Quantum Spectrometers of Electrical Noise

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Gurus: Michel Devoret, Steve Girvin, Aash Clerk

And many discussions with D. Prober, K. Lehnert, D. Esteve, L. Kouwenhoven, B. Yurke, L. Levitov, K. Likharev, …

Thanks for slides: L. Kouwenhoven, K. Schwab, K. Lehnert,…

Overview of Lectures

Lecture 1: Equilibrium and Non-equilibrium Quantum Noise in Circuits

Reference: “Quantum Fluctuations in Electrical Circuits,” M. Devoret Les Houches notes

Lecture 2: Quantum Spectrometers of Electrical Noise

Reference: “Qubits as Spectrometers of Quantum Noise,” R. Schoelkopf et al., cond-mat/0210247

Lecture 3: Quantum Limits on Measurement

References: “Amplifying Quantum Signals with the Single-Electron Transistor,” M. Devoret and RS, Nature 2000.
“Quantum-limited Measurement and Information in Mesoscopic Detectors,” A.Clerk, S. Girvin, D. Stone PRB 2003.

And see also upcoming RMP by Clerk, Girvin, Devoret, & RS

Outline of Lecture 2

- A spin or two-level system (TLS) as spectrometer

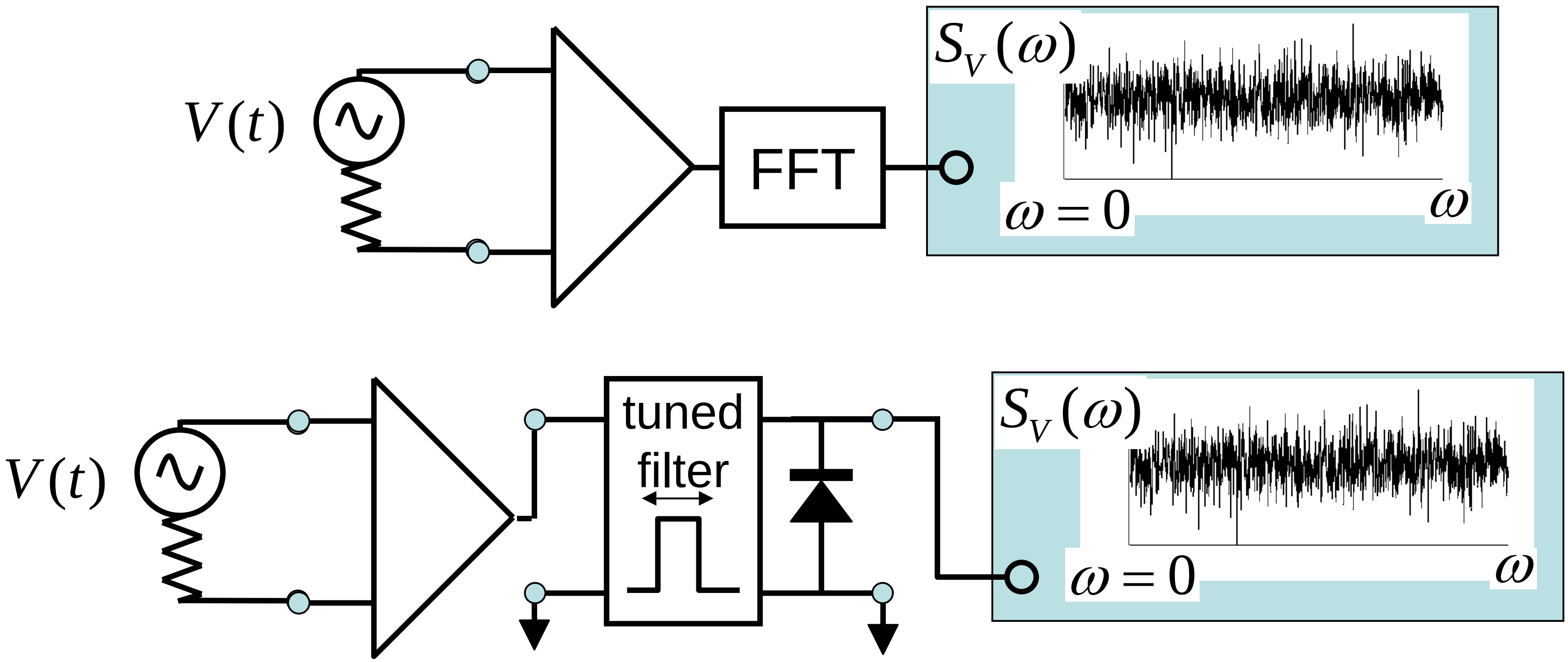
- The meaning of a two-sided spectral density

- The Cooper-pair box (CPB): an electrical TLS

- Using the CPB to analyze quantum noise of an SET
  Majer and Turek, unpub.

- Other quantum analyzers:
  - SIS junction: a continuum edge
    (DeBlock, Onac, & Kouwenhoven)
  - Nanomechanical system: a harmonic oscillator
    (Schwab et al.; Lehnert et al.)

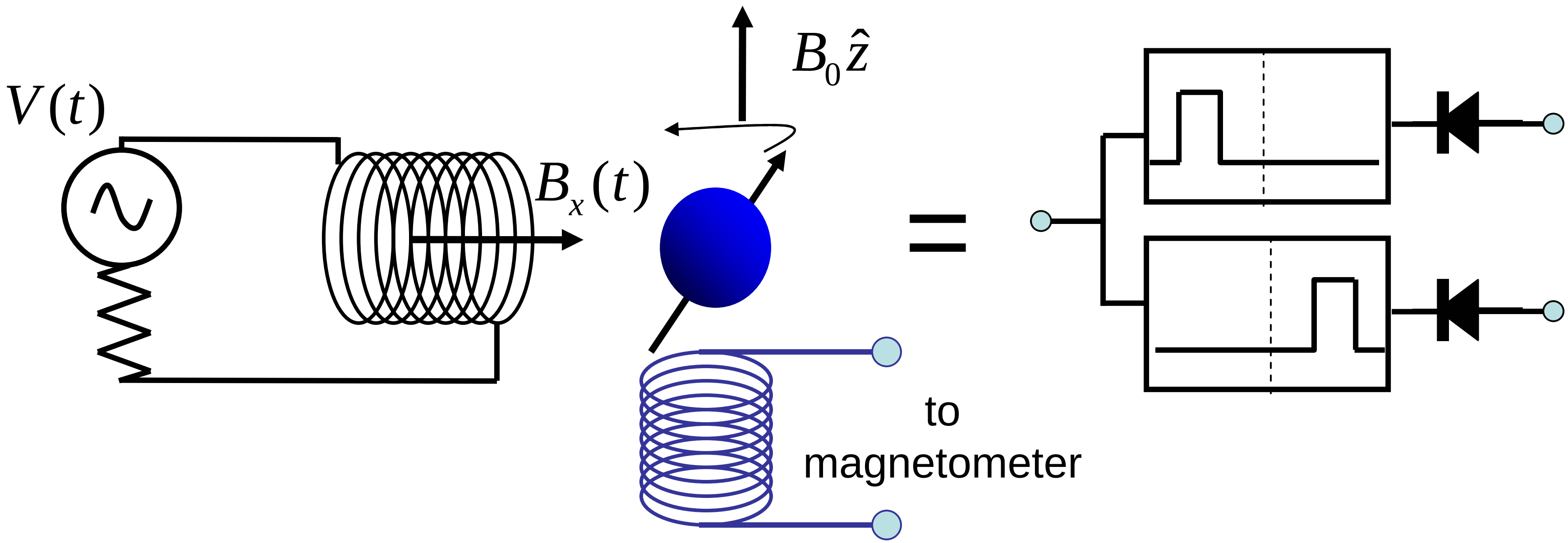
The Electrical Engineer's Spectrum Analyzers



Both measure only the ***symmetrized*** spectral density:

$$S^S_{VV}(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} \langle V(t)V(0) + V(0)V(t) \rangle$$
$$= S_{VV}(+\omega) + S_{VV}(-\omega)$$

The Quantum Mechanic's Analyzer – a Spin

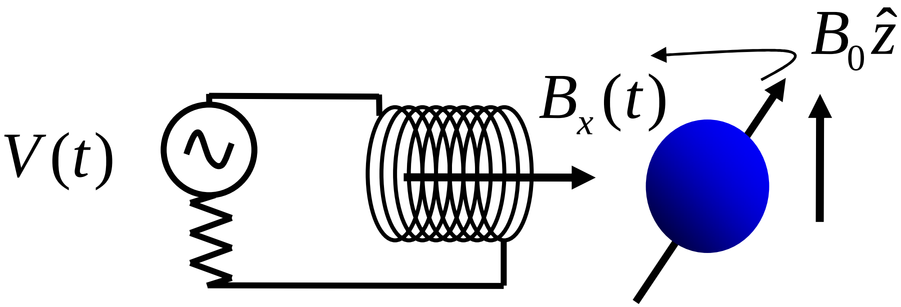


Spins only absorb at $\omega = \omega_{01} = g\mu B_0 / \hbar$ Larmor frequency

Resolution = inverse of spin coherence time

Able to measure both sides of a spectral density!

Spin as Spectrum Analyzer - II



$$H_0 = -\frac{\hbar\omega_{01}}{2}\sigma_z$$

$$H_1 = AV(t)\sigma_x$$

$$|\psi\rangle = \begin{pmatrix}\alpha_g(t)\\ \alpha_e(t)\end{pmatrix}$$

with initial condition:

$$|\psi(0)\rangle = |g\rangle$$

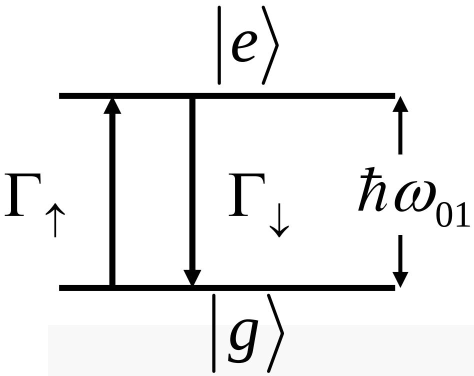
$$|\psi(t)\rangle = |\psi(0)\rangle - \frac{i}{\hbar}\int_0^t d\tau\; H_1(t)|\psi(0)\rangle$$

$$\alpha_e(t) = -\frac{iA}{\hbar}\int_0^t d\tau\,\langle e|\sigma_x(\tau)|g\rangle V(\tau) = -\frac{iA}{\hbar}\int_0^t d\tau\; e^{i\omega_{01}\tau}\;V(\tau)$$

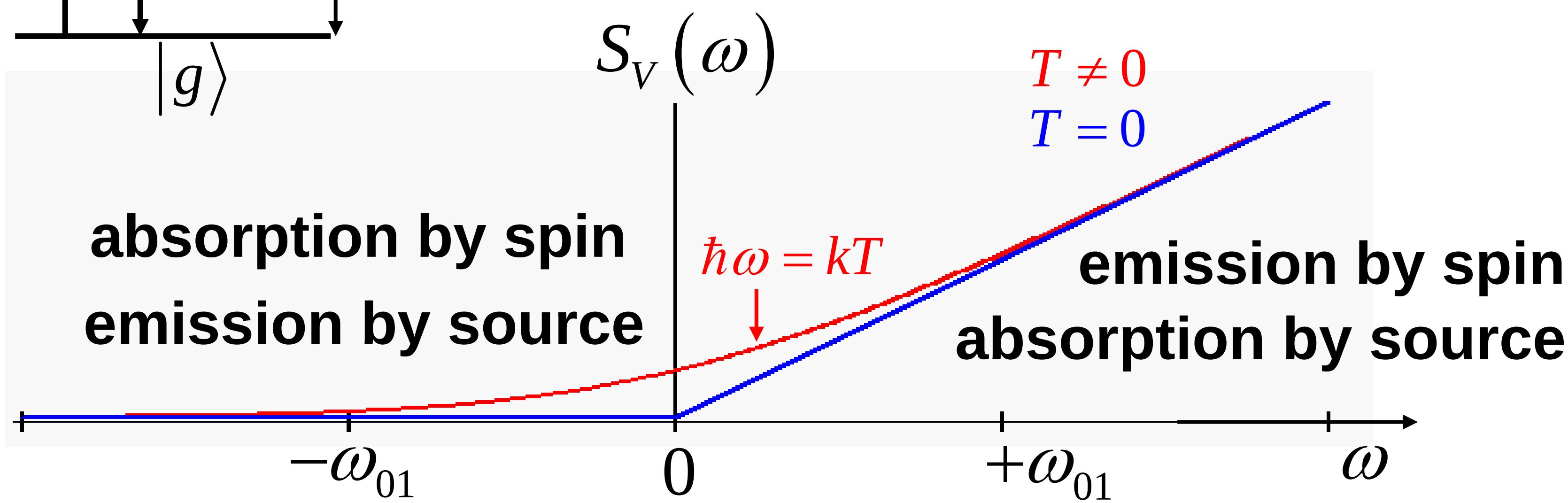
$$\langle p_e(t)\rangle = \frac{A^2}{\hbar^2}\int_0^t d\tau_1\int_0^t d\tau_2\; e^{-i\omega_{01}(\tau_1-\tau_2)}\;\langle V(\tau_1)V(\tau_2)\rangle = t\,\frac{A^2}{\hbar^2}S_V(-\omega_{01})$$

$$\boxed{\Gamma_{g\to e} = \frac{A^2}{\hbar^2}S_V(-\omega_{01})} \qquad \boxed{\Gamma_{e\to g} = \frac{A^2}{\hbar^2}S_V(+\omega_{01})}$$

Interpretation of Two-Sided Spectrum

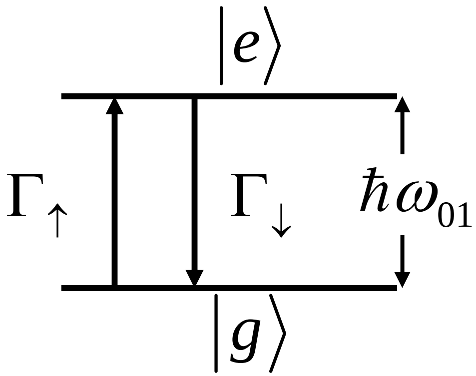


$$S_V(\omega) = \frac{2\hbar\omega R}{1 - e^{-\hbar\omega/kT}}$$



$$\Gamma_{\uparrow} \propto S_V(-\omega_{01}) \qquad \Gamma_{\downarrow} \propto S_V(+\omega_{01})$$

Polarization of Spin and Noise Spectra



$$\frac{dp_e}{dt} = p_g\Gamma_\uparrow - p_e\Gamma_\downarrow$$

$$\frac{dp_g}{dt} = p_e\Gamma_\downarrow - p_g\Gamma_\uparrow$$

Steady-state: $p_g\Gamma_\uparrow = p_e\Gamma_\downarrow$

Define polarization of spin:

$$\boxed{P = p_g - p_e}$$

If noise is truly classical,

$p_g \equiv p_e$ and no polarization!

