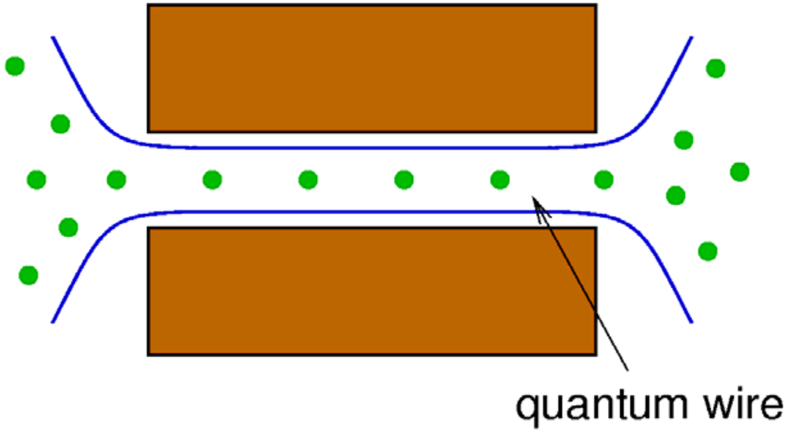
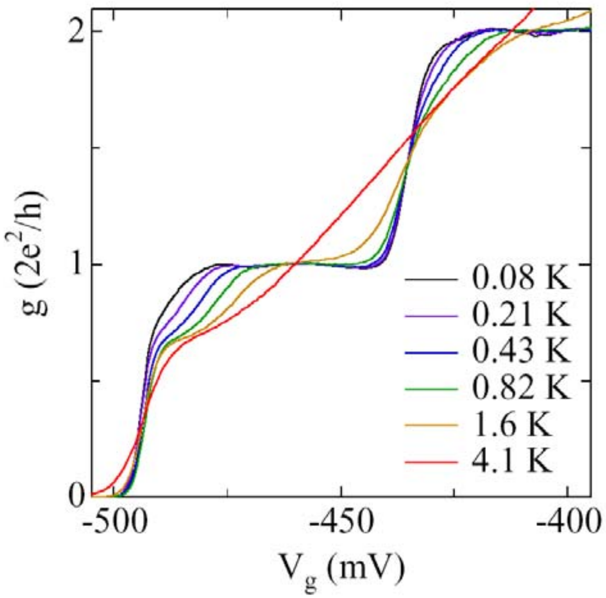
Conductance of a quantum wire at low electron density

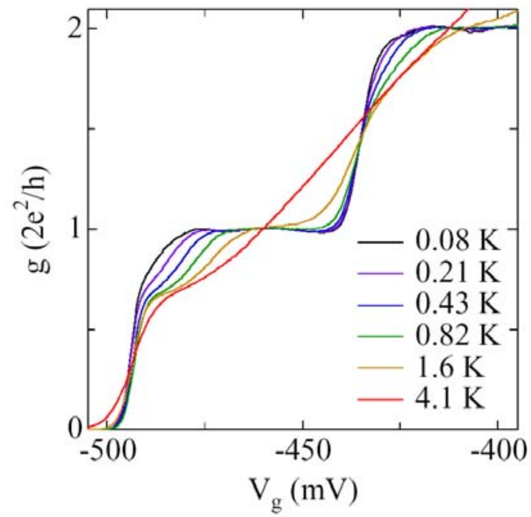
Konstantin Matveev



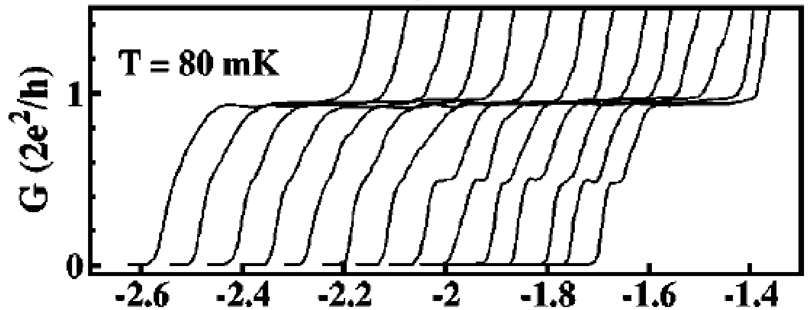


Outline

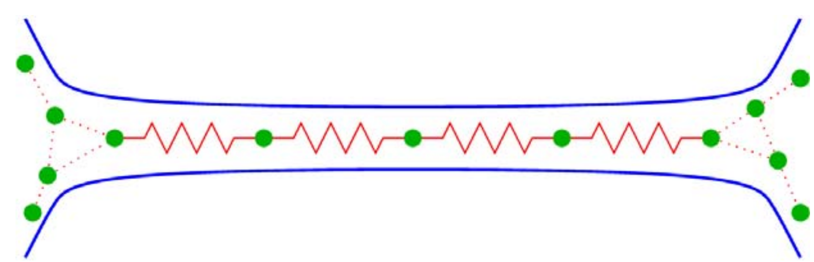
1. Quantum wires and Quantum Point Contacts



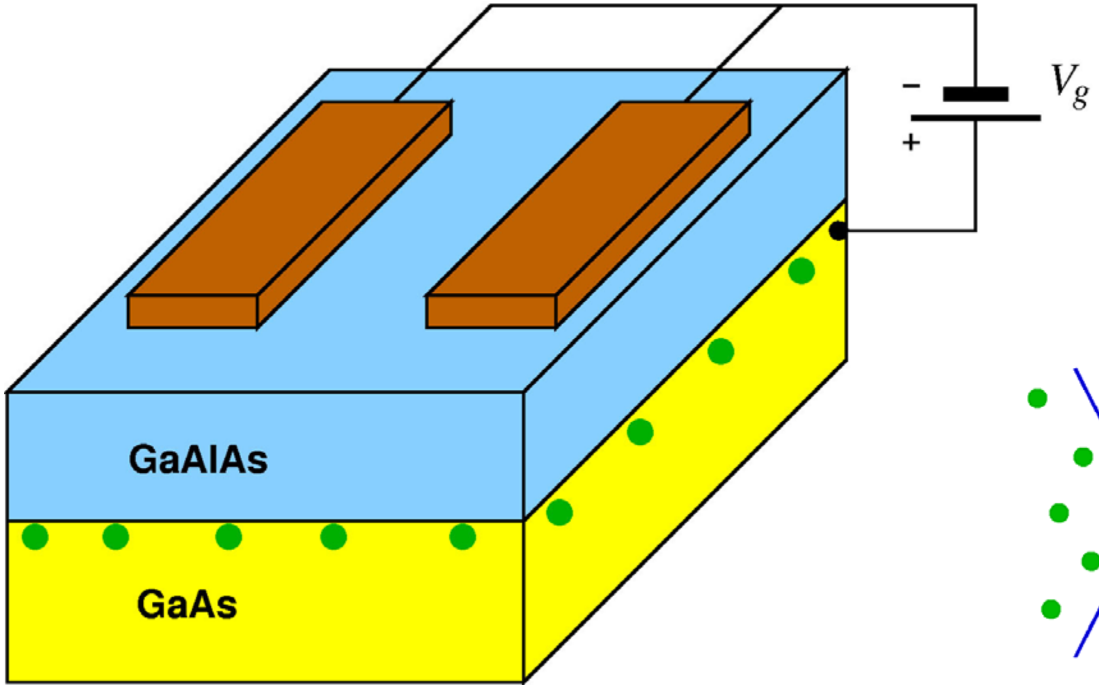
2. Experiments:
   - quantization of conductance,
   - 0.7 anomaly,
   - 0.5 plateau



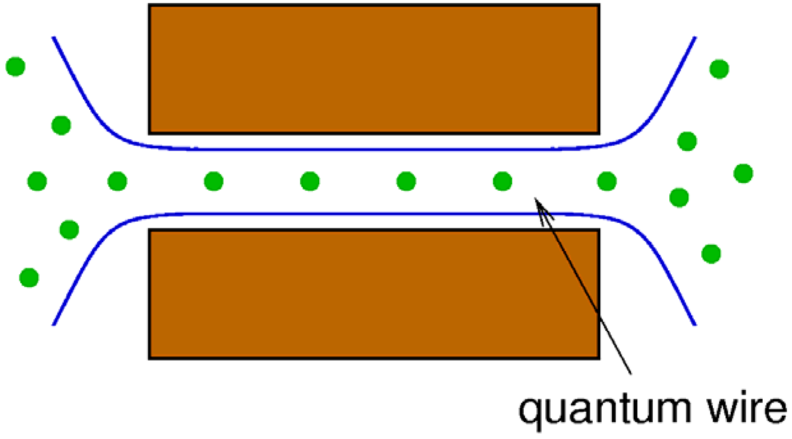
3. Theoretical ideas:
   - spin polarization,
   - Wigner crystal



