Boulder summer school, 2024: Wan - problems

1 Closed problem

Consider the Stokeslet, the fundamental solution of the Stokes equations for a delta function forcing $\mathbf{F} = F\mathbf{e}$ located at $\mathbf{x} = \mathbf{x}_0$, often expressed as

$$\mathbf{v}(\mathbf{x}) = \frac{F}{8\pi\mu}\left(\frac{\mathbf{e}}{r} + \frac{\mathbf{e}\cdot(\mathbf{x}-\mathbf{x}_0)(\mathbf{x}-\mathbf{x}_0)}{r^3}\right) := F\mathbf{G}(\mathbf{x}-\mathbf{x_0};\mathbf{e})$$

where $r = |\mathbf{x}-\mathbf{x}_0|$.

a) Plot this flow field for $\mathbf{F} = (F, 0, 0)$, $\mathbf{x_0} = 0$.

b) Now plot the flow field due to a pair of Stokeslets located along the x-axis, separated by a distance L, with equal and opposite, colinearly directed forces.

c) Derive an expression for the far-field flow (i.e. in the limit of $r \gg L$) due to the pair of Stokeslets in b), in terms of L, θ and r (hint: use polar coordinates). How does the result scale with r?

d) By linearity, the derivative of a Stokeslet is also a solution. For an arbitrary $\mathbf{F} = F\mathbf{e}$, compute the quantity $-\mathbf{d}\cdot\nabla\mathbf{G}$ (i.e. the first derivative of the Stokeslet in the $\mathbf{d}$-direction), and express the result in terms of a symmetric part and an antisymmetric part. Discuss the physical meanings of these quantities.

e) Investigate the flow field constructed from three Stokeslets. It may be helpful to visualise a *Chlamydomonas*-like swimmer. If we oscillate the 'side' Stokeslets up and down, does the whole thing swim? Suggest strategies to improve the effectiveness of the swimmer.

2 Open problem

Many organisms are capable of sensing environmental cues, such as light or chemical gradients.

a) how does detection of light differ from the detection of chemicals?

b) investigate and describe in detail some strategies by which organisms navigate towards light (consider both macroscopic and microscopic organisms), paying close attention to the role of oscillations

c) are there any fundamental limits to light perception at the smallest scales? (particularly by prokaryotic organisms)