# Boulder Summer School 2005

## Physics of Mesoscopic Systems

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## Outline of Lectures

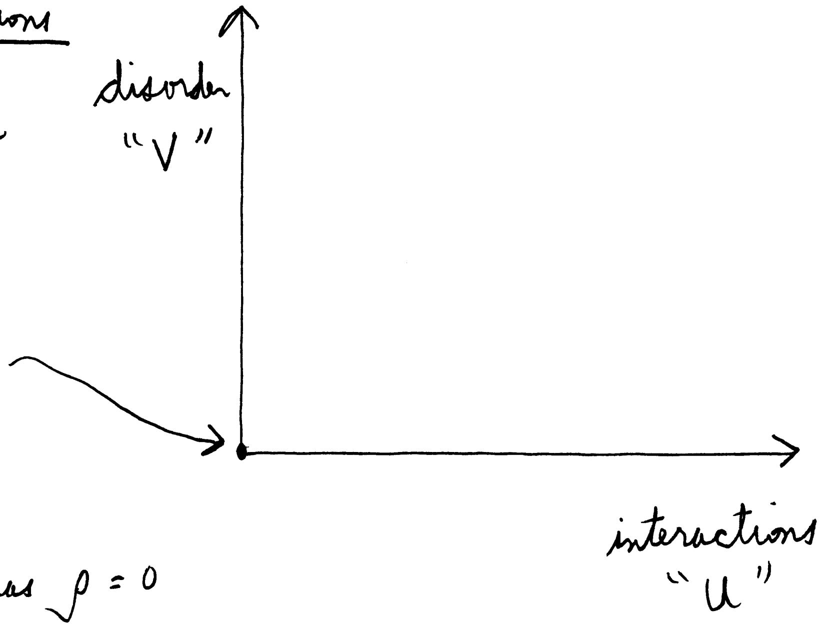
1. Review of quantum correction experiments

2 & 3. Electron dephasing and energy exchange experiments

4. UCF and 1/f noise experiments

# I. Disorder + Interactions

## The Big Picture




1. Textbook case
   $V=0$, $U=0$
   Bloch's Theorem

   a) metal at $T=0$ has $\rho=0$
   b) finite $T$, $\rho(T)$ from electron-phonon scattering

2. $V$ small, $U=0$
   $\rho(T\to 0) \equiv \rho_0 \neq 0$ from impurity scattering

3. $V=0$, $U$ small
   Fermi liquid theory of metals (Landau)
   electrons $\to$ quasiparticles: i) well-defined $\vec{p}$
   ii) $\dfrac{\hbar}{\tau(\varepsilon)} \ll \varepsilon$ as $\varepsilon \to 0$

4. $V$ large, $U=0$
   Localization of 1-particle wavefunctions
   At $V_c$, M-I transition ("Anderson transition")

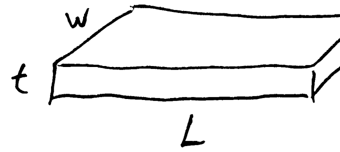
5. $V=0$, $U$ large
   Wigner crystal - electrons localized by Coulomb repulsion
   M-I transition ("Mott transition")

6. $V\neq 0$ and $U\neq 0$ $\to$ complicated!

Shocking news: the previous (incomplete) picture is valid only in $d=3$. In $d=1,2$ strange things happen.

$U=0$, arbitrary $V \neq 0$: Scaling theory of localization

conductance $G$ vs. conductivity $\sigma$




Ohm's law: $G = \frac{wt}{L}\sigma$

Hyper cube of side $L$: $G = L^{d-2}\sigma$

Good metals obey Ohm's law: $\frac{d \ln G}{d \ln L} = d-2$

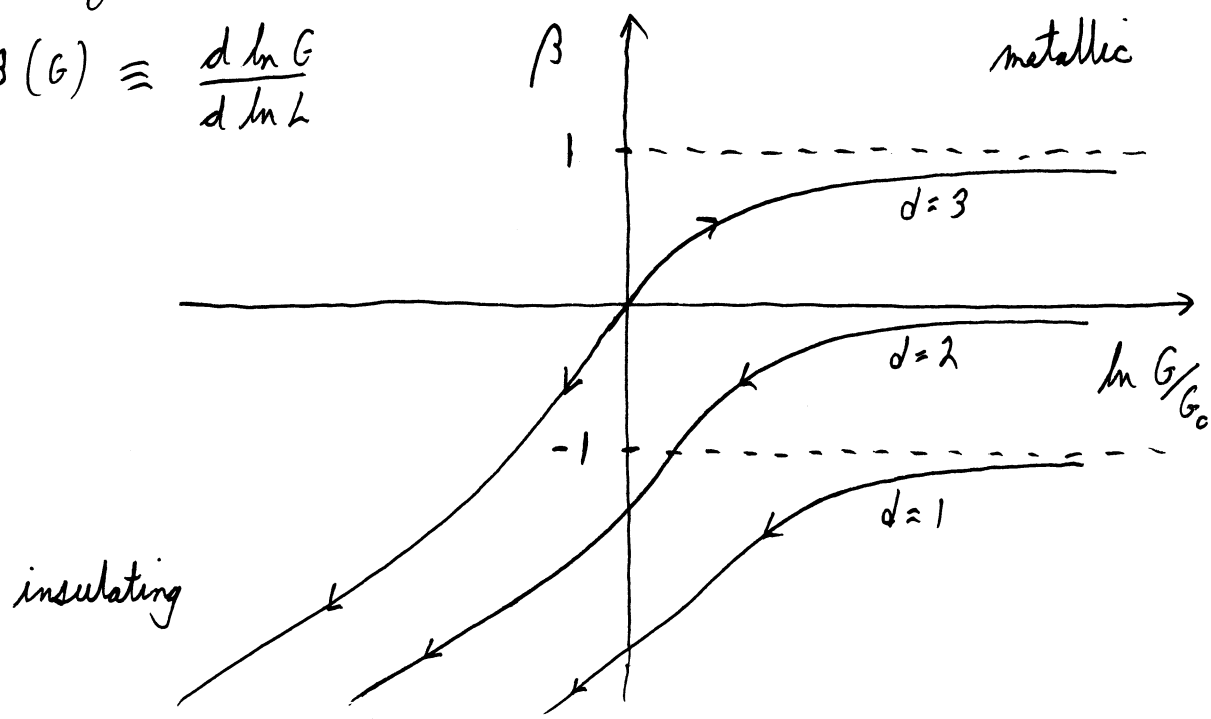
Opposite limit of strong disorder: electron states are exponentially localized $G \propto e^{-L/\xi}$