Thermal equilibrium: $p_e / p_g = e^{-\hbar\omega_{01}/kT}$

Requires particular asymmetry! $S_V(+\omega_{01}) = e^{\hbar\omega_{01}/kT} S_V(-\omega_{01})$

Polarization of Spin - II

Define steady-state polarization:

$$P_{SS}=\frac{\Gamma_{\downarrow}-\Gamma_{\uparrow}}{\Gamma_{\downarrow}+\Gamma_{\uparrow}}=\frac{S(+\omega_{01})-S(-\omega_{01})}{S(+\omega_{01})+S(-\omega_{01})}$$

due to relative asymmetry of noise

Define deviation from steady state: $\Delta P(t)=P(t)-P_{SS}$

$$\frac{d\left(\Delta P(t)\right)}{dt}=-\Delta P(t)\left(\Gamma_{\downarrow}+\Gamma_{\uparrow}\right)=-\Delta P(t)\Gamma_{1}$$

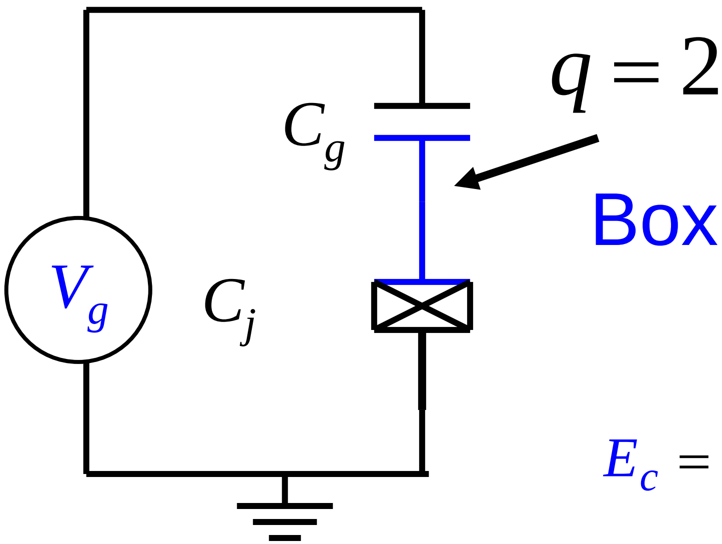
$$\Gamma_{1}=\frac{1}{T_{1}}=\frac{A^{2}}{\hbar^{2}}\left[S(+\omega_{01})+S(-\omega_{01})\right]$$

So relaxation rate (Γ_1) due to total noise (and coupling)

Ways to Characterize a Quantum Reservoir

Fermi's Golden Rule	$\Gamma_{\uparrow} = \left(\frac{A}{\hbar}\right)^2 S_V(-\omega)$	$\Gamma_{\downarrow} = \left(\frac{A}{\hbar}\right)^2 S_V(+\omega)$
Fluctuation-Dissipation Relation	$n = \frac{2\Gamma_{\uparrow}}{(\Gamma_{\downarrow} - \Gamma_{\uparrow})}$	$\mathrm{Re}\left[Z(\omega)\right] = \frac{\hbar(\Gamma_{\downarrow} - \Gamma_{\uparrow})}{A^2\omega}$
NMR	$P = \frac{\Gamma_{\downarrow} - \Gamma_{\uparrow}}{\Gamma_{\downarrow} + \Gamma_{\uparrow}}$	$T_1 = \left(\Gamma_{\uparrow} + \Gamma_{\downarrow}\right)^{-1}$
Harmonic Oscillator	$\langle E \rangle = \frac{\hbar\omega\left(\Gamma_{\uparrow} + \Gamma_{\downarrow}\right)}{2(\Gamma_{\downarrow} - \Gamma_{\uparrow})}$	$\gamma = \frac{\omega}{Q} = (\Gamma_{\downarrow} - \Gamma_{\uparrow})$
Quantum Optics	$B_{Einstein} = \Gamma_{\uparrow}$	$A_{Einstein} = \Gamma_{\downarrow} - \Gamma_{\uparrow}$

Cooper Pair Box as Two-Level System

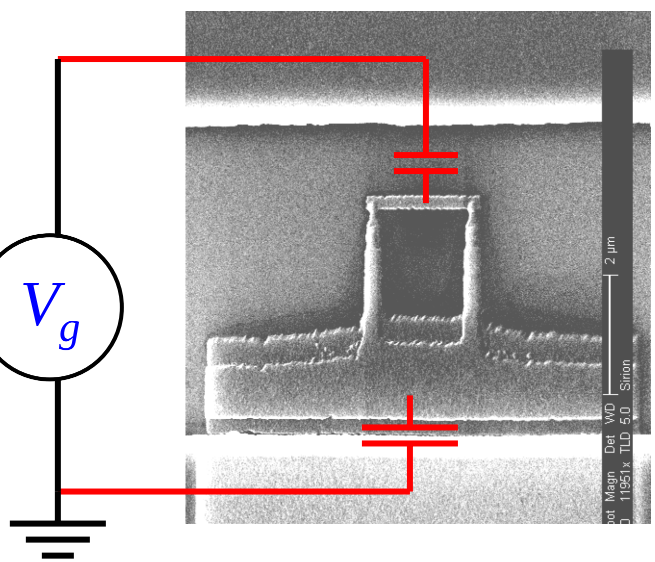


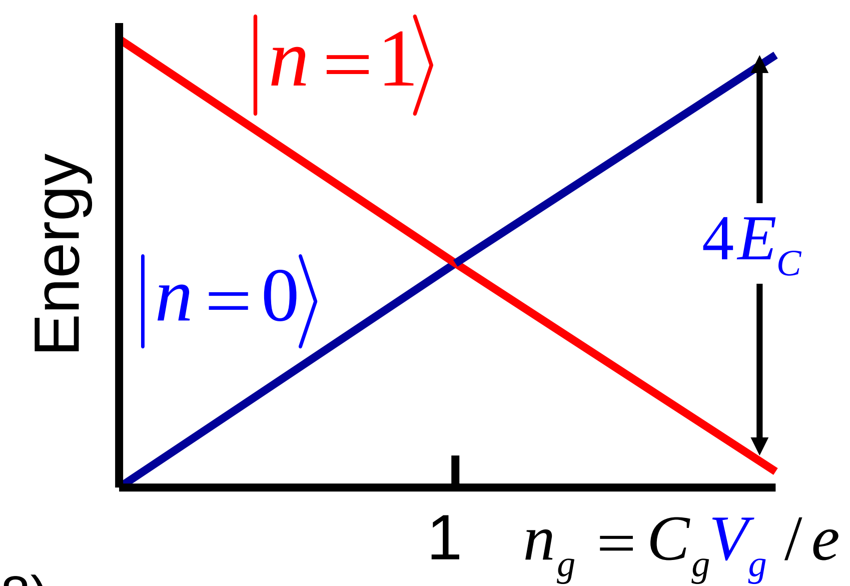
$$q = 2e\hat{n}$$

$$\hat{H}_{box} = \frac{E_{el}}{2}\hat{\sigma}_x$$

$$E_{el} = 4E_c(\hat{n} - C_g V_g / e)$$

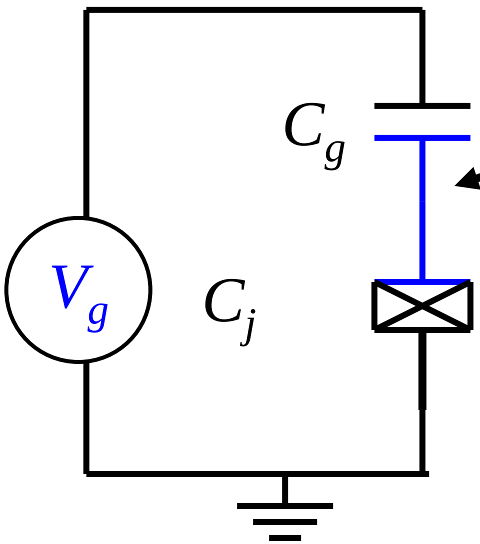
$$E_c = \frac{e^2}{2(C_g + C_j)} \sim 5\ GHz$$





(Buttiker ’87; Bouchiat et al., 98)

Cooper Pair Box as Two-Level System

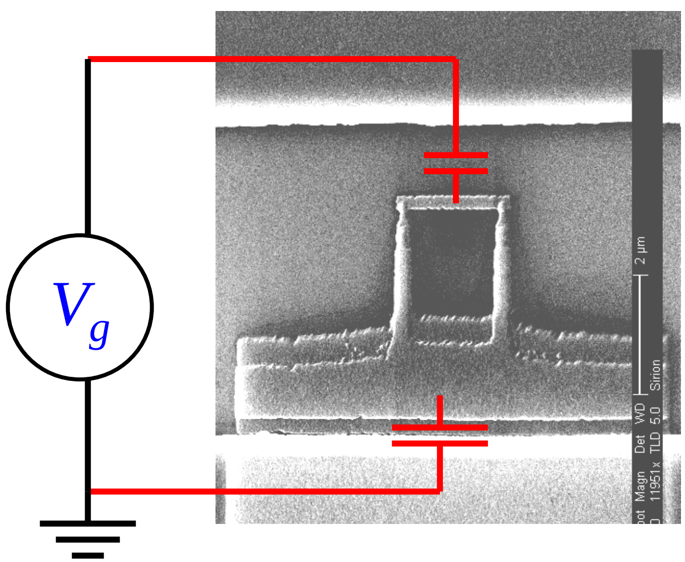


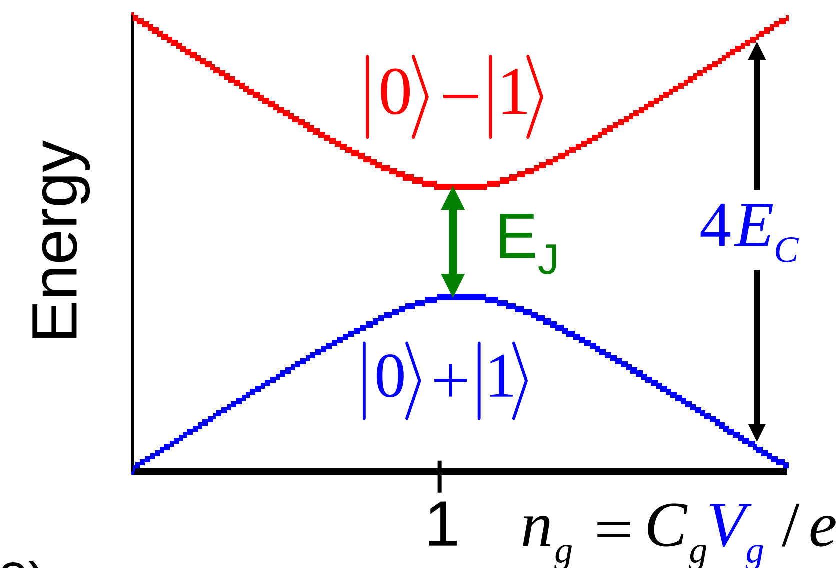
$$q = 2e\hat{n}$$

$$\hat{H}_{box} = \frac{E_{el}}{2}\hat{\sigma}_x - \frac{E_J}{2}\hat{\sigma}_z$$

$$E_{el} = 4E_c(\hat{n} - C_g V_g / e)$$

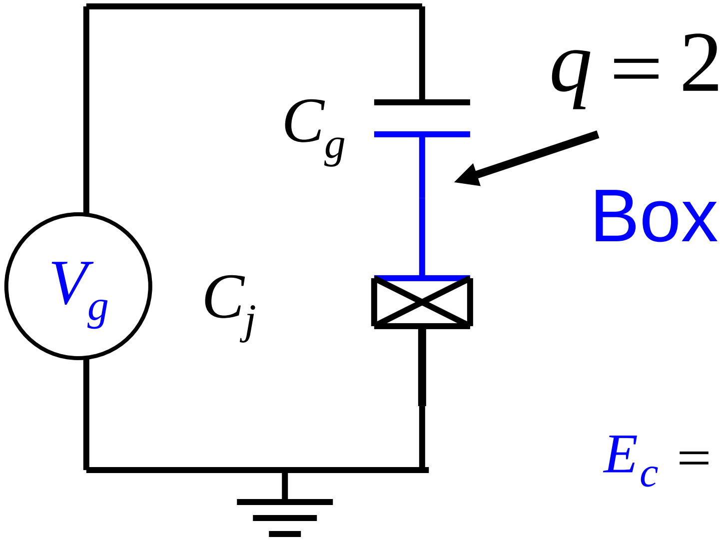
$$E_c = \frac{e^2}{2(C_g + C_j)} \sim 5\ GHz \qquad E_J = \frac{\hbar}{4}\frac{\pi\Delta}{e^2 R_J} \approx 5\ \text{GHz}$$





(Buttiker ’87; Bouchiat et al., 98)