One-dimensional conductors



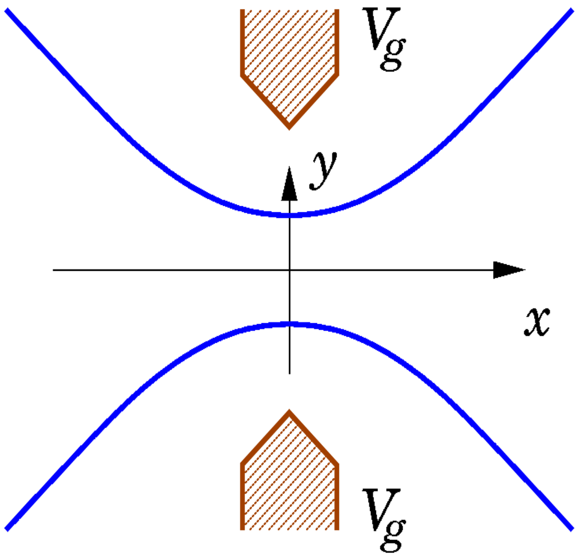
Top view:



One-dimensional electron system

Quantum Point Contacts

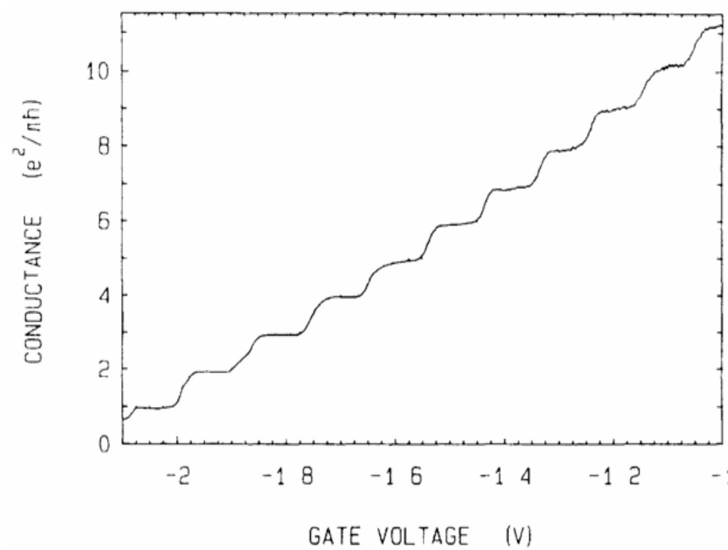
2D electron gas confined
by a split gate

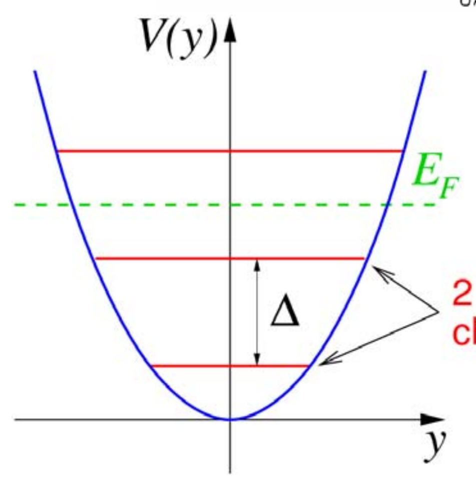


Electron motion across
the channel is quantized

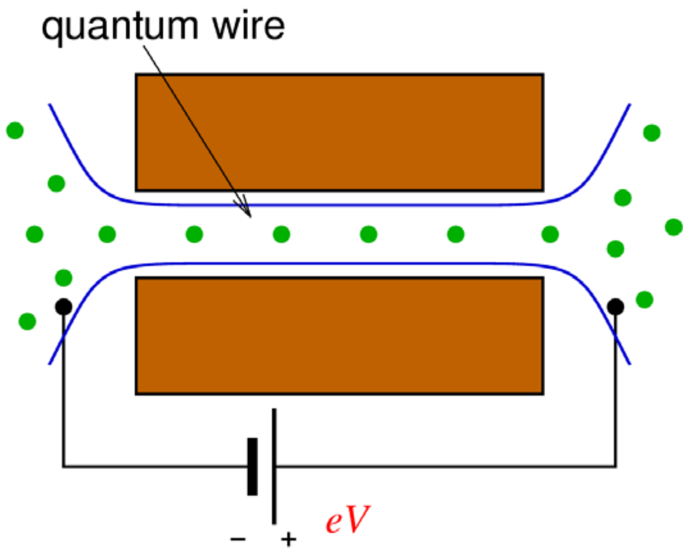
Experiment: B.J. Van Wees *et al.*, 1988;
also, D.A. Wharam *et al.*, 1988

Conductance (1/Resistance) vs. V_g



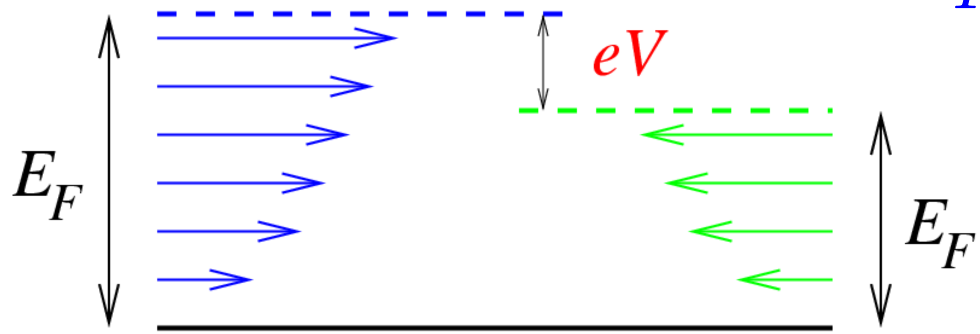


Conductance of a Quantum Wire



Applied bias results in uncompensated current

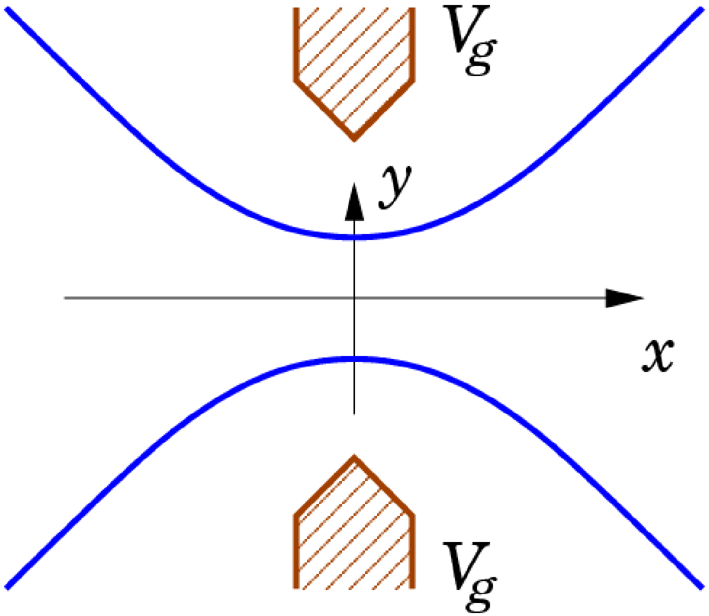
$$I = e\Delta n v_F$$



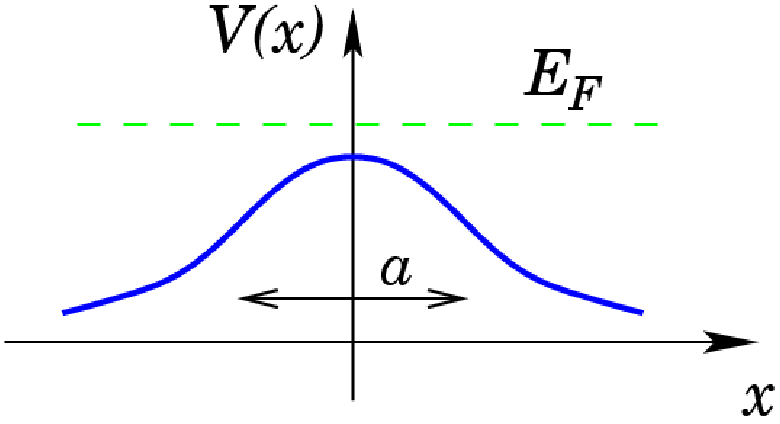
Electron density: $$\Delta n = \frac{1}{h}\Delta p_F = \frac{1}{h}\frac{dp_F}{dE_F}eV = \frac{e}{hv_F}V$$