$\xi$ = localization length

$$\frac{d \ln G}{d \ln L} = \ln \frac{G}{G_c}$$

Scaling hypothesis: $G(L)$ depends only on $G$ itself.

$$\beta(G) \equiv \frac{d \ln G}{d \ln L}$$



Consequences of scaling theory:

All states are localized in $d=1,2$ !!

There is a critical value of $G$ in $d=3$, called $G_c$.

If $G > G_c$, system is metallic as $L \to \infty$.

If $G < G_c$, system is insulating as $L \to \infty$.

Q: Why do we observe metallic behavior in $d=1,2$?

A: If $L \ll \xi$, then effects of localization are very weak.

Other strange phenomena in lower dimensions:

$d=1$, $V=0$, arbitrary $U \neq 0$: Luttinger liquid

Ground state of system is not a Fermi liquid

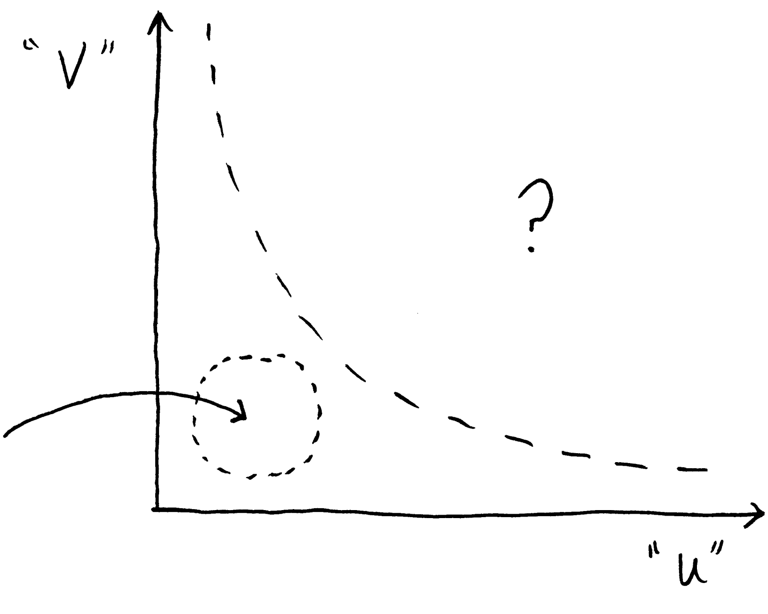
$G(V,T)$ obeys power laws as $V,T \to 0$

(talks by Hubert Saleur later this week)

The bottom line:

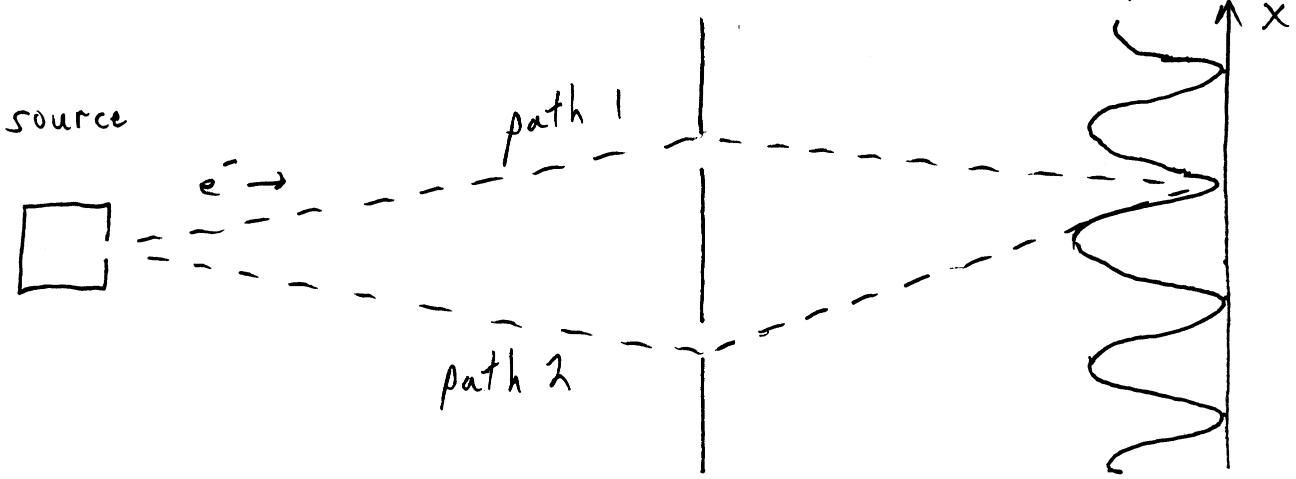
Everything to the right of the dashed line is ill-understood!