Cooper Pair Box as Two-Level System



$$\hat{H}_{box} = \frac{E_{el}}{2}\hat{\sigma}_x - \frac{E_J}{2}\hat{\sigma}_z$$

$$E_{el} = 4E_c(\hat{n} - C_g V_g / e)$$

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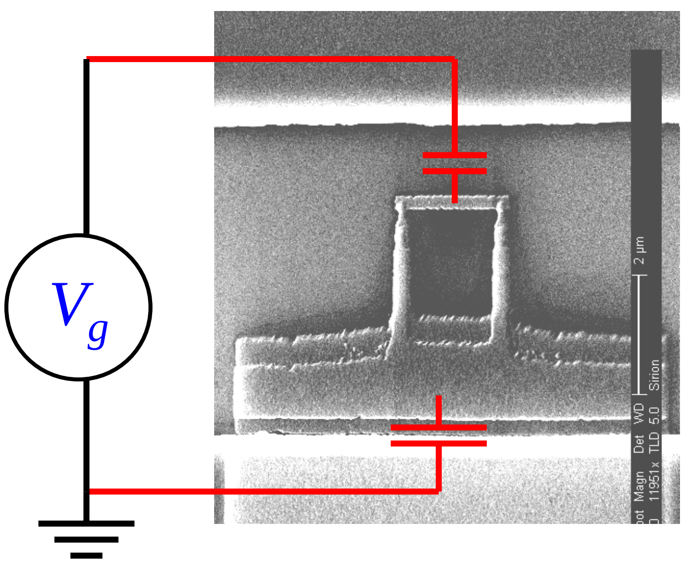
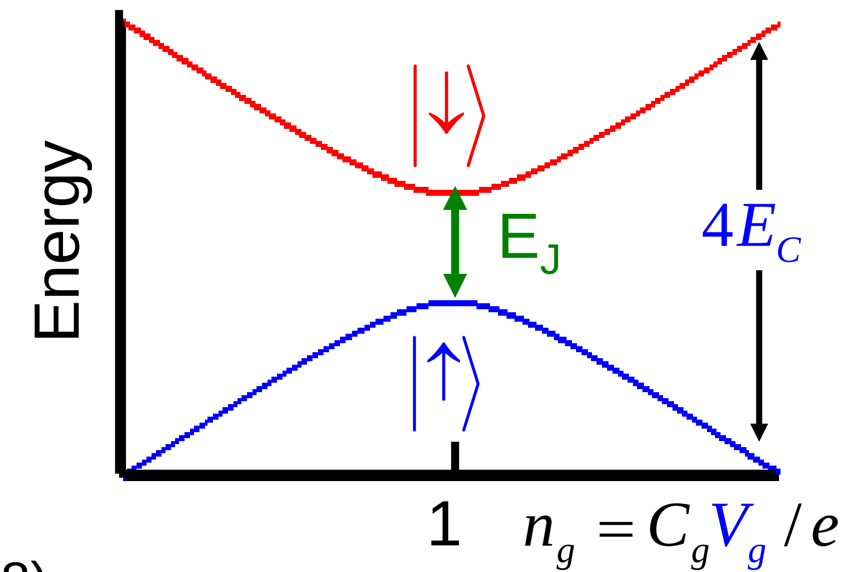
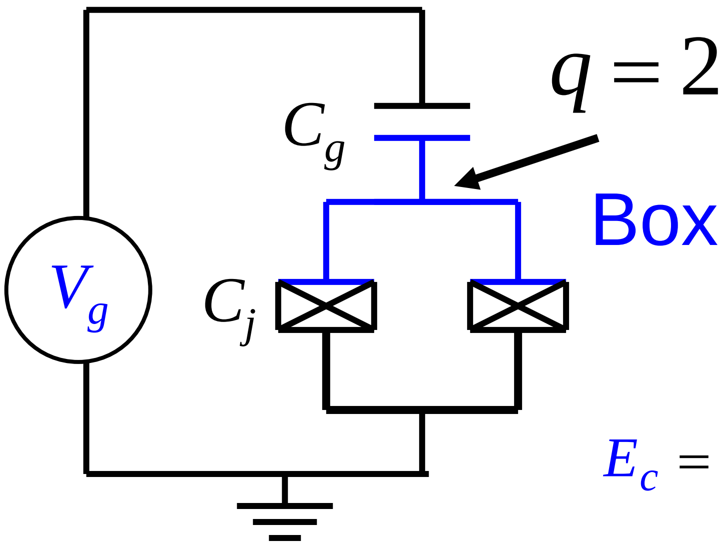




(Buttiker '87; Bouchiat et al., 98)

Cooper Pair Box as Two-Level System



$$\hat{H}_{box} = \frac{E_{el}}{2}\hat{\sigma}_x - \frac{E_J}{2}\hat{\sigma}_z$$

$$E_{el} = 4E_c(\hat{n} - C_g V_g / e)$$

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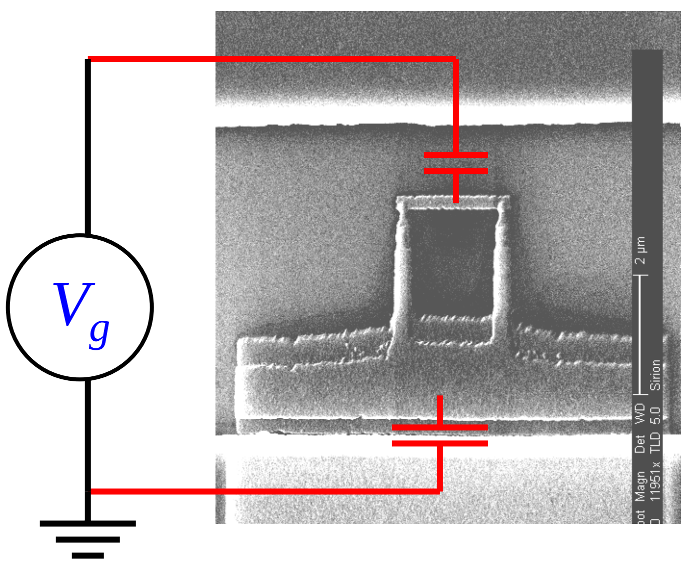
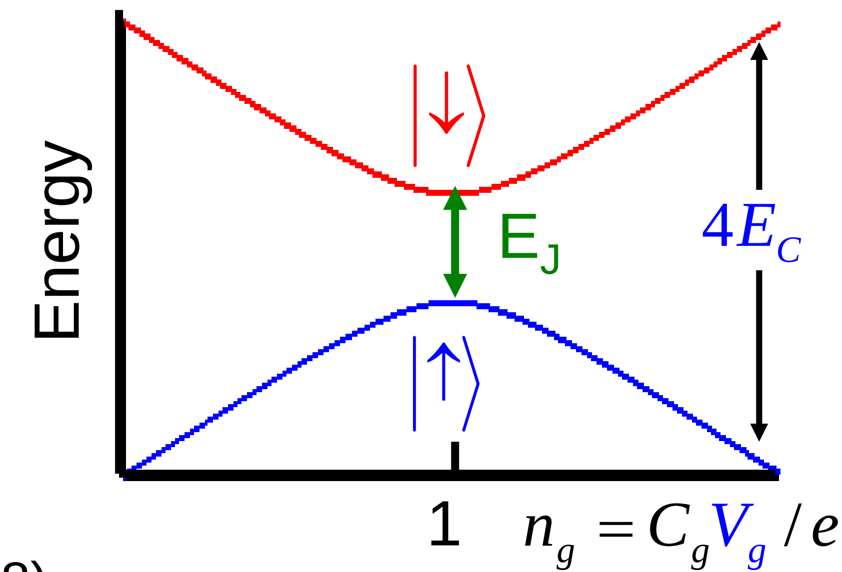
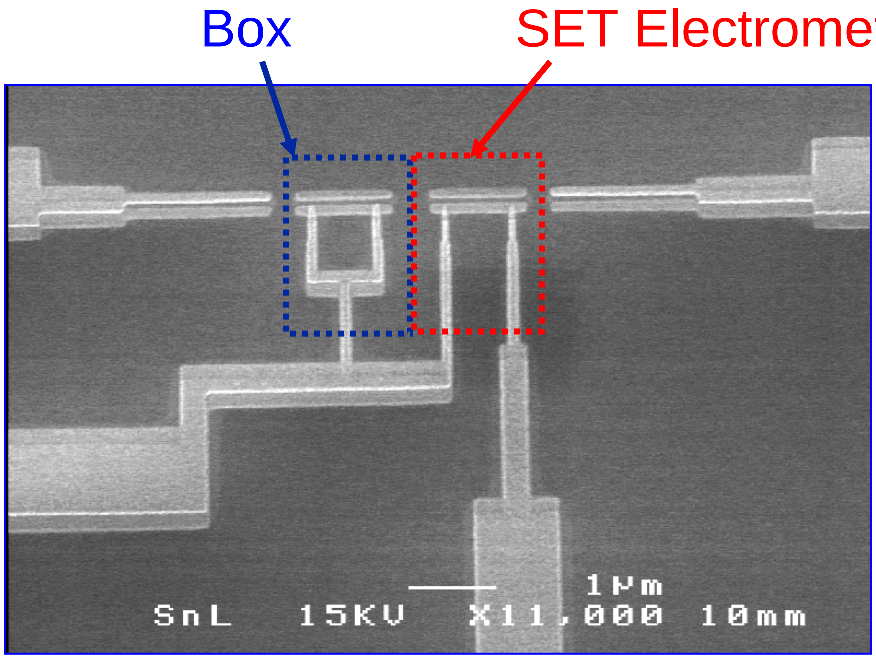
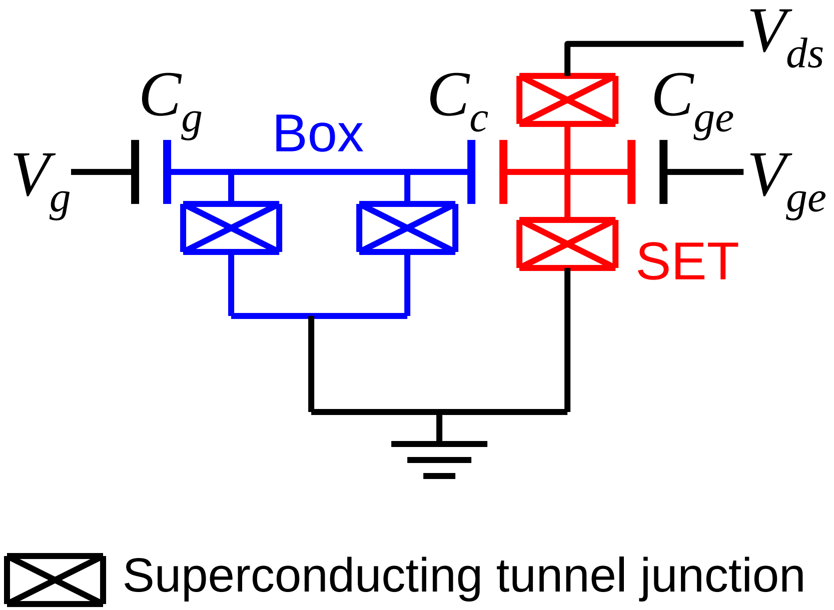




(Buttiker '87; Bouchiat et al., 98)

Cooper-pair Box Coupled to an SET





<u>Cooper-pair Box</u> | <u>SET Transistor</u>

Qubit → Quantum state readout

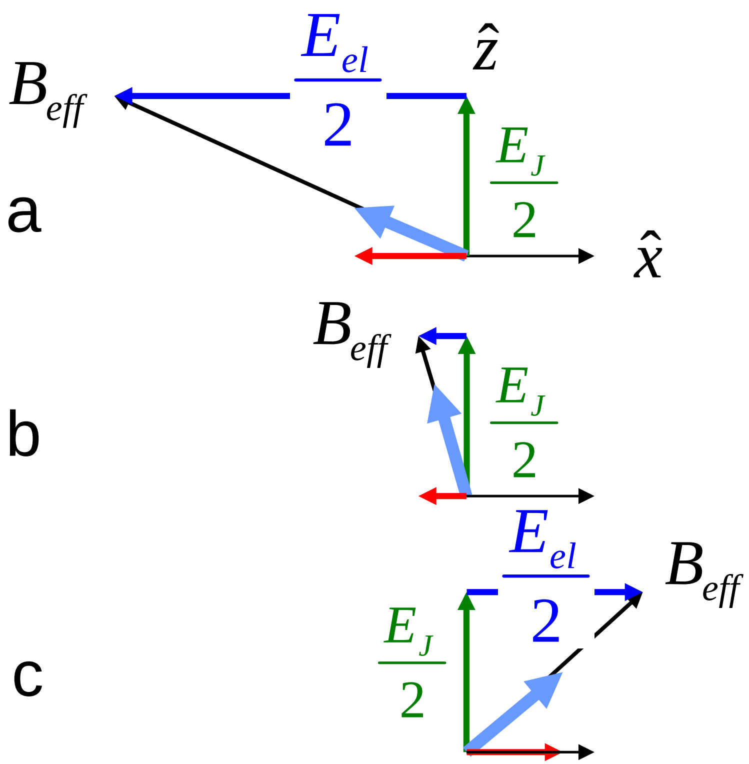
or

Quantum spectrum analyzer ← Nonequilibrium noise source

What Does SET Measure?

