidth

Beam/SET Separation: 600nm
27aF Capacitance

Resonator Response

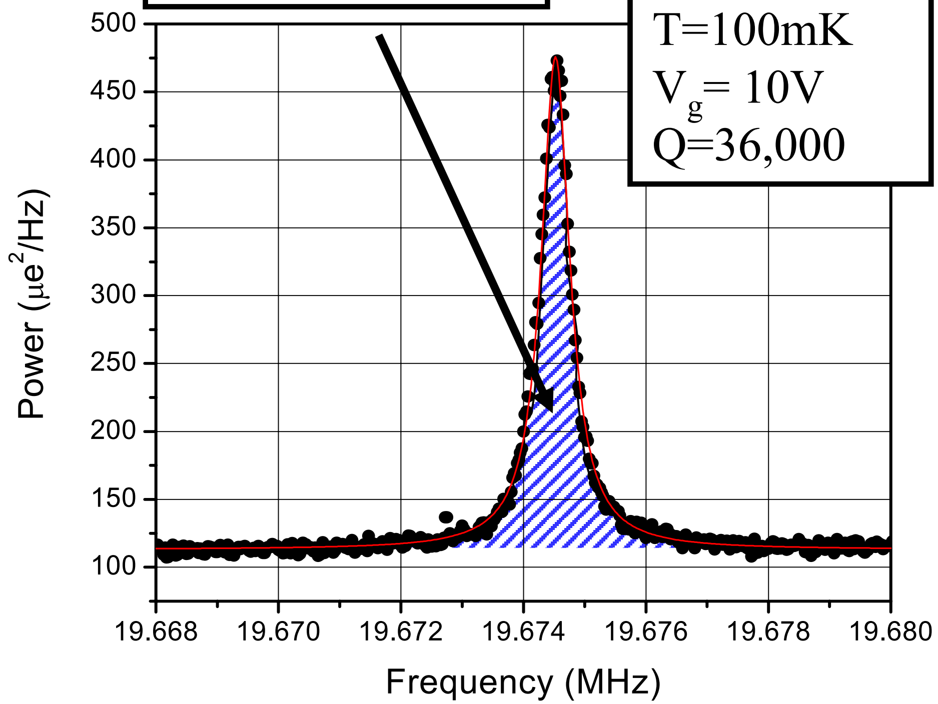
T= 30mk
V_g= 10V
Q = 54,000



Driven Response

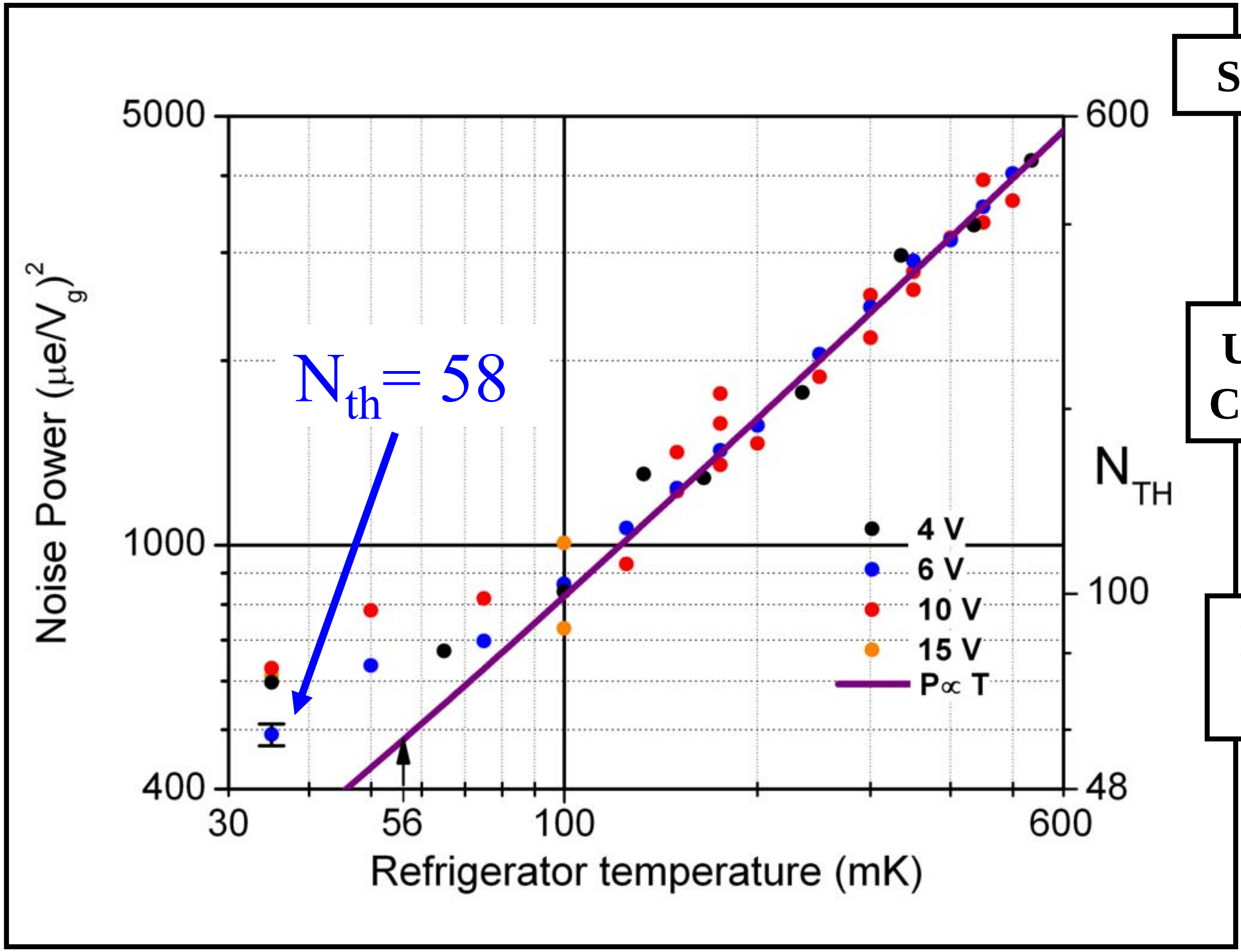
$$\frac{1}{2} m\omega_o^2 x^2 = \frac{1}{2} k_B T$$

