

Introduction to Theory of Mesoscopic Systems

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Lecture 5

Beforehand

Anderson Localization,
Mesoscopic Fluctuations,
Random Matrices, and
Quantum Chaos

Yesterday

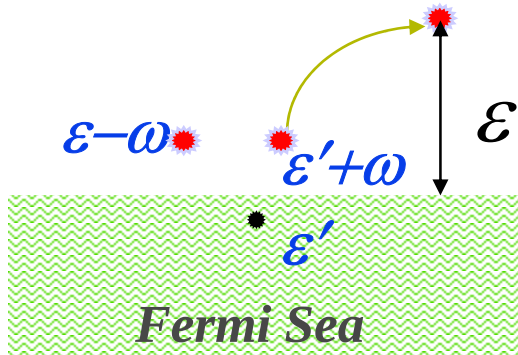
Interaction between electrons
in mesoscopic systems –
Universal Hamiltonian, 0D
Fermi liquid

Today

Continue.
Inelastic relaxation rates.

Dephasing, decoherence,...
What does it mean?

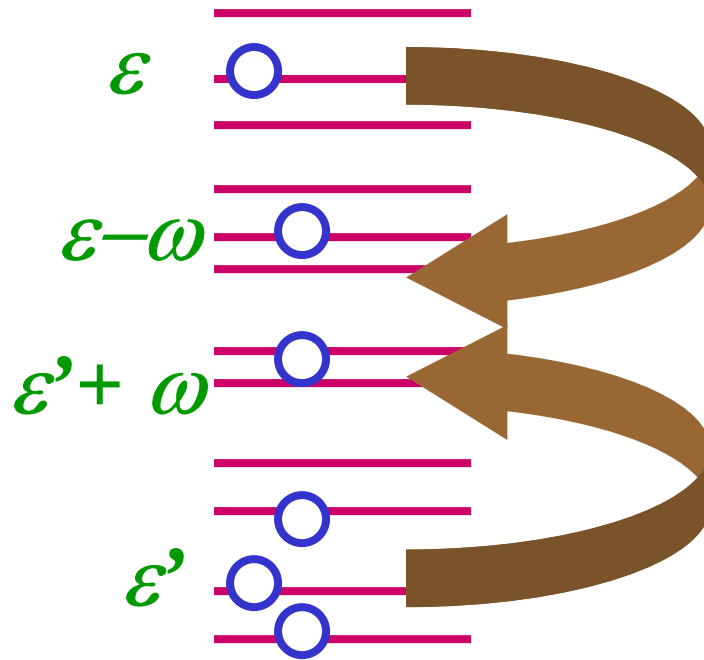
Quasiparticle decay rate at $T = 0$ in a **clean** Fermi Liquid.



	ϵ^2 / ϵ_F	$d = 3$
$\frac{\hbar}{\tau_{e-e}(\epsilon)}$	$\propto (\epsilon^2 / \epsilon_F) \log(\epsilon_F / \epsilon)$	$d = 2$
	ϵ	$d = 1$

Conclusions:

1. For $d=3,2$ from $\epsilon \ll \epsilon_F$ it follows that $\epsilon \tau_{e-e} \gg \hbar$, i.e., that the **quasiparticles** are well determined and the Fermi-liquid approach is applicable.
 2. For $d=1$ $\epsilon \tau_{e-e}$ is of the order of $\hbar$, i.e., that the Fermi-liquid approach is not valid for **1d** systems of interacting fermions.
- Luttinger liquids**



Offdiagonal
matrix
element

$$M(\omega, \varepsilon, \varepsilon') \sim \frac{\delta_1}{g} \ll \delta_1$$

Fermi
Golden
Rule

$$\frac{h}{\tau_{e-e}(\varepsilon)} \propto \sum_{0 < \omega < \varepsilon} \sum_{-\omega < \varepsilon' < 0} \frac{|M(\omega, \varepsilon, \varepsilon')|^2}{\delta_1}$$

0D case: $L < L_\varepsilon$, i.e., $\varepsilon < E_T$

$$L_\varepsilon = \sqrt{\frac{hD}{\varepsilon}}$$

• $M \propto \delta_1(L)/g$

• Each $\sum$ gives $\approx \frac{\varepsilon}{\delta_1}$

$$\frac{h}{\tau_{e-e}(\varepsilon)} \propto \delta_1 \left(\frac{\varepsilon}{E_T} \right)^2$$

(U.Sivan, Y.Imry & A.Aronov, 1994)

Quasiparticle relaxation rate in disordered conductors

$T=0$

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Golden
Rule**

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$d > 0$ case: $L > L_\varepsilon$, i.e., $\varepsilon > E_T$

$$\frac{h}{\tau_{e-e}(\varepsilon)} \propto \delta_1(L_\varepsilon)$$

At $L \approx L_\varepsilon$ the rate is of the order of the mean level spacing δ_1 . It should not change, when we keep increasing the system size, i.e. decreasing the Thouless energy

Quasiparticle relaxation rate in disordered conductors

$T=0$

**Fermi
Golden
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$d > 0$ case: $L > L_\varepsilon$, i.e., $\varepsilon > E_T$

$$\frac{h}{\tau_{e-e}(\varepsilon)} \propto \delta_1(L_\varepsilon)$$

$$\frac{h}{\tau_{e-e}(\varepsilon)} \propto \frac{\varepsilon}{g(L_\varepsilon)} \ll \varepsilon$$