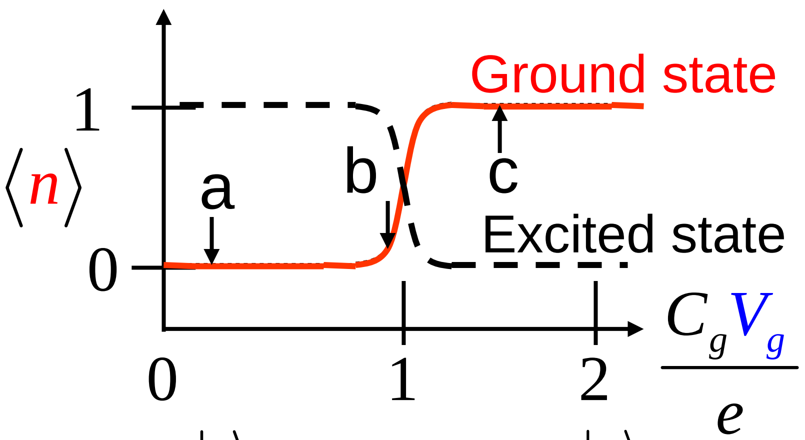
$$\hat{H} = \frac{E_{el}}{2}\hat{\sigma}_x - \frac{E_J}{2}\hat{\sigma}_z$$

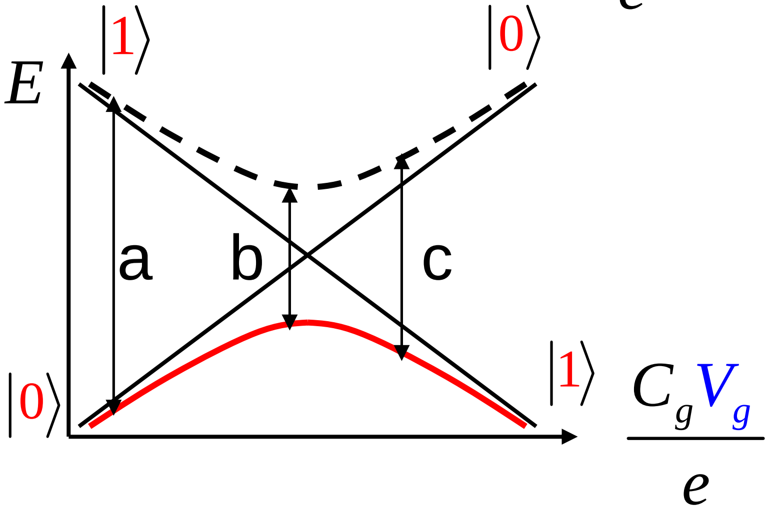
$$E_{el} = 4E_c(\hat{n} - C_g V_g / e)$$



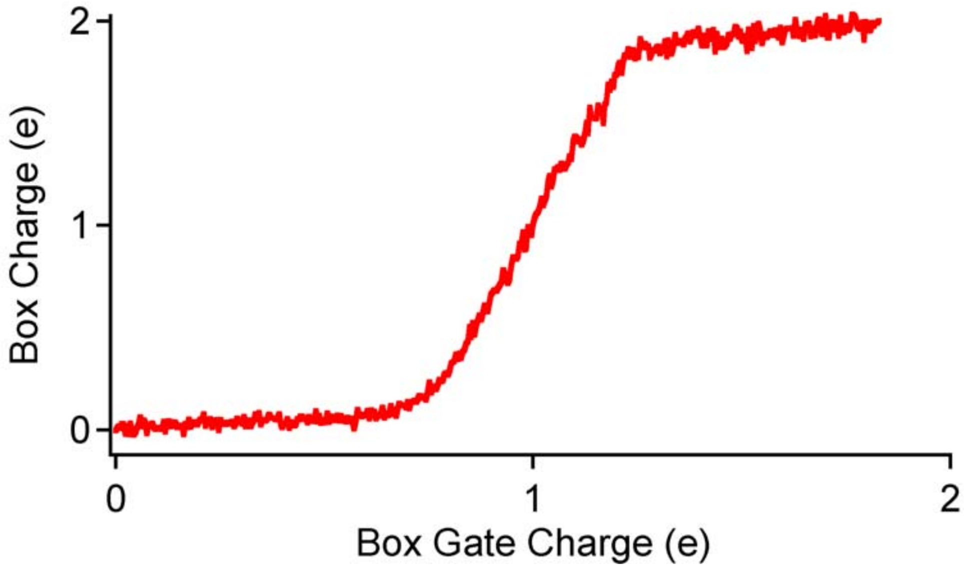
Measure box charge

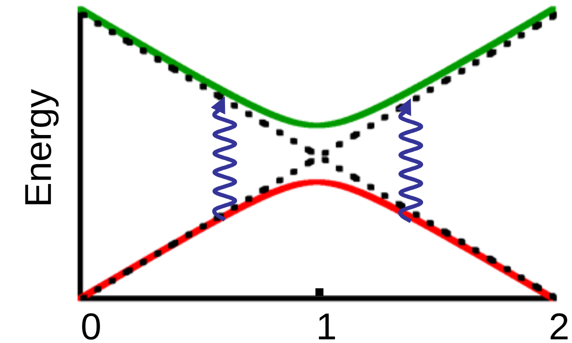
$$\langle n \rangle = \left(\langle \sigma_x \rangle + 1\right)/2$$

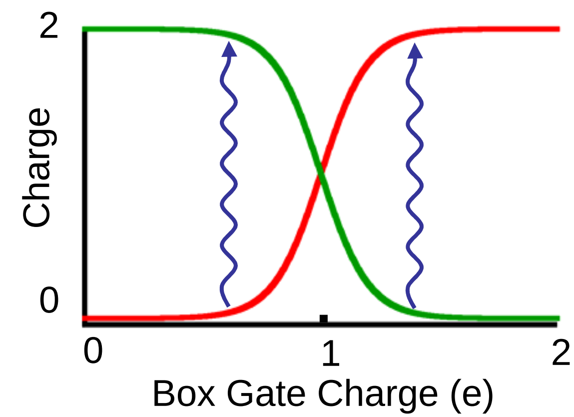




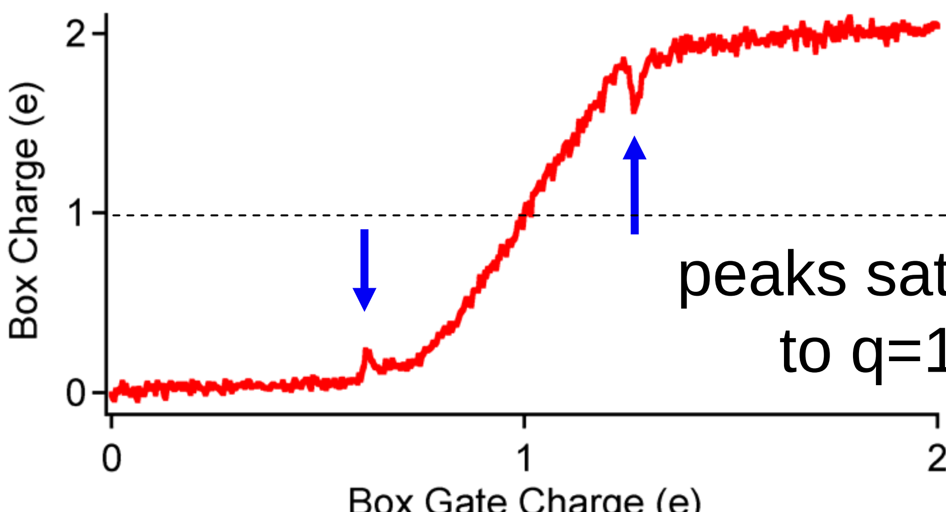
Spectroscopy of Box



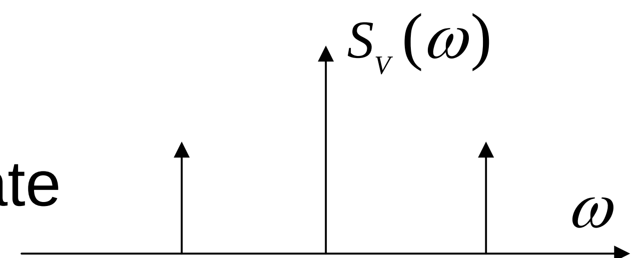




with microwave

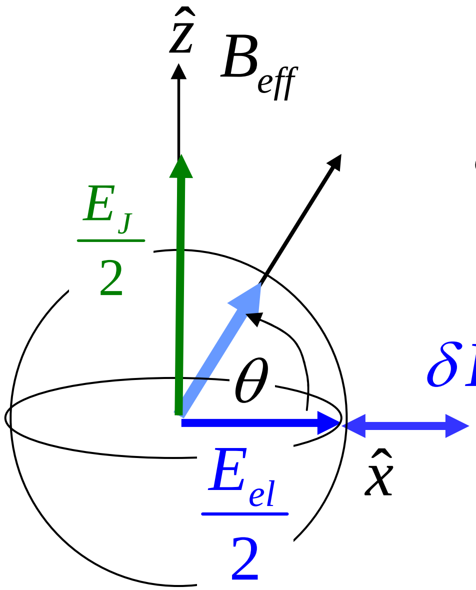


peaks saturate
to q=1e

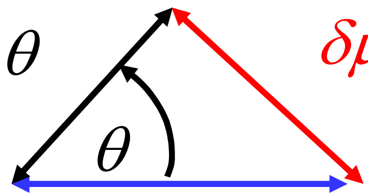


Spectrum of oscillator

Effects of Voltage Noise on Pseudo-Spin



$$\delta\mu B_{\|} = \delta E_{el}\cos\theta$$



$$\delta\mu B_{\perp} = \delta E_{el}\sin\theta$$

$$\delta E_{el} = 2e\delta V_{box}$$

slow fluctuations of $B_{\|}$ $\longrightarrow$ dephasing

$$\hbar\omega_{01} = \mu\vec{B}_{eff}$$

$$\frac{1}{T_{\varphi}} = \Gamma_{\varphi} = \left(\frac{e}{\hbar}\right)^2 S_{V_{box}}(\omega \to 0)\cos^2\theta$$

resonant fluctuations of $B_{\perp}$ $\longrightarrow$ transitions

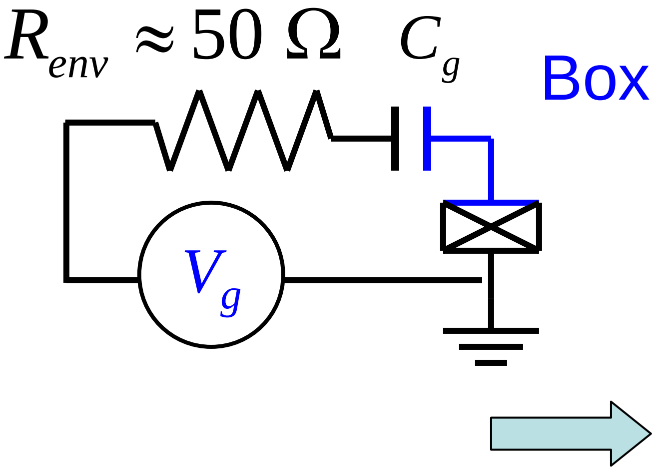
$$\sin\theta = \frac{E_J}{\hbar\omega_{01}}$$

$$\frac{1}{T_{mix}} = \Gamma_{mix} = \left(\frac{e}{\hbar}\right)^2 S_{V_{box}}(\omega_{01})\sin^2\theta$$

Noi

Spontaneous Emission of Cooper-pair Box

$$\boxed{T=0}$$



$$H_1 = AV\sigma_x$$

$$H_1 = 2E_C \frac{C_g}{e} \sin\theta\, \delta V_g\, \sigma_x$$

$$= e\left(C_g / C_\Sigma\right) \sin\theta\, \delta V_g\, \sigma_x$$

$$S_{V_{env}}(-\omega_{01}) = 0$$

$$S_{V_{env}}(+\omega_{01}) = 2\hbar\omega_{01} \times (50\ \Omega)$$

Excited-state lifetime, T_1

$$\frac{1}{T_1} = \Gamma_\downarrow = \kappa^2 \left(\frac{e}{\hbar}\right)^2 S_{V_{env}}\left(+\omega_{01}\right) \sin^2\theta$$

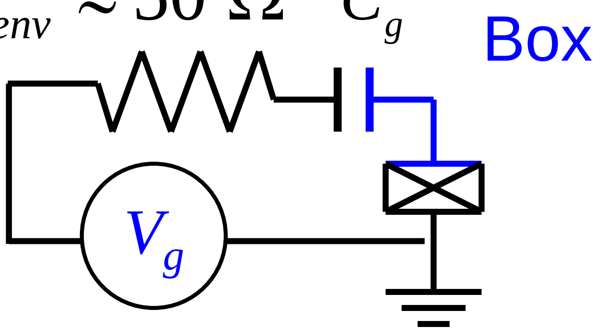
$\kappa = C_g / C_\Sigma \sim 12\%$ estimate: $\boxed{T_1 \approx 0.1 - 1\ \mu s}$

Polarization = 100%

Cooper-pair Box at Finite Temperature

$$\boxed{T > 0}$$

$R_{env} \approx 50\ \Omega \quad C_g$



$$S_{V_{env}}(-\omega_{01}) = 2\hbar\omega_{01}R\langle n\rangle$$

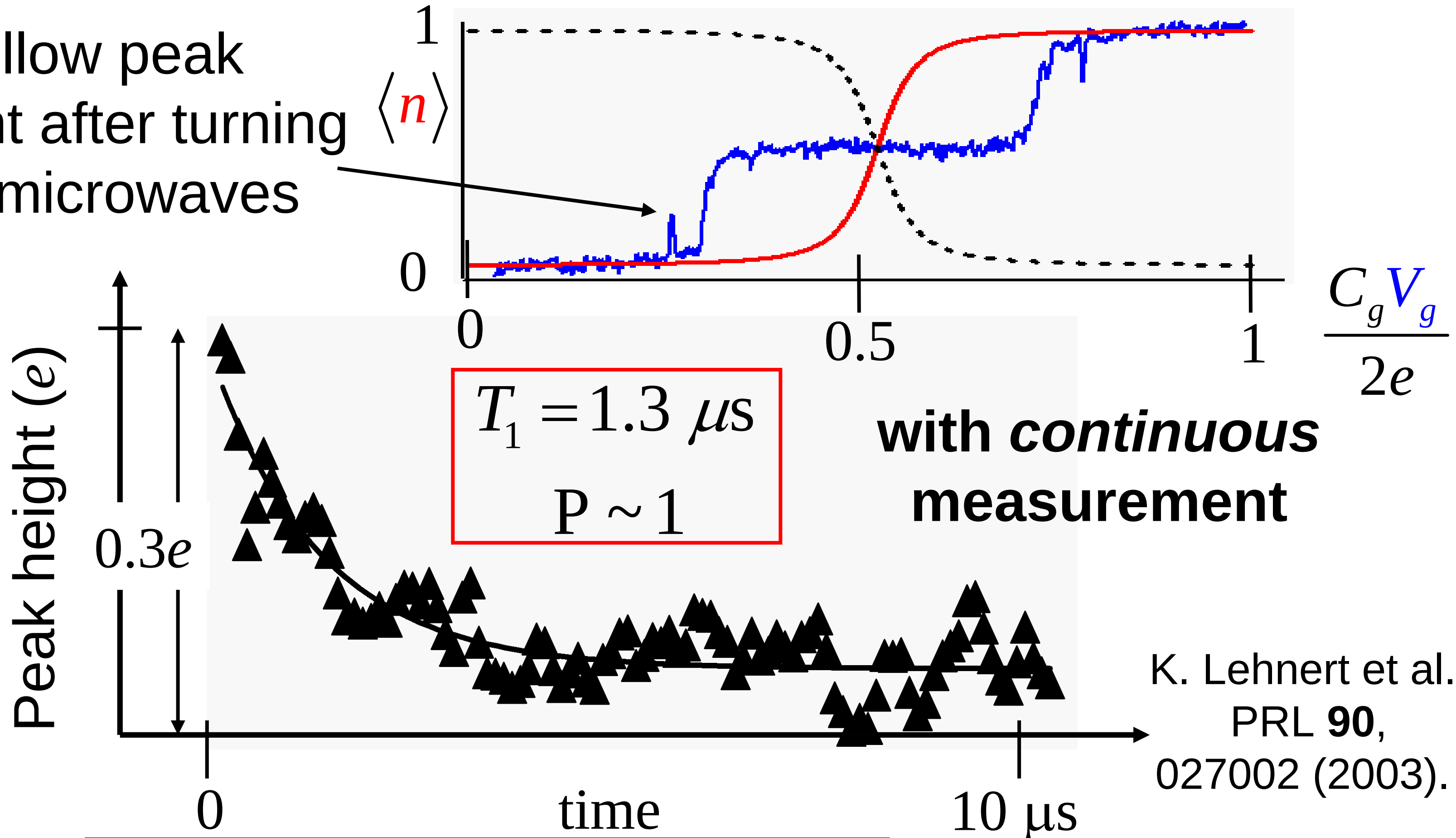
$$S_{V_{env}}(+\omega_{01}) = 2\hbar\omega_{01}R\left(\langle n\rangle + 1\right)$$