My lectures will focus on the regime of weak disorder and weak interactions.



# II. Quantum Interference and Electronic Transport

Background: The Aharonov-Bohm effect in the electron 2-slit diffraction experiment

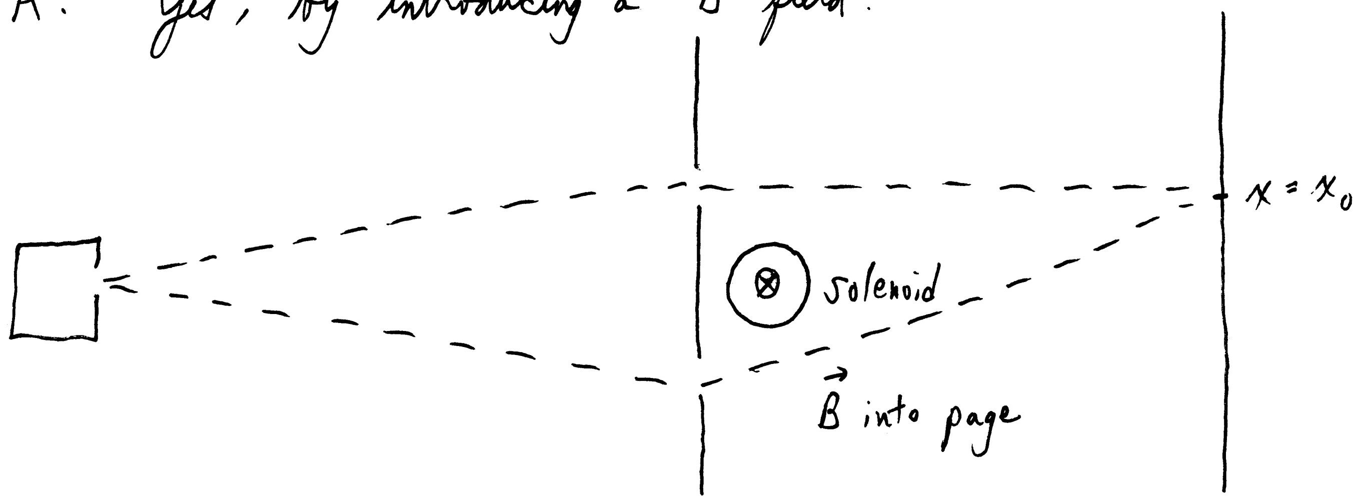



probability amplitude $A(x) = A_1 + A_2$ $\qquad A_1 = |A_1| e^{i\varphi_1}$

$$P(x) = |A(x)|^2 = |A_1|^2 + |A_2|^2 + |A_1 A_2| \cos(\varphi_1 - \varphi_2)$$

Q: Is it possible to observe the interference without moving the observation point, $x$? (i.e. use one detector)

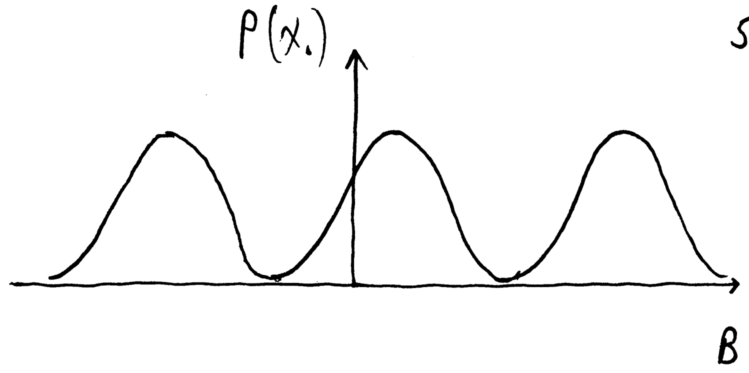
A: Yes, by introducing a $\vec{B}$ field.




$$\varphi = \frac{1}{\hbar}\int \left(\vec{p} - \frac{e}{c}\vec{A}\right)\cdot d\vec{r} \qquad\qquad \Phi_0 = \begin{cases} \frac{hc}{e} & \text{c.g.s.} \\ \frac{h}{e} & \text{S.I.} \end{cases}$$