T=100mK
V_g= 10V
Q=36,000



Thermal Response

Noise Power vs. Temperature



Saturates Below 100mK

Use Linear Data to Calibrate Below 100mK

Lowest Mode Temp Measured: T=56mk

Noise Temperature

Noise Temperature

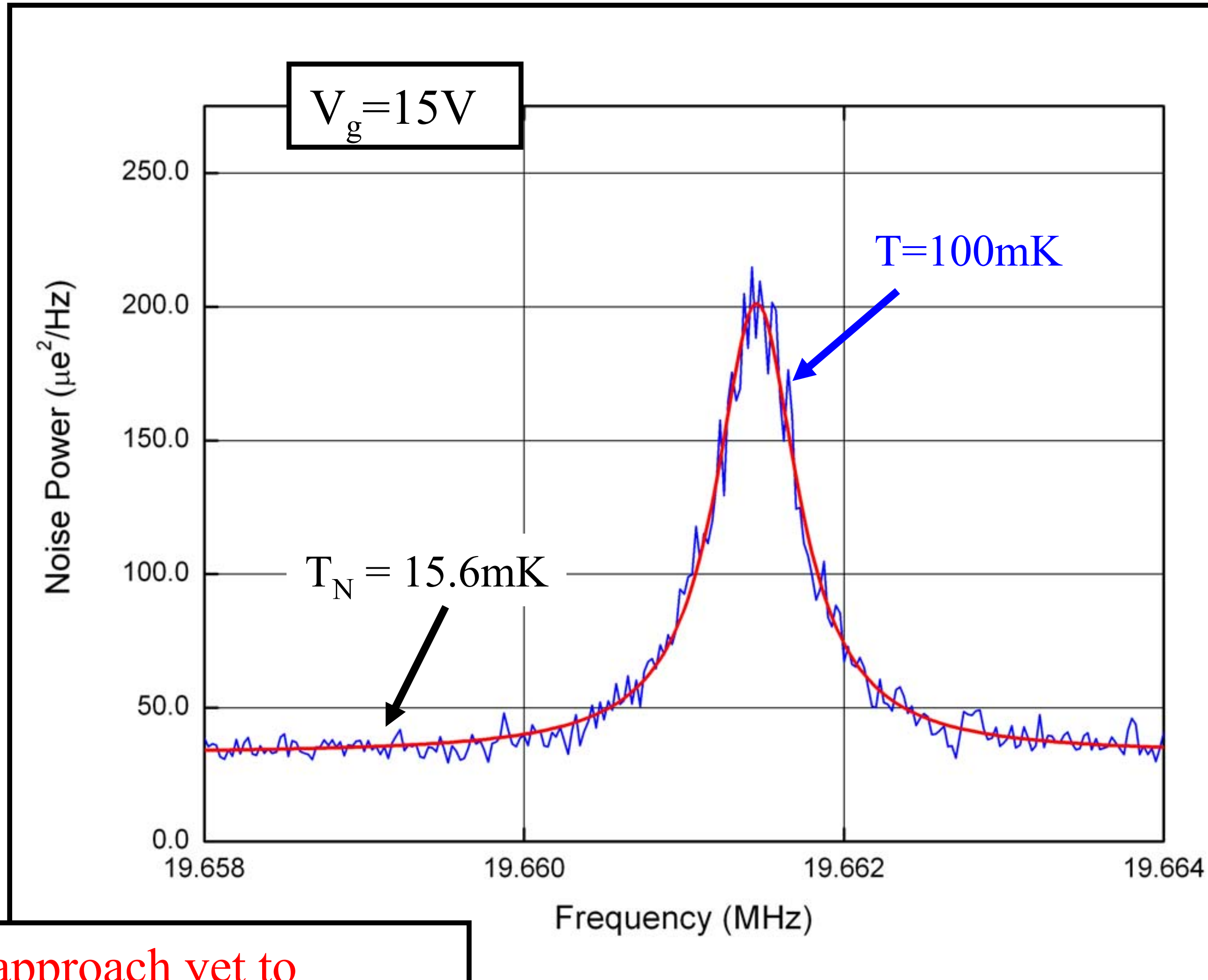