Current: $$I = \frac{e^2}{h}V$$ Conductance: $$G = \frac{e^2}{h}$$

Shape of the conductance steps



L. I. Glazman, G. B. Lesovik,
D. E. Khmel'nitskii, R. I. Shekhter, 1988



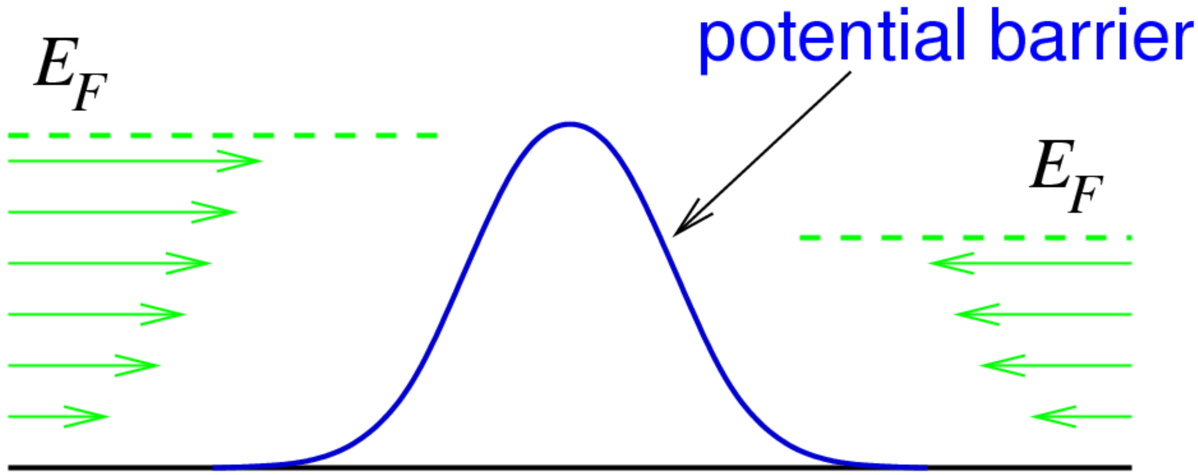
The energy of the transverse motion of electrons is inversely proportional to the square of the width $d(x)$ of the channel

$$V(x) \sim \frac{\pi^2 \hbar^2}{2md^2(x)}$$

Constriction creates a potential barrier for electrons

How does the barrier affect the conductance?

Landauer formula



Only a fraction of electrons cross the barrier!

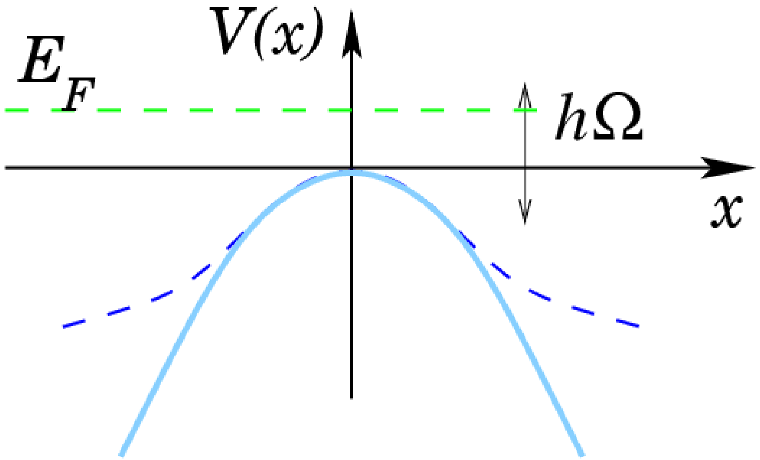
Current: $I = e\,\Delta n\, v_F\, T(E_F)$

$T(E_F)$ is the transmission coefficient of the barrier at the Fermi energy

Conductance: $G = \dfrac{e^2}{h} T(E_F)$

Transmission coefficient

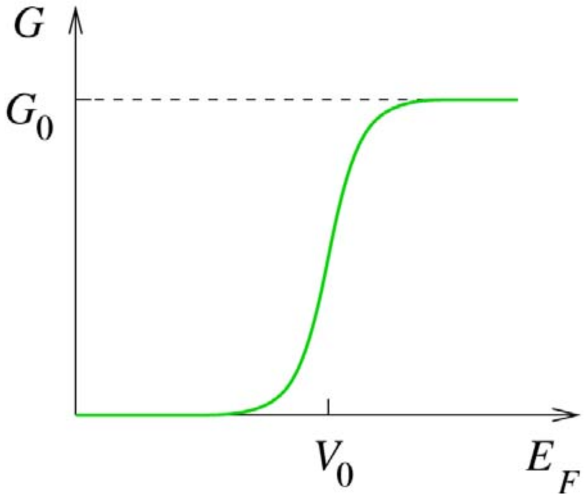
Parabolic-potential approximation: $V(x) = -\frac{1}{2} m\Omega^2 x^2$



$$T(E_F) = \frac{1}{\exp\left(-\frac{2\pi E_F}{\hbar\Omega}\right) + 1}$$

E. C. Kemble, 1935

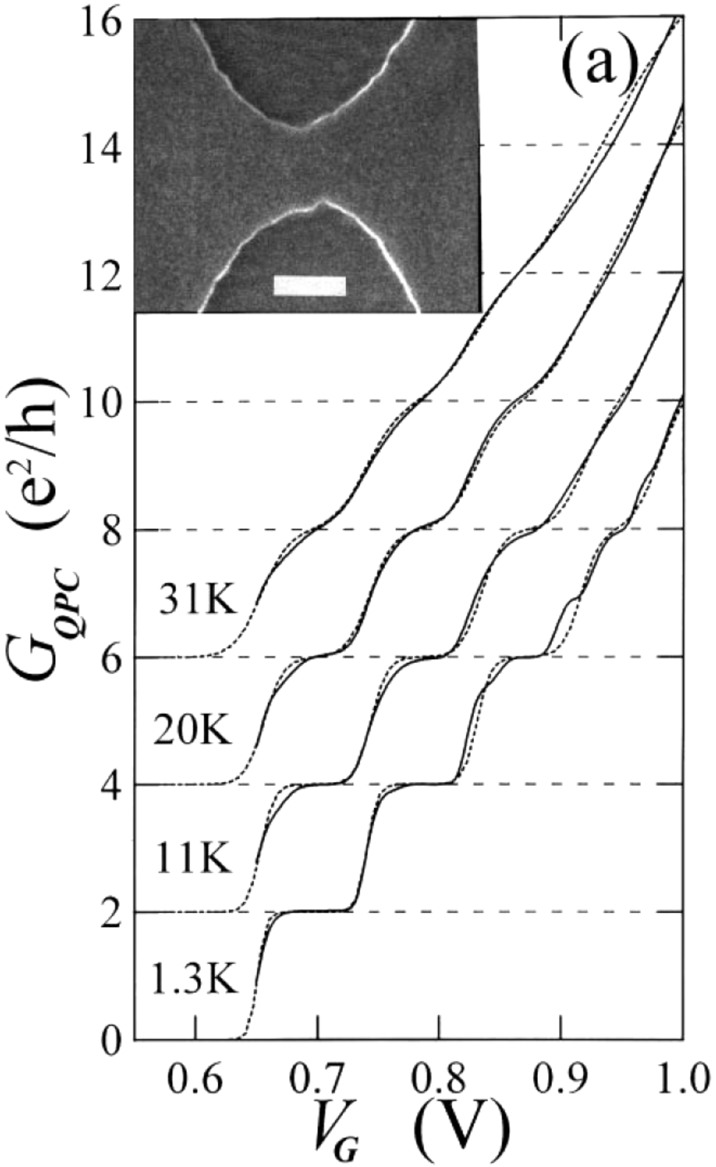
Conductance as a function of the Fermi energy



$$G_0 = 2\frac{e^2}{h}$$

2 accounts for electron spins

Shape of the conductance steps



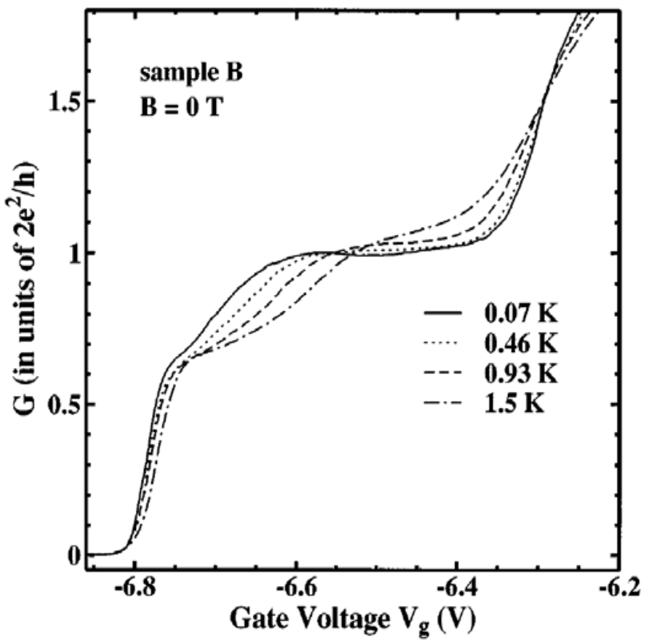
Experiment:

Kristensen *et al.*, 1998

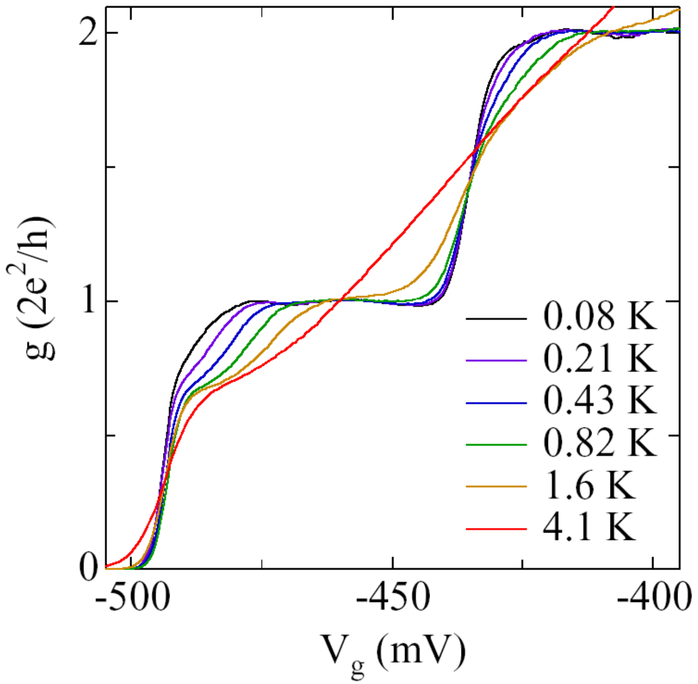
- Good agreement with theory at low temperatures;
- Noticeable deviations at higher temperatures.

0.7 Anomaly

Conductance vs. gate voltage at different temperatures:



Thomas *et al*., 1996

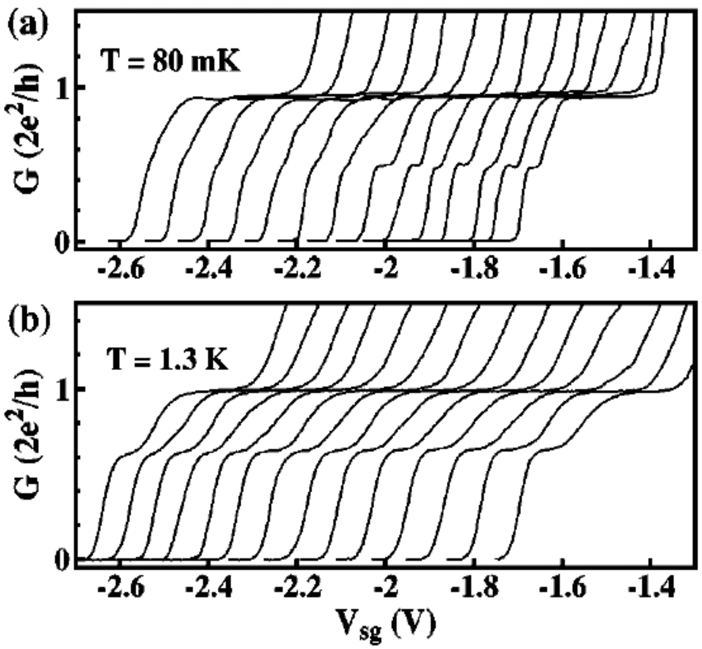


Cronenwett *et al*., 2001

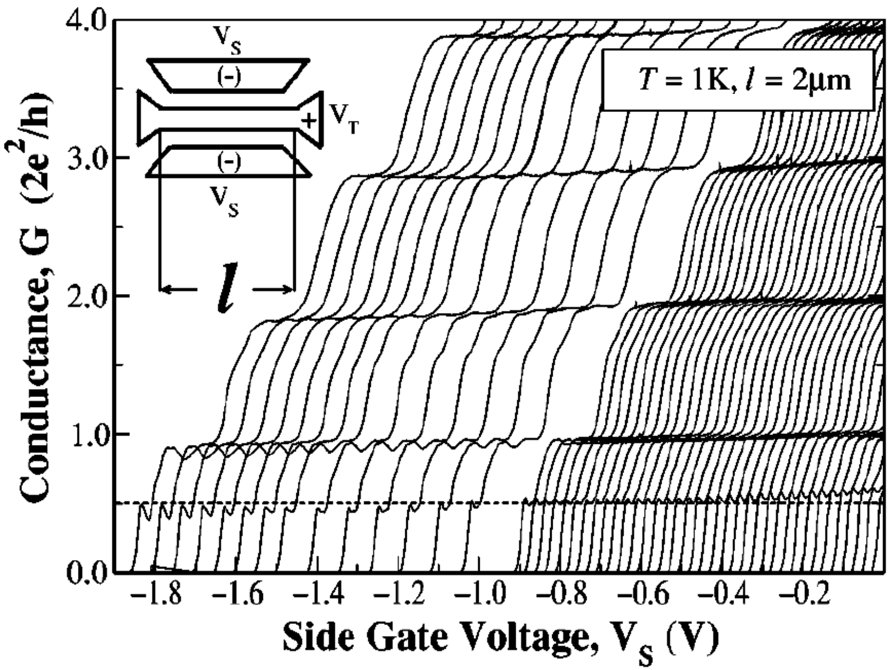
As the temperature grows, the conductance develops a shoulder near $0.7 \times \frac{2e^2}{h}$

0.5 Plateau

Thomas *et al.*, 2000



Reilly *et al.*, 2001

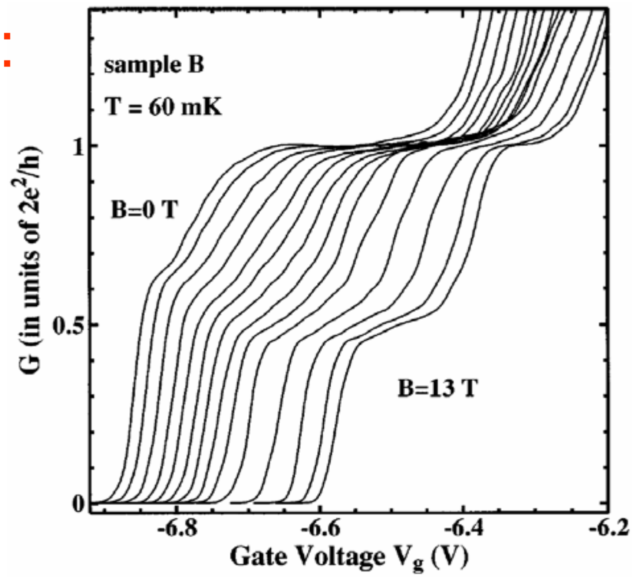


Several experiments show a plateau of conductance at $0.5 \times \frac{2e^2}{h}$

This 0.5 plateau tends to appear in longer samples

Magnetic field dependence

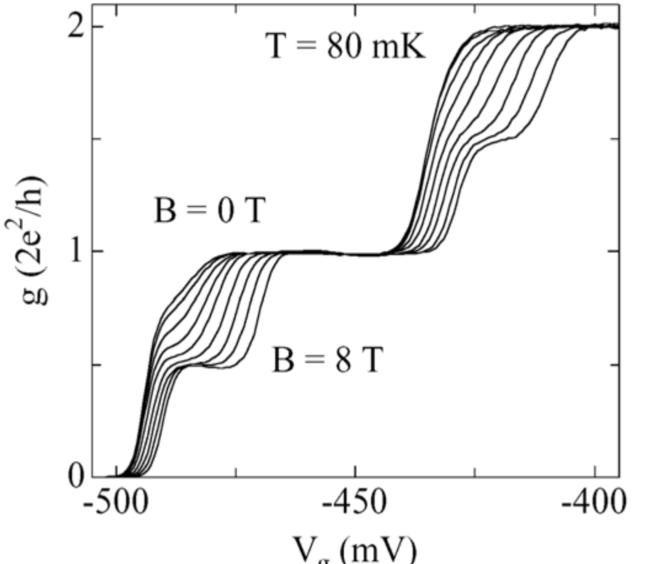
Conductance vs. gate voltage at different *B*:



Thomas *et al*., 1996

Cronenwett *et al*., 2001

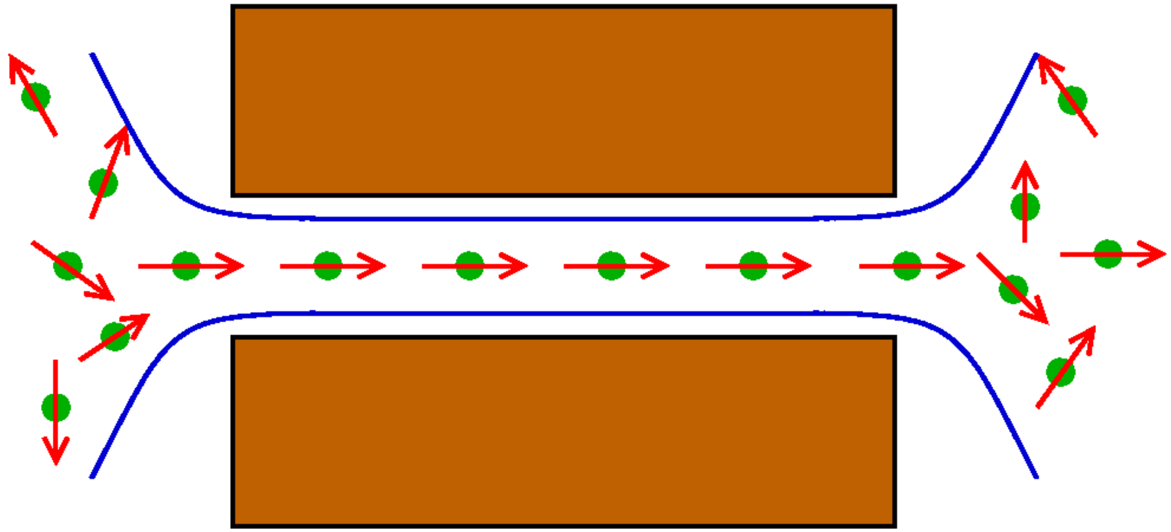
0.7-anomaly evolves toward the spin-polarized plateau at $0.5\frac{2e^2}{h}$



Spin-polarization effect?

Wang & Berggren, 1997
Spivak & Zhou, 2000
Flambaum & Kuchiev, 2000
Hirose, Li & Wingreen, 2001
...

One-dimensional ferromagnetism?



- Two-dimensional electron gas is paramagnetic.
- Is one-dimensional electron system ferromagnetic?

No! The ground state of a system of one-dimensional spin-1/2 fermions has the lowest spin possible.

E. Lieb and D. Mattis, 1962;
D. Mattis, *The Theory of Magnetism.*

Interacting one-dimensional electrons: **Bosonization**

One-dimensional electron liquid is an elastic medium

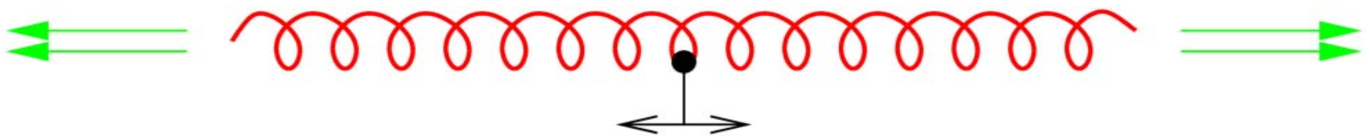
$$H = \frac{p^2}{2mn} + \frac{1}{2} mn v_F^2 \left(\frac{du}{dx}\right)^2$$

$u(x)$ is displacement of the medium, $p(x)$ is momentum density,
n is electron density.

In a non-interacting system the waves propagate at Fermi velocity v_F

In the presence of interactions v_F is replaced by plasmon velocity s

Quantized resistance



$$u(0,t) = u_0 \cos \omega t$$

Applied current: $I = en\dot{u}$

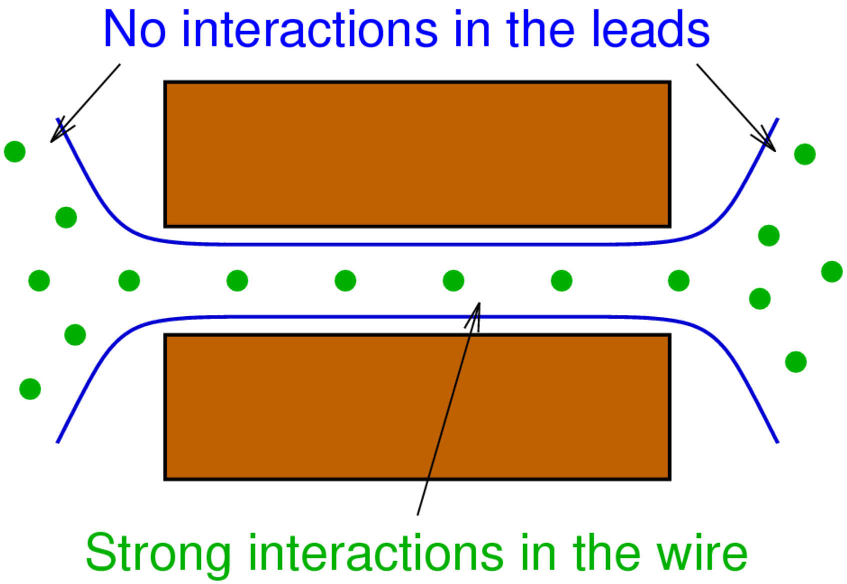
Density of elastic energy: $\langle H \rangle = \langle mn\dot{u}^2 \rangle = \frac{m}{e^2 n} I^2$

Energy carried away by plasmons in unit time:

$$W = 2\langle H \rangle v_F = \frac{2mv_F}{e^2 n} I^2 = I^2 R$$

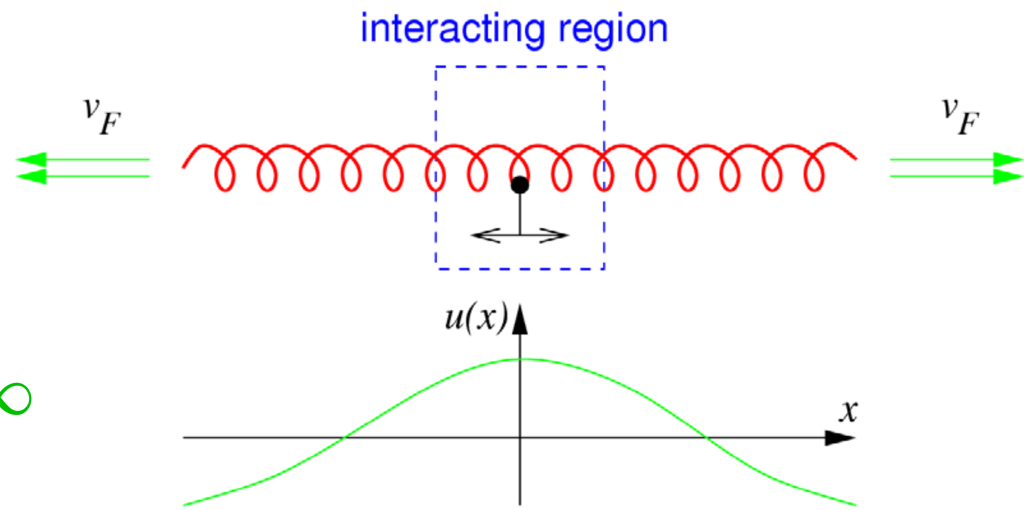
Resistance: $R = \frac{2mv_F}{e^2 n} \equiv \frac{h}{2e^2}$ $\left(n = \frac{2k_F}{\pi} \right)$