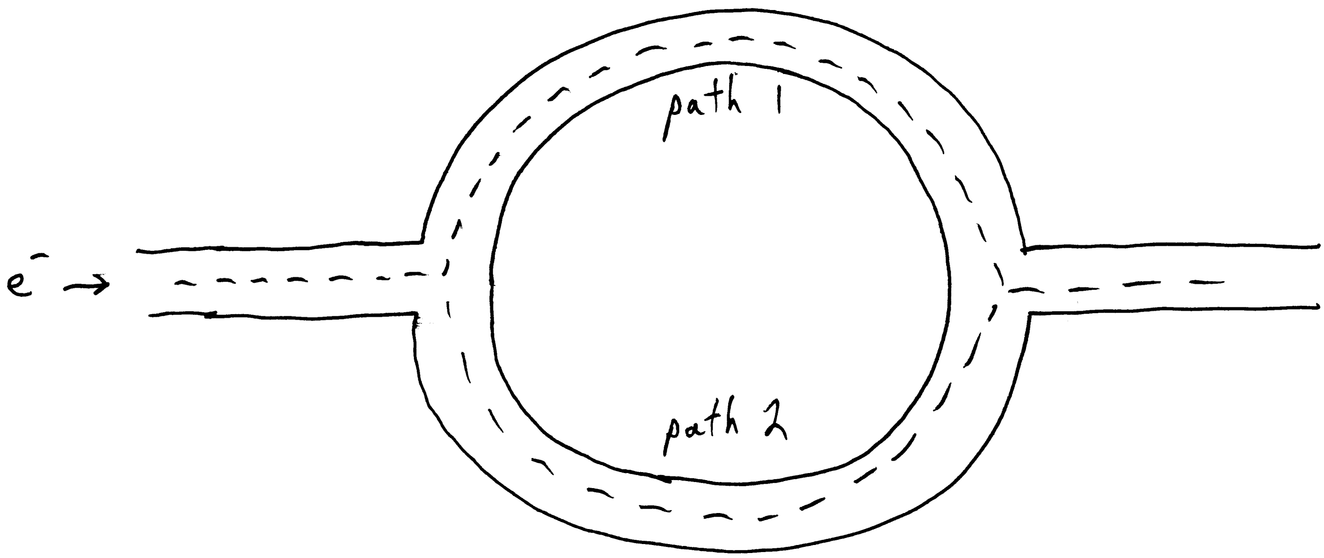
$$\varphi_1 - \varphi_2 = \frac{e}{\hbar c}\oint \vec{A}\cdot d\vec{r} = \frac{e}{\hbar c}\int \vec{B}\cdot d\vec{a} = 2\pi\frac{\Phi}{\Phi_0}$$

Measure intensity at $x = x_0$ while varying $\vec{B}$:




Since it is difficult to make microscopic solenoids, why is this useful?

Imagine performing a similar experiment inside a metal rather than in vacuum:




The only quantity you can measure is $R = G^{-1}$.

Q1: If there is interference, how would you know?

A: By applying $\vec{B}$, and observing the AB effect.

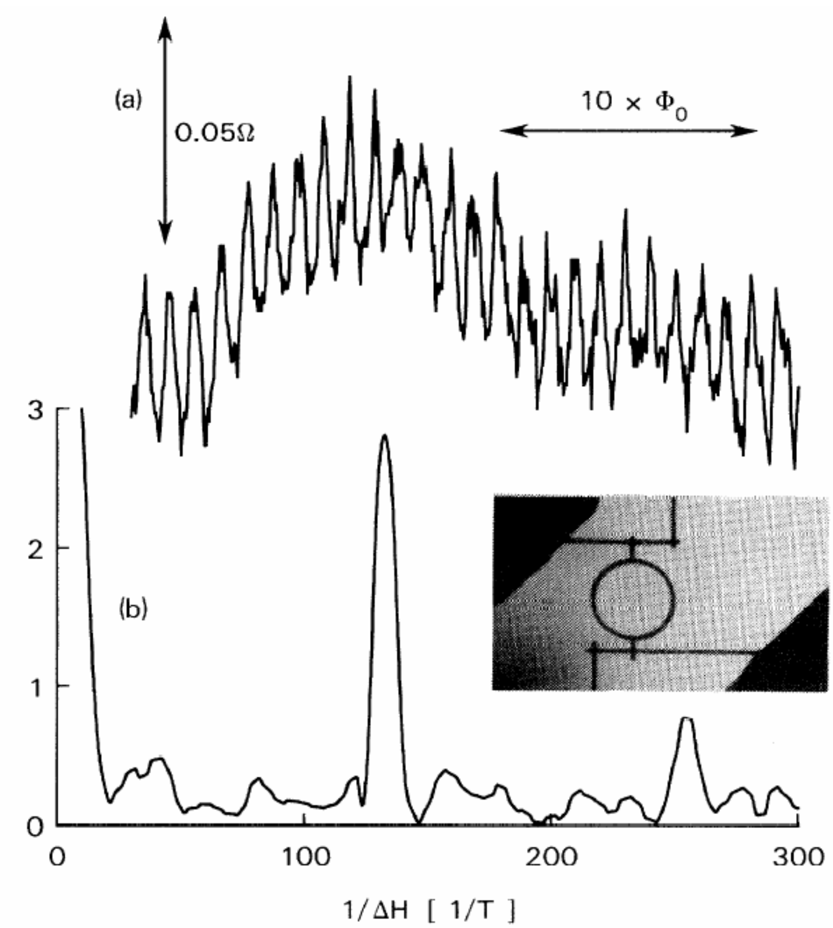
Q2: Is it really possible to see this effect in a metal, where the electrons undergo much scattering from surfaces, grain boundaries, crystalline defects, and impurities?