A. Schmid 1973

B.A. & A.Aronov 1979

Matrix elements at large, $\varepsilon, \omega \gg E_T$, energies

$$\frac{h}{\tau_{e-e}(\varepsilon)} \propto \sum_{0 < \omega < \varepsilon} \sum_{-\omega < \varepsilon' < 0} \frac{|M(\omega, \varepsilon, \varepsilon')|^2}{\delta_1}$$

$$\frac{h}{\tau_{e-e}(\varepsilon)} \propto \frac{\varepsilon}{g(L_\varepsilon)} \ll \varepsilon$$

$$|M(\omega, \varepsilon, \varepsilon')|^2 \propto \frac{\delta_1(L)^3 \delta_1(L_\omega)}{\omega^2} \propto \frac{\omega^{-2+d/2}}{D^{d/2}}$$

$$\frac{|M(\omega, \varepsilon, \varepsilon')|^2}{\delta_1(L)} \xrightarrow{\omega \rightarrow 0} \infty$$

Quasiparticle relaxation rate in disordered conductors

$T > 0$

$$\frac{h}{\tau_{e-e}(\varepsilon)} \propto \sum_{0 < \omega < \varepsilon} \sum_{-\omega < \varepsilon' < 0} \frac{|M(\omega, \varepsilon, \varepsilon')|^2}{\delta_1}$$

$T = 0$

$$\frac{h}{\tau_{e-e}(\varepsilon, T)} \propto \sum_{\omega} (1 - n_{\varepsilon - \omega}) \sum_{\varepsilon'} n_{\varepsilon' + \omega} (1 - n_{\varepsilon' + \omega}) \frac{|M(\omega, \varepsilon, \varepsilon')|^2}{\delta_1}$$

$T > 0$

$$n_{\varepsilon} = \left[\exp \frac{\varepsilon}{T} - 1 \right]^{-1} \quad \text{Fermi distribution function}$$

$$|M(\omega, \varepsilon, \varepsilon')|^2 \propto \frac{\omega^{-2+d/2}}{D^{d/2}}$$

- a) $T = 0$ -no problems:** $\sum_{\varepsilon'} \propto \omega$ and $\sum_{\omega}$ converges
- b) $T > 0$ -a problem:** $\sum_{\varepsilon'} \propto T$ and $\sum_{\omega}$ diverges !

Abrahams, Anderson, Lee & Ramakrishnan 1981

$T > 0$ - *a problem*: $1/\tau_{e-e}$ *diverges*

$$\frac{h}{\tau_{e-e}(\varepsilon, T)} \propto T \sum_{\omega} \frac{(1 - n_{\varepsilon - \omega})}{\omega^{2-d/2} D^{d/2}}$$

B.A., A.Aronov & D.E. Khmelnitskii (1983):

- Divergence of is not a catastrophe:
 $1/\tau_{e-e}$ has no physical meaning
- E.g., for energy relaxation of hot electrons processes with small energy transfer ω are irrelevant.



$$\frac{h}{\tau_{\varepsilon}} \propto \frac{\varepsilon}{g(L_{\varepsilon})}$$

Q:

Is it the energy relaxation rate that determines the applicability of the Fermi liquid approach

?

$T > 0$ - *a problem*: $1/\tau_{e-e}$ *diverges*

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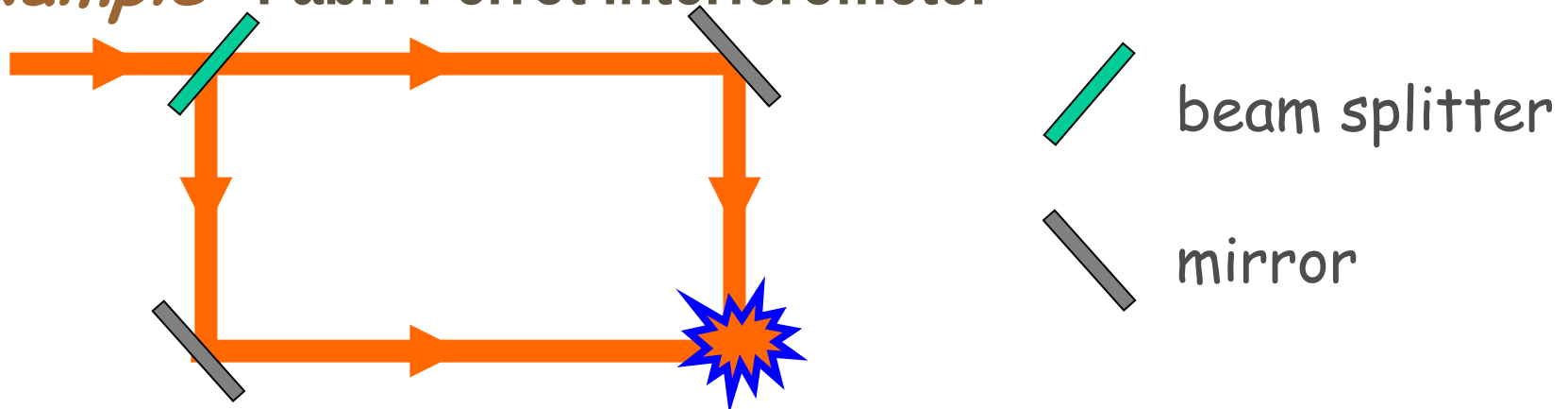
□ Phase relaxation: in a time t after a collision $\delta\varphi \approx (2\pi \omega t) / h \Rightarrow$ processes with energy transfer ω smaller than $1/\tau_{\varphi}$ are irrelevant.

$$\frac{h}{\tau_{\varphi}} \propto ?$$

What is Dephasing?