$T_N = 15.6\text{mK}$

Energy Sensitivity

$E_N \approx 17\, \hbar\omega_0$

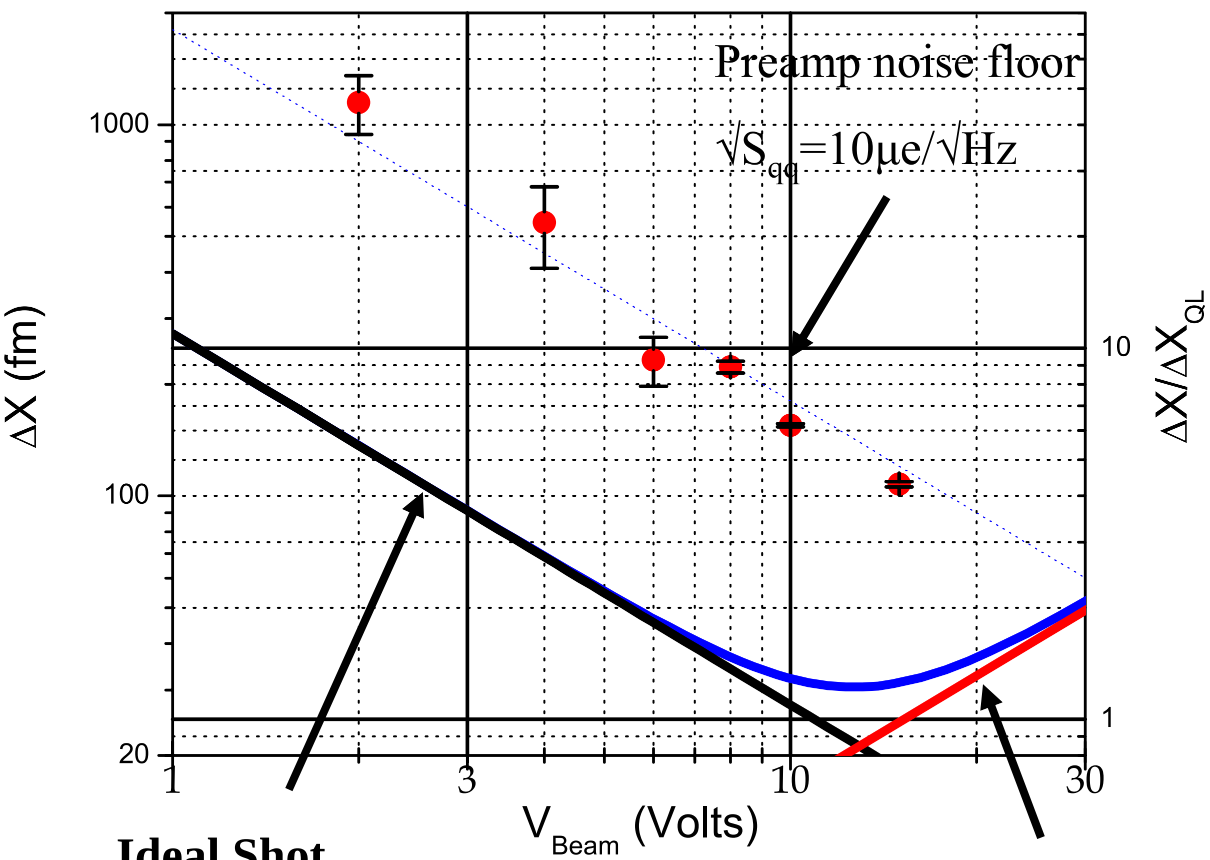
Position Sensitivity

$\Delta x = 4.3\Delta x_{QL}$



Closest approach yet to uncertainty principle limit!

How far can we push this technique ?