$\rightarrow$ Show Webb AB data on overhead projector

# Observation of h/e Aharonov-Bohm Effect in Normal Metal Rings

Webb, Washburn, Umbach, and Laibowitz,

Phys. Rev. Lett. 54, 2696 (1985)



Features in data:

1. AB oscillations are clearly visible $\qquad \Delta B = \frac{\Phi_0}{\pi r^2}$
2. Amplitude & phase vary over larger $B$ scale
3. $\delta G_{h/e} \approx e^2/h$
4. Experiments were performed at very low $T$, with $r \approx 0.4\,\mu m$

## Explanations

1. Electron scattering at low $T$ is predominantly elastic.

   Elastic scattering makes the wavefunctions look funny (they are no longer plane waves), but it does not destroy phase coherence.

4. Inelastic scattering, e.g. electron-phonon & electron-electron scattering, does destroy phase coherence.

Introduce length scales: $\qquad L$ = sample dimension

$l_e$ = elastic mean free path $\qquad l_e = v_F \tau_e$

$L < l_e \rightarrow$ ballistic transport, achievable in semiconductors but rarely in metals

$L > l_e \rightarrow$ diffusive transport

diffusion constant $D = \frac{1}{d} v_F l_e$

$L_\phi$ = inelastic or phase-breaking length $\qquad L_\phi = \sqrt{D \tau_\phi}$

$L < L_\phi \rightarrow$ phase-coherent transport

$L \gg L_\phi \rightarrow$ classical transport w/ small quantum corrections

Webb's AB experiment: $l_e \ll L < L_\phi \ll \xi$

diffusive regime  phase coherent  metallic

Other features in data:

3. $\delta G \approx e^2/h \to$ later

2. Amplitude and phase vary randomly over larger $B$ scale

The ring is not strictly 1-dimensional

electrons can take several trajectories


These paths interfere with each other, also. Changing $\vec{B}$ alters their relative phase, hence the interference pattern and the conductance.

## Universal Conductance Fluctuations (UCF)

$$G \sim \left| \sum_{\text{paths } \alpha} e^{-i S_\alpha/\hbar} \right|^2 \qquad S_\alpha = \int_\alpha \left(\vec{p} - \frac{e}{c}\vec{A}\right)\cdot d\vec{r}$$

interference terms $e^{-i(S_\alpha - S_\beta)/\hbar} = e^{-2\pi i \, \Phi_{\alpha\beta}/\Phi_0}$

contain flux enclosed between paths

$$\Phi_{\alpha\beta} \approx \begin{cases} wLB, & L < L_\phi \\ wL_\phi B, & L > L_\phi \end{cases}$$

Show Webb UCF data on overhead projector

# Observation of Universal Conductance Fluctuations in Normal Metal Rings

Umbach, Washburn, Laibowitz, and Webb,

Phys. Rev. B 30, 4048 (1984)

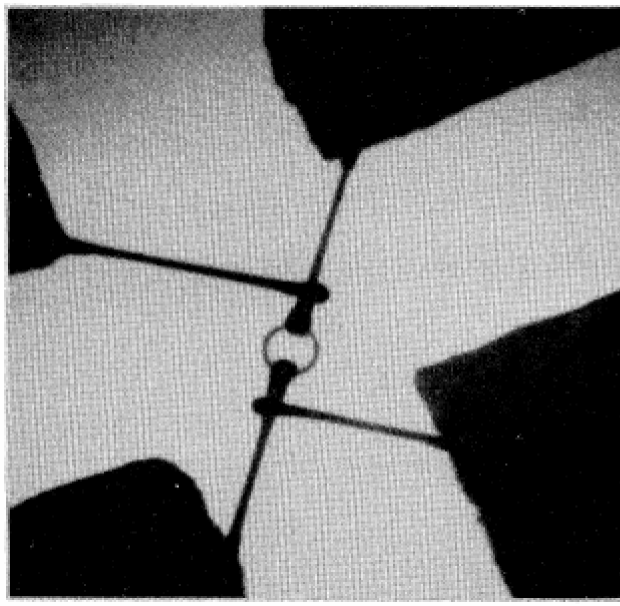
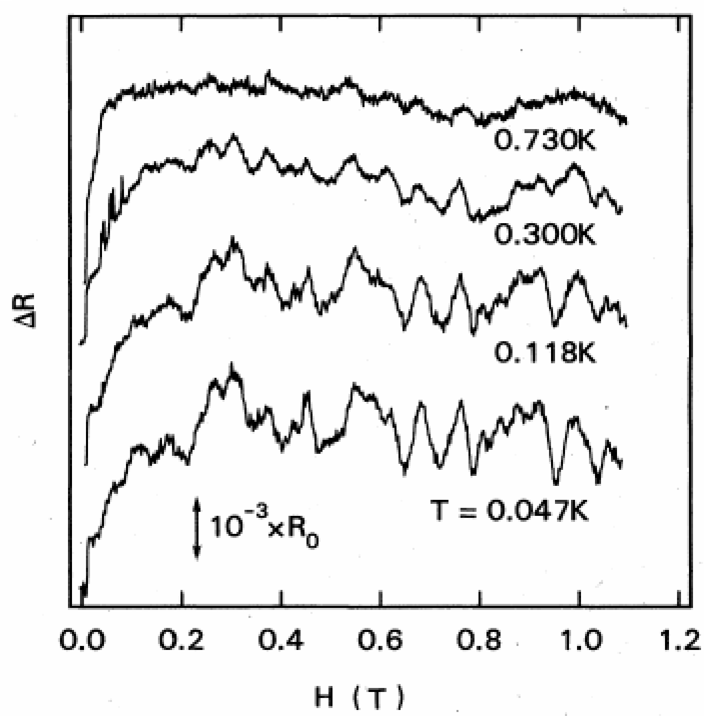


FIG. 1. Transmission electron micrograph of one of the Au rings measured in this experiment. The inside diameter of the ring was 280 nm and the width of the lines forming the ring was roughly 45 nm.