$$P = \frac{S(+\omega_{01}) - S(-\omega_{01})}{S(+\omega_{01}) + S(-\omega_{01})} = \frac{\langle n\rangle + 1 - \langle n\rangle}{2\langle n\rangle + 1} = \tanh\left[\frac{\hbar\omega_{01}}{2kT}\right]$$

$$\frac{1}{T_1} = \Gamma_\downarrow + \Gamma_\uparrow = \kappa^2\left(\frac{e}{\hbar}\right)^2 2\hbar\omega_{01}R\sin^2\theta\left(2\langle n\rangle + 1\right)$$

Excited-state Lifetime Measurement of Box

follow peak height after turning off microwaves



with *continuous* measurement

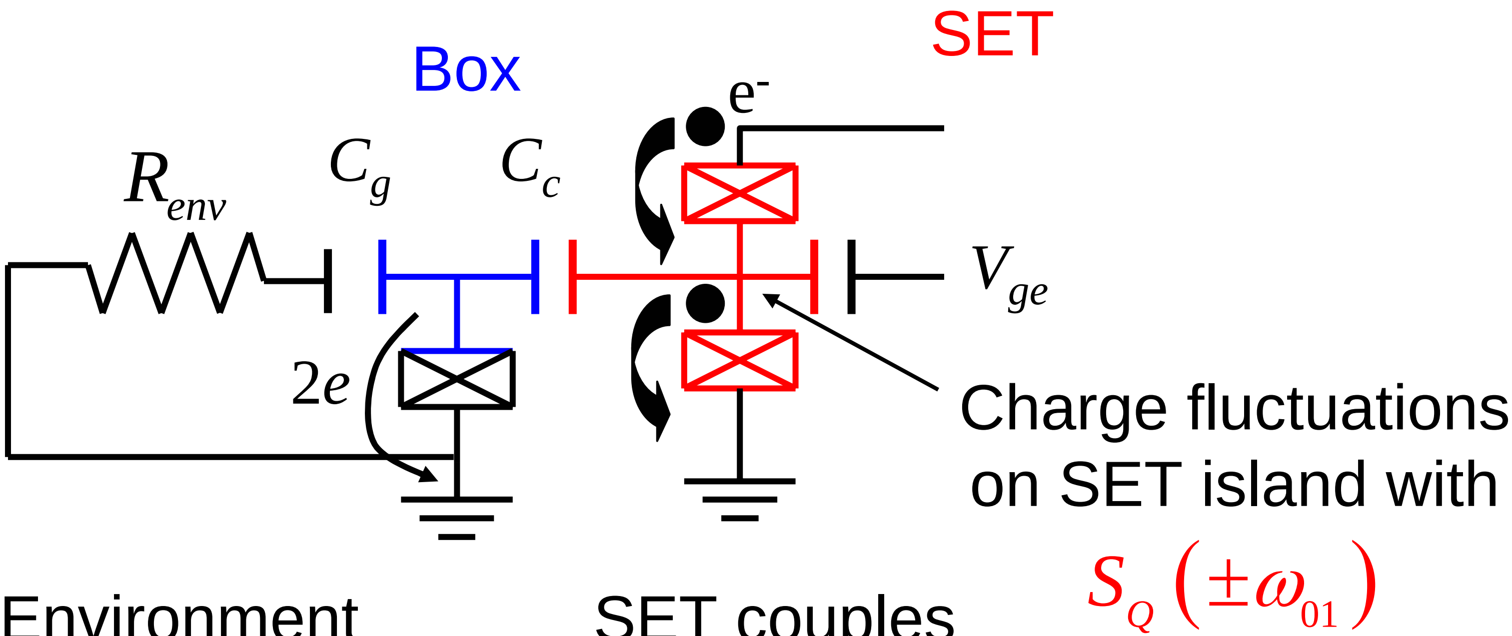
K. Lehnert et al., PRL **90**, 027002 (2003).

$$\sqrt{S_{n_{box}}(+\omega_{01})} \sim 5\times10^{-9}\ \text{pairs}/\sqrt{Hz}$$

$$\sqrt{S_{n_{box}}(-\omega_{01})} \sim 0$$

(@ 76 GHz)

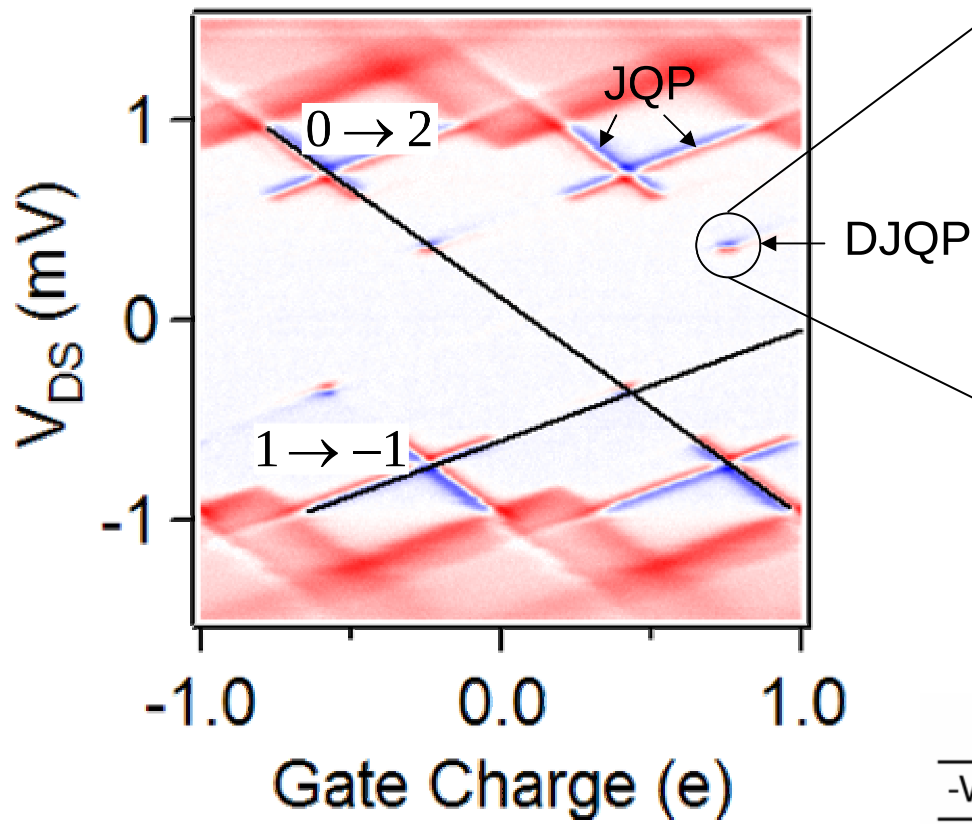
Coupling of SET Backaction to Box

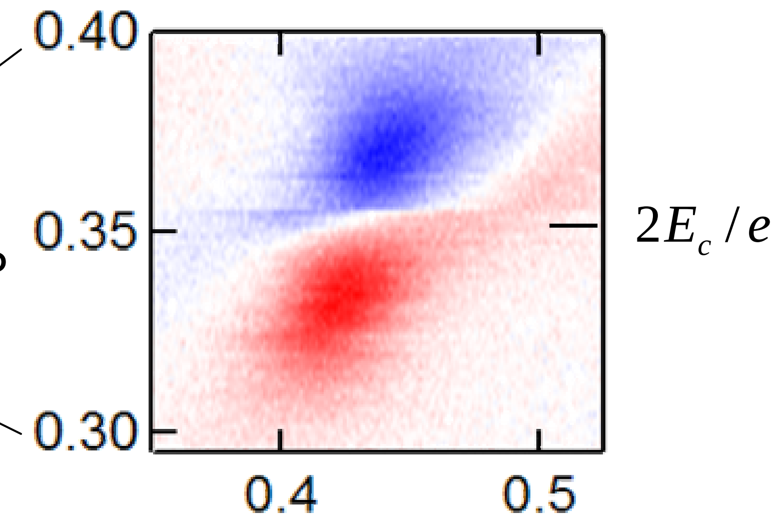


Environment relaxes box

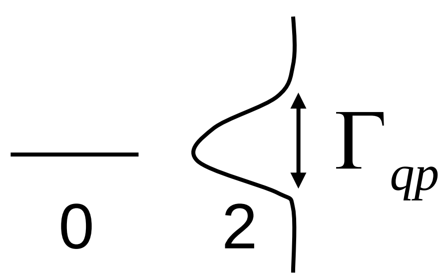
SET couples backaction to box

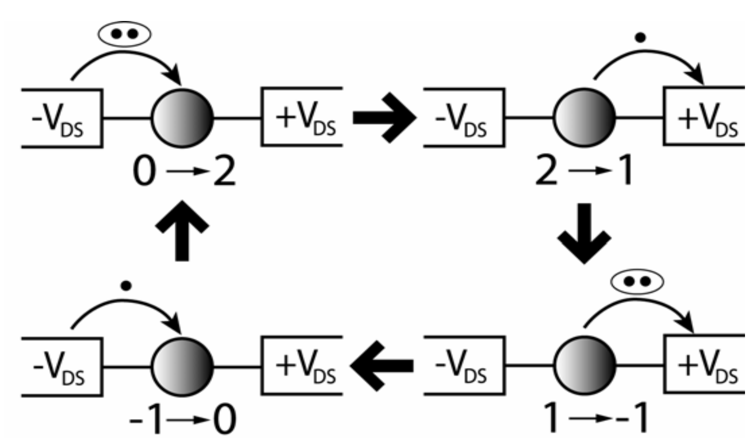
Double JQP Process in the SSET





Reflected power from RF-SET
~ Conductance of SSET

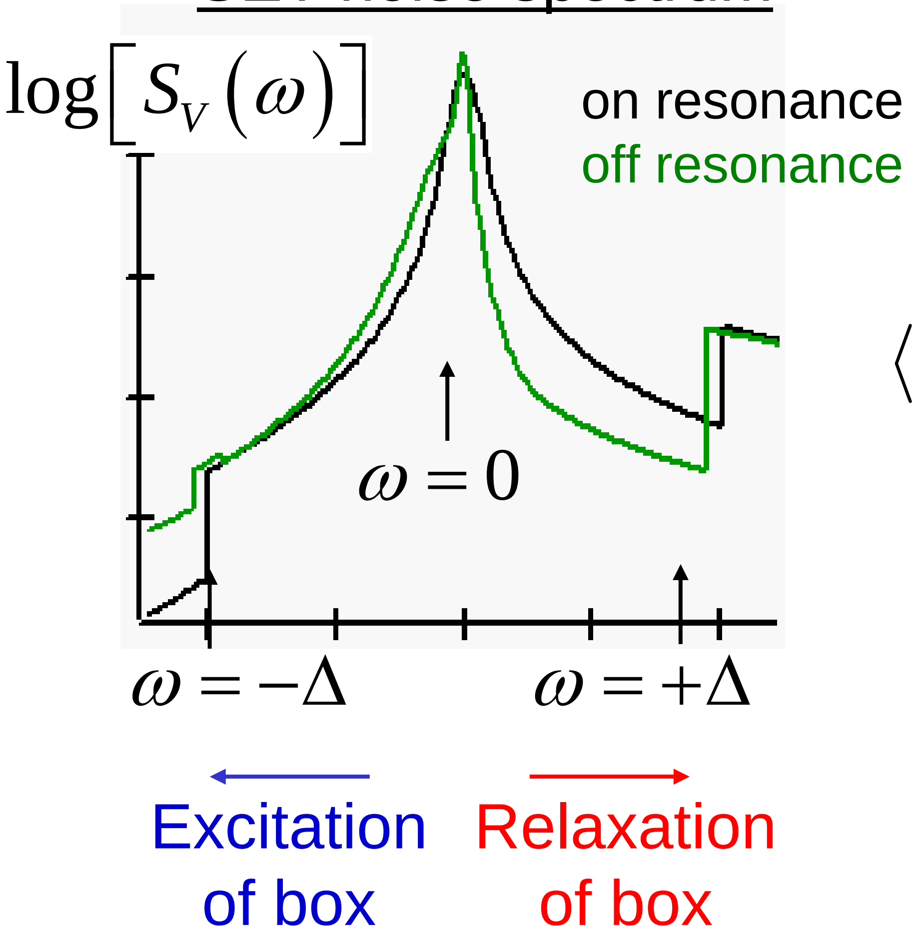




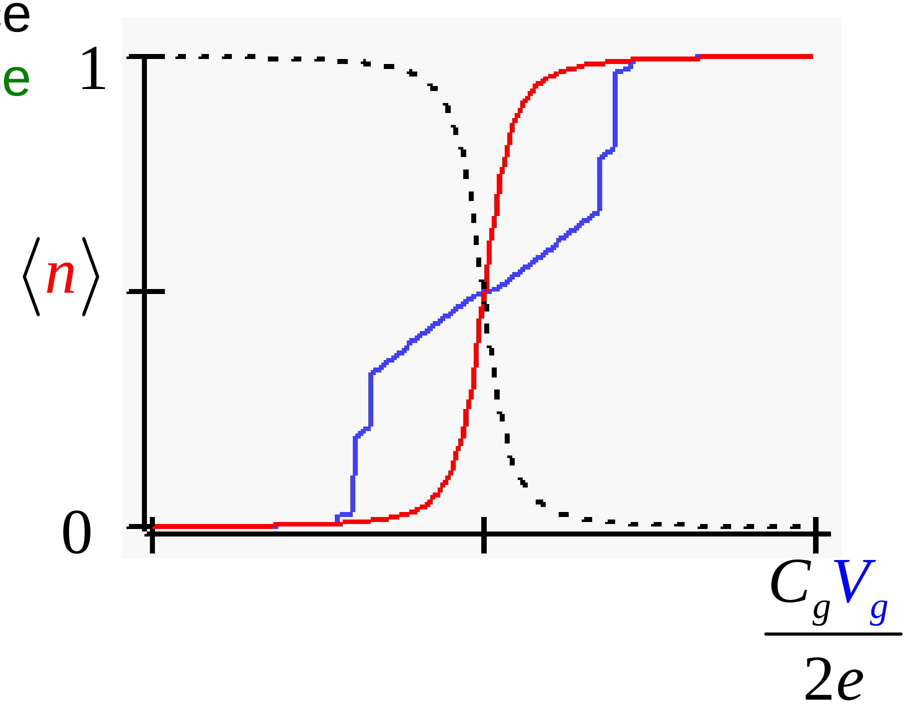
Quantum Shot Noise of DJQP* Process

*Double Josephson-quasiparticle cycle: (A. Clerk et al. PRL **89** 176804 (2002))