Ideal Shot Noise Limit

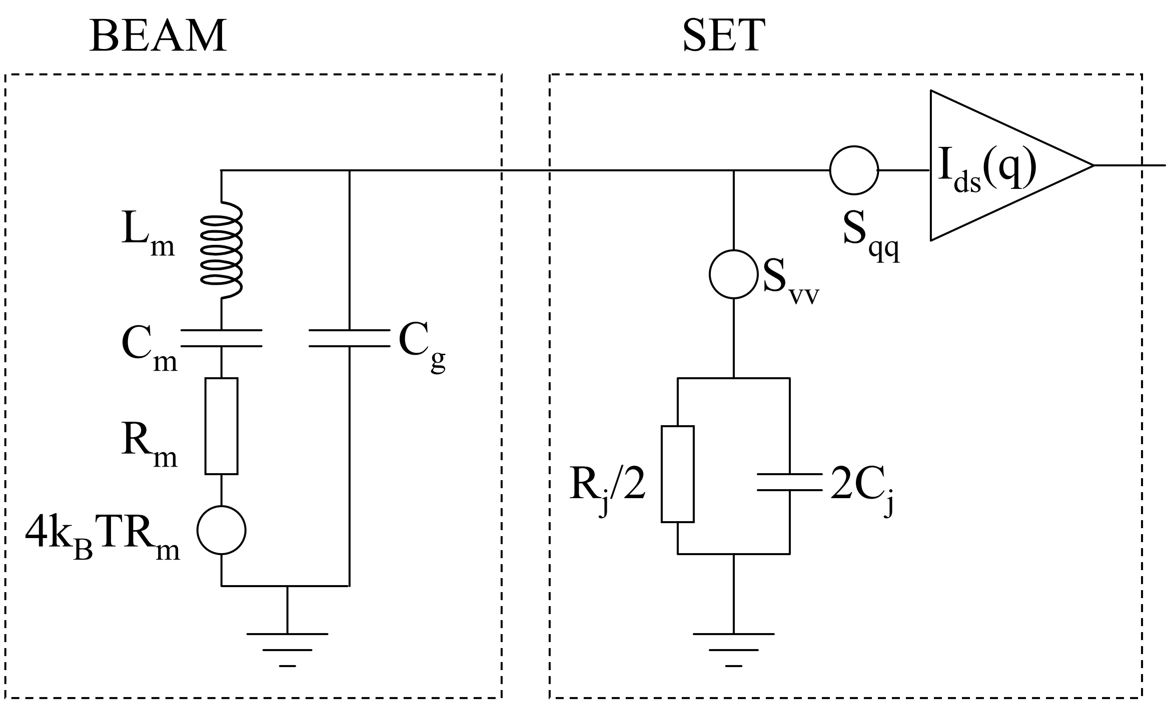
Back-Action

Induced Charge:
$\delta Q = V_g \delta C_g = (C_g V_g/d)\ \delta x$

Charge Sensitivity (forward coupling):
$S_x^{1/2} = S_{qq}^{1/2} d/(C_g V_g)$

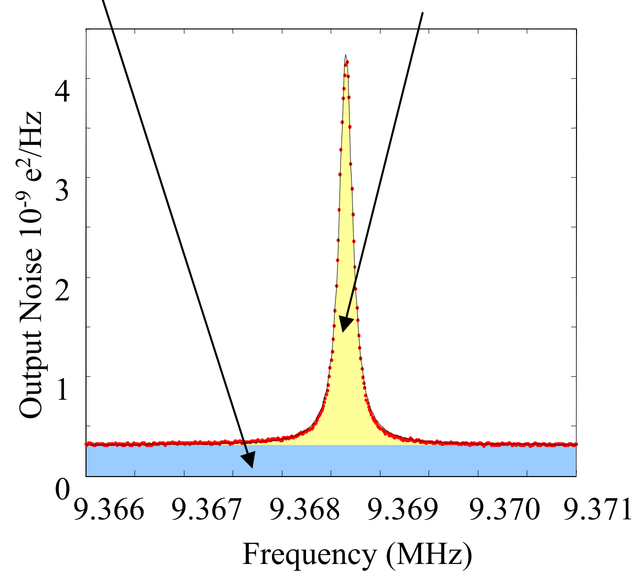
Back-Action:
$S_x^{1/2} = S_{vv}^{1/2}\ C_g V_g Q/(kd)$