<u>Properties of UCF</u>

i) no periodicity
correlation field $B_c \approx \frac{\Phi_0}{WL}$

<u>Properties of $h/e$ AB oscillations</u>

i) periodic $\Delta B = \frac{\Phi_0}{\pi r^2}$

<u>Properties of both:</u>

ii) persist to large $B$

iii) $\delta G_{UCF}$, $\delta G_{h/e} \approx e^2/h$ for $L < L_\phi$

iv) For $k_B T > E_{Th}$ where $E_{Th} = \frac{\hbar}{\tau} = \frac{\hbar D}{L^2}$

$$\delta G(T) = \delta G(T=0) \cdot \sqrt{\frac{E_{Th}}{k_B T}} = \frac{L_T}{L}\,\delta G(T=0)$$

$$L_T \equiv \sqrt{\frac{\hbar D}{k_B T}} = \text{“thermal length”}$$ energy averaging

v) For $L > L_\phi$, signals disappear by classical averaging:

$$\left(\frac{\delta G_N}{G_N}\right)^2 = \frac{1}{N}\left(\frac{\delta G}{G}\right)_{L_\phi}$$

UCF: $N = \left(\frac{L}{L_\phi}\right)^d$ = number of “coherence boxes”

$h/e$ AB: $N$ = number of rings in series or parallel

UCF and $h/e$ AB are “mesoscopic” effects:

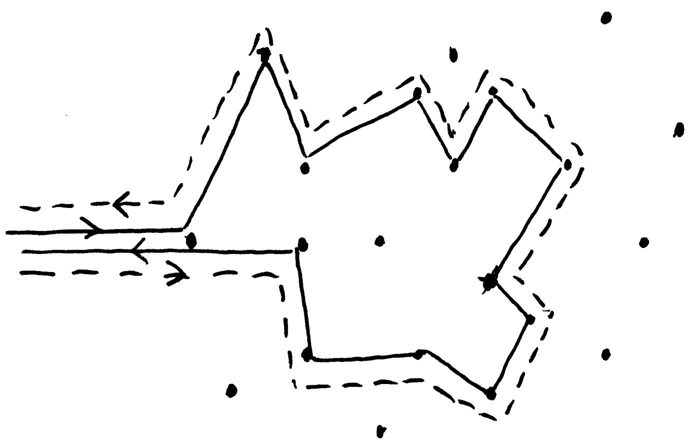
a) details are sample-specific
b) they are most visible when $L \lesssim L_\phi$
c) they do <u>not</u> survive ensemble averaging

UCF explain random amplitude + phase of $h/e$ AB oscillations

# Weak Localization

For UCF, we considered all paths traversing the sample.

Now consider only those paths that return to the origin: for example, this one:




Its time-reversed cousin interferes constructively with it.

$$\Rightarrow \quad P_{return} = \Big| \sum_{\alpha} A_{\alpha} \Big|^2 = \Big| \sum_{\substack{pairs \\ j}} \left( A^j_{cw} + A^j_{ccw} \right) \Big|^2$$

sum over paths that return to origin

$A^j_{cw} = A^j_{ccw}$ constructive interference

If there are $N$ such pairs of paths, then

$$P_{return} = \sum_{j=1}^{N} |2A_j|^2 = \sum_{j=1}^{N} 4|A_j|^2 \sim 4N|A|^2$$

If all $2N$ paths added incoherently, we would have

$$P_{return} = \Big| \sum_{j=1}^{2N} A_j \Big|^2 = \sum_{j=1}^{2N} |A_j|^2 \sim 2N|A|^2$$

$\Rightarrow$ the return probability is twice the classical prediction

# Quantitative estimate of weak localization
(see lectures by Gilles Montambaux)

We need to know the fraction of paths that return to the origin. The answer is:

$$\Delta\sigma_{WL} = \frac{-2e^2}{\pi\hbar} D \int_{\tau_e}^{\tau_\phi} dt \; w(t)$$

where $w(t) = \dfrac{1}{(4\pi D t)^{d/2}}$ is the classical return probability

Do the integral:

$$\Delta\sigma_{WL} = \begin{cases} -\dfrac{2e^2}{\pi^{3/2}\hbar}\left(L_\phi - l_e\right) & d=1 \\[2ex] -\dfrac{e^2}{\pi^2\hbar} \ln \dfrac{L_\phi}{l_e} & d=2 \\[2ex] \dfrac{-e^2}{2\pi^{5/2}\hbar}\left(\dfrac{1}{l_e} - \dfrac{1}{L_\phi}\right) & d=3 \end{cases}$$

where $L_\phi = \sqrt{D\tau_\phi}$

These results can also be obtained from the scaling theory of localization in the weak-disorder limit, with