Interacting electrons



One-dimensional model
with non-interacting leads

[Maslov & Stone; Ponomarenko; Safi and Schulz, 1995]

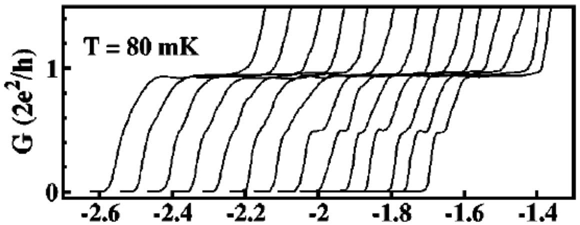


In the dc limit $\omega \to 0$
plasmon wavelength $\lambda \to \infty$

Plasmons are emitted in the non-interacting region

Resistance is still $\dfrac{h}{2e^2}$

Wigner crystal

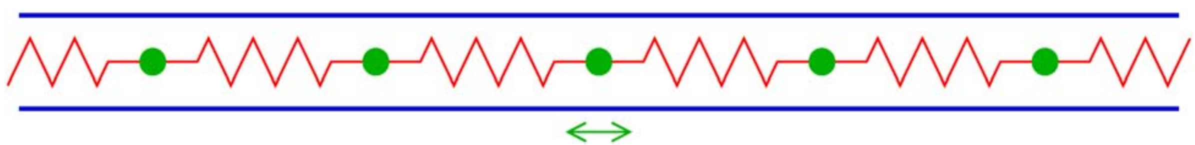


One-dimensional electrons at low density $n \to 0$

Compare kinetic and interaction energies:

$$E_{\mathsf{kin}} = \frac{\hbar^2 k_F^2}{2m} \propto n^2, \qquad E_{\mathsf{Coul}} = \frac{e^2}{r} \propto n.$$

Coulomb energy dominates:

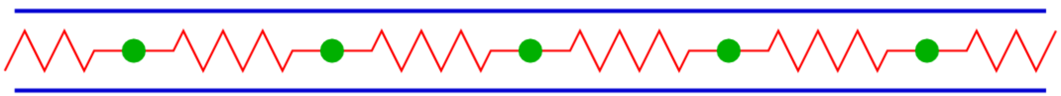


Electrons form a Wigner crystal and stay near their lattice sites

Density excitations are elastic waves in the crystal (plasmons)

$$H = \frac{p^2}{2mn} + \frac{1}{2} n m s^2 \left(\frac{du}{dx}\right)^2$$

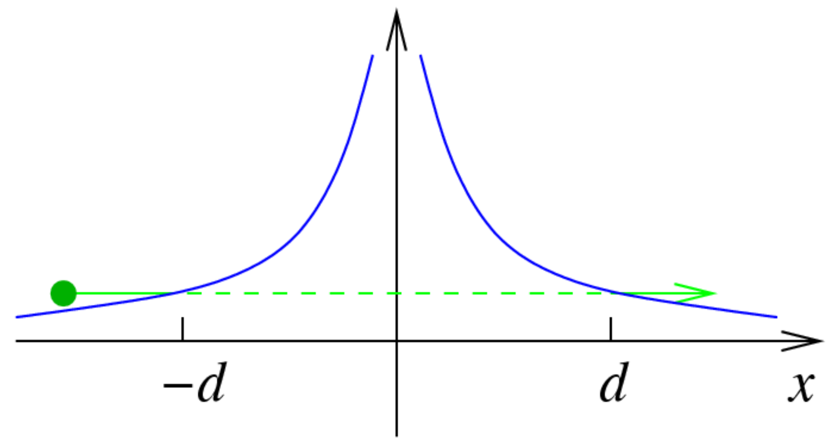
Spin coupling



To first approximation the spins do not interact

Weak exchange due to tunneling through the Coulomb barrier

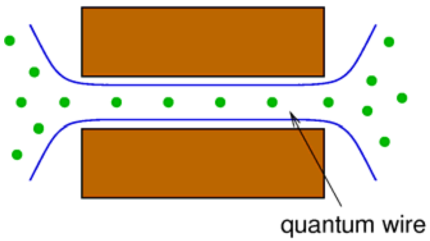
$$J \sim \exp\left(-\frac{\pi}{\sqrt{n a_B}}\right)$$

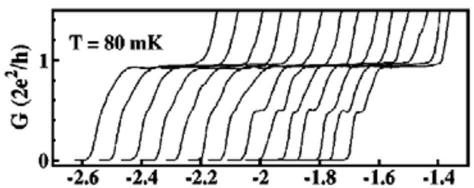


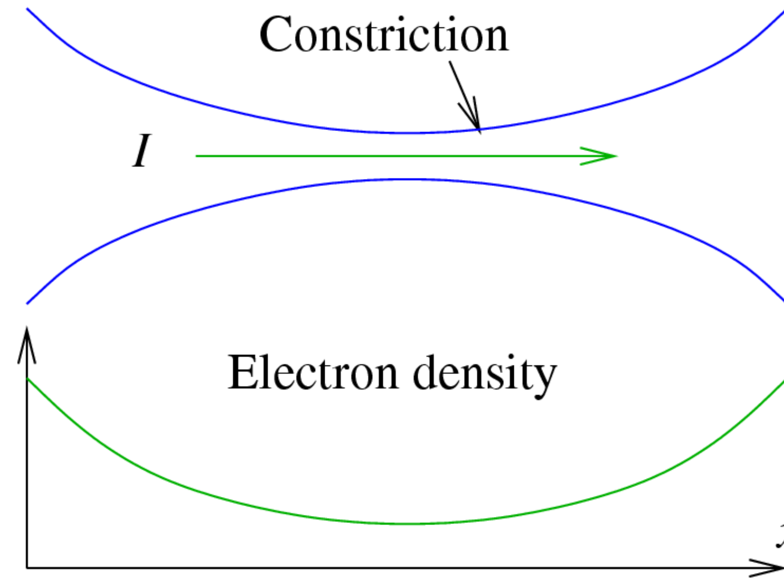
Antiferromagnetic spin chain: $H_\sigma = \sum_l J \boldsymbol{S}_l \cdot \boldsymbol{S}_{l+1}, \quad J > 0$

Note: charge and spin are not coupled!

Spin-charge separation?

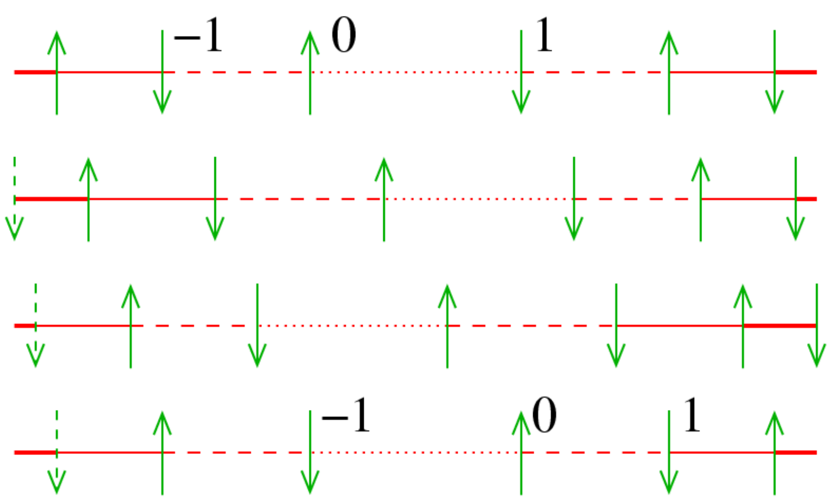






- Exchange constant depends on electron density. Thus $J = J(l)$.

- The density at site l depends on the number of electrons that moved through the wire: $J = J[l + q(t)]$.



Electric current (a charge excitation) affects coupling between the spins!