1. Suppose that originally a system (an electron) was in a **pure** quantum state. It means that it could be described by a **wave function** with a given **phase**.
2. External perturbations can transfer the system to a different quantum state. Such a transition is characterized by its **amplitude**, which has a modulus and a **phase**.
3. The **phase** of the **amplitude** can be measured by comparing it with the **phase** of **another amplitude** of the **same** transition.

Example: Fabri-Perrot interferometer



4. Usually we **can not** control **all** of the perturbations. As a result, even for fixed initial and final states, the **phase** of the **transition amplitude** has a **random** component.
5. We call this contribution to the **phase**, $\delta\varphi$, **random** if it changes from measurement to measurement in an uncontrollable way.
6. It usually also depends on the duration of the experiment, t :

$$\delta\varphi = \delta\varphi(t)$$

7. When the time t is large enough, $\delta\varphi$ exceeds 2π , and interference gets averaged out.

8. Definitions:

$$\delta\varphi(\tau_\varphi) \approx 2\pi$$

τ_φ phase coherence time; $1/\tau_\varphi$ dephasing rate

Why is Dephasing rate important?

Imagine that we need to measure the energy of a quantum system, which interacts with an environment and can exchange energy with it.

Let the typical energy transferred between our system and the environment in time t be $\delta\epsilon(t)$. The total uncertainty of an ideal measurement is

environment

$$\Delta\epsilon(t) \approx \delta\epsilon(t) + \frac{\hbar}{t}$$

quantum uncertainty

$$\left. \begin{array}{l} \delta\epsilon(t) \xrightarrow{t \rightarrow \infty} \infty; \\ \frac{\hbar}{t} \xrightarrow{t \rightarrow 0} \infty \end{array} \right\}$$

There should be an optimal measurement time $t=t^*$, which minimizes $\Delta\epsilon(t)$:

$$\Delta\epsilon(t^*) = \Delta\epsilon_{\min}$$



$$\delta\epsilon(t^*) \approx \frac{\hbar}{t^*} \Rightarrow \delta\phi(t^*) \approx 1 \Rightarrow \begin{array}{l} t^* \approx \tau_\phi \\ \Delta\epsilon_{\min} \approx \hbar/\tau_\phi \end{array}$$



Why is Dephasing rate important?

$$\delta\varepsilon(t^*) \approx \frac{\hbar}{t^*} \Rightarrow \delta\phi(t^*) \approx 1 \Rightarrow \begin{matrix} t^* \approx \tau_\phi \\ \Delta\varepsilon_{\min} \approx \hbar/\tau_\phi \end{matrix}$$

It is dephasing rate that determines the accuracy at which the energy of the quantum state can be measured in principle.

$T > 0$ - *a problem*: $1/\tau_{e-e}$ *diverges*

$$\frac{h}{\tau_{e-e}(\varepsilon, T)} \propto T \sum_{\omega} \frac{(1 - n_{\varepsilon - \omega})}{\omega^{2-d/2} D^{d/2}}$$

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- Divergence of is not a catastrophe: $1/\tau_{e-e}$ has no physical meaning
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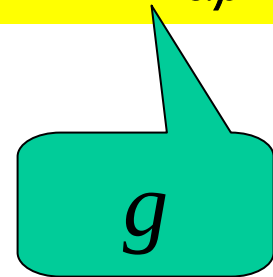
e-e interaction – Electric noise

Fluctuation- dissipation theorem:

Electric noise - **randomly** time and space - dependent electric field $E^\alpha(\vec{r}, t) \Leftrightarrow E^\alpha(\vec{k}, \omega)$
Correlation function of this field is completely determined by the conductivity $\sigma(\vec{k}, \omega)$:

$$\langle \mathbf{E}^\alpha \mathbf{E}^\beta \rangle_{\omega, \vec{k}} = \frac{\omega}{\sigma_{\alpha\beta}(\omega, \vec{k})} \coth\left(\frac{\omega}{2T}\right) \frac{k_\alpha k_\beta}{k^2} \propto \frac{T}{\sigma_{\alpha\beta}(\omega, \vec{k})}$$

Noise intensity **increases** with the temperature, T , and with resistance



$$\left\langle \mathbf{E}^\alpha \mathbf{E}^\beta \right\rangle_{\omega, \vec{k}} = \frac{\omega}{\sigma_{\alpha\beta}(\omega, \vec{k})} \coth \left(\frac{\omega}{2T} \right) \frac{k_\alpha k_\beta}{k^2} \propto \frac{T}{\sigma_{\alpha\beta}(\omega, \vec{k})}$$

$$g(L) \equiv \frac{h}{e^2 R(L)} \quad - \text{Thouless conductance} - \text{def.}$$