$\beta(g) = d - 2 - \dfrac{a}{g} \qquad a = \dfrac{1}{\pi^2}$

# Experimental Observation of weak localization:

1. T - dependence

If $\frac{1}{\tau_\phi} \propto T^p$, then $L_\phi \propto T^{-p/2}$

$\Delta\sigma_{WL}(T) \propto -T^{-p/2}$ $d=1$

Problem: e-e interactions also contribute to $\Delta\sigma(T)$

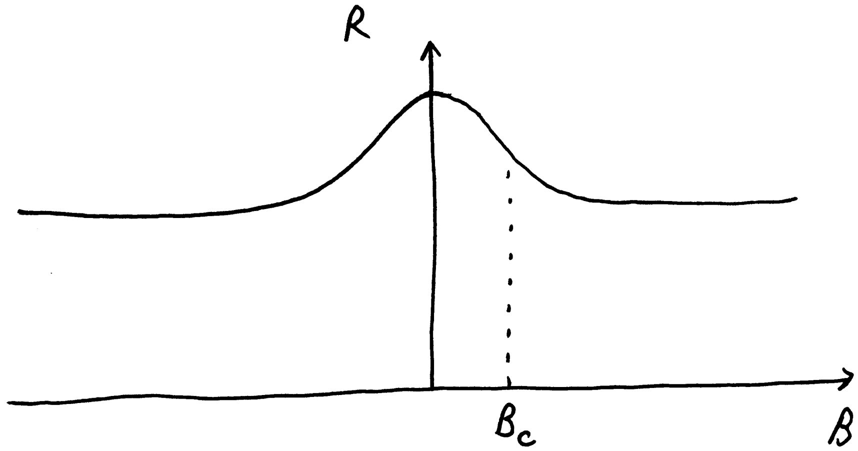
2. Magnetoresistance (MR)