SET noise spectrum

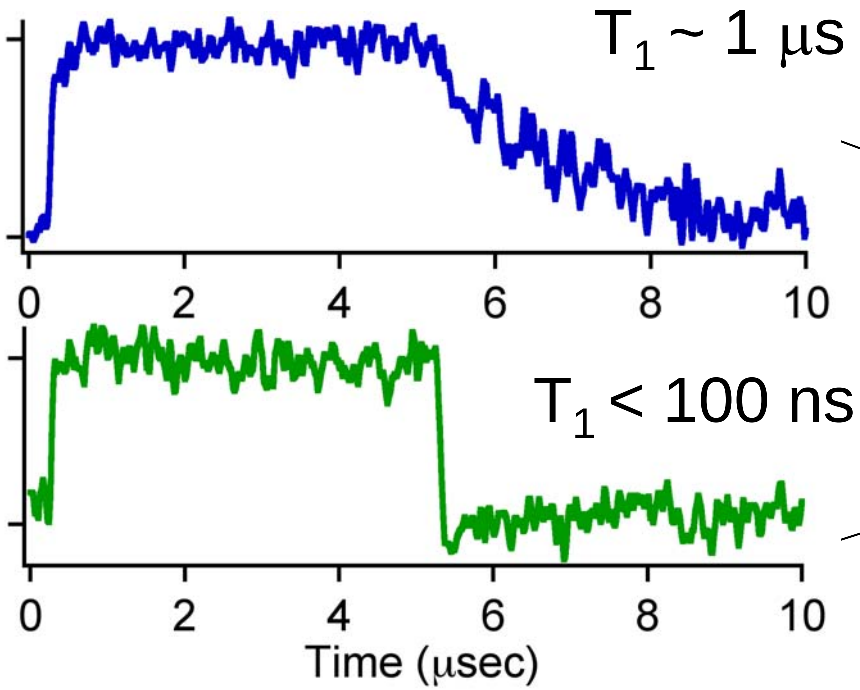


Predicted box charge



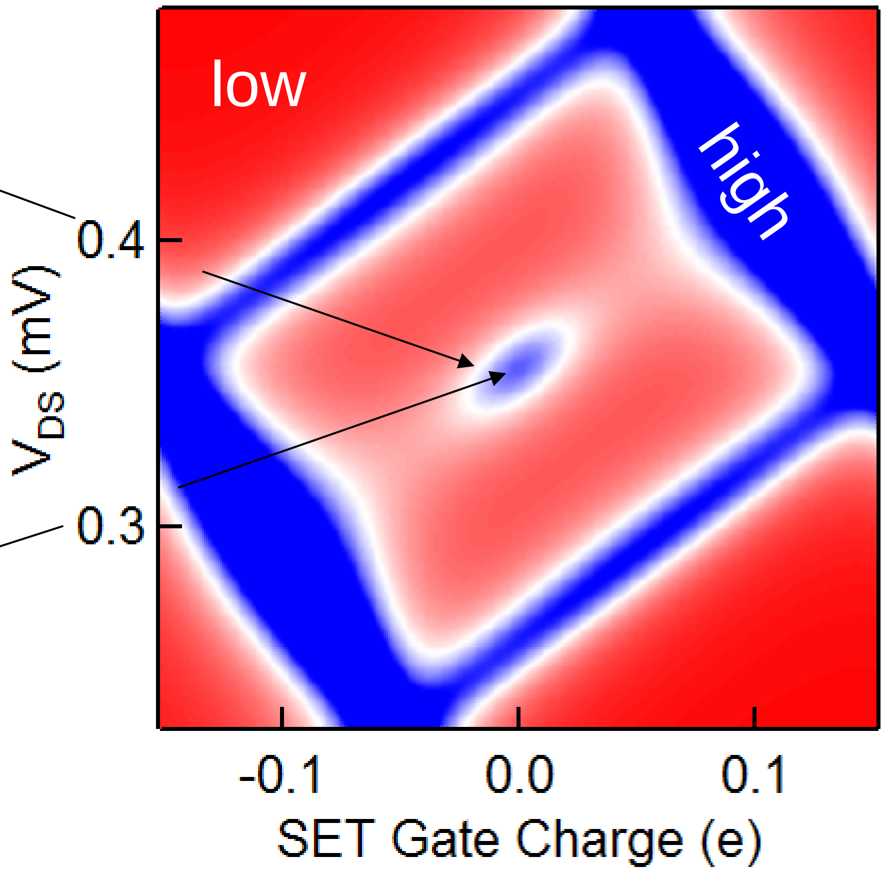
SET Determines Relaxation Time (T_1)

H. Majer and B. Turek, unpublished



Calculated **symmetric** noise at 30 GHz

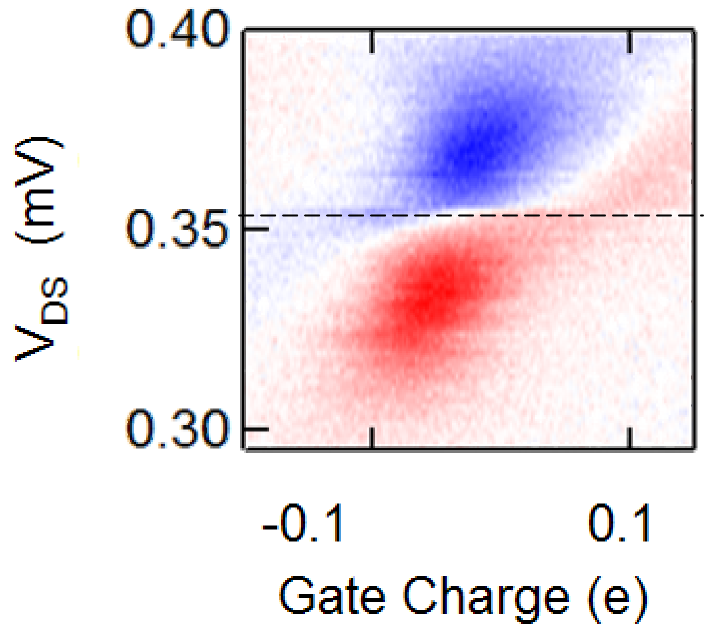
$$\Gamma_1 \propto S(+\omega) + S(-\omega)$$



Theory of quantum noise for DJQP
A. Clerk et al., PRL **89** 176804 (2002)

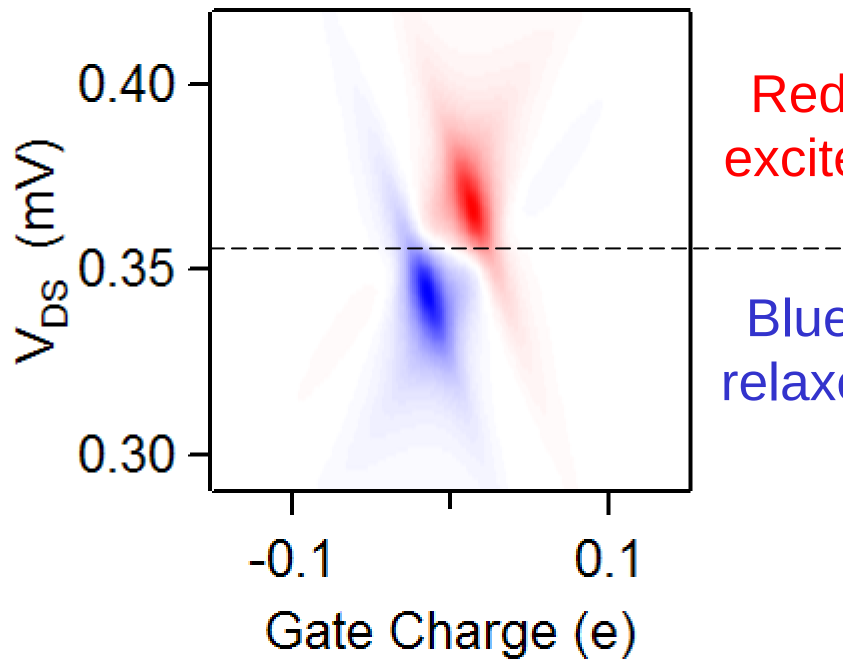
What About Asymmetry in SET Noise?

Measured reflected power



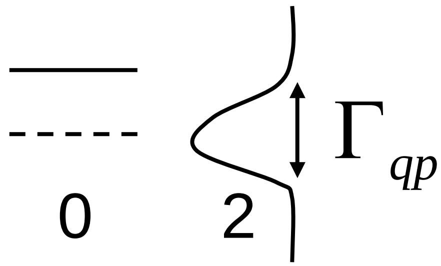
Calculated 5 GHz asymmetric noise

$$S_Q(+\omega) - S_Q(-\omega)$$



Red: excites

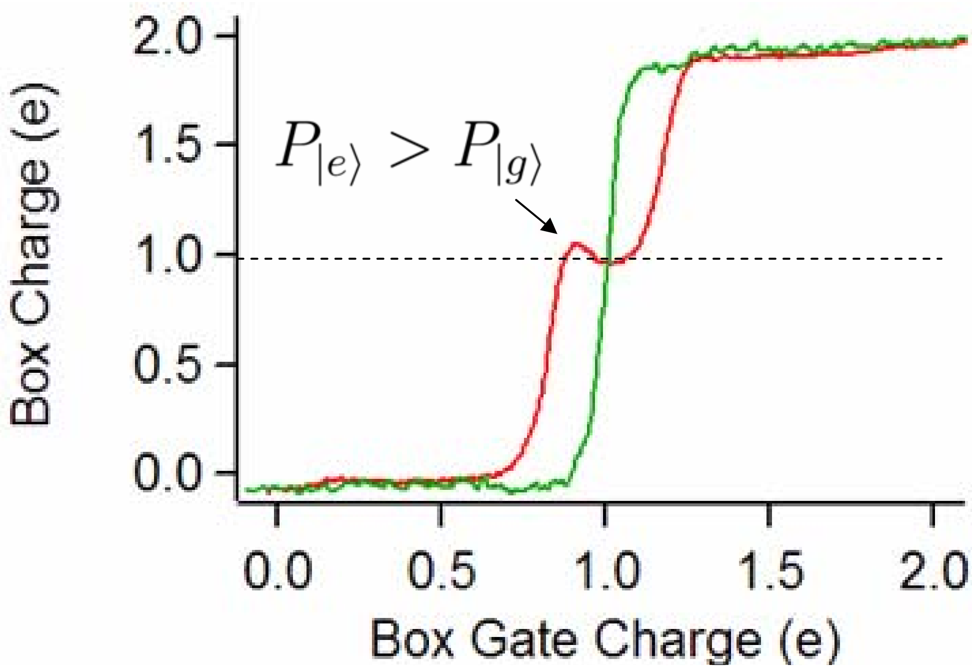
Blue: relaxes



Below resonance SET relaxes box
above resonance SET excites...