$R(L)$ - resistance of the sample with $\left\{ \begin{array}{l} \text{length (1d)} \\ \text{area (2d)} \end{array} \right\} L$

$$\frac{1}{\tau_\varphi} \propto \frac{T}{g(L_\varphi)}$$

$$L_\varphi \equiv \sqrt{D\tau_\varphi} \quad - \text{dephasing length}$$

D - diffusion constant of the electrons

$$\frac{1}{\tau_{\varphi}} \propto \frac{T}{g(L_{\varphi})}$$

This is an equation!

$$\frac{1}{\tau_{\varphi}} \propto \frac{T}{g(L_{\varphi})}$$

This is an equation!

$$g(L) \propto L^{d-2}$$

$$L_{\varphi} \propto \sqrt{\tau_{\varphi}}$$

*where d is the number of dimensions:
 $d=1$ for wires; **$d=2$** for films, ...*



$$L_{\varphi} \propto T^{-1/(4-d)}$$

$$\tau_{\varphi} \propto T^{-2/(4-d)} \propto \begin{matrix} T^{-1} & d = 2 \\ T^{-2/3} & d = 1 \end{matrix}$$

$$\frac{1}{\tau_{\varphi}} \propto \frac{T}{g(L_{\varphi})} \quad L_{\varphi} \equiv \sqrt{D\tau_{\varphi}}$$

Fermi liquid is valid (one particle excitations are well defined), provided that

$$T\tau_{\varphi}(T) > \hbar$$

$$\frac{1}{\tau_{\varphi}} \propto \frac{T}{g(L_{\varphi})} \quad L_{\varphi} \equiv \sqrt{D\tau_{\varphi}}$$

Fermi liquid is valid (one particle excitations are well defined), provided that

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1. In a purely **1d** chain, $g \leq 1$, and, therefore, Fermi liquid theory is never valid.

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Fermi liquid is valid (one particle excitations are well defined), provided that

$$T\tau_{\varphi}(T) > \hbar$$

1. In a purely **1d** chain, $g \leq 1$, and, therefore, Fermi liquid theory is never valid.
2. In a multichannel wire $g(L_{\varphi}) > 1$, provided that L_{φ} is **smaller** than the **localization length**, and Fermi liquid approach is justified

How to detect phase coherence measure the dephasing/decoherence rate ?

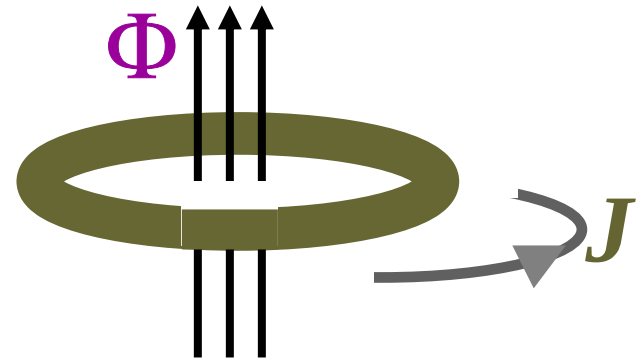
Quantum phenomena in electronic systems:

- Weak localization **yes**
- Mesoscopic fluctuations **yes**
- Persistent current **no**
-
-

Persistent current at zero temperature is a property of the ground state!

$$J \propto \frac{\partial E}{\partial \Phi}$$

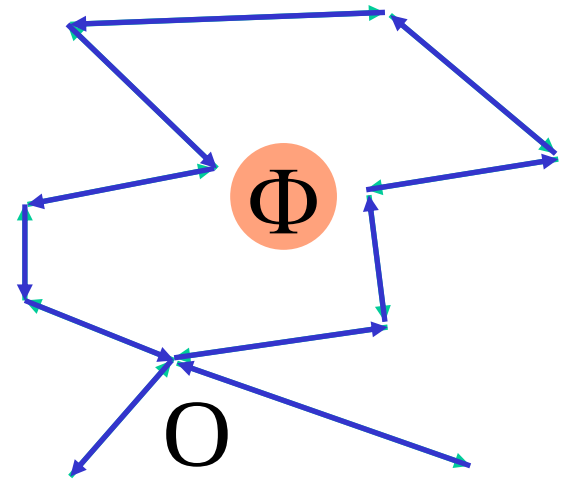
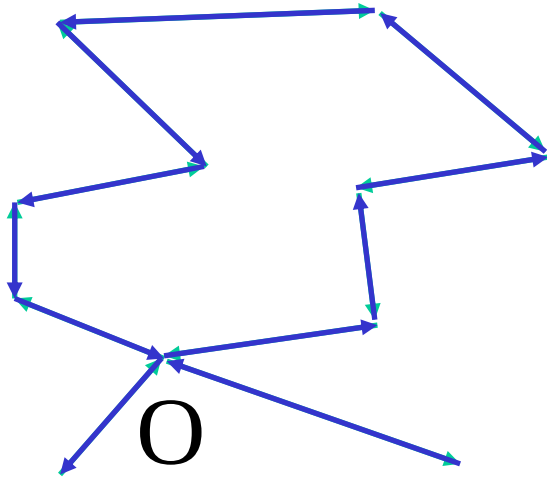
E is the ground state energy



Interaction between electrons can change both E and J , but this does not mean that there is dephasing of the ground state wave function.