Circuit Model



Total Noise Power = Gain x

$$[\underbrace{S_{qq}}\, + \underbrace{S_{thermal} + S_{vv}/|\omega Z_{in}(\omega)|^2}]$$



$C_m = C_g(C_gV_g^2/kd^2) \qquad = 0.06$ aF @ V_g=10V

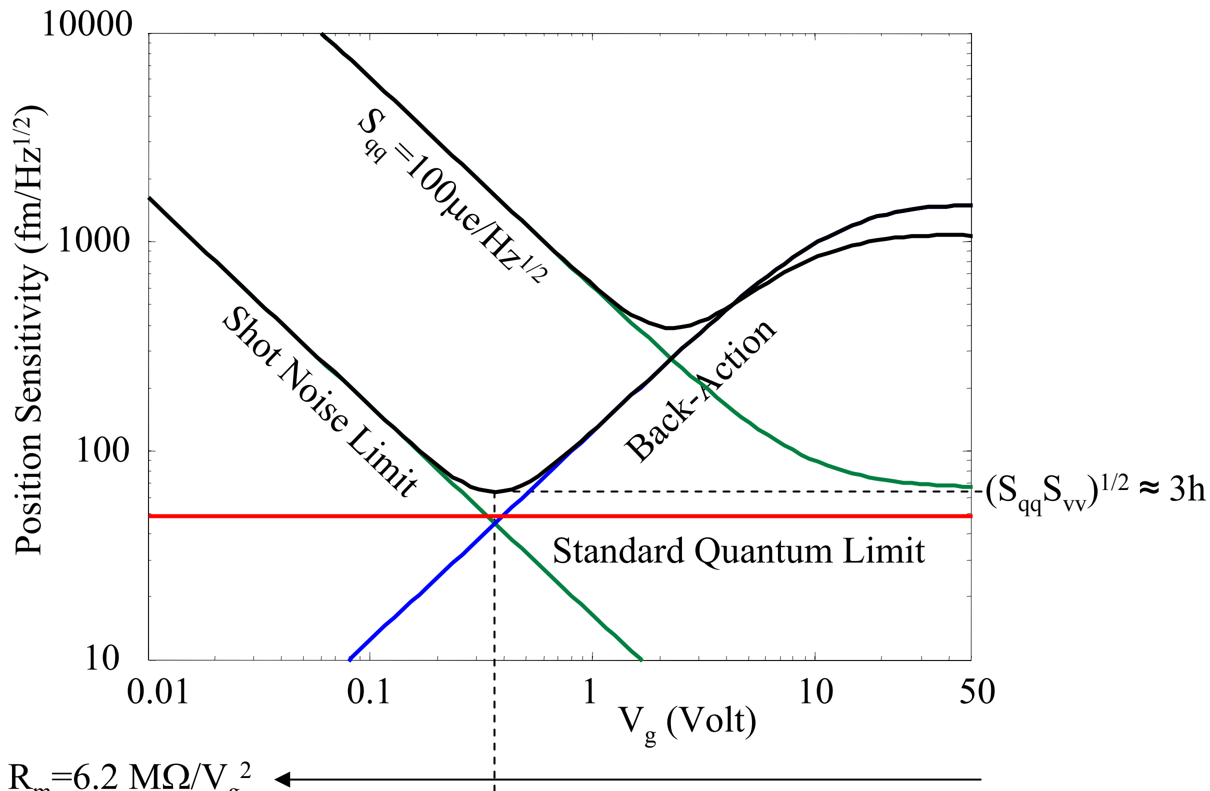
$L_m = 1/(C_g\omega^2)(kd^2/C_gV_g^2) \qquad = 4500$ H