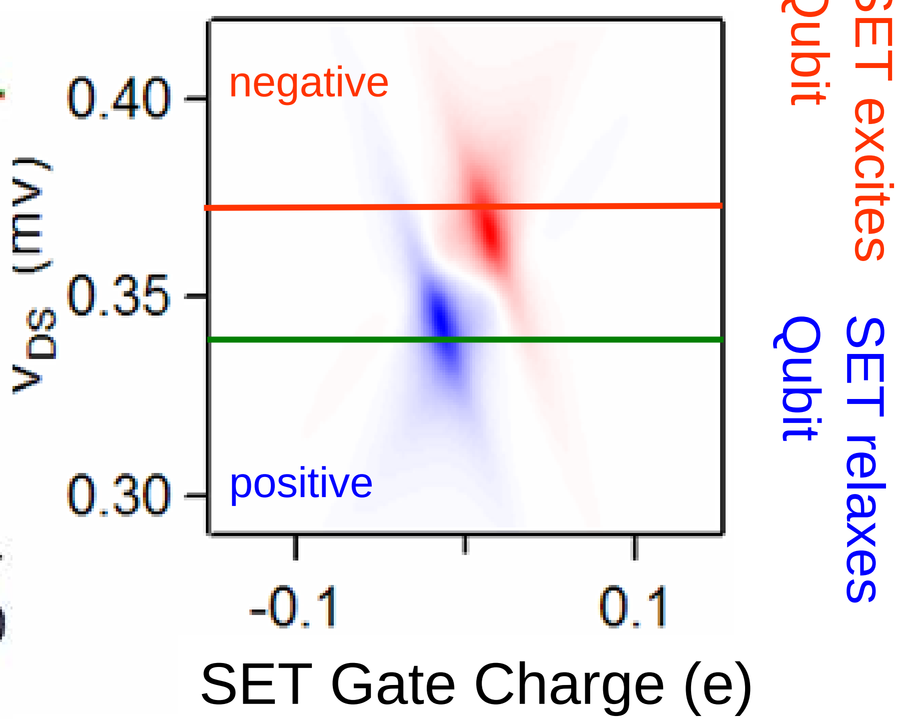
Population Inversion

Measured box charge



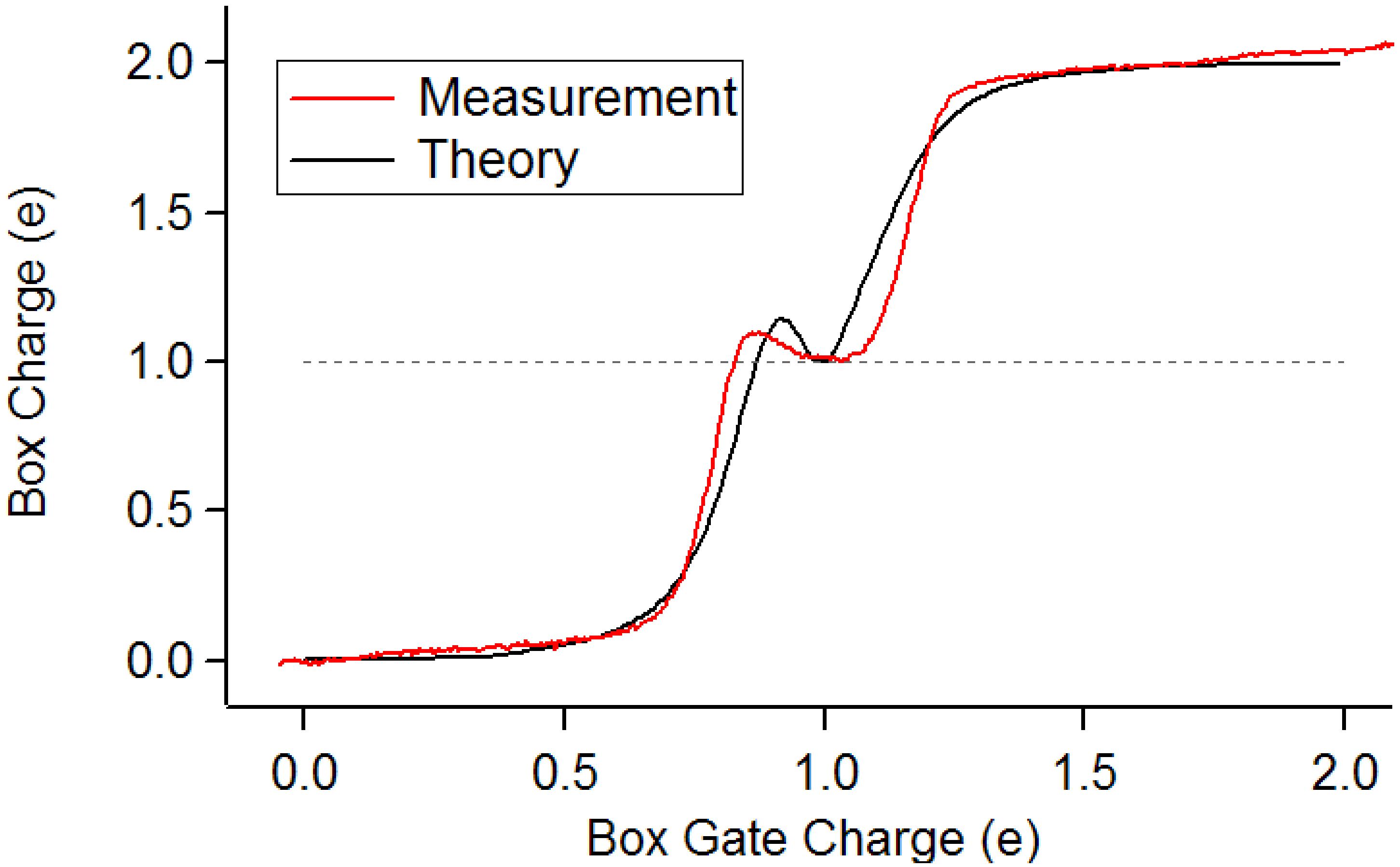
Calculated **asymmetric** noise at 5 GHz

$$S(+\omega) - S(-\omega)$$



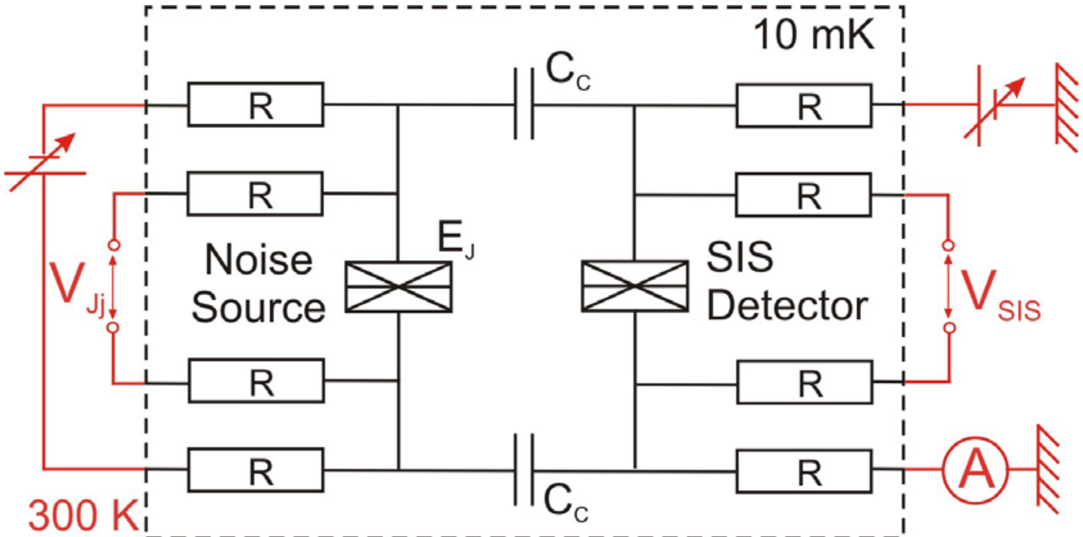
SET creates an negative effective spin temperature

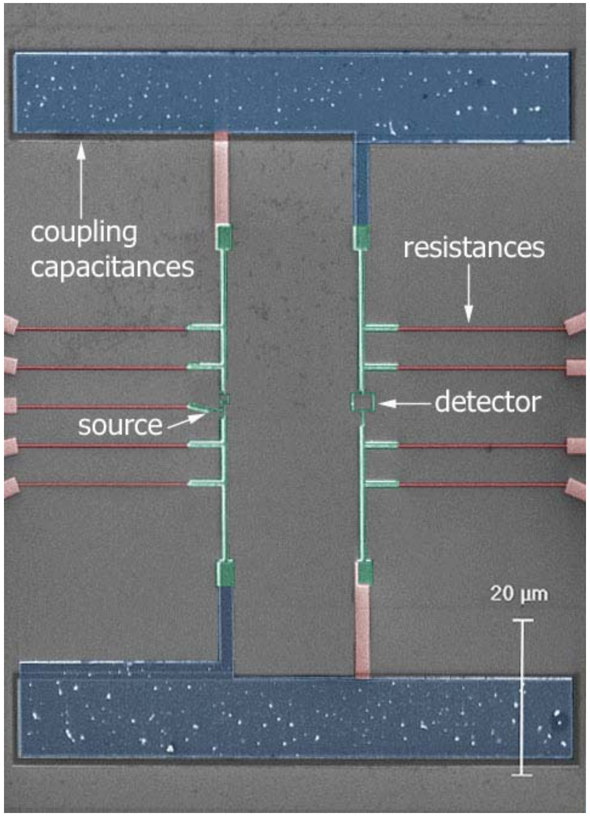
Inversion: Theory and Experiment



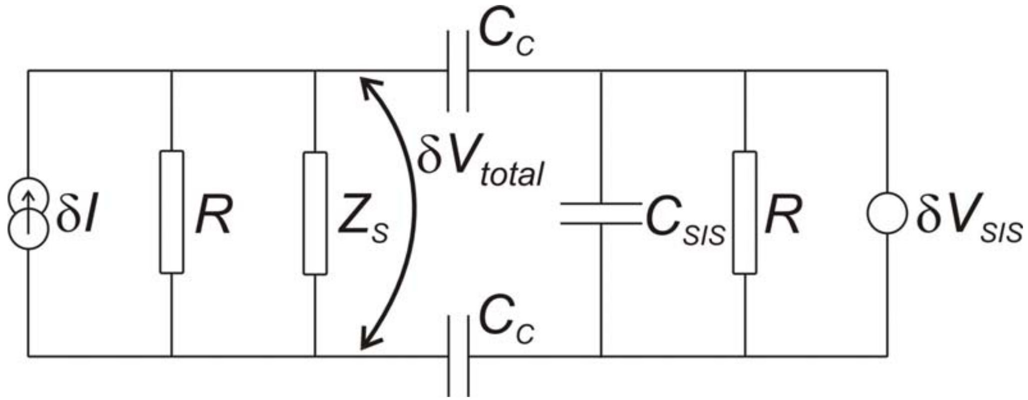
Numerical calculation including strong coupling to SET
+ environmental relaxation (A. Clerk)

Other Quantum Spectrometers – Delft Group





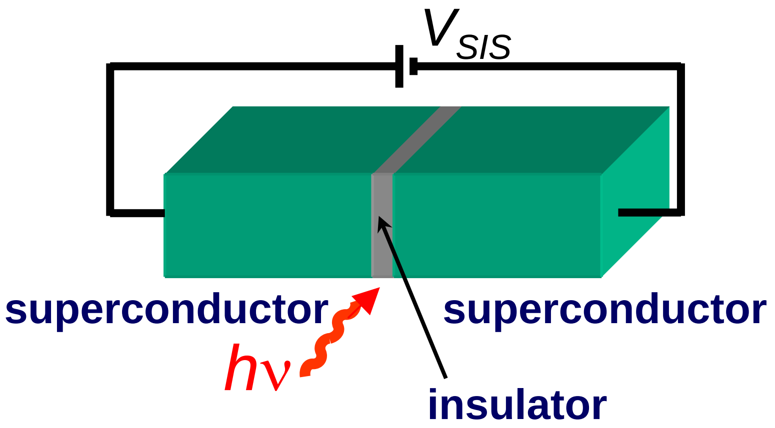
Equivalent AC circuit



Deblock, Onac, Gurevich, and Kouwenhoven,
Science 301, 203 (2003)

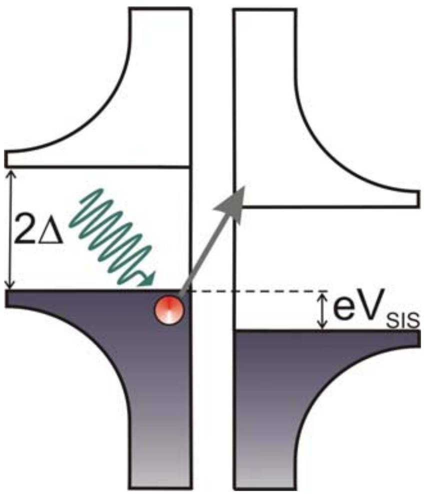
SIS detection principle

Voltage biased SIS junction

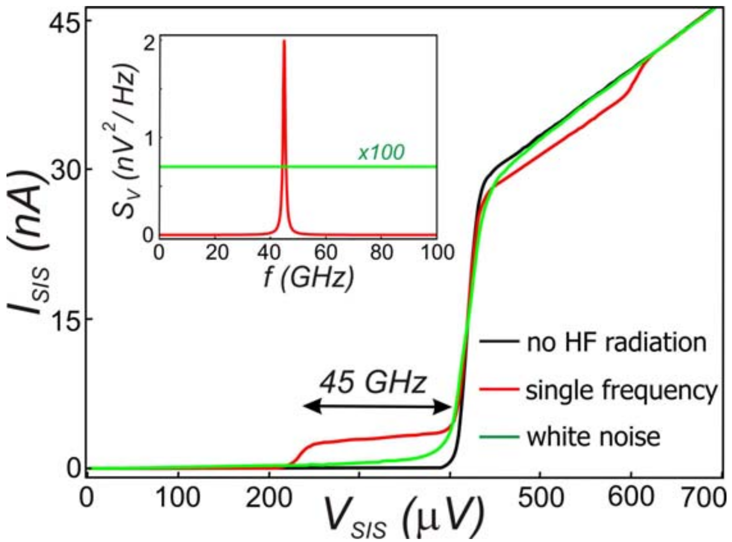


High-frequency detection based on *Photon Assisted Tunneling*

Density of States



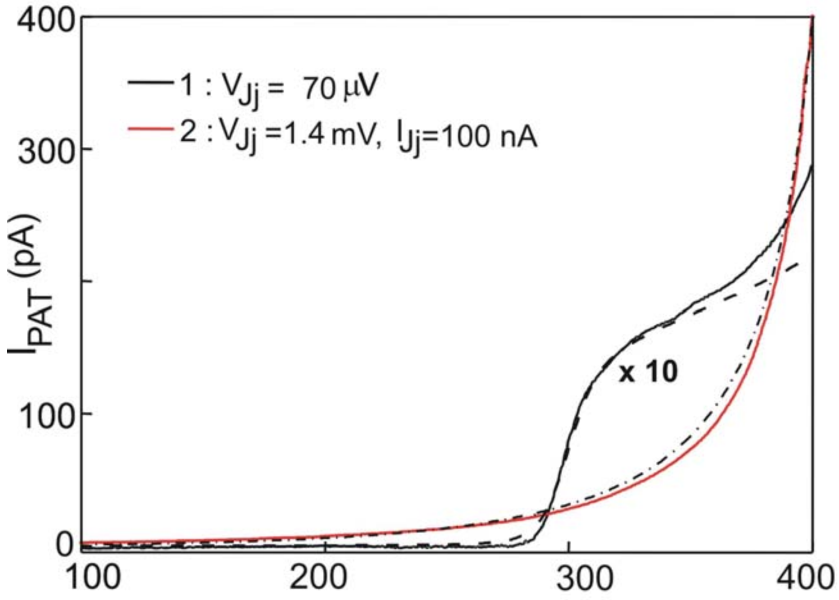
Numerical simulations



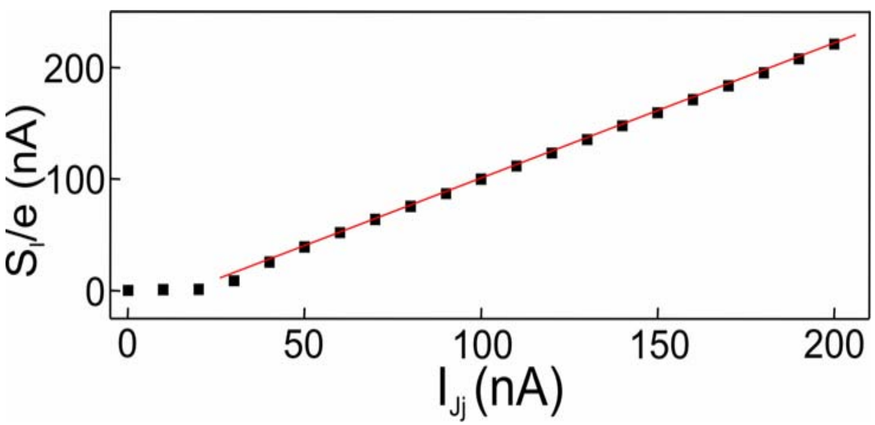
Quasi-particle shot noise

White noise fit

$$I_{PAT}(V) = \int_0^{+\infty} d\omega \left(\frac{e}{\hbar\omega}\right)^2 |Z(\omega)|^2 S_I(-\omega) I_{QP,0}\left(V + \frac{\hbar\omega}{e}\right)$$



S_I as fit parameter



Only emission part: $S_I(\omega)=eI$

Resolution: *80 fA²/Hz*
(*3mK* on a *1 kΩ* resistor)

Summary of Lecture 2

- Positive frequency noise = reservoir absorbs
- Negative frequency noise = reservoir emits

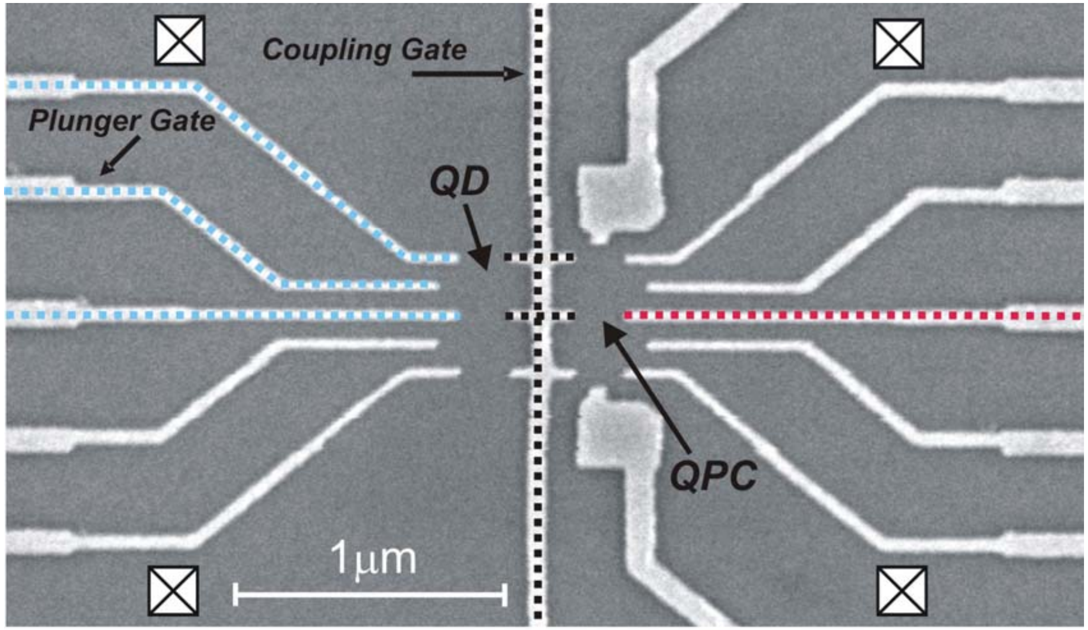
- A quantum system can act as a “quantum spectrometer,” able to measure both positive and negative frequency components of a non-classical noise source.

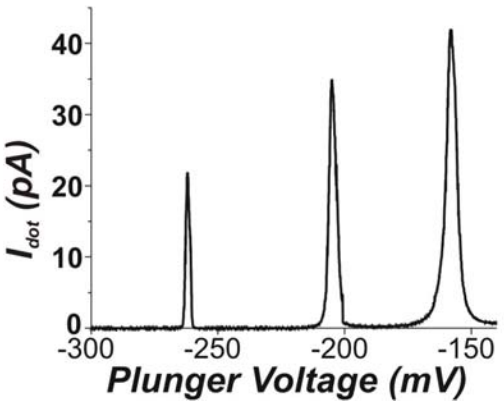
- Using CPB (= electrical TLS) as quantum spectrometer

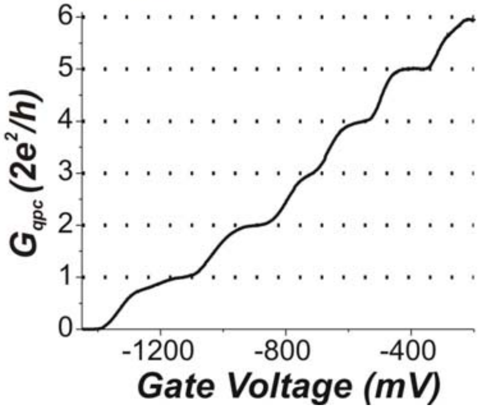
- Observed quantum noise (at > GHz) of SET backaction
  - SET controlling relaxation time of box
  - SET can create population inversion in box due to asymmetry of noise

Single Quantum Dot as Noise Detector

Device picture







Quantum Dot in CB regime:
noise detector

Quantum Point Contact:
noise source

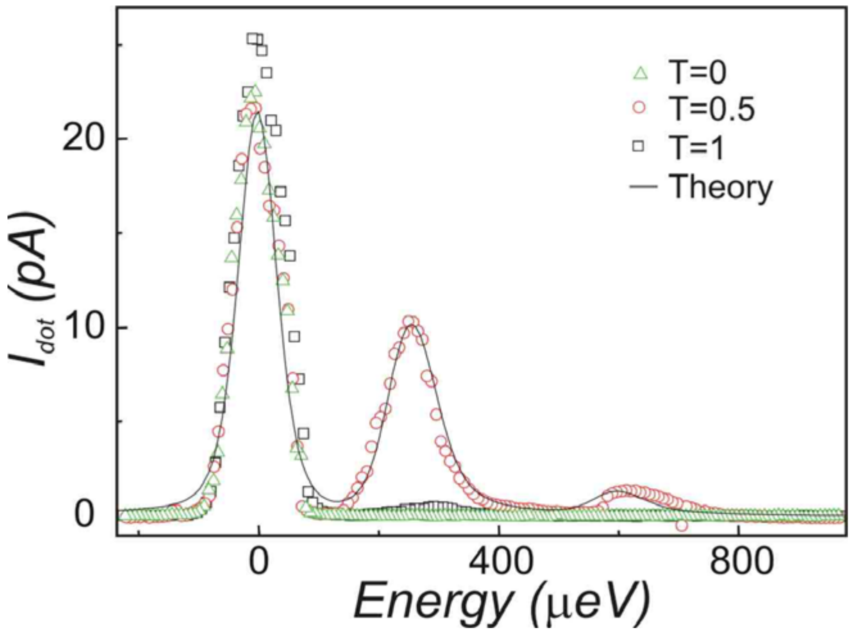
Inverse picture: QPC as a
charge detector

Detector back-action on the
studied quantum system

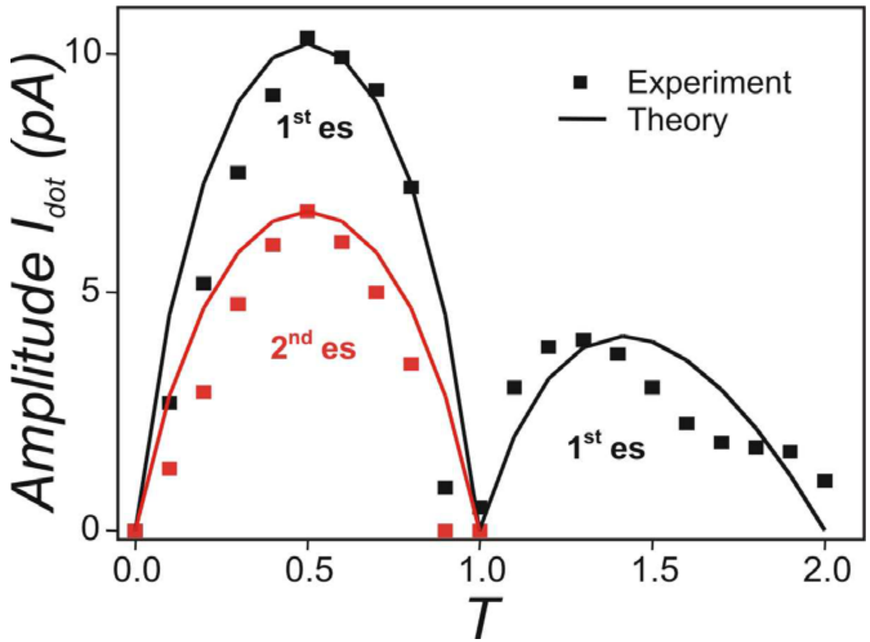
J.M.Elzerman *et.al*, Nature **430,** 431 (2004)

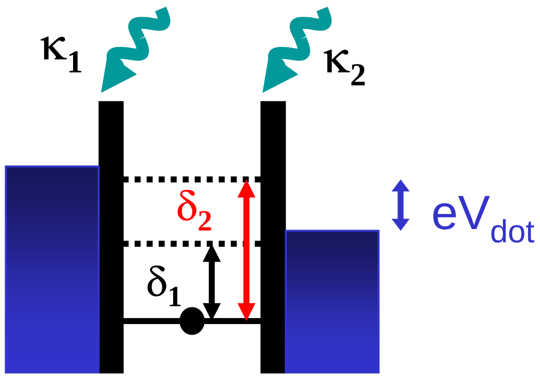
Single Quantum Dot as Noise Detector

Transport through orbital states



QPC transmission dependence





V_{QPC}= 1.27 meV
V_{dot}= 30 μeV
Temperature=200 mK
δ_1= 245 meV ~ 60 GHz
δ_2= 580 meV ~ 140 GHz
Γ_{gs} = 0.575 GHz

Set of fitting parameters

Γ_{1es} = 5.75 GHz
Γ_{2es} = 4.025 GHz
$\kappa_1 = \kappa_2$ = 0.0167

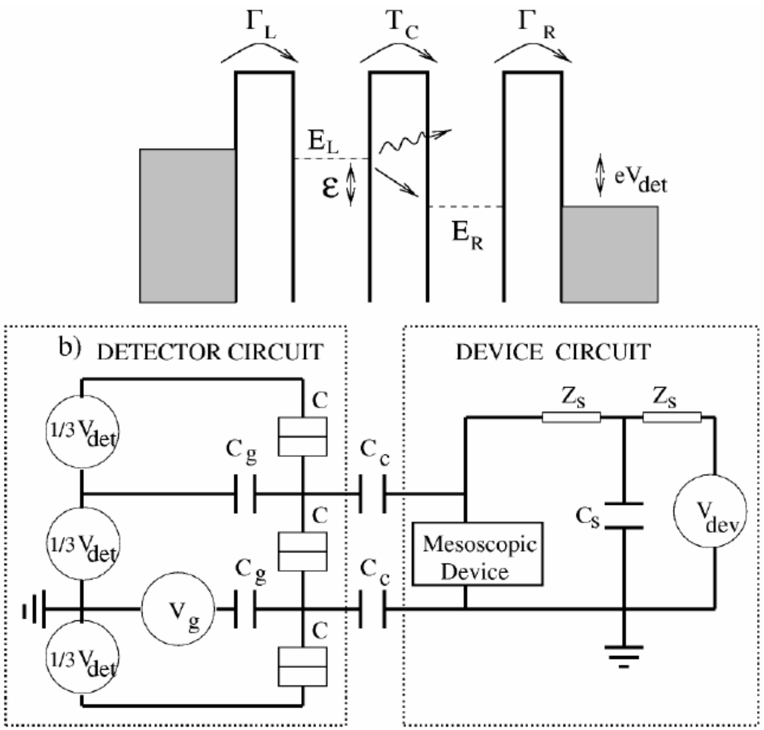
$\kappa^*_1 = \kappa^*_2$ = 0.0048

Double Quantum Dot Detection

- Non-coherent limit $\varepsilon \gg T_C$; $\Gamma_i \ll \Gamma_L, \Gamma_R$
- $I_{inel} = e/\hbar\, (\Gamma_L^{-1} + \Gamma_i^{-1} + \Gamma_R^{-1})^{-1} \approx e/\hbar\, \Gamma_i^{-1}$

$$I_{inel}(\epsilon) = \frac{e}{\hbar} T_C^2 P(\epsilon)$$

- $P(\varepsilon)$ probability for energy exchange with the enviroment



- Circuit transimpedance
  $S_V(\omega) = |Z(\omega)|^2\, S_I(\omega)$
  $|Z(\omega)|^2 = \kappa^2 R_K^2$

R. Aguado and L.P. Kouwenhoven, PRL **84,** 1986 (2000)