Spin contribution to the resistance

The spin Hamiltonian $H_\sigma = \sum J[l + q(t)]\boldsymbol{S}_l \cdot \boldsymbol{S}_{l+1}$ depends on $q(t) \propto I$. Thus the applied current excites not only charge, but also spin waves:

$$W = I^2 \frac{h}{2e^2} + I^2 R_\sigma = I^2 R$$

These processes give an additive contribution to the resistance

$$R = \frac{h}{2e^2} + R_\sigma$$

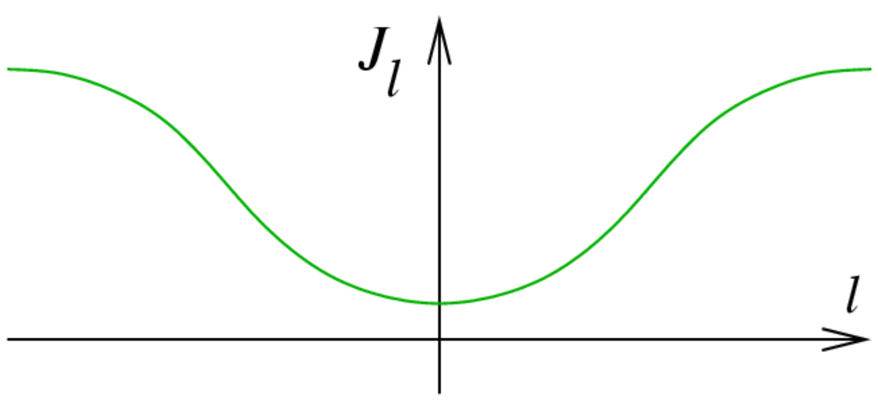
How do we find R_σ?

Approximation: XY model

$$H_\sigma = \sum_l J_l \left(S_l^x S_{l+1}^x + S_l^y S_{l+1}^y \right), \qquad J^z \equiv 0$$

The Jordan-Wigner transformation converts the problem to that of non-interacting fermions:

$$H_\sigma = \frac{1}{2} \sum_l J_l \left(a_l^\dagger a_{l+1} + a_{l+1}^\dagger a_l \right)$$



J is large in the leads, but small in the constriction

Jordan-Wigner transformation

Raising and lowering operators $S_l^{\pm} = S_l^x \pm i S_l^y$

can be expressed in terms of fermionic operators:

$$S_l^+ = a_l^\dagger \exp\left(i\pi \sum_{j=1}^{l-1} a_j^\dagger a_j \right), \qquad S_l^z = a_l^\dagger a_l - \frac{1}{2}$$

Then the Hamiltonian of a spin chain

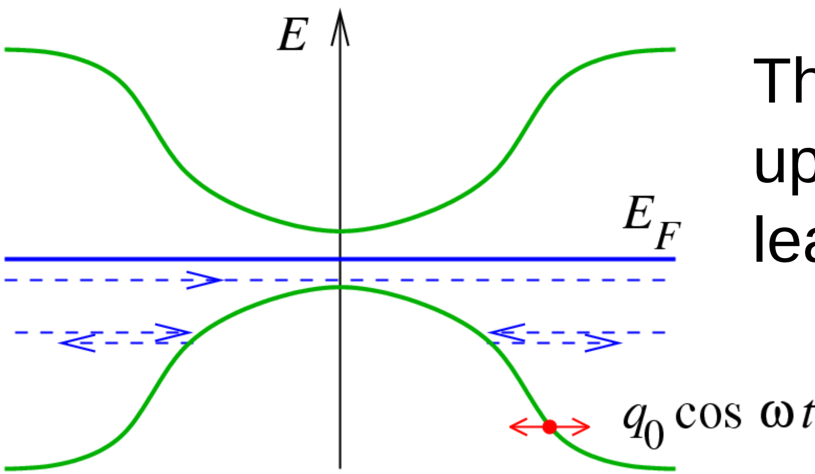
$$H = \sum_l [J_\perp (S_l^x S_{l+1}^x + S_l^y S_{l+1}^y) + J_z S_l^z S_{l+1}^z]$$

transforms to

$$H = \sum_l \frac{1}{2} J_\perp \left(a_l^\dagger a_{l+1} + a_{l+1}^\dagger a_l \right) + J_z \left(a_l^\dagger a_l + \frac{1}{2} \right) \left(a_{l+1}^\dagger a_{l+1} + \frac{1}{2} \right)$$

XY model: $J_z = 0$ $\Rightarrow$ Non-interacting fermions

Resistance due to scattering of excitations

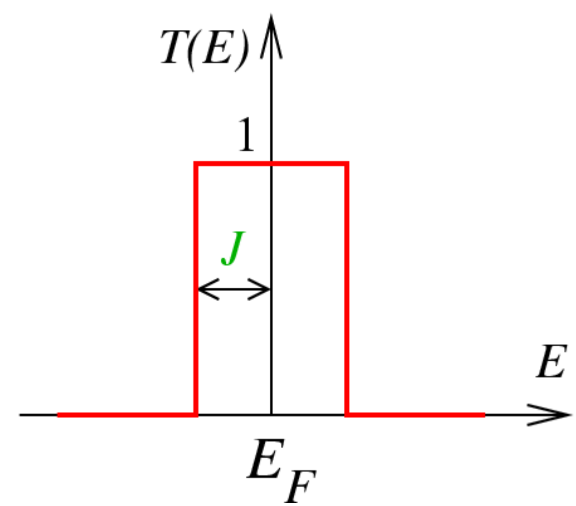


$J_l = J[l + q(t)]$

The fermions near the Fermi level pick up energy from the oscillating barrier, leading to dissipation $W = I^2 R_\sigma$

$$R_\sigma = \frac{h}{4e^2} \int \left(-\frac{\partial n_F}{\partial E} \right) [1 - T(E)] dE$$