$R_m = 1/(QC_g\omega)(kd^2/C_gV_g^2) \qquad = 2.8$ MΩ

Sensitivity Optimization

$2R_j$ = 75KΩ
$2C_j$=1.3fF
K=1.7 N/m
Q=1.5×10^5

S_{qq}=2.2μe/$Hz^{1/2}$ (shot noise)
S_{qq}=100μe/$Hz^{1/2}$ (preamp)
S_{vv}=1nV/$Hz^{1/2}$



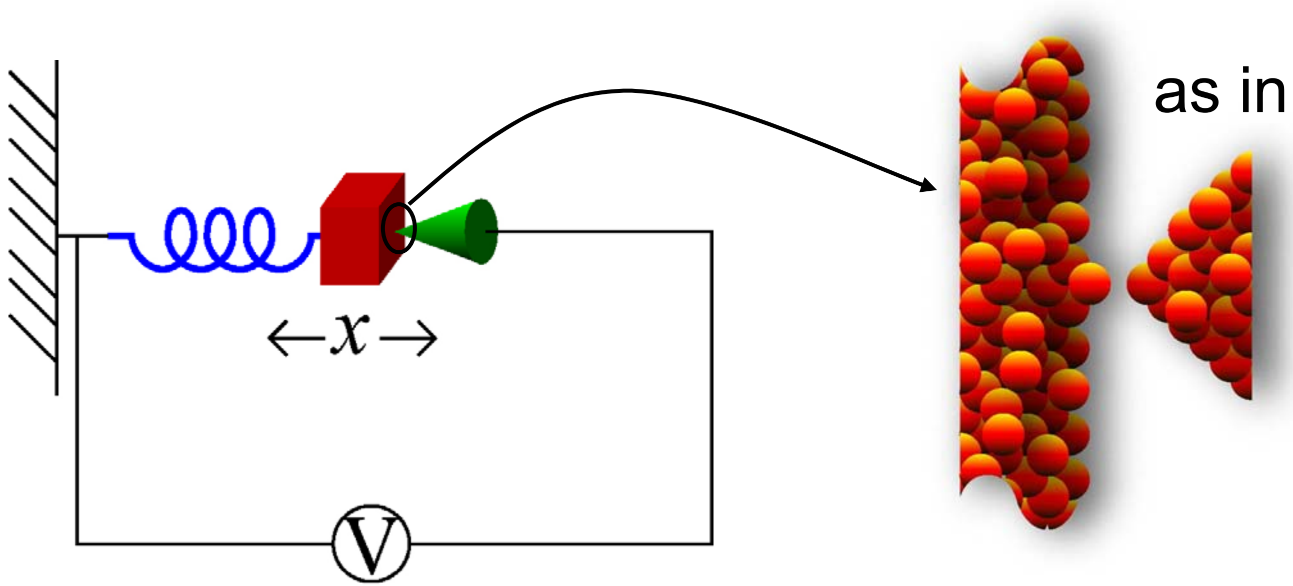
$R_{optimum} = (S_{vv}/S_{qq})^{1/2} / \omega$
$= 47\ \text{M}\Omega$

<u>Loading:</u>

$\omega = \omega_0(1- (C_g V_g^2/kd^2)\ (C_g/2C_j))^{0.5}$

$Q_{eff}^{-1} = Q^{-1} + (C_g V_g^2/kd^2)(C_g/2C_j)\omega_0(R_j C_j)$

Atomic Point Contact Displacement Detector: Lehnert group at JILA/CU



as in an STM

Infer postion from tunnel current

Sensitive: $\delta x \approx \dfrac{\lambda_e}{\sqrt{N_e / \tau}} \approx 1.2\times 10^{-15} \dfrac{\text{m}}{\sqrt{\text{Hz}}}$ with 1 nA current