Measurements of the persistent current as well as of other **thermodynamic** properties **do not** allow to extract the dephasing rate. **Only transport measurements**

Magnetoresistance



No magnetic field

$$\varphi_1 = \varphi_2$$

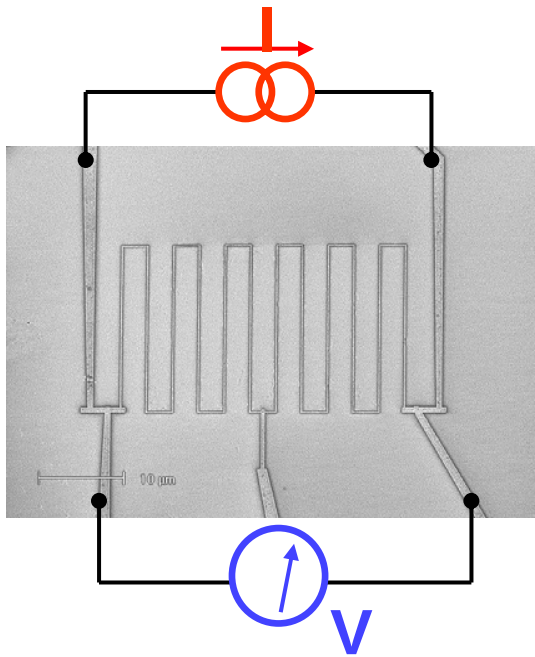
With magnetic field H

$$\varphi_1 - \varphi_2 = 2 * 2\pi \Phi / \Phi_0$$

$\Phi = HS$ - *magnetic flux through the loop*

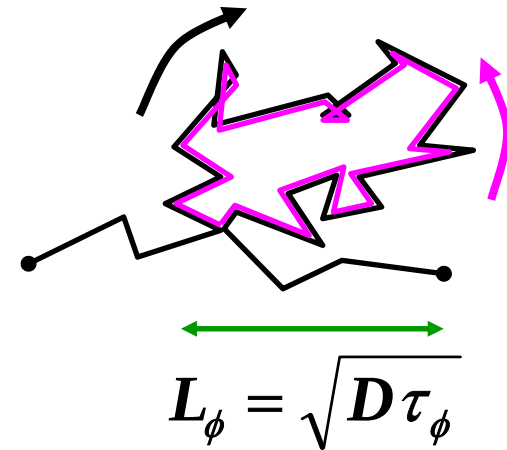
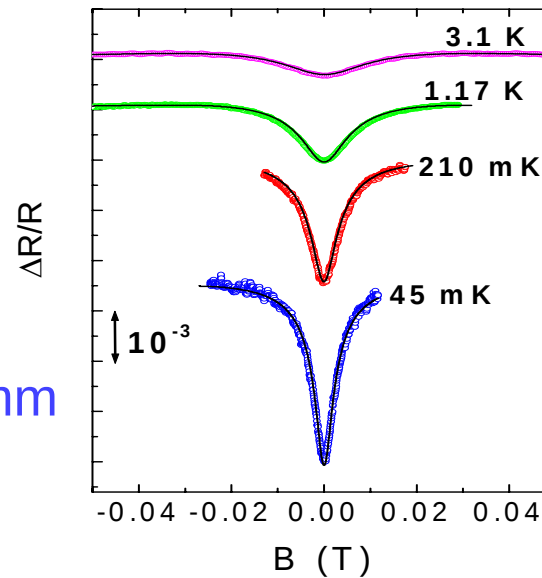
$\Phi_0 = hc/e$ - *flux quantum*

Weak Localization, Magnetoresistance in Metallic Wires



$\odot B$

$L \sim 0.25 \text{ mm}$



1d case; strong spin-orbital coupling

$$\frac{\Delta R}{R} = -\frac{h}{e^2 L} R \sqrt{\frac{1}{L_\phi^2} + \frac{1}{3} \left(\frac{A}{L_H} \right)^2}$$

$$L_H = \sqrt{\frac{h}{eH}}$$

A – area of the wire cross-section

Can we always reliably extract the inelastic dephasing rate from the experiment



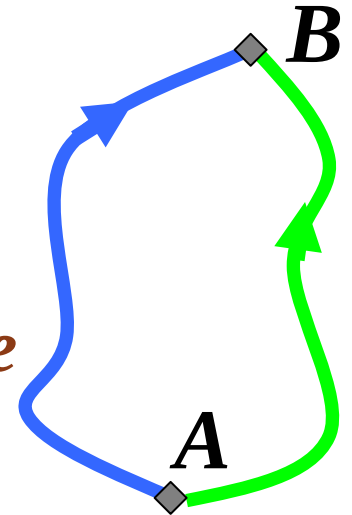
Weak
localization:

NO - *everything that violates T -invariance will destroy the constructive interference*

EXAMPLE: *random quenched magnetic field*

Mesoscopic
fluctuations:

YES - *Even strong magnetic field will not eliminate these fluctuations. It will only reduce their amplitude by factor 2.*