i) $\vec{B}$-field changes relative phase of CW and CCW paths

ii) Average over all return paths enclosing different areas

$\Rightarrow$ WL enhancement is suppressed

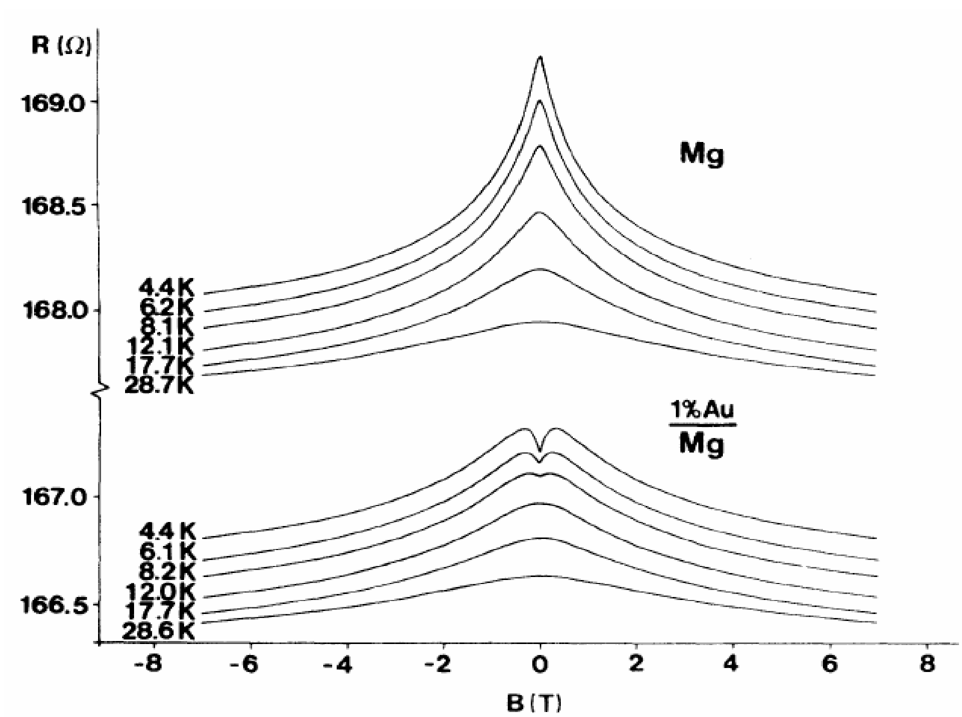
Expect negative MR

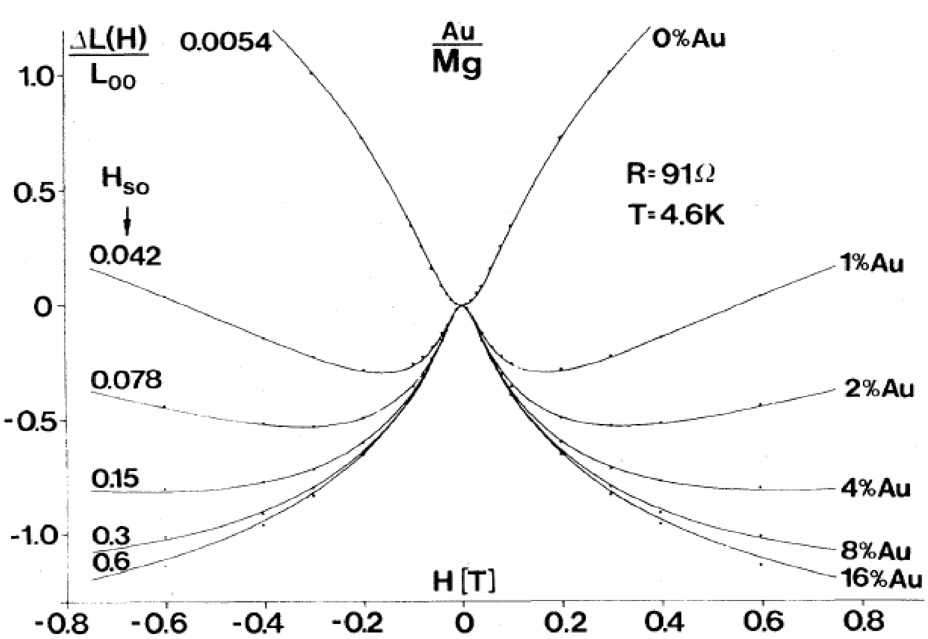



$$B_c = \begin{cases} \Phi_0 / L_\phi^2 & d=2,3 \\ \Phi_0 / wL_\phi & d=1 \end{cases}$$

# Effect of spin-orbit scattering on weak localization magnetoresistance of 2D films

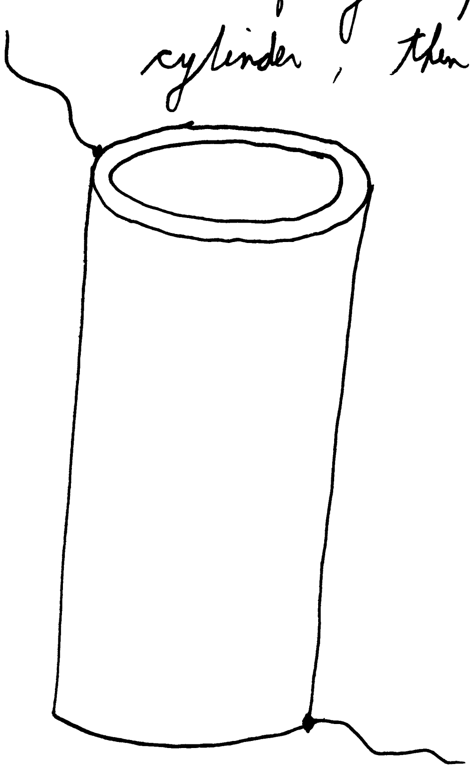
G. Bergmann, Phys. Rev. B 28, 2914 (1983)
& Phys. Rev. Lett. 48, 1046 (1982).



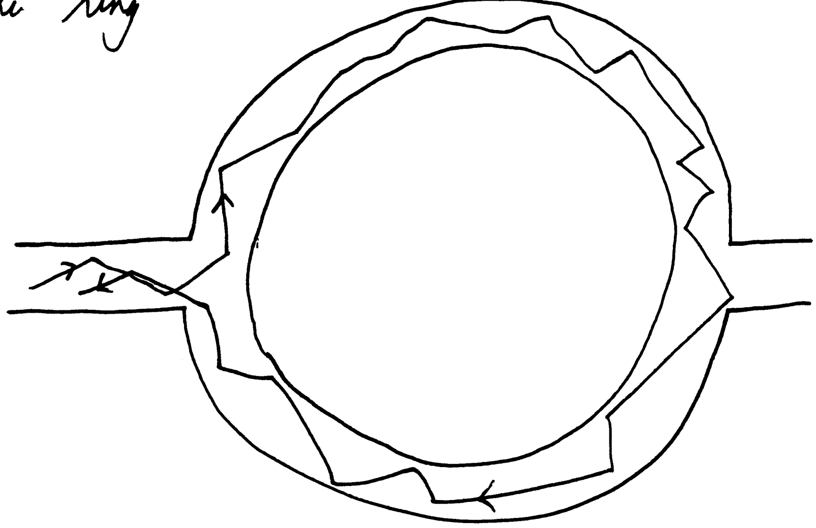
A second look at the Aharonov-Bohm effect:

If you put many rings together, or measure a cylinder, then the $h/e$ AB oscillations average to zero.



But time-reversed paths going all the way around the ring CW + CCW still interfere constructively.

They enclose twice the flux in the ring



$$\varphi_1 - \varphi_2 = 4\pi \frac{\Phi}{\Phi_0}$$

flux periodicity is $\frac{1}{2}\Phi_0 = \frac{h}{2e}$

These $\frac{h}{2e}$ AB oscillations were predicted and observed before the $h/e$ oscillations.

theory: Altshuler, Aronov + Spivak
expt: Sharvin + Sharvin

## Properties of WL MR and $h/2e$ AB oscillations

$$WL: \quad B \lesssim B_c \approx \begin{cases} \Phi/L_\phi^2 & 2D \\ \Phi_0/wL_\phi & 1D \end{cases}$$

i) observable at small $B$ only

$$h/2e \text{ AB}: \quad B \lesssim \frac{\Phi_0}{2\pi r w}$$

ii) $\delta G_{WL}, \ \delta G_{h/2e} \approx e^2/h$

iii) For $kT > E_{Th}$, no suppression by energy averaging (because time-reversed paths always interfere constructively)

iv) survive ensemble average $\Rightarrow$ observable in macroscopic samples (except $4\pi r \gtrsim L_\phi$ for $h/2e$ AB oscillations)