Long wire limit

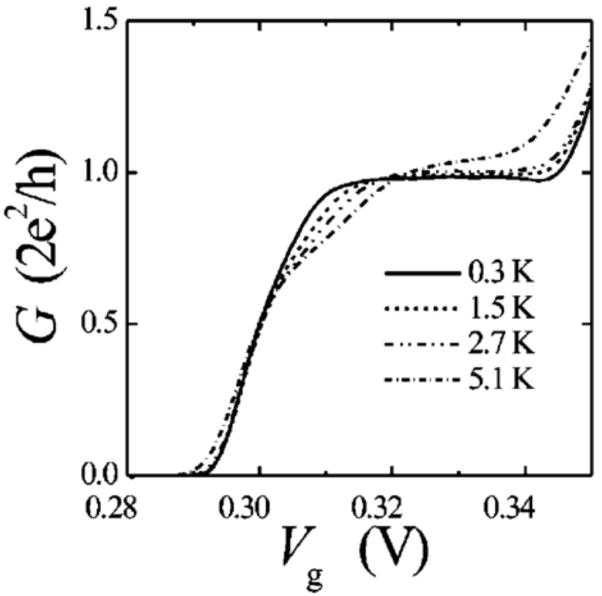


Exact result: $R_\sigma = \frac{h}{2e^2} n_F(J)$

1. At $T \ll J$ very few fermions are scattered by the barrier: $R_\sigma \propto e^{-J/T}$

2. At $T \gg J$ most excitations are backscattered, and resistance saturates: $R_\sigma = \frac{h}{4e^2}$

Activated temperature dependence

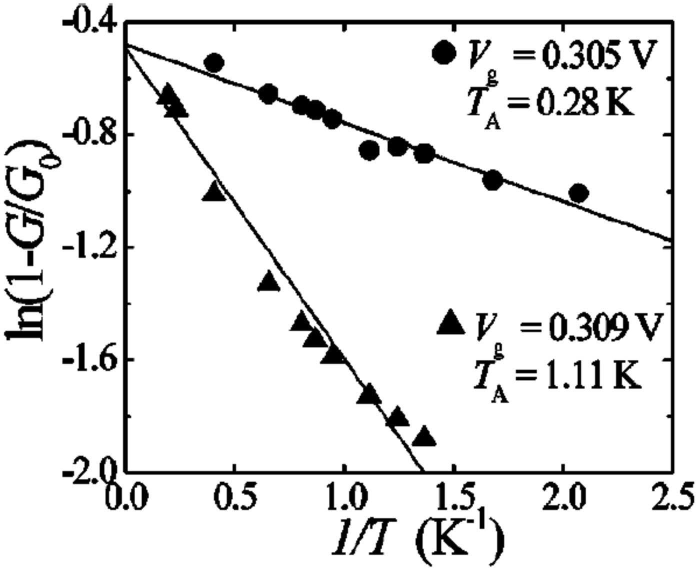


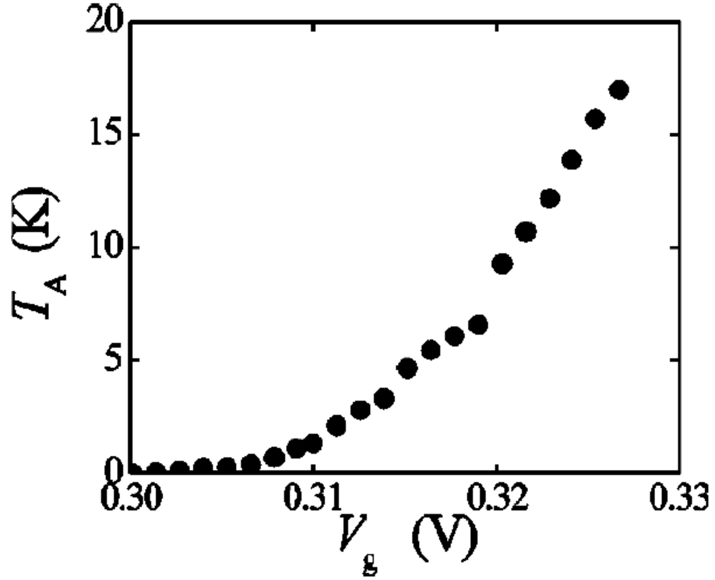
Kristensen *et al.*, 2000

The temperature dependence of 0.7 structure fits to

$$G = \frac{2e^2}{h} - G_A \exp\left(-\frac{T_A}{T}\right)$$

Activation temperature





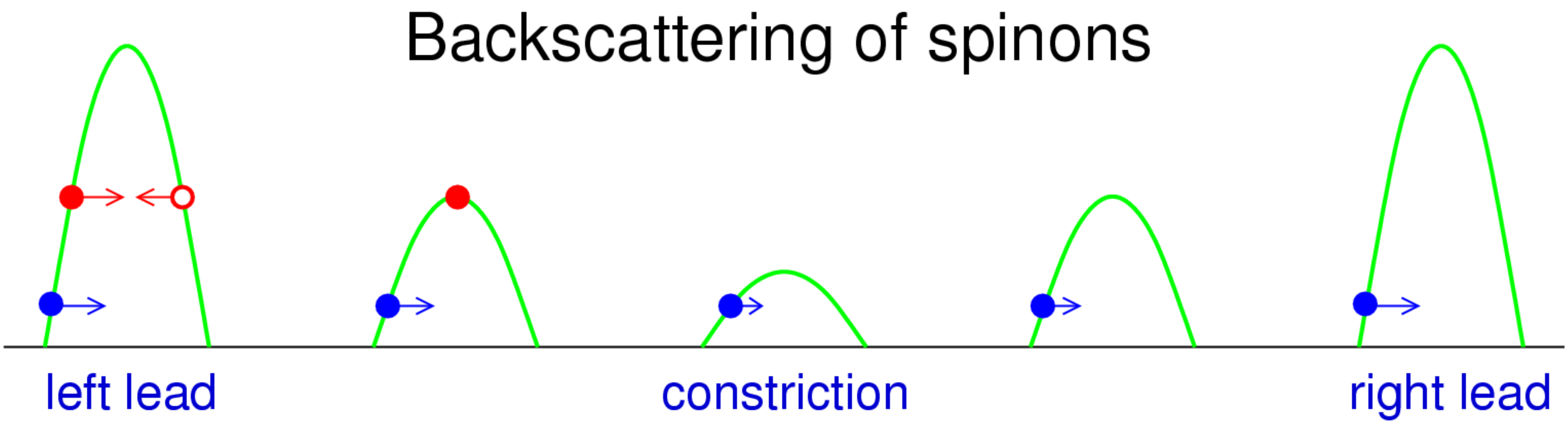
$T_A \sim 1\mathrm{K}$

Isotropic exchange

The true excitations of the spin chain are spinons, with the dispersion relation

$$\epsilon(q) = \frac{\pi J}{2} \sin q, \qquad 0 < q < \pi$$

$$J = J[l + q(t)]$$



Spinons with energies below $\pi J/2$ pass through; all others are reflected and contribute to R_σ

Conductance of a Wigner crystal wire

At $T \ll J$ only an exponentially small fraction of the spinons are reflected:

$$R_\sigma \propto \exp\left(-\frac{\pi J}{2T}\right), \quad T \ll J$$

At higher temperatures R_σ grows, and saturates at

$$R_\sigma = \frac{h}{2e^2}, \quad T \gg J$$

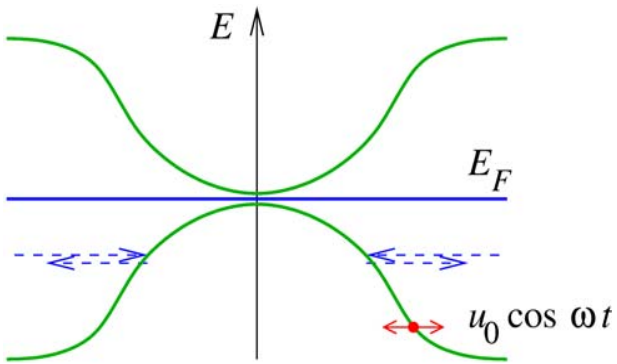
The conductance of the wire

$$G = \frac{1}{\frac{h}{2e^2} + R_\sigma}$$

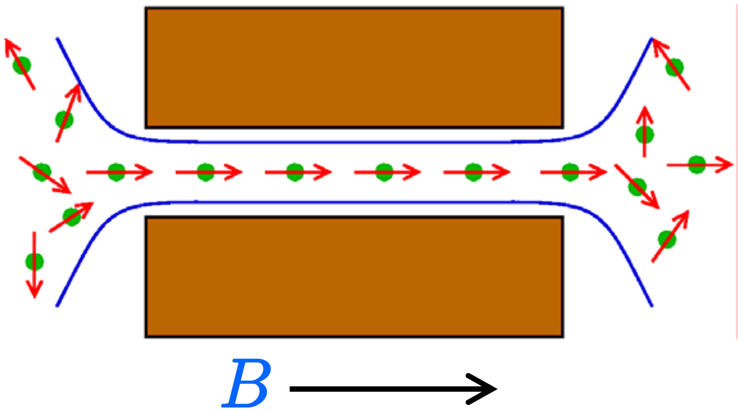
changes from $\frac{2e^2}{h}$ at $T \ll J$ to $\frac{e^2}{h}$ at $T \gg J$.

High temperature

At $J \ll T$ all spin excitations are reflected by the barrier (c.f. XY model)



Analogy: high B



When spins in the wire are polarized, no excitations pass through; expect $G = \dfrac{e^2}{h}$

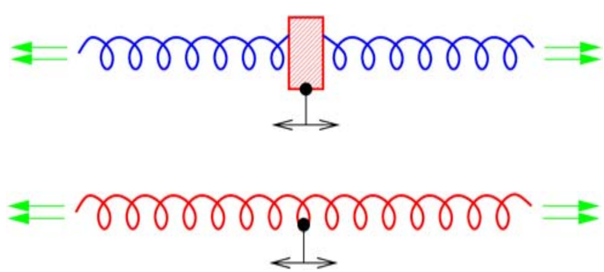
Check with bosonization:

In the leads $\psi = e^{i(\phi_\rho \pm \phi_\sigma)/\sqrt{2}}$

Boundary conditions at $x = 0$

$$\phi_\sigma = \frac{\pi q(t)}{\sqrt{2}}$$

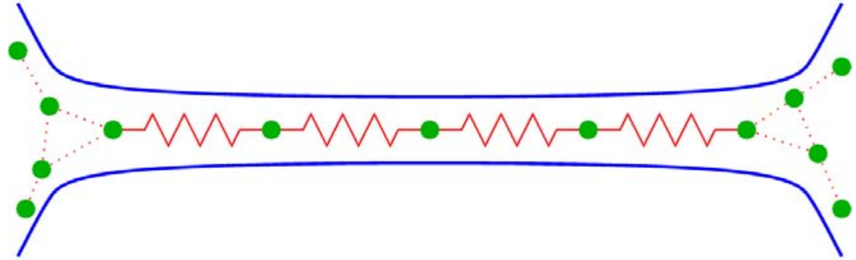
$$\phi_\rho = \frac{\pi q(t)}{\sqrt{2}}$$



$$R_\sigma = R_\rho = \frac{h}{2e^2}$$

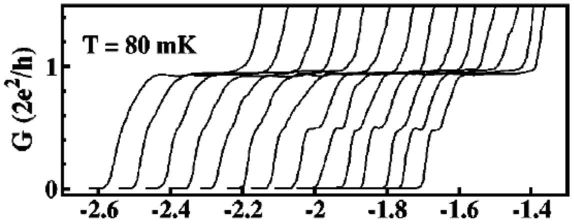
Conclusions

1. At low density of electrons in a quantum wire, they form a Wigner crystal



2. The spins of electrons in the Wigner crystal are weakly coupled, and the propagation of spin excitations through the wire is impeded.

3. As a result, the conductance of the wire decreases by a factor of 2.



4. No spin polarization is required!