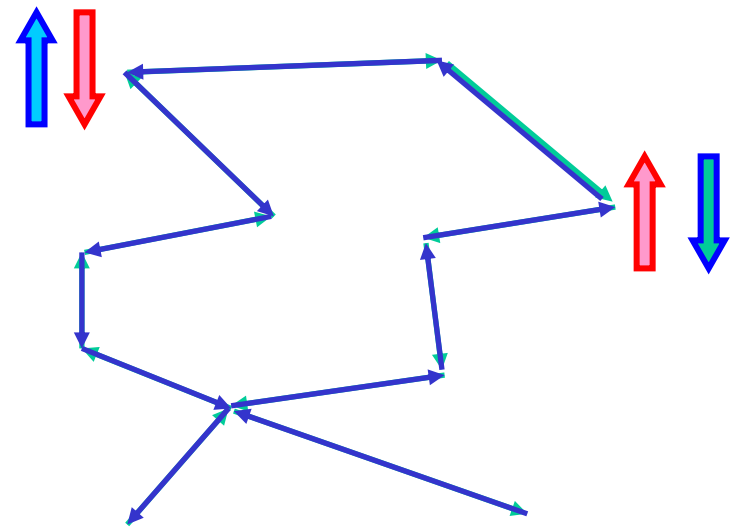
But

Slow diffusion of the impurities will look as dephasing in mesoscopic fluctuations measurements

Magnetic Impurities

↑ - before ↑ - after

T-invariance is clearly violated,
therefore we have dephasing



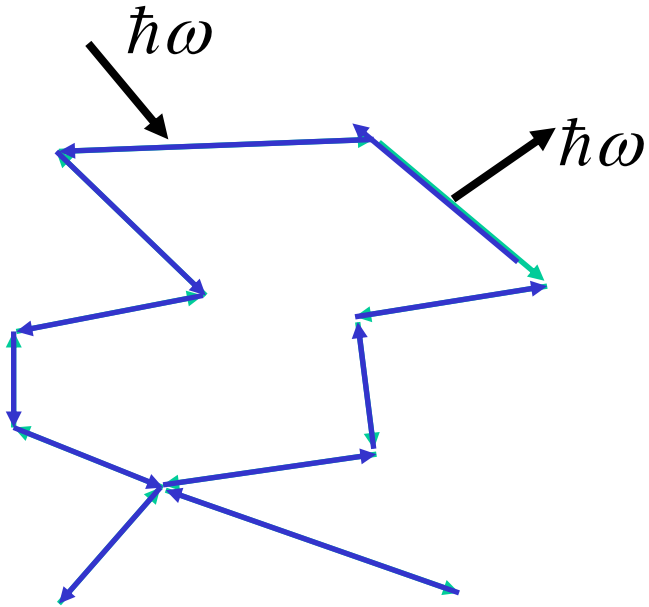
Mesoscopic fluctuations

Magnetic impurities cause dephasing only through effective interaction between the electrons.

$T \rightarrow 0$ *Either Kondo scattering or quenching due to the RKKY exchange.*

In both cases no “elastic dephasing”

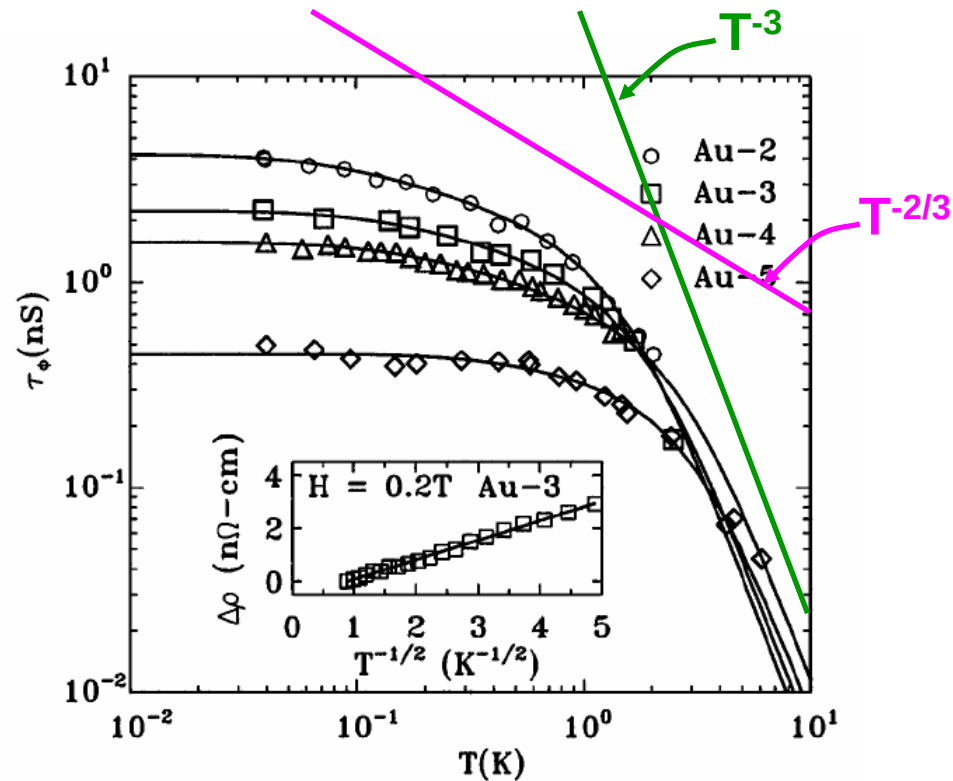
Inelastic dephasing rate $1/\tau_\phi$ can be separated at least in principle



- other electrons
- phonons
- magnons
- two level systems
-
-

THE EXPERIMENTAL CONTROVERSY

Mohanty, Jariwala and Webb, PRL 78, 3366 (1997)

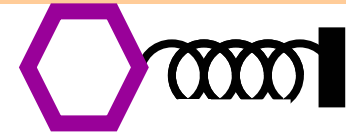


Saturation of τ_ϕ :

Artifact of measurement ?
Real effect in samples ?