Local: ideal for sub-micron objects

Atomic Point Contact Displacement Detector: Simple Noise Analysis

Imprecision
(shot noise limit)

$$\Delta x = \lambda_e \left(\frac{2e}{I} \frac{1}{\tau} \right)^{1/2}$$

$$\Delta x = \lambda_e \left(1/N_e \right)^{1/2}$$

Tunneling length scale

Counting statistics

Backaction
(momentum diffusion)

$$\Delta p = \frac{\hbar}{2\lambda_e} \left(\frac{I}{2e} \tau \right)^{1/2}$$

$$\Delta p = \frac{\hbar}{2\lambda_e} \left(N_e \right)^{1/2}$$

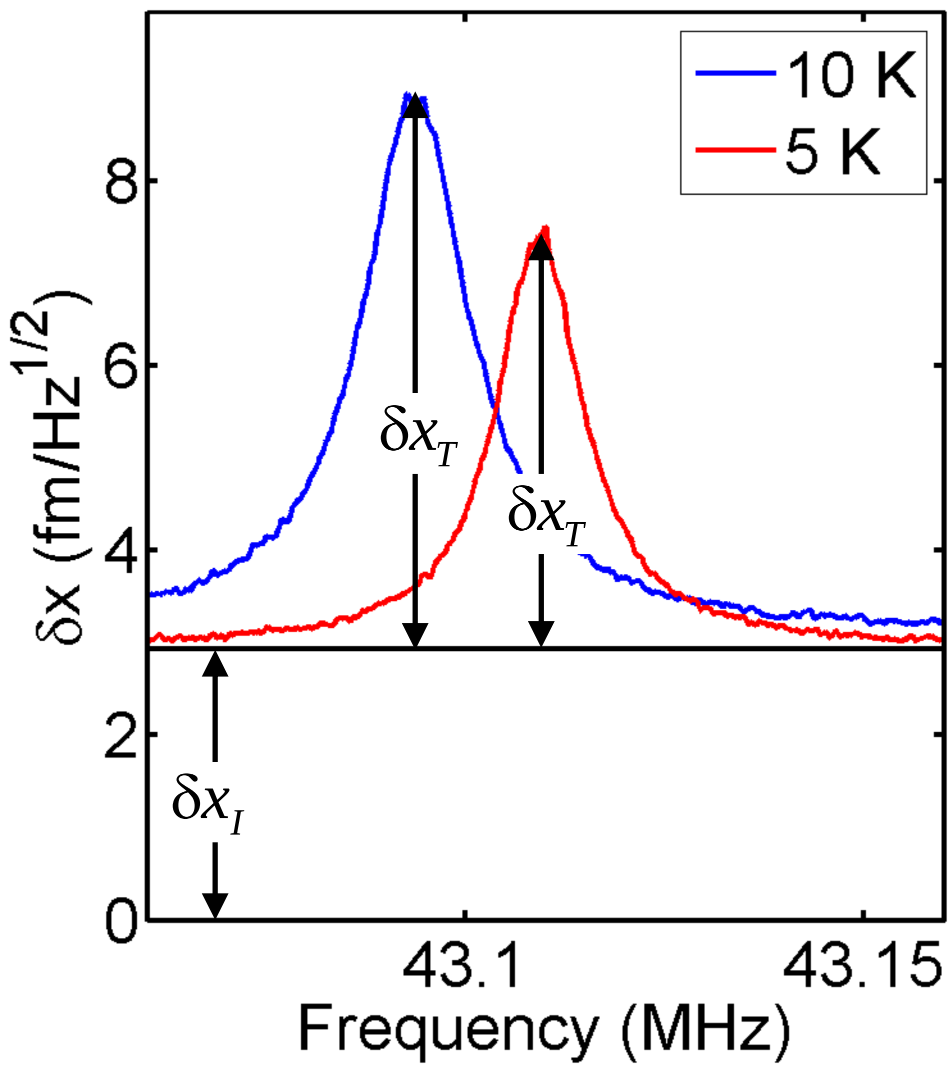
Momentum per tunneling attempt

Number diffusion

Ideal quantum displacement amplifier $\Delta x \Delta p = \frac{\hbar}{2}$

B. Yurke PRL 1990, A. A. Clerk PRB 2004

Thermal Motion at 43 MHz Resonanace



- Zero-point motion:

$$\delta x_{ZP} = \sqrt{\frac{\hbar\omega_0}{2k_s B_w}}$$

$$\delta x_{ZP} = 100 \text{ am/Hz}^{1/2}$$

$$\frac{\delta x_I}{\delta x_{ZP}} = 28$$

- Mechanical bandwidth

$$B_w \approx 9 \text{ kHz} \,;\, Q \approx 5000$$

- Sensitivity to normal coordinate