Zero-point Oscillations

Collision between the quantum particle and a harmonic oscillator



$\mathcal{E}$ - energy counted from the Fermi level

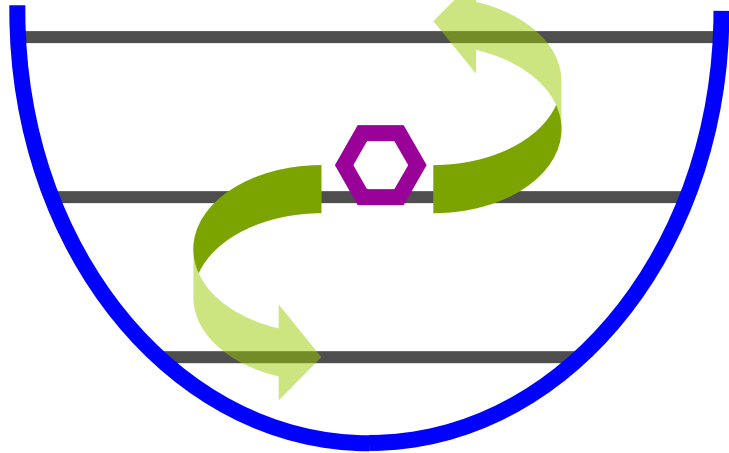
$$E_n = \hbar \omega \left(n + \frac{1}{2} \right)$$

1. $T > \omega$



$\mathcal{E} > \omega; n > 0$

The particle and the oscillator **can** exchange energy



$n=2$

$n=1$

$n=0$

Inelastic scattering



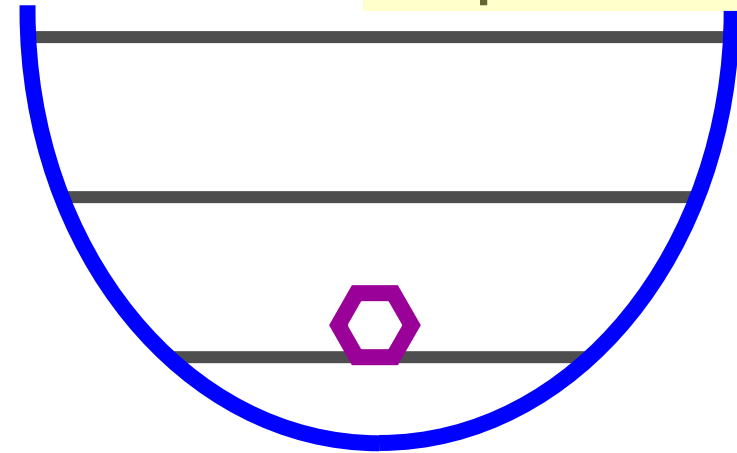
dephasing

2. $T \ll \omega$



$\mathcal{E} \ll \omega; n = 0$

No energy exchange between the oscillator and the particle



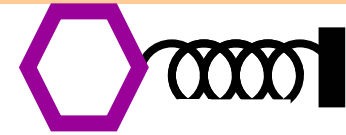
Pure elastic scattering



No dephasing

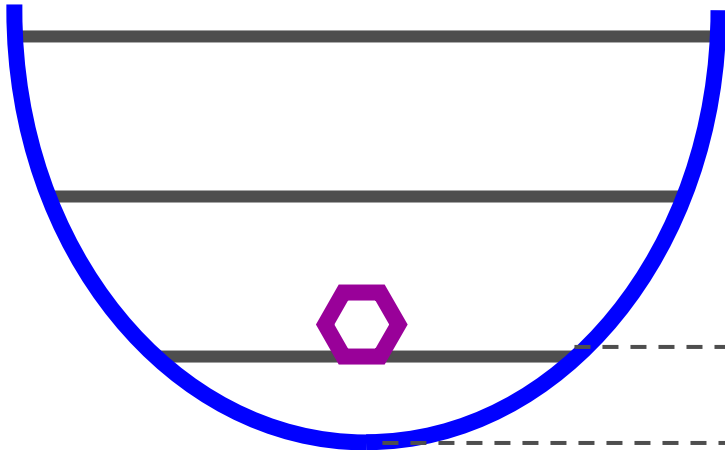
Zero-point Oscillations

Collision between the quantum particle and a harmonic oscillator



$\mathcal{E}$ - energy counted
from the Fermi level

$$E_n = \hbar \omega \left(n + \frac{1}{2} \right)$$



$$T \ll \omega$$

$$\mathcal{E} \ll \omega; n = 0$$

No energy
exchange
between the
oscillator and
the particle

$$\frac{1}{2} \hbar \omega$$

Pure elastic
scattering

No
dephasing

Zero-point
oscillations

Chaos in Nuclei – Delocalization?

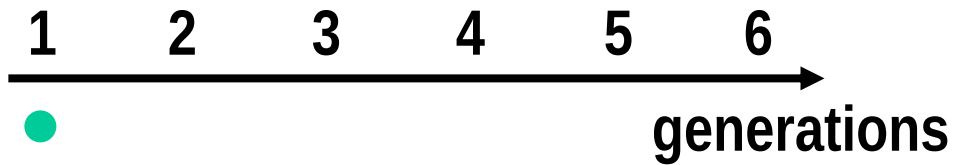


Fermi Sea

Without interactions between fermions energy of each of the particles is conserved, i.e., there are as many “integrals of motion” as there are excited particles.

For a finite Fermi Gas one should expect Poisson situation for the eigenstates of the whole system.

Chaos in Nuclei – Delocalization?

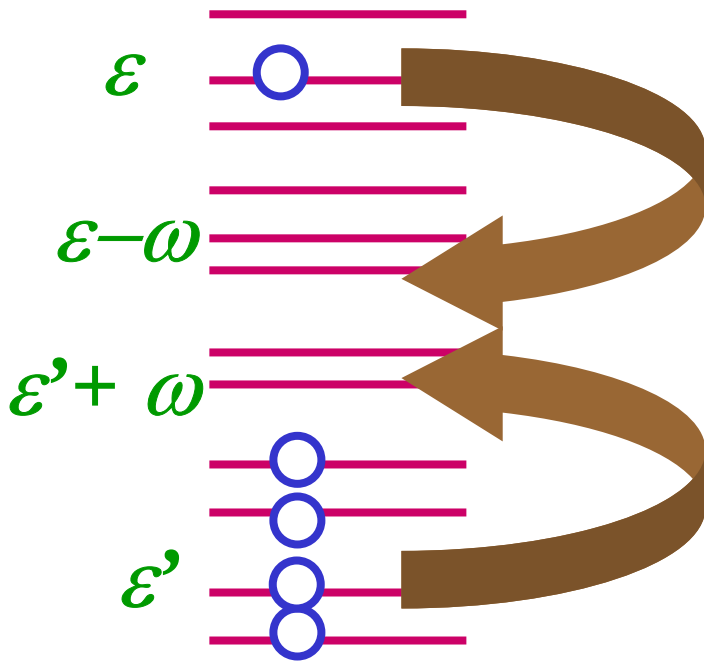


Delocalization in Fock space

Expansion of a typical eigenstate of the many-body system in the basis of states with given number of excitations involves a large number of terms

Fermi Sea

Quasiparticle relaxation rate in 0D case



$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \delta_1 \left(\frac{\varepsilon}{E_T} \right)^2$$

(U.Sivan, Y.Imry & A.Aronov,1994)

Becomes incorrect as soon as provided that

$$\varepsilon < \sqrt{E_T \delta_1} = \delta_1 \sqrt{g} \gg \delta_1$$

(B.A, Y.Gefen. A Kamenev & L.Levitov,1994)

**Metal-insulator transition
in a
weakly interacting many-electron system
with localized single-particle states**

cond-mat/0506617 v1 23 Jun 2005

D. M. Basko

Princeton University

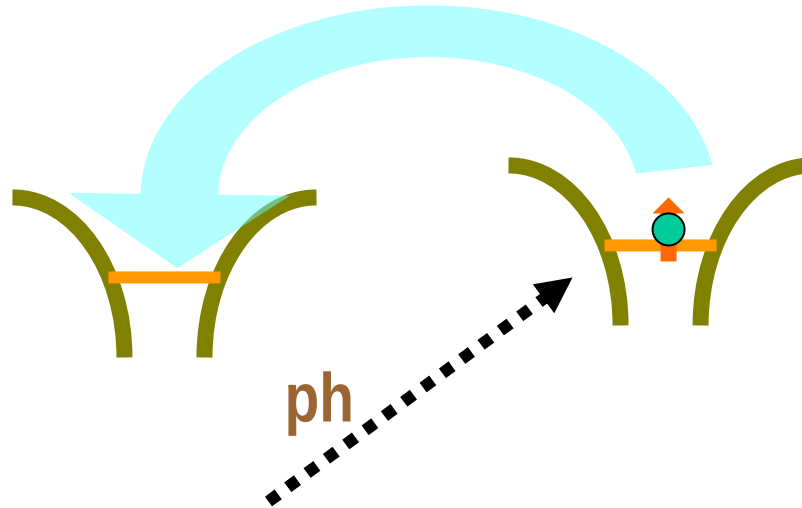
I.L. Aleiner

Columbia University

B. L. Altshuler

Princeton University, Columbia University, NEC-Laboratories America

Phonon – assisted hopping conductivity in the Anderson Insulator



Can hopping conductivity
exist **without phonons**



1. Temperature is finite
2. All one-particle states are localized
3. Electrons interact with each other
4. They are isolated from the outside world

Does DC conductivity
vanish or it is finite.



Q:

Does DC conductivity vanish or it is finite

?

A:

